LABOR MARKETS AND LABOR CONTRACTS IN A

DYNAMIC, GENERAL EQUILIBRIUM MODEL

by

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I. Introduction

According to Robert Hall [1980a], "The greatest recent progress in understanding the labor market comes from the study of long-term employment arrangements. There is no point any longer in pretending that the labor market is an auction market cleared by the observed average hourly wage." (p. 210). Most economists would be willing to accept, I think, the idea that there is more to the recorded payments from employers to their employees than the equating of marginal costs and products at each instant in time. But what are the implications of this idea? Does the fact that agents trade in terms of explicit or implicit long-term contracts render the competitive market equilibrium framework irrelevant, perhaps even providing a solid theoretical foundation for the Keynesian postulate of sticky (nominal?) wages?[1] Or, does the principal implication of contracting turn out to be, as Robert Barro [1977] conjectured, that some frequently discussed aspects of labor market behavior are a "facade," while the salient results of competitive market theory remain valid? These issues will be examined here, and the results will be seen to support Barro's view.

The analysis is carried out in the context of a particular, somewhat novel, class of economic models. Since many of the applications towards which the study of contracts is directed involve business cycle phenomena, it is essential to work with a genuinely dynamic, general equilibrium framework incorporating aggregate uncertainty. There are two types of models meeting these requirements which economists have analyzed: those with infinite horizon decision makers, and more recently, those with overlapping generations of finite horizon decision makers. There are several reasons for working with overlapping generations (OG) models, including their tractability. In the present context, it is also the case that OG models entail a natural heterogeneity which is useful in addressing some issues relating to seniority. On the other hand, while it does seem true that workers perpetually come and go (and are all, in the end, transient) in actual economies, there is this notion that employers are more or less permanent.

Without attempting to explain the evolution of infinite horizon "firms" in a population of finite lived individuals, one way hopefully to capture the impact on the employer-employee relationship is to introduce some infinite lived agents who can be identified as employers into the OG model. This is what is done here. Although formal results do not depend on this partitioning of workers and employers by their planning horizon, it helps to motivate several applications. In fact, the main reason for introducing some (actually, one representative) infinite lived agents is a practical one: it serves to restrict the set of equilibria in such a way as to guarantee that equilibrium allocations will be optimal in an appropriate dynamic sense. At the same time, there is almost no analysis in the literature of models with a combination of infinite and finite lived agents in an OG context. The only example that I am aware of is Wilson [1980], who concentrates on non-stochastic exchange economies. Hence, it will be necessary to proceed prudently with the formal specification before labor market applications can be explored.

The methodology is to begin in Section II by describing an environment; that is, by providing a complete specification of the technology, demography, endowment, information and preference structure for some artificial community. In Section III, competitive (market) equilibrium is defined, and existence and optimality are examined. The notion of (labor) contracts and a precise definition of contract equilibrium are introduced in Section IV. It is shown there that the economy with contracts in addition to markets is equivalent to the economy with no contracts, in the sense that an allocation can be supported as an equilibrium in one economy if and only if it can be supported in the other. One implication is that the existence and optimality of contract equilibria follow directly from the respective results for market equilibria. More significantly, this equivalence means that the introduction of long-term contracting does not change the set of allocations which are attainable as equilibria. However, it will be argued that the introduction of contracts can have dramatic implications for observations. Thus, the same equilibrium allocations can produce some very different looking patterns for wages and securities exchanges, depending on whether agents trade in terms of long-term contracts, or soley through markets. This is the sense in which Barro's conjecture, that contracts produce a "facade" in front of the otherwise accurate predictions of competitive theory, are supported.

Section VI contains some examples. Properties of optimal insurance agreements are characterized for the case of separable workers' preferences and a risk neutral employer, as entailing perfectly smoothed consumption but (potentially large) fluctuations in employment, or "effort," for senior workers. Sample economies are shown to generate equilibria that mimic at least some aspects of business cycles. While competitive real wages are necessarily procyclical here, contract wages (that is, employee compensation per hour worked) can display a wide variety of cyclical behavior since they include, in addition to a payment for productive services, an implicit credit and/or an implicit insurance arrangement.[2] Also, both cross-section and longitudinal wage profiles can become confounded by long-term contracts, for the same reasons.[3] Finally, the model has implications for the observation that some markets, and contingent security markets in particular, appear to be inactive in many actual economies. In (some) contract equilibria, securities do not get traded because they are unnecessary, or redundant, in the presence of long-term employment relationships. Competitive complete market theory predicts accurately the allocations of all commodities (and hence welfare) that obtain in contract equilibrium, even though it can misforecast active securities markets and some aspects of wage behavior over the bussiness cycle and over the life cycle.

II. The Environment

Here I describe in order the technology, demography, endowment, information, and preference structure. That is, I specify the environment. Time is discrete and continues forever: $t = 1,2,\ldots$. At each date t there is (ignoring leisure for the moment) one aggregate consumption good, c, produced via the stochastic aggregate production function $f(n,k;x_t)$, where n is labor input, k is capital (or land) input, and x_t is a random variable described fully below. For each x_t, f is continuously differentiable, concave, strictly increasing, and homogeneous of degree one in (n,k); also, $f(0,0;x_t) = 0$. As a convention, assume f is strictly increasing in x_t.[4]

Call x_t the state of nature at t and summarize its history to date by $H_t = (x_1,x_2,\ldots x_t)$. It will sometimes be convenient to write $H_t = (H_{t-1},x_t)$. For all H_t, x_{t+1} has finite support $\{1,2,\ldots m\}$, with $\mathrm{Prob}[x_{t+1}=j|H_t] > 0$, for $j = 1,2,\ldots m$. In Figure 2.1, each "node" H_t represents a potential course of history up to date t when $m = 2$. It will be useful to enumerate the countable set of nodes "lexicographically" in the general case as indicated for the special case in the diagram; let $\{H_t\}$ be this sequence, and refer to it as the lexicographic ordering of the nodes.[5]

There are two distinct classes of individuals in this environment. The first consists of two period lived overlapping generations of "workers," each endowed with one unit of time in each period of his life to divide between leisure and labor.[6] Let I_t be the set (generation) of workers born at t, and assume

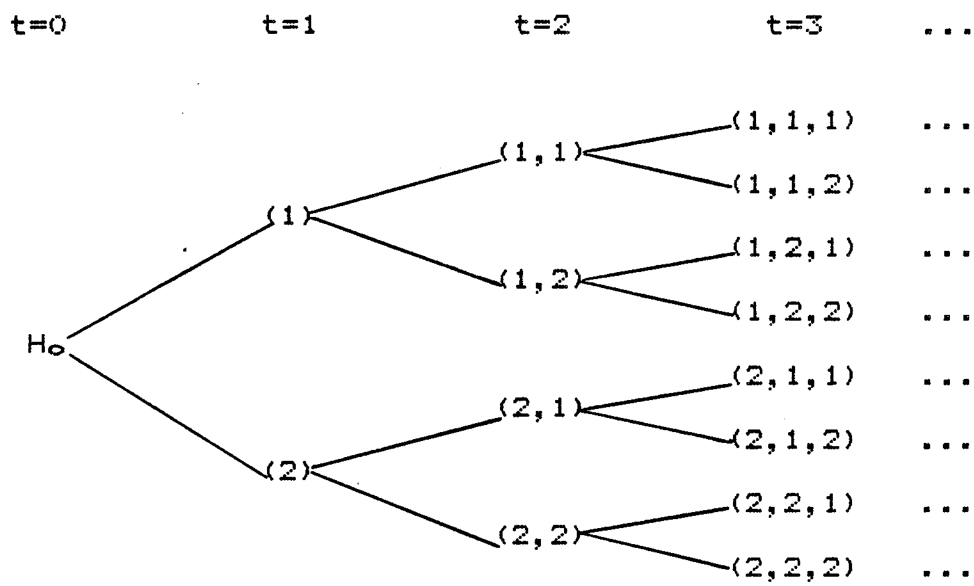

Figure 2.1: The possible evolutions of history from t=1 on, when m = 2. At each date t, there are 2^t t-tuples, $H_t = (x_1,\ldots x_t)$, called nodes. The lexicographic ordering is:

$$\{H_t\} = \{(1),\ (2),\ (1,1),\ (1,2),\ (2,1),\ \ldots\}$$

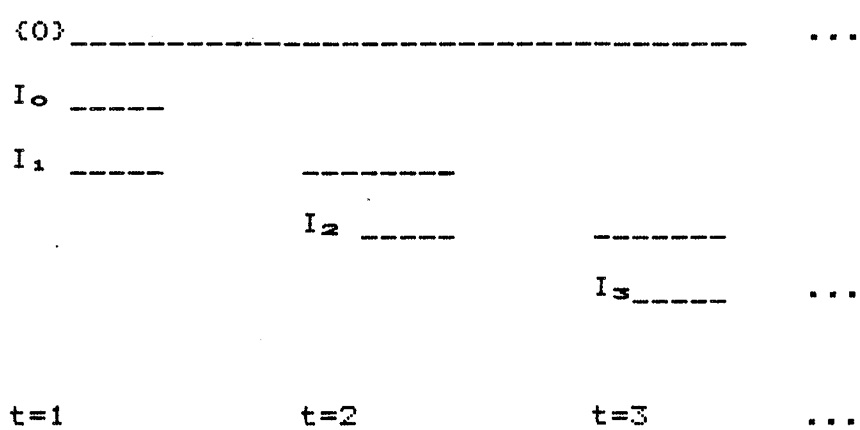

Figure 2.2: The demography: at each date t, the union of the sets in column t is the total population.

that for all t, $0 < \#I_t \leq I$ for some integer I. The second class of individuals consists effectively of a single, infinitely lived "capitalist," endowed with K units of land (or capital) each period, and referred to as agent 0.[7] Thus, for example, the set of individuals alive at date t would be $I_t \cup I_{t-1} \cup \{0\}$, consisting of some young workers, some old workers, and the capitalist (see Figure 2.2). The value of x_t is revealed simultaneously with the appearance of I_t,[8] and the entire history H_t is costlessly observed. Agents have rational expectations in the sense that while they do not know the future course of nature, they do know all prices over every possible course, as well as the transition probabilities $P[j|H_t]$.

Let $c^i(H_t)$ and $l^i(H_t)$ denote consumption and labor supplied by agent i at node H_t. Each worker has preferences over deterministic consumption-leisure pairs throughout his life given by the continuous, strictly increasing except possibly on the boundary of his consumption set, and strictly concave utility function $U^i: Z^2 \to R$, where $Z = R_+ \times [0,1]$ is his set of allowable consumption-leisure pairs at each date.[9] Agent 0 has preferences over deterministic consumption sequences given by the utility function U^0, specialized to

$$U^0(c_1, c_2, \ldots) = \lim_{T \to \infty} \sum_{t=1}^{T} b^t u(c_t),$$

where $u: R_+ \to R$ is continuous, concave, and strictly increasing, and $0 < b < 1$. All agents are assumed to maximize the expected value of U^i subject to the relevant constraints in each of the economies to be studied. This completes the description of the environment.

III. Competitive Equilibria

Here a notion of competitive (market) equilibria is presented. To begin, consider the generic worker i in I_t. At date t, upon observing H_t, he chooses a bundle $z^i(H_t)$ in the set Z^{1+m}, consisting of a current consumption-leisure pair plus a contingent consumption-leisure pair for each of the m possible states at date t+1:

$$z^i(H_t) = [c^i(H_t), 1-l^i(H_t), c^i(H_t,1), \ldots, 1-l^i(H_t,m)]$$

Agent i also chooses at date t a portfolio $s^i(H_t)$ in R^m, where the j^{th} component $s^i_j(H_t)$ is the amount of the security which pays 1 unit of c at t+1 if and only if $x_t = j$. He chooses so as to solve the following problem:

$$\begin{aligned} \text{MAX} \quad & \varphi^i(z^i(H_t)) = \sum_{j=1}^{m} \text{Prob}[j|H_t]U^i[c^i(H_t), \ldots, 1-l^i(H_t,j)] \\ \text{s.t.} \quad & c^i(H_t) \leq w(H_t)l^i(H_t) - q(H_t)s^i(H_t) \\ & c^i(H_t,j) \leq w(H_t,j)l^i(H_t,j) + s^i_j(H_t), \; j=1,\ldots m \\ & z^i(H_t) \text{ in } Z^{1+m}, \quad s^i(H_t) \text{ in } R^m \end{aligned} \qquad ..[3.1]$$

where $w(H_s)$ is the real wage and $q(H_s)$ is the vector of securities prices at H_s. At t = 1, the initial old, I_0, have already made their optimizing choices, and therefore $s^i(H_0), c^i(H_0,1), \ldots, l^i(H_0,m)$ for all i in I_0 are given.

Now consider agent 0. He must choose contingent consumption and portfolio sequences $c^0 = \{c^0(H_t)\}$ and $s^0 = \{s^0(H_t)\}$, running over all nodes H_t enumerated lexicographically. He chooses so as to solve:

$$\text{MAX} \quad \phi^o(c^o) = \sum_{t=1}^{\infty} \sum_{H_t} \text{Prob}(H_t)\ b^t u[c^o(H_t)]$$

$$\text{s.t. } c^o(H_t) \leq Kr(H_t) - q(H_t)s^o(H_t) + s^o{}_{x_t}(H_{t-1}) \qquad \text{...[3.2']}$$

$$c^o(H_t) \geq 0, \text{ for all } H_t = (H_{t-1}, x_t),\ t = 1,2,...$$

where $r(H_s)$ is the rental price of land at H_s. The initial condition $s^o(H_o)$ is given, and assumed equal to $-\sum_{I_o} s^i(H_o)$. The notation $\sum_{H_t}$ means to sum over all nodes possible at t.

It turns out that problem [3.2'] has no solution. This is a standard difficulty with setting infinite lived agents in competitive credit markets: they (believe they) are able to run up arbitrarily large debts without ever being held accountable, each period borrowing enough to refinance existing principal plus interest, or more, if desired. Formally, we have:

Lemma 3.1: Decision problem [3.2'] has no solution.

Proof: Suppose (c^o, s^o) maximizes ϕ^o subject to the recursive budget constraints

$$c^o(H_t) \leq Kr(H_t) - q(H_t)s^o(H_t) + s^o{}_{x_t}(H_{t-1}).$$

But consider the alternative sequences (c^o+c^*, s^o+s^*), where for each $H_t = (x_1,...,x_t)$

$$s^*{}_j(H_t) = -t \cdot \{R(x_1) \cdots R(x_1,...,x_t)\}$$

for $j = 1,2,...,m$, where

$$R(H_s) = \{ \sum_j q_j(H_s) \}^{-1}$$

is the (certain) gross interest rate at s. Using the budget restrictions at equality,

$$c^*(H_t) = (2t-1)\cdot\{R(x_1) \cdots R(x_1,\ldots,x_{t-1})\} > 0$$

is seen to be affordable. By monotonicity, (c^o, s^o) could not have been a solution.

Q.E.D.

There are several ways to modify [3.2'] to ameliorate the "rollover problem" spelled out in this lemma. A natural one is to restrict borrowing so that agents never end up in a state where their debt exceeds the present value of their future contingent savings plan. Define $p_c(H_1) = 1$, and for each $t > 1$ and $H_t = (x_1,\ldots,x_t)$, define

$$p_c(H_t) = q_{x_2}(x_1) \cdots q_{x_t}(x_1,\ldots,x_{t-1}). \qquad \ldots[3.3]$$

Now $p_c(H_t)$ is the cost at date 1 of a certain claim to one unit of c at node H_t, purchased via a sequence of (t-1) one-step-ahead security trades. Hence, it is exactly the present value (at H_1) of one unit of c at H_t. Let $p_c = \{p_c(H_t)\}$ enumerate these terms (lexicographically, as always).

The present value of agent 0's contingent savings plan starting at H_t up to date $T > t$ is:

$$v^o(H_t,T) = \sum_{s=t}^{T} \sum_{H_s \setminus H_t} p_c(H_s)[Kr(H_s) - c^o(H_s)],$$

where the notation $\sum_{H_s \setminus H_t}$ means to sum over all nodes at date s which are possible given H_t.[10] However, at this stage there is no reason to assume, for arbitrary sequences $r = \{r(H_s)\}$ and $q = \{q(H_s)\}$, that this summation converges as T approaches infinity, and so the present value of lifetime savings may not be well-defined.[11] Therefore, it will only be required that the present value of O's debt at any H_t, $-p_c(H_t)s^o(H_t)$, not exceed the limit infimum of this series. His set of allowable choices is then written:

$$\bar{S} = \{s^o : -p_c(H_t)s^o(H_t) \leq \liminf_{T\to\infty} v^o(H_t,T) \text{ for all } H_t\} \qquad ...[3.4]$$

When $p_c > 0$, imposing the side condition that s^o lie in $\bar{S}$ eliminates the rollover problem in [3.2'] if (and only if) liminf $v^o(H_o,T)$ is finite.

The next step is to consider production. Since technology in this environment is essentially static, optimization on the part of the aggregate "firm" involves the standard marginal conditions for profit maximization. Given instantaneous values w_t and r_t, if (n^*,k^*) maximize $f(n,k;x_t) - w_t n - r_t k$, then:

$$\begin{aligned} f_n(n^*,k^*;x_t) &\leq w_t, \quad = w_t \text{ if } n^* > 0 \\ f_k(n^*,k^*;x_t) &\leq r_t, \quad = r_t \text{ if } k^* > 0. \end{aligned} \qquad ...[3.5a]$$

Suppose, however, that instead of renting out his land, agent O operates the production process himself, hiring labor at the competitive wage, using as much of his land as he sees fit, and retaining for his income the surplus, $f(n,k;x_t) - w_t n$. If (n^{**},k^{**}) maximize this surplus then[12]

$$\begin{aligned} f_n(n^{**},k^{**};x_t) &\leq w_t, \quad = w_t \text{ if } n^{**} > 0 \\ k^{**} &= K. \end{aligned} \qquad ...[3.5b]$$

Now in the case where O rents his land out, he will do so inelastically; hence market clearing will require $k^* = K = k^{**}$, and so [3.5a] and [3.5b] then entail $n^* = n^{**}$, also. Thus, employment is the same in either case. Furthermore, an application of Euler's equation yields:

$$Kr_t = Kf_k(n^*,K;x_t) = f(n^{**},K;x_t) - n^{**}f_n(n^{**},K;x_t)$$
$$= f(n^{**},K,x_t) - n^{**}w_t$$

which implies capital (and thus labor) income is the same.

In other words, the two arrangements for production yield the same equilibrium values for employment, output and income. Formally, it is therefore possible to dispense with the notions of firms and a (physical) capital market altogether, and to pose agent 0's problem as a combined saving-consumption-hiring decision. This is the sense in which it is possible to identify 0 as the employer in what follows. To streamline the presentation, let

$$N(H_t) = \underset{n\geq 0}{\operatorname{argmax}}\{f(n,K;x_t)-w(H_t)n\} \qquad \text{...[3.6a]}$$

be the demand correspondence, with generic element $n(H_t)$, and denote the income of agent 0 by[13]

$$w^o(H_t) = \sup_{n\geq 0}\{f(n,K;x_t)-w(H_t)n\}. \qquad \text{...[3.6b]}$$

The optimal hiring decision and the rental price of land will remain implicit in much of the subsequent discussion, and agent 0's problem will be written (note s^o is restricted to lie in $\bar{S}$):

$$\text{MAX} \quad \phi^o$$
$$\text{s.t. } c^o(H_t) \leq w^o(H_t) - q(H_t)s^o(H_t) + s^o x_t(H_{t-1}) \qquad \text{...[3.2]}$$
$$\text{for all } H_t;\ c^o \geq 0,\ s^o \text{ in } \bar{S}.$$

We may now define competitive equilibrium. First, let $z(H_t) = (\ \ldots z^i(H_t)\ldots\ )$ and $s(H_t) = (\ \ldots s^i(H_t)\ldots\ )$ list the consumption-leisure and portfolio plans of generation I_t at H_t, and let $z = \{z(H_t)\}$, $s = \{s(H_t)\}$, $n = \{n(H_t)\}$, $w = \{w(H_t)\}$, $q = \{q(H_t)\}$, and $w^o = \{w^o(H_t)\}$. Then we have:

Definition 3.1: An **overlapping generations competitive equilibrium (OGCE)** is a set of (non-negative) price and quantity sequences (w,q,z,s,c^o,s^o,n) satisfying:

1.Optimization:

(a) for all t, for all i in I_t, for all H_t: given (w,q), $(z^i(H_t),s^i(H_t))$ solves [3.1].

(b) given (w,q,w^o), (c^o,s^o) solves [3.2] (where w^o is defined by [3.6b]).

(c) for all t, for all H_t, $n(H_t) \in N(H_t)$.

2.Market Clearing: for all t, for all H_t,

(a) $n(H_t) = \sum_{I_t \cup I_{t-1}} l^i(H_t)$

(b) $s^o(H_t) = -\sum_{I_t} s^i(H_t)$

(c) $c^o(H_t) + \sum_{I_t \cup I_{t-1}} c^i(H_t) = f(n(H_t),K;x_t)$.

Observe that 1 and 2 are required to hold for all H_t; thus, these conditions must be satisfied for each possible realization of the x_t process. Also, markets are required to clear exactly (strict equalities in 2), but this is unrestrictive due to the monotonicity of preferences. The following is proved in Wright [1983]:[14]

<u>Theorem 3.1</u>: For the specified environment, OGCE always exists.

The welfare properties of competitive equilibria will now be explored. To begin, some terminology must be made precise.

<u>Definition 3.2</u>: An **allocation** is a listing (z, c^o) of the consumption-leisure bundles of each i in I_t, $z^i(H_t)$, for each H_t possible at date t, $(t = 1,2,\ldots)$, plus the consumption of agent 0 at each H_t, plus the values of $[c^i(H_1), 1-l^i(H_1)]$ for the initial old I_o.

<u>Definition 3.3</u>: An allocation (z, c^o) is **feasible** if

$$c^o(H_t) + \sum_{I_t \cup I_{t-1}} c^i(H_t) \leq f(n(H_t), K; x_t),$$

where $n(H_t) = \sum_{I_t \cup I_{t-1}} l^i(H_t)$, for each H_t possible at date t $(t = 1,2,\ldots)$.

<u>Definition 3.4</u>: Given two feasible allocations, (z, c^o) and $(\bar{z}, \bar{c}^o)$, the former **dynamically Pareto dominates** the latter if at each date t, for each i in I_t, for every H_t

$$\phi^i(z^i(H_t)) \geq \phi^i(\bar{z}^i(H_t)),$$

and

$$\phi^o(c^o) \geq \phi^o(\bar{c}^o),$$

with strict inequality for at least one agent.

<u>Definition 3.5</u>: A feasible allocation (z, c^o) is **dynamically Pareto optimal (DPO)** if there exists no other feasible allocation which dynamically dominates (z, c^o).

Observe that this notion of optimality is concerned with the well being of agents <u>given their birth states</u>. An alternative version of optimality would involve the unconditional expected utilities of agents.

<u>**Definition 3.6**</u>: Given two feasible allocations, (z,c^0) and $(\bar{z},\bar{c}^0)$, the former **strongly Pareto dominates** the latter if for each date t, for each i in I_t

$$E[\phi^i(z^i(H_t))] = \sum_{H_t} \text{Prob}[H_t]\phi^i(z^i(H_t))$$

$$\geq E[\phi^i(\bar{z}^i(H_t))] = \sum_{H_t} \text{Prob}[H_t]\phi^i(\bar{z}^i(H_t)),$$

and

$$\phi^0(c^0) \geq \phi^0(\bar{c}^0),$$

with strict inequality for at least one agent.

<u>**Definition 3.7**</u>: A feasible allocation (z,c^0) is **strongly Pareto optimal (SPO)** if there exists no other feasible allocation which strongly Pareto dominates (z,c^0).

DPO is a weaker concept than SPO. If (z,c^0) is <u>not</u> DPO then there exists some reallocation satisfying the inequality on ϕ^0 and the inequalities on every $\phi^i(H_t)$ for each H_t which is possible at t in Definition 4.3 with strict inequality somewhere. This same reallocation thus also satisfies the inequality on ϕ^0 and (averaging over H_t) the inequalities on every $E[\phi^i]$ in Definition 4.5, with strict inequality somewhere, so that (z,c^0) cannot be SPO. Hence, SPO implies DPO. On the other hand, DPO allocations need not be SPO, since when the value of ϕ for two agents varies in different directions across H_t states, by concavity $E\phi$ could be increased for both by averaging their allocations across these states.[15]

The competitive mechanism has little chance of achieving SPO allocations, due to the limited participation inherent in the overlapping generations demography. There is no way for workers to insure themselves through the marketplace against contingencies (such as lower than average wages) which are revealed before they appear in the economy. However, it is shown in Wright [1983] that competiitve equilibria are necessarily DPO.

Theorem 3.2: If (q,w,z,s,c^o,s^o,n) is an OGCE then (z,c^o) is DPO.

This result is interesting because it is well known that competitive equilibria need not be optimal in OG models where all agents have finite horizons. The standard optimality proof (e.g. Debreu [1954]) fails because the value of aggregate wealth is not neccessarily finite in those economies. In the present model, the presence of some infinite horizon decision maker(s) serves to guarantee that the value of aggregate wealth is finite. It is important to emphasize that optimality has nothing to do with the presence of something called "land" or "capital" in the environment. Agent 0 need only be endowed with something of value, such as productive labor time, in all (but a finite number of) periods. Land is not a store of value in these economies, and consequently is <u>not</u> able to play the role that fiat money plays in some OG models (see, e.g., Wallace [1980]).

The fact that competitive equilibria need not be SPO should not be construed as a defect. Both Peled [1980], and Cass and Shell [1983] argue convincinly that DPO is the "correct" optimality criterion for OG environments. Furthermore, as Peled puts it:

> Whenever the optimality criteria used in evaluating allocations is stonger than that used by agents in their maximizing behavior, one should not be surprised that decentralized equilibria turn up to be non-optimal. A larger reward, and perhaps a greater challenge, lies in the extensions of the environments in which the fundamental welfare theorems hold, and the understanding of why these theorems fail in other environments. (p. 89)

IV. Contract Equilibria

A new economic object will now be introduced, the (labor) contract.[16] Agent 0 (the employer) makes the contract offers to the workers, who simply accept or reject; "bargaining" is not analyzed. All agents are bound by the terms of any arrangement they enter into; reneging on contracts is ruled out. The main results will be seen to imply that contract equilibrium is equivalent to competitive market equilibrium in a precise sense, although the obsevations generated by the alternative equilibria can be quite different.

To begin, define a contract offer by 0 to worker i at date t in state H_t to be a vector $y^i(H_t)$ in the set Y^{1+m}, where $Y = R_+ \times [-1,0]$, specifying a current compensation-(minus employment) pair, plus a contingent compensation-(minus employment) pair for each state which is possible at date t+1:[17]

$$y^i(H_t) = [d^i(H_t),-e^i(H_t),d^i(H_t,1),\ldots,-e^i(H_t,m(H_t))],$$

where $d^i(H_t)$ and $e^i(H_t)$ denote contract compensation and contract employment, respectively. Let $a^i(H_t) = 1$ if he accepts, $= 0$ if he rejects the offer, and for future reference, let $y(H_t) = (\ \ldots y^i(H_t)\ldots\ )$ and $a(H_t) = (\ \ldots a^i(H_t)\ldots\ )$ list the offers to and the decisions of generation I_t at H_t.

In addition to accepting or rejecting the contract $y^i(H_t)$, i still has the option of participating in the market place. If he chooses a market consumption-leisure bundle $z^i(H_t)$, his <u>net</u> (market plus contract) bundle will be

$$z^i(H_t) + a^i(H_t)y^i(H_t),$$

constrained to lie in the set Z^{1+m}.[18] He also chooses a portfolio $s^i(H_t)$. His decisions are made to solve the following:

$$
\begin{aligned}
&\text{MAX } \phi^i[z^i(H_t) + a^i(H_t)y^i(H_t)] \\
&\text{s.t. } c^i(H_t) \leq w(H_t)l^i(H_t) + a^i(H_t)d^i(H_t) - q(H_t)s^i(H_t) \\
&\qquad c^i(H_t,j) \leq w(H_t,j)l^i(H_t,j) \\
&\qquad\qquad\qquad + a^i(H_t)d^i(H_t,j) + s^i{}_j(H_t) \\
&\qquad z^i(H_t) + a^i(H_t)y^i(H_t) \text{ in } Z^{1+m}, \\
&\qquad s^i(H_t) \text{ in } R^m,\ a^i(H_t) \text{ in } \{0,1\}.
\end{aligned}
\qquad \text{...[4.1]}
$$

In order to formulate O's decision problem, it must be established how he will come up with contract offers for his workers. In the partial equilibrium treatments in the implicit contract literature, employers attempt to maximize their utility by offering workers compensation-employment packages subject to the constraint that the latter's utilities are not below some fixed level. This level, U*, say, is typically fixed exogenously with an appeal to "market conditions" outside the model.[19] Here, it will be assumed that O maximizes his utility by trading in the markets and/or through bilateral arrangements with workers, subject only to the constraint that a worker will not accept an offer which reduces his utility below what that worker could achieve by trading exclusively in the market place. Let $A^i(H_t)$ be the set of offers acceptable to i in I_t at H_t:

$$
\begin{aligned}
A^i(H_t) = \{y^i(H_t)\text{: } &\sup \phi^i \text{ s.t. constaints [4.1] and } a^i(H_t) = 1 \\
&\geq \sup \phi^i \text{ s.t. constraints [4.1] and } a^i(H_t) = 0\}.
\end{aligned}
$$

It should be emphasized that it is possible for agents to accept contracts and still trade in the market place, "moonlighting," if you will.[20] Let $A(H_t) = (\ldots A^i(H_t) \ldots)$, and let $A = \{A(H_t)\}$. It will be assumed that agent 0, as the employer, chooses sequences for his consumption, portfolio, and market hiring, plus a sequence of contract offers $y = \{y(H_t)\}$ in A. Denote total (market plus contract) labor demand at H_t by

$$n^*(H_t) = n(H_t) + \sum_{I_t} a^i(H_t)e^i(H_t) + \sum_{I_{t-1}} a^i(H_{t-1})e^i(H_t),$$

and the income of 0 at H_t by

$$w^*(H_t) = f(n^*(H_t),K;x_t) - w(H_t)n(H_t) - \sum_{I_t} a^i(H_t)d^i(H_t) - \sum_{I_{t-1}} a^i(H_{t-1})d^i(H_t).$$

Agent 0's choice problem may now be written:

$$\text{MAX } \phi^0(c^0) = \sum_{t=1}^{\infty} \sum_{H_t} \text{Prob}[H_t|H_1]\, b^t u(c(H_t))$$

$$\begin{aligned}
\text{s.t. } & c^0(H_t) \leq w^*(H_t) - q(H_t)s^0(H_t) + s^0 x_t(H_{t-1}) \\
& w^*(H_t) = f(n^*(H_t),K;x_t) - w(H_t)n(H_t) \\
& \qquad - \sum_{I_t} a^i(H_t)d^i(H_t) - \sum_{I_{t-1}} a^i(H_{t-1})d^i(H_t) \\
& n^*(H_t) = n(H_t) + \sum_{I_t} a^i(H_t)e^i(H_t) + \sum_{I_{t-1}} a^i(H_{t-1})e^i(H_t) \\
& c^0,\ n^* \geq 0, \quad s^0 \text{ in } \bar{S}, \quad y \text{ in } A.
\end{aligned} \qquad \ldots[4.2]$$

Notice that market hiring, $n(H_t)$, is not restricted to be non-negative -- only total (market plus contract) hiring is. Also, one can think think of agent 0 as solving [4.2] in two stages: first choosing y and n^* to maximize w^*, and then choosing c^0 and s^0 to maximize ϕ^0 taking income, w^*, as given. These facts will become important below.

It is now possible to define a notion of contract equilibrium:

<u>**Definition 4.1**</u>**:** A **contract equilibrium (CONE)** is a set of price, quantity, and contractual arrangement sequences (w,q,z,s,c^o,s^o,n,y,a) satisfying:

1. Optimization:

(a) for all t, for each i in I_t, for all H_t: given (w,q,y), $(z^i(H_t),s^i(H_t),a^i(H_t))$ solves [4.1].

(b) given (w,q,A), (c^o,s^o,n,y) solves [4.2]

2. Market Clearing: for all t, for all H_t,

(a) $n(H_t) = \sum_{I_t \cup I_{t-1}} l^i(H_t)$

(b) $s^o(H_t) = -\sum_{I_t} s^i(H_t)$

(c) $c^o(H_t) + \sum_{I_t \cup I_{t-1}} c^i(H_t) = f(n^*(H_t),K;x_t)$

$$- \sum_{I_t} a^i(H_t)d^i(H_t) - \sum_{I_{t-1}} a^i(H_{t-1})d^i(H_t).$$

Notice that conditions 2 in this definition are expressed in terms of <u>market</u> supplies and demands. For the consumption good, not all output is available for sale on the market -- some has been previously committed by contract (to some of the employees), and must therefore be subtracted from gross output on the right hand side of 2(c). For labor services, only $\sum l^i$ (respectively, n) is supplied (respectively, demanded) on the market, although potentially more is traded through contracts. By definition, there is no market clearing condition for contract trades.

The next step is to verify the existence of CONE, which turns out to follow directly from the existence and optimality of OGCE.

Theorem 4.1: For the specified environment, CONE always exists.

Proof: By Theorem 3.1, there always exists an OGCE, $(\bar{w},\bar{q},\bar{z},\bar{s},\bar{c}^o,\bar{s}^o,\bar{n})$, for this environment. Call out the same prices $(\bar{w},\bar{q})$. Then each worker can always do as well as he did in OGCE by rejecting all contracts. Hence, any y in A must make all workers no worse off than the allocation (z,c^o). By Theorem 3.2, $(\bar{z},\bar{c}^o)$ is DPO, and no reallocation can make agent 0 better off. Since he can always do as well as in OGCE, simply by offering null contracts, $y = 0$, it follows that $(c^o,s^o,n,0)$ solves [4.2]. Now $(\bar{z},\bar{s},0)$ solves [4.1] for every worker. Since markets clear in OGCE, it follows that $(\bar{w},\bar{q},\bar{z},\bar{s},\bar{c}^o,\bar{s}^o,\bar{n},0,0)$ is a CONE.

Q.E.D.

The CONE constructed in the above proof is a trivial one, in the sense that all exchanges take place through the market (only the 0 contract is ever traded), and the allocation is one which can be supported as an OGCE. That there are CONE with non-zero contracts offered (and accepted) will be demonstrated below. Of immediate concern is whether their exist CONE allocations which cannot be supported as OGCE, and the following answers in the negative:

Theorem 4.2: If (w,q,z,s,c^o,s^o,n,y,a) is a CONE, then (w,q,z^*,s,c^o,s^o,n^*) is an OGCE, where

$$z^{*i}(H_t) = z^i(H_t) + a^i(H_t)y^i(H_t)$$

and

$$n^*(H_t) = n(H_t) + \sum_{I_t} a^i(H_t)e^i(H_t) + \sum_{I_{t-1}} a^i(H_{t-1})e^i(H_t).$$

Proof: Let (w,q,z,s,c^o,s^o,n,y,a) be a CONE.

(i) Suppose that given (w,q), $(z^{*i}(H_t),s^i(H_t))$ does not solve [3.1] for some i at some H_t. Then there is some $(\bar{z}^i(H_t),\bar{s}^i(H_t))$ such that

$$\phi^i(\bar{z}^i(H_t)) > \phi^i(z^{*i}(H_t))$$
$$\bar{c}^i(H_t) \leq w(H_t)\bar{l}^i(H_t) - q(H_t)\bar{s}^i(H_t)$$
$$\bar{c}^i(H_t,j) \leq w(H_t,j)\bar{l}^i(H_t,j) + \bar{s}^ij(H_t).$$

But now $(\overline{z^i(H_t)},\overline{s^i(H_t)},0)$ satisfies the constraints in [4.1] and yields a higher value of ϕ^i than $(z^i(H_t),s^i(H_t),a^i(H_t))$. This contradicts the hypothesis of CONE (condition 1(a) in Definition 4.1), thus establishing that (z^*,s) solves [3.1] for all i at every H_t.

(ii) Now suppose that, given (w,q), n^* does not maximize profit at each H_t. Then there is some $\bar{n}$ such that

$$\begin{aligned} f(\bar{n}(H_t),K;x_t) &- w(H_t)\bar{n}(H_t) \\ &> f(n^*(H_t),K;x_t) - w(H_t)n^*(H_t) \\ &= f(n^*(H_t),K;x_t) - w(H_t)[\,\bar{n}(H_t) + \sum_{I_t} a^i(H_t)e^i(H_t) \\ &\qquad + \sum_{I_{t-1}} a^i(H_{t-1})e^i(H_t)\,] \end{aligned}$$

(using the definition of n^*) at some H_t. Rearranging and subtracting contract compensation from both sides of the inequality,

$$f(\bar{n}(H_t),K;x_t) - w(H_t)[\ \bar{n}(H_t) - \sum_{I_t} a^i(H_t)e^i(H_t)$$

$$- \sum_{I_{t-1}} a^i(H_{t-1})e^i(H_t)\ ] - \sum_{I_t} a^i(H_t)d^i(H_t) - \sum_{I_{t-1}} a^i(H_{t-1})d^i(H_t)$$

$$> f(n*(H),K;x_t) - w(H_t)n(H_t) - \sum_{I_t} a^i(H_t)d^i(H_t)$$

$$- \sum_{I_{t-1}} a^i(H_{t-1})d^i(H_t) = w*(H_t).$$

But the left hand side of the inequality is the profit that results when the same contracts are offered as in the hypothesized CONE, plus an additional

$$\bar{n}(H_t) - \sum_{I_t} a^i(H_t)e^i(H_t) - \sum_{I_{t-1}} a^i(H_{t-1})e^i(H_t)$$

is purchased on the spot market. This cannot be greater than $w*(H_t)$ in CONE, so it follows that $n*$ must in fact maximize profit at each H_t.

(iii) Suppose that given (w,q), (c^o,s^o) does not solve [3.2]. Then there is some $(\bar{c}^o,\bar{s}^o)$ such that

$\phi^o(\bar{c}^o) > \phi^o(c^o)$, and

$c^o(H_t) \leq w^o(H_t) - q(H_t)s^o(H_t) + s^o{}_{xt}(H_t)$

for all H_t, where

$$w^o(H_t) = \sup_{n\geq 0} [f(n,K;x_t) - w(H_t)n] \leq w*(H_t).$$

The last inequality follows from the fact that agent 0 can always choose $y = 0$ in [4.2]. But now $(\bar{c},\bar{s},n,y)$ satisfies the constraints in [4.2] and yields a higher value of ϕ^o, contradicting the hypothesis of CONE. This establishes that (c^o,s^o) solves [4.2].

Summarizing, it has been shown that given (w,q), all workers and agent 0 maximize their utility, and profit is maximized, at the quantities (z^*, s, c^o, s^o, n). It is easily verified that the following conditions also hold:

$$n^*(H_t) = \sum_{I_t \cup I_{t-1}} l^{*i}(H_t)$$

$$c^o(H_t) + \sum_{I_t \cup I_{t-1}} c^{*i}(H_t) = f(n^*(H_t), K; x_t).$$

The conclusion is that $(w, q, z^*, s, c^o, s^o, n^*)$ is an OGCE.

Q.E.D.

An immediate corollary is:

Theorem 4.3: If $(w, q, z, s, c^o, s^o, n, y, a)$ is a CONE then (z, c^o) is DPO.

Proof: This follows directly from Theorem 4.2, and the fact that every OGCE is DPO by Theorem 3.2.

Q.E.D.

In the proof of Theorem 4.1 it was demonstrated how one could construct a CONE from an OGCE. Theorem 4.2 shows how to go in the other direction. Hence, it has been proven that an allocation can be supported as a CONE if and only if it can be supported as an OGCE. In other words, introducing contracts into the economy has not changed the set of allocations attainable as equilibria. But if they have no implications for allocations, just what is the significance of contracts? It turns out that the presence of contract trading can have dramatic implications for the <u>observations</u> generated by the model.

For example, for any given OGCE $(w, q, z, s, c^o, s^o, n)$, one particular CONE is $(w, q, \bar{z}, 0, c^o, 0, 0, y, a)$, where for all i at H_t,

$$\bar{z}^i(H_t) = [0,1,\ldots 0,1]$$

(i.e., i purchases no consumption good and sells no labor time on the market), and

$$a^i(H_t) = 1$$

$$y^i(H_t) = z^i(H_t) - \bar{z}^i(H_t).$$

That this is a CONE can be deduced using an argument similar to the proof of Theorem 4.1. Indeed, this is the opposite extreme from the equilibrium constructed there, since here all trades are bilateral, while there all trades took place on the market.[21] An outside observer of this economy (such as an econometrician) would see worker i supplying $e^i(H_t)$ units of labor and receiving $d^i(H_t)$ units of the consumption good as compensation at H_t. Would it not be reasonable for him to conclude that the real wage being paid to worker i is $w^i(H_t) = d^i(H_t)/e^i(H_t)$?

Yet $w^i(H_t)$, in general, embodies more than a payment for labor services, since the contract is also replacing securities market trading. The competitive market wage, $w(H_t)$, always equals $f_n(n^*(H_t),K;x_t)$ when $n^*(H_t) > 0$. However, due to an implicit credit arrangement between 0 and i, $w^i(H_t)$ can be lower or higher than the competitive market wage depending on whether i is a saver or a debtor at H_t. Additionally, due to an implicit risk sharing arrangement between 0 and i in I_t, $w^i(H_t)$ might deduct an insurance premium, while $w^i(H_t,x_{t+1})$ could be higher than $w(H_t,x_{t+1})$ when the latter is below average, and vice versa.[22] For reasons such as these, the wage dispersion across individuals, as well as the profile of wages for worker i over time, can display patterns which are difficult to explain in terms of the competitive market framework. When all trades are bilateral, the economist never receives any observations on market wages, and it will be a difficult task to unscramble the multiple roles played by long-term employment contracts.

One possibly misleading observation is that, in the CONE constructed above, contingent securities are never traded. There is a crucial distinction to be made between markets which are inactive and markets which are nonexistent. The fact that contingent securities markets are inactive in this CONE has no implications for allocations or welfare; these securities are not traded because they are unnecessary, or redundant, in the presence of long term employment relationships. On the other hand, arbitrarily shutting down the securities markets can effect the equilibrium. If $s^i(H_t) = 0$ is <u>imposed</u> on agents, the economy could still operate, but of course the the resulting maket equilibria would not generally be DPO. This should be expected to have implications for contracting: employers have a greater potential to extract a "rent" from their workers in providing bilateral insurance or credit arrangements when those workers have fewer other (market) opportunities. Shutting down these markets shifts the terms of trade in favor of employers. Hence, even inactive markets, by the very possibility of their operation, can have an important role in the economy.

Now the remarks in the above three paragraphs would increase in significance if there was some reason to suspect a tendency for the equilibrium with exclusively bilateral trading, instead of the original OGCE, to obtain in actual economies. There may be good reasons for such a suspicion. To the extent that the organization and operation of markets is at all costly in the real world, one would expect to see institutions arise which minimize the necessary set of trading arrangements. It seems reasonable to conjecture that we will not observe securities plus spot trading when a small number of bilateral arrangements can support the same allocations, as would be the case when the sets I_t are not too big or too diverse. Although transactions costs have not been modeled here, the resources required for the set up and maintenance of markets have potential as one explanation of the institutional trading arrangements that tend to surface. Such an analysis, however, is beyond the scope of this study.[23]

In any case, the stand taken here is that the important implications of employment contracts are for observations, and not for allocations. This view is quite different from that taken elsewhere; Ito [1980], for example, states that "Examining the reasons why resource allocation deviates from the Arrow-Debreu allocation is the key to understanding the implications of implicit contract theory ..." (p. 10).[24] An exception to this trend in the literature is Robert Barro [1977], who was lead to conjecture the following: "In fact, the principal contribution of the contracting approach to short-run macro-analysis may turn out to be that some frequently discussed aspects of labor markets are a facade with respect to employment fluctuations." (p. 316). Although I do not think the adjective "short-run" is required, the results here clearly lend support to this general position on the implications of contracts. In the next section, examples of the "facades" which may appear are examined in detail.

V. Examples

Here some parametric examples are presented. These not only provide insight into the workings of the model, but demonstrate how some observations that economists have considered in need of explanation emerge naturally as the equilibria of simple, perfectly functioning economies. Applications to real wage movements over the business cycle, as well as some implications for seniority observations, will be discussed. First, the nature of the cycles displayed by the model must be examined.[28]

One enormous simplification is achieved by assuming that agent 0 is risk neutral:

Lemma 5.1: If agent 0 is risk neutral then securities prices in (OGCE or CONE) equilibrium with $c^0 > 0$ are given by

$$q_j(H_t) = b\ \text{Prob}[j|H_t].$$

Proof: Consider decision problem [3.2], underlying OGCE, in the case where 0 is risk neutral (i.e., $u(c) = c$):

$$\text{MAX } \phi^0 = \sum_{t=1} \sum_{H_t} \text{Prob}[H_t|H_1]\ b^t c^0(H_t)$$

$$\text{s.t. } c^0(H_t) \leq w^0(H_t) - q(H_t)s^0(H_t) + s^0_{x_t}(H_{t-1}).$$

Substituting the budget constraints at equality into the objective function, the problem becomes

$$\text{MAX} \sum_{t=1} \sum_{H_t} \text{Prob}[H_t|H_1]\ b^t[w^0(H_t) - q(H_t)s^0(H_t) + s^0_{x_t}(H_{t-1})].$$

Necessary conditions for an interior solution (i.e., one with $c^0(H_t) > 0$ for all H_t) are the "Euler equations"

$$-\text{Prob}[H_t|H_1]\ q_j(H_t) + b\ \text{Prob}[H_t, j|H_1] = 0$$

for all $H_{t+1} = (H_t, j)$. Rearranging and using the definition of conditional probability,

$$q_j(H_t) = b \; \mathrm{Prob}[j|H_t].$$

Since any CONE implies a OGCE with the same prices, by Theorem 4.2, the result also holds in CONE.

Q.E.D.

For the sake of illustration, the case where workers have log-linear preferences will be analyzed in detail, although more general specifications will be returned to. Thus, suppose every i in each generation has preferences over deterministic net (market plus contract) consumption-leisure profiles $(c_1, c_2, 1-l_1, 1-l_2)$ given by

$$U^i = \ln c_1 + \ln [1-l_1] + \ln c_2 + \ln [1-l_2].$$

Considering the case of one representative worker in each generation, and therefore deleting superscripts, by substituting the budget restrictions at equality into the objective function, problem [3.1] becomes:

$$\begin{aligned} \mathrm{MAX}\ \phi = {} & \ln [w(H_t)l(H_t) - q(H_t)s(H_t)] + \ln [1-l(H_t)] \\ & + \sum_{j=1}^{m} \mathrm{Prob}[j|H_t]\{\ln [w(H_t,j)l(H_t,j) + s_j(H_t)] \\ & \qquad + \ln [1-l(H_t,j)]\}. \end{aligned}$$

Assuming that the constraint on leisure being less than or equal to 1 is not binding (the consistency of this assumption may be checked later), necessary conditions for a solution are:

$$[w(H_t)l(H_t) - q(H_t)s(H_t)]^{-1}w(H_t) - [1-l(H_t)]^{-1} = 0$$

$$[w(H_t,j)l(H_t,j) + s_j(H_t)\}]^{-1}w(H_t,j) - [1-l(H_t,j)]^{-1} = 0 \qquad \ldots[5.1]$$

$$- [w(H_t)l(H_t) - q(H_t)s(H_t)]^{-1}q_j(H_t) + [w(H_t,j)l(H_t,j) + s_j(H_t)]^{-1}\mathrm{Prob}[j|H_t] = 0,$$

where $j = 1,2,\ldots m$.

Consider the case with $m = 2$ and transition probabilities:

$$\mathrm{Prob}[j|H_t] = P_j(H_t) = \begin{cases} P_{1j} & \text{if } x_t = 1 \\ P_{2j} & \text{if } x_t = 2 \end{cases}$$

for $j = 1,2$, where $P_{ij} > 0$ and $P_{i1} + P_{i2} = 1$. Thus, x_t follows a two state, time invariant, first order Markov process. Assuming that agent 0 is risk neutral and $c^0 > 0$, by Lemma 5.1 asset prices may be written

$$q_j(H_t) = b\ P_j(H_t) = \begin{cases} b\ P_{1j} & \text{if } x_t = 1 \\ b\ P_{2j} & \text{if } x_t = 2 \end{cases}$$

for $j = 1,2$. Now [5.1] may be solved for the decision rules, or the "demand system:"

$$\begin{bmatrix} l(H_t) \\ l(H_t,1) \\ l(H_t,2) \\ s_1(H_t) \\ s_2(H_t) \end{bmatrix} = \begin{bmatrix} \dfrac{3w(H_t) - bP_1(H_t)w(H_t,1) - bP_2(H_t)w(H_t,2)}{4w(H_t)} \\[2ex] \dfrac{-w(H_t) + b[3+P_2(H_t)]w(H_t,1) - bP_2(H_t)w(H_t,2)}{4bw(H_t,1)} \\[2ex] \dfrac{-w(H_t) - bP_1(H_t)w(H_t,1) + b[3+P_1(H_t)]w(H_t,2)}{4bw(H_t,2)} \\[2ex] \dfrac{w(H_t) - b[1+P_2(H_t)]w(H_t,1) + bP_2(H_t)w(H_t,2)}{2b} \\[2ex] \dfrac{w(H_t) + bP_1(H_t)w(H_t,1) - b[1+P_1(H_t)]w(H_t,2)}{2b} \end{bmatrix} \qquad ..[5.2]$$

Assuming agent 0 is risk neutral is attractive because it implies that $\{q(H_t)\}$ is completely determined by the stochastic process $\{x_t\}$ and the discount parametric b; agent 0 is willing to absorb entirely the risk (of low wages) in period t+1 for each member of I_t, subject only to $c^o \geq 0$. However, even for the case of log-linear workers it is difficult to explicitly compute a sequence $\{w(H_t)\}$ which equates labor supply with some element of the demand correspondence at each H_t. One strategy is to concentrate on the special technology that is linear in labor input, allowing a separation of price and quantity determination analogous to that employed in the securities market: the instantaneous wage will be identically equal to the marginal product of labor (which is constant in n, but does depend on x_t), and firms will hire all of the labor forthcoming. With wages a technological datum, the right hand side of [5.2] is completely determined. Qualitative properties of more general specifications can be inferred from the special case; making the production function concave in n typically smooths out the equilibrium time series without changing the basic properties of the model.

With technology linear in n, write $w_j = f_n(n,K;x_j)$ for $j = 1,2$ and assume $w_2 > w_1$. Since workers live two periods and x_t follows a first order process, labor supply decisions at t depend only on x_{t-1} and x_t. Let $L(i,j)$ be the aggregate labor supply when $(x_{t-1},x_t) = (i,j)$; then the equilibrium employment path is given by the stochastic process $\{L(x_{t-1},x_t)\}$. There are four combinations of (x_{t-1},x_t) that can occur; aggregating [5.2] and simplifying:

$$
\begin{aligned}
L(1,1) &= \frac{7b-1-b(1+b)P_{11}}{4b} - \frac{(1+b)(1-P_{11})}{4}\,\frac{w_2}{w_1} \\
L(1,2) &= \frac{6-bP_{22}+P_{11}}{4} - \frac{1+bP_{11}+b^2(1-P_{22})}{4b}\,\frac{w_1}{w_2} \\
L(2,1) &= \frac{6-bP_{11}+P_{22}}{4} - \frac{1+bP_{22}+b^2(1-P_{11})}{4b}\,\frac{w_2}{w_1} \\
L(2,2) &= \frac{7b-1-b(1+b)P_{22}}{4b} - \frac{(1+b)(1-P_{22})}{4}\,\frac{w_1}{w_2}
\end{aligned}
\qquad \text{...[5.3]}
$$

Given the special assumptions it is possible to analyze the dynamic behavior of this economy simply by studying the set $\{L(i,j) : i,j = 1,2\}$. The following result, depicted in Figure 5.1, is proved in Wright [1983]:

Theorem 5.1: In the case where O is risk neutral, all i are log-linear, and f is linear in n: in equilibrium with interior solutions, employment satisfies $L(2,1) < L(1,1) < L(2,2) < L(1,2)$.

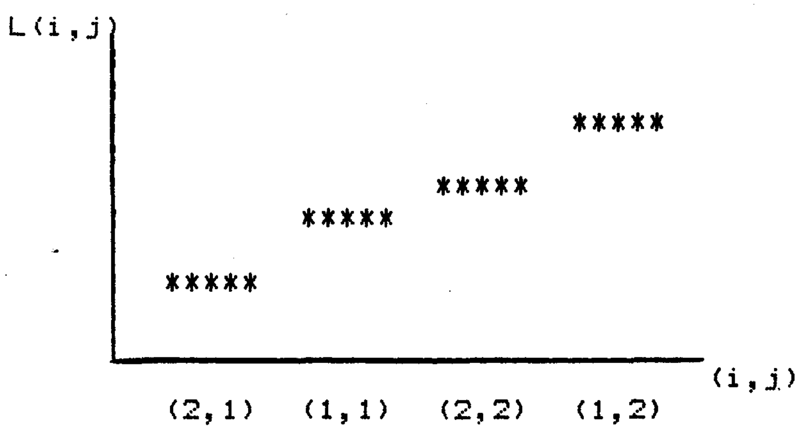

Figure 5.1: The ranking of employment states by (x_{t-1}, x_t) pairs.

Notice that the aggregate labor supply responds in the same direction as the productivity shocks: $L(x_{t-1},2) > L(x_{t-1},1)$ for any x_{t-1}. Hence, (as long as the x_t shocks do not affect the productivity of capital too much in the wrong direction), output and employment display a high degree of "coherence." Another implication is that when the previous period was bad, the labor supply curve (as a function of the contemporaneous wage rate w_t) is uniformly higher than when the previous period was good. That is, $L(1,x_t) > L(2,x_t)$ at t, for any x_t. This property actually extends to any worker preferences for which leisure is a normal good, and has important implications for the "stability" of equilibrium time series: Periods of sustained high (respectively, low) productivity will see output regress towards its trend level, as workers translate their windfall gains (losses) into a higher (lower) demand for leisure. Another way to say the same thing is that the early part of a run of high x_t's will see the largest levels of output, as workers strive to take advantage of the transitory earning opportunity, while the early part of a run of low x_t's will see the greatest reduction in output, as workers take low wages to be a signal to work less.

Hence, exogenous shocks are, symmetrically over the cycle, amplified by the endogenous labor supply decision, but this effect is only temporary and fades out as changes in wealth impinge on the demand for leisure. This is an interesting phenomenon to observe in a perfectly functioning economy, for one of the most challenging tasks of equilibrium business cycle theory is to explain the swings in employment which pervade actual economies. As Lucas [1977] puts it: "...nowhere is the 'apparent contradiction' between 'cyclical phenomena' and 'economic equilibrium' theory sharper than in labor market behavior. Why, in the face of moderately fluctuating nominal wages and prices, should households _choose_ to supply labor at sharply irregular rates through time?" (p.220). This is especially puzzling in light of the low (near zero) estimates of the long run labor supply elasticity which are generally accepted.

The answer here is a form of the intertemporal substitution model of labor market behavior.[26] Agents "choose to supply labor at sharply irregular rates through time" simply because it is efficient to specialize, working more when wages are high, and then smoothing consumption via securities trading. With time separable preferences, intertemporal substitution effects depend critically on the availability of some mechanism (such as securities markets, or contracts) through which resources can be traded across time. In fact, with log-linear preferences the absence of any such mechanism results in _constant_ employment, and therefore the cycles displayed in a market economy with securities trading ruled out will be less severe. Yet the restricted allocations are dynamically Pareto dominated by the laissez-faire equilibrium allocations. It is simply more efficient for employment to fluctuate. Labor supply responses to transitory wage shocks can be quite large, in spite of the fact that the model is consistent with a negligible long run labor supply elasticity (it is immediate from equations [5.3] that a long run increase in wages -- a proportional increase in both w_1 and w_2 -- has no impact on labor supply.[27]

To briefly investigate the implications of serial correlation in $\{x_t\}$, consider the total derivatives of equations [5.3]:

$$dL(1,1) = (1/4)(1+b)(w_2/w_1-1)\ dP_{11}$$

$$dL(1,2) = (1/4)(1-w_1/w_2)\ [dP_{11} - bdP_{22}]$$

$$dL(2,1) = (1/4)(1-w_2/w_1)\ [dP_{22} - bdP_{11}] \qquad ...[5.4]$$

$$dL(2,2) = (1/4)(1+b)(w_1/w_2-1)\ dP_{22}$$

One consequence is that employment is increasing in P_{11} and decreasing in P_{22}, the result of changes in expected wealth, which affect leisure demand. Second,

$$dP_{11} > 0 \Rightarrow dL(i,1) > 0, \quad i = 1,2$$

$$dP_{22} > 0 \Rightarrow dL(i,2) < 0, \quad i = 1,2$$

The persistence of bad shocks tends to increase employment when x_t is low, while the persistence of good shocks tends to decrease employment when x_t is high. However, it would be hasty to conclude that serial correlation in $\{x_t\}$ necessarily implies smoother time series. Equations [5.4] yield: $dP_{11} = dP_{22} > 0 \Rightarrow dL(1,1) > 0$, $dL(1,2) > 0$, $dL(2,1) < 0$, $dL(2,2) < 0$. So a symmetric increase in the serial correlation of $\{x_t\}$ spreads out the extremes in employment, while moving the intermediate states closer together.

```
L(i,j)
     |
     |                         *****          Legend:
     |                         +++++
     |                  +++++                 low serial correla-
     |                  *****                 tion in x_t:    +++++
     |             *****
     |             +++++                      high  serial corre-
     |        +++++                           lation in x_t: *****
     |        *****
     |______________________________ (i,j)

        (2,1)  (1,1)  (2,2)  (1,2)
```

Figure 5.2: The ranking of employment states with P_{11} and P_{22} both higher in the ** equilibrium than in the ++ equilibrium.

As alluded to earlier, there is a possibility of implicit contractual arrangements confounding observed real wage movements over the cycle. Kydland and Prescott [1980] suggest (but do not analyze) a "potential source of cyclical measurement bias [resulting from] the implicit employment contract" (p. 179). It is an implication of their equilibrium business cycle model, as it is of many such models, that the real wage should move procyclically; however, they indicate that the data does not unequivocally support this prediction.[28] It is therefore important to investigate any potential cyclical bias in the real wage series explicitly. To begin, consider an OGCE with a bundle for i in I_t of $z^i(H_t) = [c^i(H_t),\ldots,1-l^i(H_t,m)]$. Assume that f_n is increasing in x_t. In the equivalent exclusively bilateral CONE, as discussed in the last section, one has $d^i(H_t) = c^i(H_t)$ and $e^i(H_t) = l^i(H_t)$, and the contract wage for i, $w^i(H_t) = c^i(H_t)/l^i(H_t)$.

Risk sharing tends to smooth $c^i(H_t,x_{t+1})$ across x_{t+1}; in fact, as long as U^i is separable in old-age consumption and leisure, and agent 0 is risk neutral, it is easy to verify that $c^i(H_t,x_{t+1})$ is <u>constant</u> across x_{t+1}, although $l^i(H_t,x_{t+1})$ is not.

<u>Theorem 5.2</u>: If worker i has preferences which are separable in second period consumption and leisure, and if 0 is risk neutral, then in OGCE or CONE with interior solutions $c^i(H_t,x_{t+1})$ is constant across x_{t+1}, while $l^i(H_t,x_{t+1})$ fluctuates procyclically.

<u>Proof</u>: Let $U^i(c_1,1-l_1,c_2,1-l_2) = u_o(c_1,1-l_1) + u_c(c_2) + u_l(1-l_2)$. The problem

$$\text{MAX} \quad u_o(c(H_t),1-l(H_t)) + \sum_j P_j(H_t)\{u_c(c(H_t,j)) + u_l(1-l(H_t))\}$$

$$\text{s.t.} \quad c(H_t) = w(H_t)l(H_t) - q(H_t)s(H_t)$$

$$c(H_t,j) = w(H_t,j)l(H_t,j) + s_j(H_t)$$

has as necessary conditions for (interior) solution:

$$D_c u_o(c(H_t),1-l(H_t))\cdot q_j(H_t) = P_j(H_t)\cdot u_c'(c(H_t,j))$$

$$u_c'(c(H_t,j))\cdot w(H_t,j) = u_l'(1-l(H_t,j))$$

for $j = 1,\ldots m$. The first of these implies:

$$\begin{aligned} u_c'(c(H_t,j)) &= D_c u_o(c(H_t),1-l(H_t))\cdot q_j(H_t)/P_j(H_t) \\ &= D_c u_o(c(H_t),1-l(H_t))\cdot b \end{aligned}$$

for all j, by Lemma 5.1. Hence, the left hand side is constant in j, and so old age consumption is perfectly smoothed; say $c(H_t,j) = c*$ for all j.

The second marginal condition now implies:

$$u_l'(1-l(H_t,j)) = w(H_t,j)\cdot u_c'(c*)$$

By strict concavity, u_l' is strictly decreasing, so old-age leisure = $1-l(H_t,j)$ is strictly decreasing in $w(H_t,j)$.

Q.E.D.

At least in the separable case, then, the consumption of senior employees is perfectly smoothed, while their employment is positively correlated with productivity.[29] Thus, even though the competitive wage rate is procyclical here, by construction, observations on senior employees' contract wages will be countercyclical in the sense that when productivity goes up, $w^1 = c^1/l^1$

goes down (and vice-versa). At the same time, by specifying high enough serial correlation in the $\{x_t\}$ process, it is possible to generate equilibria where senior workers' contract wages are positively correlated with the competitive wage in steady state, as is done in the Appendix. So there is also a sense in which one can say senior contract wages sometimes move procyclically for certain parameterizations of the model. However, precisely because of the implicit insurance aspect of the long-term employment arrangement, senior contract wages move less procyclically than competitive wages, and often countercyclically.

For young workers the story is different, since by the hypothesis of overlapping generations and the timing of x_t, agents are not able to enter into contracts which smooth consumption in their youth. For the linear/log-linear economy with $P_{11} = P_{22} = .5$, the contract wage for i in I_t at H_t is (from [5.2]):

$$w^i(H_t) = \frac{c^i(H_t)}{l^i(H_t)} = \frac{w(H_t)^2 + bw(H_t)Ew}{3w(H_t) - bEw},$$

where $Ew = .5(w_1+w_2)$. As $b \to 0$, $w^i(H_t) \to w(H_t)/3$, and thus moves procyclically. Meanwhile, as $b \to 1$, it is only slightly more difficult to verify that $w^i(H_t)$ must move in the opposite direction to $w(H_t)$, i.e. countercyclically. In simulations of the linear/log-linear economy in the Appendix, there are cases where young, old, or aggregate workers' contract wages move with, and cases where they move against, the cycle.

In general, senior workers' contract wages will be more or less variable than competitive wages depending on which fluctuates more, their labor supply or the competitive wage. It is entirely possible that fluctuations in senior workers' contract wages will be small, especially in economies where employment by these workers does not vary much. However, by suitably choosing the parameters it is possible to realize values of senior employment at or as near 0 as desired in some states, and have it strictly positive in other states, resulting in arbitrarily large countercyclical variability in senior contract wages. Now of course labor statisticians may well differentiate between the hourly wages of active employees, on the one hand, and severance payments, retirement benefits, etc. on the other, and it seems reasonable to include payments to (ex ?) employees for whom $l^{1}(H_t)$ is at or near 0 in the latter category. The point being made here is that w^{1} is a biased proxy for w for all values of $l^{1}(H_t)$, whenever implicit insurance arrangements are part of the long-term employment relationship.

Cross sectional seniority observations are also influenced by long-term contracting. Recall that old workers have state independent consumption at t, while young workers' consumption, and in fact, their entire life time consumption-leisure plan, is determined by the state of the economy at which they appear. This is one sense in which one could say that senior employees are "insulated" from current economic conditions.[30] While senior employees have smoothed consumption, however, their employment may be highly variable. How can this be reconciled with the observation that more senior workers seem to have more stable employment in many actual economies? In fact, there is nothing to say that variations in l^{1} should ever get translated into labor market statistics. Quite possibly, senior workers would maintain their official relationship with the employer, continue to show up at the work place, but work less hard when x_t is low. It is a simple reinterpretation to suggest that everyone is always "fully employed" in the economy, while $l^i(H_t)$ represents the "effort" by i at H_t.

Variations in effort may well be an important aspect of the employment relationship. Hall [1980a] observes:

> Fluctuations in output have been larger proportionately than fluctuations in the total volume of work, measured as employee hours. Within the theory of long-term employment arrangements, this reflects the operation of an implicit or explicit agreement that employees work harder when there is more work to do. ... Workers put in more effort during booms and take it easy during slumps. ... These data ... lead to the conclusion that changes in the amount and intensity of effort in existing jobs are an important factor in total cyclical variations in effective labor input. (pp. 93-96)

What Hall is calling "wages" in his study seems to correspond to what I call contract compensation (which equals consumption by the relevant worker in the exclusively bilateral CONE). On this interpretation, the employment and "wages" of senior employees are smoothed, while their effort moves procyclically.

Another implication of this model is that, depending on the rates of time preference of the agents, longitudinal observations on a single employee will see his consumption changing over the life cycle even in the absence of risk. When it is assumed that workers do not discount future consumption, as in the log-linear example above, since $b < 1$, naturally worker consumption = contract compensation will rise with age. Alternatively, instead of discounting future consumption less than their employers, workers might value leisure in their old age more highly than in their youth, and to the extent that they prefer smoothed consumption, this could also induce an increasing relationship between tenure and contract wages. When rates of time preference are the same for workers and the employer, a situation investigated in the Appendix for the linear/log-linear economy by letting b -> 1, employee consumption = contract compensation is constant over the life cycle, although cross sectional differences will remain.

So whenever employees wish to transfer wealth forward through their long-term employment arrangements, the model predicts senior employees with the same productivity (by construction) appear to be paid better.[31] Now human capital also has a role to play in explaining the seniority-wage relationship, and in many instances productivity probably does rise with tenure. Nevertheless, Medoff and Abraham's [1980] study finds "The evidence ... has as its most important implication that, while greater experience moves white male managerial and professional employees toward the upper tail of the earnings distribution for their grade levels, it does not move them toward the upper tail of the relevant performance distribution." (p. 704). They conclude "the results imply that the human capital on-the-job training model cannot explain a substantial part of the observed return to labor market experience." (p. 703).

Medoff and Abraham themselves suggest that, for a variety of reasons, employees and employers may enter into implicit contracts that provide for earnings growth which is less than perfectly related to changes in productivity.[32] One aspect of seniority, one which could only have surfaced upon examining contracts in a dynamic environment, and one which should not be ignored, is the implicit credit arrangement. In conjunction with the implicit insurance arrangement, this makes it difficult to interpret recorded wage and employment data literally in terms of competitive market theory. Yet, as Medoff and Abraham suggest, the divergence of the experience-earnings and experience-performance profiles need not be inconsistent with competitive theory. For although the patterns of certain prices and securities exchanges may differ between contract and market equilibria, both ential the same values of the most important variables for many purposes -- the allocations of commodities and welfare.

VII. Summary

This paper has analysed long-term employment contracts in a dynamic, general equilibrium model, using a framework which integrates two types of models used by economists in the past: those with infinite horizon decision makers, and those with overlapping generations of finite horizon decision makers. OG models are both tractable and natural to use, but it turns out that the introduction of an infinite horizon decision maker has certain advantages: it serves to restrict the set of equilibrium allocations to those which are dynamically Pareto optimal. Labor contracts and contract equilibrium were introduced, and it was shown that the economy with contracts in addition to markets is equivalent to the economy with only markets, in the sense that an allocation can be supported as an equilibrium in one economy if and only if it can be supported in the other. However, the two economies can generate some very different observations. Real wage movements over the business cycle, and over the life cycle, can become confounded by implicit credit or insurance arrangements incorporated into the long-term employment relationships. Also, the observation that certain markets look to be inactive in many actual economies need not render the competitive market paradigm irrelevant. In (some) contract equilibria securities are not exchanged because they are redundant in the presence of long-term employment contracts; yet the allocations are the same ones generated by the competitive market model. Hence, one could say long-term employment arrangements produce a facade in front of the otherwise accurate predictions of competitive theory.

Footnotes

1. This was the view of many Keynesians when the original "implicit contract" results (e.g., Azariadis [1975]) were published. Lipsey [1979], for example, asserts that "Although wage rates may rise when there is excess demand for labor they do not fall (in the short term at least) when there is excess supply. ... This was something of a 'mystery' in Keynes' time, but downward rigidity can be shown to follow from rationally arrived at 'implicit contracts' where layoffs occur according to a seniority rule." (p. 292 and footnote 11).

2. These results formalize some conjectures of Kydland and Prescott [1980] about implicit contracts introducing a cyclical bias into real wage data.

3. These results formalize some conjectures of Medoff and Abraham [1980] about implicit contracts influencing the tenure-compensation relationship.

4. In some applications it is desirable to further assume that f and its marginal products f_n and f_k are increasing in x_t. This allows an unambiguous identification of higher values of x_t with "better economic times."

5. Formally, construct a sequence as follows: $H_s = (x_1^*, \ldots x_s^*)$ precedes $H_t = (x_1, \ldots x_t)$ if and only if:

$$s < t$$

or

$$s = t \text{ and } x_1^* < x_1$$

or

$$\cdots\cdots\cdots\cdots$$

or

$$s = t \text{ and } x_1^* = x_1 \text{ and } \ldots x_{s-1}^* = x_{t-1} \text{ and } x_s^* < x_t.$$

6. Two period lived workers are studied purely for tractability: this is the simplest specification which allows for some notion of tenure. As an additional defence of this assumption note that, with constant population, two period lived workers implies that at each date one half of the labor force consists of senior workers, and this accords reasonably well with studies of the numbers of employees involved in long-term employment relationships. Hall [1980b] reports that 58 percent of workers "are currently holding reasonably long jobs" (p.12).

7. One need not take the hypothesis of an infinite life time literally. It would suffice to have a group of agents (a family, say) which internalizes the utility of its heirs so as to behave collectively like the infinite horizon decision maker imagined here. The class of utility functions used below could be generalized; it is only critical that some decision maker has an endowment which is of some value in all but a finite number of periods. (see the discussion of optimality in Section III).

8. The timing of births and stochastic events is critical in OG models because it determines the contingencies against which agents are able to insure themselves.

9. On the boundary of the consumption set, if $c^i(H_t)$ or $c^i(H_{t+1}) = 0$, or if $l^i(H_t)$ or $l^i(H_{t+1}) = 1$, for i in I_t (for example), then U^i may not be increasing in all of its arguments. This permits, e.g., the use of Cobb-Douglas or log-linear utility.

10. For instance, given $H_2 = (1,2)$, $H_3 = (1,2,j)$ is possible for $j = 1,...m$, but $H_3 = (1,1,j)$ is not possible.

11. It turns out that for equilibrium sequences (r,q), the series in question does indeed converge. See Wright [1983], Lemma 3.6.

12. First order conditions are:

$$f_n \leq w, \quad = w \text{ if } n^{**} > 0$$

$$f_k \leq R, \quad = R \text{ if } k^{**} > 0$$

$$k^{**} \leq K, \quad = K \text{ if } R > 0$$

where R is the Lagrangian multiplier on the constraint $k \leq K$. Now $f_k > 0$ implies $R > 0$, so $k^{**} = K$, and therefore $f_k = R$.

13. Labor demand will not in general be a (single-valued) function, and in fact $N(H_t)$ could be empty for some $w(H_t)$. As long as $N(H_t)$ is not empty, $w^p(H_t)$ is a well defined (nonnegative, finite) value of wealth.

14. The existence proof proceeds by demonstrating that the economy in question is equivalent to another economy, where all trades take place at date 1 on complete contingent commodity markets (instead of on a sequence of spot plus one-step-ahead contingent securities markets), and then adapting the argument in Balasko, Cass and Shell [1980] to accomodate production and some infinite lived individual(s).

15. DPO is used by Cass and Shell [1983], as well as by Peled [1980], who refers to it (after Muench [1977]) as Conditional Pareto Optimality. Muench himself worked with what he called Equal Treatment Pareto Optimality, which conditions the expected utility of generation I_t on H_{t-1}, and which is equivalent to SPO if the distribution of states of the economy is time invariant. Lucas [1972] worked with a very weak optimality criteria in which one feasible allocation dominates another if the former is preferred by all agents, and strictly preferred by some agent, along each possible realization of the history of the economy.

16. For surveys of the implicit contract literature, see Ito [1982] or Azariadas [1979]. Cooper [1982] studies contracts in an OG environment, which has two period lived capitalists as well as workers, both of whom produce only when young and consume only when old, while inputs are supplied inelastically and all agents

save (their entire income) in the form of money - securities trading is ruled out by assumption. Farmer [1982] also studies contracts in an OG model, concentrating on the effects of incomplete information.

17. The convention of specifying <u>minus</u> employment is merely to facilitate computations below. The definition of $y^i(H_t)$ allows for several special features discussed in the implicit contract literature, including severance payments, quitting penalties, pensions, and work-sharing. The feature of "home production" could be included in the aggregate technology. Generally, different workers receive different contract offers.

18. For example, assuming momentarily (for the sake of illustration) that he saves nothing, at date t worker i would consume:

$$c^i(H_t) + a^i(H_t)d^i(H_t),$$

and have the following amount of leisure time:

$$1 - l^i(H_t) - a^i(H_t)e^i(H_t).$$

19. Holmstrom [1980] shows how to determine U* endogenously for the simple case where l^i is restricted to the set {0,1}, but this procedure does <u>not</u> generalize to the case where l^i lies in some interval. Note that much of the implicit contract literature assumes that firms "enter" until the surplus extracted from workers is driven to zero, so that effectively the workers utility is maximized subject to expected profits being nonnegative.

20. Virtually all of the literature assumes labor is traded through markets, or contracts, but not both at the same time.

21. Other CONE can be constructed by "averaging over" these two extreme cases.

22. This depends, of course, on the risk aversion of the agents in the model. There is no (decentralized) scope for sharing the risk that impinges on the <u>first</u> period of a workers life due to the OG demography and the timing of x_t.

23. In addition to transactions costs, an important consideration in the evolution of actual trading arrangements is government policy. A tax on the interest paid to securities could be totally avoided by making all intertemporal exchanges through long-term employment contracts (although whether this is beneficial or not depends on the entire tax structure in place). This is an example of the general principal that, while two equilibrium concepts may be equivalent in one context, they need not respond identically to all (policy) interventions.

24. It should be pointed out that much of implicit contract theory proceeds from the <u>assumption</u> that some (usually securities) markets do not exist. The justification for this assumption is usually not clear, and hopefully the arguments here have

convinced the reader that it is _not_ good enough simply to note that securities markets look to be inactive in many actual economies. Of course, the additional hypothesis of incomplete (including asymmetric) information limits the contingencies which can be traded against, and therefore can lead to deviations from the complete information general equilibrium allocation. But this is due to a feature of the _environment_ and is not a contribution of contracts (an economic construct) per se.

25. Some commentators question the applicability of OG models for business cycle (and other) analysis because of the implied length of each period (e.g., Tobin [1980]). Three points are important here: First, it is only analytic tractability which recommends _two_ period lived workers. Second, there may actually be an interesting economics of "long waves" (see, e.g., Kondratieff [1935]), to which a literal interpretation of the timing in two period models is relevant. But, thirdly, a literal interpretation of the timing is typically not intended. The generational fiction is simply a convenient way to formalize the fact that agents are perpetually entering and exiting actual economic processes.

26. Any form of the intertemporal substitution hypothesis is a "mixed blessing" in the modelling of business cycles. While intertemporal substitution does entail increased cyclical movements in employment, and hence does help to explain the amplitude of the cycle, it also tends to produce the wrong serial correlation in output and employment: High levels of employment in one period indicate that one should expect low levels of employment in the next period, since agents are wealthier (assuming leisure is normal), and when preferences are not time separable, because agents are "tired." With serially independant productivity shocks (i.e., $P_{11} = P_{22} = .5$), it is easily verified that

$$E[L(x_t, x_{t+1}) | L(x_{t-1}, x_t) = L_0]$$

is actually a _decreasing_ function of L_0. Hence, some form of propagation mechanism will have to be built into the system in order to generate equilibria that mimic the amplitude and the persistence of actual time series. Lucas and Sargent [1979] discuss several potential propagation mechanisms.

27. Notice the applicability of Lucas's [1976] critique of econometric policy making here: Even if historical time series indicated that for a given stochastic process on wages, employment was higher when $w(H_t)$ was above average, this would be of _no_ value in predicting the response of the economy to a change in the stochastic process itself, such as would occur with a permanent change in the income tax schedule.

28. It is not clear from reading the literature which way real wages actually move; Bodkin [1969] examines the evidence and concludes that it weakly supports procyclical movements.

29. Movements in employment are necessary in order to maintain efficiency, in the sense that the marginal rate of substitution

between leisure and consumption must equal the marginal rate of transformation between labor and output. Notice that the more concave is u_1 (i.e., the more risk averse the worker is to fluctuating leisure), the less variable employment is.

30. Ayigari and Wallace [no date] study seniority in an OG model with similar timing and are led to similar conclusions about senior workers being insulated, but since they assumed leisure was not in the utility function, their senior employees were just as well off in all states. With leisure in the utility function and separable preferences, senior employees are better off in low productivity states, since they have to work less (or less hard).

31. Hall [1980a] says that "At the simplest level, the schedule of wage payments for the duration of an unbreakable employment contract is a matter of indifference." This is so only if agents are also trading on the securities markets; in the exclusively bilateral CONE the timing of payments is determined by the relative rates of time preference of the agents.

32. Holmstrom [1980] develops a model where senior workers are better paid on average, which depends on the assumption that workers may renege on contracts ("ex post mobility"). One problem with assuming ex post mobility is that it results in equilibrium allocations which are not optimal, and can be dominated by the equilibrium allocations which arise when agents are allowed to bargain in terms of binding promises. It seems difficult to justify models that impose institutions which make everyone worse off, even when they do seem to correspond to instutions observed in (some) actual economies.

Appendix: Simulation Results

Here are contained several simulations of the linear/log-linear economy described in Chapter VI. Preferences for the one representative worker in each generation are given by:

$$U(c_1,1-l_1,c_2,1-l_2) = \ln c_1 + \ln (1-l_1) + \ln c_2 + \ln (1-l_2).$$

The $\{x_t\}$ process is described by:

$$\text{Prob}[x_{t+1}=j \mid x_t=i] = P_{ij} \; ; \quad i,j = 1,2.$$

The capitalist is risk neutral, with discount factor b and an endowment of K capital units. The production function is linear in (n,K) -- but the coefficients, as functions of x_t, are stochastic:

$$f(n,K;x_t) = A_n(x_t)\cdot n + A_k(x_t)\cdot K.$$

The economy can now be completely parameterized by the vector

$$[P_{11} \; P_{22} \; b \; K \; A_n(1) \; A_n(2) \; A_k(1) \; A_k(2)].$$

Six examples are presented after a discussion of the effects they depict. The parameterizations chosen are:

[.50 .50 0.800 1 2.0 2.2 2.0 2.2] ...(a)

[.50 .50 .9999 1 2.0 2.2 2.0 2.2] ...(b)

[.85 .85 0.800 1 2.0 2.2 2.0 2.2] ...(c)

[.71 .71 .9999 1 2.0 2.2 2.0 2.2] ...(d)

[.50 .50 0.800 1 3.0 3.3 3.0 3.3] ...(e)

[.60 .60 0.800 1 2.0 2.2 2.0 2.2] ...(f)

Example (a) is for reference; note the following features: The labor supply decision amplifies the productivity shocks, but there is negative serial correlation in employment (and here, output). Of course, employment states are ranked according to (x_{t-1},x_t) pairs as Theorem 6.1 predicts. Senior employees have higher contract wages and consumption, and work less, than their junior counterparts at all phases of the cycle.

Example (b) displays what happens as b -> 1. As can be seen by comparing a variable for young person in (x_{t-1},x_t) with the same variable for an old person in (x_t,x_{t+1}), consumption is constant, while employment moves up or down with productivity, over the life cycle. Cross sectional differences in consumption persist. Note that here, contract wages for all workers have a negative coherence (i.e., are coutercyclical), while in example (a) junior employees' contract wages had positive coherence. The third simulation provides an example where all contract wages have positive coherence, while (d) provides an example where the aggregate contract wage is virtually independant of the competitive wage.

Example (f) demonstrates how employment does not respond at all to a proportional (long run) increase in $A_n(1)$ and $A_n(2)$, in spite of the strong positive correlation between employment and wages for a given distribution of $[A_n(1), A_n(2)]$. Example (g) demonstrates the situation displayed in Figure 5.2, where $dP_{11} = dP_{22} > 0$ results in an increase (as compared to the first example) in the difference between the highest and lowest values of employment, while moving the intermediate states closer together. However, the coefficient of variation for employment is lower in (g). Note that a small increase in the serial correlation of the $\{x_t\}$ process can result in positive persistence in output, but in no case is employment positively autocorrelated, for the reasons described in the text.

Equilibrium for the values:

[P11 P22 B K AN(1) AN(2) AK(1) AK(2)] = [.5 .5 .8 1 2 2.2 2 2.2]

	(2,1)	(1,1)	(2,2)	(1,2)	average	coef.var.	autocor	coherence
w(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.000	1.000
Ly(i,j)	0.540	0.540	0.559	0.559	0.550	0.017	0.000	1.000
Lo(i,j)	0.394	0.425	0.449	0.477	0.436	0.070	-.424	0.874
L(i,j)	0.934	0.965	1.008	1.036	0.986	0.040	-.351	0.925
Yy(i,j)	1.080	1.080	1.230	1.230	1.155	0.065	0.000	1.000
Yo(i,j)	0.787	0.850	0.988	1.050	0.919	0.114	-.285	0.955
Y(i,j)	1.867	1.930	2.218	2.280	2.074	0.086	-.173	0.984
Cy(i,j)	0.920	0.920	0.970	0.970	0.945	0.026	0.000	1.000
Co(i,j)	1.212	1.150	1.213	1.150	1.181	0.026	0.000	0.000
C(i,j)	2.133	2.070	2.183	2.120	2.126	0.019	0.488	0.625
Ck(i,j)	1.735	1.860	2.235	2.360	2.048	0.126	-.235	0.970
Kr(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.000	1.000
GNP(i,j)	3.867	3.930	4.418	4.480	4.174	0.066	-.112	0.994
CWy(i,j)	1.704	1.704	1.735	1.735	1.719	0.009	0.000	1.000
CWo(i,j)	3.079	2.706	2.701	2.410	2.724	0.087	-.496	-.709
CW(i,j)	2.284	2.145	2.165	2.046	2.160	0.039	-.491	-.644

Steady state probabilities: [P(2,1) P(1,1) P(2,2) P(1,2)] = [.25 .25 .25 .25]

LEGEND: w = competitive real wage rate (maginal product of labor)
L = employment, Y = labor income, C = consumption
Kr = profits (capital X the rental rate), GNP = output
CW = contract wage (= contract compensation/contract employment)
Subscripts y, o, and k refer to the young worker, old worker, and capitalist, respectively.
Desciptive statistics are for the stationary distribution of (Xt-1,Xt).
The coefficient of variation is the standard deviation divided by the mean.
Autocor is the correlation coefficient of the variable with itself once lagged.
Coherence is the correlation with w, measuring the degree to which the variable is procyclical.

Equilibrium for the values:

[P11 P22 B K AN(1) AN(2) AK(1) AK(2)] = [.5 .5 .9999 1 2 2.2 2 2.2]

	(2,1)	(1,1)	(2,2)	(1,2)	average	coef.var.	autocor	coherence
w(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.000	1.000
Ly(i,j)	0.488	0.488	0.511	0.511	0.499	0.024	-.000	1.000
Lo(i,j)	0.462	0.487	0.511	0.534	0.499	0.054	-.400	0.894
L(i,j)	0.950	0.975	1.023	1.045	0.998	0.038	-.300	0.949
Yy(i,j)	0.975	0.975	1.125	1.125	1.050	0.071	0.000	1.000
Yo(i,j)	0.925	0.975	1.125	1.175	1.050	0.098	-.235	0.970
Y(i,j)	1.900	1.950	2.250	2.300	2.100	0.084	-.140	0.990
Cy(i,j)	1.025	1.025	1.075	1.075	1.050	0.024	0.000	1.000
Co(i,j)	1.075	1.025	1.075	1.025	1.050	0.024	-.000	0.000
C(i,j)	2.100	2.050	2.150	2.100	2.100	0.017	0.500	0.707
Ck(i,j)	1.800	1.900	2.300	2.400	2.100	0.121	-.192	0.981
Kr(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.000	1.000
GNP(i,j)	3.900	3.950	4.450	4.500	4.200	0.066	-.090	0.996
CWy(i,j)	2.102	2.102	2.102	2.102	2.102	0.000	0.000	-.978
CWo(i,j)	2.325	2.103	2.102	1.919	2.112	0.068	-.498	-.706
CW(i,j)	2.211	2.103	2.102	2.009	2.106	0.034	-.499	-.707

Steady state probabilities: [P(2,1) P(1,1) P(2,2) P(1,2)] = [.25 .25 .25 .25]

LEGEND: w = competitive real wage rate (maginal product of labor)
L = employment, Y = labor income, C = consumption
Kr = profits (capital X the rental rate), GNP = output
CW = contract wage (= contract compensation/contract employment)

Subscripts y, o, and k refer to the young worker, old worker, and capitalist, respectively.
Desciptive statistics are for the stationary distribution of (Xt-1,Xt).
The coefficient of variation is the standard deviation divided by the mean.
Autocor is the correlation coefficient of the variable with itself once lagged.
Coherence is the correlation with w, measuring the degree to which the variable is procyclical.

Equilibrium for the values:

[P11 P22 B K AN(1) AN(2) AK(1) AK(2)] = [.85 .85 .8 1 2 2.2 2 2.2]

	(2,1)	(1,1)	(2,2)	(1,2)	average	coef.var.	autocor	coherence
w(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.700	1.000
Ly(i,j)	0.547	0.547	0.553	0.553	0.550	0.005	0.700	1.000
Lo(i,j)	0.385	0.434	0.441	0.485	0.437	0.045	-.122	0.536
L(i,j)	0.932	0.981	0.994	1.038	0.987	0.022	-.073	0.628
Yy(i,j)	1.094	1.094	1.216	1.216	1.155	0.053	0.700	1.000
Yo(i,j)	0.770	0.868	0.970	1.068	0.919	0.081	0.252	0.884
Y(i,j)	1.864	1.961	2.186	2.284	2.074	0.063	0.469	0.964
Cy(i,j)	0.906	0.906	0.984	0.984	0.945	0.041	0.700	1.000
Co(i,j)	1.230	1.132	1.230	1.133	1.181	0.041	0.700	0.700
C(i,j)	2.136	2.038	2.214	2.117	2.126	0.038	0.848	0.903
Ck(i,j)	1.728	1.923	2.172	2.367	2.048	0.082	0.312	0.911
Kr(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.700	1.000
GNP(i,j)	3.864	3.962	4.386	4.484	4.174	0.055	0.577	0.988
CWy(i,j)	1.656	1.656	1.780	1.780	1.718	0.036	0.700	1.000
CWo(i,j)	3.195	2.611	2.790	2.334	2.710	0.069	0.046	0.061
CW(i,j)	2.292	2.079	2.228	2.039	2.155	0.039	0.514	0.527

Steady state probabilities: [P(2,1) P(1,1) P(2,2) P(1,2)] = [7.499999E-02 .425 .425 7.499999E-02]

LEGEND: w = competitive real wage rate (maginal product of labor)
L = employment, Y = labor income, C = consumption
Kr = profits (capital X the rental rate), GNP = output
CW = contract wage (= contract compensation/contract employment)
Subscripts y, o, and k refer to the young worker, old worker, and capitalist, respectively.
Desciptive statistics are for the stationary distribution of (Xt-1,Xt).
The coefficient of variation is the standard deviation divided by the mean.
Autocor is the correlation coefficient of the variable with itself once lagged.
Coherence is the correlation with w, measuring the degree to which the variable is procyclical.

Equilibrium for the values:

[P11 P22 B K AN(1) AN(2) AK(1) AK(2)] = [.71 .71 .9999 1 2 2.2 2 2.2]

	(2,1)	(1,1)	(2,2)	(1,2)	average	coef.var.	autocor	coherence
w(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.420	1.000
Ly(i,j)	0.493	0.493	0.507	0.507	0.500	0.014	0.420	1.000
Lo(i,j)	0.457	0.493	0.507	0.539	0.499	0.046	-.224	0.736
L(i,j)	0.950	0.986	1.013	1.045	0.999	0.028	-.119	0.838
Yy(i,j)	0.986	0.986	1.115	1.115	1.050	0.061	0.420	1.000
Yo(i,j)	0.914	0.985	1.114	1.185	1.050	0.087	0.067	0.935
Y(i,j)	1.900	1.971	2.229	2.300	2.100	0.073	0.215	0.978
Cy(i,j)	1.014	1.014	1.085	1.085	1.050	0.034	0.420	1.000
Co(i,j)	1.086	1.015	1.086	1.015	1.050	0.034	0.420	0.420
C(i,j)	2.100	2.029	2.171	2.100	2.100	0.028	0.710	0.842
Ck(i,j)	1.800	1.942	2.258	2.400	2.100	0.100	0.114	0.951
Kr(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.420	1.000
GNP(i,j)	3.900	3.971	4.429	4.500	4.200	0.060	0.298	0.992
CWy(i,j)	2.059	2.059	2.143	2.143	2.101	0.020	0.420	1.000
CWo(i,j)	2.374	2.059	2.143	1.883	2.109	0.065	-.222	-.302
CW(i,j)	2.211	2.059	2.143	2.009	2.103	0.031	0.007	0.008

Steady state probabilities: [P(2,1) P(1,1) P(2,2) P(1,2)] = [.145 .355 .355 .145]

LEGEND: w = competitive real wage rate (maginal product of labor)
L = employment, Y = labor income, C = consumption
Kr = profits (capital X the rental rate), GNP = output
CW = contract wage (= contract compensation/contract employment)

Subscripts y, o, and k refer to the young worker, old worker, and capitalist, respectively.
Desciptive statistics are for the stationary distribution of (Xt-1,Xt).
The coefficient of variation is the standard deviation divided by the mean.
Autocor is the correlation coefficient of the variable with itself once lagged.
Coherence is the correlation with w, measuring the degree to which the variable is procyclical.

Equilibrium for the values:

[P11 P22 B K AN(1) AN(2) AK(1) AK(2)] = [.5 .5 .8 1 3 3.3 2 2.2]

	(2,1)	(1,1)	(2,2)	(1,2)	average	coef.var.	autocor	coherence
w(i,j)	3.000	3.000	3.300	3.300	3.150	0.048	-.000	1.000
Ly(i,j)	0.540	0.540	0.559	0.559	0.550	0.017	0.000	1.000
Lo(i,j)	0.394	0.425	0.449	0.477	0.436	0.070	-.424	0.874
L(i,j)	0.934	0.965	1.008	1.036	0.986	0.040	-.351	0.925
Yy(i,j)	1.620	1.620	1.845	1.845	1.733	0.065	0.000	1.000
Yo(i,j)	1.181	1.275	1.481	1.575	1.378	0.114	-.285	0.955
Y(i,j)	2.801	2.895	3.326	3.420	3.111	0.086	-.173	0.984
Cy(i,j)	1.380	1.380	1.455	1.455	1.418	0.026	0.000	1.000
Co(i,j)	1.819	1.725	1.819	1.725	1.772	0.026	0.000	0.000
C(i,j)	3.199	3.105	3.274	3.180	3.189	0.019	0.488	0.625
Ck(i,j)	1.602	1.790	2.252	2.440	2.021	0.167	-.266	0.961
Kr(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.000	1.000
GNP(i,j)	4.801	4.895	5.526	5.620	5.211	0.070	-.127	0.992
CWy(i,j)	2.556	2.556	2.602	2.602	2.579	0.009	0.000	1.000
CWo(i,j)	4.619	4.059	4.052	3.614	4.086	0.087	-.496	-.709
CW(i,j)	3.426	3.218	3.248	3.068	3.240	0.039	-.491	-.644

Steady state probabilities: [P(2,1) P(1,1) P(2,2) P(1,2)] = [.25 .25 .25 .25]

LEGEND: w = competitive real wage rate (maginal product of labor)
L = employment, Y = labor income, C = consumption
Kr = profits (capital X the rental rate), GNP = output
CW = contract wage (= contract compensation/contract employment)
Subscripts y, o, and k refer to the young worker, old worker, and capitalist, respectively.
Desciptive statistics are for the stationary distribution of (Xt-1,Xt).
The coefficient of variation is the standard deviation divided by the mean.
Autocor is the correlation coefficient of the variable with itself once lagged.
Coherence is the correlation with w, measuring the degree to which the variable is procyclical.

Equilibrium for the values:

[P11 P22 B K AN(1) AN(2) AK(1) AK(2)] = [.6 .6 .8 1 2 2.2 2 2.2]

	(2,1)	(1,1)	(2,2)	(1,2)	average	coef.var.	autocor	coherence
w(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.200	1.000
Ly(i,j)	0.542	0.542	0.557	0.557	0.550	0.014	0.200	1.000
Lo(i,j)	0.391	0.428	0.447	0.480	0.436	0.066	-.334	0.809
L(i,j)	0.933	0.970	1.004	1.037	0.986	0.036	-.258	0.877
Yy(i,j)	1.084	1.084	1.226	1.226	1.155	0.061	0.200	1.000
Yo(i,j)	0.782	0.855	0.983	1.055	0.919	0.108	-.153	0.934
Y(i,j)	1.866	1.939	2.209	2.281	2.074	0.081	-.012	0.977
Cy(i,j)	0.916	0.916	0.974	0.974	0.945	0.031	0.200	1.000
Co(i,j)	1.218	1.145	1.218	1.145	1.181	0.031	0.200	0.200
C(i,j)	2.134	2.061	2.192	2.119	2.126	0.024	0.592	0.714
Ck(i,j)	1.733	1.878	2.217	2.362	2.048	0.116	-.097	0.955
Kr(i,j)	2.000	2.000	2.200	2.200	2.100	0.048	0.200	1.000
GNP(i,j)	3.867	3.939	4.409	4.481	4.174	0.064	0.067	0.991
CWy(i,j)	1.690	1.690	1.748	1.748	1.719	0.017	0.200	1.000
CWo(i,j)	3.112	2.678	2.726	2.388	2.721	0.085	-.390	-.565
CW(i,j)	2.286	2.126	2.183	2.044	2.159	0.037	-.321	-.392

Steady state probabilities: [P(2,1) P(1,1) P(2,2) P(1,2)] = [.2 .3 .3 .2]

LEGEND: w = competitive real wage rate (maginal product of labor)
L = employment, Y = labor income, C = consumption
Kr = profits (capital X the rental rate), GNP = output
CW = contract wage (= contract compensation/contract employment)
Subscripts y, o, and k refer to the young worker, old worker, and capitalist, respectively.
Desciptive statistics are for the stationary distribution of (Xt-1,Xt).
The coefficient of variation is the standard deviation divided by the mean.
Autocor is the correlation coefficient of the variable with itself once lagged.
Coherence is the correlation with w, measuring the degree to which the variable is procyclical.

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