

A hidden symmetry of crazy QCD

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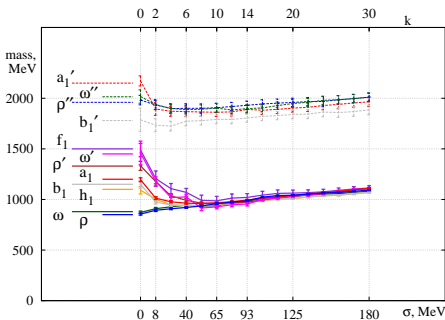
- 1 Crazy QCD
- 2 Left-right mixing. The chiral spin $SU(2)_{CS}$ and $SU(4)$ groups.
- 3 Zero modes and symmetries of Euclidean QCD
- 4 Conclusions



Crazy QCD

M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

$J = 1$ spectra upon the low-lying mode truncation



We clearly see a larger degeneracy than the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian. It makes you crazy...



Low mode truncation

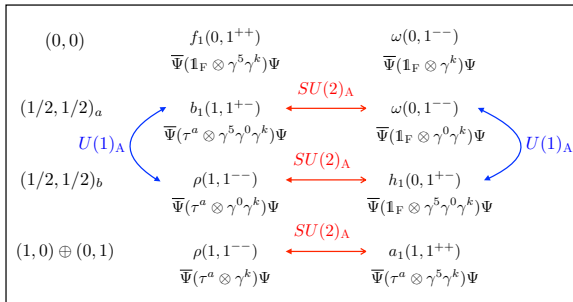
Banks-Casher:

$$\langle \bar{q}q \rangle = -\pi\rho(0).$$

What we do:

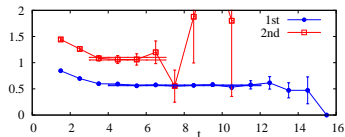
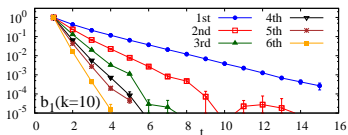
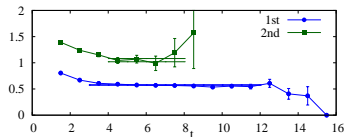
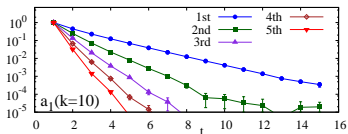
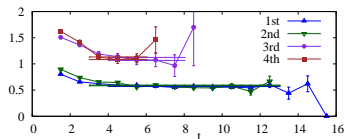
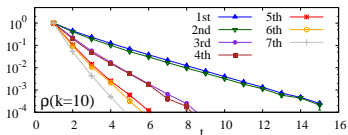
$$S = S_{Full} - \sum_{i=1}^k \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$

What one expects for $J = 1$ mesons:



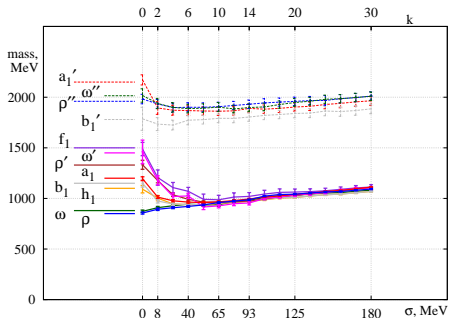
M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

Isovector $J = 1$ states.

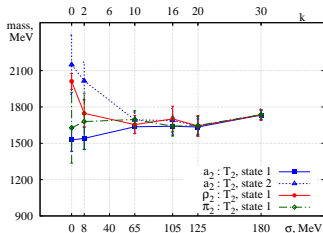
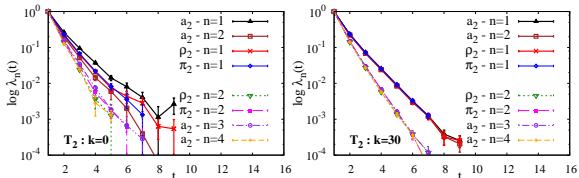


M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

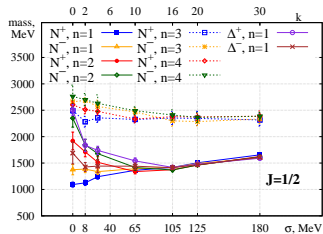
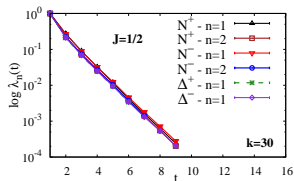
$$J = 1$$



J=2 mesons: M. Denissenya, L.Ya.G, M.Pak, PRD 91(2015)114512



$J = 1/2$ baryons: M. Denissenya, L.Ya.G, M.Pak, PRD 92 (2015) 074508



A hidden symmetry of QCD in Euclidean space

We clearly see a larger degeneracy than the

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

symmetry of the QCD Lagrangian.

What does it mean !?

There must be a hidden symmetry in Euclidean space-time.



L.Ya.G., EPJA 51(2015)27

(i) $(0,0)$:

$$|(0,0); \pm; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}R \pm \bar{L}L\rangle_J.$$

(ii) $(1/2, 1/2)_a$ and $(1/2, 1/2)_b$:

$$|(1/2, 1/2)_a; +; I=0; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}L + \bar{L}R\rangle_J,$$

$$|(1/2, 1/2)_a; -; I=1; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}\tau L - \bar{L}\tau R\rangle_J,$$

$$|(1/2, 1/2)_b; -; I=0; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}L - \bar{L}R\rangle_J,$$

$$|(1/2, 1/2)_b; +; I=1; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}\tau L + \bar{L}\tau R\rangle_J.$$

(iii) $(0,1) \oplus (1,0)$:

$$|(0,1) + (1,0); \pm; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}\tau R \pm \bar{L}\tau L\rangle_J,$$



L.Ya.G., EPJA 51(2015)27

Consider rotations in an imaginary 3-dim space of doublets constructed from the L and R bispinors

$$U = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \quad D = \begin{pmatrix} d_L \\ d_R \end{pmatrix}$$

$$U \rightarrow U' = e^{i \frac{\epsilon \cdot \sigma}{2}} U, \quad D \rightarrow D' = e^{i \frac{\epsilon \cdot \sigma}{2}} D,$$

where σ are the standard Pauli matrices: $[\sigma^i, \sigma^j] = 2i \epsilon^{ijk} \sigma^k$.

We refer to this imaginary three-dimensional space as the **chiral spin** space and denote this symmetry group as $SU(2)_{CS}$

A group that contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS}$ is $SU(4)$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix}$$



L.Ya.G., M. Pak, PRD 92(2015)016001

We can construct an explicit realization of the $SU(2)_{CS}$ algebra that acts on the Dirac bispinors.

Then the $SU(2)_{CS}$ chiral spin rotations are generated through

$$\Sigma = \{\gamma^0, i\gamma^5\gamma^0, -\gamma^5\}, \quad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk} \Sigma^k.$$

The $SU(4)$ group contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS} \supset U(1)_A$ with the fundamental vector

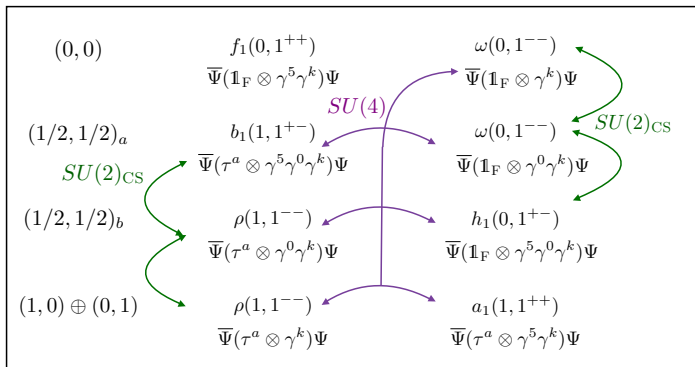
$$\Psi = \begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix}$$

and has the following set of generators:

$$\{(\tau^a \otimes \mathbf{1}_D), (\mathbf{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}$$



L.Ya.G., M. Pak, PRD 92(2015)016001



Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

The Lagrangian in Euclidean space-time for a massive quark in a given gauge configuration is:

$$\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x), \quad (1)$$

with

$$D_\mu = \partial_\mu + ig \frac{t^a}{2} A_\mu^a, \quad (2)$$

The mass term $\Psi^\dagger(x)\Psi(x)$ is invariant under $U(1)_A$ chiral transformation:

$$\Psi(x) \rightarrow e^{i\alpha\gamma_5}\Psi(x); \quad \Psi^\dagger(x) \rightarrow \Psi^\dagger(x)e^{-i\alpha\gamma_5}. \quad (3)$$

The kinetic term, $\Psi^\dagger(x)(\gamma_\mu D_\mu)\Psi(x)$, breaks this symmetry ($\gamma_5\gamma_\mu + \gamma_\mu\gamma_5 = 0$).

The same is true with respect to the axial part of $SU(N_F)_L \times SU(N_F)_R$.

What are symmetry properties of both parts under the $SU(2)_{CS}$ and $SU(2N_F)$ transformations?



Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

The Euclidean $SU(2)_{CS}$ generators:

$$\Sigma = \{\gamma^4, i\gamma^5\gamma^4, -\gamma^5\}. \quad (4)$$

Combining the $SU(2)_{CS}$ generators with the $SU(N_F)$ flavor generators into a larger algebra we arrive at the Euclidean $SU(2N_F)$ transformations.

These symmetries are missing in the Lagrangian. Why?

Because a massless "on-shell" quark is described by the zero mode of the Dirac equation

$$\gamma_\mu D_\mu \Psi_0(x) = 0. \quad (5)$$

The zero mode is chiral, L or R , depending on the topological charge $Q \neq 0$.
Atiyah-Singer:

$$Q = n_L - n_R$$

Conclusion: The zero modes break explicitly $SU(2)_{CS}$ and $SU(2N_F)$.



Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

Zero modes are absent in the $Q = 0$ sector.

$$Z_{Q=0}^V = \int D\Psi D\Psi^\dagger e^{\int d^4x \Psi^\dagger(x) (i\gamma_\mu D_\mu + im) \Psi(x)}. \quad (6)$$

The hermitian Dirac operator has in a finite volume V a discrete spectrum:

$$i\gamma_\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x). \quad (7)$$

We expand $\Psi(x)$ and $\Psi^\dagger(x)$ over a complete and orthonormal set $\Psi_n(x)$:

$$\Psi(x) = \sum_n c_n \Psi_n(x); \quad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi_k^\dagger(x) \quad (8)$$

$$Z_{Q=0}^V = \int \prod_{k,n} d\bar{c}_k d c_n e^{\sum \bar{c}_k c_n \int d^4x \bar{c}_k c_n (\lambda_n + im) \Psi_k^\dagger(x) \Psi_n(x)}. \quad (9)$$

It is precisely $SU(2)_{CS}$ and $SU(2N_F)$ symmetric:

$$(U\Psi_k(x))^\dagger U\Psi_n(x) = \Psi_k^\dagger(x) \Psi_n(x). \quad (10)$$



Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

H. Leutwyler, A. Smilga, 1992: Up to an inessential normalization factor

$$Z_{Q=0}^{V \rightarrow \infty} \sim Z_{\theta=0}^{V \rightarrow \infty}$$

The rate is $1/V$.

We conclude: **The Euclidean QCD has hidden $SU(2)_{CS}$ and $SU(2N_F)$ symmetries.**

The Banks-Casher relation:

$$\lim_{m \rightarrow 0} \langle 0 | \bar{\Psi}(x) \Psi(x) | 0 \rangle = -\pi \rho(0). \quad (11)$$

The quark condensate in Minkowski space breaks all symmetries $U(1)_A$, $SU(N_F)_L \times SU(N_F)_R$, $SU(2)_{CS}$, $SU(2N_F)$ to $SU(N_F)_V$.

A remark. In $Q \neq 0$ sectors there do appear contributions from the zero modes: $\Psi_0^\dagger \Psi_0$; $\Psi_k^\dagger \Psi_0$; $\Psi_0^\dagger \Psi_n$.

Their contributions vanish as $1/V$. **Symmetry properties of the relevant part of the partition function are the same in all topological sectors.**



Conclusions

The Euclidean QCD has hidden $SU(2)_{CS}$ and $SU(2N_F)$ symmetries.

A consequence (L.Ya.G., arXiv 1512.06703): At high temperatures $T > T_c$, where chiral and $U(1)_A$ are restored, the matter is $SU(2)_{CS}$ and $SU(2N_F)$ symmetric. There cannot be deconfined quarks and gluons.

