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"EFFICIENCY OF 25 NATIONAL UNIVERSITIES: AN APPLICATION OF DATA ENVELOPMENT ANALYSIS TO HIGHER EDUCATION"

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Abstract

Data envelopment analysis (DEA) is used to measure the relative efficiency of 25 national universities. The results indicate how DEA may be used to measure relative efficiency of these higher education institutions from commonly available "performance indicators." In addition to conventional comparative-static sensitivity analysis of inefficient universities, the effects of simultaneous perturbations to all components are examined. The sensitivity of efficient universities to perturbations which would cause the universities to become relatively inefficient is also analyzed. This methodology makes full use of the newest developments in the DEA field.

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I. Educational Efficiency and Perceived Quality

During the last several decades, administrators in higher education have become more willing to accept the notion of efficiency and its measurement in making informed managerial decisions. Problematic to viewing education in a production paradigm is the notion that several of the outputs of education are quite intangible. Moreover, some outputs like scholastic achievement can be measured by standardized exams; other outputs or goals relating to ethical and attitude development and creativity are more difficult to measure (Ray, 1991; Lindsay, 1982). Others have argued that even where measures exist, those measures are unsuitable for decision making at the institutional level (Romney, et. al., 1979). Nonetheless, few administrators can ignore external ranking analyses of their institution, either to justify past decisions or to support new policy initiatives.

One such ranking is the U.S. News (November 1991) "Top 25 National Universities" and "Best of the Rest" 179 national universities comprising 204 national universities using the guidelines established by the Carnegie Foundation for the Advancement of Teaching. In that study, five categories or attributes were collected and assigned importance weights: financial resources (18 percent); student satisfaction (7 percent); academic reputation (25 percent); student selectivity (25 percent); faculty resources (25 percent). (See Table 1.) In some cases the data used for the attributes are the most recent (e.g., fall 1991 average or midpoint SAT/ACT test scores); in

Table 1

Category	Fixed U.S. News Weights (%)	Performance Indicator	DEA
Reputation	25	(1) U.S. News Reputation Survey	
Student Selectivity	25	(1) Applicant Acceptance Rate (2) High School Class Standing (3) SAT/ACT Average or Midpoint (4) Percent in Upper 10% of High School Class	X_1
Faculty Resources	25	(1) Percent Faculty with Doctorates (2) Faculty to Student Ratio (3) Percent Faculty with Part Time Status (4) Tenured, Full Professors, Salary and Fringes	X_2 X_3
Financial Resources	18	(1) Educational and General Expenditures Per Student	X_4
Student Satisfaction	7	(1) Graduation Rate (2) Freshman Retention Rate	Y_1 Y_2
Total	100		

other cases, statistical averages over several years were used (e.g., 1983-86 freshmen who graduated within five years). Additional data which were used to calculate these attributes were also included in the ranking analyses for prospective parents' and students' comparisons in choosing a university. The Top 25 national universities were ranked individually, while the remaining 179 national universities were ranked into quartiles on the basis of these attributes.

Just as perceived quality indicators are useful in the context of choosing a college, efficiency indicators are useful in the context of administering these institutions. Presently, the existence of large federal and state deficits has created declining public financial support for institutions of higher education, and this situation has resulted in a renewed interest in efficiency and economy.

The production analysis approach views education as a process. This view combines many of the most critical variables which have been frequently perceived as "stand-alone," general performance indicators. A more comprehensive list of these indicators and their characteristics have been compiled for British primary and secondary schools and also higher education (Fitz-Gibbon, 1989). In the production context, graduation levels have been uniformly held to be the major output of education. The quality of graduates is much more problematic. Standardized achievement tests have been commonly used and criticized at the primary and secondary levels. Their use in

higher education is much more contentious. Post-graduation employment statistics and professional qualifying exam scores can be used in measuring the quality of professional programs, but these measures will only apply to a minority of programs. Arguably, the appropriateness of particular quality measures will differ between various colleges' missions within a university or for a particular college between competing universities.

Recently, retention rates have become a highly visible dimension of an institution's performance. A small number of public institutions with open admissions policies may view low retention rates as a desirable outcome of maintaining quality standards of these institutions. However, the prevailing "consumerist" view of education considers a high retention rate as a quality-outcome which is based on the university's ability to satisfy the expectations of the prospective graduates and their parents.

In the production context, the outputs, graduation and retention rates, being optimized comprise the U.S. News student satisfaction index. In a more limited sense, this model allows for the maximization of these two measures of student satisfaction, subject to the constraints imposed by the input indicators. Although this is not customary production analysis, based on input and output levels, this approach is consistent with the characteristics of the DEA model.

II. Variable Selection

Graduation and retention rates were used and reported in U.S. News and will be used as output indicators (Y_1 and Y_2) in the DEA model which will be presented. (See Table 1, p. 2.) Input indicators utilized in the DEA model include: SAT/ACT average or midpoint (X_1), percent of faculty with doctorates (X_2), faculty to student ratio (X_3), and educational and general expenditures per student (X_4). The input indicators are also illustrated in Table 1.

A regression analysis was performed to see whether the expected association between the input indicators, X_1, ..., X_4, and output indicators, Y_1 and Y_2, existed. A random sample of 25 universities was selected from the 204 national universities. The diagnostic statistics are reported for descriptive reasons only; statistical inferences would be questionable because of the extensive multi-collinearity between the independent variables. Other authors have approached the variable selection problem in a similar way (Sexton, et. al., 1986 and Bessent, et. al., 1982).

The results are reported in Table 2, p. 6. The use of these indicators is based partly upon the notion that they are easily available and are broadly recognized as general performance indicators. The estimates in Table 2 are made simply to denote that a plausible, but not proven, relationship exists between the input and output indicators. These regressions do not conform to standard production function estimates based on levels of inputs and outputs, since all of these indicators are ratios.

Table 2

Y_i / X_i	Graduation Rate (Y_1) GRAD		Freshman Retention Rate (Y_2) FRR	
	1.1	1.2	2.1	2.2
X_1 SAT/ACT	1.351***		7.754***	
X_2 FWDOC (% of Faculty with Doctorate)	-0.679**			0.346*
X_3 FSR (Faculty Student Ratio)		2.988**		
X_4 SPEND (Educational and General Expenditures per Student)		.016***		0.008***
Constant	-.250	.186**	2.143**	0.404**
R^2	.803	.712	.723	.568
F	44.940***	27.226***	60.131***	14.472***

* Statistically significant at the 0.10 level.
** Statistically significant at the 0.05 level.
*** Statistically significant at the 0.01 level.

A noticeable degree of collinearity exists between the inputs[1] indicating that they are interdependent and trade-offs between them are plausible. For this reason, several specifications of each model are shown to indicate that all of these input indicators are related to the various outputs. The signs of the variables and their statistical significance are often affected by multicollinearity.

The average or midpoint SAT/ACT score (X_1) is highly significant and uniformly positive relationship with both graduation (Y_1) and retention (Y_2) rates in equations (1.1) and (2.1). Although the level of accomplishment is likely to be the attribute of interest to employers and society, educators, especially those administering mass education in public institutions, are more likely to be concerned with increases in the level of knowledge (i.e., the value added) of their respective educational institution. If uniform achievement tests were administered to graduating seniors, an assessment of value added might be made. Short of this productivity measure, however, we simply accept the average or midpoint scores as a measure of the quality of entering freshmen to the education process.

The percent of faculty with doctorates, FWDOC, (X_2) has the lower correlations with the output indicators. The simple correlation matrix, not depicted here, shows that X_2 has the

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[1]Although the four correlation matrixes are not reported here, correlations between the X_i's are often as high as 0.7.

lowest simple correlation coefficients with the graduation rate (GRADR) and the second lowest correlation the freshmen retention rate (FRR). The negative sign in equation 2.1 indicates that the lower percentages of faculty with doctorates are associated with higher graduation and retention rates. Regression coefficients are significant at the five percent level in equation (1.2) and the ten percent level in equation (2.2), but only because of high collinearity with the other X_i's included in those specifications. For the purpose of DEA, however, increased efficiency could be achieved if this input indicator were reduced. Similarly, the faculty-student ratio is only in one specification (1.2), and arguably, increased efficiency could be achieved if this input indicator were reduced as well.

Educational and general expenditures per student, SPEND, (X_4) has positive simple correlations in all four specifications, but it achieves statistical significance in equations (1.2) and (2.2). Again, we expect that increased efficiency could result if this indicator were reduced.

The variables selected in this model bear a much more complex relationship than simple production analysis suggests. However, since satisfactory correlations exist between the input and output indicators, these ratios, as defined, are felt to bear the appropriate relationship to one another in the context of DEA.

III. DEA and Student Satisfaction in Education

Farrell (1957) proposed a method of measuring technical efficiency of organizations. This approach was extended to a linear programming technique by Charnes and Cooper (1978) and

Charnes, Cooper and Rhodes (1978, 1981) which became known as the CCR ratio model. To these were added the multiplicative model (Charnes, et. al., 1982) and later the additive model (Charnes, et. al., 1985). These models all utilize Farrell's criterion for efficiency.

In addition, DEA combines multiple inputs to produce multiple outputs or joint products and is, therefore, ideal for the evaluation of non-profit public sector services (for example, see Mayston, 1985). The results of the analysis provide a basis for classifying a decision making unit (DMU) or university as being efficient or inefficient. Moreover, sensitivity analysis techniques may be implemented to explore a given DMU's sensitivity to change with respect to reclassification.

Unique to this approach is that the weights used to determine the efficiency scores are not uniform across DMU's. The linear programming solution selects importance-weights for the DMU's set of inputs and outputs which cause the DMU to achieve an efficiency score approaching unity, unity being the maximum possible. Also, the constraints which the DEA model imposes on each university's selection of these weights assume this set of weights will not cause <u>another</u> university's efficiency score to exceed unity.

The additive model can be illustrated in the following way. Each university can be represented by an input-output vector. This vector is compared to the empirical production possibility set, PE:

$$\text{where} \quad PE = (Y^T, X^T) = \sum_{k=1}^{25} \mu_k (Y^T, X^T); \; \sum_{k=1}^{25} \mu_k = 1 \text{ and } \mu_k \geq 0,$$

and where (Y^T_k, X^T_k) is the transposed vector of outputs and inputs for university k.

An efficient university is Pareto-Koopmans efficient with respect to the best performers of this set. For all of the DEA models, including the additive model, if no input-output vector equals or exceeds all of a particular university's output levels while simultaneously not exceeding (exceeding here referring to less than) any of that university's input levels with a strict inequality in at least one input or output component, then that university is said to be efficient. For example, let:

$\begin{pmatrix} Y_o \\ X_o \end{pmatrix}$ = Particular university's input-output vector currently being classified;

e^T = The summation vector comprised of twenty-five ones which allows compatible inner products which are used to compute the efficiency score by the summation of the optimal slacks;

s^{+T} = Output shortfall slacks for two outputs, s^+_1 and s^+_2;

s^{-T} = Input excess slack for four inputs, s^-_1, s^-_2, s^-_3, and s^-_4;

Y = Matrix of universities' outputs;

X = Matrix of universities' inputs.

For a particular university (DMU_0), the test for efficiency becomes:

$$\begin{aligned} \text{Minimize} \quad & -e^T s^+ - e^T s^- \\ \text{Subject to} \quad & Y\lambda - s^+ = Y_o \\ & X\lambda + s^- = X_o \\ & e^T\lambda = 1 \\ & \lambda, s^+, s^- \geq 0. \end{aligned}$$

Note that k linear programs must be solved, since the coefficients or weights are different for each university's efficiency score. For an efficient university, the slack variables in the constraints are equal to zero and the minimized objective function is also equal to zero. For an inefficient university, positive slacks will appear and the sum of the slacks can be used to compute the efficiency score. The efficiency score, following Charnes, et. al., (1984), is calculated as the efficient projection computed from the optimal sum of the slacks:

$$Z_k = \frac{e^T Y_p + e^T X_p}{e^T Y_p + e^T X_p + e^T s^+ + e^T s^-}$$

where Y_p and X_p represent the efficient projections of an inefficient university's output and input components. Note that if the slacks are zero, $Z_k = 1$ for an efficient university. The efficient reference universities form a "facet" against which an inefficient university can be made efficient by referencing and utilizing the lambdas appearing in the optimal basis. In this way the amount which input components need to be reduced and/or output components need to be increased can be projected. This (hypothetical) university would now have new levels of input-output indicators represented by $\begin{pmatrix} Y\lambda^* \\ X\lambda^* \end{pmatrix}$ and would be efficient with respect to the empirical production possibility set.

In all DEA models, however, efficiency scores are not "units invariant." See Haag and Jaska (1992) for an example. Correction of this problem can be accomplished by prescaling the original data. One approach can be done by dividing each input and output by the corresponding component average. Alternatively

the scaling could be performed implicitly by solving the linear programs using diagonal matrices of output and input component averages. See Haag, et. al., (1992) for an explanation of this procedure. A units invariant efficiency score can then be computed from the optimal slacks and lambdas. This "implicit" approach was utilized in the computation of the scores in this paper. These scores (Z_k) are shown in Table 3.

It must be emphasized from the above discussion that, because of the scaling issues discussed in Haag and Jaska (1992), these scores can only be used to classify universities as efficient or inefficient.

IV. The Stability of the Classifications to Changes in Inputs and Outputs

In the past, stability analysis of an optimal solution employed an algebraic technique for measuring the sensitivity of input and output changes necessary to make an inefficient DMU efficient. The conventional analysis employed a comparative static approach by changing a single input or output until the DMU could be made efficient. In the analysis which follows, the newest developments in stability analysis are utilized. These contributions are possible by employing a geometric approach to sensitivity analysis. Following Charnes, et. al., (1992), a symmetric cell comprising the region of stability is computed whose length (i.e., the radius of stability) will determine the extent of input and/or output changes which would be required to change the classification. See Jaska (1991) for a graphical illustration of the properties of the various stability cells.

Table 3

DMU	Efficiency Score* Z_k	Tau or R Value*
1 Harvard	1.000	0.012
2 Dartmouth	1.000	0.015
3 Rice	1.000	0.036
4 Univ of Michigan	1.000	0.159
5 NY University	0.805	0.046
6 U. of CA, Irvine	1.000	0.042
7 U. of TX, Austin	1.000	0.041
8 George Washington U.	0.887	0.041
9 Ohio State U.	0.882	0.003
10 Stevens Inst. Tech	1.000	0.020
11 U. of Cincinnati	0.836	0.044
12 U. of Georgia	1.000	0.000
13 U. of Tulsa	0.888	0.019
14 Catholic U. of America	0.850	0.028
15 Hofstra U.	1.000	0.018
16 New School for S. Res.	1.000	0.071
17 U. of Alabama	1.000	0.057
18 U. of New Hampshire	1.000	0.035
19 Virginia Common	0.799	0.102
20 Indiana St.	1.000	0.096
21 Montana State U.	1.000	0.098
22 Old Dominion U.	1.000	0.005
23 U. of Maine	0.960	0.016
24 Univ of N. Texas	0.951	0.012
25 U. of TX, Arlington	1.000	0.055

* Shown to only three significant digits.

Two measures of the radius of stability are examined, the ∞-norm and the 1-norm. The ∞-norm is employed when simultaneous perturbations across all of the components are of interest. Moreover, this symmetric cell, whose dimension is referred to as the "radius of stability," represents the maximum simultaneous perturbation (i.e., changes to inputs and outputs) which will move the DMU under scrutiny to "virtual" efficiency. Virtual efficiency here referring to the placement of a given DMU on the efficient frontier such that any miniscule perturbation will change the DMU's classification.

However, if the responses of a select set of components are to be examined for resource reallocations, then the 1-norm should be employed. One cannot know which technique will yield the more appropriate region of stability beforehand, but it has been recommended that the ∞-norm be employed first because of computational ease. If the ∞-norm results are satisfactory, then the 1-norm computation need not be undertaken (Jaska, 1991, p. 58).

Another important application of this new approach is that the stability conditions of efficient DMU's are examined. This is critical since 16 out of the 25 universities were identified as efficient, and past comparative static techniques had no means of distinguishing which universities were "robustly" efficient and which universities were "barely" efficient. In section V both the ∞-norm and the 1-norm analysis of stability are applied to efficient universities.

V. Sensitivity Analysis of the Efficient Universities: The ∞-Norm Vector Measure.

The stability cell has as its maximum boundaries a set of points whose largest component is exactly R_∞. It is chosen as the maximum of the absolute value of its components. The selection of R_∞ is found from the solution of the following linear program:

$$\min \theta$$

$$\begin{aligned} \text{subject to:} \quad & Y^{(E)}\lambda - S^+ + \theta d_o = Y_E \\ & X^{(E)}\lambda + S^- - \theta d_I = X_E \\ & e^T\lambda = 1 \end{aligned}$$

$$\lambda,\ S^+,\ S^-,\ \theta \geq 0$$

where:

d_0^T = (1,1,...,1), transposed summation vector for outputs,

d_I^T = (1,1,...1), transposed summation vector for inputs,

Y_E = vector of outputs for efficient DMU_E,

X_E = vector of inputs for efficient DMU_E,

$Y^{(E)}$ = matrix of outputs with the output vector for DMU_E removed,

$X^{(E)}$ = matrix of inputs with the input vector for DMU_E removed.

Table 4 ranks the 16 efficient universities according to the R_∞ value from most robust efficient classification, University of Michigan (DMU_4) to the least robust, University of Georgia (DMU_{12}).

For example two efficient universities, University of Michigan (DMU_4) and University of Georgia (DMU_{12}), had extremely different and contrasting R_∞ or tau values: 0.1591 and 0.00004. See Table 4, p. 17. A scaled and simultaneous maximum perturbation of 0.1591 across all components would move the University of Michigan to "virtual" efficiency, but a change in excess of 0.1591 will change Z_k to less than unity (1). Similarly a scaled and simultaneous maximum perturbation of .00004 would move the University of Georgia to "virtual" efficiency. If a simultaneous change occurs, however slightly beyond 0.00004, the University of Georgia will fall below the efficient frontier.

The 1-Norm Radius of Stability for an Efficient DMU.

While the ∞-norm defines the maximum simultaneous perturbation to all components to reach virtual efficiency, the 1-norm defines the minimum total change beyond which an efficient classification can no longer be guaranteed. The stability cell for the 1-norm with radius R_1 has, at its boundary, the set of points whose coordinates sum (in absolute value) to R_1.

In practical terms, the 1-norm defines a set of components and the sum of the perturbation values (R_1) that should not be exceeded in that set of components. An efficient DMU will become inefficient if the R_1 perturbation value is exceeded in the 1-norm defined set of components. An efficient DMU's classification may also change if the R_1 perturbation value is exceeded in a set of components not defined by 1-norm, but that

Table 4
Efficient Universities (Z_k=1)

DMU	National Universities	R_{oo} or Tau Value of Robustness
4	U. of Michigan	.1591
21	Montana St. U.	.0983
20	Indiana St. U.	.0960
16	New School for S.R.	.0706
17	U. of Alabama	.0566
25	U. of Texas, Arlington	.0552
6	U. of California, Irvine	.0424
7	U. of Texas	.0413
3	Rice U.	.0359
18	U. of New Hampshire	.0349
10	Stevens Inst. of Tech.	.0196
15	Hofstra	.0178
2	Dartmouth	.0153
1	Harvard	.0123
22	Old Dominion U.	.0050
12	U. of Georgia	.0000

* Shown only to four significant digits.

reclassification is not guaranteed by the 1-norm sensitivity analysis. An efficient DMU will retain its classification if the total perturbation to the DMU is less than R_1, regardless of the set of components perturbed.

The computation of R_1 is found by solving the following linear program for each efficient DMU:

$$\min \; e^T w^+ + e^T w^-$$

$$\begin{aligned}
\text{s.t. } & Y^{(E)}\lambda - s^+ + w^+ = Y_E \\
& X^{(E)}\lambda + s^- - w^- = X_E \\
& e^T\lambda = 1 \\
& \lambda,\ s^+,\ s^-,\ w^+,\ w^- \geq 0,
\end{aligned}$$

where previously undefined vectors are:

w^+ = vector of output change of w_{y1} and w_{y2},

w^- = vector of input change of w_{x1}, w_{x2}, w_{x3}, and w_{x4}.

The R_1 value is the sum of the absolute values of the component changes found in the vectors w^+ and w^-. Unlike the R_∞ measure, where all components must be perturbed simultaneously, the individual components (and their perturbation summation) of the R_1 measure demonstrate the fastest way in which a DMU (university) could lose its "efficient" classification.

The R_1 sensitivity analysis is exampled by evaluating its application to the University of Michigan (DMU_4), the most "robustly" efficient university in terms of the R_∞ analysis. See Table 5 for the numerical analysis of this university utilizing R_1.

Table 5

R_1 Analysis for the University of Michigan
and the University of Georgia

The University of Michigan (DMU_4)

Inputs	Actual Value (1)	R_1 Value (2)	R_1 Adjustment (3)	Adjusted Value (4)	Percentage Movement (5)
X_1 SAT/ACT	1175.000	N/A	N/A	1175.000	N/A
X_2 FWDOC	98.000	N/A	N/A	98.000	N/A
X_3 FSR	0.053	N/A	N/A	0.053	N/A
X_4 SPEND	14237.000	N/A	N/A	14237.000	N/A
Outputs					
Y_1 GRAD	80.000	0.298	17.61	62.39	22.01
Y_2 FRR	95.000	0.070	5.68	89.32	5.98

$R_1 = 0.368$

The R_1 perturbation value for the University of Michigan is 0.368 (i.e., $R_1 = W_{y1} + W_{y2} = 0.298 + 0.070 = 0.368$). The University of Michigan will remain efficient if any scaled sum of perturbations to any combinations of components is less than 0.368. Conversely, the University of Michigan will no longer be guaranteed of its efficiency status if a scaled sum of perturbations exceeds 0.368. Specifically, it is known that the University of Michigan will become virtually efficient if the graduation rate is adjusted by (W_{y1} =) 0.298 (i.e., GRAD falls by 17.61 percent, from 80.00 percent to 62.39 percent), and the freshmen retention rate is adjusted by (W_{y2} =) 0.070 (i.e., FRR falls by 5.68 percent, from 95.00 percent to 89.32 percent). Additionally, the University of Michigan will become inefficient if the scaled sum of perturbations for GRAD and FRR (Y_1 and Y_2) exceeds 0.368, irrespective of the allocation of perturbation sum within the two outputs. By using the R_1 analysis, the impact (to efficiency classification) of the trade-offs between components that are management controlled and those which are not controllable could be assessed.

VI. Sensitivity Analysis of the Inefficient Universities: The ∞-Norm Vector Measure.

Unlike the comparative statics of the conventional algebraic stability analysis, the R_∞ measure of stability shows the minimal simultaneous perturbations (decrease) in inputs and (increase) in outputs that would move an inefficient DMU to virtual efficiency.

The selection of R_∞ is found from the solution of the following linear program:

$$\text{Maximize } \theta$$

$$\begin{aligned} \text{subject to:} \quad & Y\lambda - s^+ - \theta d_o = Y_o \\ & X\lambda - s^- + \theta d_I = X_o \\ & e^T\lambda = 1 \\ & \lambda,\ s^+,\ s^-,\ \theta \geq 0. \end{aligned}$$

For example, two inefficient universities, Ohio State University (DMU_9) and Virginia Commonwealth University (DMU_{19}), have very different R_∞ or tau values: 0.0031 and 0.1018. See Table 6, p. 22 for a ranking of R_∞ values from least robust (nearly efficient) to most robust (largest adjustments to attain efficiency). A scaled and simultaneous decrease in inputs and increase in outputs of .0031 would allow Ohio State to become virtually efficient. Inasmuch as Ohio State requires such a small adjustment in inputs and outputs, one could conclude that it is "nearly" virtually efficient. In other words, it almost lies on the efficiency frontier.

In contrast to Ohio State, Virginia Commonwealth has a large R_∞ value, illustrating its relatively "large" simultaneous perturbation distance from the efficiency frontier. A scaled and simultaneous perturbation of 0.1018 would allow Virginia Commonwealth to become virtually efficient. Clearly, Ohio State is practically on the frontier, while Virginia Commonwealth has considerable adjustments to make in order to become efficient.

Table 6

Inefficient Universities ($Z_k < 1$)

DMU	National University	R_∞ or Tau Value of Robustness*
9	Ohio State U.	0.0031
24	U. of N. Texas	0.0120
23	U. of Maine	0.0155
13	U. of Tulsa	0.0187
14	Catholic U. of America	0.0281
8	George Washington U.	0.0412
11	U. of Cincinnati	0.0440
5	New York U.	0.0463
19	Virginia Commonwealth U.	0.1018

* Shown only to four significant digits.

What minimal specific adjustments could be made to approach the frontier can best be seen by examining the R_1 radius of stability.

The 1-Norm Vector Measure for Inefficient Universities.

Similar to conventional analysis, a comparative static approach is employed in evaluating the components to determine how much adjustment to each component individually is necessary in order for an inefficient DMU to approach the efficiency frontier. In other words, for each individual component, the R_1 or radius of stability represents the minimum required adjustment between an inefficient DMU and one (perhaps hypothetical) which is virtually efficient. Inasmuch as two outputs and four inputs are utilized, six linear programs are solved for inefficient university I. Consider six linear programs which:

$$\text{maximize } e_i^T w$$

$$\begin{array}{lllll} \text{subject to:} & Y\lambda & -w^+ & = Y_I \\ & X\lambda & +w^- & = X_I \\ & e^T\lambda & & = 1 \\ & & \lambda,\ w & \geq 0. \end{array}$$

where: e^T = vector of zeroes with a '1' in the position of the ith input or output w;

w = R_1 value for the ith input or output.

Note that six w values will be determined, one for each input and output. An examination of these values will suggest to the policy maker what changes in individual components are necessary for a university to approach relative efficiency.

The sensitivity of the components to approaching relative efficiency is illustrated in Table 7 for Ohio State University (DMU_9) and Virginia Commonwealth University (DMU_{19}). The scaled R_1 values for Ohio State are shown in column (2). The minimum percentage changes required in order for Ohio State to achieve virtual efficiency are for SAT/ACT scores (X_1) to fall by approximately 7/10 of a percent (from 995 to 988.3) or for freshmen retention rate FRR (Y_2) to rise by 6/10 of a percent (from 83.00 to 83.50 percent). On the other hand, a very substantial percentage drop in per student expenditures (X_4) of approximately 33 percent (column 5) would have to occur for Ohio State to become virtually efficient. Note that the R_1 values in column (2) represent raw adjustment values. These must be multiplied by the mean of the corresponding components in order to obtain the R_1 adjustment shown in column (3). For X_1, a decrease in SAT/ACT scores of 6.67 (column 3), holding all other components constant, would move Ohio State to virtual efficiency. At this point, statements regarding which of the individual component perturbations should be attempted to achieve virtual efficiency cannot be made. That decision must be left to the policy makers. Indeed, the policy makers of Ohio State University may decide that it would be more appropriate to adjust per student spending by 33 percent rather than to target a group of potential incoming students that have an approximate 7/10 of a percent decrease in current levels of SAT/ACT scores.

Table 7

R_1 Analysis for Ohio State University and Virginia Commonwealth University

Ohio State University (DMU_9)

Inputs	Actual Value (1)	R_1 Value (2)	R_1 Adjustment (3)	Adjusted Value (4)	Percentage Movement (5)
X_1 SAT/ACT	995.000	0.006	6.667	988.333	0.67
X_2 FWDOC	95.000	0.143	12.805	82.195	13.48
X_3 FSR	0.067	0.199	0.014	0.053	21.14
X_4 SPEND	10961.000	0.302	3584.466	7376.534	32.70
Outputs					
Y_1 GRAD	46.000	0.151	8.919	54.919	19.39
Y_2 FRR	83.000	0.006	0.500	83.500	0.60

Virginia Commonwealth University (DMU_{19})

Inputs					
X_1 SAT/ACT	980.000	0.158	170.000	810.000	17.35
X_2 FWDOC	87.000	0.167	15.000	72.000	17.24
X_3 FSR	0.071	0.390	0.028	0.044	38.61
X_4 SPEND	11163.000	0.574	6816.001	4346.999	61.06
Outputs					
Y_1 GRAD	35.000	0.518	30.604	65.604	87.44
Y_2 FRR	63.0000	0.239	19.375	82.375	30.75

In contrast, the R_1 perturbations for Virginia Commonwealth University demonstrate relatively large percentage perturbations to the individual components to achieve virtual efficiency. The minimum percentage changes required for Virginia Commonwealth University to achieve virtual efficiency are for SAT/ACT scores (X_1) to fall by 17.4 percent from 980 to 810 or to reduce percent of faculty with doctorates (X_2) by 17.2 percent from 87 percent to 72 percent. The largest percentage perturbation for Virginia Commonwealth to achieve virtual efficiency is to increase graduation rate (Y_2) by 87.44 percent, from 35 percent to approximately 65 percent. Again, the perturbations defined in the R_1 analysis are for each component taken individually while holding all others at current levels.

VII. Summary and Conclusions

From a random sample of 25 out of 204 national universities, a units invariant DEA analysis of the relative efficiency was performed. Of this group, 16 universities were classified as efficient and 9 were classified as inefficient. The R_∞ and the R_1 radii of stability were calculated for simultaneous and individual perturbations required for both efficient and inefficient reclassifications. Moreover, the practical implications of these techniques were demonstrated.

Several caveats concerning the interpretation and application of this data must be added. The DEA model is sensitive to the specification of the components. Inefficient universities may always claim, and perhaps correctly so, that it

performs a "special" mission and this mission is not measured in the specification of outputs. If the level of this output can be quantified across the other institutions, then it should be included as an output, and a determination could be made whether the efficiency classifications remain consistent.

In addition, the model includes a sample of twenty-five of the national universities. If different institutions were chosen as part of the sample, the efficiency frontier would change and so would the efficiency classifications. For this reason a group of mutually agreed upon set of peer institutions might be chosen beforehand, so that the implications of the results could be agreed upon once the analysis is performed.

As these caveats should demonstrate, the analysis only points out where deficiencies exist for a DMU relative to its reference group, but it cannot determine why they exist. Judicious interpretation of the results is a must if DEA is to be effectively utilized.

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