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A SIMULTANEOUS HEDONIC MODEL OF PRICES AND ASSESSMENTS OF SINGLE FAMILY HOMES+

by

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I. INTRODUCTION

Property tax reform in various states and localities has heightened the need to provide both theoretical and empirical models which will predict what the impacts of reform might be (i.e., who will benefit and who will lose). In this paper the focus is on one type of tax reform - the elimination of intentional or unintentional over or underassessment of properties. While the model and empirical data to be used are for single-family homes, the method to be proposed could be extended to other forms of property as well.

Earlier studies or models with a similar purpose (e.g. Berry and Bednarz [2] and Oldman and Aaron [9]) have been proposed to look for or measure "systematic inconsistencies"[1] in property tax assessment. In most studies the assumption is that tax reform will replace the current assessment procedure with one in which assessments are a constant proportion of market value.[2] Hence, their purpose is to identify those people or properties who will stand to benefit (i.e., receive lower assessment) or lose (i.e., receive higher assessment) as a result of a change in the assessment procedure.

In this paper an attempt will be made to develop a model which will not only identify the properties (e.g. new homes or those in a black neighborhood) that will benefit or lose from a change but which will be able to measure the dollar effect (i.e., change in price) on individual properties of such a change. This latter aspect will be accomplished with a model which can measure the capitalized value of current over or underassessment that would be eliminated by tax reform. Put another way, the model will measure the current (capitalized) value which homeowners are willing to pay to obtain a house with a low assessment.

The capitalization feature of the property tax has only recently been analyzed starting with the work of Mieszkowski [7] . He suggested that variations in tax rates (e.g., across localities or activities) would be capitalized into the value of assets. Most extensions of this theory have focused on urban interjurisdictional differences in the property tax on a single-family home, or the Tiebout Hypothesis. For example, assuming an equal distribution of local public services and a proportional property tax, Hamilton [5] has developed a theoretical model which suggests that it is a difference in tax rates relative to the public sector benefits (or fiscal surplus), not just differences in taxes, which are capitalized in property values or prices. Though various theoretical models have been developed based on different assumptions, there has been little econometric testing to verify the existence of such capitalization effects on property values. In this paper a model will be proposed which provides the means of estimating the capitalized value of intentional or unintentional under or overassessment of properties in a single jurisdiction which might likely be eliminated with tax reform.

Special consideration will be given to estimating the capitalization effects of the "limited market value" system being used in Minnesota. This system, which is a deliberate form of over and underassessment,[3] has recently been overturned in the state courts. Consequently, legislation has been passed to eliminate the system though no attempt has been made to measure, as will be done in this paper, capitalization effects of such a change on property values.

II. A SIMULTANEOUS HEDONIC MODEL

The model to be offered in this section is an extension of the hedonic price technique (e.g., Griliches [4] and Rosen [10]) which has been applied to housing markets in many studies (e.g., Ball [1], Goodman [3], Kain and Quigley [6], Nelson [8], and Witte, Sumka and Erekson [12]). In these studies the dependent variable has been the price (P) of a home or property with the independent variables being various characteristics or attributes of the home or property. While various theoretical issues remain unsettled concerning the technique, the general interpretation is that the coefficients (for each attribute) are "shadow prices" which measure the stream of returns from each characteristic of the home. The attributes considered usually are housing or structural characteristics (e.g., square feet, number of bathrooms) and neighborhood or locational characteristics (e.g., accessibility, environmental quality and school quality), the latter having been obtained from census information for the tract or block in which a property is located. Hence, the hedonic model is of the general stochastic form:

(1) $$P = f(H, N, e_1)$$

where H and N are vectors and e_1 is a random disturbance term satisfying the usual assumptions. Various functional forms (e.g., linear or log-linear) have been estimated, usually on the basis of which provides a better fit since there is no firm theoretical basis for any particular specification.

The hedonic method has also been applied to assessor records with assessment (A), rather than price (P), as the dependent variable in a few studies (e.g., Berry and Bernarz [2] and Sumka [11]). These models attempt to find the characteristics, or independent variables (e.g. H or N), which determine or explain the assessed value of a property (A). While there is some question as to whether such a model, or hedonic equation, satisfies the theoretical "market" conditions underlying the hedonic approach, the coefficients are interpreted analagously to those in equation (1). Again, the functional form may vary in estimation but the general hedonic assessment equation would be:

(2) $$A = f(H, N, T, e_2)$$

where H, N are vectors defined earlier, though the elements may differ in each equation, and e_2 is a random distrubance as noted earlier. T is a vector of factors (e.g., transfer history of property or "limited market" status) which may be considered in arriving at the assessed value (A), but are generally not determinants of price (P) in the market. It is by comparing the coefficients for various independent variables (e.g., age of house) in the two equations[4] that Berry and Bednarz [2] measured the effects of tax reform in Chicago.

In estimating the system of equations, (1) and (2), Berry and Bednarz [2] used Ordinary Least Squares (OLS) since each equation has only exogenous variables (H,N,T) on the right-hand side. This model might be considered the reduced form of the simultaneous equation model which will now be proposed in which P and A (or AL, AB) are considered endogenous, or simultaneously determined, variables. This simultaneious system takes the following general form:

(3) $$P = f(A, H, N, e_3)$$

(4) $$A = f(P, H, N, T, e_4)$$

where all variables are as previously defined, e_3 and e_4 are again random disturbances, and H, N, and T are vectors of characteristics or attributes.

The system, (3) and (4), will be estimated using Two-Stage-Least-Squares (TSLS), an appropriate econometric procedure provided the equations are overidentified. Using the order condition equation (3) is overidentified by the vector T and (4) will be overidentified since N will contain more elements in (3) than in (4).[5] In the next section both OLS and TSLS estimates of (3) and (4) will be provided for comparison purposes though the TSLS results are "correct."

Equation (4) may be termed the hedonic assessment, or assessor's, equation and determines factors which influence or determine assessed value (A). It can be used to "look" for systematic inconsistencies in a jurisdiction. Specifically, if proportional assessment was being used, then P would be the only significant variable in (4). In fact, if the assessor could somehow instantaneously assess each property at its current selling price (P), the fit would be perfect (i.e. $R^2 = 1$), though the regression coefficient for P in equation (4) might be less than 1 if the assessor were to deliberately assess properties at a fraction of their selling price (e.g., coefficient would be .50 if assessment were 1/2 of selling price).

If other variables (e.g. H or T) in equation (4) prove significant, this would indicate some sort of systematic inconsistency (over or underassessment). For example, if the age variable of a home (one element of vector H) were to be positive in equation (4) this would indicate that, given a selling price (P), a house will be assessed more the older it is. The coefficients for other variables, aside from P, in equation (4) can be used in a similar way to detect other forms of systematic inconsistency. Furthermore, the coefficients may be used to determine in terms of dollars of assessment (A), what effects tax reform might have on an individual property.

Equation (3) is the hedonic price, or market, equation and determines which attributes affect the market price (P) of a house or property. As noted in the last section, many studies have estimated an equation like this using various independent variables. The specific variables (vectors H and N) to be used in this paper will be described in the next section. What makes the specification of (3) in this paper unique is the inclusion of A as a right-hand variable. It is expected that the coefficient for A will be negative[6] if homeowners have capitalized the value of over or underassessment into the market price of the property. That is, a negative coefficient for A in equation (3) would verify that properties sell (P) at a premium the lower their assessed value (A), all other things (H,N) equal. Furthermore, the size of the coefficient for A in equation (3) would measure the extent of such capitalization. Many factors would effect the size of the coefficient including the number of years and certainty with which purchasers expect to maintain their low assessment in the future and the rate of discount. The longer and more certain the underassessment (i.e., the lower the probability of tax reform) and the lower the discount rate, the larger would be the coefficient for A in equation (3). However, to confirm these propositions would require time series data in order to have adequate variation in uncertainty and the discount rate. Finally, a two variables will be included in (3) to represent the "limited market" status of properties to determine in a similar way if such status has any capitalized value (i.e., price higher if property has "limited market" status) which will be lost as a result of tax reform in Minnesota (i.e., elimination of "limited market" system).

III. ESTIMATION RESULTS

In order to test the simultaneous hedonic model as specified in the last section, a geographically (by census tract) stratified random sample of 200[7] single family homes sold in Duluth, Minnesota in November, 1978 was taken. Information on price (P) and housing characteristics (H) were obtained from Multiple Listing Service books for Duluth and were verified by the Duluth City Assessor's office. The assessor's records also provided specific information on the tax and transfer history (T) of each property. Finally, the neighborhood characteristics (N) for each property were obtained from 1970 census data for the tract in which the property was located. The various elements of the vectors H, N and T are listed in Table 1 along with the names assigned to these variables in the regression results. The variable D, distance from downtown, is a measure of accessibility which has been included as a part of N. This variable, along with the other elements of N and H have been widely used in hedonic models of the

TABLE 1: LIST OF VARIABLES

Endogenous Variables	
P	Price of house
A	Total assessed (market) value of home
Housing (H) Characteristics Vector	
AGE	Number of years since built
LOTSZ	Lot size (square feet)
HSESZ	House size (square feet)
DHMSTD	= 1 if homestead status as of November, 1978
GAR	Number of car garage
BATHS	Number of baths
DGAS	= 1 if gas heat
DFBSMT	= 1 if finished basement
DGARB	= 1 if garbage disposal
Neighborhood (N) Characteristics Vector[1]	
INC	Median income
SINGUN	Proportion of structures which are single family
MIG	Proportion of people in tract who resided elsewhere in 1965
DIS	Distance to downtown
Tax Assessor's (T) Characteristics Vector	
CHANGE	Number of times sold since 1974
LASTS	Month of last sale (January, 1974 = 1, November, 1978 = 59)
LASTA	Month of last on-site assessment (January, 1974 = 1, November, 1978 = 59)
LIMIT	= 1 if home has limited market status as of November, 1978
DIFF	Difference between A and limited market value

[1]These characteristics are for census tract in which house is located.

housing market. Most of these models also include racial and ethnic variables as part of N but since these are all near zero for the census tracts in Duluth such variables are not included. The tax assessor (T) vector includes not only transfer history elements but "limit market" status variables. The former elements are included in equation (4), while the latter appears in equation (3).

Since the model being tested is a system of simultaneous equations, with P and A the endogenous variables, it is necessary to estimate the system using TSLS, as explained in the last section. Along with the TSLS results, Table 2 provides the corresponding OLS results for comparison purposes. Specifically, where differences (e.g., in signs for coefficients of independent variables) exist, the TSLS are considered "correct" while the OLS results would be misleading since such a procedure considers all right-hand variables exogenous. While Table 2 provides comparable OLS and TSLS for each equation and independent variable, R^2 values are only provided for the OLS results since the TSLS procedure does not provide comparable values.[7]

Before evaluating the model as a whole, in this section the estimation results for each equation will be examined in turn. An important aspect of the model proposed is that the price equation (3) can hopefully provide a way of estimating the capitalization effects of the property tax which may be eliminated by tax reform. The negative coefficient for A in equation (3) does suggest the market will value a home higher, the lower its assessment, given other things equal. Furthermore, the correct sign (negative) for the coefficient of A in equation (3) is only found when TSLS is used (i.e., coefficient of A in equation (3) is incorrect (positive) when OLS is used). While this change in signs between OLS and TSLS is important, it must be noted that these variables have low

TABLE 2: ESTIMATION RESULTS

Estimation Method:	Ordinary Least Squares (OLS)		Two-Stage-Least Squares (TSLS)	
Dependent Variable:	Price (P)	Assessment (A)	Price (P)	Assessment (A)
Equation:	(3)	(4)	(3)	(4)
Endogenous Variables:	Regression Coefficients[1] (t values in parentheses)			
Price (P)		.12		1.27
(t - value)		(1.91)		(5.72)
Assessment (A)	.18		-.024	
(t - value)	(.88)		(-.02)	
House (H) Variables:				
LOTSZ	.25	.25	.27	-.15
(t - value)	(2.44)	(4.20)	(1.09)	(-1.69)
HSESZ	17.93	14.13	21	-10
(t - value)	(2.71)	(3.98)	(.97)	(-1.87)
AGE	-26	-44	-32	-15
(t - value)	(-.93)	(2.76)	(-.56)	(-.97)
BATHS	9086	-2400	8828	-12713
(t - value)	(2.67)	(-1.11)	(2.21)	(-4.68)
GAR	1611	2086	1993	-72
(t - value)	(.85)	(1.86)	(.55)	(-.07)
DGAS	-1290		-1195	
(t - value)	(-.49)		(-.44)	
DGARB	1673	5501	2662	1432
(t - value)	(.46)	(2.59)	(.76)	(.71)
DFBSMT	-633		-478	
(t - value)	(-.20)		(-.14)	
Neighborhood (N) Variables:				
INC	1.65	.75	1.93	-1.32
(t - value)	(2.08)	(1.62)	(.82)	(-2.34)
SINGUN	-143	33.2	-149	204
(t - value)	(-1.33)	(.50)	(-1.28)	(3.07)
MIG	-188	103	-196	306
(t - value)	(-.93)	(.81)	(-.92)	(2.58)
DIS	5.73	-.95	6.92	-9
(t - value)	(.41)	(-.10)	(.41)	(-1.12)

TABLE 2: (continued): ESTIMATION RESULTS

Estimation Methods:	Ordinary Least Squares (OLS)		Two-Stage-Least Squares (TSLS)	
Dependent Variable:	Price (P)	Assessment (A)	Price (P)	Assessment (A)
Equation:	(3)	(4)	(3)	(4)
Tax Assessor (T) Variables:				
CHANGE		313		5344
(t - value)		(.15)		(2.51)
LASTS		38.7		-60
(t - value)		(.59)		(-.98)
LASTA		-14		-46
(t - value)		(-.44)		(-1.55)
LIMIT	-2671		-2233	
(t - value)	(-.55)		(-.37)	
DIFF	.45		.68	
(t - value)	(1.12)		(.41)	
DHMSTD	3907		3952	
(t - value)	(.81)		(.82)	
R^2	.69	.74		

[1]Each regression equation included a constant (intercept) which has not been provided in this table.

t-values. The low t-values for these and other independent variables are caused, in part, by the presence of multicollinearity, which is a common problem in hedonic estimation using housing data.[8] Equation (3) also contains three other variables (LIMIT, DIFF and DHMSTD) which have capitalization implications. The homestead status variable, DHMSTD, has the expected positive coefficient and is large since this status gives tax advantages with no uncertainty. The variable LIMIT has an unexpected or incorrect sign, perhaps because over 90% of homes in the sample had this status. However, the other variable measuring "limit market" status, DIFF, does have the correct positive sign, indicating that such status has been capitalized into the price of a home. Having established the existence, by correct or expected signs in estimation results, of capitalization effects no attempt will be made to measure the extent (i.e., interpret the magnitude of coefficients) of such capitalization since this would require assumptions with respect to uncertainty and the discount rate, as explained in the last section.

The coefficients for the elements of H in equation (3) are as expected and similar to the results in previous hedonic price studies. Similarly, the elements of N are as expected with the possible exception of DIS. The positive coefficient for DIS is contrary to well-known theoretical urban economic models which are based on accessibility to the center of the city. However, such a positive coefficient has been found in several empirical studies and has recently prompted alternative theories which use different assumptions to derive or develop a model of intraurban location with a positive, rather than negative, rent gradient. For all independent variables in equation (3) the TSLS and OLS results yield coefficients of the same sign with the important exception noted, A.

Turning next to the hedonic assessment equation (4), both the TSLS and OLS estimation results are provided in Table 2. It was explained earlier that the primary interest in equation (4) is that the estimation results can reveal "systematic inconsistencies" in assessment. Overall, such inconsistencies do not appear to be a major concern since P is the most significant (t-value = 5.72) variable in the TSLS estimation results for equation (4). The coefficient for P is greater than one since the constant, or intercept which is not presented in Table 2, is a large negative number. Had the regression been forced through the origin, the coefficient for P would be less than one. However, since P is not the only explanation of A (i.e., R^2 1 for a simple regression A or P), there are some indications of "systematic inconsistency".

Among the housing (H) variables, the negative coefficients for LOTSZ, HSESZ, BATHS and GAR (for TSLS results) would be incorrect if price (P) had not been included (i.e., if equation (2) specification had been used). With price included, the negative coefficients can be interpreted as forms of "systematic inconsistency". That is, given a price, a house will be assessed less the larger its lot, its size, its baths and garage size. One possible interpretation, which could be verified with additional data, is that this means homes of higher quality are assessed more than those of lower quality since the former would sell at a higher price, given any dimensions (LOTSZ, HSESZ, BATHS, and GAR). While no explicit measure of quality of construction has been used (e.g., brick vs. wood), the positive coefficient for DGARB may be serving as a partial proxy for quality. In general, the inconsistencies for the housing (H) variables appear to be inconsequential.

Turning to the neighborhood (N) variables the TSLS estimation results for equation (4), inconsistencies appear to be more significant (i.e., t-values higher than for H variables). The negative coefficient for income (INC) suggests higher assessments in low income neighborhoods, which is similar to the regressive conclusion reached by Oldman and Aaron [9] for the Boston property tax. The positive coefficient for SINGUN suggests there are lower assessments in areas with multiple family dwellings but this cannot be directly confirmed since data were only collected for single-family homes. DIS has a negative coefficient which suggests some tendency for assessments to rise with distance from the city center. The positive coefficient for MIG and CHANGE both suggest, respectively, that neighborhoods and homes which are changing owners are systematically overassessed. Finally, the last two tax (T) variables, LASTS and LASTA, suggest that, if anything, homes which have recently sold or been inspected are assessed less. The first of these variables, LASTS, and CHANGE were included in the model as a means of detecting "spearing"; the practice of increasing assessments on homes when they are sold. However, the coefficients are contradictory (i.e., positive coefficients for CHANGE suggests the presence of "spearing" but the negative coefficient for LASTS denies "spearing" exists) and so no clearcut indication of "spearing" has been detected.

In the discussion of equation (4) the focus has been on the TSLS estimation results since they are "correct" given the simultaneous nature of the model. The corresponding OLS results are presented in Table 2 and while they are similar (in signs) it should be restated that if differences exist, the OLS results may be misleading.

IV. CONCLUSION

In this paper a hedonic model has been offered which is novel because it treats the price and assessment of single-family homes as simultaneously determined, whereas previous hedonic models have not. The primary motivation for this new approach is to be able to establish empirically the existence of property tax capitalization effects which are the focus of recent theoretical studies in public finance. Specifically, it was hypothesised that the market will price a home higher the lower its assessment, given other things equal. This was confirmed in the estimation results when assessment (A) was found to have a negative coefficient in the hedonic price equation (3).

The existence of such effects indicate that changes in assessment practices, a form of tax reform, will not only change current taxes on homes but also change the price of the homes. For example, owners of homes which had been relatively underassessed, whether intentionally or unintentionally, will find their taxes rising and the market price of their home falling as a result of reform. In this paper two specific forms of intentional underassessment in Minnesota, homestead and "limited market" status, were found to have been capitalized in the price of a home and if these systems were to be eliminated through tax reform, this would be reflected in higher prices for such homes.

The other part of the model, the hedonic assessment equation (4), can be used to detect "systematic inconsistencies" which would usually be considered unintentional, rather than intentional, forms of under or overassessment. Some such inconsistencies were found for the data base used to estimate the model and these too would have been capitalized into the price of a home. Should tax reform lead to assessment based solely on market price (i.e., the elimination of "systematic inconsistencies"), then owners of properties which had been over-assessed by such inconsistency will gain twice in that both their taxes will fall and the price of their home will rise because of the capitalization effects.

Given the limited data base, both in terms of variables and observations, the estimation results cannot be considered definitive. However, the results do confirm the simultaneous approach proposed in this paper and suggest that further testing will provide even greater insight as to what the effects of property tax reform might be. At this point, it seems clear that such reform will not only directly effect current taxes but will also lead to indirect changes in price through the capitalization effect. In contemplating property tax reform policymakers and others would be remiss to consider only the former, and not the latter, aspect of their actions.

FOOTNOTES

1. Such inconsistencies might involve systematic over or under assessment of high or low value houses (e.g., Oldman and Aaron [9] found that in Boston assessments were regressive, or relatively lower on expensive homes, while Berry and Bednarz [2] found the opposite to be true in Chicago) or consistent over assessment in certain neighborhoods (e.g., Berry and Bednarz found assessments higher in black neighborhoods of Chicago).

2. The movement to such a system using market value has become possible in recent years through the computerization of assessor's records in many areas (e.g., St. Paul and Minneapolis, Minnesota).

3. The limited market value system started in 1973 for the purpose of easing the effects of inflation. For example, if a house was sold for $50,000 and the assessor's market (or assessed) value for the home was only $30,000, then rather than raise the assessed value immediately to $50,000 it would be done over a period of three years. During this time it would be assessed according to its limited market value until this was eventually (i.e., in three years) raised to the full market value. Unfortunately, the rationale for the system was based on an assumption of constant prices and as a result of inflation, the limited market value never reached the current market value. The system resulted in generally higher taxes for new and larger homes which were not eligible for limited market status.

4. In fact, Berry and Bednarz [2] also had two separate equations for land assessment (AL) and building or improvement assessment (AB), as well as an equation for total assessment (A). The independent variables, or characteristics, used varied from equation to equation with, in general, H attributed to AB and N to AL.

5. An alternative specification, using AL and AB defined in footnote 4, would be a three equation model with P, AL, and AB considered endogenous:

$$(5) \quad P = f(AL, AB, H, N, e_5)$$

$$(6) \quad AL = f(P, HL, N, T, e_6)$$

$$(7) \quad AB = f(P; HB. T, e_6)$$

where HL and HB are subsets of H containing land (e.g., lot size) and building (e.g., house size) characteristics, respectively, and vector T may contain different elements in (6) and (7).

6. On the other hand it is expected that P will be positive in equation (4), or the assessment will increase with selling price, all other things (H, N, T) equal. The simple correlation between P and A is positive and this direct relationship would carry over to equation (3) if it were estimated using ordinary least squares, rather than two-state-least-squares. The latter econometric estimation procedure is appropriate, as explained in the next section, for the simultaneous equation system, equations (3) and (4).

7. Sometimes R^2 values are given for the first stage (reduced form) equations, but since these are not of interest in this paper no information will be provided about them.

8. Multicollinearty can be minimized by selective elimination of independent variables which measure the same thing. While such elimination will generally increase the significance (t-values) of the remaining variables, it will not improve the overall fit (R^2) and, hence, this procedure has not been employed in this paper.

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