Primordial Gravitational Waves Enhancement

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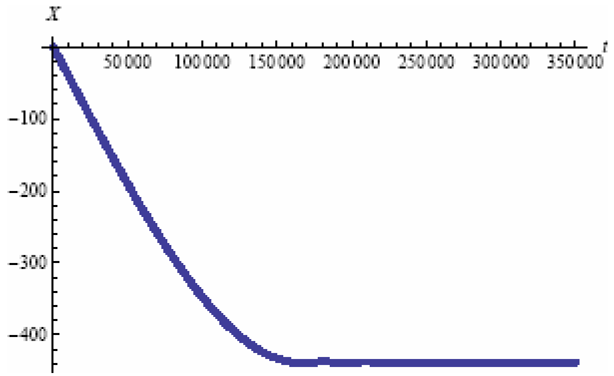
Question: What happens to gravitons in QG inflation?

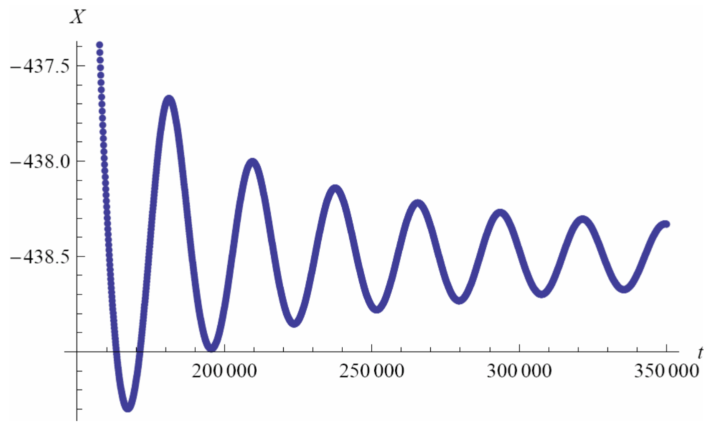
- QG Inflation
  - $G\Lambda \sim 10^{-6}$ starts inflation
  - QG back-reaction stops inflation
- Eff. Field Eqns: $G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}[g]$
  - $T_{\mu\nu}[g] = (\rho+p)u_\mu u_\nu + p g_{\mu\nu}$
  - $p[g] = \Lambda^2 f(-G\Lambda \square^{-1} R)$
    - f(x) grows w/o bound
  - $\rho[g]$ & $u_\mu[g]$ from conservation

Numerical Results for $G\Lambda = 1/300$ and $f(x) = e^x - 1$

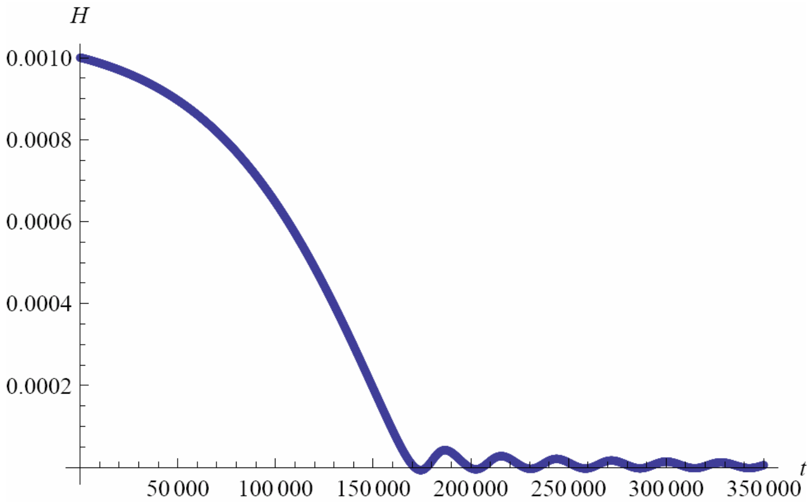
- $X = -\int^t dt'\, a^{-3} \int^{t'} dt''\, a^3 R$
- Criticality

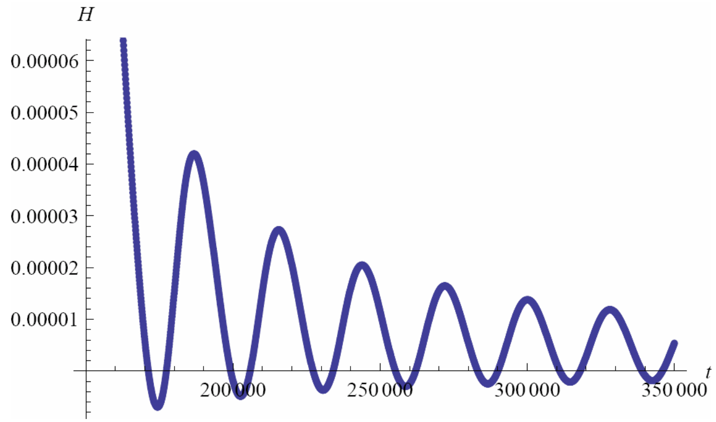
  $p = \Lambda^2 f(-G\Lambda X) = \Lambda/8\pi G$
- Evolution of $X(t) = \Box^{-1} R$
  - Falls steadily to X_{cr}
  - Then oscillates with constant period and decreasing amplitude
  - Generic for any f(x) growing w/o bound

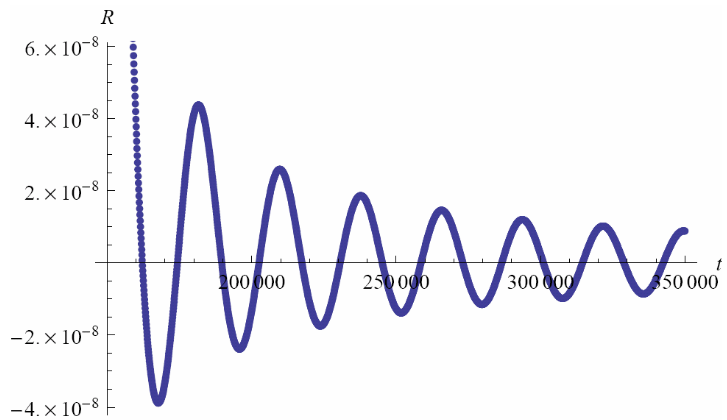




Inflation Ends, H(t) goes < 0, R(t) oscillates about 0







Generic Expansion Histories with only $\omega^2 = 24\pi f_{cr}'(G\Lambda)^2\Lambda$

Near End of Inflation

- $a(t) = a_{cr}\, e^{-N}$
- $H^2(t) \approx \omega^2/9\,(4N + 4/3)$

Brief Oscillatory phase in $\Delta t = t - t_{cr}$

- $a(t) \approx a_{cr}\,[6 + \omega\Delta t + \sqrt{2}\sin(\omega\Delta t)]/6$
- $H(t) \approx \omega\,[1 + \sqrt{2}\cos(\omega\Delta t)]/[6a(t)/a_{cr}]$

Radiation Domination

- $R = 6\dot{H} + 12H^2 \rightarrow 0 \;\Rightarrow\; p = \text{const}$
- $-\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}[g] \rightarrow 0 \;\Rightarrow$ usual evolution

Tensor Perturbations

- Usual eqn for u(t,k)

  $$\partial_t^2 u + 3H \partial_t u + k^2/a^2 u = 0$$

- But H(t) oscillates & goes negative!
  - Hubble "friction" becomes anti-friction
  - Increases $|\partial_t u(t,k)|$
- Effect on $M(t,k) = |u(t,k)|^2$ unclear
  - One solution surely enhanced
  - But what is its coefficient?

Better to evolve $M(t,k) = |u(t,k)|^2$ directly

- $M(t,k) = u(t,k)\, u^*(t,k)$
  - $\partial_t M = (\partial_t u)\, u^* + u\, \partial_t u^*$
  - $\partial_t^2 M = (\partial_t^2 u)\, u^* + 2\, (\partial_t u)\, \partial_t u^* + u\, \partial_t^2 u^*$
- Now use $\partial_t^2 u + 3H\, \partial_t u + (k/a)^2\, u = 0$
  - $\partial_t^2 M = -3H\, \partial_t M + 2(k/a)^2\, M + 2\, (\partial_t u)\, \partial_t u^*$
- Now use $u\, \partial_t u^* - (\partial_t u)\, u^* = i/a^3$
  - Implies $(\partial_t u)\, \partial_t u^* = [(\partial_t M)^2 + a^{-6}]/4M$
- $\partial_t^2 M + 3H\, \partial_t M + 2(k/a)^2\, M = [(\partial_t M)^2 + a^{-6}]/2M$

Great analytic control of M(t,k)

- Before 1st crossing
  - Bunch-Davies ➔ M(t,k) → $1/2ka^2(t)$
  - $2ka^2M = 1 + (1-\frac{1}{2}\varepsilon)(Ha/k)^2 + O(H^4a^4/k^4)$
  - Breaks down for large $\varepsilon(t)$
- During radiation domination
  - Exact solution: M(t,k)

    $= [2ka^2(t)]^{-1} [C^2 \sin^2(k/Ha) + C^{-2} \cos^2(k/Ha)]$
  - C large for $k/a(t) \ll H(t)$ ➔ M ~ const
  - C large for $k/a_{cr} \sim \omega$ ➔ $2ka^2\langle M\rangle \sim$ const

How much energy is there?

- $E(t,k) = \frac{1}{2}a^3\dot{x}^2 + \frac{1}{2}ak^2x^2$
  - $x(t,k) = u(t,k)\alpha(k) + u^*(t,k)\alpha^\dagger(k)$
  - $\alpha(k)|\Omega\rangle = 0$
  - $\langle\Omega|E(t,k)|\Omega\rangle = \frac{1}{2}a^3|\partial_t u|^2 + \frac{1}{2} k^2a|u|^2$
- $|u|^2 = M$ & $|\partial_t u|^2 = [(\partial_t M)^2 + a^{-6}]/4M$

  ➔ $\langle E\rangle = \frac{1}{2}a^3[(\partial_t M)^2 + a^{-6}]/4M + \frac{1}{2}k^2aM$
- Subtract off $\frac{1}{2}\hbar\omega = \frac{1}{2}k/a(t)$

  → $\Delta E = [a^6(\partial_t M)^2 + (1-2ka^2M)^2]/8a^3M$

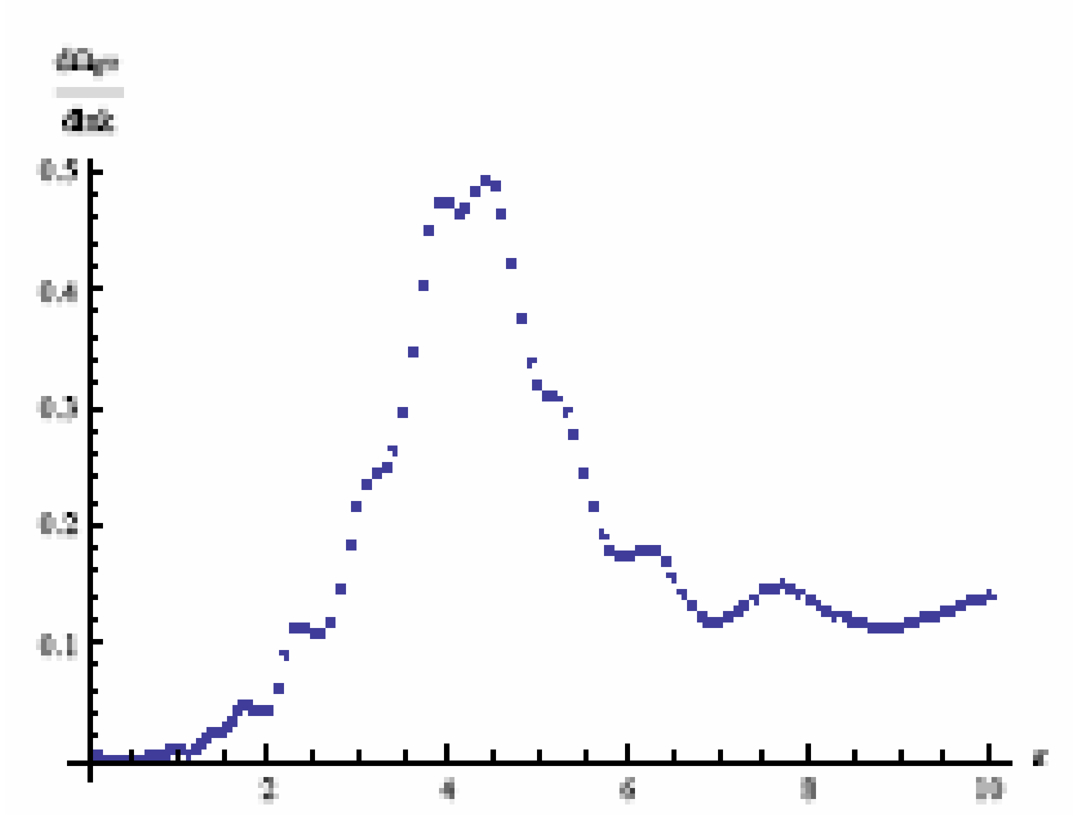
Gravity wave people want $d\Omega_{gw}/d\ln(k)$

- $d\rho(t,k) = 4\pi k^2 dk/[2\pi a(t)]\ \Delta E(t,k)$
- $d\Omega_{gw}/d\ln(k) = [8\pi G/3H^2(t)]\ d\rho(t,k)\ k/dk$

  $= (4/3\pi)\ Gk^4/H^2a^4\ [a^6(\partial_t M)^2+(1\text{-}2ka^2M)^2]/8ka^2M$

  - M-terms ~ constant
  - H^2a^4 constant during radiation domination
  - $1/H^2a^4$ falls like $1/a(t)$ during matter dom.
- $Gk^4/H^2a^4 = G\omega^2\ (k/\omega a)^4\ (\omega/H)^2$

Bump in the gravity wave background



Conclusions

- Oscillations at end of inflation
  - Little effect of cosmological perturbations
  - Pulse of gravity waves at $\omega a_{cr}/a_0 \sim 10^{10}$ Hz
    - Not currently detectable, maybe in future
- Eqn for $M(t,k) = |u(t,k)|^2$
  - Excellent analytic control
  - Works for $Q'' + [k^2 - \theta''/\theta]Q = 0$ with ANY θ
    - Astro-ph/0306602