Conformality Lost

Crossing the conformal boundary

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to be published

Motivation

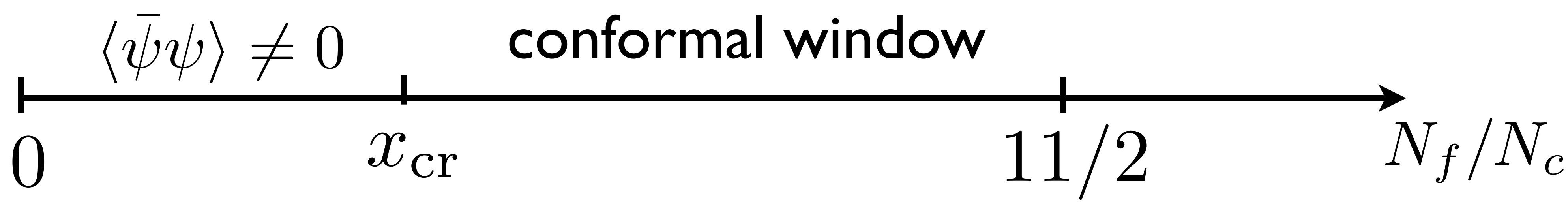
Consider large N_c , large N_f QCD

N_f/N_c can be changed continuously

$\frac{N_f}{N_c} = \frac{11}{2} - \epsilon$ Banks-Zaks fixed point, CFT

$\frac{N_f}{N_c}$ small Confinement, chiral symmetry breaking

There exists a critical N_f/N_c where transition happens



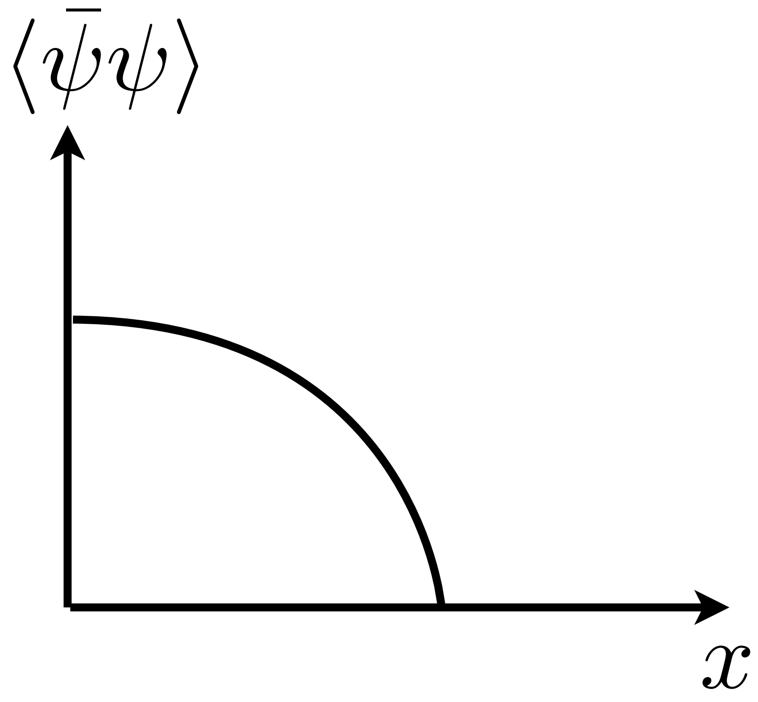
What happens slightly below the left edge of the conformal window? asked Armoni Shifman Veneziano ~ 2004

Possibilities:

- Power-law?

$$\langle \bar{\psi}\psi \rangle \sim (x_{\rm cr} - x)^{\beta}$$

$$x = \frac{N_f}{N_c}$$



typical of a 2nd order phase transition

But: in a 2nd order pt, conformal symmetry only at the phase transition

Here the system should be conformal for any $x > x_{\rm cr}$

- Another possibility:

a Berezinskii-Kosterlitz-Thouless phase transition

BKT scaling:

$$\xi^{-1}(T) = \begin{cases} \exp\left(-\frac{\#}{\sqrt{T-T_{\rm cr}}}\right) & T > T_{\rm cr} \\ 0 & T < T_{\rm cr} \end{cases}$$

If this is the case: chiral condensate goes to zero exponentially, with all derivatives vanishing as $x \to x_{\rm cr}$

Also: in BKT phase transition: power-law correlations (CFT) for all $T < T_{\rm cr}$

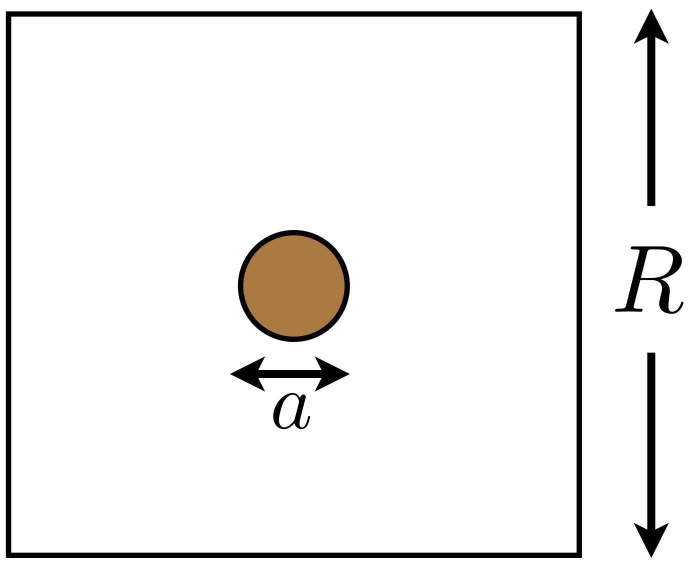
But the BKT phase transition is very specific for 2D

The BKT phase transition

Consider 2D theory with "spontaneously broken" $U(1)$

Vortex: energy logarithmically divergent with system size

$$E_{\text{vortex}} \sim v^2 \ln \frac{R}{a}$$



It can be anywhere in the box:

$$S \sim \ln \frac{R}{a}$$

Free energy: competition between two effects: $F = E - TS$

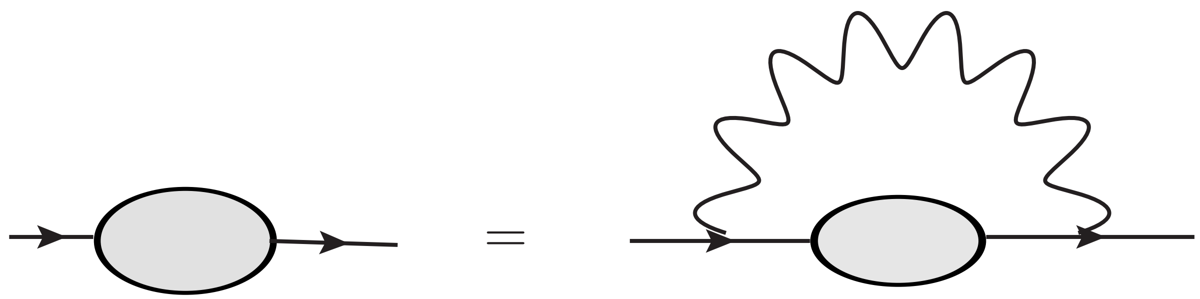
$T < T_{\text{cr}}$ vortex has infinite energy, confined in pairs

$T > T_{\text{cr}}$ vortices are unbound

All that is very good, but: QCD is a 4D theory

No vortices

Nevertheless: Schwinger-Dyson approach gives BKT scaling



Miransky 1985

Appelquist, Terning, Wijerwardhana 1996

This approach involves uncontrolled approximation:

- Truncate SD equation (summing up only rainbow diagrams)
- Breaking gauge invariance; Laudau gauge usually used

Critical N_f/N_c is clearly unreliable

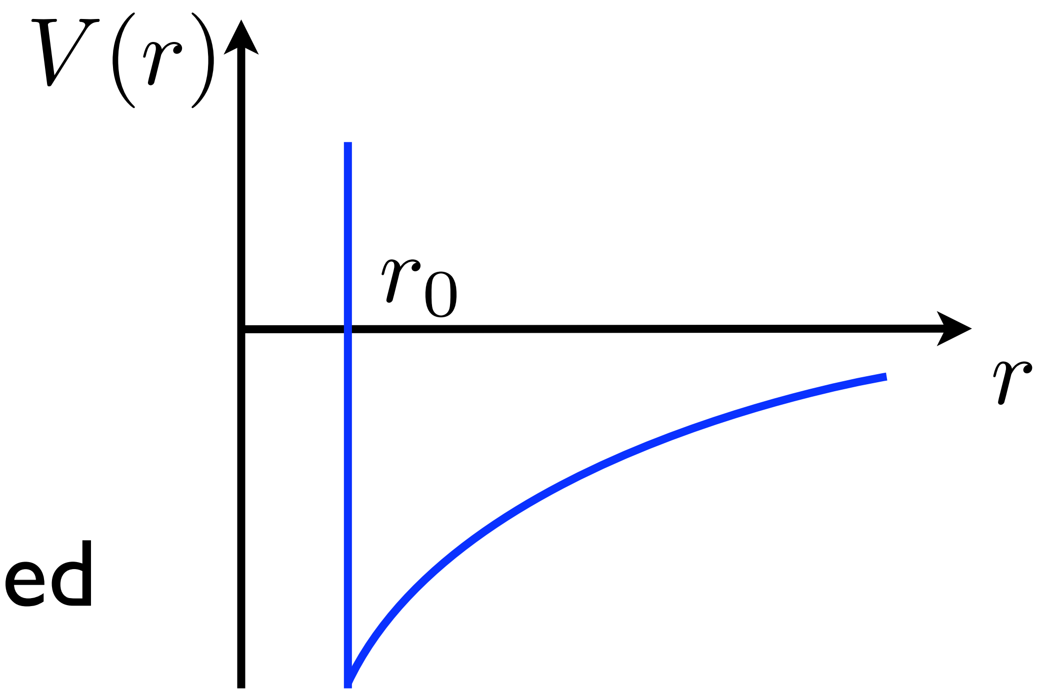
May the scaling be right?

Quantum mechanics with $1/r^2$ potential

Schrödinger equation $$-\psi'' - \frac{2}{r}\psi'(r) + \frac{\alpha}{r^2}\psi(r) = E\psi(r)$$

Short-distance behavior:

$$\psi(r) \sim \frac{1}{r^\nu} \qquad \nu = \frac{1}{2}(1 \pm \sqrt{1+4\alpha})$$



$\alpha > -1/4$ conformal QM

$\alpha < -1/4$ nonconformal: cutoff needed

Let us put an infinite repulsive core for $r < r_0$

The potential always has bound state, and the energy is

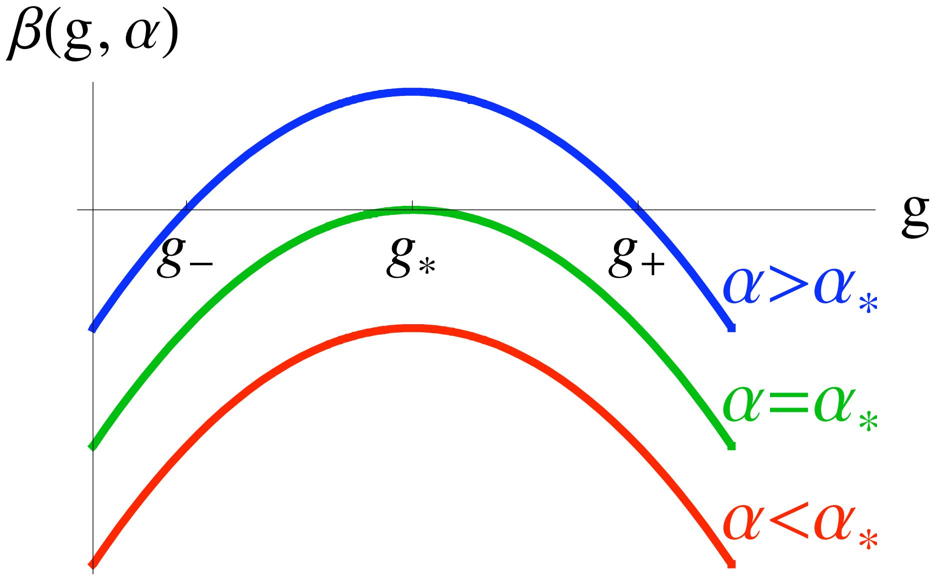
$$\exp\left(-\frac{2\pi}{\sqrt{-1/4-\alpha}}\right)$$

BKT scaling again!

- Three transitions have the same scaling:
  - The BKT phase transition
  - The transition in QM with $1/r^2$ potential
  - Chiral phase transition in SD approach
- Pure coincidence, or there is a deeper reason?

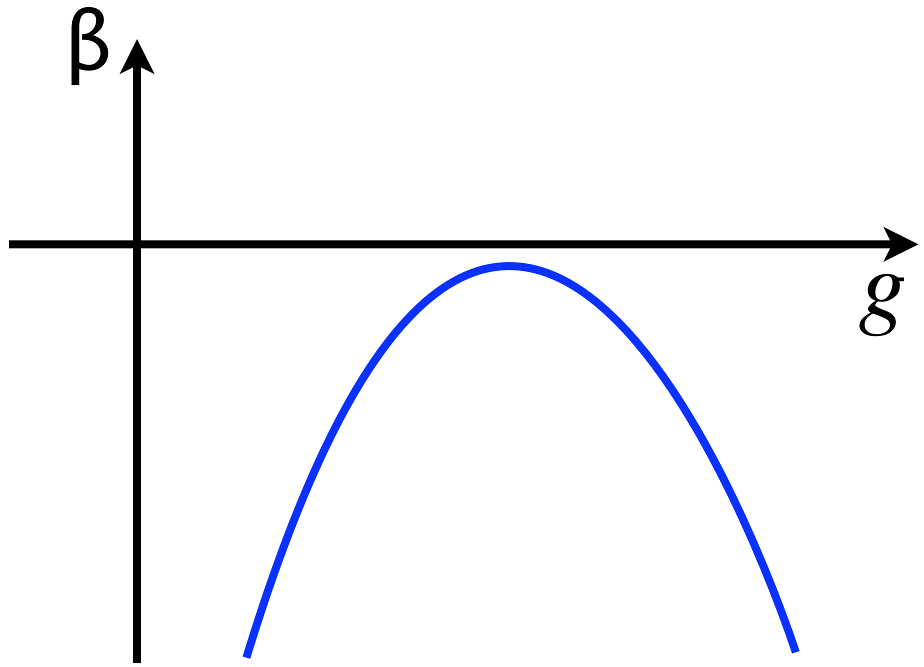
In the language of renormalization group, conformality may be lost due to

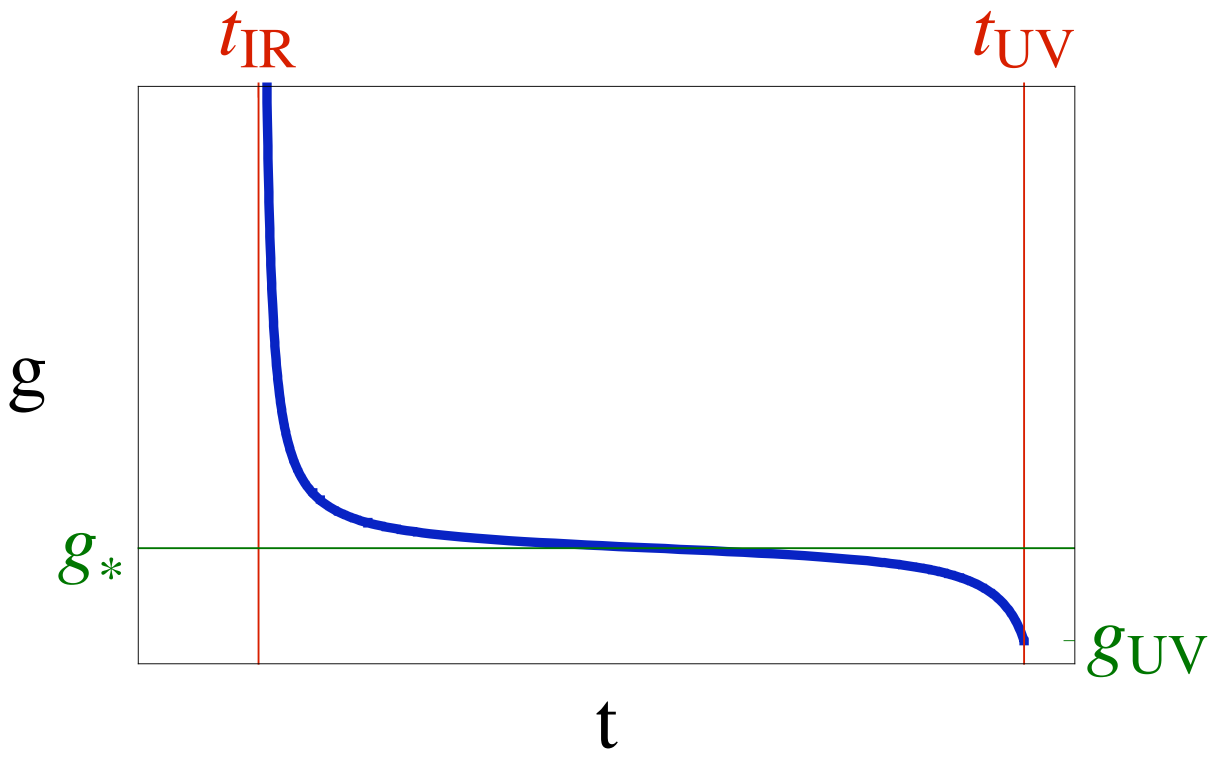
- Fixed point moving to zero SQCD $N_f/N_c = 3$
- Fixed point moving to infinity SQCD $N_f/N_c = 3/2$?
- Fixed point merger and annihilation



$$\beta(g;\alpha) = \frac{\partial g}{\partial t} = (\alpha - \alpha_*) - (g - g_*)^2$$

Running of coupling for $\alpha = \alpha_* - \epsilon$





$$\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}} = \exp\left[t_{\rm IR} - t_{\rm UV}\right] = \exp\left[\int_{g_{\rm UV}}^{g_{\rm IR}} \frac{dg}{\beta(g;\alpha)}\right] \simeq e^{-\pi/\sqrt{(\alpha_* - \alpha)}}$$

This may be the explanation!

If that picture is correct, what do we predict:

Two fixed points for $\alpha > \alpha_*$

Marginal deformation (logarithmic running) $\alpha = \alpha_*$

A situation familiar from AdS/CFT correspondence:

Operator-field mapping: $\Delta(\Delta - d) = m^2 R^2$

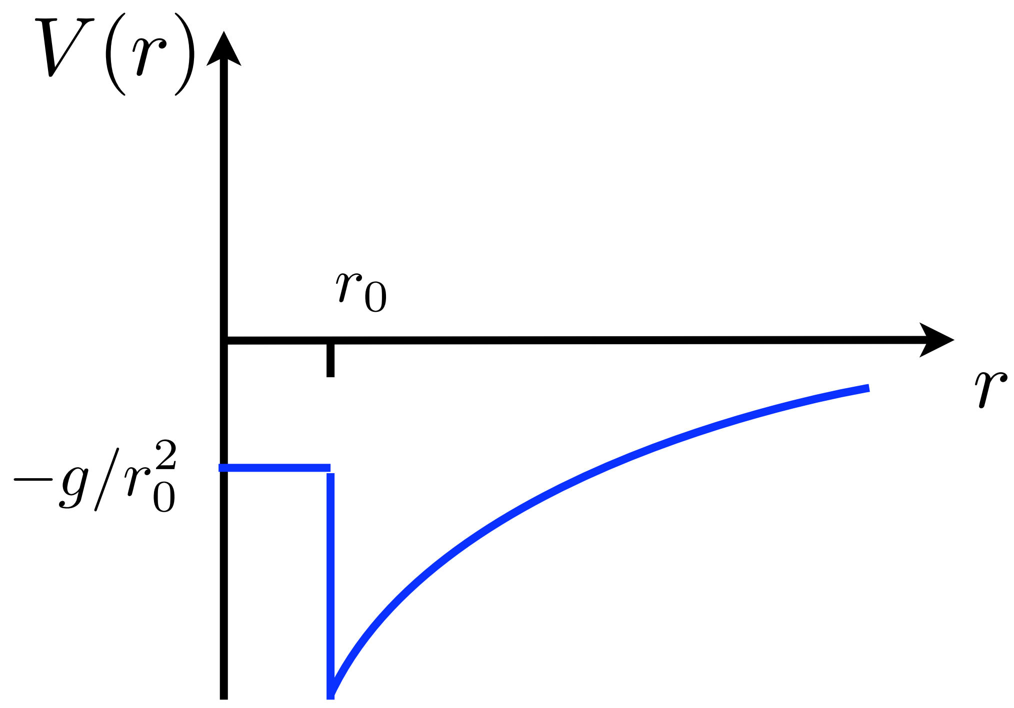
For a range of m^2: two different boundary theories

$$\phi(z) \;=\; c_+ z^{\Delta_+} \;+\; c_- z^{\Delta_-} \qquad \Delta_+ + \Delta_- = d$$

source	vev
vev	source

Lost of conformality: m^2 drops below the Breitenlohner-Freedman bound $m^2_{\rm BF} = -d^2/4$

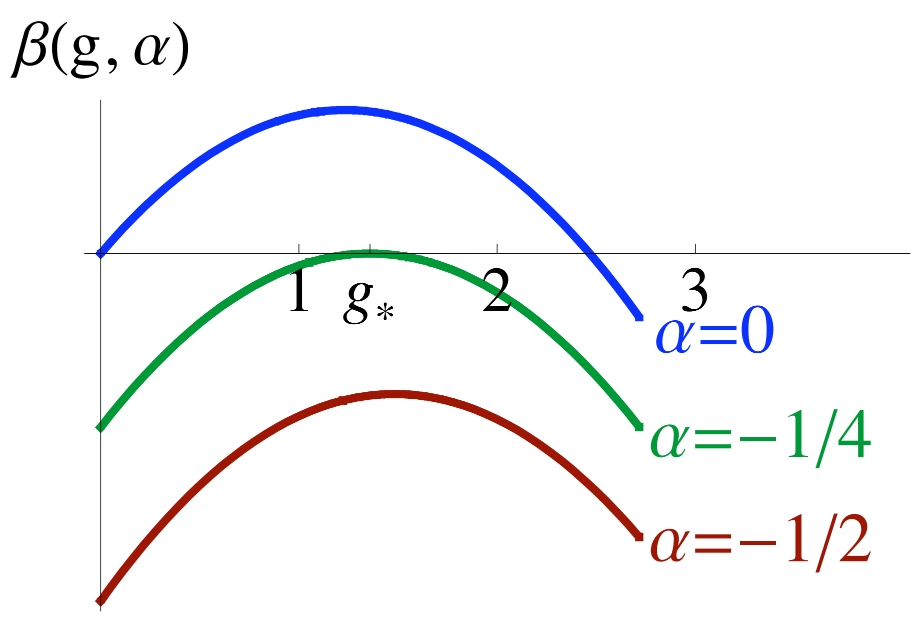
RG for quantum mechanics with $1/r^2$ potential



Regularize potential by a square-well core

Change g and r_0, preserving low-energy physics

Get beta function



$$\beta(g) = \frac{2\sqrt{g}\left(\alpha + \sqrt{g}\cot\sqrt{g} - g\cot^2\sqrt{g}\right)}{-\cot\sqrt{g} + \sqrt{g}\csc^2\sqrt{g}}$$

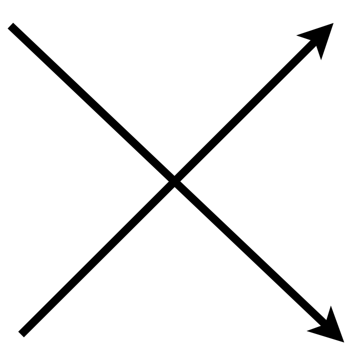
QM through ε expansion

$$d = 2 + \epsilon$$

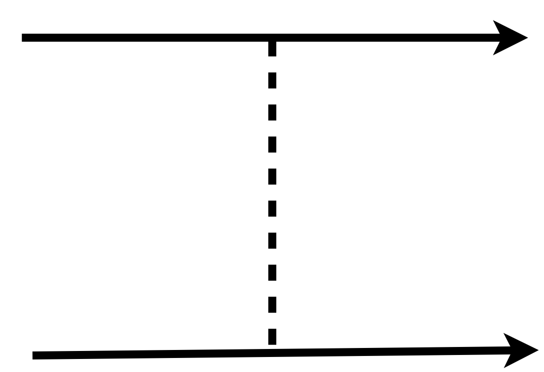
$$S = \int dt\, d^d\mathbf{x} \left(i\psi^\dagger \partial_t \psi - \frac{|\nabla\psi|^2}{2m} + \pi \frac{g}{4} \psi^\dagger\psi^\dagger\psi\psi \right) - \int dt\, d^d\mathbf{x}\, d^d\mathbf{y}\, \psi^\dagger(t,\mathbf{x})\psi^\dagger(t,\mathbf{y}) \frac{\alpha}{|\mathbf{x}-\mathbf{y}|^2} \psi(t,\mathbf{y})\psi(t,\mathbf{x})$$

Feynman rules:

$$\frac{i}{\omega - \mathbf{p}^2/2m}$$



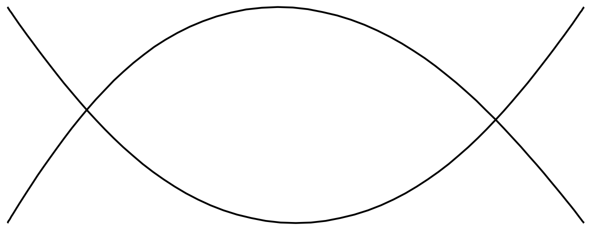
$$i\pi g \mu^{-\epsilon}$$



$$\frac{2\pi i\alpha}{\epsilon} \frac{1}{|\mathbf{q}|^\epsilon}$$

QM beta function from diagrams

$\beta(g) = \frac{1}{\epsilon}$ pole in



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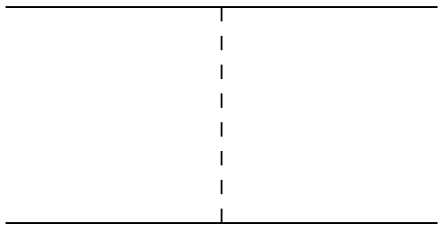


tree, but contains 1/ε pole from Fourier transform of $1/x^2$

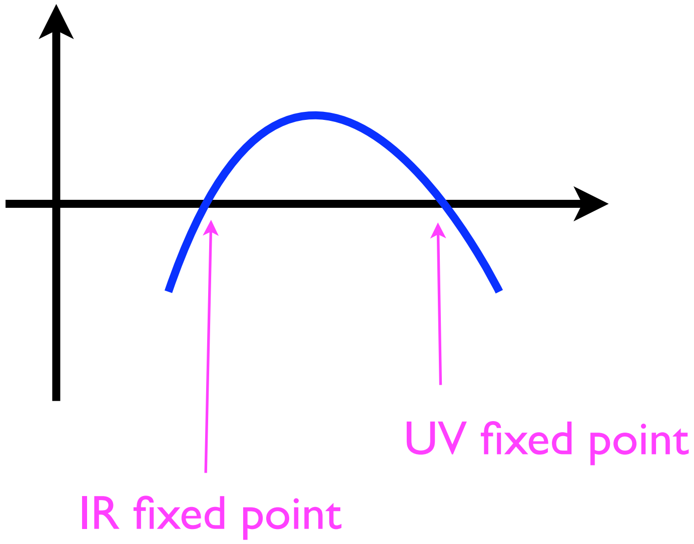
$$\beta(g) = \epsilon g - \frac{g^2}{2} + 2\alpha$$

Two zeros when $\alpha > \alpha_* = -\frac{\epsilon}{4}$

Zeros merging: $g_\pm = g_* = \epsilon$ at $\alpha = \alpha_*$

No zeros at $\alpha < \alpha_*$

What are the two fixed points?



The Schrödinger equation

$$-\psi'' - \frac{2}{r}\psi' + \frac{\alpha}{r^2}\psi = E\psi$$

has two solutions at small r: $\psi = \frac{c_+}{r^{\nu_+}} + \frac{c_-}{r^{\nu_-}}$ $\nu = \frac{1}{2}(1 \pm \sqrt{1+4\alpha})$

c_+/c_- depends on the short-range part of the potential

Choosing $c_+ = 0$ or $c_- = 0$ conformal theory

Generic short range potential "flows" to $c_+ = 0$

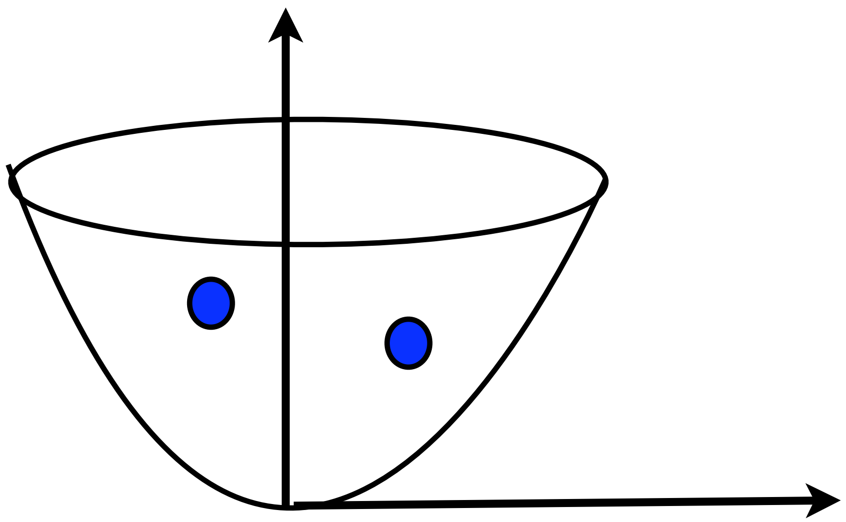
Fine-tuning required to achieve $c_- = 0$

Operator dimensions at fixed points

If AdS/CFT intuition is correct: there is an operator that has different dimensions at two fixed points

Nonrelativistic conformal symmetry:

dimensions of primary operators = Eigenstates in harmonic potentials, in unit of oscillator frequency ω



In our case: the operator $\psi\psi$

corresponds to ground state of 2 particles in harmonic potential

$$\psi(\mathbf{x},\mathbf{y}) = \frac{e^{-\omega(x^2+y^2)/2}}{|\mathbf{x}-\mathbf{y}|^{\nu_\pm}}$$

$$\Delta_\pm = \frac{E_\pm}{\omega} = \frac{d+2}{2} \pm \sqrt{\alpha - \alpha_*}$$

$$\Delta_+ + \Delta_- = d + 2$$

space (→ d) NR time (→ 2)

BKT phase transition

Can be interpreted as the merging of fixed points

The XY model, with vortices, is equivalent to sine-Gordon model

$$L = \frac{T}{2}(\partial_\mu \phi)^2 - 2z\cos\phi$$

$$u = 1 - 1/8\pi T$$
$$v = 2z/T\Lambda^2$$

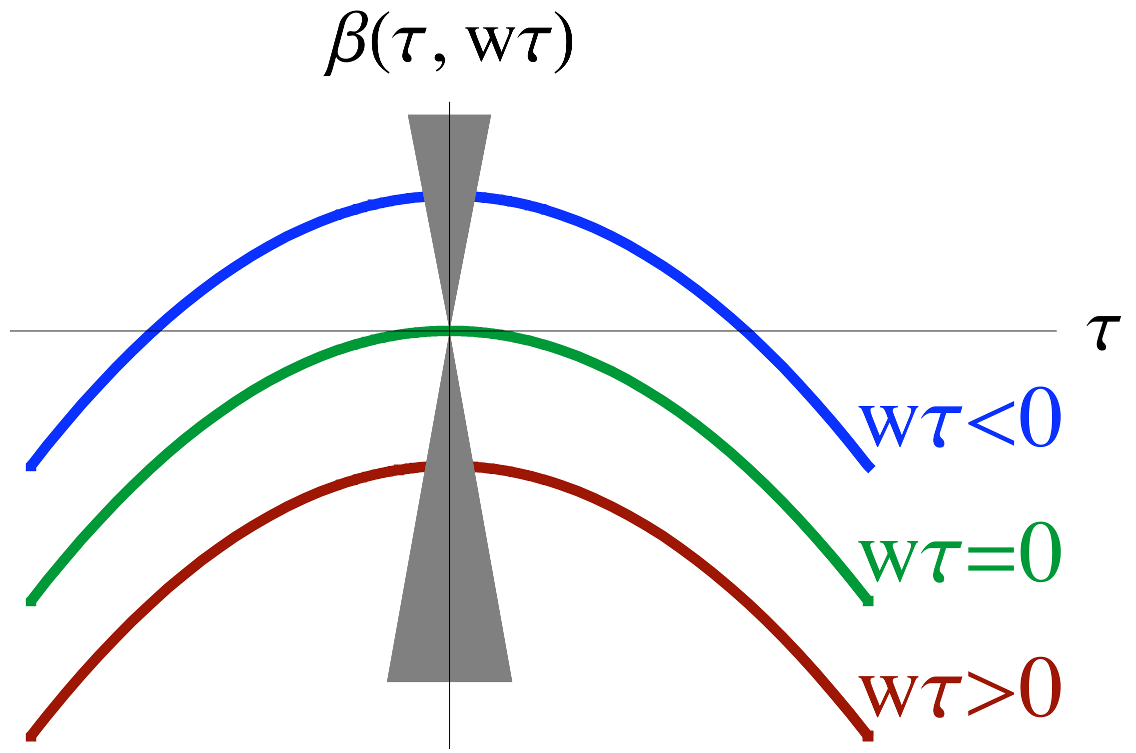
$$\beta_u = -2v^2$$
$$\beta_v = -2uv$$

$$v + u = \tau$$
$$v - u = 2w$$

τw invariant

$$\beta(\tau; w\tau) = -2w\tau - \tau^2$$

$\beta(\tau)$ has or does not has zero depending on the sign of $w\tau$

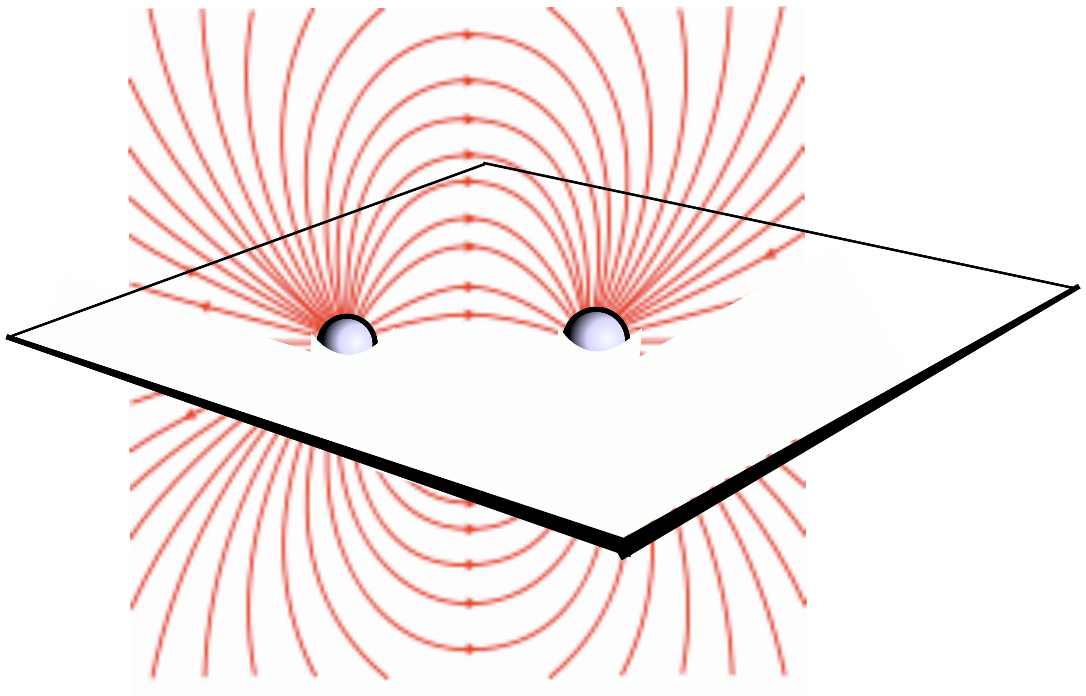


$$v + u = \tau$$
$$v - u = 2w$$

gray region not reliable: w→∞

A solvable relativistic model

Inspired by graphene: fermions in d<3 spatial dimensions coupled with nonabelian gauge field in 3+1 dimensions



$$S = \int d^d x \, i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4g^2}\int d^4 x \, F^a_{\mu\nu}F^a_{\mu\nu} + \cdots$$

Take (3+1)D theory to be N=4 SYM: no running of g

We expect a phase transition as one changes g

"Chiral symmetry breaking" at large g

No χSB at small g

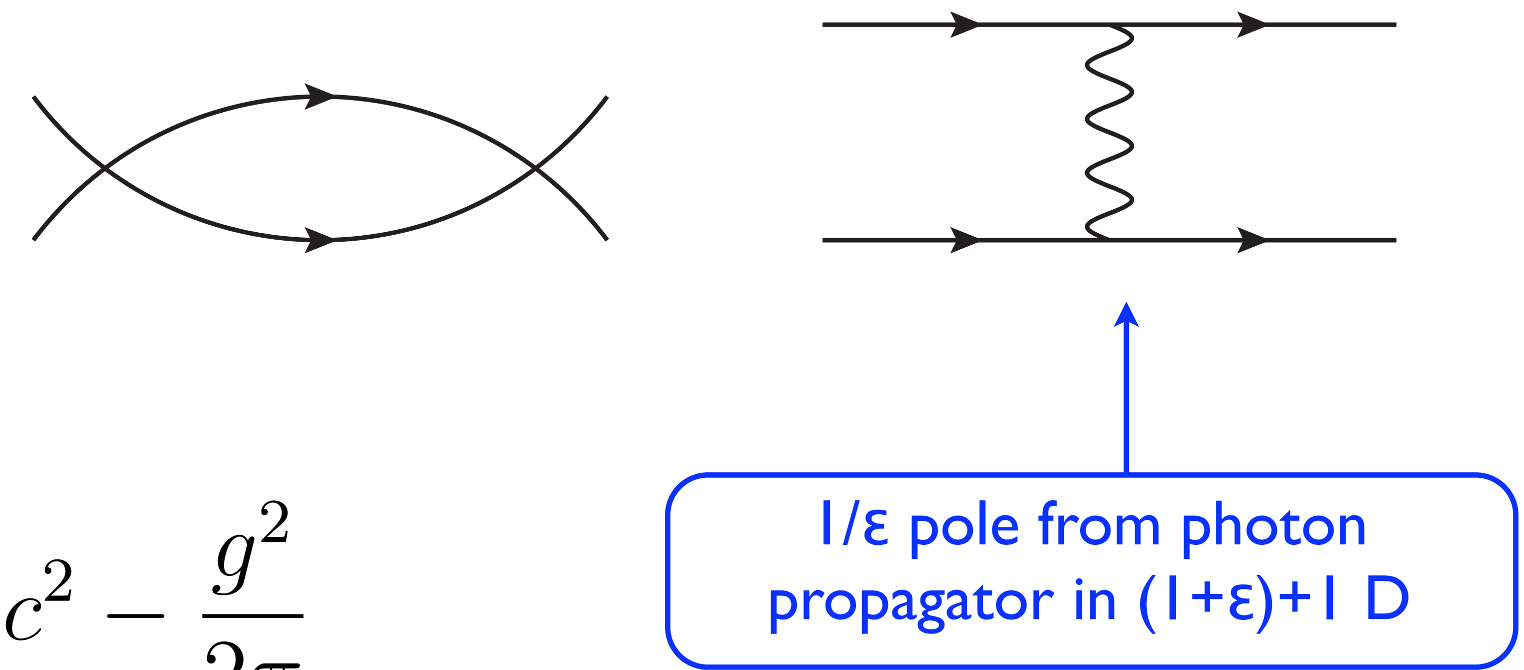
Phase transition occurs in perturbative regime for d=2+ε

RG approach

Four-fermi interaction is generated

$$L = \cdots - \frac{c}{2}(\bar{\psi}\gamma^\mu t^a \psi)^2$$

$$\beta(c) =$$



$$\beta(c) = \epsilon c - \frac{N_c}{2\pi}c^2 - \frac{g^2}{2\pi}$$

Fixed point merging at $g_* = \dfrac{\pi^2 \epsilon}{N_c}$

Gap equation: gives qualitatively the same result (BKT scaling etc)

Lessons from the defect QFT model

- Both gap equation and RG gives BKT scaling
- RG is systematic, gap equation sums up only a subset of important diagrams (rainbow)
- Gap equation becomes reliable at large N in the small ε regime
- At small N gap equation is not quantitatively reliable, but gives the correct scaling near critical coupling

Back to QCD

- Perhaps the situation in QCD is similar to that of the defect QFT
- Imagine that the four-fermi coupling c runs

$$\beta(c) = (2-\gamma)c - c^2 - \alpha_s N_c$$

dimension

Iterating 4-fermi interaction

One-gluon exchange generates four-fermi coupling

at some value of gauge coupling there is no fixed point

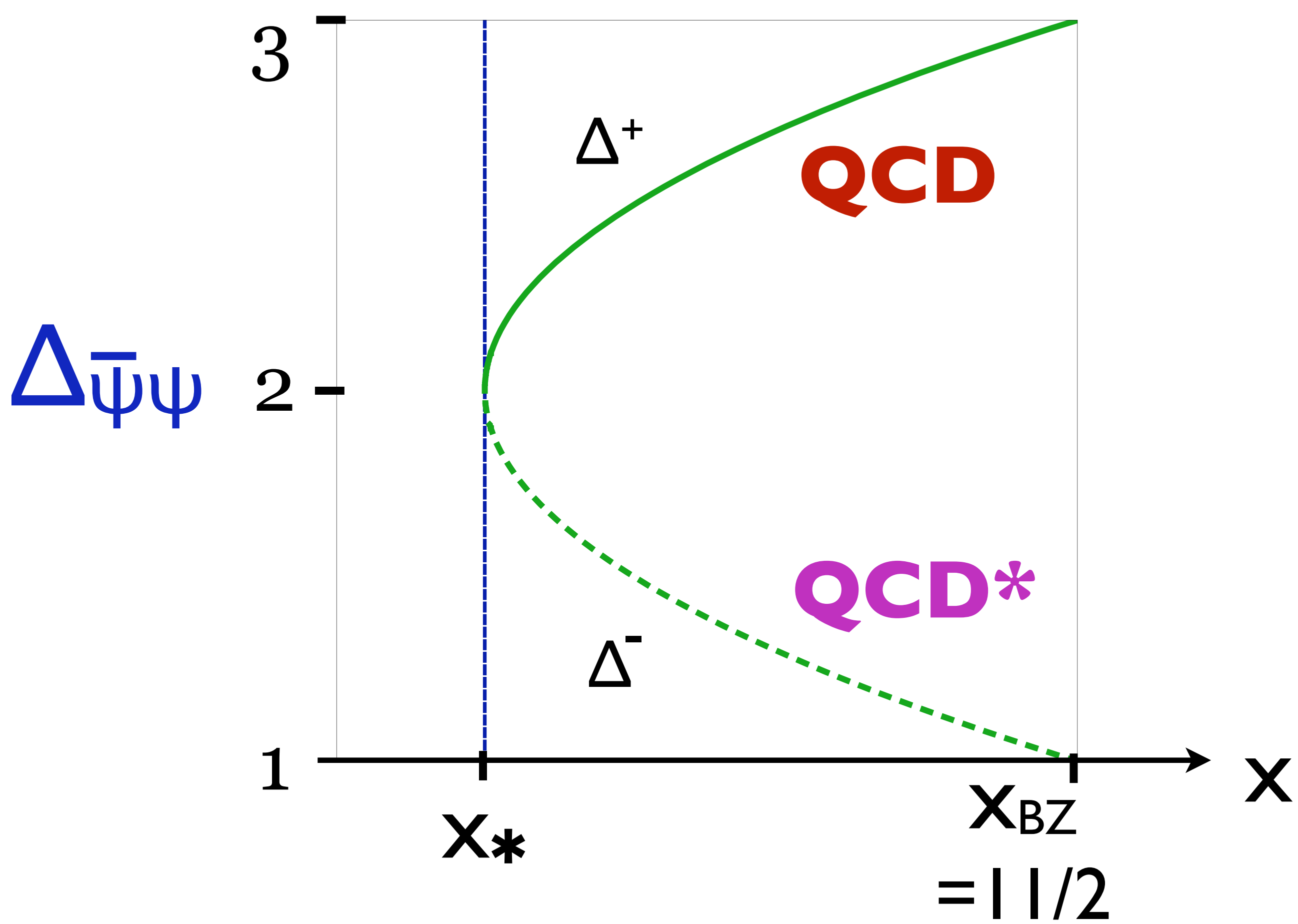
- It is also not surprising that the gap equation gives BKT scaling: same for the defect QFT

- But that means that near the lower end of the conformal window, there is a UV fixed point in addition to the usual IR fixed point

- In this new fixed point the fermion bilinears have dimensions different from their dimension at the IR fixed point

- It would be nice (though not guaranteed) that this fixed point can be found in the weak coupling regime $N_f/N_c = 11/2 - \epsilon$

- We begin gently: can one find a perturbative fixed point where one fermion bilinear changes dimension?



$\Delta_{\bar{\psi}\psi}$
3
2
1
Δ^+
QCD
QCD*
Δ^-
x
x_*
x_{BZ}
=11/2

Model A

Start with perturbative Banks-Zaks fixed point,

$$\Delta(\bar{\psi}\psi) = 3 - \#\lambda$$

In the UV fixed point: we should have a scalar with $\Delta = 1 + \#\lambda$, almost a free scalar

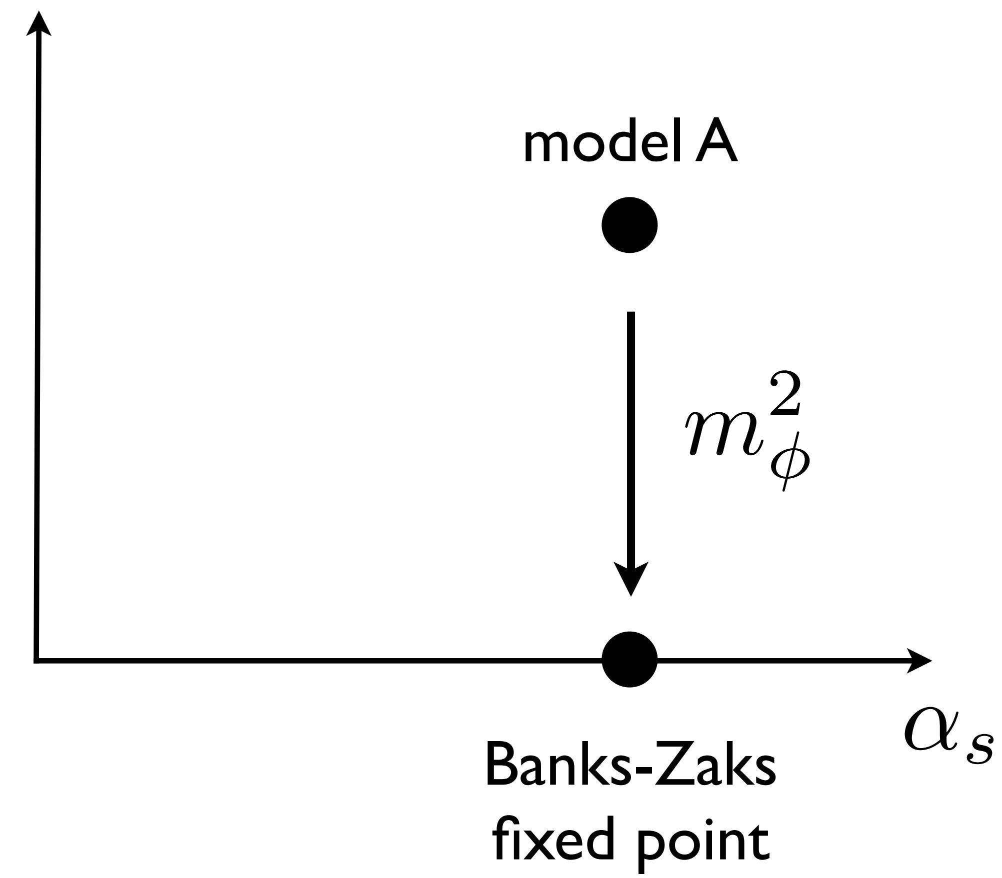
This suggests how to construct such a theory

$$L = L_{\rm QCD} + \frac{1}{2}(\partial_\mu \phi)^2 - y\bar{\psi}\psi\phi - \frac{\lambda}{24}\phi^4$$

$$\beta_y = \frac{y}{16\pi^2}(y^2 N_f N_c - 3g^2 N_c)$$

Fixed point for y and λ exists

Running of α unaffected


model A
m_ϕ^2
α_s
Banks-Zaks
fixed point

Model C

Previous model does not have chiral symmetry: cannot be a candidate for the UV fixed point

To restore chiral symmetry: introduce O(Nf^2) scalars

$$L = L_{\mathrm{QCD}} - y(\bar{\psi}t^A\psi\phi^A + i\bar{\psi}t^A\gamma^5\psi\pi^A)$$ + action for scalar

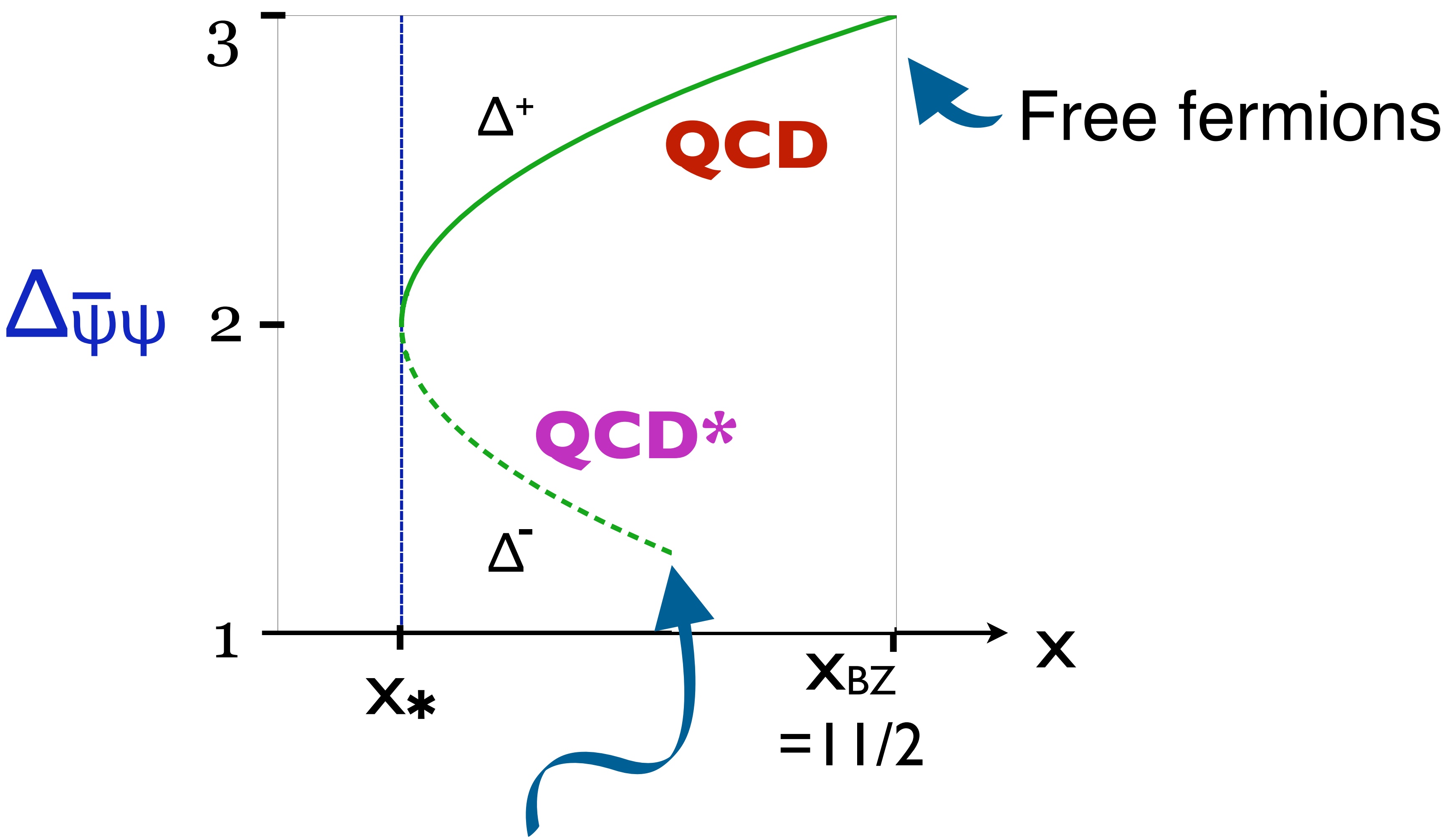
But now the running of α is affected: too many scalars

This model turns out not to have perturbative fixed point

Perhaps the UV fixed point exists only sufficiently close to the critical Nf/Nc?

This fixed point should be looked for on the lattice

QCD* ?



UV fixed point starts at strong-ish coupling?

Model D

Preserve $\mathrm{SU}(M) \times \mathrm{SU}(M),\ M = N_f/k$

$$L = L_{\mathrm{QCD}} - y\bar{\psi}_i^{\alpha} t^A_{\alpha\beta}(\phi^A + i\gamma^5\pi^A)\psi^i_\beta + \text{scalar terms}$$

$$\alpha, \beta = 1 \ldots M \qquad A = 1 \ldots M^2 - 1$$

$$i = 1 \ldots k$$

Nontrivial perturbative fixed point exists for $k > 1$

But now $\Delta[\phi]_{\text{model D}} + \Delta[\bar{\psi}\psi]_{\text{BZ}} = 4 + O(M^2/N_c^2)$

AdS/CFT interpretation: AdS curvature changes when boundary condition for O(N^2) scalars is changed

Conclusions

- Merger and annihilation of fixed points explains the loss of conformality in a variety of systems
- The scaling of the IR scale near the phase transition coincides with that of a Berezinskii-Kosterlitz-Thouless phase transition
- It is conceivable that the chiral phase transition in Nf/Nc also has BKT scaling
  - a UV fixed point in QCD with fine-tuned four-fermi interaction
- Loss of conformality ~ violation of the Breitenlohner-Freedman bound in gravity dual
  - implies $\Delta[\bar{\psi}\psi] = 2$ exactly at the phase transition

Maybe soon we can answer Misha's question

Happy birthday, Misha