A COMPETITIVE MARKET MODEL FOR "FUTURES" PRICE DETERMINATION

by

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A. Market prices and quantities with no intervening markets or information

Consider a decision maker faced with specifying his demand at any given price for the following commodity: each unit of the commodity yields $x(\omega)$ dollars. $x(\omega)$ is a random variable whose value depends on which state of the world $\omega \varepsilon \Omega$ occurs. The decision maker will purchase his desired amount of these "futures", y, then the state of the world $\hat{\omega}$ will be observed, and $yx(\hat{\omega})$ will be his income.

Now consider a market where <u>n</u> decision makers trade these futures. Each of these traders makes decisions which obey the axioms given by Herstein and Milnor [3] for choices under uncertainty. Thus we assume:

U1) An absolutely continuous utility index over alternatives for each trader exists and is unique up to positive linear transformations.

U2) Each trader makes choices which maximize the expected value of utility.

The alternatives in the futures market are uncertain amounts of dollars; we will denote trader i's utility function for certain wealth as $U_i(M)$. His utility index for uncertain amounts of wealth can then be found using the Expected Utility Theorem U2. Further, for reasons that will become apparent, we assume:

U3) $U_i(M)$ is bounded for $M \to \infty$, $M \to 0$.

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U4) $U_i'(M)$ exists and is continuous for $M \varepsilon (0, \infty)$.

$U_i'(M) > 0$, and $U_i'(M) \to \infty$, $M \to 0$.

U5) $U_i''(M)$ exists for $M \varepsilon (0, \infty)$. $U_i''(M) < 0$.

(U3) is the standard boundedness condition to rule out behavior of the St. Petersburg Paradox type. (U4) requires that the "marginal utility of wealth" be positive and change only continuously, with small increments of wealth very important for small wealth. (U3) and (U4) together require that $U_i'(M) \to 0$, and $M \to \infty$. Before trading in futures, each trader is assumed to have wealth $M_i > 0$.

The demand for futures y_i^* by trader i at any price r may now be given:[1]

$$y_i^* \text{ maximizes } V_i(y_i, r) \quad ;$$

$$V_i(y_i, r) = E_x^i \{U_i[M_i + y_i (x - r)]\} \quad .$$

The expectation operator E_x^i is the expectation with respect to the i-th trader's probability distribution. Conditions (U1)-(U5) guarantee that this expectation always exists, and that a unique regular maximum for V_i is obtained.

Demand y_i^* as a function of r is given by the first-order conditions:

$$E_x^i \{(x - r) U_i'[M_i + y_i^* (x - r)]\} = 0 \quad . \tag{1}$$

Note that formula (1) involves interchanging the expectation operator E_x^i with the partial differentiation $\frac{\partial}{\partial y_i}$. This is possible when r is restricted so that the solution y_i^* to the above equation obeys $|y_i| < \infty$.

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1. The problem of timing consumption between two different periods is ignored in this analysis. In a commodity futures market, this hedging behavior is important. See Sandmo [10] for one of the numerous discussions of this problem.

This will be true if U_i is absolutely continuous, and condition (P1) holds.

(P1) r is restricted to lie in the region $\underline{r} \le r \le \bar{r}$, where for all traders i, $\text{Prob}_i \{x < \underline{r}\} > 0$, $\text{Prob}_i \{x > \bar{r}\} > 0$.

Furthermore, demand y_i^* is a continuous function of r . This can be seen by noting that $V_i(y_i, r)$ is a continuous function of y_i and r . Thus the particular y_i which maximizes V_i for a given r , say $y_i^*(r)$, is an upper-semicontinuous correspondence, and in this case, a continuous function.

Let us call $y_i^*(r)$ trader i's demand function for futures. Note that demand depends on the expectation, operator E_x^i , and thus on trader i's subjective probability distribution of payoffs. The derivative dy_i^*/dr is not necessarily negative. This is because the effect of a change in price may be separated into two effects, just as in the certainty case. There is a substitution effect in favor of futures when their price falls considering expected wealth constant, and a wealth effect, since a change in price changes expected wealth. This can be easily demonstrated. Leaving out trader subscripts and remembering this analysis is for one trader only: $V_y(y^*, r) = 0$ $\Rightarrow V_{yy}dy + V_{yr}dr = 0$.

$$\left.\frac{\partial y^*}{\partial r}\right|_{M\,=\,\text{constant}} = -\frac{V_{yr}}{V_{yy}} \tag{2}$$

$$V_{yy} = E_x\{(x - r)^2\, U''[M + y(x - r)]\} \quad , \quad \text{and}$$

$$V_{yr} = E_x\{-U'[M + y(x - r)] - y(x - r)\, U''[M + y(x - r)]\} \quad .$$

$$\left.\frac{\partial y^*}{\partial r}\right|_{M\,=\,\text{constant}} = -\frac{E_x\{U'[M+y(x-r)]\}}{E_x\{(x-r)^2U''[M+y(x-r)]\}} - \frac{E_x\{y(x-r)U''[M+y(x-r)]\}}{E_x\{(x-r)^2U''[M+y(x-r)]\}} \quad (3)$$

The first term on the right is the substitution effect, while the second term on the right is the income effect.

Consider constraining the trader so that the final optimized level of utility, V, is constant. To do this, initial wealth M must vary. $V_y = 0$ is still the first order condition. Total differentiation yields

$$V_{yy}d_y + V_{yr}dr + V_{yM}dM = 0 \quad ,$$

while the V = constant constraint gives

$$V_yd_y + V_rd_r + V_MdM = V_rd_r + V_MdM = 0 \quad .$$

Some rearrangement yields:

$$\left.\frac{\partial y^*}{\partial r}\right|_{V\,=\,\text{constant}} = \frac{V_r}{V_M}\frac{V_{yM}}{V_{yy}} - \frac{V_{yr}}{V_{yy}} \quad .$$

Note that if M changes while r does not,

$$V_{yy}d_y + V_{yM}dM = 0 \;; \quad \left.\frac{\partial y^*}{\partial M}\right|_{r\,=\,\text{constant}} = -\frac{V_{yM}}{V_{yy}} \quad .$$

After noting that $\frac{V_r}{V_M} = -y^*$, we finally have

$$\left.\frac{\partial y^*}{\partial r}\right|_{M\,=\,\text{constant}} = \left.\frac{\partial y^*}{\partial r}\right|_{V\,=\,\text{constant}} - y^*\left.\frac{\partial y^*}{\partial M}\right|_{r\,=\,\text{constant}} \quad (4)$$

The first terms on the right in (3) and (4) are equal, as are the second right-hand terms in each equation. (3) and (4) are seen as the Slutsky equation.[2]

Two conditions can be stated which insure that demand for futures is downward sloping:

U6) [Arrow].[3] Risky assets are not an inferior good if absolute risk aversion $A(M) = \frac{-U''(M)}{U'(M)}$, is a strictly decreasing function of M.

U6') [Stiglitz].[4] $\left.\frac{\partial y^*}{\partial r}\right|_{M = \text{constant}} < 0$ if relative aversion $R(M) = \frac{-MU''(M)}{U'(M)}, < 1$.

<u>Proof</u>: (U6')

$$\left.\frac{\partial y^*}{\partial r}\right|_{M = \text{constant}} = \frac{-V_{yr}}{V_{yy}}$$

But $V_{yy} = E_x\{(x - r)^2 U''[y(x - r)]\} \leq 0$, so $\left.\frac{\partial y^*}{\partial r}\right|_{M = \text{constant}}$ has the same sign as $V_{ry} = E_x\{-U'[y(x - r)] - y(x - r)U''[y(x - r)]$.

$V_{ry} = E_z\{-U'(z) - zU''(z)\}$ where z is net payoff. But if

$$\frac{-zU''(z)}{U'(z)} < 1 \ , \ -1 \frac{-zU''(z)}{U'(z)} < 0 \ ,$$

$$\Rightarrow -U'(z) - zU''(z) < 0 \ ,$$

and the result follows.

(U6) For $w > 0$

$$A(z + w) < A(z) \Rightarrow U''(z + w) > -A(z)U'(z + w)$$

$$wU''(z + w) > -wA(z)U'(z + w)$$

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2. This analysis is implicit in Arrow [1], Chapter 3.

3. Arrow [1], page 119.

4. Stiglitz [11].

which is true for all w .

Now let z be some fixed benchmark payoff, and w be the random element in payoff;

$$E_w\{wU''(z+w)\} > -A(z)\ E_w[wU'(z+w)] = 0 \quad .$$

$\therefore E_z\{zU''(z)\}$ is positive for any z , and the result follows.

To understand why an equilibrium price exists, it is instructive to notice that the following lemma is true:

Lemma 1. [Arrow).[5] The sign of $\frac{\partial}{\partial y} V_i(o, r)$ is the same as the sign of $E_x^i(x) - r$.

Proof:
$$\frac{\partial}{\partial y} V_i(y, r) = E_x^i\{(x - r)\ U_i'[M_i + y_i(x - r)]\} \ .$$

$$\frac{\partial}{\partial y} V_i(o, r) = E_x^i\{(x - r)\ U_i'(M_i)\}$$

$$= [E_x^i(x) - r]\ U_i'(M_i) \quad .$$

$U_i'(M_i) > 0$ by assumption, and the assertion is true.

For notational purposes, define $\max\limits_i E_x^i(x) = \bar{x}$ and $\min\limits_i E_x^i(x) = \underline{x}$. So that there will be some trading in futures, assume $\bar{x} > \underline{x}$.

If either (U6) or U6') is met for each trader i , then each of the demand functions $y_i^*(r)$ is monotonic decreasing, and the following holds:

Proposition 1: Assume that all traders have decreasing absolute risk aversion, or relative risk aversion less than one. Further, assume that (P1) holds for $\bar{r} = \bar{x}$, $\underline{r} = \underline{x}$. Then excess demand $D(r) = \sum\limits_i y_i^*(r)$ is a continuous strictly monotonic decreasing function of r . Thus, there exists a unique r^* such that $D(r^*) = 0$.

Proof: $D(\bar{x}) < 0$, while $D(\underline{x}) > 0$. Since $D(\cdot)$ is obviously continuous,

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5. Arrow [1], page 100. This is his proposition that a small part of a favorable risk will always be taken.

$D(r^*) = 0$ for some r^* such that $\bar{x} > r^* > \underline{x}$. Since $D(\cdot)$ is strictly monotonic, r^* is unique.[6] It is crucial that all traders think there is some chance that a realization of x could lie outside the range $[\underline{x}, \bar{x}]$. Otherwise the existence of a "sure thing" gamble for one or more traders could cause $D(\cdot)$ to be discontinuous. Without continuity, equilibrium might not exist.[7]

Our analysis thus yields an equilibrium price r^* for futures that lies somewhere between the expected outcomes $E_x^i(x)$ for the various traders. This competitive price is some kind of average of traders' expectations. The exact averaging procedure is determined by the traders' subjective probability distributions, and by their preferences for risk.

B. Market prices and quantities when new information will be available, and recontracting is possible

So far the analysis has allowed traders to gamble according to their judgment about various outcomes, but no appreciation or depreciation in terms of price has been allowed. The market takes place, and binding contracts for dollar claims contingent on various states of the world are made. The realization of the state-of-the-world process is then observed, and claims are paid. Let us now extend the model to two sequential markets.

The first market is conducted in a manner similar to that in part A; with excess demand set at zero, and all traders behaving as price takers. Then some given period of time elapses, and a second competitive market takes place. The same futures are again traded, and a (possibly new)

6. Debreu [2] proves a more general theorem on competitive equilibrium with uncertainty in Chapter 7, but his weaker assumptions do not imply uniqueness.

7. See Green [3] for a proof of a very similar proposition under less restrictive conditions.

competitive price is found. After this second market, the state of the world $\hat{\omega} \varepsilon \Omega$ is observed, and the claims contracted by the trade in futures are paid off. The first market will be called the "initial market", while the second market will be called the "intermediate market". Traders in the initial market realize that they will have a chance to recontract in the intermediate market. To simplify matters, time preference is neutral for all traders.

So far, there is nothing in the two-market model to make the equilibrium prices in the two markets differ. The timeless preferences of the traders will make the second market identical to the first. This is unrealistic; with the passage of time, information becomes available. Let us formalize this by saying that in the intervening time between the initial and intermediate markets, traders will observe event $\sigma \varepsilon S$. This σ contains information about the state of the world ω that will eventually occur. Each trader assigns a joint subjective probability measure to $\Omega \times S$. If events in Ω are not independent of events in S , the realization $\hat{\sigma}$ occurring between the two markets could change the equilibrium price on the second market.

The returns to futures traders in the initial market are determined by the equilibrium price in the intermediate. Only to the extent that the intermediate price depends on the distribution of $x(\omega)$ will the initial price be dependent on Prob $\{x(\omega)\}$. To determine their demand functions for futures in the initial market, traders must first consider the price that will prevail in the intermediate market.

B1. Trader j's estimate of the intermediate price r

Each trader must calculate the equilibrium intermediate price r^* as a function of the information σ which will occur before the intermediate market takes place. Let trader j's estimate of trader i's demand function conditional on σ be shown as

$$y^*_{ij}(r_j, \sigma) .$$ [8]

(r_j is trader j's estimate of the price of futures in the intermediate market.)

Trader j's estimate of the equilibrium price, r^*_j , is thus defined by the market clearing condition:

$$\sum_i y^*_{ij} \; (r^*_j) = 0 \quad . \tag{5}$$

The r^*_j , which are functions of the event σ , are the payoffs in the initial market. Note that differences of opinion may exist in the initial market not only about the relevant probabilities Prob (σ) , but also about the rewards $r^*_j(\sigma)$. This element was not present in the analysis with no recontracting.

B2. Equilibrium price for the initial market when an intermediate market exists

The r^*_j derived above in B1 are functions of σ , the state of the world (or state of information) that is observed between the initial and intermediate markets. Thus each trader perceives the payoffs for purchasing a future in the initial market as $r^*_j(\sigma)$. If we call s the price in the

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8. Note that trader j's task in estimating y^*_{ij} (r_j, σ) is an enormous task, since y^*_{ij} (r_j, σ) depends both on trader i's aversion to risk, and his probability distribution for payoffs $x(\omega)$ given that σ occurs.

initial market, w_j the amount of futures bought by trader j in the initial market, and E_σ^j trader j's expectation operator with respect to the state σ, then j's preferences are given by

$$V_j(w_j, s) = E_\sigma^j\{U_j[M_j'(\sigma) + w_j(r_j^*(\sigma) - s)]\} \quad . \tag{6}$$

The term $M_j'(\sigma)$ has replaced the M_j for trader j's initial wealth, and reflects the possibility that events σ give the trader information about his expected wealth after ω is observed. $M_j'(\sigma)$ is, then, trader j's expected wealth just after event σ has been observed.

Following the analysis in part A, there is a w_j^* such that

$$\frac{\partial V_j}{\partial w_j} = E_\sigma^j\{r_j^*(\sigma) - s)\; U_j'[M_j'(\sigma) + w_j^*(r_j^*(\sigma) - s)]\} = 0 \quad . \tag{7}$$

Again we are assuming that a condition analogous to (P1) holds, and no trader has an unbounded demand due to what he considers to be a sure thing. The market clearing condition

$$\sum_j w_j^* \; (s^*) = 0 \tag{8}$$

thus determines the equilibrium price s^* in the initial market. Since we assume either (U6) or (U6') to obtain downward sloping demand curves, we obtain the following proposition:

<u>Proposition 2</u>: For an initial market in futures, with one intermediate market scheduled to take place after some information σ about the final state of the world (and payoff vector) is received by all traders, an equilibrium price s^* is determined by (7) and (8). $r_j^*(\sigma)$, a parameter in these equations, is determined by (5).

This analysis is put forward to answer the following question: What effect does speculation have on the market price for futures? To answer this question, we must compare the price s^* to some other price determined in a less speculative manner. The price proposed for this comparison is the equilibrium price for the initial market when no intermediate market is held. The price for this market should be an average of traders' expectations about the final state of the world ω . It will be speculative in the sense that traders will be betting on this final state, but traders will not try to outguess each other about the intermediate state σ . Traders will incorporate their expectations about σ into their demand functions, but only to the extent that it relates to ω . Thus the traders are constrained to trade for "long run" gains or losses only, since "short run" gains (buying in the initial market and selling in the intermediate market, say) are excluded.

B3. Equilibrium in the initial market when no intermediate market exists

This market price is determined exactly like the one in A. Expectations over σ will be taken to find each trader's probability distribution for $x(\omega)$ before the realization $\hat{\sigma}$ occurs. For each trader i , the expectation functional

$$E_x^i\ (g(x)) \quad \text{is given by} \quad \int_x (g(x) \int_s dF_x^i(\sigma) \tag{9}$$

where $F_x^i(\sigma)$ is the i-th trader's distribution function of x conditional on σ .[9]

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9. Note that x is a random variable depending on ω , so that the joint probability measure on $\Omega \times S$ represents a trader's expectations about states of the world.

If we call the price in this market t, and the amounts demanded z_i, then the traders' demand functions $z_i^*(t)$ are given by:

$$E_x^i \{(x - t) U_i' [M_i + z_i^*(x - t)]\} = 0 \quad , \tag{10}$$

and the equilibrium price t^* is determined by

$$\sum_i z_i^* (t^*) = 0 \quad . \tag{11}$$

Equations (9), (10) and (11) give the unique equilibrium price t^* when no recontracting in an intermediate market is allowed. Again "sure thing" problems are ruled out by an assumption like (P1). We now turn to the relation of s^*, the speculative price, to t^*, a less speculative one.

C. <u>The relationship of t^* to s^*</u>

In general t^* and s^* will be different. To discover what might cause them to be different, it is instructive to list some conditions under which they will be the same.

<u>Proposition 3</u>: The equilibrium price with recontracting s^* (determined according to equations (5) - (8)), and the equilibrium price without recontracting, t^*, (determined according to equations (9) - (11)) are equal if any of the following are true:

a) Realization $\sigma \varepsilon S$ has no effect on traders' expectations about ω. This is true if σ and ω are considered independent by all traders. This is the trivial case where no new information will be available to distinguish the intermediate market from the initial one.

b) At equilibrium market price s^*, each trader's demand for lottery tickets with payoffs $x(\omega)$ contingent on ω is the same as his demand for lottery tickets with payoffs $r(\sigma)$ contingent on σ.

c) Individual demands at price s^* for $x(\omega)$ tickets and $r(\sigma)$ [10] tickets are different, but both add to zero at s^*. In this case, s^* and t^* are "accidentally" the same.

Case b) is the most interesting one. If each individual trader i could calculate an equilibrium market price $r_i(\sigma)$ as a function of the information σ, then it would be most unlikely that these would be just the numbers to equate his demand for $x(\omega)$ tickets and $r(\sigma)$ tickets. Notice that the calculation of $r_i(\sigma)$ is a very involved one. If traders have <u>neutral short run expectations</u>, however, and ignore the intermediate market completely, s^* and t^* will obviously be identical. The difficulties of getting good estimates $r_i(\sigma)$ have been erased.

This is a case where no decision is a decision; "total ignorance" of $r_i(\sigma)$ is the same as assuming that $r_i(\sigma)$ is any of a class of random variables that will equate short-run demand and long-run demand. But there is a very reasonable sort of guess, when faced with calculation difficulties regarding intermediate prices. Consider the following example:

> <u>Example</u>: A trader i with initial wealth W_o is faced with choosing the number of lottery tickets which maximizes his expected utility. Contingent on σ, <u>net</u>

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10. An "$x(\omega)$ ticket" means a contract which pays $x(\omega)$ to its holder contingent on event $\omega \varepsilon \Omega$.

payoffs for these tickets and his associated judgmental probabilities are:

$\sigma = 0$		$\sigma = 1$	
$x(\omega) = -1$	prob $= \frac{1}{2}$	$x(\omega) = 0$	prob $= \frac{1}{2}$
$x(\omega) = +1$	prob $= \frac{1}{2}$	$x(\omega) = 4$	prob $= \frac{1}{2}$

The trader's utility function is $U(M) = \log M$, and he thinks the outcomes $\sigma = 0$ and $\sigma = 1$ are equally likely (probability $= \frac{1}{2}$ for each). If the trader ignores any market that may exist after σ has been observed, he will invest $W_o/2$ in lottery tickets. If we let a be the payoff when $\sigma = 0$, and b be the payoff when $\sigma = 1$, then if $a = \frac{-b}{b+1}$, and $0 > a > -2$, ignoring any intermediate market will have been rational. Note that if other traders' expectations are not radically different from those of the trader under discussion, then some of these values for a and b are reasonable guesses about intermediate prices.[11]

D. Efficiency of futures prices

In a world of certainty, the price of a product is the unique indicator of its relative scarcity and cost of production. A decision maker consuming it or using it as a factor of production need not consider the methods for producing it. The price carries with it all the relevant information about economic efficiency.[12] In a world of uncertainty, this is no longer true. If (using the notation of the above model) $x(\omega)$ is the uncertain spot

11. Neutral short-run expectation, then, is the behavioral assumption that traders base their expectations on real (long-run) considerations rather than guess about other traders' misinterpretations in the short run. The model applies to a trader on the stock exchange who invests in a company whose earnings he expects to increase. It does not apply to considerations of today's stock price versus tomorrow's, which are both "short run".

12. This is a very "perfect" world.

price of a commodity, then reliance on the futures price t^* may be costly. By using only t^* as a decision variable, the decision maker essentially relies on traders' expectations about $x(\omega)$, and does not take into account that their risk aversion may be different from his. If $x(\omega)$ is important to the decision maker, it will usually be worth his time to find out about the production of the commodity, and form his own opinion about $x(\omega)$.

Note, however, for the decision makers who do use futures prices as decision variables, t^* is a more efficient indicator than s^* . To the extent that decision makers rely on futures prices, and traders have definite opinions about short-run prices that conflict with their long-run expectations, introduction of short-run markets will result in bad decisions and economic inefficiency.

It seems unlikely, however, that in the absence of some exogenous information that is known to all traders that many short-run expectations may be different from long-run ones, that s^* and t^* will be very different. Just the calculation difficulties speak eloquently for this point.

E. The models of Keynes and Radner

J. M. Keynes, in his General Theory, discusses a model of speculative price determination that is very similar to the one proposed above. In Chapter 12, "The State of Long-Run Expectation", Keynes wrote of investment on a speculative market:

> "..... professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor

> whose choice most nearly corresponds to the average preferences of competitors as a whole, so that each competitor has to pick, not those faces which he himself finds prettiest, nor even those which average opinion thinks is prettiest."[13]

This "beauty contest" is exactly the problem facing a speculator in the initial market when an intermediate market exists. Keynes' abstraction is complicated by the fact that wide divergences of opinion about beauty are possible, while the $x(\omega)$ final outcomes in our model are assumed agreed upon by all parties. The computational difficulties in calculating an average of other contestants' preferences argues for ignoring them; neutral short-run expectations indicate that any trader who is fairly sure which six contestants are the prettiest should make them his choice.

It is clear that the stocks of industrial corporations have final outcomes that are somewhere between those of a beauty contest and exact payments contingent upon observable events. Sales and earnings reports provide evidence about a company's value. On the other hand, corporations are not liquidated at any given date, so that the final outcome is always based on opinion and judgment.

Keynes makes a convincing case that stock prices are not ruled by long-run considerations, that is, s^* and t^* may be radically different. It is hard to argue with a man whose investments made fortunes for himself and King's College. But any theory that attempts to explain short-run expectations and price formation must be based on a model of mass psychology and external stimuli. If it is well known that in certain situations many traders will hold erroneous beliefs, then it is in the interest of the rational trader to

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13. Keynes [5], page 156.

capitalize on this information. Consider for instance, a plainclothesman at a demonstration. If the crowd panics and runs at the sight of approaching police units, it is in his interest to stay with the crowd, rather than stand and be trampled, even though he knows that police have orders forbidding violence.

In the absence of the external stimulus (police units), or knowledge of it, it would have been ridiculous for the plainclothesman to try to predict the behavior of the crowd and move in advance of it. Any theory of short-run expectations must take account of widespread misinterpretation of universally observable stimuli. This would be a study of mass hysteria rather than rationality.

Radner[14] has studied price determination in an uncertain world from a more abstract point of view. He starts with the Arrow-Debreu model,[15] which assumes the existence of a market for any commodity contingent on any possible state of the world. Such a world is not an evolutionary one; all prices may be determined at the beginning of time. Then as time passes, states of the world are observed, and sellers of commodities in unrealized states are winners, while those who sold contingent on realized states are usually losers.

An Arrow-Debreu formulation of the model presented in Section B would consist of contracts for dollars contingent on states of the world, $\sigma \varepsilon S$, and states of the world $\omega \varepsilon \Omega$. The markets for the contracts would be held simultaneously, and there would be one market for each possible outcome. A price for dollars contingent on any state σ would be determined by the

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14. See Radner [6], [7], [8], [9].

15. See Debreu [2], Chapter 7.

competitive mechanism. Note that for this world, the information variable must be perceived identically by all traders. If there is any outcome of the information process which yields disagreement about the realization $\hat{\sigma}$, no contracts may be made contingent upon this outcome.

In Radner's terms, information may be subjective.[16] The model in Section B may be characterized by saying that all $\sigma \varepsilon S$ are not objectively verifiable, or that it is too costly to decide what state of the world has actually occurred. An example of such a σ might be "economic activity is improving". The equilibrium price s^* depends on actions (expectations coupled with attitudes towards risk) of the other traders. This was recognized by Radner in both [6] and [7].

It is interesting, however, to analyze this dependence and see the complexity of the calculation it entails. It is this author's contention that this complexity makes a very good case for neutral short-run expectations in cases where correlated group misinterpretation of information is absent.

16. Radner [7], page 11.

BIBLIOGRAPHY

[1] Arrow, Kenneth J. Essays in the Theory of Risk-Bearing. (Chicago: Markham, 1971.)

[2] Debreu, Gerard. Theory of Value. (New York: Wiley, 1959.)

[3] Green, Jerry R. "Temporary General Equilibrium in a Sequential Trading Model with Spot and Futures Transactions." CORE Discussion Paper No. 7127, August 1971.

[4] Herstein, I. N., and John Milnor. "An Axiomatic Approach to Measurable Utility." Econometrica, April 1953, pages 291-297.

[5] Keynes, J. M. The General Theory of Employment, Interest and Money. (New York: Harcourt, Brace, 1936.)

[6] Radner, Roy. "Competitive Equilibrium Under Uncertainty." Econometrica, January 1968, pages 31-58.

[7] ___________ "Equilibre des Marchés à Terme et au comptant en Cas d'Incertitude." Cahiers d'Económétrie. (Paris: Centre National de la Recherche Scientifique, 1967.)

[8] ___________ "Problems in the Theory of Markets under Uncertainty." American Economic Review, May 1970, pages 454-460.

[9] ___________ Existence of Plans, Prices and Price Expectation in a Sequence of Markets. Unpublished.

[10] Sandmo, Agnar. "Capital Risk, Consumption, and Portfolio Choice." Econometrica, October 1969, pages 586-599.

[11] Stiglitz, Joseph. "The Effects of Income, Wealth, and Capital Gains Taxation on Risk-Bearing." Quarterly Journal of Economics, May 1969.