The dynamic structure factor of XXZ chain

Alex Kamenev

UNIVERSITY OF MINNESOTA

in collaboration with

Leonid Glazman, Yale
Maxim Khodas, U of M
Michael Pustilnik, Georgia Tech

PRL **96**, 196405 (2006);
PRB **76**, 155402 (2007);
PRL **99**, 110405 (2007);
arXiv:0710.2910

AFM Spin Chain Compounds

R. CACIUFFO *et al.*

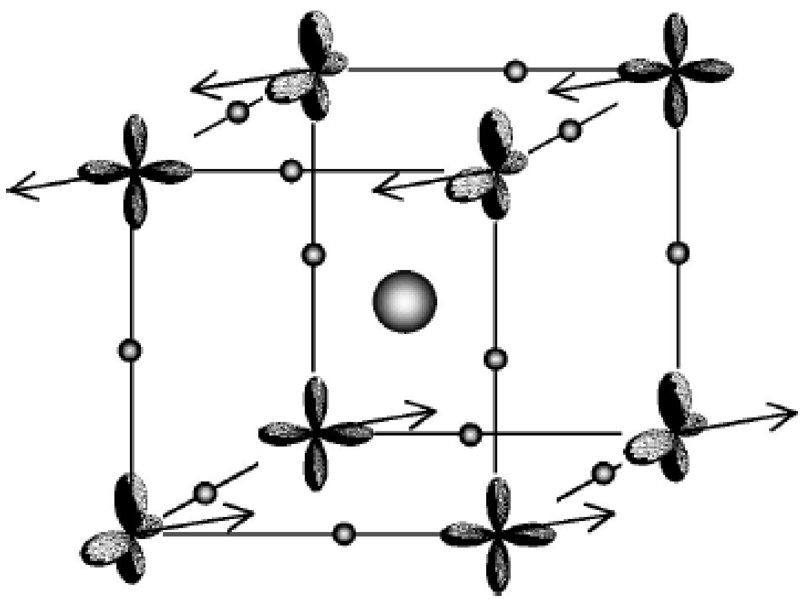
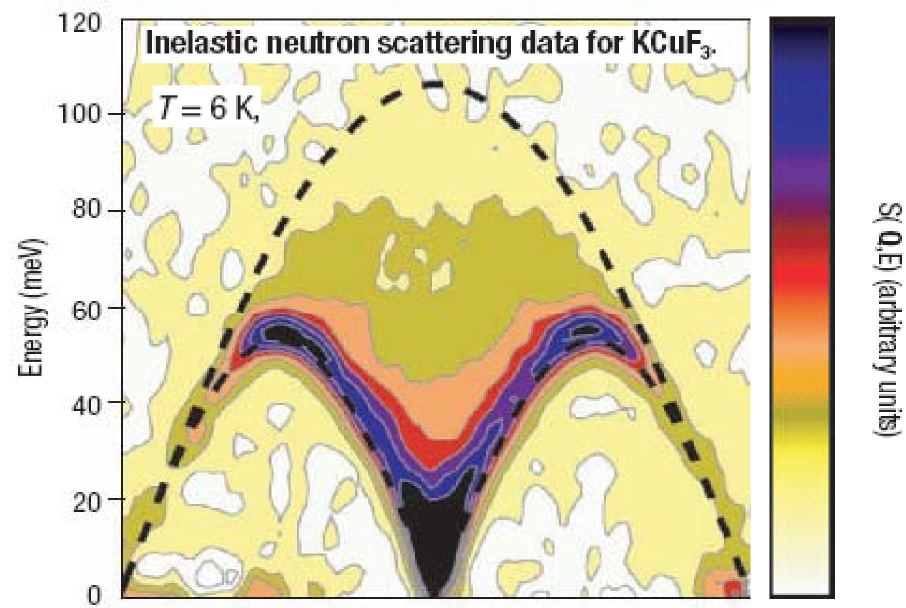


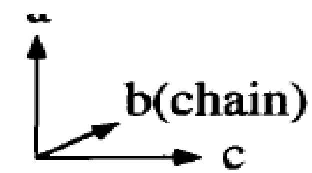
FIG. 8. The *type a* crystal structure of $KCuF_3$. A possible ordered pattern of the Cu 3*d* orbitals is schematically shown. The large circle represents the K ions, the small, full circles are the F ions. The arrows indicate the direction of the magnetic moments in the ordered phase.

Sr_2CuO_3



BELLA LAKE[1,2*†], D. ALAN TENNANT[2,3‡], CHRIS D. FROST[3] AND STEPHEN E. NAGLER[1]

nature materials | VOL 4 | APRIL 2005



XXZ chain

$$H = J \sum_{n=1}^{N} \left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z - h S_n^z \right)$$

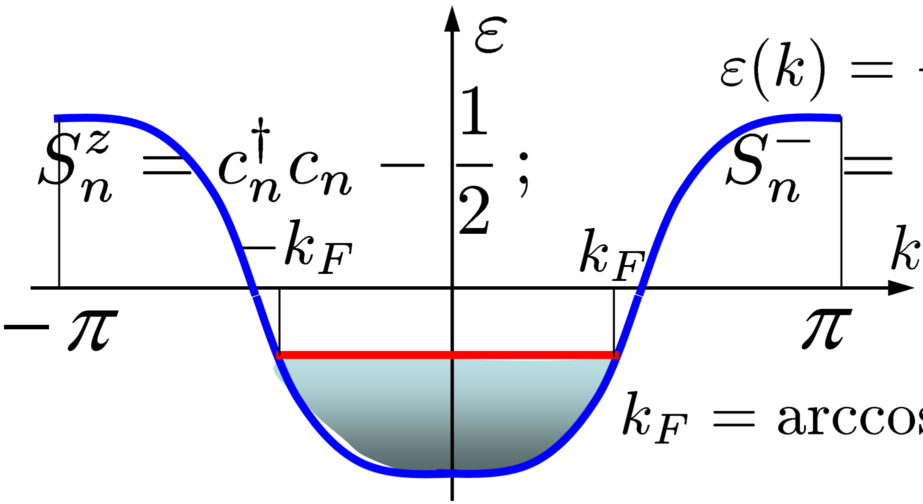
$$-1 \le \Delta \le 1$$

Gapless phase

$$S^{zz}(q,\omega) = \sum_{n=1}^{N} e^{-iqn} \int dt\, e^{i\omega t} \left\langle S_n^z(t) S_0^z(0) \right\rangle$$

$$-\pi \le q \le \pi$$

Jordan-Wigner Transformation



$$\varepsilon(k) = -J\cos k$$

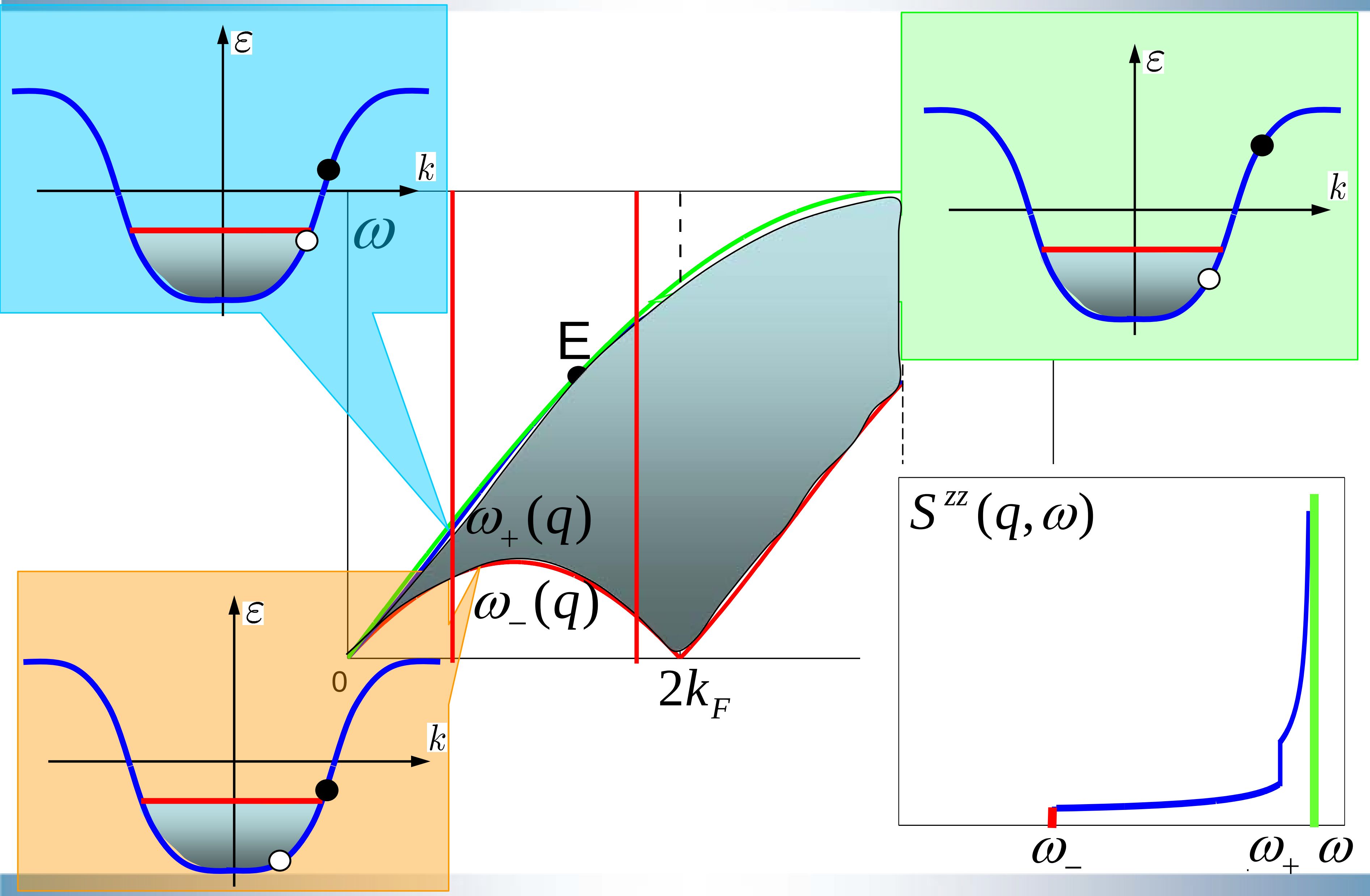
$$S_n^z = c_n^\dagger c_n - \frac{1}{2}\,; \quad S_n^- = (-1)^n c_n \exp\left\{-i\pi \sum^{n-1} c_{n'}^\dagger c_{n'}\right\}$$

$$U(q) = 2J\Delta\cos q$$

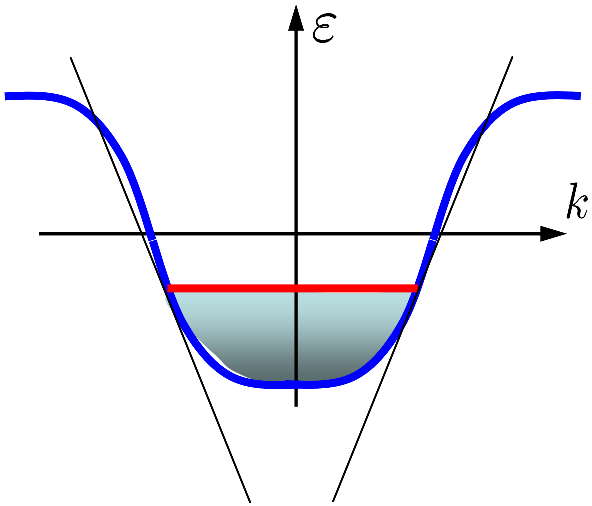
$$k_F = \arccos(-h)$$

$$H = -\frac{J}{2}\sum_{n=1}^{N}\Big[c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n + 2hc_n^\dagger c_n - 2\Delta\left(c_n^\dagger c_n - \frac{1}{2}\right)\left(c_{n+1}^\dagger c_{n+1} - \frac{1}{2}\right)\Big]$$

XX model $\Delta = 0$



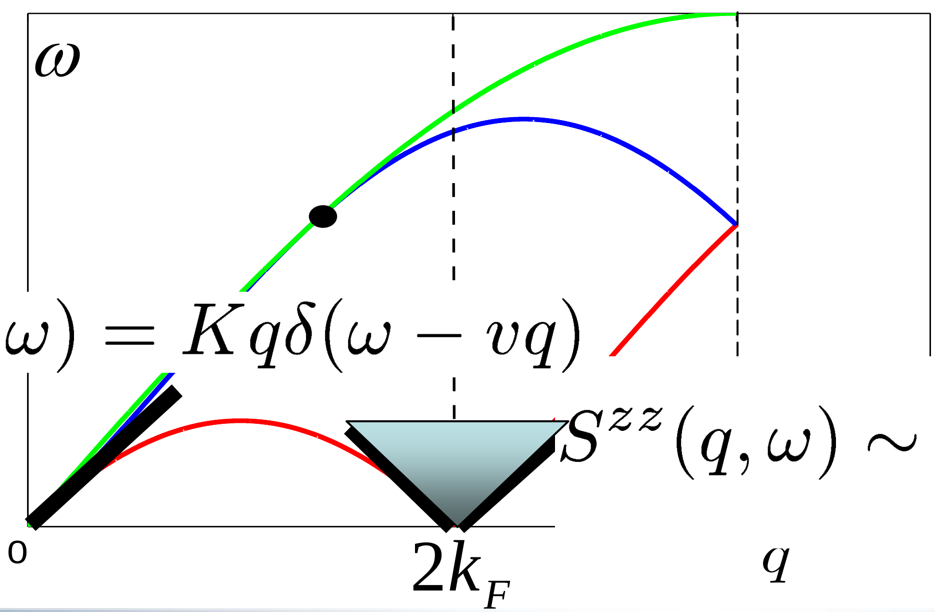
Bosonization



$$H_L = \frac{v}{2\pi}\int dx\left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right]$$

$$v \approx v_F\left(1 + \frac{2\Delta}{\pi}\sin k_F\right)$$

$$K \approx 1 - \frac{2\Delta}{\pi}\sin k_F$$



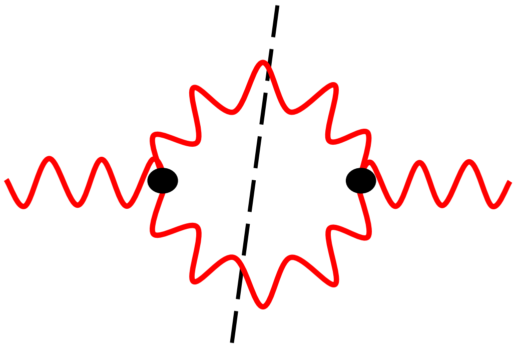
$$S^{zz}(q,\omega) = Kq\delta(\omega - vq)$$

$$S^{zz}(q,\omega) \sim \frac{1}{(\omega \mp v(q-2k_F))^{1-K}}$$

Bosonisation + Curvature

$$H = H_L + \frac{1}{6m} \int dx \left[3(\partial_x \theta)^2 (\partial_x \phi) + (\partial_x \phi)^3\right] \qquad \frac{1}{m} = \varepsilon''(k_F)$$

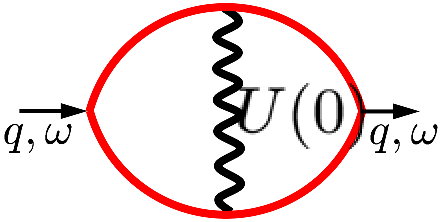
See R.G. Pereira et al. 2007 for expression of the coupling constants through Bethe ansatz.

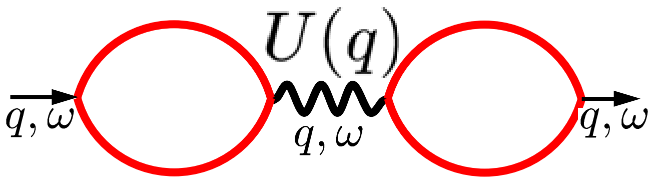


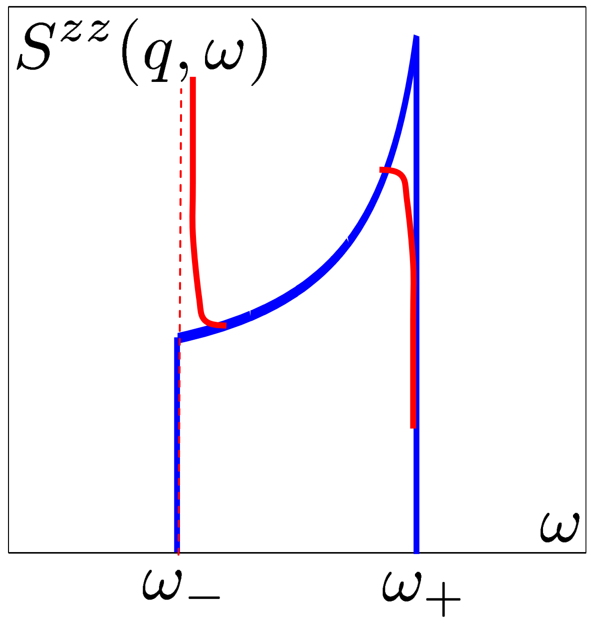
- ✓ Divergent at the mass shell, due to the linear dispersion relation of bosons.
- ✓ No resummation method available

Interaction as a perturbation

1-st order corrections:







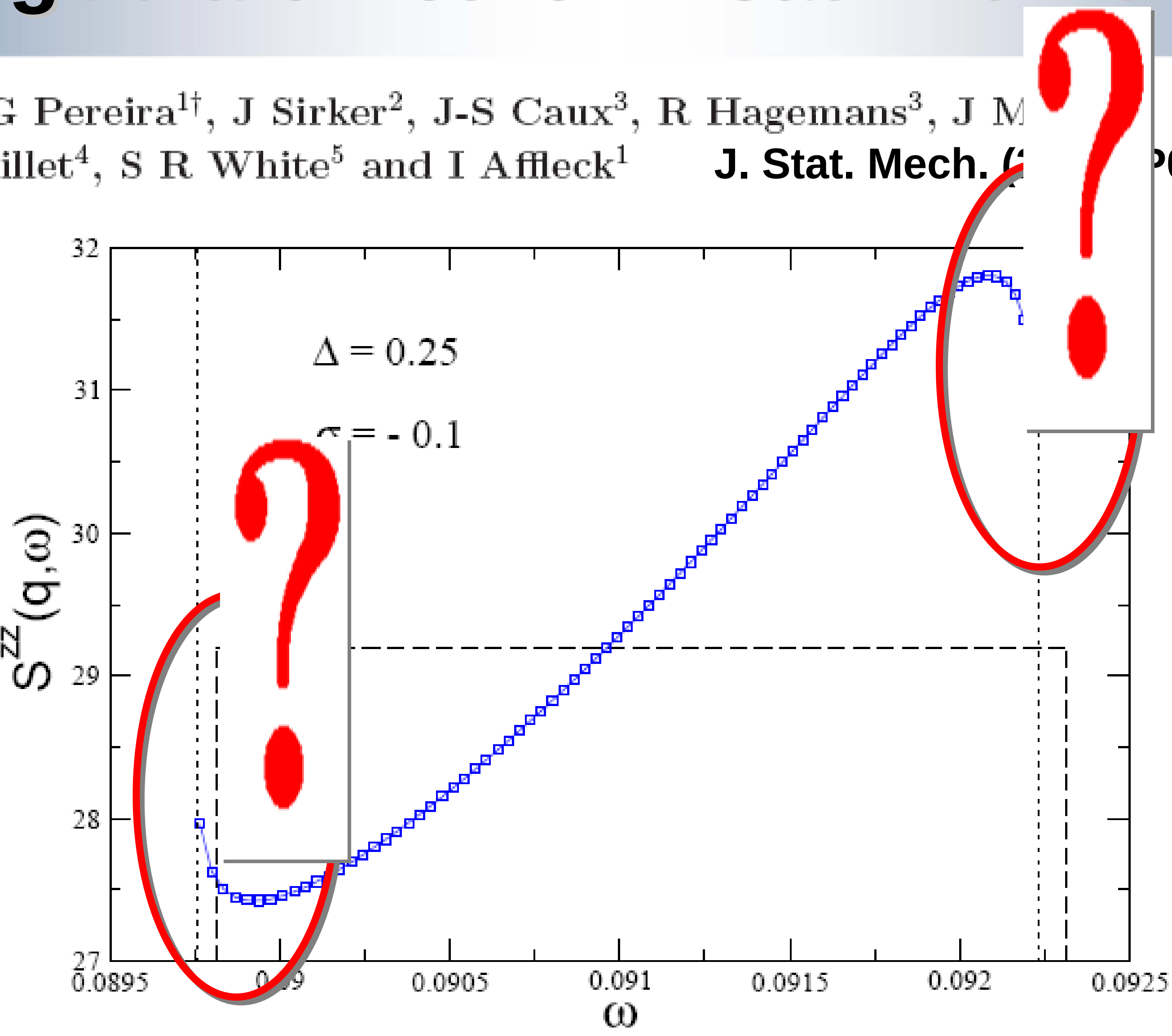
$$\delta S \propto (U(0) - U(q))\, \mathrm{Im}\chi_0(q,\omega)\, \mathrm{Re}\chi_0(q,\omega)$$

Kramers-Kronig

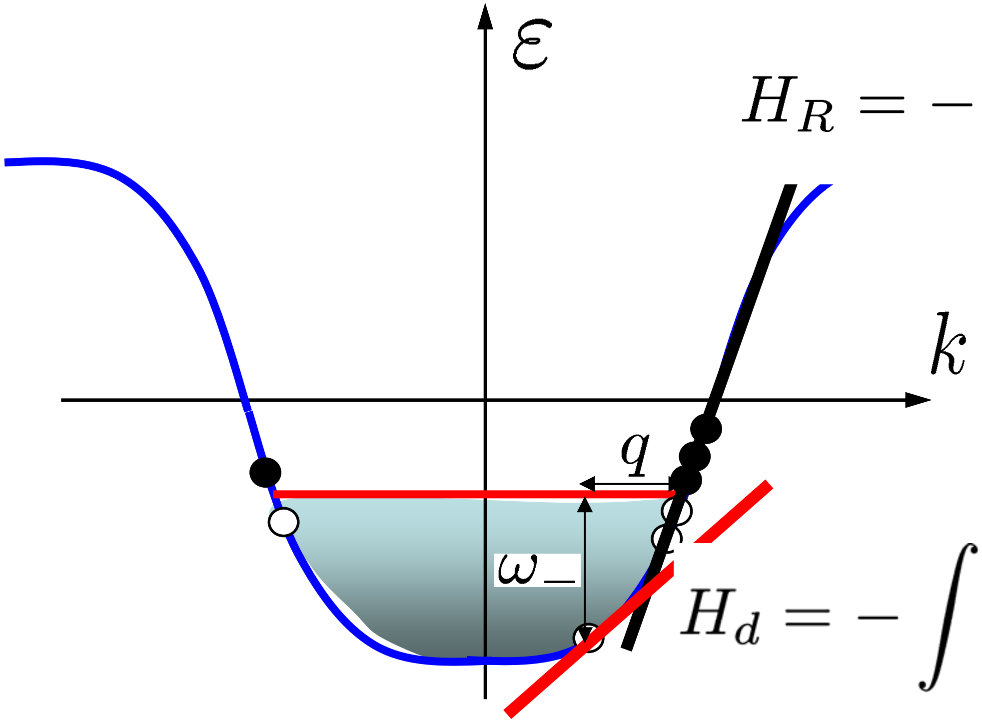
$$\delta S \sim (U(0) - U(q))\, \ln\left[\frac{\omega_+ - \omega}{\omega - \omega_-}\right]$$

Algebraic Bethe Ansatz Numerics

R G Pereira[1†], J Sirker[2], J-S Caux[3], R Hagemans[3], J M
Maillet[4], S R White[5] and I Affleck[1] **J. Stat. Mech. (2 P08022**



Lower Spectral Edge

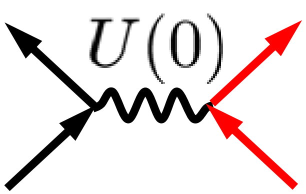


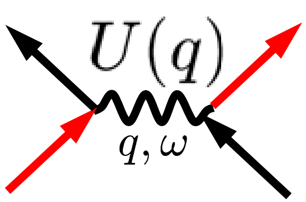
$$H_R = -\int dx\, \Psi_R^\dagger(x) v_F i\partial_x \Psi_R(x)$$

$$v_d(q) = \varepsilon'(k_F - q)$$

$$H_d = -\int dx\, d^\dagger(x)\left[\omega_-(q) + v_d i\partial_x\right] d(x)$$

$$H = H_R + H_d + (U(0) - U(q)) \int dx\, (d^\dagger d)(\Psi_R^\dagger \Psi_R)$$





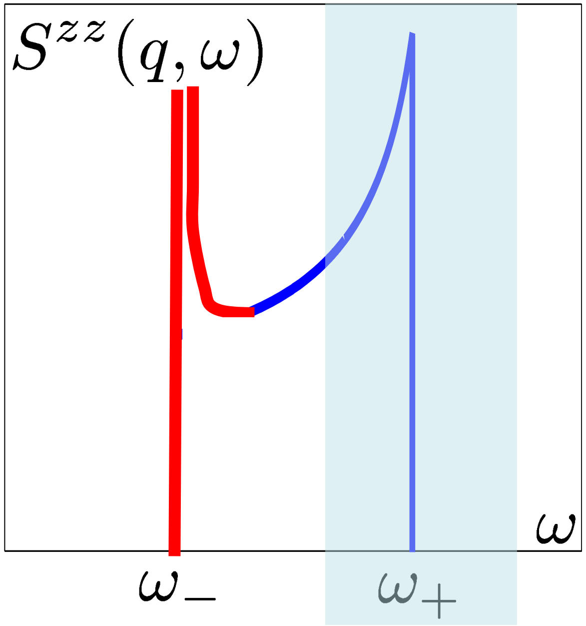
$$U(q) = 2J\Delta \cos q$$

Lower Spectral Edge (cont)

$$\langle S_n^z(t) S_0^z(0)\rangle = \langle \Psi_R^\dagger(x,t) d(x,t) d^\dagger(0,0)\Psi_R(0,0)\rangle$$

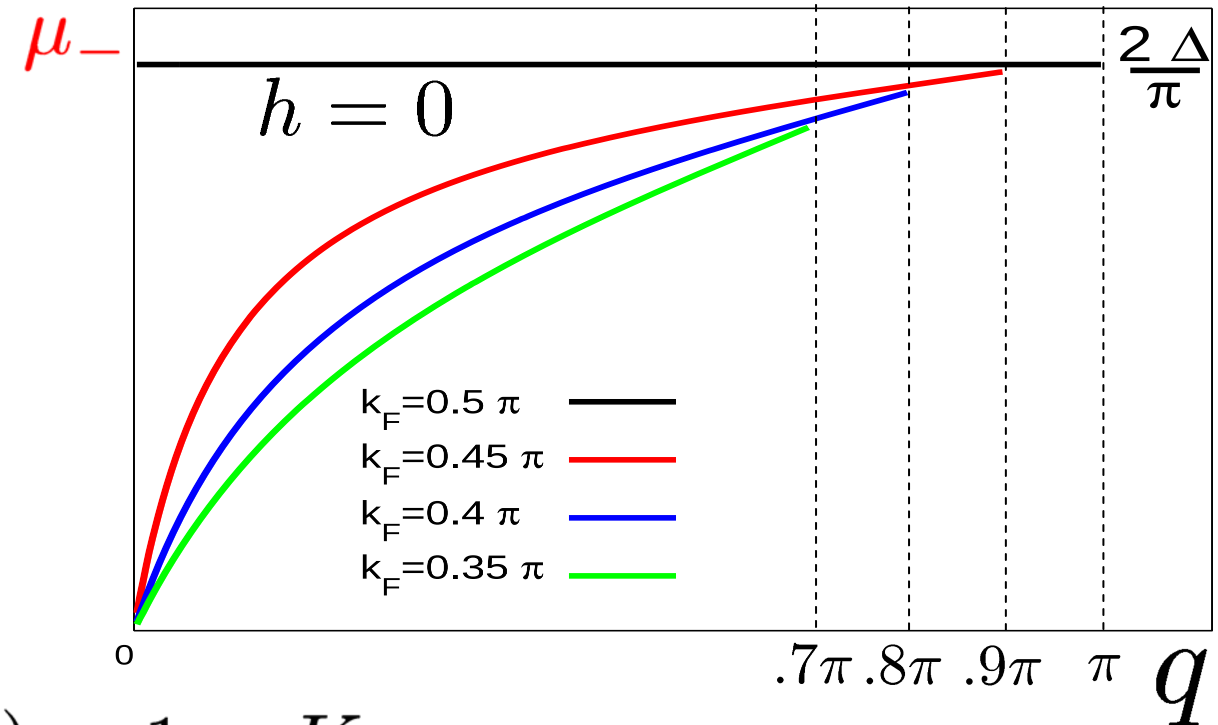
$$S^{zz}(q,\omega) \sim \frac{1}{[\omega - \omega_-(q)]^{\mu_-(q)}}$$

$$\mu_-(q) = \frac{1}{\pi}\frac{U(0)-U(q)}{v_F - v_d(q)}$$



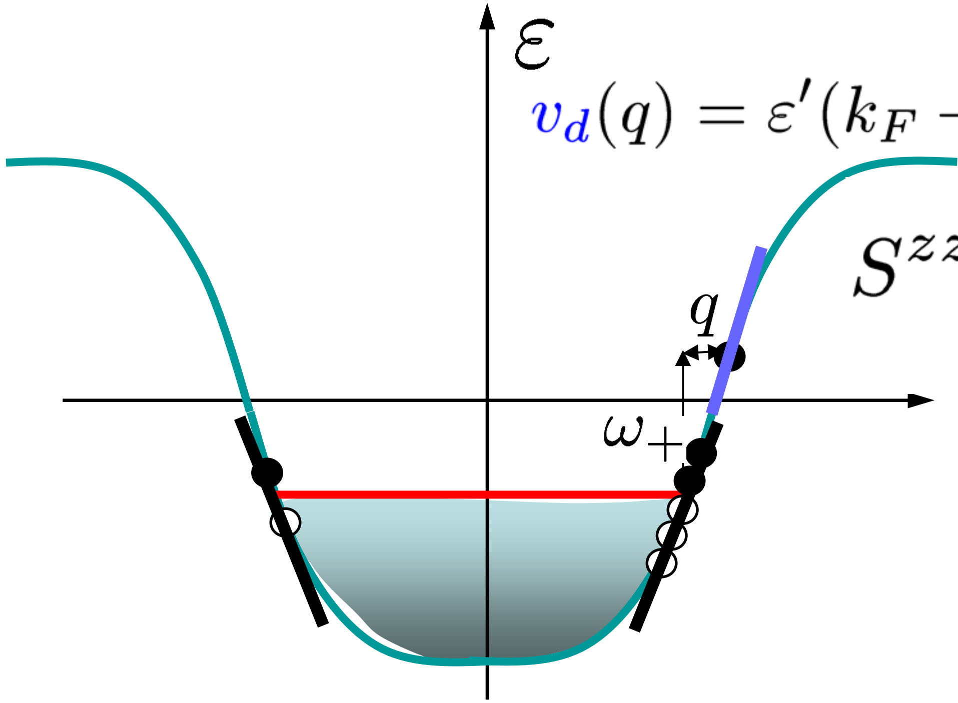
Lower Spectral Edge (cont)

$$\mu_-(q) = \frac{1}{\pi}\frac{U(0)-U(q)}{v_F - v_d(q)} = \frac{2\Delta}{\pi}\frac{\sin(q/2)}{\cos(k_F - q/2)}$$



$$\mu_-(2k_F) = 1 - K$$

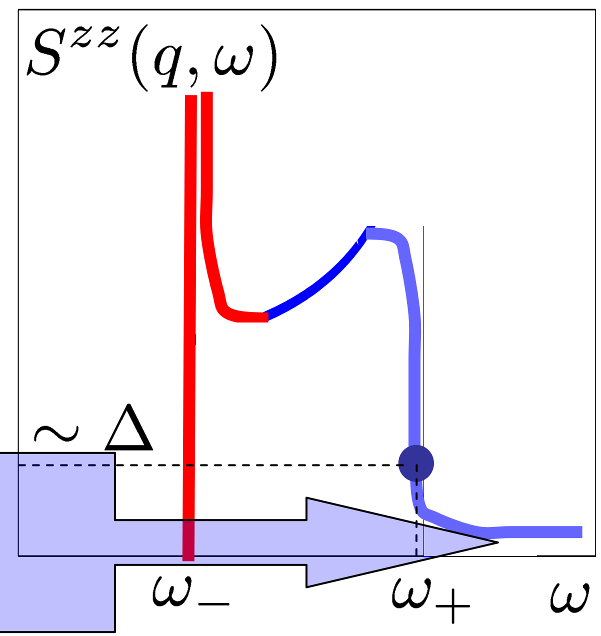
Upper Spectral Edge



$$v_d(q) = \varepsilon'(k_F + q)$$

$$S^{zz}(q,\omega) \sim \frac{1}{|\omega_+(q) - \omega|^{\mu_+(q)}}$$

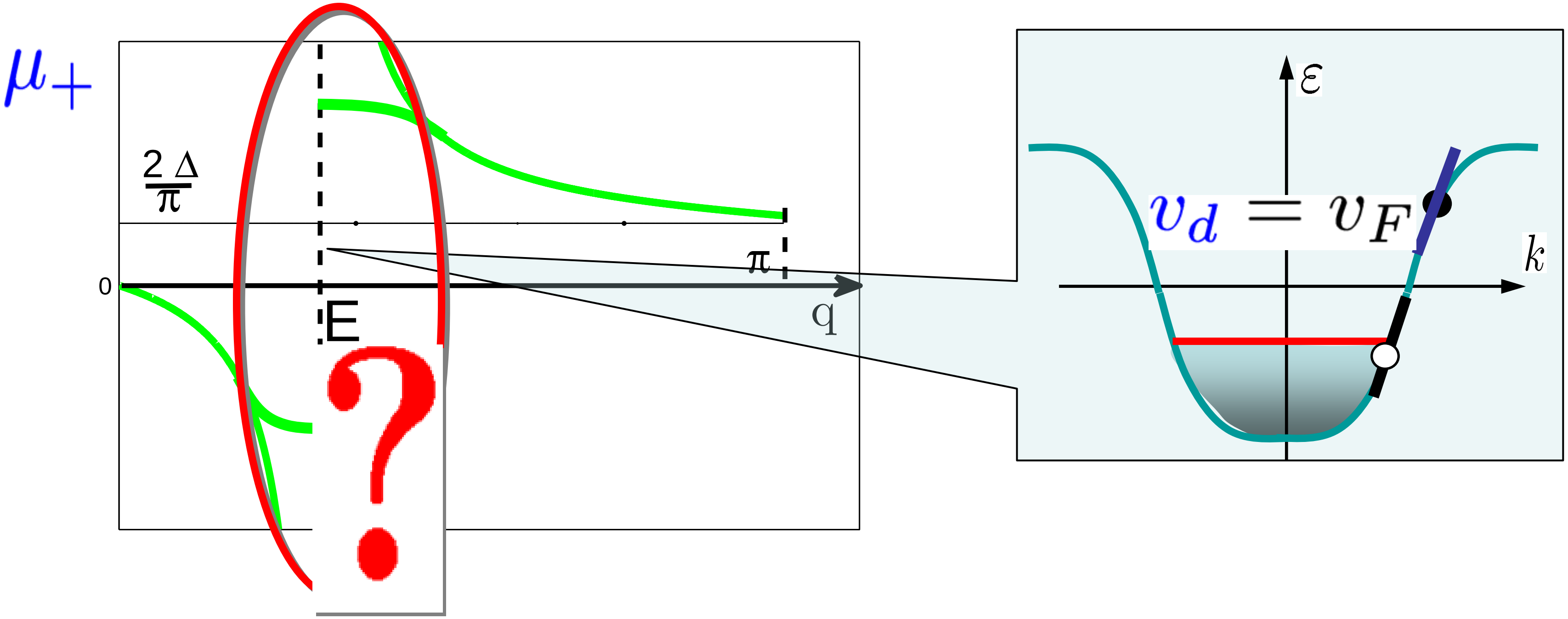
$$\mu_+(q) = \frac{1}{\pi}\,\frac{U(0) - U(q)}{v_F - v_d(q)}$$



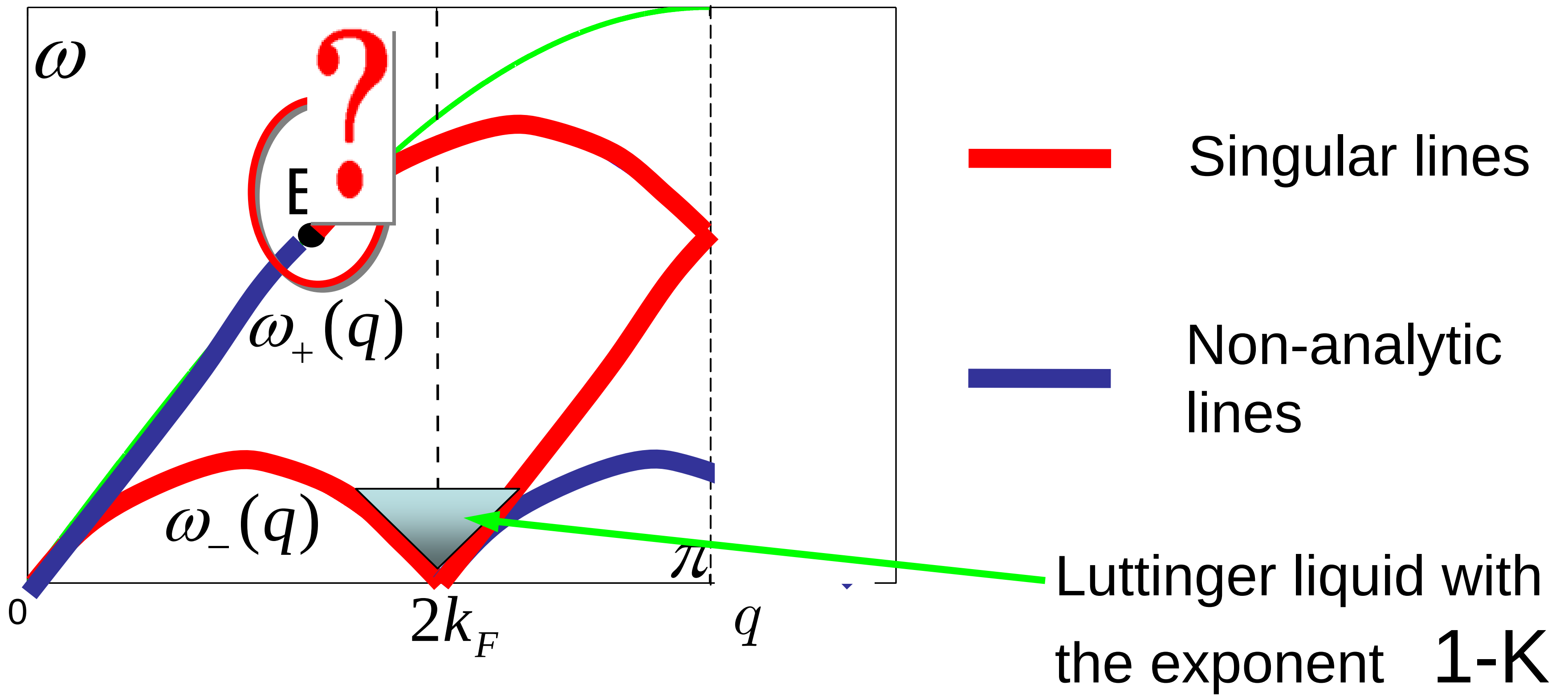
M. Pustilnik, et al 2003,
S. Teber 2006
R. G. Pereira, et al 2006.

Upper Spectral Edge (cont)

$$\mu_+(q) = \frac{1}{\pi}\frac{U(0)-U(q)}{v_F - v_d(q)} = -\frac{2\Delta}{\pi}\frac{\sin(q/2)}{\cos(k_F+q/2)}$$

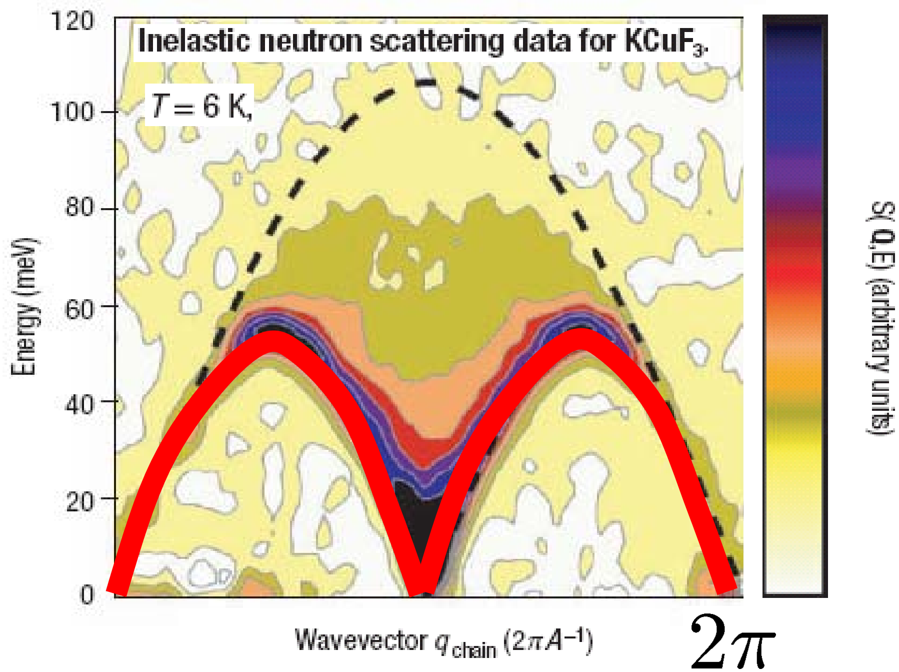


Summary



✓ Exponents are in general q—dependent

Zero Magnetic Field   h=0



BELLA LAKE[1,2*†], D. ALAN TENNANT[2,3‡], CHRIS D. FROST[3] AND STEPHEN E. NAGLER[1]

nature materials | VOL 4 | APRIL 2005

$$\omega_-(q) = \omega_+(q)$$

$$\mu_-(q) = \mu_+(q) = \mathrm{const}$$

$$\Delta \ll 1\,; \qquad \mu_\pm = \frac{2\Delta}{\pi}$$

$$\Delta = 1\,; \qquad \mu_\pm = \frac{1}{2}$$

Muller ansatz, 1981

Strong Interactions $\Delta \approx 1$

Pereira, White, Affleck, arXiv 0709.0960
Cheianov, Pustilnik, arXiv 0710.3589

$$H = H_{Luttinger} + H_d(q) + \int dx\, (d^\dagger d)(\beta_R\, \partial_x \varphi_R + \beta_L\, \partial_x \varphi_L)$$

$v_d(q)$ and $\beta_{R,L}(q)$ are to be determined through Bethe Ansatz

L