Entrepreneurship and the Macroeconomy

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Dedications

To my Mother who has always encouraged me in my pursuits and who supported me all the way in reaching my dream and succeeding in this degree program.

To my Sister who substituted for me in my absence from the family over these long five years.

To my step Father, a talented scientist.

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Part I

Introduction

This thesis focuses on entrepreneur as an owner of a privately held business. As the name "privately held" suggests, there are no readily available markets for trading risks associated with entrepreneurs' investments in their businesses. On the contrary, investments in publicly traded companies provide opportunities for diversification of risks through trade in public equity markets. I study the characteristics of these alternative investments and their effect on decisions of entrepreneurs to invest in a portfolio of private and public equities. First, I compare empirically returns to an index of all non-traded entrepreneurial equity and to an index of publicly traded equity in the United States. Second, I study the aggregate effects of improving investment conditions in public equity markets through their influence on entrepreneurs' individual portfolio decisions. I provide explicit quantitative measures of associated changes in the welfare of entrepreneurs and levels of aggregate economic activity using data from Ecuador and Chile.

In part 2, I revisit the finding of Moskowitz and rgensen (2002) on the relative returns to public and all non-publicly traded equity in the United States. Based on the data from the Survey of Consumer Finances (SCF) the two authors found that over the period from 1989 to 1998 the average returns to private equity were not higher than returns to public equity. This suggested that entrepreneurs were not rewarded for holding undiversified entrepreneurial risk giving rise to the "private equity premium puzzle". I test the sensitivity of this finding to the years 1989-1998 under consideration. First, I replicate the original set of findings from SCF for this period using the authors' methodology. I then extend the period under consideration with subsequently released data from the waves of SCF for 2001, 2004 and 2007. I find that the "private equity premium puzzle" is not a robust feature of the data and does not survive beyond the period of high public equity returns in the 1990s. Returns to entrepreneurial equity remain largely unaffected when public equity returns plunge between 1999 and 2001, and display a substantial premium over public equity in the following years as well. While the original results are established using data from SCF, I find they are supported in other data sources as well. In particular, returns in the private equity industry reported in VentureXpert database of Thomson Reuters yield the same comparison with the public equity returns. For the latter, I use CRSP value-weighted returns as in the benchmark calculation in MVJ, and alternative measures of

S&P composite index.

Next, I consider differences in private and public equity in terms of the opportunities for diversification of investment risks they provide. Owners of privately held businesses cannot trade away risks associated with their investments. Public equity does allow this opportunity, but operating publicly traded companies is costly. In part 3 of the dissertation, I ask the following question: "What are the quantitative aggregative and distributive effects of reducing the costs of operating a publicly traded firm?" To provide an answer I use an open economy extension of Angeletos (RED, 2007). In the model, entrepreneurs solve portfolio investment problem. They can invest in their privately held firms facing idiosyncratic productivity shocks, and in publicly traded firms facing operating costs but no risk. Arguably, different policies can affect the size of the costs paid by publicly traded companies and lead to their variations across countries. I consider the experiment of exogenous reduction of the operating cost. To obtain explicit quantitative measures of the effects of the experiment I parameterize the model using company-level and aggregate data from Ecuador and Chile. I obtain parameters of the idiosyncratic productivity process with the approach to production function estimation from plant-level data due to Olley & Pakes (1996). This approach deals explicitly with the issue of endogeneity in input choices. I use aggregate data to discipline the counterfactual reduction of public equity costs in Ecuador to their Chilean level. Within this parameterized framework, a 15% decrease in public equity costs results in an increase in capital stock of 5.4% and corresponding increases in wages and output of 1.9%. I also find large differential welfare effects. Entrepreneurs experience an aggregate welfare loss of 9.9% of permanent consumption, and the welfare gain for the workers constitutes 3.2%. These aggregative and distributive effects are tightly linked. The intuition for the welfare effects is best explained in the benchmark case of a small open economy. Reduction in public equity costs raises economy's demand for capital and labor. With fixed labor supply and perfectly elastic supply of capital at the world interest rate, the capital-labor ratio and wages of the workers both increase. Workers consume their current labor income and can only benefit from the wage increase. Entrepreneurs face two returns - on investment in own firm and on public equity investment. With higher wages, the cost of labor in own firms of entrepreneurs increases and so returns fall. At the same time, public equity returns do not change pinned down by the world interest rate. Thus, entrepreneurs' investment opportunities worsen and they cannot be better off.

Part II

Private Equity Premium Puzzle Revisited

1 Introduction

In this paper, I extend the results of Moskowitz and rgensen (2002) (henceforth MVJ) on the returns to entrepreneurial investments in the United States. First, I replicate the original set of findings in Moskowitz and rgensen (2002) using the authors' methodology and data from the four Surveys of Consumer Finances available for the period from 1989 to 1998. I then incorporate into analysis subsequently released waves of SCF 2001, 2004 and 2007 and assess the robustness of the findings to this extension.

I focus on the results regarding the composition of entrepreneurial portfolios and relative returns to private equity. In the extended period, I find an equally high concentration and poor diversification in entrepreneurial portfolios documented in MVJ. [1] When comparing returns to the indices of entrepreneurial and publicly traded equity (given by both CRSP market index and S&P 500 index return), I also find that entrepreneurial equity significantly outperformed public equity in the later period. In particular, in 1999-2001 the average public equity returns fell dramatically to non-positive values reflecting the poor performance of public equity markets, while returns to entrepreneurial equity remained largely unaffected. This is in sharp contrast to the original finding of Moskowitz and rgensen (2002) that over the period from 1989 to 1998 private equity returns were no higher than returns to public equity.

From an accounting perspective, this result is due to the continued loss in the value of outstanding public equity between 1999 and 2002. The value of private equity over the same period, if anything, has grown. This raises the question of the accuracy of owners' valuations of non-traded private equity in the absence of active markets as in the case of public equity. To address this issue, I validate my findings using returns of investors in private equity industry. The original finding by Moskowitz and Vissing-Jørgensen seemed to suggest a "far worse risk-return trade-off in the privately held firms of entrepreneurs relative to public equity" and has been termed the "private equity premium puzzle." This "puzzle" is not a robust feature of the data and does not survive beyond the 1990s, a period

[1] See Table 6 for a comparison of statistics on the concentration of entrepreneurial investments in their firms and public equity ownership concentration for SCF 1995, 2004 and 2007.

of extraordinarily high public equity returns.

The paper is organized as follows. Section 2 provides an update from SCF 2001, 2004 and 2007 on the basic facts regarding the relative size of public and private equity, as well as the concentration of entrepreneurial portfolios in the United States, and presents extended series of returns to public and private equity for the period from 1989 to 2007. Section 4 validates the results using different measures of investment returns in private equity industry. Section 5 provides the discussion of the assumptions behind the original MVJ methodology and their alternatives. It also reports on the robustness of the results to them. Section 14 concludes.

2 Performance of Private and Public Equity Beyond 1989-1998

In this section, I report extended series for the statistics pertinent to the household's sector aggregate public and private equity holdings and their investment performance in the aftermath of the 1990s.

2.1 Aggregate Values of Private and Public Equity

The private equity refers to owner's capital in non-traded *unincorporated* businesses such as sole proprietorships, general and limited liability partnerships, limited liability limited partnerships, limited liability companies, and non-traded shareholders' equity in *incorporated businesses* such as subchapter S & C corporations. In turn, the public equity refers to direct and indirect share holdings in publicly traded companies. Table 1 reports the total values of private[2] and public equities owned by households in SCF, as well as the value of public equity from CRSP between 1989 and 2007. Public equity had grown substantially over this time period gradually overtaking private equity. Since 1998 some of these gains had been reversed, however. The series in Table 3 show that from 1999 to 2002 public equity had been losing in value every year. With its value in 2001 only slightly above that in 1998, the average annual growth rate of public equity over this period was well below its 1989-1998 average. On the contrary, as reported in Table 1 the value of private equity has been growing at a fairly constant rate throughout. These changes in the aggregate values

[2] With respect to the value of private equity, the precise formulation of the question included in the SCF codebook is as follows: For how much could you sell your (share of) business?

are important in accounting for the relative performance of the public and private equity over the extended period.

The benchmark results in the paper use values of public equity from CRSP. Estimates from SCF are reported for reference. The public equity holdings in the latter exclude a number of categories included in CRSP, which explains the systematic downward bias in their values in Table 1.[3] An additional source of discrepancies between CRSP and SCF comes from the reporting of indirectly held equities in mutual, defined contribution retirement plans, and trusts, etc. These often combine bonds and stocks investments, and the split reported in SCF may not be entirely accurate.

In terms of the time series behavior, the SCF value of public equity appears to diverge substantially from CRSP between 1998 and 2004. The household sector's public equity holdings in CRSP are obtained by multiplying its total value by 0.7, which is its average share that has remained rather stable over time. Given this, the value of public equity in household's sector would follow closely its total. In CRSP, the market value of public equity increased slightly between 1998 and 2001 and then more dramatically between 2001 and 2004. In SCF, on the contrary, the value of public equity experienced significant growth between 1998 and 2001 and then declined somewhat between 2001 and 2004. Given this inconsistency, I include in Table 1 the alternative series for the public equity series from SCF, which use the information on the behavior of total public equity from CRSP. For the value of public equity in 2001 they use the weighted average of SCF values in 1998 and 2004 with the weights reflecting the percentage of the equity growth from 1998 to 2004 that occurred between 1998 and 2001 in CRSP.[4]

The other statistics reported in Table 1 include various measures of profits and labor compensation of privately held businesses and the dividend distributions of the publicly traded companies used in the construction of returns to public and private equity in Section 2.2.

[3]Excluded from SCF holdings are public equities owned by defined benefit retirement plans, including state and local government retirement plans, or by nonprofit organizations, insurance companies, and foreigners. More systematically, comparison of SCF with other sources of aggregate statistics is addressed in Antoniewicz (2000).

[4]This is similar to the imputation used by MVJ with respect to the 1995 value of profits of proprietorships and partnerships.

2.2 Aggregate Returns to Private and Public Equity

I follow MVJ methodology to replicate returns to an index of private equity for the period 1989-1998 and to extend the series to 2007. The SCF is a cross-sectional survey and is carried out triennially. Figure 1 shows data that are available for the construction of returns over any three-year period between the two surveys, e.g., SCF 1989 and 1992. The values of equity are concurrent with the year of the survey, and the values of profits (net income) correspond to the preceding year.

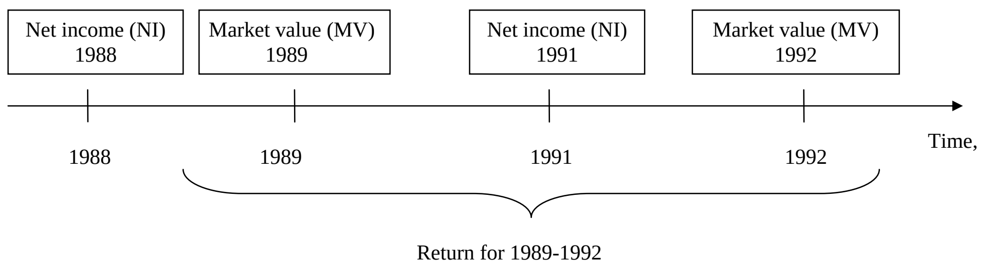

Figure 1: Time line for computing returns

The private equity return R over the period 1989-1992 is computed as the geometric average of returns R_1 and R_2, where:

$$R_1 = (\frac{\text{MV}_{1992} + 3 \cdot \text{NI}_{1988}}{\text{MV}_{1989}})^{\frac{1}{3}} \tag{1}$$

$$R_2 = (\frac{\text{MV}_{1992} + 3 \cdot \text{NI}_{1991}}{\text{MV}_{1989}})^{\frac{1}{3}} \tag{2}$$

$$R = (\sqrt{R_1 \cdot R_2} - 1) \cdot 100\% \tag{3}$$

The raw returns to an index of all private equity and to indices of corporate and non-corporate private equity constructed using (1)-(2) are reported in lines 1-3 in Table 2. In the remainder of Table 2, I report raw returns with adjustments in stages for (*i*) the profit tax; (*ii*) retained earnings; (*iii*) labor income of entrepreneurs managing their own businesses; (*iv*) firm birth and new equity investment; and (*v*) the exit of firms through IPO, mergers and acquisitions, and liquidation. These adjustments apply both to the market value of equity (MV) and to net income (NI) in (1)-(2). Lines 7-24 of Table 2, provide comparison of replicated and original returns for the period 1989-1998 after each stage of adjustment. The originals are reported in parentheses, and replicated returns for all private equity and corporate/noncorporate equity mirror them very closely.[5] The final returns to all private equity are reported in lines 23 and 24 (the originals) of Table 2.

For the benchmark comparison, I use CRSP market index return reported in line 30 of Table 2. This is a value-weighted portfolio return for all available stocks in the trading period.[6] The number reported for each three-year period is the geometric average of the annual returns aggregated from the monthly returns computed in CRSP. In the original period 1989-1998, returns to aggregate private equity and to an index of public equity appear to co-move closely. The returns to aggregate private equity are also not higher than returns to public equity. Consequently, with the extension of the period under consideration into 2000s the close co-movement of the returns weakens and private equity displays a substantial premium over public equity. Over this period, the public equity returns plunge to near zero values reflecting the poor performance of public equity markets, whereas returns to private equity remain largely unaffected. Line 2 of Table 3 reports annual returns to CRSP index. In the period from 1999 to 2001, two of the three returns are highly negative, driving the average net return close to zero. Relatively low returns from 2002 to 2004 are also aided by negative returns in 2002. As noted above the returns to public equity index are driven by unfavorable changes in the value of equity over this period. These are reported in line 1 of Table 3. The large negative capital gains appear to be only partly offset by the dividend distributions of publicly traded corporations. Together these lead to an overall poor performance of public equity and its unfavorable comparison with private equity.

Alternative measures of public equity performance used for comparison include the CRSP S&P holding return, and Shiller's S&P 500 index return as reported in lines 32 [7] and 33 respectively. They deliver qualitatively similar comparison with private equity returns. I also use public equity returns from SCF reported in line 35 of Table 2, that are constructed in the same way as their private equity counterparts. At the household

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[5] It would be impossible to replicate these results exactly, since the SCF is edited periodically and the old versions of the survey data are replaced by the new ones. For a list of all changes to the data, see http://www.federalreserve.gov/PUBS/oss/oss2/changes.html.

[6] The weights applied to the individual stock returns are given by the market value of the individual stock outstanding at the end of the previous trading period in the total value of the stock.

[7] As well as in line 3 of Table 3.

level, "equity" variable from the SCF public extract is used for the value of household public equity holdings[8] and the "dividend income" variable from the full dataset - for the value of dividends distributed to the household.[9] The aggregate values for public equity and the dividend distributions are computed by summing up the values for each household multiplied by the SCF non-response adjusted sample weights. As in the case of private equity, I report the geometric average of the two returns ($R1$ and $R2$) computed using two alternative values of aggregate dividend income. The returns to public equity in SCF are somewhat higher than the returns obtained in CRSP, but display the same large negative premium relative to private equity returns.

2.3 SCF 2007 Returns to Private Equity

SCF 2007 does not appear to have captured the onset of economic slow-down in August 2007. The returns to all private equity and its components (PP and SC) for the period 2004-2007 compare rather favorably to previous periods.

As with public equity returns, they can be mechanically accounted for by the behavior of equity values and profits. Table 1 reports a large increase in the total value of private equity from 2004 to 2007, largely due to an almost two-fold increase in the value of unincorporated equity. This category includes sole proprietorships, general partnerships, limited partnerships, and limited liability companies[10] following the the Flow of Funds definition of non-corporate sector based on the type of tax return filed.[11] Non-corporate profits experienced a similarly large increase contributing to the capital gain part of non-corporate equity returns. The rate of growth of corporate equity and corporate returns at the same time remained largely unchanged. Therefore, the total equity returns reported for 2005-

[8] As mentioned previously, the household public equity holdings include direct holdings of stocks, mutual funds, defined contribution retirement plans, trusts and annuities.

[9] The question corresponding to this variable in SCF is: "In total, what was your (family's) annual income from dividends in ... (year), before deductions for taxes and anything else?". The hint provided to respondents refers to the values reported on the IRS form 1040 line number 9a, which includes only the dividends of publicly traded companies.

[10] The limited liability companies and limited liability partnerships are grouped together for the public dataset. The LLC business type combines the corporate characteristic of limited liability for all owners with the pass-through tax treatment of partnerships, and offers more organizational flexibility than S-corporations.

[11] In particular, partnerships (general and limited) and limited liability companies file the IRS 1065 form, sole proprietorships file IRS schedule C or Schedule C-EZ. Non-corporate equity for the purposes of this paper excludes individual owned equity in rental residential properties, which is part of the FFA/NIPA definition. The owners of these properties file IRS schedule E.

2007 in Line 25 of Table 2 reflect the changes in the aggregate and corporate/noncorporate values and profits.[12] To partially validate these results, the same Table 2 reports equity returns using data from FFA/NIPA with the average non-corporate return for 2005-2007 of 20%. The value of equity in non-corporate sector includes both market and non-market components computed as the sum of tangible assets (real estate at estimated market value, equipment, software, and inventories all at replacement/current), financial assets minus liabilities. It does not include the value of intangible assets, and therefore the FFA/NIPA P&P return may be an underestimate of the return computed in SCF.

The FFA/NIPA report the aggregate statistics for the unincorporated sector as a whole, without the breakdown by business type, in particular for the LLP/LLC which weigh most heavily in the value of non-corporate equity in SCF, and thus account for the large part of its returns. Since the original source of data for unincorporated sector in FFA/NIPA is the individual tax returns filed with the Internal Revenue Service (IRS), some measure of investment returns can be alternatively constructed using IRS's Statistics of Income (SOI). It reports aggregate net income, business receipts, and the balance sheet value of total assets of the LLP/LLC.[13] Since total assets is the only balance sheet item available, I use returns on assets (rather than equity) constructed as the change in the value of total assets plus the net income normalized by the initial value of total assets.[14] I then compare them with the equity returns from SCF. Table 4 reports returns on assets computed from the SOI for the time periods available for all partnerships and separately for the limited liability companies.

[12]The values of corporate and non-corporate equities relative to the value of total private equity serve as weights in the computation of the overall private equity return.

[13]In particular, I use SOI publication of "Partnership statistics by sector or industry" which covers all partnerships, incl. LLC which file the partnership return form 1065, for the years 1993-2006. I also look separately at the "Limited Liability Company" statistics for 1999-2006, since the largest growth in the value of unincorporared equity over the period 2004-2007 has occurred for this particular type of business.

[14]Given that statistics on net income come from tax returns, there is potentially a problem of tax income underreporting. To deal with this bias as in the case of unincorporated returns from FFA/NIPA, I adjust the SOI net income for misreporting using 1.75 ratio (this means that for each 1$ of net income reported to the IRS, the adjusted net income used in the computation is 1.75$). I also apply the labor income adjustment of 6.5%. FFA/NIPA value of total assets used in the computation of returns does not include any adjustment, and so the total value of assets reported on the tax returns will remain unadjusted as well.

3 Historical Comparison of Private and Public Equity Returns

Table 9 and Figure 2 compare returns to unincorporated equity (PP) and returns to value-weighted CRSP market equity index over a longer time period from 1953 (1963) to 2007 than is available in SCF.[15] Table 9 reports geometric and arithmetic average returns, as well as their standard deviations. In the table unincorporated equity returns from FFA has both a higher mean and a lower standard deviation. This higher volatility can be eye-balled in Figure 2 as well. Figures 3 and 4 compare FFA/NIPA non-corporate returns with the total equity and non-corporate equity returns from SCF over the period from 1989 to 2007 for which they are available. The two return series from SCF and FFA/NIPA unincorporated equity returns track each other closely suggesting that the latter could be used in place of SCF private equity returns in the long-term comparison with public equity returns from CRSP.

3.1 Mergers and Acquisitions

MVJ report their most accurate estimates of private equity returns with the mergers and acquisitions adjustment, which is an important channel for movements in and out of private equity in addition to firm births and deaths through liquidations and bankruptcies. Recomputed returns after M&A adjustment together with the originally reported are included in lines 25 and 26 of table 2. The data for M&A adjustment come from the SDC Platinum database of Thomson Reuters. It is a commercial use database which is regularly updated as additional information for past transactions becomes available, in particular, for missing deal values.[16] MVJ report about 50% of transactions with missing deal values, and address this issue by using regression method to impute these values. For large part

[15] Table 9 also reports returns for the 1953-1999 (1963-1999) period under consideration in MVJ. The replicated market equity returns from CRSP are the same as originally reported. However, due to regular revisions in FFA/NIPA (usually every quarter) it is absolutely impossible to obtain time series reported in MVJ either from the printed or electronic sources. This the FFA/NIPA returns differ somewhat. The COMPUSTAT data on publicly traded companies are also subject to revisions and corrections, unlike for example CRSP, which leads to similar departures from the originally reported returns.

[16] The database contains a special variable which records the date when the transaction was updated last. Many transactions continue being updated many years after their effective date and the date of the original posting in the database. In particular, often updated fields include the total value of the deal, sources of financing (debt or equity or both) and their values. Thomson Reuters, which owns the database, collects its data from many different sources, including prospectuses, newspapers, experts, etc., so the information may become available in parts and find its way into database over time.

of these transactions the missing deal values were updated over the number of years. Thus the discrepancy in returns may reflect systematic upward or downward bias created by the imputation procedure. Comparison of returns from Table 2 reported for the same period likely points to the downward bias in MVJ procedure, as recomputed returns are higher than the original ones. Another source of discrepancy is associated with classification of transactions by sources of financing. For the purpose of M&A adjustment, the relevant division is into transactions financed with debt, including internal funds, and equity. While some transactions can be classified as debt or equity only, others are financed from both sources. The discrepancies in the numbers of transactions reported in Table 5 of MVJ and Table 8 suggest that for these mixed transactions all of the deal value may have been assigned to either debt or equity category and not split up accordingly. An additional source of discrepancies might be associated with pure input errors. For example, some of the acquirors/targets in SDC have been mistakenly coded as both private and public.[17] That returns in lines 25 and 26 are substantially different can therefore be attributed to the increased coverage of transactions, reclassification, and other changes to the database. The returns reported in line 25 are significantly higher for all subperiods of 1989-1998, but in particular for 1995-1998. Mergers and acquisitions also produce an upward adjustment to the total private equity returns from 1999 to 2007 period and contribute positively to the premium of private over public equity over this period.

4 Private Equity Fund Returns

One way to validate results presented in Section 2.2 is to compare returns to a index of all entrepreneurial equity with private equity industry returns. The industry is largely composed of venture capital funds and buyout funds. Venture capital funds undertake investments in companies that have undeveloped or developing products. The different stages of venture financing refer to seed, start-up, expansion, and capital replacement. Buyout funds typically target mature companies and primarily finance changes of ownership in these companies. The bulk of the investments of the venture and buyout funds falls into the high-tech, consumer, communications and other service sectors.[18] All of the results

[17]These discrepancies and inaccuracies were established with the help of Thomson's Help Desk.

[18]Thomson Financial reports show that in 1998 76% of all private equity industry investments were attracted by these sectors. For comparison, the share of these three sectors in private equity of US households reported in SCF constitutes a somewhat smaller but comparable number of 66%, so that there do not appear compositional biases in the results reported.

below are reported for the private equity industry as a whole, and for the venture capital and buyout funds separately.[19]

The private equity industry is organized primarily through partnerships, which involve multiple actors - limited partners (investors), general partners (managers), and entrepreneurs (scientists and executives). Thus it is important to specify whose returns are used in the comparison and why. The general partners and entrepreneurs in the partnership are the suppliers of human capital, who earn return on their human capital investment.[20] The entrepreneurs provide the original idea and "an intensive flow of new thinking and problem-solving as the company develops", while the general partners perform a large range of services related to the management of the portfolio companies. The limited partners in turn are the investors, who supply all of the capital and earn financial returns.

The average entrepreneur in SCF combines the functions performed by distinct patries in the venture and buyout partnerships. In particular, large share of total private equity in SCF is associated with businesses that are started and managed by entrepreneurs themselves[21], and have high ownership stakes of entrepreneurs resulting from their own equity investments.[22] The raw returns reported in Table 2 therefore represent the return to financial investment of entrepreneur and payment for hours spent working in (managing) the business. The adjustment in Line ... in Table 2 removes this labor component with the remaining part representing return to financial investment of entrepreneur.[23] This return has been used in the comparison with public equity returns, and its private equity industry counterpart is the return of limited partners-investors (rather than entrepreneurs-scientists) in the private equity partnership.[24]

[19]One possible reason for separating these two types of private equity funds is the different structure of financing, with buyout firms very highly leveraged, which may affect the computation of the IRR-based measure of private equity industry performance. Another reason is the differences in the valuation of portfolio investments in venture capital and buyout funds as described below.

[20]Human capital here refers to the present discounted value of the labor input - time adjusted for effort - of entrepreneur.

[21]Rather than acquired, inherited, etc. These reflect the human capital investment of entrepreneur.

[22]This reflects financial investment of entrepreneur, as additional investments are accompanied with increases in the share of the company owned.

[23]After additional adjustments for survivorship and other biases.

[24]Hall and Woodward (2009) provide estimates of the rewards to both venture entrepreneurs and limited partners. They use data from Sand Hill Econometrics on the venture-supported investments which record their outcomes. This allows them to overcome the common problem of survivorship bias in other data sources on all entrepreneurial businesses, which leads to the estimation of the conditional distribution of their returns. Hall and Woodward (2009) estimate the unconditional distribution of human capital returns of entrepreneurs in their venture-backed start-ups. Unlike Hall and Woodward (2009), MVJ and this paper deal with entrepreneurial investments more broadly and by using the index they are able to overcome this

The venture capital and buyout funds represent a rather small but growing part of the total private equity. The level of activity in the industry is measured by total resources (commitments) raised from investors by general partners and disbursements (investments) from the funds to their portfolio companies. Table 5 summarizes aggregate statistics for the US private equity industry from Thomson Reuters' VentureXpert database[25] which had indeed experienced substantial growth over the period from 1989 to 2007. These statistics reflect traditional valuation practices adopted by the industry. Venture capital financing takes place in rounds, and investments are valued at the last round of financing.[26] The values in non-venture or buyout funds are set at the purchase of portfolio companies and do not change subsequently until the fund exit. These investments represent cash outflows, and receipts from companies' exits - cash inflows, which enter the computation of returns in private equity industry.

The standard measure of the industry's performance has been the pooled internal rate of return[27] of the limited partners (investors) produced by the VentureXpert of Thomson Reuters.[28] This return is net of the management fees and the carry interest of the general partners as well as cash payments to entrepreneurs according to their share of company's

survivorship bias as well. However, unlike Hall and Woodward (2009) this approach allows them to estimate only the mean of the unconditional distribution of returns without being able to characterize it further. This would require data on exiting as well as surviving entrepreneurial businesses. The index of equity can be viewed as the diversified portfolio of investments held by a single entrepreneur, and together with the public equity index it is subject only to the aggregate systemic risk.

[25]As reported by Thomson Reuters, these data are collected from quarterly surveys of private equity firms, government filings, public news releases, etc. And Kaplan, Steven N., and Antoinette Schoar (2005) provide an estimate from VentureXpert that 70% of the overall private equity market is covered by the database.

[26]Hall et al. (2008) discuss endogeneity of these financing events, and propose the correction to returns of investors that would take this endogeneity into account. They propose to use linear interpolation to impute values in between rounds of financing over the regular valuation intervals.

[27]Internal rate of return is defined as return at which net present value of investment becomes 0.

[28]In particular, the returns from VentureXpert have been used by Kaplan, Steven N., and Antoinette Schoar (2005) as the aggregate measures of private equity industry performance in the 1980s and 1990s. Hall and Woodward (2007) do not use venture index returns produced by VentureXpert, but compute their own measures from alternative data sources and compare them with VentureXpert. Their data are from Sandhill Econometrics. The unit of observation in these data is a venture-supported investment (company-round) rather than venture fund in VentureXpert database. Hall and Woodward (2009) compute venture index returns using two methods. First, they construct returns using company-level flows and the share of ownership of the limited investors adjusting for preferences, dilution and GP charges. Second, they calculate returns using the net amounts directly received by limited partners. They find that the two measures move "reasonable close" with each other, and with the VentureXpert reported returns. The findings in Hall and Woodward (2007) and previous studies justify using readily available index returns from VentureXpert. I will use the return to an index of all private equity, incl. both venture and buyout funds, and for the two types of private equity separately.

ownership.[29] In its construction all funds are pooled together to form a single "fund", similar to the index of all private equity in MVJ.[30] More specifically, the IRR is calculated as an annualized effective compounded rate of return using cash flows to and from investors together with the residual value of the fund which is treated as a terminal cash flow to investors. This residual value is net of the current liabilities, and includes cash, short-term investments, long-term equity investments, outstanding loans, and other assets.[31] For the purpose of comparison with returns to entrepreneurial equity in SCF and public equity returns, Line 3 in Table 5 reports average pooled IRR for the 3-year intervals. An alternative measure of returns is included in line 4 of Table 5. It is the average one-year performance horizon IRR. The return in each period uses the fund's net asset value (i.e. the value of the fund's investments) at the beginning of the horizon period as an initial cash outflow, residual value at the end of the period as the terminal cash flow, and the value of any cash flows from and to investors during the horizon period. These returns are then averaged over the three-year intervals. The two IRR returns thus largely differ in how much of the underlying values are actual values obtained at exit vs. book values on the fudns' balance sheets. In the case of venture capital funds, investments are revalued at new financing rounds when the price per share and ownership of shares are negotiated between the entrepreneurs and venture partners. Then the performance horizon return which relies on these balance sheet values of the portfolio companies, likely underestimates the actual returns of the investors. Thus I use pooled IRR as the benchmark return for comparison. The two series of returns to private equity reported in lines 23 of Table 2 and line 3 of Table 5 follow each other closely.

I also report on what can be thought of as the SCF counterpart of private equity industry returns. The limited partners of the venture partnerships are the owners of their

[29]The management fees of the general partners are paid annually for the duration of the life of the fund as a fixed percent of the commitments or investments under their management still in the fund. So they may be inversely related to the performance of the fund - with poor performance portfolio companies may require longer to exit, and as long as they remain in the fund, the partners would receive their management fee. The carry interest constitutes a fixed percent (usually about 20%) of the difference between the initial investment of the limited partners and the exit value of the investment (i.e. the net cash flow generated by the investment). The carry interest constitutes the performance-based part of the general partner's compensation and accrues on the fund as a whole rather than on successful exits.

[30]Recall, its construction involves the total values of privately held equity and net income.

[31]The advantage of this measure is that it does take the scale and timing of cash flows of large and small scale funds into consideration. The disadvantage is that larger cash flows will be given more weight given by the value of the investment, so that in a composite portfolio of small early stage funds and large later stage or buyouts funds the larger funds will have more influence on the performance than the smaller funds.

portfolio companies but do not have a say in either their management or day-to-day operations. The SCF assigns these limited partners in the category of owners of non-actively managed businesses, i.e. owners who do not have an active management role in their businesses. It also includes angel investors, and other wealthy individuals with equity in non-publicly traded companies, but no active management role. In the absence of active management role the returns to passive owners constitute pure financial investment returns. These returns are computed from the values of the non-actively managed equity and associated profits using the formula in Section 2.2. While the labor income adjustment does not apply to these returns, several other adjustments are not possible due to less detailed coverage in SCF of this equity category.[32] Consequently, the final returns are net of the tax and retained earnings adjustments only. Lines 27 and 28 of Table 2 report returns to passive investors with and without any other active investments respectively.[33] Different measures of private equity industry returns reported in Table 5 therefore appear to support the earlier conclusion that the private equity premium puzzle is not a robust feature of the data and does not survive beyond the period of high public equity returns in the 1990s.

As a last consideration, the Capital Asset Pricing Model provides a natural framework for the analysis of incentives to invest of an average entrepreneur choosing between fully diversified portfolios of public and private equity given by the CRSP and SCF indices respectively. However, data limitations make the formal analysis in this framework quite difficult, as the SCF is conducted only every three years and the sample of private equity returns is very short. Hall and Woodward (2007) focus on venture capital, which constitutes a fraction of the total entrepreneurial equity, and apply CAPM to the times series from 1989 to 2005 of monthly venture index and public equity index returns. The construction of monthly returns creates investment performance measures for the venture index at regular fixed intervals, thus eliminating the problem of using CAPM with endogenous holding period returns directly available in the data.[34]. CAPM regressions in Hall and Woodward

[32]As before, non-actively managed businesses include both corporate and non-corporate ones, with respondents self-reporting the values of their equity holdings.

[33]I.e. some of these passive investors also report owner's interest in the businesses where they do have an active management rile. The returns to an index of all private equity previously reported in Section 2.2 include both actively and non-actively managed equity.

[34]Hall and Woodward (2007) use interpolation technique to obtain values of the venture investments between the periods for which actual valuations are available. The returns are constructed as a proportional change in the value of venture positions over the month, which is analogous to the standard stock-market index returns.

(2007) deliver a positive estimate of alpha which measures the excess return of investments into venture index over public equity index over the years taken together. This result for venture equity which represents a fraction of the total entrepreneurial equity may provide some support for the main finding in this paper as well.

5 Robustness Checks: Alternative Assumptions

In this section, I describe an alternative set of assumptions for the computation of returns compared to that originally proposed by MVJ.

Retained earnings adjustment. The purpose of the retained earnings adjustment is to exclude from computation of returns any part of net income retained in the business. As pass-through entities, partnerships, proprietorships and S corporations do not have undistributed income. The total owners' equity in these businesses can be increased only through new equity investment. Thus, the application of the retained earnings adjustment in MVJ for partnerships, proprietorships, and S corporations is unnecessary and leads to a downward bias in returns. It is possible, however, for the part of the distributed net income of an S-corporation to be reinvested in it later. The reinvested amount of net income would be added to equity and would be taken into account when computing equity returns for the corresponding period. This is similar to the way returns to venture capital investments are computed. Venture capital financing is carried out in rounds, and so new investments are added to the equity holdings of venture and would be reflected in equity returns for the period investment is undertaken. The final returns without the retained earning adjustment for the non-corporate sector are reported in line 3 of Table 10.

Labor income adjustment. The purpose of the labor income adjustment is to separate the investment returns of entrepreneurs from returns to their human capital. To do that, MVJ assume that (*i*) net income of the business is distributed pro rata the ownership share; (*ii*) reported wages represent a fair remuneration for the services performed;[35] (*iii*) there is no double counting of the reported wages in the net income. Assumption (*i*) is very restrictive; however, one cannot do better due to data limitations in the SCF.[36] It can lead to both upward and downward biases in returns, and the net effect cannot be readily

[35] In the case of the owners who provide services in their businesses organized as S corporations, for example, the law requires them to be paid fair market wages. So it is assumed that the wages reported represent the market value of the services provided and need not be adjusted.

[36] E.g. in the partnership, the ownership shares and distribution shares do not have to coincide.

determined. The alternative versions of assumptions (*ii*) and (*iii*) in the data can be formulated as follows. When asking about the net income of the business, the SCF questionnaire specifically refers to IRS tax returns filed by individual business owners. Given that owners of unincorporated businesses cannot pay themselves salary, the net income reported on these forms would also include any amounts that might be due as labor compensation. Thus for the entrepreneurs who do report wages in the employment section of SCF, the amount of net income reported would not necessarily be exclusive of wages. Thus in the labor adjustment for unincorporated businesses it may be necessary to take out both paid and unpaid wages of entrepreneurs reported for PP in Table 1. This would in turn lower the returns to unincorporated equity. To deal with a potential problem of underreporting in (*ii*), wages of all owners of unincorporated equity and owners of S corporations may need to be imputed using the regression approach. The results for this adjustment alone are reported in line 6 of Table 10.

Labor income adjustment: imputation regression. In the regression model used to impute unreported wages, only observed demographic characteristics were included as regressors. In the estimation, the decision to become self-employed was treated as exogenous. The same regression model instead can be estimated on the sample of active business owners, who report paid employment outside of their business as their main occupation. This may give a more accurate prediction of the opportunity cost of self-employment. The final returns obtained with the alternative samples for imputation regressions are presented in lines 4 and 5 of Table 10.

The results for the joint labor income (imputation regression and non-corporate sector labor income) and retained earnings adjustments are presented in lines 7-9 of Table 10.

6 Conclusions

In this paper, I have constructed an extended series of returns to the private equity index for the period 1989-2007 using the original methodology of Moskowitz and Vissing-Jørgensen. I find that the "private equity premium puzzle" is not a robust feature of the data and does not survive beyond the period of high public equity returns in the 1990s. In particular, the returns to entrepreneurial equity index remain largely unaffected when public equity returns plunge between 1999 and 2001. This result is validated using different measures of the private equity industry returns and against other studies in the literature. These findings suggest that private equity as an asset class might serve as a hedge against public equity.

Table 1: The Size of Private and Public Equity (1989-2007), $ bln.

Survey year	1989	1992	1995	1998	2001	2004	2007
Statistics from SCF							
Private Equity Values							
Total equity [1]	3,680	3,740	4,290	5,710	7,830	9,650	14,700
	(525)	(298)	(277)	(427)	(484)	(521)	(859)
Equity in PP	2,020	1,980	1,990	2,510	3,570	4,310	8,260
	(397)	(143)	(174)	(208)	(272)	(299)	(626)
Equity in SC	1,660	1,770	2,300	3,200	4,260	5,350	6,460
	(242)	(223)	(241)	(348)	(358)	(411)	(560)
Public Equity Values							
Original series	1,770	2,230	3,640	7,430	11,400	10,800	13,700
Revised series[2]	1,770	2,230	3,640	7,430	8,010	10,800	13,700
Equities ratio	2.08	1.67	1.18	0.77	0.98	0.89	1.08
Profits							
Pretax profits PP	335	433	460	543	787	842	1,480
After tax profits SC	266	287	341	489	677	696	863
Profits - RE PP [3]	268	347	368	434	629	673	1,180
After tax profits - RE SC	175	194	246	351	479	530	641
Labor income							
Total paid wages	141	191	246	292	331	336	419
Unpaid wages PP	152	148	181	172	214	245	334
Unpaid wages SC	27	36	24	53	54	56	53
FFA/NIPA							
Private Equity Values							
Equity in noncorporate business	2,966	2,980	3,487	4,121	4,821	6,359	8,347
Minus Value of 1-4 family rental properties	916	982	1,109	1,235	1,441	1,756	1,991
Equals Proprietors and partnerships (market value)	2,050	1,998	2,377	2,886	3,380	4,602	6,356
S and C corporations (market value) (estate multiplier=2) [4]	1,412	1,622	1,921	2,540	2,098	2,535	-
S and C corporations (market value) (estate multiplier=3)	2,117	2,433	2,887	3,811	3,147	3,803	-

Table 1: The Size of Private and Public Equity (1989-2007), $ bln. (continued)

Survey year	1989	1992	1995	1998	2001	2004	2007
FFA/NIPA							
Income and dividends							
Proprietors' income [5]	363.3	427.6	492.1	627.8	771.9	911.6	1056.2
Adjusted proprietors' income-RE	187.7	216.3	283.3	514.6	514.7	557.3	678.1
Dividends, S and C private corporations [6]	165.2	177.2	249.0	387.6	406.4	466.0	–
Dividend income, public corp.	66.4	69.2	100	98.3	108	107	148
Statistics from CRSP							
Public equity in CRSP [7,8]	3,306	4,396	6,785	13,288	13,829	16,458	19,851
	(3,292)	(4,376)	(6,734)	(13,217)			
Public equity in CRSP x 0.7 [9]	2,314	3,078	4,750	9,301	9,681	11,521	13,896
		1992	1995	1998	2001	2004	2007
SDC M&A equity adjustment [10]		284.4	441.6	846.2	1,162	1,061	1,941

Note 1: For private equity statistics, included in parentheses are standard errors.
Note 2: This revision as described in the text.
Note 3: RE refers to retained earnings adjustment.
Note 4: The market values for CRSP are obtained from WRDS version of CRSP from Monthly Stock Market Indices database and correspond to the variable "Total Market Value". It reports monthly total market value for a given market for all non-ADR securities with valid prices. The values reported are those for the end of calendar year.
Note 5: Numbers reported here refer to the value of closely held equity at date of death, and the year refers to the year of death of decedents. MVJ originally reported numbers from tax returns for the year of filing. Given that the majority of returns are filed a year after the decedent's death, appropriate adjustment for the year must be made.
Note 6: Proprietors' income as reported in NIPA with the inventory valuation (IVA) and capital consumption adjustment (CCAdj).
Note 7: Dividend distributions of publicly traded corporations include include only distributions carried out from the current net income and do not include liquidating and other distributions.
Note 8: The values in brackets are those originally reported in MVJ.
Note 9: CRSP value multiplied by 0.7 corresponds to the total value of public equity held by households, as they are reported to own about 70% of total publicly traded equity.
Note 10: M&A adjustment is reported as the aggregate values for periods 1990-92, 1993-95, 1996-98, 1999-2001, 2002-04, 2005-07.

Table 2: The Returns to Private Equity (1989-2007)

	90 − 92	93 − 95	96 − 98	99 − 01	02 − 04	05 − 07	Line
Total Private equity in SCF							
Unadjusted returns							
All	17.4	21.9	26.4	27.7	22.7	29.0	1
PP	15.7	19.0	26.3	30.3	23.7	39.5	2
SC	19.5	25.0	26.5	25.5	21.8	19.1	3
Income taxes							
All	15.8	20.7	25.4	26.5	21.8	28.5	4
PP	15.7	19.0	26.3	30.3	23.7	39.5	5
SC	16.0	22.5	24.5	23.38	20.2	18.1	6
Retained earnings							
All	12.3	17.2	22.1	23.1	18.9	25.8	7
	(12.3)	(17.0)	(22.2)				8
PP	12.8	15.7	23.1	27.1	20.6	36.7	9
	(12.6)	(15.6)	(23.0)				10
SC	11.7	18.7	21.1	19.8	17.2	15.4	11
	(12.0)	(18.5)	(21.4)				12
Labor income							
All	8.2	13.3	18.6	20.2	16.2	23.5	13
	(8.2)	(12.7)	(18.4)				14
PP	6.5	9.2	16.9	22.1	16.0	33.0	15
	(6.4)	(9.4)	(15.9)				16
SC	10.2	17.5	20.0	18.6	16.3	14.6	17
	(10.9)	(16.9)	(20.6)				18
Firm birth							
All	7.0	11.8	16.1	18.2	14.5	17.4	19
	(7.5)	(11.6)	(16.4)				20
PP	5.3	7.5	13.8	20.4	13.9	26.3	21
SC	9.2	16.3	18.0	16.4	15.0	9.0	22
IPO [1]							
All	7.4	12.3	16.7	18.8	14.7	17.6	23
	(7.8)	(12.1)	(17.0)				24

Table 2: The Returns to Private Equity (1989-2007) (continued)

	90 − 92	93 − 95	96 − 98	99 − 01	02 − 04	05 − 07	Line
Total Private equity in SCF							
M&A							
All	9.6	15.4	21.4	23.5	18.0	22.3	25
	(8.2)	(13.0)	(19.4)				26
Non-actively managed private equity in SCF [2]							
All owners	37.5	10.5	26.8	15.1	15.4	22.7	27
Non-active owners	36.0	21.7	40.2	13.3	5.54	36.8	28
Private equity in FFA/NIPA							
PP	5.4	13.2	17.3	16.9	19.6	19.5	29
Public equity in CRSP							
CRSP value-wgtd index updated [3]	11.0	14.5	24.5	−0.4	6.0	10.2	30
	(11.0)	(14.6)	(24.7)				31
CRSP S&P index holding return	7.2	11.2	25.9	−2.3	1.8	6.6	32
S&P index, Shiller [4]	7.9	12.1	26.1	−4.2	0.1		33
Public equity in SCF							
Index	11.2	20.4	28.5	16.4	−0.8	9.2	34
Index, revised	11.2	20.4	28.5	3.8	11.5	9.2	35

Note 1: For the IPO adjustment I use values for the aggregate proceeds of IPOs by year reported by Jay Ritter at the following link: http://bear.cba.ufl.edu/ritter/Moneybyyear.pdf

Note 2: "All owners" refers to entrepreneurs with and without other actively managed closely held businesses; "Non-active owners" refers to entrepreneurs who do not have interest in businesses where they have an active management role.

Note 3: For the CRSP value-weighted index returns in brackets are those originally reported in MVJ.

Note 4: S&P index return for the period 2004-2007 is unavailable on the Shiller's web-site and is left blank.

Table 3: Public Equity Market Values and Returns (1998-2007)

Year	1998	1999	2000	2001	2002	Line
CRSP market value, bln. $	13,288	17,009	13,829	11,034	14,585	1
CRSP index return, %	22.3	25.3	−11.1	−11.3	−20.8	2
CRSP S&P return, %	26.7	19.5	−10.1	−13.0	−23.4	3
Year	2003	2004	2005	2006	2007	Line
CRSP market value, bln. $	16,458	17,384	19,548	19,851	11,697	1
CRSP index return, %	33.1	13.0	7.3	16.2	7.3	2
CRSP S&P return, %	26.4	9.0	3.0	13.6	3.5	3

Table 4: Unincorporated Asset Returns from Statistics of Income (All Partnerships and Limited Liability Companies)

Year	1996 – 98	1999 – 01	2002 – 04	2005 – 07	Line
PP returns, %					
Unadjusted returns	28.5	22.4	14.8	26.3	1
Tax misreporting and labor income adjustment	31.6	25.2	17.0	29.4	2
LLC returns, %					
Unadjusted returns			27.5	27.9	3
Tax misreporting and labor income adjustment			29.1	30.4	4

Table 5: Private Equity Industry Characteristics and Returns, 1989-2007

Year	90 – 92	93 – 95	96 – 98	99 – 01	02 – 04	05 – 07	Line
Aggregates, mln. $							
Total resources	61,841	137,174	362,162	802,647	446,440	1,390,315	1
Total investments	13,461	22,789	67,812	287,907	120,923	165,859	2
Returns, %							
All Private Equity							
Pooled IRR	13.57	14.90	16.67	18.03	14.2	14.1	3
1-yr hrz IRR	9.3	23.6	25.4	14.7	6.9	13.0	4
Venture Capital							
Pooled IRR	9.76	12.03	14.93	19.23	16.43	15.90	5
Buyout Funds							
Pooled IRR	21.32	19.63	19.80	17.25	12.50	13.27	6

Table 6: Private Equity (PE) and Own-Company Stock Ownership

Measure	1995	2004	2007
Private Equity (PE) Ownership			
% of total PE owned by households with:			
more than 25% of net worth in PE	93.3	92.4	93.5
more than 50% of net worth in PE	77.1	73.7	79.0
more than 75% of net worth in PE	50.6	41.9	51.4
Mean % of net worth invested in PE for households with positive private equity and net worth:			
SCF weights only	36.9	37.5	37.6
net worth weights	45.7	41.4	46.0
Mean % of PE held in one actively managed firm for households with positive PE:			
SCF weights only	82.5	85.4	85.3
private equity weights	73.9	73.8	72.2
Own Company Stock Ownership in Public Firms			
% of total public equity owned by households with:			
more than 25% of public equity in own company	11.0	9.9	12.6
more than 50% of public equity in own company	6.5	4.6	6.7
more than 75% of public equity in own company	3.1	1.2	2.7
Mean % of net worth invested in own-company stock for households with positive own-company stock and net worth:			
SCF weights only	11.1	9.0	8.9
net worth weights	10.4	8.3	11.4
Mean % of directly held public equity in own-company stock for households with positive own-company stock:			
SCF weights only	70.1	69.7	73.3
directly held public equity weights	48.4	47.0	57.4

Table 6: Private Equity (PE) and Own-Company Stock Ownership (continued)

Measure	1995	2004	2007
Own Company Stock Ownership in Public Firms			
Mean % of total public equity held in own-company stock for households with positive own-company stock:			
SCF weights only	44.2	32.2	35.3
total public equity held weights	29.4	20.9	29.1

Note: This table is an analogue of Table 2 in MVJ (p. 751). For comparison purposes, I only report the statistics for years 1995 (included in MVJ), 2004 and 2007.

Table 7: SUMMARY STATISTICS ON ENTREPRENEURS FROM THE SURVEY OF CONSUMER FINANCES

Section A			Percentile				
CHARACTERISTIC	Mean	Std. dev.	10th	25th	Median	75th	90th
SCF 1995							
	46.5	12.6	31	37	45	54	65
Entrepreneur age							
Firm age	11.9	11.6	2	4	8	16	26
Market equity	289,256	1,664,984	0	5,000	30,000	129,000	481,000
Sales	6,994,702	217,000,000	0	4,200	30,000	130,000	700,000
Profits	94,173	1,646,680	0	520	6,500	29,000	80,000
Net worth	774,471	3,365,676	32,000	80,600	182,000	489,550	1,377,660
	86.45	26.42	100	100	100	100	100
Share owned, %							
Employees [1]	12.1	123.8	1	1	2	4	12
SCF 2007							
	49.4	11.9	34	41	49	58	65
Entrepreneur age							
Firm age	12.7	10.6	2	4	10	18	28
Market equity	751,893	4,592,438	0	11,000	80,000	350,000	1,200,000
Sales	6,521,583	101,000,000	1,200	15,000	70,000	373,000	1,500,000
Profits	147,231	994,537	0	3,000	20,000	75,000	200,000
Net worth	2,066,981	8,291,575	64,030	178,300	515,820	1,519,000	4,647,790
	82.9	29.2	33	50	100	100	100
Share owned, %							
Employees [1]	28.5	272.9	1	1	2	6	15

Table 7: Summary Statistics on Entrepreneurs from the Survey of Consumer Finances (continued)

Section B	
SCF 2007	
Industry	% PE
Agriculture [2]	10.2
Construction/mining [3]	17.5
Manufacturing [4]	6.5
Retail, wholesale [5]	13.7
FIRE/communications [6]	14.7
Services [7]	37.4

Note 1: The numbers reported include entrepreneur working in the business.

Note 2: Agriculture, forestry, fishing, hunting, veterinary and landscaping services;

Note 3: Oil and gas extraction, coal mining, quarrying, supporting services for mining, and construction;

Note 4: Manufacturing, newspaper and other publishing, independent artists, spectator sports;

Note 5: Wholesale and retail trade, restaurants and food service;

Note 6: Finance and insurance, real estate, automotive rental and leasing, software publishing, data processing, hosting and related, automotive repair and maintenance;

Note 7: Commercial, industrial and other intangible assets rental and leasing; employment and business support services; broadcasting and telecommunications; other administrative and support services; health, education and social services; arts, and entertainment services; recreation, accommodation and food services; personal services, etc.

Table 8: Merger and Acquisition Activity in Private and Public Firms (1989-2007), $ bln.

Survey year	1990 – 1992			1993 – 1995			1996 – 1998		
Acquiror:	Pub	Priv	Priv	Pub	Priv	Priv	Pub	Priv	Priv
Target:	Priv	Priv	Pub	Priv	Priv	Pub	Priv	Priv	Pub
All acquirors, all targets									
Value	220.6	192.9	69.7	363.8	84.4	247.2	756.5	219.9	446.1
Number of deals	4,715	3,701	528	7,746	626	4,911	11,794	1,026	7,093
All acquirors, domestic targets									
Value	94.3			201.4			424.9		
Number of deals	2,209			4,313			6,417		
Domestic acquirors, domestic targets debt or cash funded									
Value		69.9			101.9			157.1	
Number of deals		1,208			1,615			2,107	
Foreign acquirors, domestic targets									
Value		13			9.3			16	
Number of deals		143			95			167	
Domestic acquirors, foreign targets equity funded									
Value		0.7			0.6			2.1	
Number of deals		5			12			14	
Domestic acquirors, all targets equity funded									
Value			2.7			6.9			24.8
Number of deals			21			12			37
Total change		284.5			441.7			846.2	

Table 8: Merger and Acquisition Activity in Private and Public Firms (1989-2007), $ bln. (continued)

Survey year	1990 − 1992			1993 − 1995			1996 − 1998		
Acquiror:	Pub	Priv	Priv	Pub	Priv	Priv	Pub	Priv	Priv
Target:	Priv	Priv	Pub	Priv	Priv	Pub	Priv	Priv	Pub
All acquirors, all targets									
Value	1,126	400.9	635.4	723.6	287.3	470	1,241	1,327	1,488
Number of deals	11,588	1,590	8,133	9,025	1,253	7,592	12,765	1,606	9,670
All acquirors, domestic targets									
Value	534.1			330.1			497.4		
Number of deals	4,670			3,056			3,866		
Domestic acquirors, domestic targets debt or internally funded									
Value		181			230.8			457.7	
Number of deals		1,966			1,959			2,308	
Foreign acquirors, domestic targets									
Value		22.4			18.7			82.2	
Number of deals		222			150			265	
Domestic acquirors, foreign targets equity funded									
Value		0.4			2.8			2	
Number of deals		19			11			8	
Domestic acquirors, all targets equity funded									
Value			7.9			9.3			45.1
Number of deals			33			14			10
Total change		1,162			1,061			1,941	

Table 9: THE RETURNS TO PRIVATE EQUITY (1953-2007)

ANNUALIZED RETURNS	Arithm. average	Std. dev.	Geom. average	Line
A. Private Equity Returns (from the FFA/NIPA)				
Proprietors and partnerships, equity returns: 1953-1999	14.6	6.0	14.4	1
Proprietors and partnerships, equity returns: 1963-1999	14.8	6.4	14.6	2
Proprietors and partnerships, equity returns: 1953-2007	15.2	6.2	15.0	3
Proprietors and partnerships, equity returns: 1963-2007	15.5	6.7	15.3	4
B. Public Equity Returns				
Value-weighted index, CRSP market equity returns: 1953-1999	14.0	17.0	12.7	5
Value-weighted index, CRSP market equity returns: 1963-1999	13.7	16.0	12.5	6
Value-weighted index, CRSP market equity returns: 1953-2007	12.6	17.0	11.2	7
Value-weighted index, CRSP market equity returns: 1963-2007	12.0	16.0	10.8	8
Value-weighted index, Compustat book returns [1]: 1964-2007	16.1	8.0	15.8	9
Value-weighted index, Compustat book returns: 1964-1999	17.0	7.3	16.8	10

Note 1: As in MVJ the book equity construction follows Fama and French, which define it as the book of value of stock holders' equity, plus balance sheet deferred taxes and investment tax credit, minus the book value of preferred stock. In the reported results preferred stock is at its redemption value. The dividend distributions are those paid to common equity holders. The results for preferred stock using liquidation and par values differ only in decimals, and so are not reported here.

Figure 2: The Returns to Private and Public Equity (1963-2007)

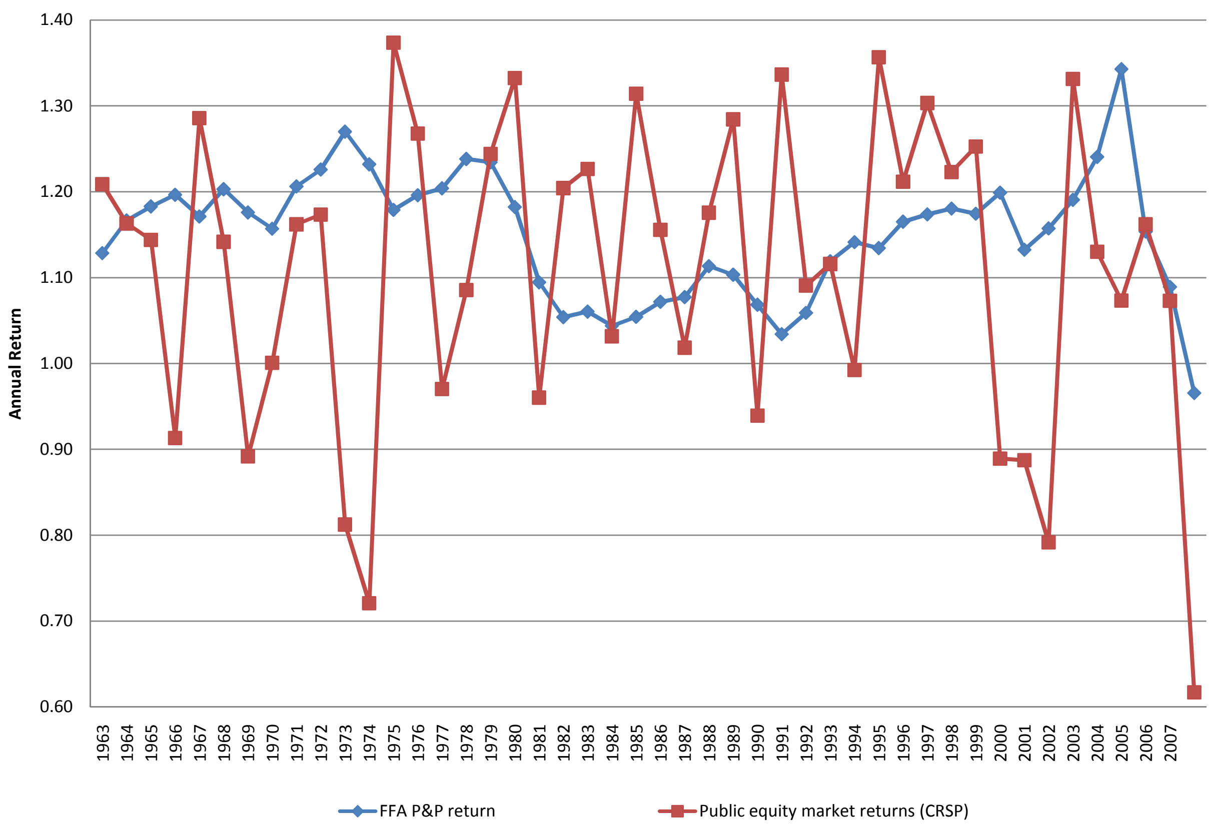


Figure 3: The Average Annualized Returns to All Private and Public Equity (1963-2007)

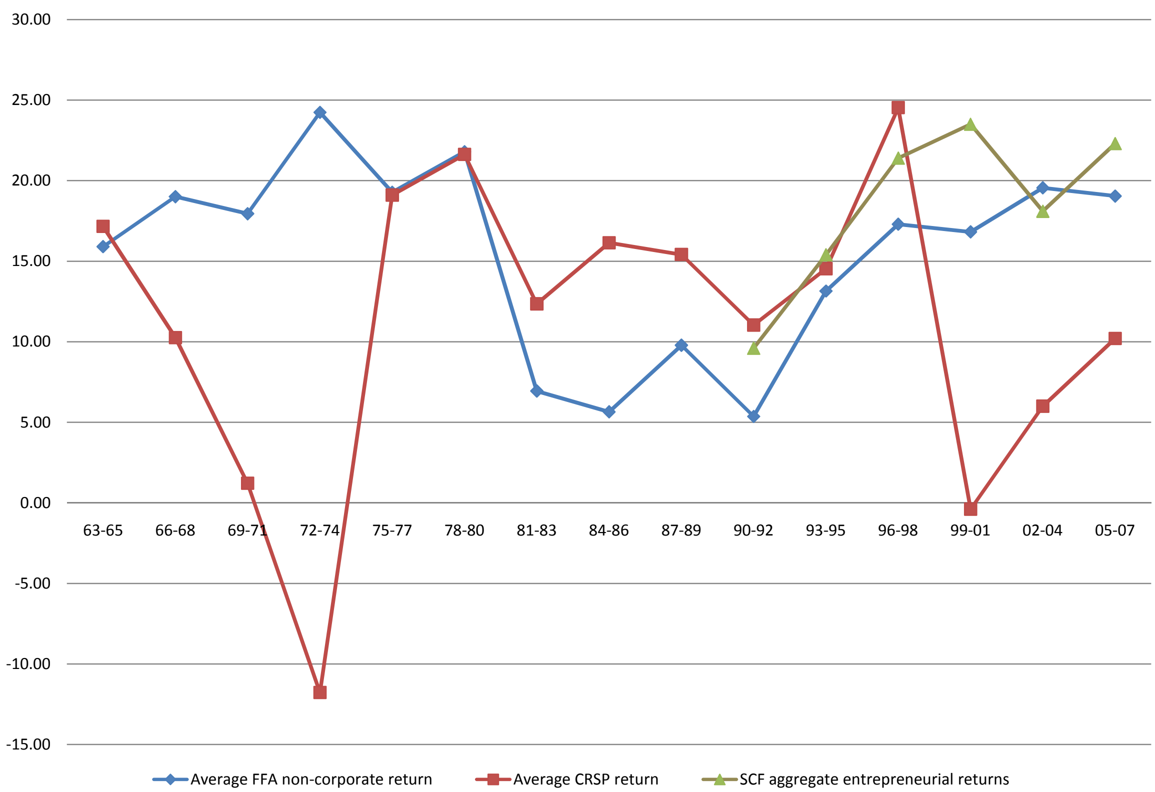


Figure 4: The Average Annualized Returns to Unincorporate Private and Public Equity (1963-2007)

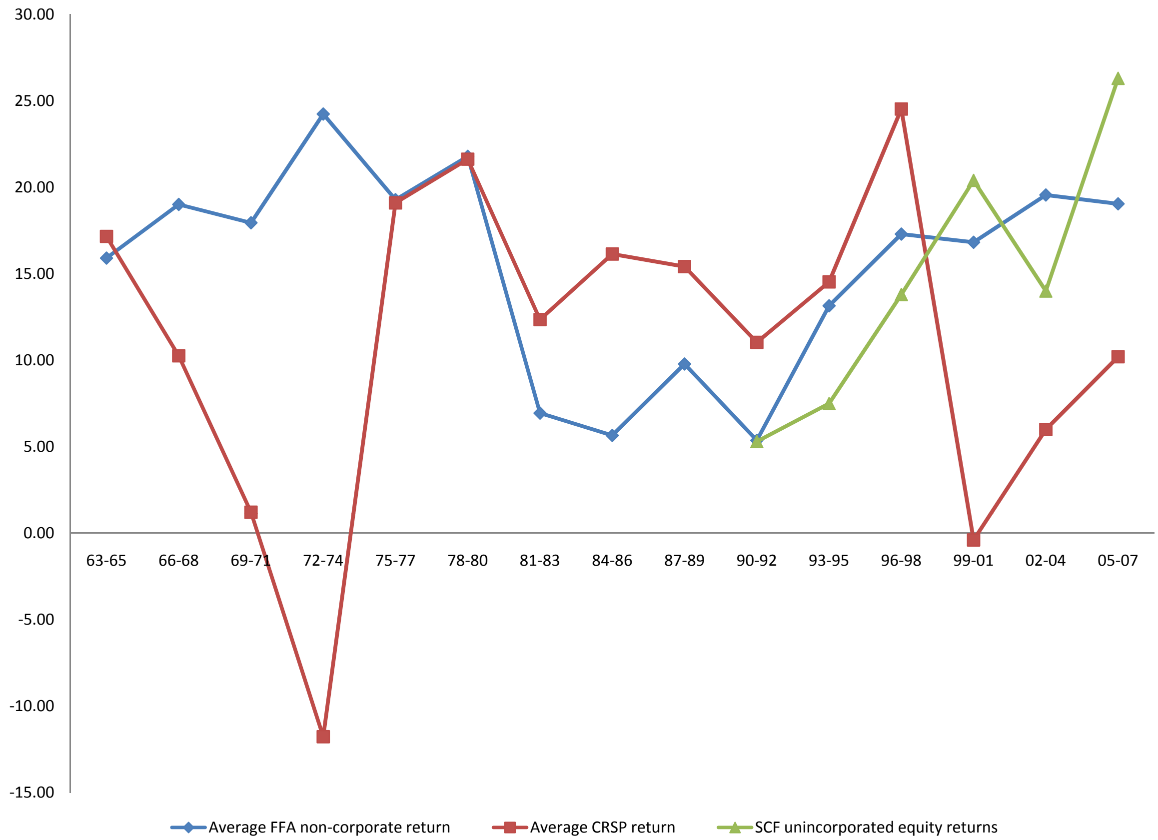


Table 10: ROBUSTNESS CHECKS FOR THE PRIVATE EQUITY RETURNS (1989-2007)

	90 – 92	93 – 95	96 – 98	99 – 01	02 – 04	05 – 07	Line
Total Private equity in SCF							
MVJ ASSUMPTIONS							
ALL	9.6	15.4	21.4	23.5	18.0	22.3	1
	(8.2)	(13.0)	(19.4)				2
ALTERNATIVE ASSUMPTIONS							
RETAINED EARNINGS							
ALL [1]	10.4	16.3	22.1	24.1	18.8	22.9	3
LABOR INCOME							
Imputation Regression							
ALL, REGRESSION 1 [2]	8.6	14.5	20.3	22.5	17.4	21.7	4
ALL, REGRESSION 2 [3]	8.3	14.4	20.5	22.7	17.3	21.6	5
Non-corporate Labor Income							
ALL [4]	8.7	14.5	20.4	22.6	17.5	21.7	6
LABOR INCOME & RE							
Imputation Regression							
ALL, REGRESSION 1	9.5	15.4	21.1	23.2	18.1	22.3	7
ALL, REGRESSION 2	9.1	15.4	21.3	23.4	18.0	22.3	8
Non-corporate Labor Income							
ALL	9.6	15.5	21.2	23.3	18.2	22.4	9

Note 1: This calculation eliminates the 20% retained earnings adjustment for the non-corporate sector.
Note 2: The imputation regression here uses the sample of respondents who own a business and have an active management role in this business, report being self-employed and receive positive wages.
Note 3: The imputation regression here uses the sample of entrepreneurs with active management role in their business who at the same time report working for some-one else and receive positive wages associated with this employment.
Note 4: This calculation subtracts from the non-corporate sector net income the total of paid and imputed unpaid wages.

Part III

Improving Public Equity Markets: No Pain, No Gain

7 Introduction

The relationship between financial and economic development has been commonly studied in cross-country regressions. These regressions have found a positive association between different measures of financial development and macroeconomic outcomes.[37] However, using these regression analyses it is difficult to establish the direction or magnitude of any causal link between the two. Public equity represents one form of financial development. It provides the benefit of better sharing of production risks in the economy. At the same time, operating publicly traded firms involves costs. Arguably, different policies can have an effect on the size of the costs and lead to their variations across countries, in particular with higher costs in less financially developed countries. In this paper, I address the following question: "What is the quantitative aggregative and distributive effects of reducing the costs of operating a publicly traded firm?"

To study the effects of lowering public equity costs, I use an open economy version of Angeletos (2007). The economy is populated with workers and entrepreneurs. This division allows for differential welfare effects for the owners of capital and labor factors. Entrepreneurs face idiosyncratic productivity risk in privately held firms. They can diversify away from this risk by investing in public equity. However, these investments have effectively lower average productivity compared to private equity investments. The difference in productivities is meant to capture the many costs associated with operating a public firm, which include but are not limited to costs of communication with investors, accounting, auditing, and financial reporting costs, as well as costs of legal services, etc. Private equity has higher average return and is risky, while public equity has lower average return and no associated risk. Equity investments of entrepreneurs are then determined as part of a portfolio decision problem taking as given the amount of risk in private equity and the cost of public equity.

[37]See Demirguç-Kunt and Levine (2008) for an excellent summary of the past and more recent findings in the empirical literature.

The focus of the quantitative part is on Ecuador and Chile. One reason for this is the availability of data for the key parameters in the model. In particular, the production data for privately held firms needed to estimate the productivity process of entrepreneurs are not widely available; however, both Ecuador and Chile have these data. And the difference in the relative sizes of their public equity markets can be used to impose discipline on the counterfactual experiment of the reduction in public equity costs in Ecuador to their Chilean level. Thus in the quantitative part, I use two sets parameters - one for Ecuador and one for Chile.

For the microestimation of productivity process I cannot use ordinary least squares (OLS). This is due to the fact that productivity realization in each period fully translates into the entrepreneur's choice of labor input given capital installed. To deal with the issue of endogeneity, I use the modification of the proxy variable approach suggested by Olley and Pakes (1996) and Levinsohn and Petrin (2003). I take estimated sample variance of the predicted residuals from this procedure as the measure of the productivity risk in entrepreneurial firms. Since direct measures of public equity costs are unavailable, I use the model relationships to infer the costs of operating a publicly traded firm from the relative size of the publicly traded and entrepreneurial sectors in the aggregate data for the two countries.

Qualitatively, I find a positive effect of lower public equity costs on macroeconomic aggregates, and differential effect on the welfare of workers and entrepreneurs. The aggregative and distributional effects are tightly linked. The intuition for both is best illustrated in the simple benchmark case of a small open economy, and is as follows. Reduction in public equity costs raises demand for capital and labor in the economy. With fixed labor supply and elastic supply of capital at the world interest rate, the capital-labor ratio and wages of the workers increase. Workers consume their current labor income and can only benefit from the wage increase. Entrepreneurs face two returns: return on investment in their own firm and return on their public equity investment. With higher wages, the cost of labor hired by entrepreneurs in their own firms increases while returns fall. At the same time, public equity returns do not change pinned down by the world interest rate. Thus, with lower returns in their own firms, entrepreneurs face worse investment opportunities and cannot be better off. Note that society as a whole benefits from lower public equity costs with more investment in the publicly traded equity. None of this gain, however, accrues to entrepreneurs. The scarce labor factor gets all of the benefits.

Quantitatively, the 15% reduction in public equity costs in Ecuador to their Chilean level is associated with an increase in the aggregate capital stock by 5.4%. At the same time, aggregate wages and output increase by 1.9%. This is also the magnitude of the increase in the aggregate consumption of workers. The welfare gain of workers from the reduction in public equity costs is equal to 3.2% in lifetime consumption equivalents. The welfare loss of entrepreneurs assuming the same initial wealth level is about 9.9%. Both the sign and the size of the welfare effects of entrepreneurs is independent of their position in the initial distribution.

Related Literature. This paper is most closely related to the literature that studies the effects of better risk diversification associated with financial development on economic development, including but not limited to papers by Greenwood and Jovanovic (1990), Obstfeld (1994), and Devereux and Smith (1994). In particular, Greenwood and Jovanovic (1990) consider an AK model of endogenous growth with risky *individual* production technology and costly financial intermediation. As in this paper, the agents in the economy face a trade-off between risk and the cost of diversification. Since the focus in the model is on endogenous growth and intermediation, the framework does not allow its straightforward application in quantitative analysis. In particular, in an AK permanent growth environment, the non-intermediated sector would be predicted to become extinct over a sufficiently long time period as the relative cost of intermediation falls. For quantitative aggregative evaluation one would at least need to modify its production structure, possibly in line with Atje and Jovanovic (1993), who provide its empirical test. With aggregate uncertainty and idiosyncratic shocks, however, this would make the framework analytically intractable. In terms of welfare implications, the equilibrium in Greenwood and Jovanovic (1990) is efficient, and conducting the experiment of lowering intermediation costs similar to this paper would probably have trivial welfare outcomes. For these reasons, the framework in this paper might be better suited for quantitative evaluation, but at the expense of endogenous financial intermediation. Unlike Greenwood and Jovanovic (1990), Obstfeld (1994) and Devereux and Smith (1994) consider representative agent economies where the only source of investment uncertainty is at the aggregate level.

In the model of this paper, markets are incomplete with respect to *idiosyncratic* investment risk. Covas (2006) and Meh and Quadrini (2006) consider similar environments but ask different questions. In particular, in Meh and Quadrini (2006) markets are endogenously incomplete due to private information about the realization of the shocks. In the presence of these informational asymmetries, they focus on the macroeconomic and welfare implications of different risk-sharing arrangements. Covas (2006) does not address the source of market incompleteness, but rather focuses on its effects on capital accumulation with borrowing constraints and persistent investment shocks. He also considers a version of the model with a corporate (traded) sector. This sector is introduced to quantify more accurately the effect of risk on capital accumulation. In the model the two sectors differ technologically, with the corporate sector employing both capital and labor of the workers. Since the entrepreneurial sector uses only capital, unlike in this paper, returns of entrepreneurs will be unaffected by any changes in economy-wide wages. The entrepreneurial technology also exhibits decreasing returns to scale. With the scarce entrepreneurial factor in the experiment of lowering public equity costs considered here, entrepreneurs would be able to claim part of the benefits accruing to the workers. This paper also bears a relationship to the literature on differential taxation of corporate and non-corporate sectors going back to Harberger (1962). Recently Meh (2008) has added entrepreneurial risk and liquidity constraints to this literature. In the occupational choice framework, he focuses on the trade-off between variability of income in the entrepreneurial sector and double taxation of income in the corporate sector. The two papers, however, are very different in the treatment of entrepreneurial risk and in the resulting trade-off studied.

In its focus on exogenous policy changes, this paper is closely related, e.g., to Antunes, Cavalcanti, and Villamil (2008). In a model of occupational choice, its authors investigate the effects of exogenous differences in intermediation costs and contract enforcement in credit markets on output per capita, total credit, and income inequality. Their analysis focuses on the cross section of developed and developing economies. The benchmark model is calibrated to the United States, and the only difference between countries under consideration is in terms of the size of their intermediation costs and degree of contract enforcement. These differences are then mapped into the long-run measures of economic development. The occupational choice approach is also used by Giné and Townsend (2003). Similar to this paper, the authors perform their quantitative analysis for Thailand - a country of interest for which extensive micro and macro data are available - and assess the aggregative and distributional consequences associated with the process of financial liberalization. An alternative growth accounting approach is pursued in Jeong and Townsend (2007). The focus of these papers, however, is on the extensive margin of participation in the financial sector. Financial liberalization relaxes regulatory and other legal requirements and allows access to the financial system for a segment of the population that would have otherwise been excluded.[38] And finally, a number of papers, e.g. Shleifer and Wolfenzon (2002)

[38]More specifically, they focus on reforms that lead to less restricted licensing requirements for financial

and Castro, Clementi, and MacDonald (2004), study the effects of exogenous changes in investor protection on economic development. These authors do not study the quantitative implications of their models, resorting instead to testing their consistency with some empirical regularities.

The paper is organized as follows. Section 8 introduces the model. Section 9 characterizes the main features of equilibrium. Section 10 describes the setup for the experiment and describes the qualitative effects of a reduction in public equity costs on macroeconomic aggregates and welfare in a simple benchmark case. The sections that follow deal with parameterization of the model. Section 11 provides a description of the data and the procedure for estimation of the productivity process. Section 12 summarizes the choice of functional forms and the parameterization of the model. Section 13 summarizes the quantitative effects of the experiment on the aggregates and individual welfare. Section 14 concludes. The details on the construction of variables in micro estimation and proofs are included in the Appendices.

8 Model

The model is an open-economy extension of Angeletos (2007). Time is discrete and infinite, $t \in \{0, 1, ...\}$. There are two types of agents in the economy: a measure χ of entrepreneurs, and a measure $1-\chi$ of workers. There is no occupational choice decision, and the division into workers and entrepreneurs is exogenous.

8.1 Entrepreneur's Problem

Entrepreneurs' preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{{c_t^i}^{1-\gamma}}{1-\gamma}, \tag{4}$$

where β is the discount factor and E_0 is the expectation as of date 0 with respect to uncertainty facing entrepreneurs. At each date t, entrepreneur i chooses consumption, c_t^i, capital investment, k_{t+1}^i, investment in public equity shares, s_{t+1}^i, and foreign asset

institutions (both foreign and domestic), the reduction of excess capitalization requirements, and enhanced ability to open new branches, which are modeled as resulting in greater access to deposits and credit for the population.

holdings, b^i_{t+1}.[39] Entrepreneurs employ capital k^i_{t+1} in their privately held firms with production technology given by

$$F(k^i_t, n^i_t, \eta^i_t) = \eta^i_t (k^i_t)^\alpha (n^i_t)^{1-\alpha}. \tag{5}$$

Here, k^i_t and n^i_t are physical inputs, and η^i_t is the idiosyncratic productivity shock. The shocks are assumed to be i.i.d. over time and over agents, with continuous p.d.f. $g : R_+ \rightarrow R_+$.[40] At any date t, capital k^i_t is a fixed input in the production technology, and n^i_t is a variable input optimally chosen at the beginning of the period in response to the realization of shock η^i_t given k^i_t. Due to the timing investment in k^i_t is risky, and in the absence of markets for equity claims in privately held firms, entrepreneurs bear the investment risk in full. They can diversify away from their firms by investing in public equity capital, x^i_{t+1}, and foreign assets, b^i_{t+1}, both of which are assumed to be riskless.[41] Entrepreneurs receive residual income in their privately held firms, π^i_t, defined below and investment income from their net safe asset holdings. The budget set of entrepreneur is given by

$$c^i_t + k^i_{t+1} + b^i_{t+1} + x_{t+1} \leq \pi^i_t + R_t b^i_t + R_{xt} x^i_t \tag{6}$$

$$\pi^i_t = \eta^i_t (k^i_t)^\alpha (n^i_t)^{1-\alpha} - w_t n^i_t + (1-\delta) k^i_t$$

$$c^i_t, k^i_{t+1} \geq 0$$

$$\bar{b}^i_{t+1} \equiv b^i_{t+1} + x^i_{t+1} \geq -k^i_{t+1}, \tag{7}$$

[39] Here, $b^i_{t+1} < 0$ refers to a loan from a foreign bank or financial institution. Alternatively, one could interpret $b^i < 0$ as the issuance of fixed income claims in foreign bond markets. The former interpretation would be preferred, however, given that *private* borrowers are disproportionately important to the market in international bank loans, while *sovereigns and other government borrowers* primarily rely on the bond market (see Eichengreen and Mody (1999)). In the case of private borrowers, this is particularly true for the manufacturing sector, to which the model will be parameterized later. This relationship with foreign intermediaries is described in greater detail in Section 8.5.

[40] In the quantitative part the shocks will be assumed to be ln $\eta^i_t \sim (0, \sigma^2_\eta)$.

[41] Regarding the assumption of riskless public equity note the following. First, public equity does not insure against aggregate uncertainty, but does allow diversification of idiosyncratic investment risks. Second, in the absence of aggregate uncertainty in the model and with trade in public equity idiosyncratic risks associated with public equity investments would indeed be diversified.

where R_t is the interest rate associated with foreign asset holdings and R_{xt} is the return to public equity. With returns on safe assets equalized in the absence of arbitrage opportunities, in equilibrium safe investments can only be pinned down in the aggregate, but not at the individual level of entrepreneur. The borrowing constraint of entrepreneurs in (7) is not binding.

8.2 Worker's Problem

Workers' preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \frac{{c_t^i}^{1-\gamma}}{1-\gamma}$$

There is no expectation as in (1) since the workers in the model do not face any uncertainty. Unlike entrepreneurs, workers do not have access to production technology in privately held firms. They can hold positive amounts of assets abroad and invest in public equity. Workers receive labor income from selling one unit of time endowment in the competitive labor market, and investment income from asset holdings. They choose how much to consume/save, and their portfolio holdings subject to budget feasibility:

$$c_t^i + x_{t+1}^i + b_{t+1}^i \leq R_{xt} x_t^i + R_t b_t^i + w_t$$

$$c_t^i,\, b_{t+1}^i,\, x_{t+1}^i \geq 0$$

Without any source of uncertainty, in the steady-state equilibrium of this incomplete markets economy with $\beta R < 1$,[42] workers choose to consume all of their income and do not save/invest.

8.3 Returns to Public Equity Investments

While entrepreneurs are able to diversify away productivity risk by investing in publicly traded firms, these investments have effectively lower average productivity compared to private equity investments. The difference in productivities is meant to capture the many costs associated with operating a public firm, which include but are not limited to costs of

[42] In equilibrium $R_{xt} = R_t$.

communication with investors, accounting, auditing, and financial reporting costs, as well as costs of legal services, etc. Arguably, different policies can have an effect on the size of these costs, and result in their variations across countries.

Formally, the relationship between average productivities of investments in private and public equity is given by:

$$R_{xt} = \frac{F_{Kt} + 1 - \delta}{1 + \tau}$$

where F_{Kt} is the average productivity of the private equity investment, and τ measures the cost associated with public equity investment, which reduces its average productivity. Higher τ results in lower average productivity and lower returns to public equity.

8.4 Entrepreneur's Trade-off

Equity investments of entrepreneurs are determined as part of a portfolio decision problem. Private equity has higher average return and is risky (with $\sigma_\eta^2 > 0$), while public equity has lower average return (with $\mu_x > 0$) and no associated risk. The portfolio shares of these investments depend on the relationship between $\sigma_\eta^2 > 0$ and equity cost $\mu_x > 0$.

8.5 Foreign sector

There is a continuum of foreign agents,[43] who supply resources according to the following schedule:

$$R = \begin{cases} g(\mid \frac{B}{Y} \mid) & \text{if } B < 0 \\ \\ \mathrm{R}_w & \text{if } B \geq 0 \end{cases} \tag{8}$$

where B $= \int_{i\in[0,1]} b_t^i di$ denotes the economy's net foreign assets, $\frac{B}{Y}$ is the net asset-to-income ratio, and $g(\cdot)$ is increasing in $\frac{B}{Y} < 0$; and R_w is base interest rate.[44] In this specification, R

[43] These are banks or financial institutions more generally.

[44] R_w does not have to be constant and can vary with changes in the credit market conditions, e.g., many international loans are priced off LIBOR, which fluctuates over time. R_w can be interpreted as the world interest rate as well.

is external to the individual decisions of the agents.[45] When the economy is a net creditor, i.e., $B > 0$, it faces a constant base rate R_w. When the economy is a net debtor, i.e., $B < 0$, its borrowing rate is increasing with the amount of total debt normalized by income.[46] The special case of a small open economy obtains when $g(\mid \frac{B}{Y} \mid) = R_w$ for $\forall \mid \frac{B}{Y} \mid$.

9 Equilibrium Characterization

9.1 Equilibrium Definition

Definition 1. *Given initial asset holdings of workers and entrepreneurs, the equilibrium of the economy is defined as deterministic sequences of wages* $\{w_t\}_{t=0}^{\infty}$*, interest rates* $\{R_t\}_{t=0}^{\infty}$*, share prices* $\{p_t\}_{t=0}^{\infty}$*, aggregate quantities* $\{C_t, C_t^w, K_{t+1}, X_{t+1}, Y_t, B_{t+1}, N_t, L_t\}_{t=0}^{\infty}$*, deterministic plans* $\{c_t^{i,w}, \bar{b}_{t+1}^{i,w}\}_{t=0}^{\infty}$ *for workers* $i \in [\chi, 1]$*, and collections of contingent plans* $\{c_t^i, \bar{b}_{t+1}^i, n_t^i,$
$k_{t+1}^i\}_{t=0}^{\infty}$ *for entrepreneurs* $i \in [0, \chi]$*, such that the following conditions are satisfied:*

(i) Given sequences of wages $\{w_t\}_{t=0}^{\infty}$ *and interest rates* $\{R_t\}_{t=0}^{\infty}$ *and initial endowment* $b_0^i + x_0^i$ *and* k_0^i*,* $\{c_t^i, n_t^i, \bar{b}_{t+1}^i, k_{t+1}^i\}_{t=0}^{\infty}$ *solves entrepreneur i's problem for* $\forall i \in [0, \chi]$*;*

(ii) Given sequences of wages $\{w_t\}_{t=0}^{\infty}$ *and interest rates* $\{R_t\}_{t=0}^{\infty}$*, and initial endowment* $b_0^{i,w} + x_0^{i,w}$*, deterministic plans* $\{c_t^{i,w}, \bar{b}_{t+1}^{i,w}\}_{t=0}^{\infty}$ *solve worker i's problem for* $\forall i \in [\chi, 1]$*;*

(iii) Aggregate accounting identities hold.

(iv) Labor market clears, $\forall t \geq 0$*:*

$$N_t + L_t = 1 - \chi \ ;$$

(v) Pairs $\{R_t, \frac{B_t}{Y_t}\}_{t=0}^{\infty}$ *satisfy the foreign lending schedule;*

(vi) Aggregate net foreign assets evolve in $\forall t \geq 0$ *according to:*

$$C_t + C_t^w + K_{t+1} + X_{t+1} + B_{t+1} = F(K_t, N_t, \bar{\eta}) + F(X_t, L_t, \bar{\eta}) + (1 - \delta)(K_t + X_t) + R_t B_t$$

[45] In economies like Ecuador, for example, the interest rate may depend not so much on the individual borrower characteristics, but on the source of funds - banks or bond markets.

[46] The functional relationship between debt-to-income ratio $\frac{B}{Y} < 0$ and the interest rate would be specified in Section 12. In particular, it captures the debt elastic external interest rate in the small open economy literature.

In this definition, $F(K_t, N_t, \bar{\eta})$ is the output produced in the entrepreneurial sector, and $F(X_t, L_t, \bar{\eta})$ is the output in the publicly traded sector.[47]

9.2 Individual Behavior

Due to timing, the profit maximization problem of an entrepreneur in a privately held firm is static. Capital is a fixed factor of production and labor is a static variable input, which does not have an effect on future profitability of the firm. Given prices, the profit-maximizing choice of labor, n_t, and resulting profits, π_t, are linear in capital k_t:

$$n_t = n(\eta_t, w_t)k_t \text{ and } \pi_t = (R_k(\eta_t, w_t) + 1 - \delta)\, k_t \tag{9}$$

where $n(\eta, w) = \arg\max_l[F(\eta, 1, l) - wl]$ and $R_k(\eta, w) = \max_l[F(\eta, 1, l) - wl]$ are labor input and profit per unit of capital, respectively. The individual private returns (profits per unit of capital) are independent of the amount invested.

With entrepreneurial wealth at date t $\omega_t \equiv R_{kt}k_t + R_t(b_t + x_t)$ linear in returns and homothetic preferences, optimal consumption, investment in risky private equity, k_{t+1}, and safe (public equity and foreign) asset holdings, $\bar{b}_{t+1}$, are linear in wealth:

$$c_t = (1 - \varsigma_t)\omega_t \tag{10}$$

$$k_{t+1} = \varsigma_t \phi_t \omega_t \tag{11}$$

$$\bar{b}_{t+1} = \varsigma_t(1 - \phi_t)\omega_t, \tag{12}$$

where ς_t is the savings rate, and ϕ_t and $1 - \phi_t$ are portfolio shares of risky and risk-free assets. All agents choose the same fraction of wealth for consumption, $1 - \varsigma$, and the same portfolio allocation, ϕ. Logarithm of effective wealth follows a random walk at steady state prices, and individual variables are highly persistent.

The optimal decision rules in (10)-(12) help highlight the difference between the incomplete markets model of this paper and the class of Aiyagari/Bewley models. In Aiyagari/Bewley models, shocks to labor income enter additively similar to endowment shocks.

[47]Total output in the economy is: $Y = F(X_t, L_t, \bar{\eta}) + F(K_t, N_t, \bar{\eta}) = \bar{\eta}(K_t + X_t)^{\alpha}(1 - \chi)^{1-\alpha}$.

With standard CRRA preferences, in particular, agents' marginal propensity to consume declines with the level of wealth and the optimal consumption function is concave. This reflects strictly positive precautionary demand for savings due to uncertainty associated with future labor income. Concavity implies that the aggregate dynamics depend on the whole wealth distribution in the economy preventing the use of closed-form solutions.

In the model of this paper, uncertainty is associated with the rate of return to investment (no labor income risk). With the same standard CRRA preferences the consumption function is linear in wealth.[48] Entrepreneurs choose to save the same fraction of their wealth and allocate the same fraction of the portfolio to risky assets. With linearity in the optimal decision rules, all of the aggregates in the economy are independent of the distributions. With exact aggregation results the model is highly analytically tractable, unlike the class of Aiyagari/Bewley models.

9.3 Aggregate Steady State

The aggregate steady-state quantities and prices in the open economy facing the foreign lending schedule in (8) are uniquely determined for a given vector $\{\alpha, \sigma_\eta^2, \delta, \beta, \gamma, \chi\}$ of parameters of technology, preferences, and population structure, as a function of the proportional public equity cost τ.

While the steady state is well defined in terms of the aggregates, there is no stationary distribution for the cross section. At steady-state prices, the log of wealth of the agents follows a random walk. To get around the problem of non-stationarity one can introduce a positive probability of death of entrepreneurs. In each period entrepreneurs who die are replaced with an equal mass of new agents, and their assets are distributed uniformly among the newly born. Since the qualitative findings in this paper would not be affected by this modification and it will not be pursued here.

[48] For example, see the discussion of the precautionary savings motives in Ljungqvist and Sargent (2000). The two well- known cases when the marginal propensity to consume out of wealth is independent of wealth are: (*i*) the CARA utility if all of the risk is to labor income (no rate-of-return risk), and (*ii*) CRRA utility if all of the risk is rate-of-return risk (no labor-income risk). This model is an example of (*ii*).

10 Qualitative Results

10.1 Aggregate Implications and Welfare Effects

Benchmark case. One can use the case of a small open economy ($g(\cdot) = const$) to determine the sign of the differential welfare effects associated with lower public equity costs for workers and entrepreneurs, and derive a response in the aggregates in closed form.

Aggregate implications. Consider a change in $\tau = \overline{\tau}$ by $1+\epsilon$, ϵ small, in the economy facing a constant world interest rate. In equilibrium, returns on safe assets are equalized:

$$R_w = \frac{F_k + 1 - \delta}{1 + \overline{\tau}(1+\epsilon)},$$

where F_k is the marginal product of capital of the publicly traded firm.[49]

With $\mathrm{d}\epsilon < 0$, the percentage decrease in the marginal product of capital and the aggregate return in the entrepreneurial sector are given by

$$\mathrm{dlog\ F_k} = \frac{R_w \overline{\tau}}{R_w(1 + \overline{\tau}(1+\epsilon)) + \delta - 1}\mathrm{d}\epsilon.$$

where $R_w(1 + \overline{\tau}(1+\epsilon)) + \delta - 1 > 0$.

At the same time, with $\alpha < 1$ and $\mathrm{d}\epsilon < 0$, the aggregate capital stock increases by

$$\mathrm{dlog\ (K + X)} = \frac{\mathrm{dlog\ F_k}}{\alpha - 1}\mathrm{d}\epsilon. \tag{13}$$

Then, with fixed labor supply, the aggregate output increases by fraction α of the increase in the aggregate capital, and so do the wages of the workers, which they consume in equilibrium.

Welfare effects. The welfare effects of lower public equity costs are tightly linked to the aggregate effects discussed above. Theorem 1 shows that the sign of the effects is different for workers and entrepreneurs. Its proof is included in the Appendix; only the intuition is provided in this section.

Theorem 1. *Consider two economies with the same vector of parameter values of preferences, technology, and population structure facing the world interest rate, R_w. Let the*

[49] Note that in equilibrium, aggregate capital-labor ratios in publicly traded and entrepreneurial sectors would be the same and would also be equal to the aggregate capital-labor ratio in the economy.

initial distribution of wealth of entrepreneurs $\{k_0^i, x_0^i, b_0^i\}_{i\in[0,\chi]}$ *be the same, but public equity costs different with* $\overline{\tau} > \underline{\tau}$*. Then, in the economy with* $\underline{\tau}$ *entrepreneurs are worse off, while workers are better off.*

The intuition for the results in Theorem 1 is straightforward. Both economies with $\underline{\tau}$ and $\overline{\tau}$ face the same world interest rate R_w. With aggregate capital increase given in (13) and constant labor supply, the capital-labor ratio and wages will be higher in the economy with low proportional cost $\underline{\tau}$. Since in equilibrium workers choose to consume all of their labor income in both economies, they will be strictly better off in the economy with $\underline{\tau}$.

Entrepreneurs face two returns: return on investment in own firm and return on their public equity investment. With higher wages, the cost of labor hired by entrepreneurs in their own firms is higher and returns are lower in the $\underline{\tau}$ economy. At the same time, public equity returns do not differ across the two economies pinned down by the world interest rate. Thus with lower returns in their own firms, entrepreneurs face worse investment opportunities in the $\underline{\tau}$ economy and cannot be better off.

Alternatively, consider any allocation chosen by the entrepreneur in the economy with low proportional cost, $\underline{\tau}$. This allocation would cost strictly less in the economy with high equity cost $\overline{\tau}$ due to its lower price of labor. Entrepreneurs in the economy with high public equity cost can improve on this allocation by increasing their consumption and will be better off.

Theorem 1 implies that in the small open economy with $\overline{\tau}$, a reduction in the proportional cost to $\underline{\tau}$ would be associated with welfare losses of entrepreneurs and welfare gains of the workers.

Alternative scenarios for welfare effects. Unlike the case of the small open economy, when $g(\cdot)0$ is an increasing function, the supply of capital at each interest rate is limited. Therefore, a reduction in the public equity cost from $\overline{\tau}$ to $\underline{\tau}$ increases both demand for and the cost of capital. Other things equal, the capital-labor ratio and wages of the workers increase by less than in the case of the small open economy.

For entrepreneurs, a wage increase is associated with a reduction in returns in their own firms, but this reduction is smaller than in the case of the small open economy. At the same time, returns to public equity increase somewhat. When the elasticity of interest rate is small, entrepreneurs would still be left worse off. However, the loss in welfare would be smaller.

Welfare outcomes in the cross section. The sign and magnitude of the welfare effects of entrepreneurs is independent of their position in the initial distribution due to homotheticity of preferences.

Quantitative results. To quantify the qualitative findings above, among other parameter values one needs to know: (i) the variance of the productivity process in entrepreneurial firms, σ_η^2, (ii) the magnitude, ϵ, of the plausible reduction in proportional public equity cost, and (iii) the shape of the foreign lending schedule. The estimates of (i)-(iii) are obtained from micro and macro data detailed in Sections 11-12.

11 Micro Estimation

This section provides detailed information about the estimation of the income shares and the productivity process in the data.

11.1 Description of the Data

In the estimation, I use data from the non-publicly traded sector of Ecuador and Chile. They come from two sources: Ecuador's Manufacturing, Services, and Retail Trade Survey and Chile's Manufacturing Census. The Chilean data are well known and have been widely used in the development and industrial organization literatures. Its full description is available elsewhere;[50] therefore, only the necessary detail will be repeated here. Data for Ecuador are new and will be the focus of this section.

Ecuador. Data for Ecuador come from the Instituto Nacional de Estadistica y Censos (INEC, National Institute for Statistics and Census). Its collection by INEC has been mandated by law, and the first three representative sectoral samples for manufacturing, services, and retail trade were compiled in 1995.[51]

The type of data is survey with the company as the primary sampling unit. At the start the sample included around 3,000 companies, a significant fraction of which has remained. In the case of termination of activities, companies from the original sample were replaced by others with similar characteristics. At the same time, the new companies were added to the sample in the way that preserves its representative nature. In this unbalanced panel, the annual company data are available from 1995 to 2006.

The survey questionnaire includes detailed questions about the production activities

[50] The original source for the description of the data and construction of all the variables is Liu (1991).
[51] For the purpose of comparability, I will use only manufacturing sample.

Table 11: Aggregate statistics for company and plant level data for Ecuador and Chile

	Chile	Ecuador
Capital-value added ratio	1.47	1.24
Share of publicly traded capital in total capital	0.30	0.03
Depreciation rate, %	8.0	12.0
Investment rate, %	10.0	13.0

of the company,[52] both primary and secondary, capital stock, fixed and inventory investments, employment, remuneration, including wages and contributions to social security for employees, number of owners of the firm, and so on.[53] All of the companies are registered with the authorities for tax purposes and pay employee social security contributions. Therefore, these data do not cover the informal sector of the economy.[54] The sample includes both privately held and publicly traded companies. The capital share of publicly traded companies, however, constitutes a mere 3%, which reflects the small size of public equity markets in Ecuador.[55] About 2/3 of the sample are corporations, limited liabilities companies, and about 1/3 are general and limited partnerships.

The survey includes about 1,500 manufacturing companies per year on average. Table 11 summarizes some descriptive statistics for the companies in the dataset, and Table 12 provides details on the sample selection process.

The variables from the survey used in estimation include value added, net investment, employment, and capital stock. To exploit the panel component of the data, the values of these variables have been converted into constant prices using the methodology of Liu

[52]That is, output produced, intermediate consumption, and value added.

[53]Note that for the purposes of the model, it does not matter whether the non-publicly traded company has multiple owners. The main criterion is whether these ownership or equity shares are liquid and readily transferrable. In the case of non-traded companies, they are not.

[54]The percentage of economic activity in the informal sector in Ecuador is estimated at ... in terms of employment.

[55]Capital is reported at its end-of-period value, which includes inflationary adjustment. From the descriptions of the data, it is hard to determine whether it is reported at true replacement cost. There is no variable with the traded status of the company. To determine which companies are traded I match their identification numbers in the dataset with the identification numbers for the publicly traded companies in the Ecuador's Superintendency of Companies, which keeps a registry of all publicly traded companies.

(1991) as described in the Appendix. This methodology was originally applied to the Chilean data, which are briefly discussed next.

Chile. The Chilean data are collected annually by the Instituto Nacional de Estadistica (INE). By design, the Census includes *all* of the manufacturing *plants* with at least 10 workers.[56] Its data coverage is similar to Ecuador's Manufacturing Survey.[57]

The plants in the sample belong to limited liability companies, corporations, cooperatives, public enterprises, and others. The capital share of plants owned by publicly traded companies is approximately 10 times that in Ecuador, or about 30%.[58] The descriptive statistics for the plants are provided in table 11.

Ecuador's data do not report the number of plants per company. At the same time, it is not possible to link different plants in the Chilean data to one company. Given this drawback of the data, I choose to treat Chilean plant-level data as company data. I justify this choice based on the US data, where, for example, across all employer firms in 2002 the average number of plants was 1.25 plants per firm, and across all firms - 1.05.[59]

Table 12: Sample selection for Ecuador and Chile

	Ecuador	Chile
Original sample	6,860	12,916
Excluded		
Missing log capital	108	5,413
Missing log labor	9	0
Missing log value added	546	385
Missing log investment	3,106	7,145
Final sample	3,376	3,405

11.2 Estimation Procedure

This section describes the estimation procedure for the income shares and parameters of the productivity process in the individual production technology of entrepreneurs. It is

[56]There are plants in the sample which employ less than 10 workers as well.

[57]For details of the data coverage and construction of the variables, please see Liu (1991).

[58]These capital stocks are reported on the plants' balance sheets. I classify plants as publicly traded if they have corporate form of organization and pay dividends in the current year. Then I average the share of publicly traded capital in total capital and report it in Table 11.

[59]Source: Census Bureau, Statistics about Business Size, http://www.census.gov/epcd/www/smallbus.html.

based on the proxy variable approach to production function estimation, which corrects for endogeneity in firm input choices and allows for time-varying heterogeneity. This approach was originally suggested by Olley and Pakes (1996, henceforth OP) and later modified by Levinsohn and Petrin (2003, henceforth LP). In what follows I will discuss its assumptions and the implementation in the context of the model. I will use the sample variance of the residuals from the production function estimation as a measure of productivity risk in entrepreneurial firms in the model.

Recall the structure of the entrepreneur's problem associated with the privately held firm. The capital of the firm in period t, k_t, is determined as part of entrepreneurial portfolio choice at date $t-1$. It is a fixed input in the firm's production technology. Given k_t, the variable labor input, n_t, is chosen by the entrepreneur in response to realization of i.i.d. productivity shock η_t at the start of period t. At t, the entrepreneur also decides how much to invest in a privately held firm for the next period. With i.i.d. assumption, the expectation of future productivity is not affected by the current realization. Other things equal, investment depends on the current realization of productivity through its effect on the total resources of the entrepreneur. In particular, entrepreneurs with different realizations of current productivity choose to invest the same portfolio share in privately held firms.

The above discussion can be translated into the following structural assumptions: (*i*) k_t is a predetermined input in the production technology; (*ii*) $n_t = n(\eta_t, w_t)k_t$ is an endogenous input, and η_t fully transmits into n_t; (*iii*) k_t and η_t are contemporaneously uncorrelated; (*iv*) the current period investment in the privately held firm is a monotone function of the current productivity shock, η_t. Both labor input in (*ii*) and investment in the privately held firm in (*iv*) respond to realization of the productivity shock. However, unlike labor, investment does not enter the production function estimation. The proxy variable approach uses assumed investment demand relationship to substitute for the unobservable productivity and therefore to help identify the coefficient on the labor input.[60] Assumption (*iv*) is the key technical assumption on the demand relationship. It ensures that the investment function is invertible, and observed investment perfectly proxies the productivity shock. In the Appendix I verify that (*iv*) is satisfied in the model with respect to entrepreneurial investments in their privately held firms. More generally, assumptions

[60] The assumption on the form of the demand relationship in the structural proxy variable approach to production function estimation also allows one to obtain the "true" productivity as opposed to its noisy counterpart delivered by the dynamic panel methods.

(i)-(ii) and (iv) are the same, and in (iii) I depart from OP, who assume the first-order Markov process on productivity[61]. I modify the proxy variable approach appropriately to estimate the production function given assumption (iii). I describe the approach and the modifications introduced next.

Starting with specification of the data- generating process in logged variables:[62]

$$\tilde{y}_t = \beta_0 + \beta_k \tilde{k}_t + \beta_n \tilde{n}_t + \tilde{\eta}_t + \tilde{u}_t \tag{14}$$

where $\tilde{\eta}$ is the part of the shock transmitted into firm's decision, and $\tilde{u}$ is the untransmitted part, the estimation proceeds in two stages[63]:

Stage 1: Estimate using OLS labor coefficient β_n substituting for the productivity shock η_t the inverse of the investment function.

Stage 2: Write down partial productivity equation with dependent variable $\tilde{y}_t - \hat{\beta}_n \tilde{n}_t$ using the estimate of β_n from stage 1 and obtain consistent estimate of β_k using OLS.

Stage 1 of the estimation uses a non-parametric form of the investment function[64] given by

$$\tilde{i}_t = f(\tilde{k}_t, \tilde{\eta}_t). \tag{15}$$

So that under the assumption of monotonicity $\tilde{\eta}_t$ can be mapped to observed $\tilde{i}_t$ using the inverse function f^1:

$$\tilde{\eta}_t = f^{-1}(\tilde{k}_t, \tilde{i}_t) \equiv g(\tilde{k}_t, \tilde{i}_t). \tag{16}$$

[61]With the first-order Markov process, capital k_t would be correlated with the persistent part of the productivity, but uncorrelated with the current period innovation.

[62]I do not impose the constant returns to scale assumption, and test their presence in the data postestimation. I find that they are not rejected in the data with the proxy variable approach, but not with OLS, which gives inconsistent estimates of the coefficients when labor input is endogenous.

[63]Recently, a number of papers, notably Wooldridge (2005) and Ackerberg, Caves, and Fraser (2005), have suggested alternative estimation procedures based on the ideas of OP and LP. In particular, Wooldridge suggests using a generalized method of moments estimation as opposed to the two-step estimation approach in both LP and OP, on the grounds of efficiency and potential problems with identification of the first-stage coefficients in LP. These issues are addressed in Section 18

[64]It is also possible to determine whether the model is consistent with the data by testing this particular functional form; however, this will not be pursued at this point.

I substitute (16) into (14) and rewrite:

$$\tilde{y}_t = \tilde{\beta}_0 + \beta_k \tilde{k}_t + \beta_n \tilde{n}_t + g(\tilde{i}_t, \tilde{k}_t) + \tilde{u}_t. \tag{17}$$

Note that coefficient β_k cannot be identified in (17), since capital k_t enters twice in $\beta_k \tilde{k}_t$ and $g(\tilde{i}_t, \tilde{k}_t)$. Thus, to obtain consistent estimate of β_n define

$$\varphi_t(\tilde{i}_t, \tilde{k}_t) \equiv \tilde{\beta}_0 + \beta_k \tilde{k}_t + g(\tilde{i}_t, \tilde{k}_t). \tag{18}$$

Following OP I use a series polynomial estimator of the non-parametric function φ. It is given by an order three polynomial in $\tilde{i}$ and $\tilde{k}$:

$$\varphi(\tilde{i}_t, \tilde{k}_t) = \sum_{l=0}^{3} \sum_{j=0}^{3-l} \delta_{lj} \tilde{i}_t^l \tilde{k}_t^j, \tag{19}$$

where δ_{00} is the constant term. This gives the estimable version of (17), which is a function of observables only:

$$\tilde{y}_t = \beta_n \tilde{n}_t + \sum_{l=0}^{3} \sum_{j=0}^{3-l} \delta_{ij} \tilde{i}_t^l \tilde{k}_t^j + \tilde{u}_t. \tag{20}$$

Since $\tilde{u}_t$ is the component of productivity disturbance that does not transmit into any decisions of the firm (either $\tilde{n}_t$, $\tilde{k}_t$, or $\tilde{i}_t$), OLS provides unbiased and consistent estimates of β_n and coefficients $\{\delta_{lj}\}$ of the φ function. This completes stage 1 of the estimation.

Under the i.i.d. assumption on the shocks, one only needs coefficient $\hat{\beta}_n$ to write down the partial productivity equation in stage 2. The dependent variable in this equation is the output net of labor's contribution, $\tilde{y}_t - \hat{\beta}_n \tilde{n}_t$, and the independent variable is capital:[65]

$$\tilde{y}_t - \hat{\beta}_n \tilde{n}_t = \beta_k \tilde{k}_t + \tilde{\eta}_t + \tilde{u}_t \tag{21}$$

This equation can also be estimated using OLS, which gives consistent and unbiased estimates of β_k. This is the case, since $\tilde{k}$ and $\tilde{\eta}$ are contemporaneously uncorrelated and $\tilde{u}$ is an untransmitted component.[66]

[65] See Griliches and Mairesse (1995) for the discussion of this case.

[66] With a first-order Markov process assumed in OP, the partial productivity equation in the 2nd stage is more complicated, taking into account productivity dynamics. In particular, the second stage estimation

Figure 5: Results of the Estimation

	Chile	Ecuador
Income shares		
Capital	0.32(0.16)	0.36(0.10)
Labor	0.67(0.04)	0.69(0.03)
Productivity shock		
σ_η^2	0.21	0.16
Number of obs.	3405	3376

Using $\hat{\beta}_n$ and the coefficients of the φ function one can obtain consistent predictions of the sample residuals:

$$\widehat{\tilde{\eta}}_t \equiv \hat{\varphi}_{it} - \hat{\beta}_n \tilde{n}_t. \tag{22}$$

The estimated sample variance of these residuals constitutes the measure of the productivity risk in the model.

Table 5 summarizes the values of the estimated income shares and of the variance of the productivity shocks for the benchmark parameterization of the model. OLS coefficients are included to illustrate biases introduced into the estimation when endogeneity of the labor decision in the firm's problem is ignored. The standard errors reported in the table are obtained using a bootstrap method for the two-stage estimation. This procedure closely follows Levinsohn and Petrin (2003).

Values for σ^2 reported in Table 5 for Chile and Ecuador suggest a lot of uncertainty associated with operating individual production technology in entrepreneurial firms. These values, however, cannot be compared directly to those available in the literature.[67] They

also uses coefficients $\{\delta_{lj}\}$ from the approximation of the investment function to define the persistent component of productivity.

[67]See, e.g., Davis, Haltiwanger, Jarmin, and Miranda (2006), Campbell, Lettau, Malkiel, and Xu (2001), and others. Davis, Haltiwanger, Jarmin, and Miranda (2006) compute measures of volatility and dispersion of the employment and sales growth rates of publicly traded and privately held firms. Dispersion reflects

differ both in the data sources and in the assumptions underlying the methodology used. More importantly, given these differences the values obtained elsewhere in the literature might not be used for the purposes of this paper either.[68]

12 Parameterization

This section summarizes all the choices of the functional form and parameter values in the model, including those estimated in the micro data directly in Section 11. I compare statistics from the model to the aggregate statistics for the manufacturing sector reported in 11.

Figure 6: Parameterization

	Ecuador	Chile	Source
γ	1	1	range
β	0.95	0.95	range
δ	0.12	0.08	INEC-Ecuador and INE-Chile
ψ	0.008	0.008	EMBI, GDF, BIS
	[0.004,0.017]	[0.004,0.017]	
χ	0.30	0.24	INEC-Ecuador and INE-Chile
R_w	2.6%	2.6%	Average lending rate
ν	1	1	range
α	0.36	0.32	INEC-Ecuador and INE-Chile
σ_η	0.16	0.21	INEC-Ecuador and INE-Chile
τ	0.12	0.10	target $\kappa^{Ecuador} = 0.97$, $\kappa^{Chile} = 0.7$

Preferences: γ, β. Preferences are assumed to be of standard CRRA form.[69] In the benchmark parameterization, γ is set to 1 with logarithmic period utility of workers and

year-to-year between-firm variation in growth rates, whereas volatility refers to within-firm variation. The employment-weighted standard deviation of privately held firm growth rates is reported between 0.6 and 0.85.

[68]The issue of using the estimates available in the literature in specific models is more general, as the match between parameter estimates in the literature and the model's input requirements seldom exists.

[69]The assumption of Epstein-Zin preferences would not affect the qualitative findings in this paper. At the same time, this preference specification would require additional parameter values. They would have to be taken from the literature, as the data available do not allow estimating them directly.

entrepreneurs. The discount factor for both workers and entrepreneurs β is set to 0.95.[70] I perform a sensitivity analysis with respect to both of these parameters, for a range of acceptable values.

Technology and shocks: α, σ_η^2, and δ. Using previously obtained estimates, the share of capital in the individual and aggregate production technology α is set at 0.36 for Ecuador and 0.32 for Chile. The variance σ_η^2 parameterizes the distribution of the productivity shocks, and it is taken as 0.16 for Ecuador and 0.21 for Chile. The depreciation rates of physical capital are set to their estimated values of 0.12 and 0.10 for Ecuador and Chile respectively.

Foreign bank lending schedule: ψ and R_w. Recall the lending schedule specified in Section 8.5:

$$R = \begin{cases} \mathrm{R}_w + (\exp\{\psi \mid \frac{B}{Y} \mid\} - 1) & \text{if } B < 0 \\ \\ \mathrm{R}_w & \text{if } B \geq 0 \end{cases}$$

The parameter of interest here is ψ. I choose the range of values of ψ between 0.004 and 0.017. This range includes estimates from the Bank of International Settlements (Altunbaş and Gadanecz (2003)) obtained in the regression of log of drawn *fees* on macroeconomic and microeconomic variables.[71] In particular, the coefficient on aggregate debt-to-GDP ratio from this regression is reported to be positive[72] and statistically significant with the point estimate of 0.008. The shape of the lending schedule for the point estimate of ψ and its upper and lower bounds is given in Figure 7. I take an interval around this estimate and do robustness checks. For the base rate R_w, I take the sum of the average lending rate for manufacturing loans[73] of 2.6%. I also choose other values of R_w to check the sensitivity of the quantitative findings.

Population structure: χ. The fractions of workers and entrepreneurs are determined in the same plant-level data. "Entrepreneurs" are defined as non-wage earners and include company/plant owners and family workers. "Workers" are defined as waged and salaried

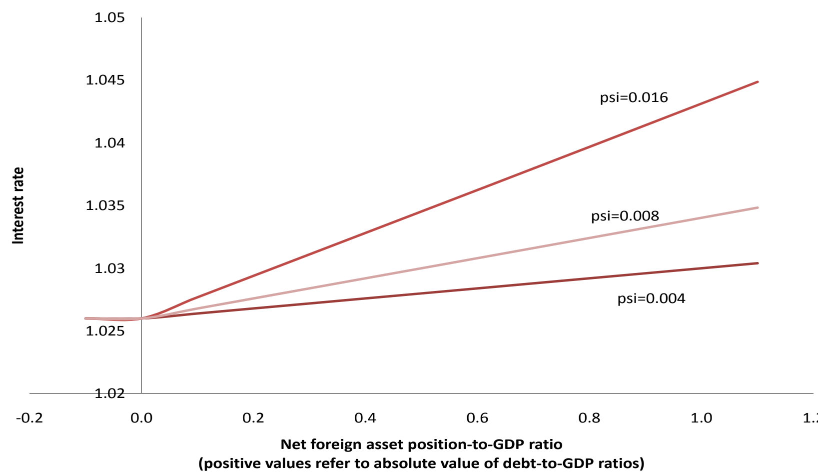
[70] Assuming a growth rate for the economy of 2.2%, and taking the standard value of $\beta = 0.98$, the "detrended" value of β is given by $\frac{0.98}{(1+0.02)^1} \approx 0.95$.

[71] Altunbaş and Gadanecz (2003) consider a sample of 5,000-plus loans to developing countries and estimate their econometric model on this sample. They do not report the results by sector, but together manufacturing and financial sectors account for the large fraction of these loans.

[72] This means that higher fees are associated with higher debt-to-income ratios.

[73] Reported by Altunbaş and Gadanecz (2003)

Figure 7: Foreign Lending Schedule



earners, which include

(i) Blue- and white- collar workers (production/non-production)

(ii) Sales personnel on commission

The fractions of entrepreneurs in Ecuador and Chile then are given by 0.3 and 0.24 respectively.

Time endowment: ν. In the benchmark parameterization, the time endowment of workers is set to 1, and I use a range of values for the endowment of time of entrepreneurs, $\nu \in [0, 1]$. With $\nu > 0$ entrepreneurs have an additional source of income, which enters the right-hand side of their budget constraint. The general form of this budget constraint is given by

$$c_t^i + k_{t+1}^i + b_{t+1}^i + x_{t+1} \leq \pi_t^i + R_t b_t^i + R_{xt} x_t^i + w_t n_t^i.$$

In terms of the welfare effects, with positive time endowment, entrepreneurs would also be able to benefit from higher wages. The overall effect for entrepreneurs in this case is the net of the welfare gains associated with higher wages and the losses associated with changes in entrepreneurial returns.[74] For different values of their time endowment, entrepreneurs

[74]Recall that changes in wages and returns have the opposite sign with the percentage change in wages

would still experience a welfare loss as shown in Section 13.3. The welfare loss is non-decreasing in the time endowment of the entrepreneur.

Public equity share: $1 - \kappa$. I use the values reported previously in Table 11. Public equity shares are set to 0.03 and 0.30 for Ecuador and Chile, respectively.

Public equity cost: τ. Reliable direct estimates of public equity costs are not available for either country, so I use the model for indirect inference instead. In particular, I construct measure of relative size of public equity both in the data and in the model as:

$$\kappa = \frac{K}{K + X}$$

Then using the values of the vector of parameters $\{\alpha, \sigma_\eta^2, \delta; \beta, \gamma; R_w, \psi; \chi\}$ and the steady state system of model relationships I choose τ to match the values of κ in the manufacturing sector of the two countries: $\kappa^{\text{Chile}} = 0.7$ and $\kappa^{\text{Ecuador}} = 0.97$.

13 Quantitative Results

13.1 Model and Data

With preferred parameterization from Table 6 in Section 12, Table 8 reports the steady-state values of ratios in the data and those generated by the model. The model matches capital-to-output and investment-to-output ratios in the aggregate micro data reasonably well when τ is calibrated to the fraction of aggregate capital in the publicly traded sector. The next subsection reports quantitative findings from the counterfactual experiment of reduction in public equity costs performed in this calibrated framework.

13.2 Counterfactual Experiment

Consider the following counterfactual experiment of reducing proportional cost τ of operating a publicly traded firm. Start with the steady state of Ecuador's economy with high proportional cost $\tau = \overline{\tau}$. Let $\overline{\tau}$ in Ecuador decrease overnight to its Chilean level, $\overline{\tau}$. Follow the path of Ecuador's economy with $\underline{\tau}$ to its new steady- state, and assess associated changes in aggregates and individual welfare.

smaller than the change in returns.

Figure 8: Model and data: steady-state comparison

	Ecuador		Chile	
	Model	Data	Model	Data
Aggregate ratios				
Capital-output	1.24	1.29	1.47	1.45
Investment rate	0.13	0.16	0.10	0.11

13.3 Quantitative Assessment of the Experiment

The quantitative results in this section are reported for the overnight reduction in public equity costs in Ecuador of $\approx$ 15%. In terms of the aggregate dynamics, in the period after this reduction aggregate capital stock, wages, output, debt-to-income ratios all increase, and converge slowly from above to their new steady-state values. In particular, in the long run the capital stock increases by 5.4%, and the corresponding increase in output constitutes $\approx$ 1.9%. This is also the magnitude of the permanent increase in output per capita. Given the observed gap in income levels in the two countries, this increase can be considered rather small. Same conclusion applies to the contribution of the gap in financial development (proportional public equity costs) to income differences more generally. In terms of the effects on prices, a reduction in the public equity costs increases the wages of the workers by $\approx$1.9%. This increase in wages leads to reduction in the returns of entrepreneurs in their privately held firms by $\approx$ 3.2. These aggregate effects in turn translate into the following welfare outcomes.

Assuming the same level of wealth, the welfare loss of entrepreneurs from a reduction in public equity costs amounts to about 9.9% in lifetime consumption equivalents. At the same time, the gain for the workers constitutes about 3.2% in lifetime consumption equivalents.

The comparative statics with respect to other parameters in the model suggest that entrepreneurial welfare losses are non-increasing in parameter ψ in the foreign lending schedule and in the amount of entrepreneurial risk σ_η^2. More generally, it might be interesting to quantify the effects of cross-country differences in the values of these parameters as well.

14 Conclusions

The cross-country and panel regressions have contributed to our understanding of the relationship between financial and economic development by providing a broad set of correlations between the two. The regression approach, however, does not distinguish between different sources of financial development and does not quantify their contribution. It also cannot determine associated distributional effects. Other approaches in the literature that have been better suited for these purposes include microeconomic studies and quantifiable economic models.

The microeconomic studies have directly tested the mechanisms suggested by the theory using the industry/company/plant level data. Their examples include Rajan and Zingales (1998) and Demirguç-Kunt and Maksimovic (1998). In particular, Demirguç-Kunt and Maksimovic (1998) use firm-level data to test whether greater financial development removes constraints to profitable growth opportunities of the firms. Using a similar approach, Rajan and Zingales (1998) test whether industries with higher requirements for external finance grow faster in economies with better developed equity markets and quantify the effect. Unlike microeconomic studies, the economic modeling approach relates primitives, such as preferences, technologies, and so on, and policies to quantifiable predictions. It allows one to conduct *controlled* experiments where *natural* experiments are either unavailable or few.

This paper uses an explicit economic model to design a realistic policy experiment of exogenous reduction in public equity costs, and evaluate its quantitative aggregative and distributive impact. It determines the magnitude of this exogenous reduction by choosing two sets of parameter values, one to match the economy of Chile with low public equity costs, and another to match the economy of Ecuador, where public equity costs are high. In this way, the reduction in public equity costs in Ecuador is disciplined, and its effects on the aggregates and individual welfare can be evaluated as close to a natural policy experiment. In particular, reduction in public equity costs produces differential welfare effects for workers and entrepreneurs. Entrepreneurs who would have been expected to benefit from this reduction are actually made worse off, while the workers are better off. Increase in the aggregate output associated with these distributional effects is consistent with statistical relationships established in the regression analyses, however, the distributional effects are overlooked in this approach. Treating Ecuador and Chile as representative of the countries with different levels of financial development this experiment might be of interest to policymakers more broadly.

15 References

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16 Appendix A: Variables Construction

To exploit in estimation the panel structure of the data, the values of the variables in current prices need to be converted into constant prices. For this purpose, I follow the deflation procedure using the aggregate price indices suggested by Liu (1991).[75]

The variables used in estimation include:

- real value added
- real capital stock
- labor input
- real gross investment

I first define the variables in **current prices**. At the firm level, output is given by the sum of the following components in current prices:

- production of items for sale (+)
- resale without processing (−)
- cost of resales (+)
- income from other activities and services (+)
- production of assets for own use (+)
- change in inventories of goods in process (end-of-period minus beginning-of-period value)

Intermediate consumption in current prices is the sum of:

- primary goods and auxiliary materials (+)
- main operating and production costs (cost of goods sold plus operating and other administrative expenditures) (+)
- other operating and production costs (+)

Then, value added in current prices = nominal output in current prices - intermediate consumption in current prices.

Value of the capital stock is the sum of:

- machines and equipment (+)
- buildings and structures (+)

[75] This procedure has been used by Liu (1991) in application to the data from the Chilean Manufacturing Census.

- transport (+)
- office equipment (+)
- other capital goods (+)

Gross investment for each capital asset is constructed as the value of new and used investment net of the value of assets sold in current prices (including replacement of depreciated assets).

Variables in constant prices

The values of variables in constant prices have been constructed using aggregate deflators from the input-output tables. The input-output tables use the two-digit sectoral classification of the system of national accounts (SNA). The firms in the dataset report their industry according to the International Standard Industrial Classification (ISIC). I use concordance tables from the National Bank of Ecuador to map the ISIC and SNA classifications.

First, deflators for each of the two-digit sectors in SNA are constructed as the ratio of the value of gross output of the sector in current basic prices to its reported value in constant prices (with year 1995 as the base year):

$$p_j = \frac{\text{gross output in current basic prices}}{\text{gross output in base year constant prices}}$$

Then, the value of output in current prices of each firm is divided by the corresponding output deflator to get the output in constant prices.

The construction of the intermediate consumption deflators for the two- digit sectors is more involved. I first construct expenditure shares, s_{ij}, for each two-digit sectors i. More specifically, sector i's expenditure share on input of sector j is given by the ratio of the total expenditure on input from sector j to the total expenditure on intermediate consumption of sector i:

$$s_{ij} = \frac{\text{total expenditure on input from sector } j}{\text{total expenditure on intermediate consumption of sector } i}$$

The intermediate consumption deflator for SNA industry i, c_i, is constructed as the sum of the gross output deflators, p_j, using expenditure shares, s_{ij}, as weights:

$$c_i \equiv \sum_j p_j s_{ij}$$

The value of intermediate consumption of each firm in sector i in constant prices is in turn obtained by dividing its value in nominal prices by the corresponding intermediate consumption deflator, c_i.

Value added in constant prices for each firm is simply the difference between the gross output of the firm and its intermediate consumption, both in constant prices.

The capital stock in the dataset is reported at its end-of-period value in current prices. To convert these values into constant prices, I use information about capital stock in the initial year, values of investments in current prices in each year, and investment goods deflators.[76] More specifically:

(i) The initial capital stock in constant prices is just the value of capital stock in current prices in the base year (1995).

(ii) New[77] capital investment in constant prices is its value in current prices deflated by the appropriate capital goods deflator.

(iii) Used capital investment in constant prices is its value in current prices deflated by the wholesale price index. The same deflator is used for the value of capital sold and depreciation.

(iv) Net investment in constant prices is new investment in constant prices (from (ii)) + used investment in constant prices - value of capital goods sold in constant prices (from (iii)).

(v) Beginning-of-period value of capital stock in constant prices for each year t is the value of the initial capital stock in (i) + cumulative value in (iv) for all years t - value of depreciation in constant prices.

Employment is constructed as the year-average number of employees, including both white- and blue-collar employees (measured in man-years). This measure does not take into account differences in the efficiency of the two types of workers. To incorporate these differences, efficiency of blue-collar workers can be normalized to 1. Then, white-collar man-years can be converted into efficiency units using the ratio of the total wage bill of

[76] I construct separate deflators for four categories of capital goods: machines and equipment, buildings and structures, transport, and office equipment and other capital goods.

[77] The investments into new and used capital goods are usually reported separately.

the white- to blue-collar workers. The sum of the two gives an alternative measure of the total employment in efficiency units.

17 Appendix B: Proofs

Lemmas 1-4 are auxiliary, used in different stages of the characterization and not formulated as separate results in the body of the paper.

Lemma 1. *Under assumption of constant returns to scale:*

$$n_t = n(\eta_t, w_t)k_t \text{ and } \pi_t = (R_k(\eta_t, w_t) + 1 - \delta)\, k_t,$$

where $n(\eta, w) = arg\ max_l[F(\eta, 1, l) - wl]$, $R_k(\eta, w) = max_l[F(\eta, 1, l) - wl]$ *are labor input and profit per unit of capital, respectively.*

Proof:

From the first order conditions:

$$w_t = A_t(1-\alpha)(\frac{n_t}{k_t})^{-\alpha}$$

$$\frac{n_t}{k_t} = \left(\frac{A_t(1-\alpha)}{w_t}\right)^{1/\alpha}$$

$$n_t = \left(\frac{A_t(1-\alpha)}{w_t}\right)^{1/\alpha} k_t$$

And defining

$$n(A_t, w_t) \equiv \left(\frac{A_t(1-\alpha)}{w_t}\right)^{1/\alpha}$$

$$n_t = n(A_t, w_t)k_t$$

$$w_t = n(A_t, w_t)^{-\alpha} A_t(1-\alpha)$$

Linearity of profit function:

$$w_t n_t = n(A_t, w_t)^{-\alpha} A_t (1-\alpha) n(A_t, w_t) k_t = n(A_t, w_t)^{1-\alpha} A_t (1-\alpha) k_t$$

$\Rightarrow$

$$r(A_t, w_t) k_t = n(A_t, w_t)^{1-\alpha} A_t k_t - n(A_t, w_t)^{1-\alpha} A_t (1-\alpha) k_t = n(A_t, w_t)^{1-\alpha} A_t \alpha k_t$$

which is linear in k_t. $\square$

Lemma 2. *Given prices, optimal consumption, investment in private and public equity, and bond holdings are linear in wealth:*

$$c_t = (1 - \varsigma_t)\omega_t$$

$$k_{t+1} = \varsigma_t \phi_t \omega_t$$

$$\bar{b}_{t+1} \equiv b_{t+1} + x_{t+1} = \varsigma_t (1 - \phi_t)\omega_t$$

where ς_t *is the savings rate, and* ϕ_t *and* $1 - \phi_t$ *are portfolio shares of risky and risk-free assets defined by:*

$$\phi_t = arg\ max_{\varphi \in [0,1]}\ E_t[\varphi_t R_k(\eta_{t+1}, w_{t+1}) + (1 - \varphi_t) R_{t+1}]^{1-\gamma} \tag{23}$$

$$\rho_t \equiv\ E_t[\phi_t R_k(\eta_{t+1}, w_{t+1}) + (1 - \phi_t) R_{t+1}]^{1-\gamma} \tag{24}$$

$$(1 - \varsigma_t)^{-1} = 1 + \beta^{\frac{1}{\gamma}} \rho_t^{\frac{1}{\gamma}} (1 - \varsigma_{t+1})^{-1} \tag{25}$$

Proof:

Let $\omega_t \equiv R_{kt} k_t + R_t b_t + R_{xt} x_t$.

The Bellman equation for the entrepreneur's problem is given by

$$V(\omega; t) = \max_{c,k,b,s}\{u(c) + \beta \mathrm{E}_t V(\omega'; t+1)$$

s.t.

$$c + k' + (b' + x') = \pi + Rb + R_x x$$

$$\pi = F(k, n, \eta) - wn + (1 - \delta)k$$

$$c \geq 0,\ \ k' \geq 0,\ \ b' + x' \geq -k'$$

Then propose the following solution:

$$V(\omega; t) = \frac{(a_t \omega_t)^{1-\gamma}}{1-\gamma} \tag{26}$$

$$c(\omega; t) = (1 - \varsigma_t)\omega_t$$

$$k(\omega; t) = \varsigma_t \phi_t \omega_t$$

so that

$$(x_{t+1} + b_{t+1})(\omega; t) = \varsigma_t (1 - \phi_t)\omega_t$$

FOC with respect to x_{t+1}, k_{t+1}, and b_{t+1} are given by

$$E_t R_{xt+1} \lambda_{t+1} = \lambda_t$$

$$E_t R_{t+1} \lambda_{t+1} = \lambda_t \tag{27}$$

$$E_t R_{kt+1} \lambda_{t+1} = \lambda_t \tag{28}$$

In equilibrium:

$$R_{t+1} = R_{xt+1}$$

Then combining (27) and (28), and using the recursion for individual wealth,

$$\omega_{t+1} = \omega_t(\phi_t R_{kt+1} + (1-\phi_t)R_{t+1}), \tag{29}$$

one gets an equation to solve for the fraction share ϕ_t:

$$\mathrm{E}\{R_{kt+1} - R_{t+1}\}(\phi_t R_{kt+1} + (1-\phi_t)R_{t+1})^{-\gamma} = 0 \tag{30}$$

Note that since this equation is independent of any individual characteristics, all agents will choose the same portfolio fraction ϕ.

Using the envelope condition:

$$V'(\omega_t;t) = U'(c_t)$$

or equivalently,

$$a_t^{1-\gamma}\omega_t^{-\gamma} = c_t^{-\gamma}$$

Multiplying (27) by $1-\phi_t$ and (28) by ϕ_t and summing them up, one gets the following:

$$\beta(1-\varsigma_{t+1})^{-\gamma}\mathrm{E}(\phi_t R_{kt+1} + (1-\phi_t)R_{t+1})^{1-\gamma} = (\frac{1-\varsigma_t}{\varsigma_t})^{-\gamma} \tag{31}$$

Defining:

$$\rho_t \equiv \mathrm{E}(\phi_t R_{kt+1} + (1-\phi_t)R_{t+1})^{1-\gamma}$$

$$\beta(1-\varsigma_{t+1})^{-\gamma}\rho_t = (\frac{1-\varsigma_t}{\varsigma_t})^{-\gamma} \tag{32}$$

or

$$1 - {\varsigma_t}^{-1} = 1 + (1-\varsigma_{t+1})^{-1}\rho_t^{1/\gamma}\beta^{1/\gamma}$$

Then applying recursion:

$$\varsigma_t = [1 + \left(\sum_{\tau=t}^{\infty} \prod_{j=t}^{\tau} \beta^{1/\gamma} \rho_j^{1/\gamma} \right)^{-1}]^{-1}$$

Then verify that $c_t > 0$, $k_{t+1} > 0$ and $b_{t+1} + x_{t+1} > -k_{t+1}$. And last, verify that (26) solves the Bellman equation. $\square$

Theorem 1. Consider two economies with the same vector of parameter values of preferences, technology, and population structure facing the world interest rate, R_w. Let the initial distribution of wealth of entrepreneurs $\{k_0^i, x_0^i, b_0^i\}_{i\in[0,\chi]}$ be the same, but public equity costs different with $\overline{\tau} > \underline{\tau}$. Then, in the economy with $\underline{\tau}$ entrepreneurs are worse off, while workers are better off.

Proof:

Recall the entrepreneurial budget constraint:

$$c_t^i + k_{t+1}^i + (b_{t+1}^i + x_{t+1}^i) \leq \eta_t^i (k_t^i)^{\alpha} (n_t^i(\eta_t^i))^{1-\alpha} - w_t n_t^i(\eta_t^i) + R_t(b_t^i + x_t^i)$$

where $R_t = R_w$ in the case of the small open economy, and define

$$\omega_t^i \equiv \eta_t^i (k_t^i)^{\alpha} (n_t^i(\eta_t^i))^{1-\alpha} - w_t n_t^i(\eta_t^i) + R_t(b_t^i + x_t^i)$$

The decision rules of the entrepreneur are given by:

$$c_t^i = (1 - \varsigma_t)\omega_t^i,\ k_{t+1}^i = \varsigma_t \phi_t \omega_t^i,\ \bar{b}_{t+1}^i \equiv b_{t+1}^i + x_{t+1}^i = \varsigma_t(1 - \phi_t)\omega_t^i$$

where ς_t is the savings rate out of total resources of entrepreneur ω_t^i, and ϕ_t and $1 - \phi_t$ are portfolio shares of private equity and safe investments in public equity and bond, respectively.

At date 0, since $\{x_0^i, k_0^i, b_0^i\}_{i\in[0,\chi]}$ are the same, so are X_0, K_0, and B_0.

With the same supply of labor, $1 - \chi$, the capital-labor ratio in both economies at date 0 is the same, and so are wages w_0. Therefore, date 0 total resources of entrepreneurs with the same realization of the shock η_0^i, $\{\omega_0^i\}_{i\in[0,\chi]}$, would be the same.

Consider an economy with low public equity cost $\underline{\tau}$. Let $\{c_t^i, k_{t+1}^i, \bar{b}_{t+1}^i\}_{t=0}^{\infty}$ be a sequence chosen by individual i, which satisfies his budget feasibility and gives the maximum utility level given the initial resources $\{\omega_0^i\}$.

For both economies it must be true that $\forall t > 0$:

$$R_w = \frac{F_{kt+1} + 1 - \delta}{1 + \tau}.$$

With $\underline{\tau} < \overline{\tau}$, and constant world interest rate for each $t > 0$: $F_{kt+1}(\underline{\tau}) > F_{kt+1}(\overline{\tau})$. This implies that for each $t > 0$, $w_t(\overline{\tau}) < w_t(\underline{\tau})$.

With $\{w_t(\overline{\tau})\}_{t=0}^{\infty} < \{w_t(\underline{\tau})\}_{t=0}^{\infty}$, and same initial distribution $\{k_0^i, x_0^i, b_0^i\}_{i \in [0,\chi]}$ it must be true that the sequence chosen by entrepreneur i in the economy with low public equity cost $\underline{\tau}$ would cost strictly more than in the economy with high public equity cost $\overline{\tau}$ due to higher wages. Therefore, the budget constraint of the entrepreneur in the economy with high public equity cost will be slack, and extra available resources can be used to increase the consumption of the entrepreneur in the economy with high public equity cost.

Thus, the entrepreneur in the high public equity cost economy can improve on the sequence of entrepreneur in the economy with low public equity, and must be better off. $\square$

Lemma 3.

Assuming that the steady state exists, a vector of parameters $\{\alpha, \sigma_\eta^2, R_w, \psi, \beta, \gamma, \chi\}$ of technology, foreign bond supply curve, preferences, and population structure, uniquely determines the aggregate steady-state quantities and prices as a function of the public equity cost τ.

Let $\lambda = \{\alpha, \sigma_\eta^2, \mathrm{R_w}, \psi, \beta, \gamma, \chi\}$.

Determination of the steady-state equilibrium:

(i) For given values of parameters, the safe interest rate in the steady state of the open economy R and the portfolio share of private equity, ϕ, are uniquely determined by the system:

$$E_\eta \left(\frac{\eta}{\bar{\eta}} \phi R_k + (1 - \phi) R \right)^{-\gamma} \left(\frac{\eta}{\bar{\eta}} R_k - R \right) = 0 \tag{33}$$

$$\varsigma(\phi R_k + (1 - \phi) R) = 1, \tag{34}$$

where $\varsigma = \beta^{\frac{1}{\gamma}} \rho^{\frac{1}{\gamma}}$ and $\rho \equiv E \left(\frac{\eta}{\bar{\eta}} \phi R_k + (1 - \phi) R \right)^{1-\gamma}$.

With $R_k = R(1 + \tau)$, equation (33) defines ϕ as a function of R given λ, $\phi(R; \lambda)$. Substituting $\phi(R; \lambda)$, equation (34) is a function of R only given λ.

(ii) Given parameters of the bond supply curve (R_w, ψ), the ratio of the net foreign asset-to-income, $\frac{B}{Y}$, is determined by:

$$R = R_w + (\exp\{\psi \mid \frac{B_t}{Y_t} \mid\} - 1)$$

with R^{open} from (i).

This result follows from the equilibrium condition that values $(R, \frac{\text{B}}{\text{Y}})$ must be on the foreign bond supply curve.

(iii) For given values of technological parameters in λ, and R^{open} from (i), the steady-state wages, labor-capital ratio, and return to aggregate private equity are given by

$$nw = nw(R^{\text{open}}; \lambda) = \left(\frac{R(1+\tau) - 1 + \delta}{\bar{\eta}\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$w = w(R^{\text{open}}; \lambda) = (1-\alpha)\left(\frac{R(1+\tau) - 1 + \delta}{\bar{\eta}\alpha}\right)^{\frac{-\alpha}{1-\alpha}}$$

$$R_k = R_k(R^{\text{open}}; \lambda) = R(1+\tau)$$

where $\bar{\eta} = \int_\eta \eta^{\frac{1}{\alpha}} g(\eta) d\eta$.

This result follows from the fact that, in equilibrium, returns to riskless foreign bond and riskless public equity must be the same.

(iv) The steady- state capital stocks X, K and the fraction of aggregate capital allocated to the publicly traded sector, $1 - \kappa = \frac{X}{K+X}$, are uniquely determined by the system:

$$K + X = nw^{-1} \tag{35}$$

$$\frac{K}{X+B} = \frac{\phi}{1-\phi} \tag{36}$$

Equations (35)-(36) is the system of two linear equations in two unknowns, K and X. Thu,s its solution is unique, and so is the value of $\kappa = \frac{K}{K+X}$.

Lemma 4.

Investment of entrepreneurs in their privately held firms is an increasing function of the realization of the productivity shock.

Proof:

Recall, investment in the entrepreneurial firm is given by

$$i_t = k_{t+1} - (1-\delta)k_t$$

where $k_{t+1} = \phi_t \varsigma_t \omega_t$ is determined in the solution to the entrepreneur's problem in (11). Substituting for k_{t+1} and simplifying:

$$i_t = \phi_t \varsigma_t (R_{kt}(\eta_t, w_t)k_t + R_t(b_t + x_t)) - (1-\delta)k_t =$$

$$\phi_t \varsigma_t (R_{kt}\eta_t k_t + R_t \frac{1-\phi_t}{\phi_t} k_t) - (1-\delta)k_t$$

$$\varsigma_t k_t (\phi_t R_{kt}\eta_t + R_t 1 - \phi_t) - (1-\delta)k_t$$

$$k_t(\varsigma_t(\phi_t R_{kt}\eta_t + R_t 1 - \phi_t) - (1-\delta))$$

$$i_t = k_t \left[\varsigma_t(\phi_t R_{kt}\eta_t + R_t(1-\phi_t)) - (1-\delta)\right] \tag{37}$$

The right-hand side of (37) defines investment function $i(k_t, \eta_t)$. It is continuous and strictly increasing in η_t for given k_t. Therefore, function $i(k_t, \cdot)$ is invertible:

$$\eta_t = \frac{1}{R_{kt}\phi_t}\left([\frac{i_t}{k_t} + 1 - \delta]\frac{1}{\varsigma_t} - (1-\phi_t)R_t\right)$$

□

18 Appendix C: Alternative Procedure for the Estimation of Productivity Shocks

18.1 Collinearity

One concern about the two-stage estimation technique using OP/LP is the identifiability of the coefficient on labor in the first stage of estimation. The non-parametric function for investment is given by $\tilde{i}_{it} = f(\tilde{\eta}_{it}, \tilde{k}_{it})$, so that $\tilde{\eta}_{it} = f^{-1}(\tilde{i}_{it}, \tilde{k}_{it})$. Even with no specific functional form assumptions on the production function, the most obvious formulation for the demand for labor input is also a function of $\tilde{\eta}_{it}$ and $\tilde{k}_{it}$, i.e. $\tilde{n}_{it} = h(\tilde{\eta}_{it}, \tilde{k}_{it})$.[78] This means that $\tilde{n}_{it}$ does not vary independently of the non-parametric function being estimated leading to collinearity and potential non-identifiability of coefficient on labor β_l. Several approaches have been proposed to address the issue of collinearity. Most prominently, by Wooldridge (2005), Ackerberg, Caves, and Fraser (2005), and Gandhi, Navarro, and David (2009). Below I describe two of the methods for getting around collinearity in Wooldridge (2005) and Gandhi, Navarro, and David (2009).

The approach suggested by Wooldridge (2005) appropriately modified for our purposes uses the following estimating equations and the orthogonality conditions:

Equation 1:

$$\tilde{y}_{it} = \tilde{\beta}_0 + \beta_k \tilde{k}_{it} + \beta_n \tilde{n}_{it} + g(\tilde{i}_{it}, \tilde{k}_{it}) + \tilde{u}_{it}. \tag{38}$$

with orthogonality condition:

$$E\{\tilde{u}_{it} | \tilde{n}_{it}, \tilde{k}_{it}, \tilde{n}_{it-1}, \tilde{k}_{it-1}, ..., \tilde{n}_{i1}, \tilde{k}_{i1}\} = 0 \tag{39}$$

Equation 2:

$$\tilde{y}_{it} = \tilde{\beta}_0 + \beta_k \tilde{k}_{it} + \beta_n \tilde{n}_{it} + \tilde{v}_{it}. \tag{40}$$

where $\tilde{v}_{it} = \tilde{\eta}_{it} + \tilde{u}_{it}$ with orthogonality condition:

$$E\{\tilde{v}_{it} | \tilde{k}_{it}, \tilde{n}_{it-1}, \tilde{k}_{it-1}, ..., \tilde{n}_{i1}, \tilde{k}_{i1}\} = 0 \tag{41}$$

In other words, in (39) and (41) one can use the contemporaneous variable for capital,

[78] Where f and h may not be the same functions.

and any lags and functions thereof of the labor input as instrumental variables. One can also add further proxy variables. In particular, in (38) the lag of labor input can be used as an instrument for $\tilde{n}_{it}$ to deal with the issue of collinearity. This allows to identify the coefficient on labor input β_n and the coefficients of non-parametric function $\beta_k \tilde{k}_{it} + g(\tilde{i}_{it}, \tilde{k}_{it})$. As mentioned previously specifying the input demand equation allows one to separate $\tilde{\eta}_{it}$ from $\tilde{u}_{it}$ in $\tilde{v}_{it}$, and obtain estimates of productivity $\tilde{\eta}_{it}$. Equation (40) can be used to estimate coefficient on capital β_k when $\tilde{n}_{it}$ is instrumented with its lag to deal with endogeneity of the labor input.

An alternative approach is suggested by Gandhi, Navarro, and David (2009). They use explicitly the first order condition of the profit-maximizing firm with respect to the variable input traded in the competitive market together with the production function to get around the issue of collinearity. Their system of equations is given by:

$$ln(\frac{w_{it} n_{it}}{y_{it}}) = \beta_n - \tilde{u}_{it} \tag{42}$$

$$\tilde{y}_{it} = \tilde{\beta}_0 + \beta_k \tilde{k}_{it} + \beta_n \tilde{n}_{it} + \tilde{\eta}_{it} + \tilde{u}_{it} \tag{43}$$

where $w_{it} n_{it}$ corresponds to the labor expenditures, and $1 \cdot_{it}$ to the output revenue, where price p_{it} is normalized to 1.

Letting $\tilde{s}_{it} = ln(\frac{w_{it} n_{it}}{p_{it} y_{it}})$ and $\tilde{x}_{it} = (\tilde{n}_{it}, \tilde{k}_{it})$, the above system can be expressed as:

$$\begin{pmatrix} \tilde{s}_{it} \\ \tilde{y}_{it} \end{pmatrix} = \Upsilon(\tilde{x}_{it}, \tilde{\eta}_{it}, \tilde{u}_{it})$$

For any realization of the data $D_{it} = (\tilde{s}_{it}, \tilde{y}_{it}, \tilde{k}_{it}, \tilde{n}_{it})$ and value of the parameter vector (β_k, β_n), one can uniquely solve the two equation system for the two unobservables for the econometrician $\tilde{\eta}_{it}$ and $\tilde{u}_{it}$, as it is a simple triangular system in $(\tilde{\eta}_{it}, \tilde{u}_{it})$. To estimate the parameter vector, one can use the following moment conditions: $E\{\tilde{u}_{it}\} = 0$ to estimate coefficient β_n on labor, and $E\{\tilde{\eta}_{it} \tilde{k}_{it}\} = 0$ to get the coefficient on capital, since capital is assumed to be a fixed input into the production function. Note that the system in two equations is needed here to be able to identify not only the coefficients on the fixed and variable inputs, but also to separate $\tilde{\eta}_{it}$ and $\tilde{u}_{it}$ from each other, as we want to know both the distribution of productivities $\tilde{\eta}_{it}$ as well as the coefficients of the production function

β_n and β_k.

18.2 Robustness

There are several potential sources of bias in the estimation originating from the adopted assumptions in the model, that need to be addressed with respect to their effect on the obtained estimates:

(i) Assumption of zero exit probability;

(ii) Assumption of i.i.d. productivity shocks;

(iii) Semi-parametric estimation with productivity shocks only a function of current firm investment and capital stock, $\tilde{\eta}_{it} = g(\tilde{k}_{it}, i_{it})$

(iv) Assumption of no investment adjustment costs

These are addressed in turn.

(i) The fact that firm's probability of exit is non-zero leads to downward bias in the estimate of the coefficient on capital, β_k. This is the case due to the negative correlation between capital stocks and productivity, as firms with larger capital stocks will be better suited to survive very low productivity realizations than their counterparts with smaller capital stocks. This negative correlation in turn would lead to a downward bias.

(ii) Persistence in productivity shocks would result in an upward bias in the estimate of the capital coefficient, as it would confound effect of capital on output with the effect that higher productivity realizations today would lead to expectation of better productivity realization tomorrow and larger capital investment to take advantage of this good productivity draw. This positive correlation between $\tilde{k}_{it+1}$ and $\tilde{\eta}_{it}$ which would be ignored in the second stage of estimation under assumption of i.i.d. shock would result in a positive bias in the capital coefficient.

Thus with assumptions (i) and (ii) resulting biases work in the opposite direction, so that the estimate of the coefficient in the model which abstracts from these assumptions may produce an estimate that is close to the actual. As shown in Olley and Pakes (1996) also, the point estimates of the upwardly biased capital coefficient in the model with idiosyncratic shocks and the downwardly biased coefficient from the model with no exit are very close to each other.

Assumptions $(i) - (ii)$ above primarily affected estimates of the coefficient on capital obtained in the second stage of OP/LP estimation procedure. Assumption (iii) may potentially affect both estimates of the labor input coefficient in the first and capital coefficient in the second stage of estimation. In (iii) investment demand is assumed to be a function of capital and productivity, so that unobserved productivity in turn can be expressed as a function of observables - capital and investment.

(iii) The assumption that investment demand can be expressed as a function of capital and productivity leads directly to the estimate of the labor coefficient in the first stage of the estimation procedure, and this in turn affects the second stage estimation where the new variable net of labor's contribution is formed to estimate coefficient on capital input.

In particular, if the estimated coefficient b_n in the first stage is not the true coefficient β_n, the new dependent variable, $\tilde{y}_t - b_n \tilde{n}_t$, would not be free of the effect of labor on output, leaving $(\beta_n - b_n)\tilde{n}_t$ unaccounted for. Then if one were to regress the "net" output $\tilde{y}_t - b_n \tilde{n}_t$ on labor $\tilde{n}_t$ with coefficient γ_n, the resulting estimate would be statistically significantly different from 0 signalling that the investment function was misspecified.

Since $\tilde{n}_t$ endogenously responds to $\tilde{\eta}_t$, to test the investment function assumption one would instrument $\tilde{n}_t$ with $\tilde{n}_{t-1}$, which is a valid instrument: (i) $\tilde{n}_t$ is highly correlated with $\tilde{n}_{t-1}$, and (ii) $\tilde{n}_{t-1}$ is a static input (does not have dynamic implications), and thus is not correlated with $\tilde{\eta}_t$. In Olley and Pakes (1996) the estimate of γ_n above is not significant, and the other coefficients are barely changed from their previous values.

(iv) Introduction of adjustment costs is meant to capture lumpiness, spikes, zero investment levels and other features of the data on investment. By themselves, zero investment observations affect only efficiency of estimation, as they have to be omitted from the first stage. Consider the general investment cost function $c(i_t)$ where $i_t = g(k_t, \eta_t)$. Let there be a maximal level of investment $\bar{i}_t$ above the cut-off productivity level $\bar{\omega}_t$. This could imply, for example, that investment costs are infinite above this level of investment. Then, in the first stage of estimation only part of productivity realization would be recovered using the mapping between investment and productivity. As a result, the source of endogeneity would not be eliminated completely. Part of the error term which cannot be inverted would still be correlated with labor input. The same problem would not arise in the case of convex adjustment cost function, so introducing convex adjustment function would not affect the estimation. More specifically, non-convex adjustment costs are important to slow down the response of investment to productivity shocks by more than is possible with convex adjustment costs. Also, they are important to generate negative investment spikes, which would not be produced by convex adjustment costs. They also help to increase the frequency of inaction - i.e. zero investment observations.