Bulletin of University of Minnesota

ENGINEERING EXPERIMENT STATION

O. M. LELAND, Director

BULLETIN NO. 5

TURNS AND PHASES IN SQUIRREL CAGE WINDINGS

BY

GEORGE F. CORCORAN, M.S.(E.E.)

AND

HENRY R. REED, M.S.(E.E.)

Department of Electrical Engineering

UNIVERSITY OF MINNESOTA LIBRARY

Vol. XXX No. 51 May 31 1927

MINNEAPOLIS

Entered at the post-office in Minneapolis as second-class matter
Minneapolis, Minn.
Acceptance for mailing at special rate of postage provided for in section 1103,
Act of October 3, 1917, authorized July 12, 1918

UNIVERSITY OF
MINNESOTA
LIBRARY

CONTENTS

U OF M BINDERY
FEB 28 '30
NOV 21 '27
39877

ILLUSTRATIONS

OSCILLOGRAMS

TURNS AND PHASES IN SQUIRREL CAGE WINDINGS

INTRODUCTION

The number of phases and the number of series turns per phase in the squirrel cage have been a source of speculation. It is important that the designer know these factors for any given cage in order that the reactance of the cage may be accurately predetermined. Analytical and experimental proof is given to substantiate the writers' conception of the number of phases and the number of series turns per phase that exist in the squirrel cage.

No experimental data have been discovered with which to correlate the results of the present investigation. For that reason, reactance formulas, based upon the results of this study, are set forth in detail. Heretofore, reactance formulas have been employed which gave satisfactory results only when used on a restricted class of machines. Much speculation has been given to finding the constants which are used in the factor which converts the reactance of the squirrel cage secondary to the primary.

The conversion factor herein employed is not restricted to any certain class of machines. No arbitrary constants are needed since it is based upon fundamental notions. Predetermined reactances are compared with actual tested values of reactances for a wide variety of machines. On pages 30, 38, and 42 are to be found comparisons between calculated and tested values of reactances of 37 different types of machines, ranging in size from 15 to 2,000 h.p., from 120 to 1,800 r.p.m., and from 220 to 6,600 volts at both 25 and 60 cycles.

The writers wish to express their appreciation to Professor J. H. Kuhlman for suggestions given during the compilation of the reactance formulas, to the Electric Machinery Manufacturing Company for their courtesy in furnishing many of the designs and tests of the synchronous motors, and to Mr. I. C. Benson for his assistance in checking the reactances of several of the machines.

A HYPOTHESIS

A quantitative treatment of the squirrel cage requires that its number of phases and turns in series per phase be known. The fact remains to be shown that a squirrel cage with its apparent numerous combinations of electric circuits may be reduced to a simple winding. By reducing the cage to a simple winding, its reactance and resistance may be readily determined.

A hypothesis concerning the equivalent circuit of the squirrel cage must be advanced in order to have a working basis for the analysis. The hypothesis: "Two bars in the squirrel cage, a pole pitch apart, function as a single turn coil and they are not influenced in their functioning, as a single coil, by reason of being electrically connected to all the other bars of the cage."

If the above statement is to be accepted unconditionally and an equivalent circuit derived, a rigid proof is required. The proof rests principally upon two facts. First, the predetermined current flow in a cage bar, under a variety of conditions, is identical with the *actual* flow of current, in this bar, as recorded by the oscillograph. Second, the reactance, calculated by means of formulas which are derived with this hypothesis as a basis, checks very closely with the reactance as obtained by test on the finished machine.

PREDICTION OF CURRENT FLOW IN THE SQUIRREL CAGE, ASSUMING THE EXISTENCE OF INDIVIDUAL POLE PITCH COILS

POLYPHASE PRIMARIES

The current induced in the equivalent secondary winding[1] by the rotating magnetic field of a polyphase primary is evident. A sinusoidally distributed rotating field generates a recurrent sine wave current in each of the conductors of the equivalent winding. The current in a given pole pitch coil depends directly upon the magnitude and wave form of the rotating field and upon the relative motion of the secondary and the moving field. The magnitude of the current will be the same in each of the secondary coils. The time phase of the currents in two adjacent coils depends upon their space phase.

SINGLE PHASE PRIMARY—SYNCHRONOUS SPEED

The reasons for predicting current flow in the equivalent secondary winding when placed in the unbalanced field of a single phase primary are:

1. Certain interesting facts concerning the secondary current of the single phase motor are revealed.

2. Several exacting requirements are set up, which, if met by actual test, practically prove the hypothesis concerning the squirrel cage.

The cross field theory of the single phase induction motor is employed as a basis for determining current flow in the secondary. This method permits the breaking up of the resultant magnetic field into its two quadrature components. The magnetic field established by a

[1] By "equivalent secondary winding" is meant a winding which is composed wholly of individual pole pitch coils (Fig. 1).

single phase primary is, of itself, only pulsating. At standstill, transformer action takes place between the primary and secondary. Current flow in the secondary is in such a direction that it sets up an opposing field, i.e., one which reacts directly upon the primary. The "main" field, along the axis of the primary winding, is thus established.

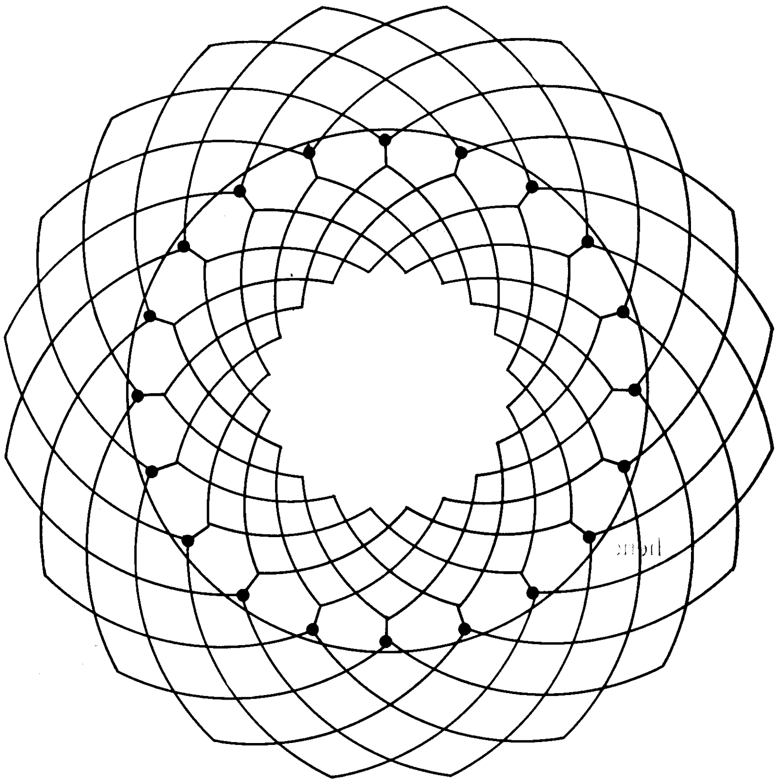


FIGURE I. SHORT CIRCUITED PITCH WINDING

If the secondary member has motion relative to the primary, an additional magnetic field is established by reason of the secondary conductors cutting the "main" field. This additional magnetic field is called the "cross" field. The ideal "cross" field is one which is equal in magnitude to, displaced ninety degrees in space from, and in time quadrature with, the "main" field. The "cross" field depends directly upon the relative motion of the primary with respect to the secondary for its value; hence, is proportional to speed, being equal to the product of the "main" field and per cent synchronous speed, as a decimal. Definite relationships between the "main" field and the "cross" field, for various conditions of operation, are given later.

A given secondary coil is acted upon by both fields so far as transformer action is concerned. The coil, in turn, has an e.m.f. generated in it by reason of its motion through each field. The result is that four component voltages must, in effect, be considered if the resultant voltage induced in a given coil is to be determined. The derivation of the equations for the four component voltages follow.

Let $i_p = I_{p\ max} \sin \omega t$ (instantaneous primary current)

Φ_m = the flux set up by i_p, together with that part of the magnetic field set up by the secondary current along the Φ_m path. Φ_m is assumed to be sinusoidally distributed in space and varying in time with i_p.

Φ_c = the flux set up by the secondary current which is in space quadrature with Φ_m. Φ_c lags behind Φ_m by θ degrees in time.

$\theta = \tan^{-1} \frac{X_c}{R_c}$ where X_c is the total reactance of the Φ_c circuit, including its leakage reactance, and R_c is the resistance of the secondary winding. Certain calculations, herein, are based upon $\theta = 75°$. This is an extremely low value of θ as compared with that of commercial machines. However, comparison is to be made between predicted values of secondary current and experimentally determined values, hence θ is chosen to correspond to the ratio of reactance to resistance of the test machine.

b_m = instantaneous value of the flux density of the magnetic field, Φ_m, at any point along the air gap, Figure 2.

b_c = instantaneous value of the flux density of the field, Φ_c, at any point along the air gap, Figure 2.

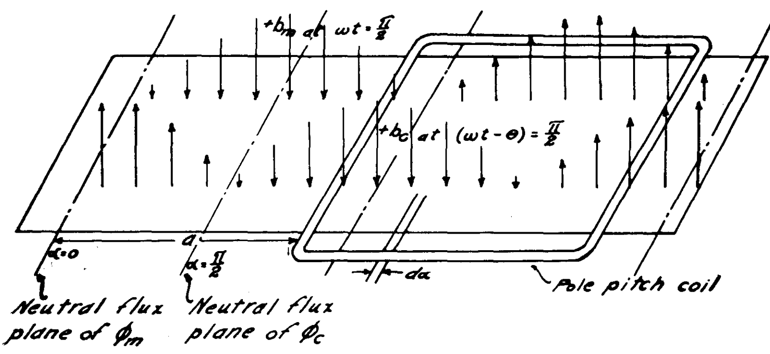


FIGURE 2. PICTORIAL REPRESENTATION OF FLUXES IN SINGLE PHASE MOTOR

A two-pole primary is assumed unless otherwise stated.

$B_{m\ avg}$=average flux density of Φ_m embraced by a pole pitch coil at any time, t.

$B_{c\ avg}$=average flux density of Φ_c embraced by a pole pitch coil at any time, t.

$b_m = B_{max} \sin(\omega t) \sin \alpha$.

$b_c = B_{max} \sin(\omega t - \theta) \sin(\alpha - \pi/2)$ where B_{max} is the maximum flux density that occurs at $\alpha = \pi/2$ when $\omega t = \pi/2$.

α=angular measure from the neutral flux plane of Φ_m.

a=the angular displacement of the first coil edge (of any given pole pitch coil), measured from α=o. (Figure 2.)

$$B_{m\ avg.} = \frac{B_{max} \sin(\omega t)}{\pi} \int_{a}^{(a+\pi)} \sin \alpha \, d\alpha$$

$$= \frac{2 B_{max}}{\pi} \sin(\omega t) \cos a$$

$$B_{c\ avg.} = \frac{B_{max} \sin(\omega t - \theta)}{\pi} \int_{a}^{(a+\pi)} \sin\left(\alpha - \frac{\pi}{2}\right) d\alpha$$

$$= \frac{2 B_{max}}{\pi} \sin(\omega t - \theta) \sin a$$

Multiplying the area of the coil, which is ($\pi\ r_c\ l$) by the average density, the amount of flux embraced by a pole pitch coil is determined.

$\Phi_{m\ coil} = 2\ r_c\ l\ B_{max} \sin(\omega t) \cos a$

$\Phi_{c\ coil} = 2\ r_c\ l\ B_{max} \sin(\omega t - \theta) \sin a$

where

r_c=radius of the secondary core.

l=axial length of the secondary core.

The e.m.f.[2] induced in a given secondary coil which is rotating in the complex magnetic field of the single phase machine can be determined by separating the resultant e.m.f. into its four component parts, namely,

$E_{t\Phi m}$=that component which results from the straight transformer action of Φ_m upon the given coil.

$E_{c\Phi m}$=that component due to the cutting action of the coil as it moves through Φ_m.

$E_{t\Phi c}$ =that component due to the transformer action of Φ_c.

$E_{c\Phi c}$ =that due to the cutting of Φ_c.

Formulas for the four component voltages:

Transformer action, $E_t = -\frac{d\Phi_{coil}}{dt} \, 10^{-8}$

Cutting action, $E_c = 2blv \, 10^{-8}$ (for the two coil sides)
where v=velocity=$r_c\omega$

$$E_{t\Phi m} = -K\cos(\omega t)\cos a \qquad (1)$$
$$E_{c\Phi m} = +K\sin(\omega t)\sin a \qquad (2)$$
$$E_{t\Phi c} = -K\cos(\omega t-\theta)\sin a \qquad (3)$$
$$E_{c\Phi c} = -K\sin(\omega t-\theta)\cos a \qquad (4)$$

where

$$K = 2r_c \, l \, B_{max}\omega 10^{-8}$$

$B_{c\ max}$ and $B_{m\ max}$ have been considered as equals. This condition exists only at synchronous speed. Account will be taken of the reduction in the cross field, Φ_c, which accompanies a reduction in speed when these cases are considered.

Angle "a" is a measure of the position of the coil with reference to the neutral flux plane of Φ_m. As such, it is a function of time, being equal to $(\omega t+e)$[3] at synchronous speed. "e" is the angular displacement of the first coil edge, of a given coil, in the direction of motion, from the neutral flux plane at t=o.

At synchronous speed, equations (1), (2), (3), and (4) may be written as follows:

$$E_{t\Phi m} = -K\cos(\omega t)\cos(\omega t+e) \qquad (a)$$
$$E_{c\Phi m} = +K\sin(\omega t)\sin(\omega t+e) \qquad (b)$$
$$E_{t\Phi c} = -K\cos(\omega t-\theta)\sin(\omega t+e) \qquad (c)$$
$$E_{c\Phi c} = -K\sin(\omega t-\theta)\cos(\omega t+e) \qquad (d)$$

[2] It is understood, that with a common system of units, all expressions for voltage represent induced e.m.f. in volts. Likewise any expression for current represents amperes since the impedance of the coil is understood to be in ohms.

[3] The method of separating the resultant voltage into its component parts permits the substitution of $(\omega t+e)$ for "a" even tho "a" has previously been differentiated as a constant.

Combining (a), (b), (c), and (d) linearly gives the resultant voltage induced in a pole pitch coil as it travels synchronously through the magnetic field of the single phase machine. Let the resultant voltage be E_r, then

$$I_{coil} = \frac{E_r}{Z_{coil}}$$

I_{coil} may be calculated in terms of K′, where

$$K' = \frac{2\, r_c\, l\, B_{max}\, \omega}{Z_{coil}} 10^{-8} = \frac{K}{Z_{coil}}$$

Instantaneous values of the current flow in a short circuited pole pitch coil have been calculated in terms of K′ and are plotted for one cycle of ωt in Figure 3. Current flow for two values of θ, namely, θ equal to 80 degrees and θ equal to 75 degrees, are shown. A pole pitch coil on the secondary of a single phase induction motor will have induced in it, at synchronous speed, a double frequency current as shown in Figure 3.

SINGLE PHASE PRIMARY —93.3 PER CENT SYNCHRONOUS SPEED

If the "equivalent winding" moves through the field of the single phase induction motor at a speed less than synchronism, "a" can no longer be set equal to (ωt+e). "a" becomes (sωt+e), where "s" is per cent synchronous speed, as a decimal.

The case of 0.933 synchronous speed has been chosen since it results in "a" progressing 336 degrees while ωt completes one cycle. Calculations of the instantaneous values of I_{coil} are thus simplified. The velocity being decreased, $E_{c\Phi m}$ and $E_{c\Phi c}$ will be decreased proportionally since cutting voltage is proportional to speed. Φ_c will also be decreased practically in proportion to the decrease in speed since Φ_c is established by virtue of the speed.

Equations (1), (2), (3), and (4) take the following form for any speed:

$$E_{t\Phi m} = -K \cos(\omega t) \cos a \qquad (1')$$
$$E_{c\Phi m} = +s\, K \sin(\omega t) \sin a \qquad (2')$$
$$E_{t\Phi c} = -s\, K \cos(\omega t - \theta) \sin a \qquad (3')$$
$$E_{c\Phi c} = -s^2\, K \sin(\omega t - \theta) \cos a \qquad (4')$$

for s=0.933

$$E_{t\Phi m} = -K \cos(\omega t) \cos a \qquad (a')$$
$$E_{c\Phi m} = +0.933\, K \sin(\omega t) \sin a \qquad (b')$$
$$E_{t\Phi c} = -0.933\, K \cos(\omega t - \theta) \sin a \qquad (c')$$
$$E_{c\Phi c} = -0.933^2\, K \sin(\omega t - \theta) \cos a \qquad (d')$$

where

$$a = (0.933\, \omega t + e)$$

Instantaneous values of I_{coil} have been calculated for the present case and are shown in Figure 4. It is seen that I_{coil} is not recurrent at $\omega t = 2\pi$. In fact I_{coil} will not be recurrent until the secondary coil has slipped through an angular distance equal to that of a pair of primary poles. The calculated wave shape over a longer period of time is a double frequency wave superimposed on a "slip frequency" wave. The form of the "slip frequency" wave follows the curve

$$y = \sin(2\pi F t)$$

where F=(r.p.s. slip) × pairs of poles.

SINGLE PHASE PRIMARY—57 PER CENT SYNCHRONOUS SPEED

The prediction of what the current flow will be in each of the coils of the equivalent winding is made for the exaggerated case of 57 per cent synchronous speed by means of formulas (1′), (2′), (3′), and (4′). At this speed the component voltages may be expressed as follows:

$$E_{t\phi m} = -K \cos \omega t \cos a \qquad (a'')$$

$$E_{c\phi m} = +0.57\, K \sin(\omega t) \sin a \qquad (b'')$$

$$E_{t\phi c} = -0.57\, K \cos(\omega t - \theta) \sin a \qquad (c'')$$

$$E_{c\phi c} = -0.57^2\, K \sin(\omega t - \theta) \cos a \qquad (d'')$$

"a" being equal to $(0.57\ \omega t + e)$, that is, "a" progresses 204 degrees while ωt completes one cycle.

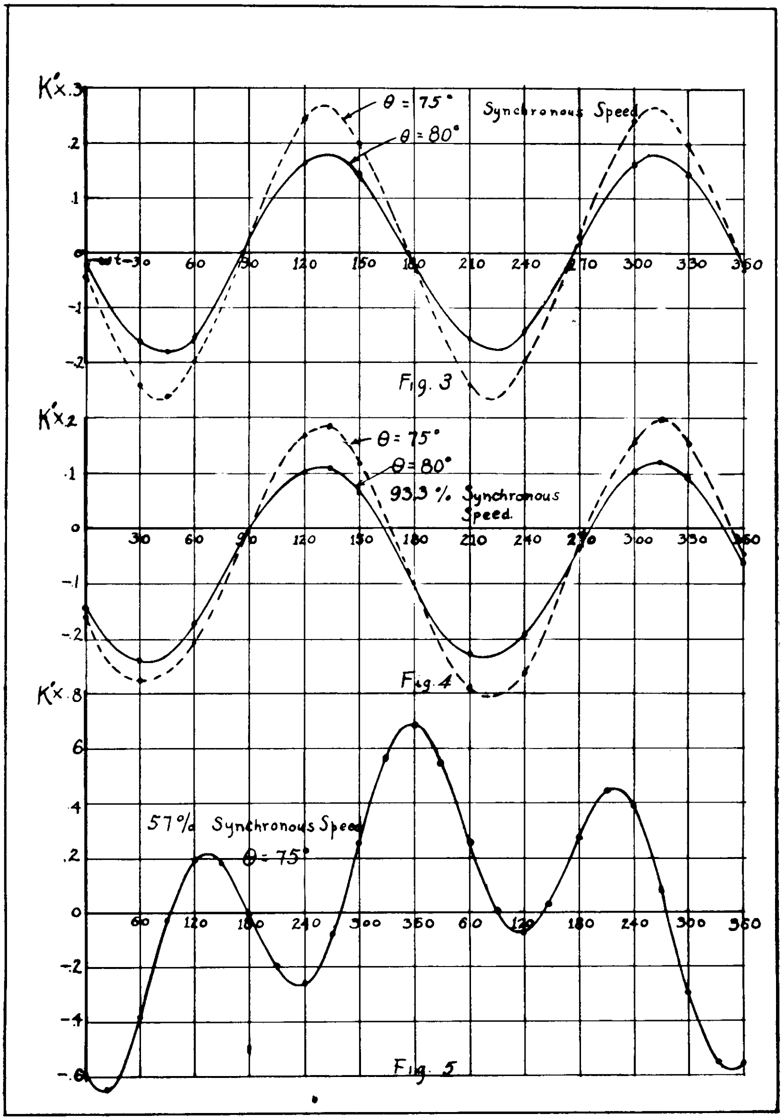
Instantaneous values of I_{coil} have been calculated for two complete cycles of ωt. The graph of these values is shown in Figure 5. The wave shape is seen to be very distorted and as such furnishes a most exacting requirement to be met by the experimentally determined wave shape.

MEASUREMENT OF SQUIRREL CAGE CURRENTS

It is thought that the present oscillographic analysis of the actual current flow in a squirrel cage is the first that has been made. Certain difficulties, which are not present in other types of motor windings, accompany the measurement of squirrel cage currents. Several methods of leading out an "IR" drop which would be indicative of the flow of current in a revolving squirrel cage were abandoned in favor of a specially constructed stationary cage.

THE EXPERIMENTAL MOTOR

The construction of the machine, showing the squirrel cage which occupies the stator, is shown in Figures 6 and 7. The bars of the cage were made of 3/16″ brass and a section 1-1/2″ in length, was reduced to 3/32″.



FIGURES 3, 4, AND 5. INDUCED SECONDARY CURRENT AS CALCULATED BY EQUATIONS

The IR drop across the reduced section of the bar was used to actuate the oscillographic element. The resistances of the bars were practically the same since they were made exactly equal in the over-all

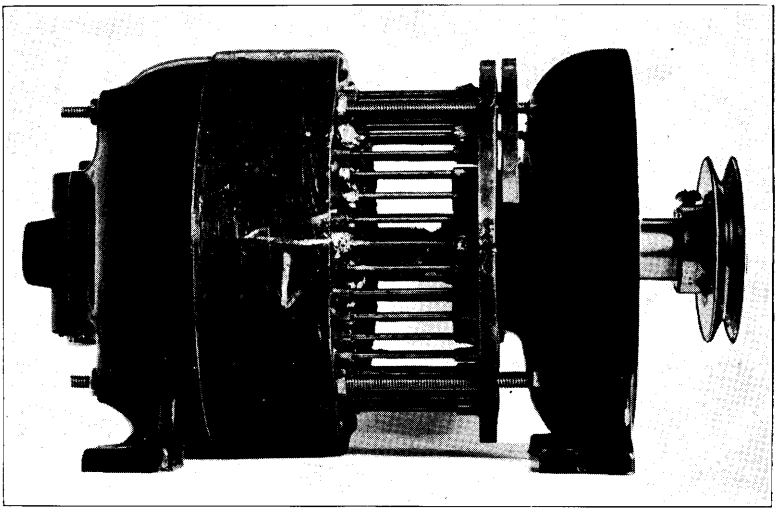


FIGURE 6. EXPERIMENTAL MOTOR

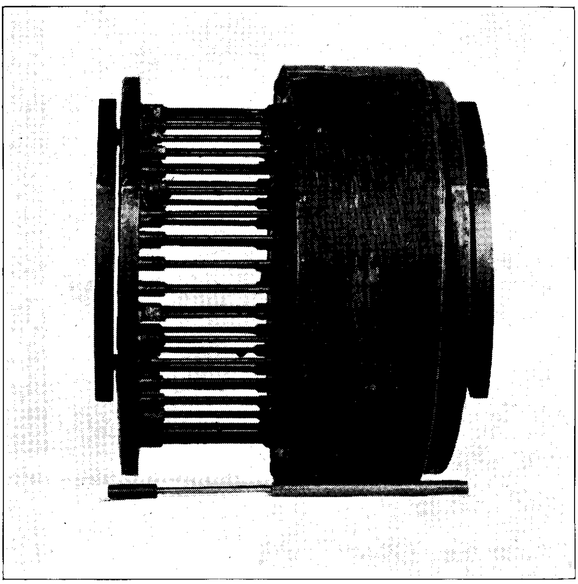


FIGURE 7. STATIONARY SQUIRREL CAGE OF EXPERIMENTAL MOTOR

dimensions and were accurately machined in their reduced sections. This somewhat higher bar resistance, over the usual resistance when copper bars are employed, was offset to some extent by heavy end rings made from 1/4″ copper plate.

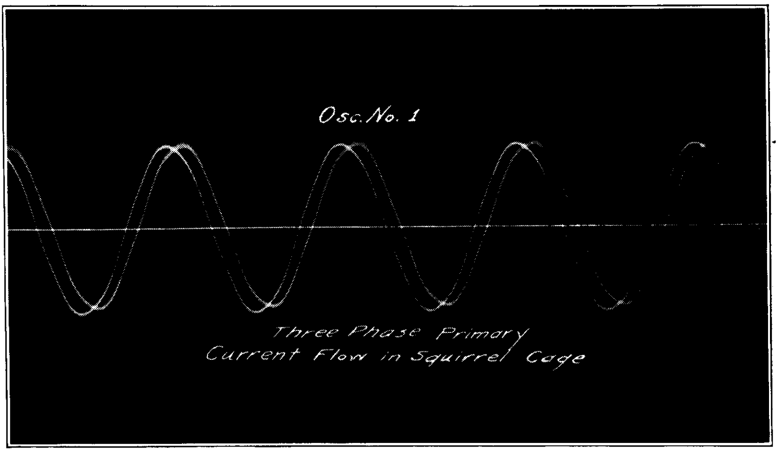
Because of the fact that there was little or no insulation between the bars and cage there were small leakage currents flowing from bar to bar, of different potentials, through the iron core. Consequently, two bars, existing under identical conditions may not have given the same IR drops across their reduced sections. However, the errors introduced by these leakage currents were not large and their presence was neglected.

COMPARISON OF MEASURED CURRENT FLOW IN THE SQUIRREL CAGE WITH PREDICTED CURRENT FLOW IN THE EQUIVALENT WINDING

1. *Polyphase primaries.*—Oscillogram 1 shows the current flow in two cage bars which are 32.7 electrical degrees from one another. The conditions under which Oscillogram 1 was taken were:

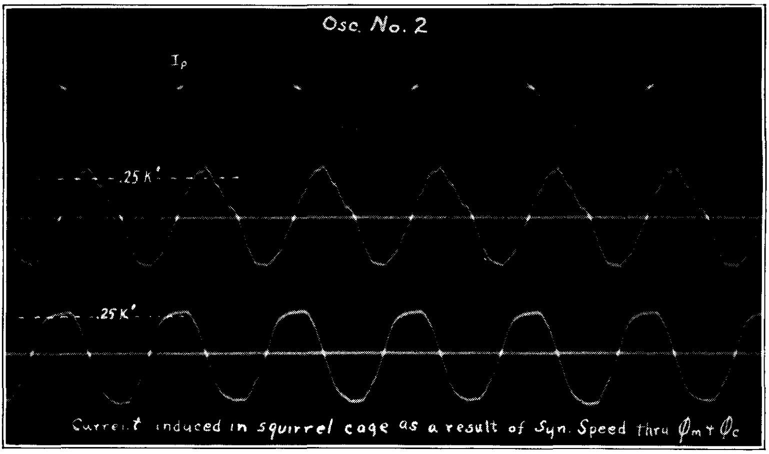
a. Locked rotor.

b. Rotating field established by a three phase primary.



It is seen that the magnitudes of the two currents are equal and that their time phase is dependent upon their location with reference to one another in the cage. Current flow in the squirrel cage, under these conditions, is identical with the predicted current flow in the equivalent winding.

2. *Single phase primary—synchronous speed.*—Oscillogram 2 shows the current flow in two squirrel cage bars which are displaced from one another by 81.75 electrical degrees.



The conditions under which Oscillogram 2 was taken were:

a. Synchronous speed.

b. Single phase primary.

The value of K′ was determined from an oscillogram of the current induced in a pole pitch coil due to the transformer action of the pulsating flux set up by the single phase primary. As was shown by equation (1),

$$E_{t\phi m} = -K \cos(\omega t) \cos a.$$

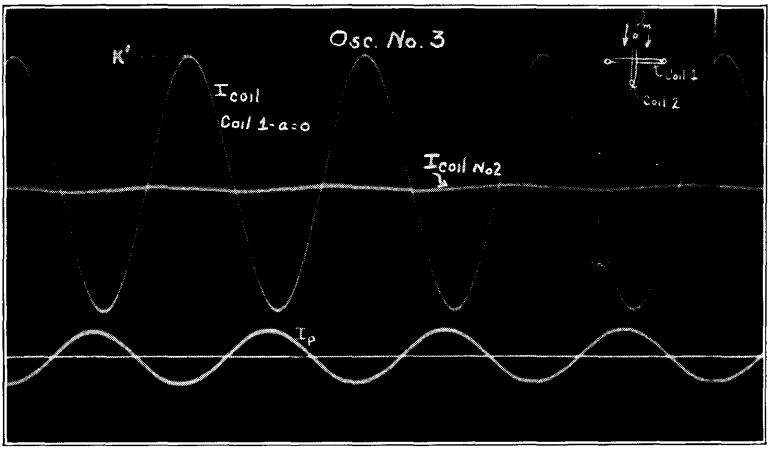
Placing the short circuited coil at "a"=o, the maximum ordinate of the oscillogram of $\frac{E_{t\phi m}}{Z_{coil}}$ is equal to K′.

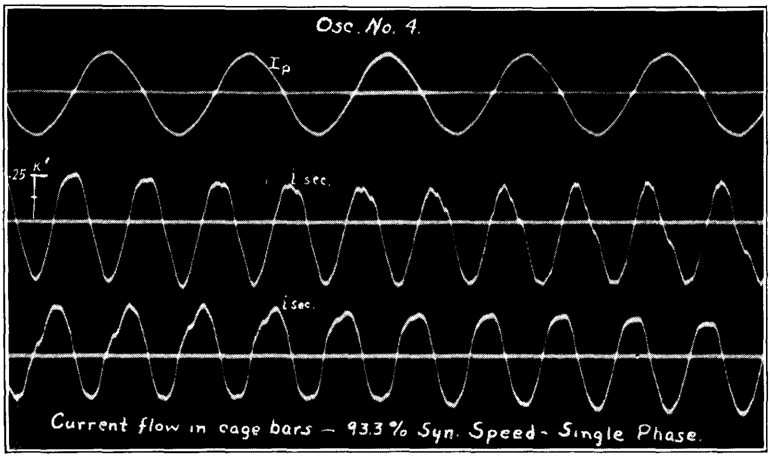
Oscillogram 3 shows I_{coil} for the coil at the position "a"=o.

Comparison of Figure 3 with Oscillogram 2 reveals the fact that the same double frequency current which was predetermined for the equivalent winding is present in the actual squirrel cage at synchronous speed.

3. *Single phase primary—93.3 per cent synchronous speed.*—The current which flows in the actual squirrel cage of a single phase induction motor operating at 93.3 per cent synchronous speed is shown in Oscillogram 4. Current flow in two bars which are 81.75 electrical degrees apart, together with the current in the primary winding, is shown. It is seen that the current in the squirrel cage is of a double

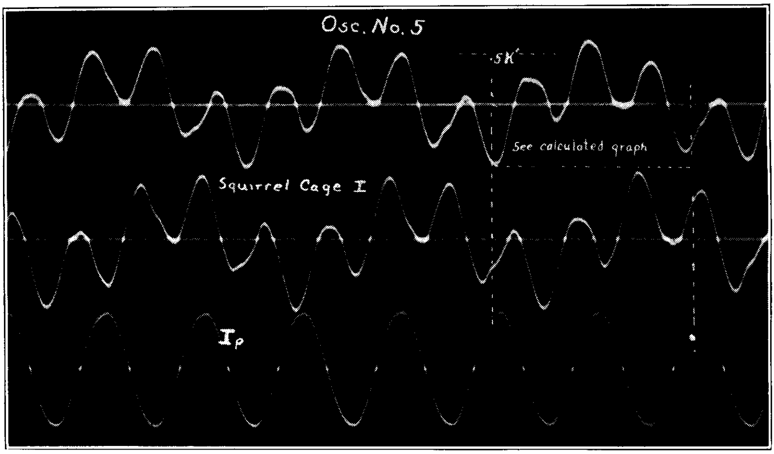
frequency wave shape superimposed on a "slip frequency" wave. The current flow in the squirrel cage is identical with the predetermined current flow in the equivalent winding for this condition of operation. See Figure 4 for the predetermined current flow for this case.





4. *Single phase primary—57 per cent synchronous speed.*—Oscillogram 5 shows the current flow in two bars of the squirrel cage of a single phase induction motor operating at 57 per cent synchronous speed. Comparison of the predetermined wave shape of the current flow in the equivalent winding, (Figure 5), with the current flow in the

actual squirrel cage for this condition of operation reveals the correlation which exists between the two windings in so far as their functioning is concerned.



ANALYSIS OF THE SQUIRREL CAGE

Since it has been proved explicitly that the current flow in the cage bars, as recorded by the oscillograph, is identical with that as predetermined from the design, it may be assumed that an experimental analysis of the squirrel cage will not be fallacious.

The analysis of the squirrel cage winding is carried out by: first, recording the current flow in various bars with the motor standing still—transformer action alone acting; second, running the motor and noting the combined effects of the transformer and cutting actions.

A single phase rotor was first used and the transformer action of the single phase primary upon the squirrel cage was determined by measuring the flow of current in the bars. This flow was measured in those bars covering a pole pitch and a half—there being a recurrence of conditions for each pole pitch.

A three phase rotor (primary) was substituted for the single phase primary. Since this rotor produced a gliding field, the only variation in current flow in the cage that occurred throughout a pole pitch was at those places where a rotor tooth was opposite a stator slot and where a rotor slot was opposite a stator slot. Such positions made the reluctances of the flux paths different which in turn caused small variations in the secondary current.

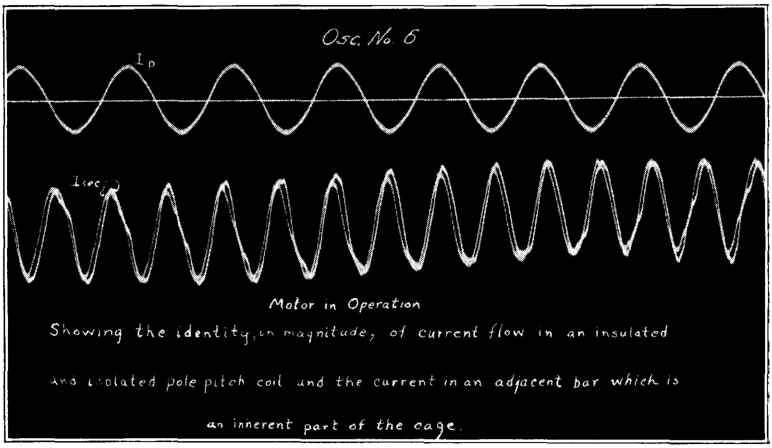
A single bar was isolated from the cage. This bar was closed on itself, thus forming an exploring coil about the secondary core. By means of this exploring coil, the current flow in a single bar, as it took different positions relative to the primary field, could be determined. What would be the relation between the current flow in the isolated bar and the current flow in any given cage bar? The oscillograph showed that the current flow in this bar, at standstill, was just one half of that which existed in a cage bar when the cage bar took the same position relative to the primary.

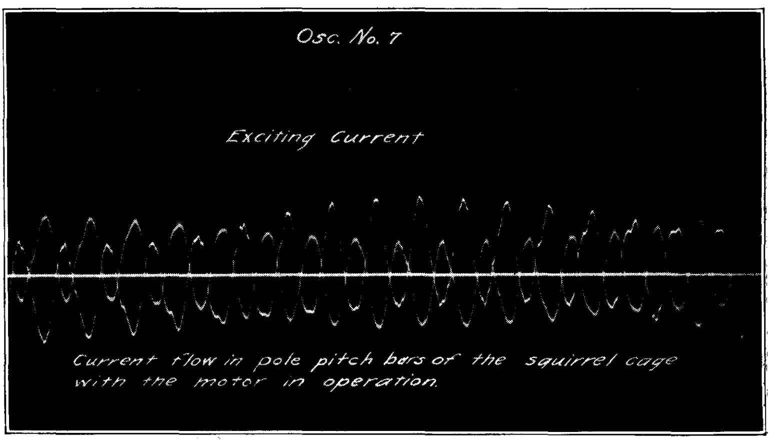
The fact is now in evidence that this bar acts together with another bar or combination of bars to produce the double current flow. There must exist another seat of induced e.m.f. which acts with a given cage bar, but where is this seat of induced electromotive force? This e.m.f. might be the result of one bar or the result of one of the many possible combinations of bars. If it is the result of one bar, this bar may be located at a pole pitch distance or it may be symmetrically located with respect to the neutral flux plane passing between two adjacent poles. Any of these conceptions are possible at standstill, but some may lead to very difficult explanations of the current flow when there is motion of the secondary relative to the primary field.

From the results of the experimental work and the mathematical analysis of the current flow, the existence of pole pitch coils seemed logical. Accordingly, two bars, a pole pitch apart, were isolated from the cage as shown in Figure 7. These bars were completely insulated from the cage so that their current flow would not be affected by any current flow in the other cage bars. If the cage reacted to the change in flux as tho it were composed of pole pitch coils, then the current flow in the bars which make up this coil would not be materially different from the current flow in the cage bars adjacent thereto.

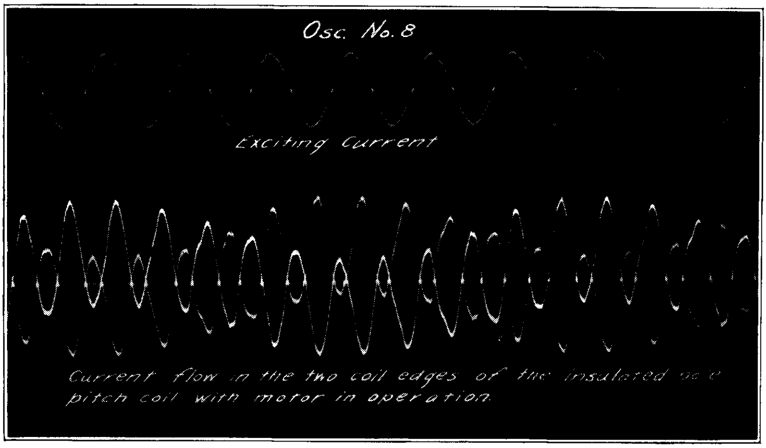
The current flow was again recorded under the conditions enumerated above, with both types of rotors, and the flow in the isolated coil was found to be the same (within limits of probable experimental error) as the flow in any bar in the cage (when operating under the same conditions). When using the three phase rotor, the current flow in the pole pitch coil was identical with the current flow in the cage bars (except for the phase displacement).

Since these tests were conducted with the rotor at rest in the various positions, the question might arise as to the existence of the same conditions when the motor was running. Accordingly, measurements were taken of the current flow in the pole pitch coil and in cage bars when the motor was operating. Oscillogram 6 shows the identity, in magnitude and wave form, of the current flow in the insulated and isolated pole pitch coil and the current flow in an adjacent bar which is an inherent part of the cage. Oscillogram 7 shows the wave forms of the current flow in cage bars a pole pitch apart with the motor in operation. The fact that the current flow is exactly opposite and of the same magnitude in these two cage bars, together with the favorable comparison they make with Oscillogram 8 (flow of current in a pole pitch coil) is conclusive proof that two cage bars a pole pitch apart function as a single turn.





If squirrel cage bars acted together as shown in Figure 8 (which is a common conception of the current flow) the current flow in the pole pitch coil and in the cage bar could not be as they are shown to be by Oscillogram 6. Obviously, if there be motion of the secondary with respect to the primary, the absolute values of the induced e.m.f.s will vary continually, resulting in a rather complex combination of e.m.f.s, since the absolute value of each e.m.f. depends upon the strength of the primary field and the position of the bar in that field.



Consider Figure 8. It is evident that, if there be motion of the squirrel cage secondary relative to the primary, no two coils such as those shown in that figure could have the same current flow (that is, no two coils within a given pole pitch). The reason is obvious; the coils (if the two bars whose current flow is shown to be continuous are to be considered as forming a coil) span different portions of the core and hence embrace different quantities of primary flux. The induced voltage, and therefore the current, is dependent on the rate of change of flux *through the coil.* If the amounts of flux embraced by two short circuited coils are different, a given rate of change of that flux will necessarily produce different induced currents in those two coils. From this it is easily seen that, if conditions actually existed as shown in Figure 8, the curve of the current flow in the isolated pole pitch coil would be very much distorted. Since there is no distortion of the current wave, that differs from that in the bar in the cage, the theory of the current flow as depicted in Figure 8 cannot hold for the running condition. The theory of pole pitch coils holds equally well for both the standstill and the running conditions.

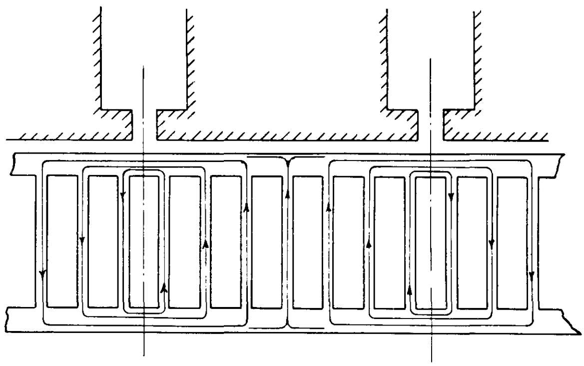


FIGURE 8. COMMON CONCEPTION OF CURRENT FLOW IN SQUIRREL CAGE WINDINGS

SHORT CIRCUITED PITCH WINDING INDUCTION MOTOR

In order to prove conclusively that the squirrel cage winding consists of pole pitch coils a special squirrel cage stator was made that consisted entirely of isolated pole pitch coils. An assembled view of this machine is shown in Figure 9 and the winding diagram in Figure 1.

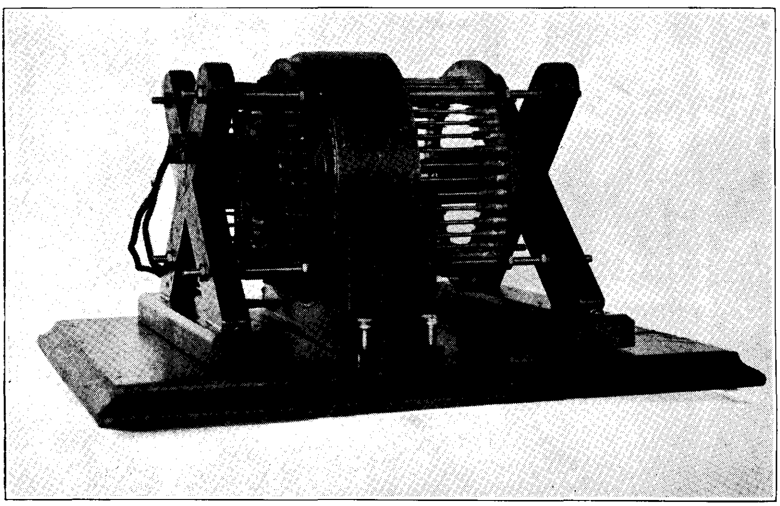


FIGURE 9. SHORT CIRCUITED PITCH WINDING INDUCTION MOTOR

The winding put on the stator, which formerly contained the squirrel cage, was a short circuited pitch winding. It was made by isolating pitch bars in the form of coils. Double connections had to be made to each bar, one connection going to the corresponding bar a pole pitch to the left and the other a pole pitch to the right, in order to obtain the requisite current flow at all times. The action herein produced is the same as would be met with if two bars were put into one slot, making two coil sides per slot.

Figure 9 shows that the insulated end connections for the pitch bars appear only on one end of the machine while a copper ring is used on the other end. On first thought this might bring up the argument that the cage is still a standard cage. It must be remembered, however, that all the bars are completely insulated from the core and all of the end connections are insulated from each other. If Kirchhoff's current law is applied to these circuits or to any part of the whole, it is readily seen that the function of the copper end ring is that of completing the circuit for each of the pole pitch coils established by the front connections shown in Figure 10.

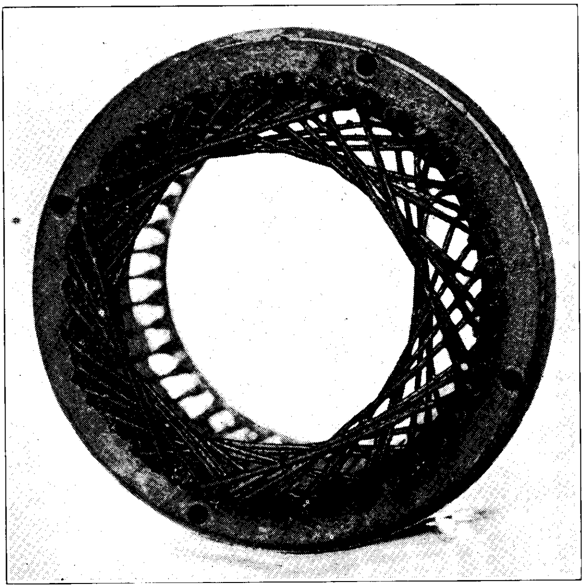
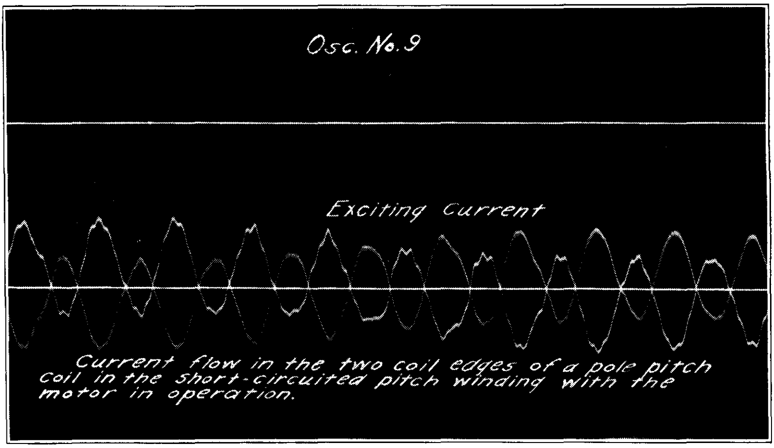


FIGURE 10. END VIEW OF SHORT CIRCUITED PITCH WINDING

Oscillogram 9 shows the current flow in the two coil edges of a pole pitch coil in the short circuited pitch winding under running conditions. These curves compare very favorably with the Oscillograms 7 and 8. Both machines were operating at practically the same slips when the oscillograms were taken. Close observation of the curves, Oscillograms 6, 7, 8, and 9, shows that even the harmonics produced in the two windings are exactly the same.



RESULTS OF THE ANALYSIS

PHASES IN THE SQUIRREL CAGE

In one pair of poles there will be as many coils as there are bar pairs, as seen from the foregoing analysis. What will be the phase relationships existing between these coils?

In a polyphase motor the squirrel cage winding will be a polyphase winding. This is due to the time displacement of the fluxes and the space displacement of the coils in the cage winding. The phase displacement will then be a function of the number of bars per pole. In each pair of poles, coils which are similarly placed will be undergoing exactly the same phenomena and hence the phase relationships of all of these coils will be the same. Therefore, the number of phases in a squirrel cage winding is equal to the number of bars per pole; that is, per half wave length of the field. Let Φ_2 be the number of phases in the squirrel cage, p the number of poles in the machine, and N_s the total number of bars in the cage. Then $\Phi_2 = N_s/p$.

The phase angle between adjacent coils, in radians, will be π/Φ_2 or $\pi p/N_s$. In electrical degrees, this phase angle will be $\frac{\pi p}{N_s} \times \frac{180}{\pi}$. If there are three bars per pole, equally spaced in the pole pitch, the phase displacement between the adjacent coils will be 60 electrical degrees. Equal distribution of the bars in a pole pitch is not encountered in all

A.C. motors using squirrel cages, such as the synchronous motor, and hence the above formula can only be approximate in those cases.

TURNS IN SERIES PER PHASE

The turns in series per phase are those coil turns which have the same phase relationships and which constitute a seat of induced e.m.f. which reacts simultaneously on the primary. In the production of an e.m.f. acting on the primary, all the coils of one phase act together and hence may be considered as acting in series. Since each pair of poles contains one coil of the same phase relationship as that in another pair of poles the turns in series per phase are equal to the number of pairs of poles in the machine.

DISTRIBUTION AND CHORD FACTORS

In armature windings which have more than one slot per pole per phase the instantaneous values of the induced e.m.f.s in the armature conductors in the different slots are not in phase. This phase displacement corresponds to the angular displacement of the armature slots, so that the induced e.m.f. per phase is the resultant vectorial sum of the induced e.m.f.s of the individual conductors. The ratio of the geometrical sum to the arithmetical sum of the induced e.m.f.s of the individual conductors is called the distribution factor. The foregoing analysis of the squirrel cage shows that the slots per pole per phase is unity so that the distribution factor of the winding is unity.

When the sides of the coils of an armature winding lie less than a full pole pitch apart, the flux embraced by the coils will be less than the flux per pole, and if they lie more than a pole pitch apart they will embrace the full flux of one pole plus a part of the flux of an adjoining pole of opposite polarity and the resultant flux will be less than the flux of one pole. A pole pitch coil will then have the maximum possible voltage induced and its chord factor will be one. For a chorded coil, the chord factor will be the ratio of the induced e.m.f. in the chorded coil to the induced e.m.f. in the pole pitch coil. Since the squirrel cage winding was found to consist entirely of pole pitch coils the chord factor of this winding is unity.

SPECIAL CASE OF AN ODD NUMBER OF BARS IN THE SQUIRREL CAGE

In synchronous motor work, the number of bars in the amortisseur winding is always an integral multiple of the number of poles. But this is not always the case in the induction motor. Here the number of bars may be an odd number wherein it would seem that the above theory would not hold. Nevertheless, the above deductions will hold true here also. With an odd number of bars there will always be one bar that is acting together with two other bars. This bar will not be the same one for all positions of the rotor but will vary depending on the relative positions of the stator and rotor. This extraneous or "floating bar" will progress around the machine depending on the speed of rotation. The error introduced by this one bar, in the number of turns in series per phase, will decrease as the number of bars per pole increases, and it may be neglected in computing the turns and phases without much loss in accuracy.

With an odd number of bars in the cage, the chord factor is not exactly unity since the coils will be chorded very slightly. Since this chording is very small, decreasing as the number of bars on a given gap diameter increases, it may for all practical purposes of calculation be assumed to be unity.

The oscillograms of the squirrel cage currents which have been shown thus far were taken on a forty-four bar cage.

Tests were made on a squirrel cage having thirty-seven bars and the curves which were obtained compared very favorably with those previously taken. The magnitudes of the waves were not the same for the thirty-seven and the forty-four bar cages because the same rotor was used with different air gap lengths. Nevertheless, the curves showed that pole pitch coils still exist in the cage having an odd number of bars.

The results of this analysis will now be applied to the derivation of reactance formulas for polyphase A.C. motors and the proof of the existence of pole pitch coils further substantiated through this medium.

The formulas for the component reactances which follow are a composite[4] of previous works on the subject, except for the conversion factor whereby the rotor reactances are converted to the stator.

LEAKAGE REACTANCE

SELF-STARTING SYNCHRONOUS MOTORS WITH OPEN FIELD WINDING

When voltage is applied to the armature of a synchronous motor the armature ampere-turns produce a magnetic flux which passes through the air gaps, the adjacent north and south poles of the rotor, and the core to complete its magnetic path. Most of this flux follows

[4] C. A. Adams, *Trans. A.I.E.E.* Vol. 24. C. J. Fechheimer, *Trans. A.I.E.E.* Vol. 31, Part I. Doherty and Shirley, *Trans. A.I.E.E.* Vol. 37, Part II. Arnold, *Wechselstromtechnik.* Vol. 6. Berlin: Julius Springer. Alexander Gray, *Electrical Machine Design.*

the paths of the main field fluxes but some of it leaks across the tooth tips, slots, and around the end connections. The interlinkages of the armature conductors with this leakage field, set up by the armature winding, induces a voltage of self-induction which is 90° out of phase with the armature current. This leakage flux is the basis of the armature leakage reactance. The reactance in ohms is computed from the relation, $X=2\pi fL$, after determining the inductance in henries due to this leakage flux.

The leakage flux may be divided into its separate parts and the reactance of each calculated. The total reactance of the machine will then be the summation of the component parts, all in terms of the stator reactance. In the following discussion, the armature leakage flux will be considered as made up of four parts and the reactance due to each flux calculated separately. All rotor reactances will be calculated as such and these will be converted into terms of the stator reactances later.

The slot leakage flux is that part of the leakage flux which passes straight across the slots and returns through the armature core as shown in Figure 11.

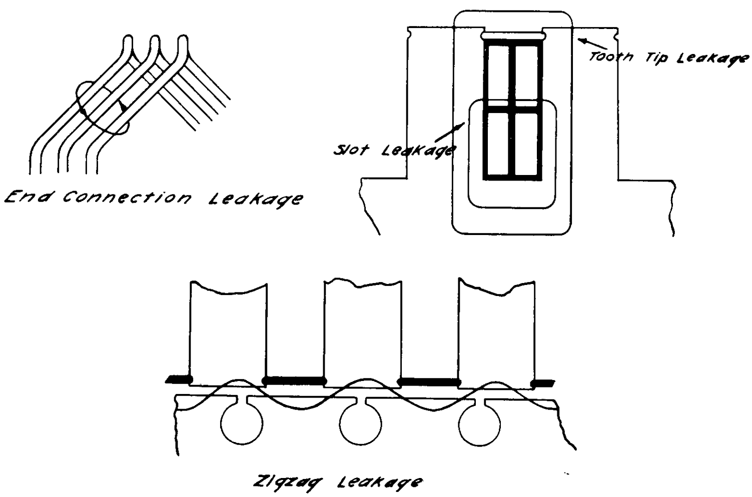


FIGURE 11. FLUX LEAKAGE PATHS

The tooth tip leakage is that component of the leakage flux which passes from one armature tooth to the next through the air gap and returns through the armature teeth. This is also shown in Figure 11. This leakage flux occurs only between those teeth which are in the interpolar space and hence the tooth tip reactance must be multiplied by the ratio, $(\tau - B)/\tau$, after calculating it as existing between all the teeth. τ is the pole pitch in inches and B is the pole arc. The reluctance of the iron part of the paths of these fluxes is very small as compared to that of the air path across the slots and gap and may be neglected.

The end connection leakage is that flux which links with the end connections of the coils. It depends principally upon the length around the belt of end connections and is present in both the stator and rotor. An accurate calculation of this leakage is not possible because of the fact that the path is chiefly in air and that considerable masses of iron are near.

The flux which zigzags from the teeth of the stator to the teeth of the rotor without interlinking with any of the conductors, as shown in Figure 11, is called the zigzag leakage flux. This leakage flux occurs only between those teeth which are opposite the rotor poles and consequently the formulas must be multiplied by the ratio B/τ after calculating the reactance as existing between all the teeth.

The zigzag leakage flux is divided into stator and rotor components since part of it completes its path through the stator iron and the remaining portion through the rotor poles.

In synchronous and induction motors, the air gap surfaces of both the stator and rotor have slot openings both of which increase the air gap reluctance. The factors C_1 and C_2, in the zigzag leakage formulas, are the Carter coefficients of the stator and rotor and these may be readily calculated by a formula due to Dr. Arnold, namely,

$$C = \frac{\lambda}{t + \delta \Lambda}$$

where λ is the tooth pitch and "t" the width of the tooth at the surface. Λ is a factor which depends upon the ratio of slot opening to the length of the air gap. The effect of the stator slot openings can be determined upon the assumption that there are no slots in the rotor and the effect of the rotor slots on the assumption that there are no stator slot openings.

The total reactance per phase is the summation of the separate reactances, all in terms of the primary.

$$\Sigma X = X_{ss} + X_{tt} + X_{se} + X_{zzs} + M(X_{zzr} + X_{rs} + X_{re})$$

where M is the multiplying factor whereby the reactances in the secondary are converted to the primary.

SYNCHRONOUS MOTOR REACTANCE FORMULAS

Let $A_1 = \frac{2\pi f \cdot s_1 \cdot a_1^2 \cdot p \cdot \ell_1}{C^2} 10^{-8}$ and $A_2 = \frac{2\pi f \cdot s_2 \cdot a_2^2 \cdot p \cdot \ell_2}{C_r^2} \cdot 10^{-8}$

Stator Slot:

a. Open Slot

$$X_{ss} = 3.2 \cdot A_1 \cdot K \cdot \left(\frac{r}{3 \cdot b_{s1}} + \frac{r_9}{b_{s1}} \right)$$

b. Partly Closed Slot

$$X_{ss} = 3.2 \cdot A_1 \cdot K \cdot \left(\frac{r_{10}}{3 \cdot b_{s1}} + \frac{r_9}{b_{s1}} + \frac{2 \cdot r_7}{r_6 + b_{s1}} + \frac{r_8}{r_6} \right)$$

Stator Tooth Tip:

$$X_{tt} = 3.2 A_1 \left[C_a \cdot \log\left(1 + \frac{\pi \cdot t_m}{b_{s1}}\right) + C_b \cdot \log\left(1 + \frac{\pi \cdot t_m}{2 \cdot b_{s1} + t_m}\right) \right] \frac{\tau - B}{\tau}$$

Stator ZigZag:

$$X_{zzs} = \frac{0.26 \cdot A_1 \cdot \lambda_1}{\delta} \left(\frac{1}{C_1} + \frac{1}{C_2} - 1 \right)^2 \frac{B}{\tau}$$

Stator End Connection:

$$X_{se} = \frac{C_e \cdot A_1 \cdot s_1 \cdot q \cdot \sqrt{\tau}}{\ell_1}$$

Rotor Slot:

a. Round Slot

$$X_{rs} = 3.2 \cdot A_2 \cdot \left(0.62 + \frac{N_1 \cdot r_{41}}{N_p \cdot r_{11}} + \frac{N_2 \cdot r_{42}}{N_p \cdot r_{12}} \right)$$

b. Rectangular Slot

$$X_{rs} = 3.2 \cdot A_2 \cdot \left(\frac{r_5}{3 \cdot b_{s2}} + \frac{2 \cdot r_2}{b_{s2} + r_1} + \frac{\delta}{r_1} \right)$$

Rotor Zig Zag:

$$X_{zzr} = \frac{0.26 \cdot A_2 \cdot \lambda_2}{\delta} \left(\frac{1}{C_1} + \frac{1}{C_2} - 1 \right)^2 \frac{B}{\tau}$$

Rotor End Connection:

$$X_{re} = \frac{1.83 \cdot 2\pi f \cdot (D_r - u)}{N_s \left(\sin \frac{\pi \cdot p}{2 \cdot N_s} \right)^2} \cdot \log \left[\frac{2.4\,(D_r - u)}{u + v} \right]$$

	2 Phase	3 Phase
C_a	.665	1.0
C_b	1.330	2.0
C_e	2.50	4.5

CONVERSION FACTOR

Since the polyphase A.C. motor, when under short circuit conditions, is essentially a transformer with a high leakage, the multiplying factor, to convert the rotor reactances into terms of the stator reactances, is essentially the turn ratio of the primary to the secondary, squared, or

$$\frac{\Phi_1(W_1 \times f_{w1} \times f_{ch1})^2}{\Phi_2(W_2 \times f_{w2} \times f_{ch2})^2}$$

The distribution and chord factors of the primary and secondary enter into the above since the windings will, in general, have more than one slot per pole per phase and since the turns may not be full pitch. Φ_1 and Φ_2 account for the difference in the number of phases of the two windings.

Complete formulas may now be set up for the calculation of the reactances of polyphase motors. The formulas as given for the synchronous motor, the squirrel cage induction motor, and the wound rotor induction motor have been simplified so that rapid and accurate calculations may be carried out.

DISTRIBUTION FACTOR TO BE USED WITH FRACTIONAL SLOT WINDINGS

When a fractional slot winding is used the effect of the winding on the voltage wave form is the effect of a large number of slots per pole. In other words, a large number of slots per pole per phase may be replaced by a fractional slot winding with equivalent effect on the wave form. The relationship is expressed by the equation:

$$n+\frac{x}{y}=y(n-1)+x+y \quad \text{(equivalent slots p.p.p.p.)}$$

Example:

$1+\frac{3}{4}=4(1-1)+3+4=7$ slots per pole per phase or 21 slots per pole, for a three phase motor. Thus the distribution factor to be used in the formulas is the factor corresponding to the equivalent slots per pole and not the actual slots per pole.

FORMULAS FOR CALCULATION OF REACTANCE

LEAKAGE REACTANCE OF SALIENT POLE SYNCHRONOUS MOTORS

STATOR:

$$K_a = \frac{20.1 \times f \times G^2}{\phi_1 \times C^2} \times 10^{-8}$$

$$K_S = K_a \frac{l_1}{S_1 \times p \times \phi_1} = K_d \frac{l_1}{S_S}$$

SLOT:

1. $K\left(\frac{r}{3 \cdot b_{S1}} + \frac{r_3}{b_{S1}}\right)$

TOOTH TIP:

2. $C_a\left[\log\left(1+\frac{\pi \cdot t_m}{b_{S1}}\right)\right]\frac{T-B}{T}$

3. $C_b\left[\log\left(1+\frac{\pi \cdot t_m}{2 b_{S1}+t_m}\right)\right]\frac{T-B}{T}$

ZIGZAG:

4. $\frac{.0813 \cdot \lambda_1}{\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2\frac{B}{T}$

COIL END:

5. $\frac{C_d \times S_1 \times q\sqrt{T}}{l_1}$

$$X_S = \Sigma X_S \times K_S$$

ROTOR:

$$K_r = K_a \frac{f_{W1}^2 \times l_2 \times f_{Ch1}^2}{N_S}$$

SLOT

1. $\left(0.62 + \frac{r_4}{r_1}\right)$

2. $\left(0.62 + \frac{N_1 \cdot r_{41}}{N_p \cdot r_{11}} + \frac{N_2 \cdot r_{42}}{N_p \cdot r_{12}}\right)$

Note: Use equation 1. when all slots have same r_4/r_1; use 2. when slots have different width and depth of opening.

ZIGZAG:

3. $\frac{\lambda_2}{\lambda_1} \cdot$ Stator ZigZag

CAGE END RING:

4. $\frac{.572(D_r-u)}{l_2 \cdot p \cdot N_S\left(\sin\frac{\pi p}{2 \cdot N_S}\right)^2}\log\left[\frac{2.4(D_r-u)}{u+v}\right]$

$$X_R = \Sigma X_r \times K_r$$

$$\Sigma X = X_S + X_R$$

$$\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2 = \left(\frac{t_t^1}{\lambda_1}+\frac{(\lambda_2-r_1)^1}{\lambda_2}-1\right)^2$$

	2 PHASE	3 PHASE
C_a	.665	1.0
C_b	1.330	2.0
C_d	.781	1.41

Sample Calculation

Salient Pole Synchronous Motor

55 H.P. 3 Phase 60 Cycle 240 Volts 1200 R.P.M. 100 Per Cent P. F.

b_{s1} = 0.438″		r = 1.188″	
B = 6.50″		r_1 = 1/16″	
c = 1		r_3 = 0.266″	
D_d = 5/16″		r_4 = 0.066″	
D_r = 15.31″		s_1 = 2.5	
f = 60		t_m = 0.820″	
f_{ch1} = 0.866		t_t = 0.712″	
f_{w1} = 0.956		u = 1.0″	
G = 270		v = 0.31″	
K = 0.78		δ = 0.0938″	
l_1 = 5.0″		Φ_1 = 3	
l_2 = 5.0″		λ_1 = 1.15″	
N_s = 42		λ_2 = 1.0″	
p = 6		τ = 8.65″	
q = 0.667			

LEAKAGE REACTANCE CALCULATION

SALIENT POLE SYNCHRONOUS MOTORS

LEAKAGE REACTANCE CALCULATION	Size: 6 Pole H.P. 55 P.F. 100% 3 PH. 60 CYC. 240 VOLTS 1200 R.P.M.

STATOR

Slot:

$$K\left(\frac{r}{3 \cdot b_{s1}} + \frac{r_3}{b_{s1}}\right)$$

$$.78\left(\frac{1.188}{3 \times .438} + \frac{.266}{.438}\right) \quad 1.180$$

Tooth Tip:

$$C_a\left[\log\left(1+\frac{\pi \cdot t_m}{b_{s1}}\right)\right]\frac{T-B}{T}$$

$$1 \times \left[\log\left(1+\frac{\pi \times .820}{.438}\right)\right]\frac{8.64-6.5}{8.64} \quad .208$$

$$C_b\left[\log\left(1+\frac{\pi \times t_m}{2 \cdot b_{s1}+t_m}\right)\right]\frac{T-B}{T}$$

$$2 \times \left[\log\left(1+\frac{\pi \times .820}{2 \times .438+8.20}\right)\right]\frac{8.64-6.5}{8.64} \quad .199$$

ZigZag:

$$\frac{.0813 \times \lambda_1}{\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2\frac{B}{T}$$

$$\frac{.0813 \times 1.15}{.0938}\left(\frac{1}{1.25}+\frac{1}{1.01}-1\right)^2\frac{6.5}{8.64} \quad .466$$

Coil End:

$$\frac{C_d \times S_1 \times q\sqrt{T}}{l_1} = \frac{1.41 \times 2.5 \times .667 \times 2.94}{5.0} \quad 1.382$$

$$\sum X_s \quad 3.435$$

ROTOR

Slot:

$$\left(0.62+\frac{r_4}{r_1}\right)$$

$$\left(0.62+\frac{.066}{1/16}\right) \quad 1.671$$

$$\left(0.62+\frac{N_1 \cdot r_{41}}{N_p \cdot r_{11}}+\frac{N_2 \cdot r_{42}}{N_p \cdot r_{12}}\right)$$

$$\left(0.62 + \text{———} + \text{———}\right)$$

ZigZag:

$$\frac{\lambda_2}{\lambda_1} \cdot \text{Stator ZigZag}$$

$$\frac{1.0}{1.15} \times .466 \quad .405$$

Cage End Ring:

$$\frac{.572\,(D_r-u)}{l_2 \times p \times N_s\left(\sin\frac{\pi p}{2 \cdot N_s}\right)^2}\log\left[\frac{2.4(D_r-u)}{u+v}\right]$$

$$\frac{.572 \times 14.81}{5 \times 6 \times 42\left(\sin\frac{\pi \times 6}{2 \times 42}\right)^2}\log\left[\frac{2.4 \times 14.81}{1.0+.31}\right] \quad .202$$

$$\sum X_r \quad 2.278$$

$$K_a = \frac{20.1 \times f \times G^2}{\phi_1 \times C^2}10^{-8} = \frac{20.1 \times 60 \times 270^2}{3 \times 1}10^{-8} = .293$$

$$K_s = K_a\frac{l_1}{S_1 \times p \times \phi_1} = .293 \times \frac{5.0}{2.5 \times 6 \times 3} = .0326$$

$$K_r = K_a\frac{f_{w1}^2 \times l_2 \cdot f_{cn1}^2}{N_s} = .293 \times \frac{.956^2 \times 5.0 \times .866^2}{42} = .0240$$

Test		
Z .207 R .125	STATOR REACT. $= X_s = K_s \times \sum X_s = .0326 \times 3.435$	.112
X .165	ROTOR " $= X_r = K_r \times \sum X_r = .0240 \times 2.278$	.055
	TOTAL " $= X = X_s + X_r = .112 + .055$	.167

COMPARISON OF CALCULATED AND TEST VALUES OF REACTANCE

Salient Pole Synchronous Motors

Poles	R.P.M.	Freq.	H.P.	Volts	Amps.	Reactance Calc.	Reactance Test
6	1200	60	55	240	110.6	0.167	0.165
6	1200	60	850	6600	58.3	8.99	9.80
8	900	60	850	6600	58.2	8.66	10.78
14	514	60	125	220	268.0	0.133	0.132
14	214	25	300	2300	60.5	6.91	6.37
24	300	60	60	220	130.5	0.240	0.260
24	300	60	60	440	66.0	1.29	1.26
26	277	60	25	440	28.5	3.046	3.460
26	277	60	125	2300	25.6	14.22	17.70
28	257	60	40	220	89.2	0.366	0.378
28	257	60	188	550	204.0	0.409	0.458
28	257	60	50	2200	11.1	30.28	30.22
28	257	60	190	2300	49.0	6.87	7.10
28	257	60	190	2400	37.3	11.18	10.91
40	180	60	40	440	45.0	3.33	3.20
40	180	60	60	440	67.0	1.380	1.380
40	180	60	70	2200	15.5	27.62	29.30
44	164	60	100	2200	22.0	27.60	27.00
52	138	60	75	2300	15.9	34.64	32.40
60	120	60	125	220	276.0	0.149	0.145
60	120	60	200	550	173.0	0.665	0.690
60	120	60	150	4000	18.0	45.2	47.8

LEAKAGE REACTANCE
SELF-STARTING SYNCHRONOUS MOTORS WITH CLOSED FIELD WINDING

REACTANCE AND RESISTANCE OF THE FIELD WINDING

The calculation of the leakage reactance of synchronous motors as given previously in this paper is under the conditions of an open circuit field winding. With the field winding closed upon itself or through an external resistance, the conditions existing in the machine are much different from those outlined above.

The stator winding of the synchronous motor is usually of the two or three phase type, while the squirrel cage winding has a large number of phases depending upon the number of bars in the cage and the number of poles in the machine. When the field winding is closed, a single phase winding is added to this polyphase squirrel cage winding. Since the motor consists of a combination of single phase and polyphase windings the characteristics of the machine must be computed on the basis of equivalent single phase resistances and reactances. A diagrammatic representation of these three windings is given in Figure 12.

The equivalent single phase reactance and resistance of a polyphase winding is equal to its phase reactance and resistance. The calculations of these constants for the stator and squirrel cage windings are as given previously, since the phase values were calculated.

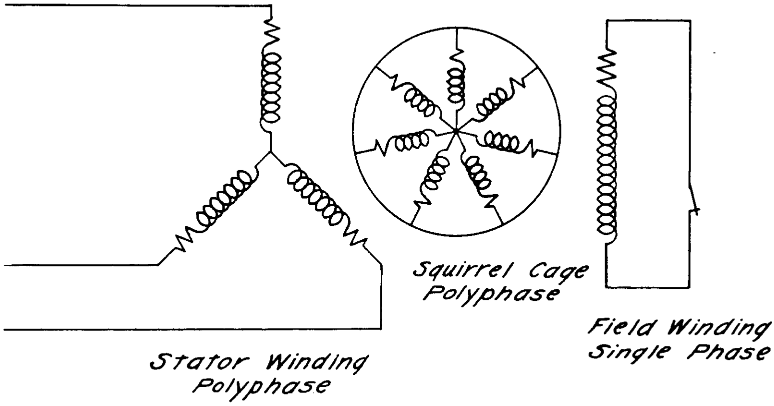


FIGURE 12. WINDINGS OF SYNCHRONOUS MOTORS

The reactance of the field winding is

$${}_F X_F = 2\pi f \times {}_s L_f$$ [5]

where ${}_s L_f$ is the self-inductance of the winding in henries, or

$${}_s L_f = p \times N_f^2 \times l_2 \times L_s \times 10^{-8}$$

and N_f is the field turns per pole.

The factor L_s depends upon the physical dimensions of the pole and its position relative to the stator. The dimensional units used in the following formula are given in Figure 14.

$$L_s = \left[3.6\frac{h}{m} + 0.04\left(\frac{m}{h}\right)^5 + 4\log\left(1 + \frac{\pi n}{e}\right) \right.$$
$$\left. + 12.8\frac{d_p}{e}\left\{1 + 0.62\frac{e}{l_2}\log\left(1 + \frac{\pi b}{4e}\right)\right\} + 1.4\frac{d}{l_2}\right]$$

$d_p = d_t - e/4$

The reactance of the field winding in terms of the primary,

$$X_F = {}_F X_F \times \frac{(\text{Equiv. single phase primary turns})^2}{(\text{Effective secondary turns})^2}$$

[5] Doherty and Shirley, Reactance of Synchronous Machines and Its Applications. *Trans. A.I.E.E.* Vol. 37, Part II.

The equivalent single phase primary turns are equal to $\sqrt{3}$ times the turns in series per phase of the stator winding, or

$$\frac{\sqrt{3}\times G\times f_{ch_1}\times f_{w_1}}{2\times c\times \Phi_1}.$$

The secondary winding is here a single phase winding so that the "turns" will be the turns on the field or $N_f\times p$. Since all of the flux produced by the stator winding does not cut the field turns, because of the choking effect of the damper bars and the distribution of the field coils, the effective secondary turns are not equal to the actual turns on the field winding. The effective secondary turns are greater than the actual turns so that the field turns must be divided by a factor, β, which is less than unity, to obtain the effective turns. This factor represents the ratio of the flux set up by the stator to the flux which cuts the turns on the field coils. It varies principally with the number of bars in the rotor pole face and with the space displacement of the field coils.

The reactance of the field winding in terms of the primary is

$$X_F = 2\pi f\times {}_sL_f\times\left(\frac{\sqrt{3}\times G\times f_{ch_1}\times f_{w_1}}{2\times c\times \Phi_1}\right)^2\times\left(\frac{\beta}{N_f\times p}\right)^2$$

$$=\frac{4.71\times f\times l_2\times G^2\times f_{ch_1}{}^2\times f_{w_1}{}^2\times\beta^2\times L_s}{c^2\times\Phi_1{}^2\times p\times 10^8}\text{ ohms}$$

The resistance of the field winding (plus any external resistance) in terms of the stator is

$$R_F = {}_FR_F\times 0.75\left(\frac{G\times f_{ch_1}\times f_{w_1}\times\beta}{c\times\Phi_1\times N_f\times p}\right)^2$$

where ${}_FR_F$ is the resistance of the field winding.

The magnetizing reactance of the synchronous motor is given approximately by

$$X_m=\frac{E}{\sqrt{3}\times I\times \text{S.C.R.}}$$

where E is the terminal voltage, I the rated full load current and S.C.R. is the short circuit ratio of the motor. X_m is assumed equal to Z_m since the watt component of the magnetizing current is very small and may be assumed equal to zero for all practical purposes of calculation.

Sample Calculation

55 H.P. 3 Phase 60 Cycle 240 Volts 1200 R.P.M. 100 Per Cent P. F.

b = 6.50″ I = 111.0

d	= 3.50″	m	= 2.80″
d_p	= 0.28″	n	= 1.50″
d_t	= 0.65″	N_f	= 363
e	= 1.50″	$_F R_F$	= 10.7
E	= 240	S.C.R.	= 1.17
h	= 2.75″	β	= 0.80

Calculations are made for the conditions of a short circuited field. The factor, β=0.80, is based upon special tests which were performed upon this motor

$$X_F=\frac{4.71\times60\times5\times270^2\times0.866^2\times0.956^2\times0.64\times L_s}{9\times6\times10^8}=0.00840\times L_s$$

$$L_s=\left[\frac{3.6\times2.75}{2.80}+0.04\left(\frac{2.80}{2.75}\right)^5+4\log\left(1+\frac{\pi\times1.50}{1.50}\right)\right.$$
$$\left.+\frac{12.8\times0.28}{1.50}\left\{1+\frac{0.62\times1.50}{5}\times\log\left(1+\frac{\pi\times6.50}{4\times1.50}\right)\right\}+\frac{1.4\times3.50}{5}\right]$$
$$=9.70$$

$$X_F=0.00840\times9.70=0.0815\text{ ohms}$$

$$R_F=10.7\times0.75\times\left(\frac{270\times0.866\times0.956\times0.80}{1\times3\times363\times6}\right)^2$$
$$=0.00603\text{ ohms}$$

$$X_m=\frac{240}{\sqrt{3}\times111.0\times1.17}=1.07\text{ ohms}$$

The equivalent circuit diagram of a synchronous motor is given in Figure 13.

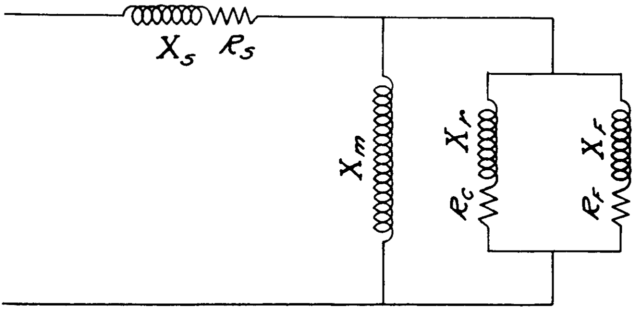


FIGURE 13. EQUIVALENT CIRCUIT DIAGRAM OF SYNCHRONOUS MOTOR

X_s	= 0.1120 ohms	X_F	= 0.0815 ohms
R_s	= 0.0549 ohms	R_F	= 0.00603 ohms
X_r	= 0.0550 ohms	X_m	= 1.070 ohms
R_c	= 0.0636 ohms		

The parallel circuit is reduced to an equivalent series circuit and the reactance and resistance thus obtained is added to X_s and R_s giving the effective reactance and resistance of the motor.

$R = R_s + r_o = 0.074$ ohms
$X = X_s + X_o = 0.151$ ohms

FIELD OPEN CIRCUITED

	X	R	Z
Calculated	0.167 ohms	0.112 ohms	0.201 ohms
Test	0.165 ohms	0.125 ohms	0.207 ohms

FIELD SHORT CIRCUITED

	X	R	Z
Calculated	0.151 ohms	0.074 ohms	0.168 ohms
Test	0.155 ohms	0.091 ohms	0.181 ohms

LEAKAGE REACTANCE
SQUIRREL CAGE INDUCTION MOTORS

Since the salient pole synchronous motor with the open field winding, is essentially a squirrel cage induction motor, the formulas as developed for synchronous motors can be modified for squirrel cage induction motors. The main difference between the two types of motors, during the starting period, is due to the salient pole construction of the first and the continuous rotor of the second.

The stator and rotor slot leakages for the squirrel cage induction motor will be the same as those for the synchronous motor. Likewise the formulas for the end connection leakages, both stator and rotor, will also hold for either type of motor since the two have similar mechanical constructions. The stator end connection leakage as herein given is for the double layer or barrel type of winding which is almost universally used in the United States.

The induction motor does not have a tooth tip leakage such as is encountered in synchronous machines. This is due to the continuous rotor construction. Since the air gap is usually very small in induction motors, any flux which tends to leak from tooth tip to tooth tip through the air gap, immediately links with the zigzag leakage because of the lower reluctance of the flux paths through the iron.

The formula for the zigzag leakage will be the same for the induction motor as it is for the synchronous machine except that the factor B/τ, as explained under the synchronous motor, will now be unity.

FORMULAS FOR CALCULATION OF REACTANCE

LEAKAGE REACTANCE—SQUIRREL CAGE INDUCTION MOTOR

STATOR:

$$K_a = \frac{20.1 \times f \times G^2}{\Phi_1 \times C^2} \times 10^{-8}$$

$$K_s = K_a \frac{l_1}{S_s}$$

Slot:

$$1.\quad K\left(\frac{r}{3 \times b_{s1}} + \frac{r_3}{b_{s1}}\right)$$

Zig Zag:

$$2.\quad \frac{.0813 \times \lambda_1}{\delta}\left(\frac{1}{C_1} + \frac{1}{C_2} - 1\right)^2$$

Coil End:

$$3.\quad \frac{C_d \times S_1 \times q \times \sqrt{T}}{l_1}$$

$$X_s = \Sigma X_s \times K_s$$

ROTOR:

$$K_r = K_a \frac{f_{w_1}^2 \times f_{ch_1}^2 \times l_2}{N_s}$$

Round Slot:

$$\left(0.62 + \frac{r_4}{r_1}\right)$$

Rectangular Slot:

$$\left(\frac{r_5}{3 \times b_{s2}} + \frac{2 \times r_2}{b_{s2} + r_1} + \frac{\delta}{r_1}\right)$$

Zig Zag:

$$\frac{\lambda_2}{\lambda_1} \times \text{Stator Zig Zag}$$

Cage End Ring:

$$\frac{.572\,(D_r - u)}{l_2 \times p \times N_s \left(\sin \frac{\pi \times p}{2 \times N_s}\right)^2} \log\left[\frac{2.4\,(D_r - u)}{u + v}\right]$$

$$X_r = \Sigma X_r \times K_r$$

	2 PHASE	3 PHASE
C_d	.781	1.41

$$\Sigma X = X_s + X_r$$

SAMPLE CALCULATION

Squirrel Cage Induction Motor

25 H.P. | 3 Phase | 60 Cycle | 550 Volts | 720 Syn. R.P.M.

b_{s1}	$= 0.27''$	r_1	$= 0.07''$
b_{s2}	$= 0.29''$	r_2	$= 0.09''$
c	$= 1$	r_3	$= 0.31''$
D_r	$= 13.94''$	r_5	$= 0.35''$
f	$= 60$	s_1	$= 3$
f_{ch_1}	$= 0.940$	S_s	$= 90$
f_{w1}	$= 0.956$	t_t	$= 0.219''$
G	$= 1080$	u	$= 0.40''$
K	$= 0.851$	v	$= 2.25''$
l_1	$= 8.5''$	δ	$= 0.028''$
l_2	$= 8.5''$	Φ_1	$= 3$
N_s	$= 69$	λ_1	$= 0.489''$
p	$= 10$	λ_2	$= 0.635''$
q	$= 0.778$	τ	$= 4.40''$
r	$= 1.04''$		

LEAKAGE REACTANCE CALCULATION

SQUIRREL CAGE INDUCTION MOTORS

LEAKAGE REACTANCE CALCULATION

Size: 10 P. H.P. 25 Ph. 3

Cyc: 60 Volts 550 Syn. R.P.M. 720

STATOR

Slot:

$$K\left(\frac{r}{3\cdot b_{s1}}+\frac{r_3}{b_{s1}}\right)$$

$$.851\left(\frac{1.04}{3\times .27}+\frac{.31}{.27}\right) = 2.070$$

ZigZag:

$$\frac{.0813\times\lambda_1}{\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2$$

$$\frac{.0813\times .489}{.028}\left(\frac{1}{1.60}+\frac{1}{1.04}-1\right)^2 = .485$$

Coil End:

$$\frac{C_d\times S_1\times q\times\sqrt{T}}{l_1}$$

$$\frac{1.41\times 3\times .778\times 2.10}{8.5} = .812$$

$$\Sigma X_s = 3.367$$

ROTOR

Round Slot:

$$\left(0.62+\frac{r_4}{r_1}\right)$$

$$\left(0.62+\frac{\quad}{\quad}\right)$$

Rectangular Slot:

$$\left(\frac{r_5}{3\times b_{s2}}+\frac{2\times r_2}{b_{s2}+r_1}+\frac{\delta}{r_1}\right)$$

$$\left(\frac{.35}{3\times .29}+\frac{2\times .09}{.29+.07}+\frac{.028}{.07}\right) = 1.302$$

ZigZag:

$$\frac{\lambda_2}{\lambda_1}\times \text{Stator ZigZag}$$

$$\frac{.635}{.489}\times .485 = .630$$

Cage End Ring:

$$\frac{.572\,(D_r-u)}{l_2\cdot p\cdot N_s\left(\sin\frac{\pi\cdot p}{2\cdot N_s}\right)^2}\log\left[\frac{2.4\,(D_r-u)}{u+v}\right]$$

$$\frac{.572\times 13.54}{8.5\times 10\times 69\left(\sin\frac{\pi\times 10}{2\times 69}\right)^2}\log\left[\frac{2.4\times 13.54}{.40+2.25}\right] = .028$$

$$\Sigma X_r = 1.960$$

$$K_a=\frac{20.1\times f\times G^2}{\phi_1\times C^2}\times 10^{-8}=\frac{20.1\times 60\times 1080^2}{3\times 1}\times 10^{-8}=4.69$$

$$K_s=K_a\frac{l_1}{S_s}=4.69\times\frac{8.5}{90}=.442$$

$$K_r=K_a\frac{f_{w1}^2\times f_{ch1}^2\times l_2}{N_s}=4.69\times\frac{.915\times .884\times 8.5}{69}=.466$$

Test: Z 2.50 R .365 X 2.47

Stator React.	$= X_s = K_s\times\Sigma X_s = .442\times 3.37$	1.49
Rotor "	$= X_r = K_r\times\Sigma X_r = .466\times 1.96$	.92
Total "	$= X = X_s + X_r = 1.49 + .92$	2.41

COMPARISON OF CALCULATED AND TEST VALUES OF REACTANCE

Squirrel Cage Induction Motors

Poles	R.P.M.	Freq.	H.P.	Volts	Amps.	Reactance Calc.	Reactance Test
4	1800	60	75	440	88.0	0.573	0.524
6	1200	60	15	550	15.5	3.08	3.09
6	1200	60	35	550	33.6	1.22	1.26
8	900	60	150	2200	35.0	5.20	5.67
10	720	60	25	550	27.3	2.41	2.47
12	600	60	25	550	28.0	2.20	2.22

LEAKAGE REACTANCE
WOUND ROTOR INDUCTION MOTORS

The stators of both the squirrel cage and the wound rotor induction motors have the same type of construction both electrically and mechanically; hence, the formulas for the stator reactances of the squirrel cage motor are directly applicable to the wound rotor induction motor.

The rotor winding is similar to the stator winding except that it is usually put in the partially closed type of slots while the stator winding uses the open slots, except on the small capacity machines. The formula for the stator slot leakage, as given, is for the open type of slot. When the closed slot is used the portion of the equation in the brackets, given under rotor slot leakage, is to be used, with the appropriate dimensions.

The zigzag leakages are similar for both the squirrel cage and wound rotor motors.

The rotor coil end leakage given in these formulas is for the double layer or barrel type of winding. With the mush and concentric or chain type of winding on the rotor and the barrel winding on the stator the end leakage is relatively higher than with the barrel winding on both the stator and rotor. Accidental circumstances influence the end connection leakage which makes an accurate determination of this leakage impossible. Thus, the nearer the rotor and stator windings lie together, so as to neutralize each other in the creation of a magnetic field, the less will be the end leakage. Also, the further the windings project from the core the greater will be the leakage.

FORMULAS FOR CALCULATION OF REACTANCE

LEAKAGE REACTANCE—WOUND ROTOR INDUCTION MOTOR

STATOR

$$K_b = \frac{20.1 \times f \times \sigma^2}{\Phi_1^2 \times p \times C^2} \times 10^{-8}$$

$$K_s = K_b \times \frac{\ell_1}{S_1}$$

Slot:

$$K\left(\frac{r}{3 \times b_{s1}} + \frac{r_3}{b_{s1}}\right)$$

ZigZag:

$$\frac{.0813 \times \lambda_1}{\delta}\left(\frac{1}{C_1} + \frac{1}{C_2} - 1\right)^2$$

Coil End:

$$\frac{C_d \times S_1 \times q \times \sqrt{T}}{\ell_1}$$

$$X_s = \Sigma X_s \times K_s$$

ROTOR

$$K_r = K_b \frac{f_{w_1}^2 \times f_{ch_1}^2 \times \ell_2}{f_{w_2}^2 \times f_{ch_2}^2 \times S_2}$$

Slot:

$$K'\left(\frac{2 r_{10}}{3 \times b_{s2}} + \frac{2 r_9}{b_{s2}} + \frac{2 \times r_2}{r_1 + b_{s2}} + \frac{r_4}{r_1}\right)$$

ZigZag:

$$\frac{\lambda_2}{\lambda_1} \times \text{Stator ZigZag}$$

Coil End:

$$\frac{C_d \times S_2 \times q_2 \times \sqrt{T_2}}{\ell_2}$$

$$X_r = \Sigma X_r \times K_r$$

	2 PHASE	3 PHASE
C_d	.781	1.41

$$X = X_s + X_r$$

Sample Calculation

Wound Rotor Induction Motor

2000 H.P. 3 Phase 25 Cycle 6600 Volts 214 Syn. R.P.M.

b_{s1}	$= 0.70''$	r	$= 3.24''$
b_{s2}	$= 0.50''$	r_1	$= 0.07''$
c	$= 7$	r_2	$= 0.27''$
D_r	$= 103.72''$	r_3	$= 0.21''$
f	$= 25$	r_4	$= 0.04''$
f_{ch1}	$= 0.905$	${}_2r_9$	$= 0.14''$
f_{ch2}	$= 1.00$	${}_2r_{10}$	$= 1.75''$
f_{w1}	$= 0.955$	s_1	$= 6$
f_{w2}	$= 0.955$	s_2	$= 8$
G	$= 14112$	t_t	$= 0.60''$
K	$= 0.816$	δ	$= 0.14''$
K'	$= 1.00$	Φ_1	$= 3$
l_1	$= 20''$	λ_1	$= 1.30''$
l_2	$= 20''$	λ_2	$= 0.97''$
p	$= 14$	τ	$= 23.35''$
q	$= 0.722$	τ_2	$= 23.30''$
q_2	$= 1.00$		

LEAKAGE REACTANCE CALCULATION

WOUND ROTOR INDUCTION MOTOR

LEAKAGE REACTANCE CALCULATION	Size 14 Pole H.P. 2000 Ph. 3
	Cyc: 25 Volts 6600 Syn.R.P.M. 214

STATOR		ROTOR	
Slot: $K\left(\frac{r}{3\times b_{s1}}+\frac{r_3}{b_{s1}}\right)$ $.816\left(\frac{3.24}{3\times .70}+\frac{.21}{.70}\right)$	1.504	**Slot:** $K'\left(\frac{2r_{10}}{3\times b_{s2}}+\frac{2r_9}{b_{s2}}+\frac{2\times r_2}{r_1+b_{s2}}+\frac{r_4}{r_1}\right)$ $1.0\left(\frac{1.75}{3\times .50}+\frac{.14}{.50}+\frac{2\times .27}{.07+.50}+\frac{.04}{.07}\right)$	2.965
Zig Zag: $\frac{.0813\times\lambda_1}{\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2$ $\frac{.0813\times 1.30}{.14}\left(\frac{1}{1.41}+\frac{1}{1.01}-1\right)^2$	.367	**Zig Zag:** $\frac{\lambda_2}{\lambda_1}\times$ Stator Zig Zag $\frac{.97}{1.30}\times .367$	.274
Coil End: $\frac{C_d\times S_1\times q\times\sqrt{T}}{l_1}$ $\frac{1.41\times 6\times .722\times 4.82}{20}$	1.471	**Coil End** $\frac{C_d\times S_2\times q_2\times\sqrt{T_2}}{l_2}$ $\frac{1.41\times 8\times 1.0\times 4.82}{20}$	2.720
ΣX_s	3.342	ΣX_r	5.959

$$K_b=\frac{20.1\times f\times G^2}{\Phi_1^2\times p\times C^2}\times 10^{-8}=\frac{20.1\times 25\times 14112^2}{9\times 14\times 49}10^{-8}=.1619$$

$$K_S=K_b\times\frac{l_1}{S_1}=.1619\times\frac{20}{6}=.539$$

$$K_r=K_b\times\frac{f_{w1}^2\times f_{ch1}^2\times l_2}{f_{w2}^2\times f_{ch2}^2\times S_2}=.1619\times\frac{.912\times .820\times 20}{.912\times 1.0\times 8}=.332$$

Test:		
Z 3.50 R .234 X 3.49	Stator React: $=X_S=K_S\times\Sigma X_S=.539\times 3.34$	1.80
	Rotor ״ $=X_r=K_r\times\Sigma X_r=.332\times 5.96$	1.98
	Total ״ $=X=X_S+X_r=1.80+1.98$	3.78

COMPARISON OF CALCULATED AND TEST VALUES OF REACTANCE

Wound Rotor Induction Motors

Poles	R.P.M.	Freq.	H.P.	Volts	Amps.	Reactance Calc.	Reactance Test
4	1800	60	250	2200	56	3.51	3.60
6	500	25	300	550	276	0.218	0.213
6	1200	60	400	2200	90.5	2.49	2.16
8	900	60	50	550	49	1.15	1.12
12	250	25	500	2200	114	2.33	2.09
14	214	25	2000	6600	155.5	3.78	3.49
18	400	60	125	550	90	0.677	0.674
20	360	60	125	550	133	0.763	0.680
26	277	60	400	2200	102	3.68	3.45

SYMBOLIC NOTATIONS

(All dimensions given in inches)

a_1	conductors per slot in stator
a_2	conductors per slot in rotor
b	dimension of pole punching, Figure 14
b_{s1}	width of stator slot, Figure 14
b_{s2}	width of rectangular rotor slot, Figure 15
B	pole arc
c	number of circuits in multiple in armature winding
c_r	number of circuits in multiple in rotor winding
C_1	$\lambda_1 \div (t_t + \delta\Lambda_1)$ Carter coefficient of stator
C_2	$\lambda_2 \div [(\lambda_2 - r_1) + \delta\Lambda_2]$ Carter coefficient of rotor
d	dimension of pole punching, Figure 14
D_r	diameter of rotor
d_t	dimension of pole punching, Figure 14
e	dimension of pole punching, Figure 14
E	terminal voltage
f	frequency of current in stator winding, cycles per second
f_2	frequency of current in rotor winding, cycles per second
f_{ch1}	chord factor of stator winding
f_{ch2}	chord factor of rotor winding
f_{w1}	distribution factor of stator winding
f_{w2}	distribution factor of rotor winding
G	total number of armature conductors
h	dimension of pole punching, Figure 14
I	rated full load current
K	reduction factor for short pitch of stator winding (Doherty and Shirley, *Trans. A.I.E.E.* Vol. 37, Part II, p. 1226)
K′	reduction factor for short pitch of rotor winding
l_1	axial gross length of stator iron (iron & ducts)
l_2	axial gross length of rotor iron (iron & ducts), length of rotor pole (synchronous motors)

${}_sL_f$	self-inductance of field winding
m	dimension given in Figure 14
n	dimension of pole punching, Figure 14
N_f	field turns per pole
N_p	number of bars per pole
N_s	total number of rotor bars
N_1 & N_2	number of bars of types 1 and 2 per pole
p	number of poles
q	per cent pitch of stator winding, as a decimal
q_2	per cent pitch of rotor winding, as a decimal
r	distance from top of copper to bottom of copper in armature winding for open type of stator slot
r_0	equivalent resistance of parallel circuit, Figure 13
r_1	opening at top of rotor slot
r_2 to r_{10}	slot dimensions, Figure 15
r_3	distance from top of copper to armature face
r_4	depth of parallel sides of slot opening of rotor
r_{10}	distance from top of copper to bottom of copper in armature winding for partly closed type of stator slot
R	equivalent resistance of motor
R_c	resistance of squirrel cage, in stator
R_F	resistance of field winding, in stator
${}_FR_F$	resistance of field winding
R_s	resistance of stator winding
s_1	stator slots per pole per phase
s_2	rotor slots per pole per phase
S_s	total stator slots
S.C.R.	short circuit ratio, no load
t_m	mean width of stator tooth
t_t	width of stator tooth at armature face
t'_t	stator tooth plus its magnetic fringe
u	mean width of end ring; perpendicular to shaft
v	mean thickness of end ring; parallel to shaft or distance from pole to outside of end ring when bars project from the pole
W_1	effective turns in series per phase of stator
W_2	effective turns in series per phase of rotor
X	equivalent reactance of motor
X_F	reactance of field winding, in stator
${}_FX_F$	reactance of field winding
X_m	magnetizing reactance
X_0	equivalent reactance of parallel circuit, Figure 13
X_r	reactance of rotor, in stator
X_s	reactance of stator
Z	equivalent impedance of motor
β	factor to obtain "effective" field turns
δ	air gap length
Φ_1	stator phases
Φ_2	rotor phases

λ_1 stator tooth pitch
λ_2 rotor tooth pitch
$(\lambda_2 - r_1)'$ rotor tooth plus its magnetic fringe
τ pole pitch in inches, stator
τ_2 pole pitch in inches, rotor

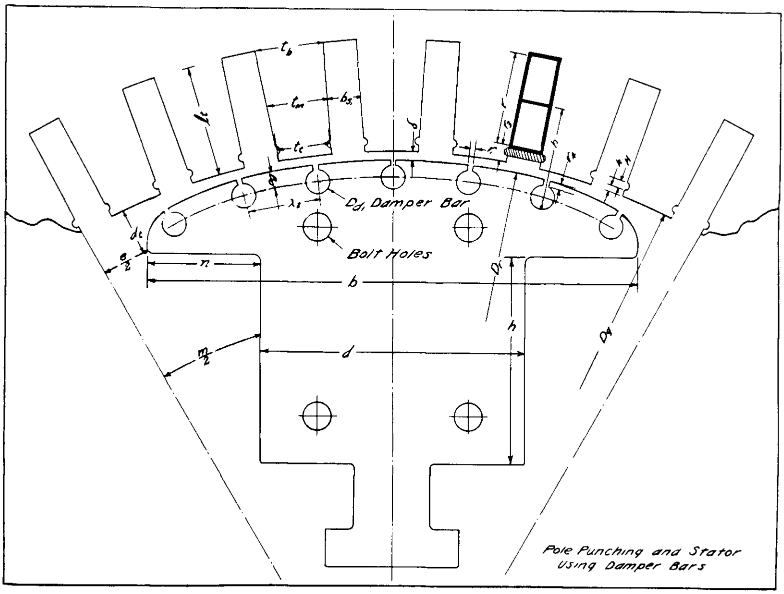


FIGURE 14. POLE PUNCHING AND STATOR OF SYNCHRONOUS MOTOR

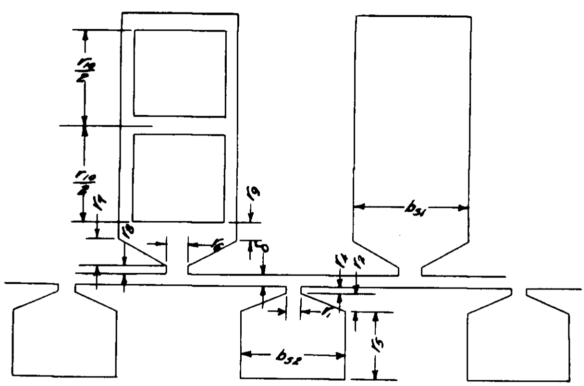


FIGURE 15. PARTLY CLOSED STATOR AND ROTOR SLOTS

SUMMARY

Three methods have been given in the proof of the fact that *two bars in the squirrel cage, a pole pitch apart, function as a single turn coil and they are not influenced in their functioning, as a single coil, by reason of being electrically connected to all the other bars of the cage.*

This reduction of the squirrel cage to its equivalent short circuited pitch winding leads to several fundamental notions which have heretofore been speculative:

The number of phases present in a squirrel cage is equal to the number of bars per pole.

The series turns per phase in the cage are equal to the number of pairs of poles of the machine.

The distribution factor of this winding is unity since there is one slot per pole per phase, and the chord factor is also unity since pole pitch coils exist.

The conversion factor for obtaining the cage reactance in terms of the stator is $\frac{\Phi_1(W_1 \times f_{w_1} \times f_{ch_1})^2}{\Phi_2(W_2 \times f_{w_2} \times f_{ch_2})^2}$. The commercial application of this conversion factor has been given a thoro test. The reactances of thirty-seven different types of machines have been calculated by means of the composite reactance formulas previously referred to together with the conversion factor. The calculated values of reactances have been found to agree very closely with the test values for these machines.