THEORY OF ONE DIMENSIONAL ADAPTIVE GRID GENERATION

By

Prabir Daripa

IMA Preprint Series # 612

February 1990

Theory of One Dimensional Adaptive Grid Generation

Prabir Daripa
Division of Applied Mathematics
Department of Mathematics
Texas A&M University
College Station, TX 77843

Abstract

A numerical method for generating one dimensional adaptive grids has been developed. It is based on approximating the grid spacing ratio on discrete lattice points by a continuous variable. For the numerical solution of a partial differential equation on an adaptive grid to be stable, the grid spacing ratio is required to be within some suitable bounds. The method makes it possible to specify apriori the extreme values of the grid spacing ratio and to generate the grid in a direct manner. We outline the theory in this paper.

1. Introduction: Adaptive grids are used when the solution of a partial differential equation exhibits singular or near-singular behavior. Examples occur in shock waves, traveling waves, and boundary layers. A correct selection of adaptive grids will produce a highly accurate solution at an optimal cost. However, they may increase the level of algorithmic complexity. The overall advantages of using adaptive grids compensates for the inconvenience involved in generating and using these grids.

Essentially the idea of adaptive grid computation is as follows (see [1]-[15]). A partial differential equation

$$u_t = L(u), \tag{1.1}$$

is transformed into

$$u_T - u_x x_T = L(u), \tag{1.2}$$

by means of the transformation $\zeta = \zeta(x,t)$, $T = t$. In (1.1) and (1.2), $L(u)$ is a differential operator which may depend on x, u_x, u_{xx}, and higher derivatives. The mapping is carefully constructed so that in the (ζ, T) system, the function $u(\zeta, T)$ is slowly varying in T and ζ. The discrete analog of the system (1.2) is written as

$$Au^{(n+1)} = B(u^n), \tag{1.3}$$

which advances the solution from time level n to time level $n+1$. (The form of equation (1.3) depends on the numerical method used.) In (1.3), A is a matrix that depends on u^{n+1} for a nonlinear problem. Matrices A and B in (1.3) depend on the adaptive grids, and if the grid spacings (i.e. the distance between consecutive grid points,) change too rapidly, obtaining an accurate solution of (1.3) may be difficult. For nonlinear cases, there may be convergence and/or stability problems. Therefore, the problem of adaptive grid generation is very important.

It should be noted that there are two aspects to the grid generation problem: (i) generating an adaptive grid and (ii) moving these grids in time. The grid movement is associated with the second term in (1.2). At times, it is preferable to generate adaptive grids at each instant instead of moving the grids. This amounts to setting $x_T = 0$ in (1.2). Thus, the new coordinate ζ is a function of x only where the grids are constructed to adapt some appropriate properties of the solution u at each instant. This paper develops a new numerical method to generate adaptive grids. In this method, a desired bound on the grid spacing ratio can be obtained without iteration. Other advantages of this method are addressed in [5]. The theory of this method is outlined in this paper.

In §2 we define the properties of adaptive grids and construct approximations to these properties by continuous variables. Using this continuous description and a suitable definition for the mapping function, the relation between grid spacing ratio and the function to be adapted is established in §3. The properties of the adaptive grids can be inferred directly from the function and can be controlled. In §4, we briefly discuss the adaptive grid generation algorithm.

2. Definition and Properties of Adaptive Grids: Let Ω_ζ and Ω_x be intervals in ζ and x space respectively. Consider a one dimensional monotonic mapping $x(\zeta) : \Omega_\zeta \to \Omega_x$, with the property that uniformly spaced ζ-grid points (grid points on ζ axis will be refered as ζ-grid points) are mapped to nonuniformly spaced x-grid points. The mapping $x(\zeta)$ will be constructed so that rapidly varying functions in x map into slowly varying functions in ζ. The desired adaptivity is incorporated in the choice of $x(\zeta)$. Let us denote by h_ζ the spacing between two consecutive ζ-grid points and $x_i = x(\zeta_i)$. The x-grid points constitute an adaptive grid. It is convenient to introduce the following two properties.

Resolution (S): The definition of resolution is motivated by the requirement that grid point concentration should be higher at locations of rapid variation in the function values. Thus, a convenient definition seems to be

$$S(x) = \frac{d\zeta}{dx}. \tag{2.1}$$

Grid Spacing Ratio (r): The grid spacing ratio at the grid point x_i is defined as the ratio of the two adjacent grid spacings, i.e.

$$r_i = \frac{x_{i+1} - x_i}{x_i - x_{i-1}}. \tag{2.2}$$

Notice that $r_i > 0$ for a monotonic mapping. This definition can be extended as a continuous variable. Below, two second order accurate continuous analogs of (2.2) have been derived. It follows from (2.2) that

$$r(x) - 1 = h_\zeta \frac{D_+ D_- x}{D_- x}; \text{ and } r(x) + 1 = 2\frac{D_0 x}{D_- x}, \tag{2.3}$$

where D_+, D_- and D_0 are the forward, backward and central difference operators respectively. For notational convenience, we have suppressed the subscript 'i ' in (2.3). These difference operators are related to their corresponding differential operators by

$$x_{\zeta\zeta} = D_+ D_- x + O(h_\zeta^2), \tag{2.4}$$

and

$$x_\zeta = D_- x + O(h_\zeta) = D_+ x + O(h_\zeta) = D_0 x + O(h_\zeta^2). \tag{2.5}$$

Using (2.3)-(2.5), we have

$$r(x) - 1 = h_\zeta \frac{x_{\zeta\zeta} + O(h_\zeta^2)}{x_\zeta + O(h_\zeta)} = h_\zeta \frac{x_{\zeta\zeta}}{x_\zeta} + O(h_\zeta^2). \tag{2.6}$$

Alternatively this can be written as,

$$r(x) = 1 + \alpha + O(h_\zeta^2), \tag{2.7}$$

where

$$\alpha = h_\zeta \frac{x_{\zeta\zeta}}{x_\zeta}. \tag{2.8}$$

Therefore, to second order accuracy, we define

$$R_1(x) = 1 + \alpha, \tag{2.9}$$

which is the approximation to $r(x)$ due to neglect of $O(h_\zeta^2)$ terms in (2.7). Thus, equation (2.9) is an appropriate second order accurate continuous extension of equation (2.2), provided that $x_\zeta \neq 0,\ \forall\ \zeta \in \Omega_\zeta$. In fact, this is guaranteed for a one-to-one smooth mapping (see §3). Addition of higher order terms in (2.2) does not alter the order of accuracy. It is possible to derive other second order accurate continuous analogs of (2.2). For example, it follows from (2.3) and (2.4) that

$$\frac{r(x)+1}{r(x)-1} = \frac{2}{h_\zeta}\frac{D_0 x}{D_+ D_- x} = \frac{2}{h_\zeta}\frac{x_\zeta + O(h_\zeta^2)}{x_{\zeta\zeta} + O(h_\zeta^2)}. \tag{2.10}$$

Notice that we have again suppressed the subscript 'i '. An easy algebriac manipulation gives

$$r(x) = \frac{2+\alpha}{2-\alpha} + O(h_\zeta^2), \tag{2.11}$$

where α is again given by (2.8). Therefore, we define

$$R_2(x) = \frac{2+\alpha}{2-\alpha}, \tag{2.12}$$

which is another second order accurate approximation to $r(x)$. Expanding $R_2(x)$ in α and dropping second and higher order terms in α, we recover $R_1(x)$. The numerical robustness of these two formulae in comparison with (2.2) is justified in a forthcoming paper [5]. The grid generation equations are discussed in the next section.

3. Equations of Grid Generation: Consider the generation of one dimensional adaptive grids by direct integration of

$$x_\zeta = cw(x;k) \text{ on } x_l \leq x \leq x_u, \tag{3.1}$$

which maps $\Omega_\zeta \to \Omega_x$. The adaptive grids are generated on the x axis. Without loss of generality, the indices of the adaptive grid points on the x axis are taken as the coordinates of the corresponding ζ-grid points, i.e. $\Omega_\zeta : [\zeta_0 = 0, \zeta_N = N] \to \Omega_x : [x_0 = x_l, x_N = x_u]$. Thus the uniform spacing $h_\zeta = 1$. This convention gives a domain Ω_ζ that is dynamically increasing as the number of x-grid points increases. Thus, the two second order accurate approximations of $r(x)$ in the previous section should be interpreted as $O(\frac{1}{N^2})$ accurate, which is the order of the derivatives hidden in the $O(h_\zeta^2)$ terms in §2.

In (3.1), 'c ' is a constant and $w(x;k)$ is allowed to have the form:

$$w(x;k) = \tilde{w}(x, f(x), f_{xx}; k). \tag{3.2}$$

Thus, $w(x;k)$ may depend on x implicitly. Here, k is a parameter which may be varied to attain desired bounds on grid spacing ratio. The required adaptivity is dictated by the choice of $w(x;k)$ [10]. However, this choice is constrained by the following requirements:

(i) The mapping must be one-to-one.

(ii) The grid spacing ratio is bounded below by r_l and above by r_u, i.e.

$$r_l \leq r \leq r_u, \tag{3.3}$$

where r_l and r_u are given.

The following theorem guarantees monotonicity of the mapping and hence the requirement (i) is satisfied, provided that $w(x;k)$ is positive in Ω_x.

THEOREM 3.1. If $w(x) > 0$, $\forall\ x \in [x_l, x_u]$ and $x_u > x_l$, then $x_\zeta > 0$, (i.e. the mapping is monotonic).

Proof. Since $0 \le \zeta \le N$, it follows that

$$(3.4) \qquad N = \int_{x_l}^{x_u} \zeta_x dx = \frac{1}{c} \int_{x_l}^{x_u} \frac{dx}{w(x;k)}.$$

Because $w(x;k) > 0, \;\; c > 0$. Therefore,

$$x_\zeta = cw(x;k) > 0, \; \forall \, x \in [x_l, x_u]. \qquad []$$

Thus, the form of $\tilde{w}(.\,;k)$ is to be chosen so that it is positive in $[x_l, x_u]$.

We require that (2.2) be satisfied to second order accuracy, (i.e. modify requirement (ii) and use relation (3.3) to obtain (3.5))

$$(3.5) \qquad r_l \;\le R \;\le r_u,$$

where R can be either R_1 or R_2, given by (2.9) and (2.12) respectively. Relation (3.5) will impose constraints on $w(x;k)$. To derive these constraints, we will need the following lemma.

LEMMA 3.2. Given r_l, and r_u, then α must satisfy

$$(3.6) \qquad a \;\le \alpha(x;k) \le b,$$

where

$$(3.7) \qquad a = \begin{cases} r_l - 1, & \text{for} \quad R = R_1, \\ 2\frac{r_l - 1}{r_l + 1}, & \text{for} \quad R = R_2, \end{cases}$$

and

$$(3.8) \qquad b = \begin{cases} r_u - 1, & \text{for} \quad R = R_1, \\ 2\frac{r_u - 1}{r_u + 1}, & \text{for} \quad R = R_2. \end{cases}$$

Proof. This follows easily from (2.9), (2.12) and (3.5). []

The constraint on $w(x,k)$ is embedded in the following theorem.

THEOREM 3.3. Given r_l, and r_u, the $w(x;k)$ must satisfy

$$a \le \frac{1}{N} g(x_u;k) w_x(x;k) \le b, \tag{3.9}$$

where $g(x;k)$ is defined as

$$g(x;k) = \int_{x_l}^{x} \frac{dy}{w(y;k)}, \tag{3.10}$$

and the constants a and b are given in (3.7) and (3.8).

Proof. From (3.1) we have,

$$x_{\zeta\zeta} = c w_x x_\zeta. \tag{3.11}$$

With $h_\zeta = 1$, α reduces to (see (2.8))

$$\alpha(x;k) = c w_x(x;k). \tag{3.12}$$

Using (3.4) and (3.10), we have

$$c = \frac{1}{N} g(x_u;k). \tag{3.13}$$

It follows from (3.12) and (3.13) that

$$\alpha(x;k) = \frac{1}{N} g(x_u;k) w_x(x;k). \tag{3.14}$$

Relation (3.9) follows from lemma 3.23 and (3.14). []

3.1 Parametric Control of (3.9): Theorem 3.3 provides the basic relation (3.9) that must be satisfied by a proper choice of the mapping function $w(x;k)$ and N, since r_l and

r_u (and hence a and b) are specified. However, in practice it is convenient to vary one or both of the free parameters k and N for a fixed form of $w(x;k)$. Below we give details on how to vary N and k for various combinations of r_l and r_u.

A set of specified values of r_l and r_u can be categorized into two basic groups: (i) $r_l < 1; r_u > 1$, and (ii) $r_l \geq 1; r_u > 1$.

Remark: The case $r_l = 1,\ r_u = 1$, corresponds to uniformly spaced grids. The other two combinations i.e. $r_l > 1,\ r_u < 1$, and $r_l < 1,\ r_u \leq 1$ are reflections of (i) and (ii) respectively under the mapping $x = -x$ (see relation (2.2) or (2.8).) []

In terms of the constants a and b defined in (3.7) and (3.8), these two basic groups correspond to (i) $a < 0,\ b > 0$, and (ii) $b > a \geq 0$ respectively. The functional form of $w(x;k)$ is usually fixed by the desired adaptivity [10]. We now discuss how to select k or N in case (i). Case (ii) can be handled by minor modification of case (i).

3.1.1 Fixed k, Adjustable N: Since $a < 0$ and $b > 0$, it follows from (3.9) that N must be chosen so that

$$N = \max\left(\frac{g(x_u;k)}{b} \max_x \ w_x, \frac{g(x_u;k)}{a} \min_x \ w_x\right). \tag{3.15}$$

However, this means that the number of grid points may change. This may be inconvenient and undesirable. This is certainly true in time dependent problems. An alternative procedure is to keep N fixed and vary the parameter k.

3.1.2 Fixed N, Adjustable k: To select proper values of k, ($a < 0$ and $b > 0$), so that (3.6) is satisfied, we need some results.

Hereafter, we assume that $w(x;k)$ is of the following form [10]:

$$w(x;k) = (k + v(x))^{\frac{-1}{2}} \tag{3.16}$$

where

$$v(x) = \tilde{v}(x, f_x, f_{xx}, ..). \tag{3.17}$$

LEMMA 3.4. The zeroes, x^*, of $\alpha(x;k)$ are independent of k and $v_x(x^*) = 0$.

Proof. From (3.12) and (3.6),

$$\alpha(x;k) = -\frac{c}{2}w^3(x;k)v_x(x). \tag{3.18}$$

According to theorem 3.1, $w(x;k) > 0,\ \forall\, x \in \Omega_x$ and $c > 0$. It follows from (3.18) that

$$v_x(x) = 0, \tag{3.19}$$

at zeros of $\alpha(x;k)$. The lemma follows. []

Suppose α^M and α^m are the sets of local maxima and local minima of $\alpha(x;k)$ respectively, and

$$X^M = \{x : \alpha(x) \in \alpha^M\}; \quad X^m = \{x : \alpha(x) \in \alpha^m\}. \tag{3.20}$$

Let $X = X^M \cup X^m$ and $\Gamma = \alpha^M \cup \alpha^m$. (Notice that Γ is set of all extrema of α.) Then the following result holds.

LEMMA 3.5. $x \in X$ are the roots of

$$v_{xx} - \frac{3}{2}(k + v(x))v_x^2 = 0. \tag{3.21}$$

(Note that $\{x : v_x = 0,\ v_{xx} = 0\} \subset X$.)

Proof. Since

$$\alpha_x = 0,\ \forall\, x \in X, \tag{3.22}$$

(3.12) and (3.22) give

$$w_{xx} = 0, \quad \forall\, x \in X. \tag{3.23}$$

Using (3.16), (3.23) and $w(x;k) > 0$, $\forall\ x \in \Omega_x$, we have

$$(3.24) \qquad v_{xx} = \frac{3}{2} w^2 {v_x}^2.$$

Hence, (3.21) follows from (3.16) and (3.24). []

LEMMA 3.6. For all $x(k) \in X$,

$$(3.25) \qquad \alpha(x,k) = \begin{cases} \gamma \frac{2c}{9} \left(\frac{v_{xx}^{\frac{3}{2}}}{v_x^2}\right), & \text{for} \quad v_x \neq 0, \\ 0, & \text{for} \quad v_x = 0, \end{cases}$$

where $\alpha(x,k) \in \Gamma$ and $\gamma = -sign(v_x)$. (Note from (3.13) that the constant 'c' depends on k and v_x, v_{xx} depend on k through lemma 3.5).

Proof. For $x \in X$, and $v_x \neq 0$, the first expression in (3.25) follows from (3.18) and (3.24). For $x \in X$, and $v_x = 0$, the second expression in (3.25) is a consequence of lemma 3.4. []

LEMMA 3.7. If the function $v(x)$ is essentially flat (i.e. all the derivatives of $v(x)$ are zero) at a point, so is $\alpha(x;k)$ at that point. (Note that $\alpha(x;k) = 0$ at such a point from lemma 3.4.)

Proof. The result follows upon repeated differentiation of (3.21). []

THEOREM 3.8. The variation of nonzero α with the parameter k is given by

$$(3.26) \qquad \frac{\partial \ln|\alpha|}{\partial k} = h(x;k),$$

where

$$(3.27) \qquad h(x;k) = -\frac{1}{2g(x;k)} \int_{x_l}^{x_u} \frac{[3w^2(x;k) + w^2(t;k)]}{w(t;k)} dt.$$

(Note that $h(x;k)$ is negative).

Proof. Using (3.14) and simple calculations,

(3.28)
$$\alpha_k = \frac{1}{N}\int_{x_l}^{x_u} \frac{W(w_x(x;k), w(t;k))}{w^2(t;k)}dt,$$

where W is the Wronskian and α_k is the partial derivative of α with respect to k. Using (3.16), expression (3.28) reduces to

(3.29)
$$\alpha_k = -\frac{\alpha}{2g(x;k)}\int_{x_l}^{x_u} \frac{[3w^2(x;k) + w^2(t;k)]}{w(t;k)}dt.$$

Expression (3.26) follows. []

Remark: For $\alpha = 0$, we have $\alpha_k = 0$ from (3.29) which is also asserted in lemma 3.4.

Theorem 3.9 is the key to finding a suitable value of k so that (3.6) is satisfied. It is clear that (3.6) can always be satisfied by choosing a large value of k. However, we would like to find a value of k so that (3.6) is satisfied and the extreme values of k are approximately equal to a and b, as the case may be.

If for an initial choice of $k = k_0$, the relation (3.6) is not satisfied, then k_0 is updated through the iteration

(3.30)
$$k_{i+1} = k_i + (\beta'(k_i))^{-1}(\beta^\star - \beta(k_i)), \quad i = 0, 1, 2, ...$$

where we have defined

(3.31)
$$\beta = \ln|\alpha|,$$

and,

(3.32)
$$\beta^\star = \begin{cases} \ln|a|; & \text{if } \alpha(k_i) < a \\ \ln b; & \text{if } \alpha(k_i) > b. \end{cases}$$

The relation (3.30) is obtained by retaining only first order terms in the Taylor series expansion of $\beta(k_i + 1)$ about k_i and setting $\beta(k_i + 1)$ to be equal to the appropriate maximum desired value as given by (3.32). In (3.30), β' denotes the partial derivative of β with respect to k. In (3.30)-(3.32), the dependency of β on x is to be understood. The k_{i+1} in (3.30) are to be evaluated for each $x \in X$ and the maximum of these updated values of k is to be taken as the new updated value.

4. Numerical Algorithm: For given Ω_x, N, r_l, and r_u, the appropriate value of k is obtained by the method outlined at the end of the previous section. Then $g(x_u; k)$ and the constant c are obtained by using (3.10) and (3.13) respectively. Inverting $g(x_1) = c$ gives x_1. The grid spacing ratio is computed at i=1,2,...,N-2 by using either (2.9) or (2.12) which is then used in (2.2) to generate the adaptive grids x_{i+1} for i=1,2,...,N-2.

Its implementation and numerical results will be reported elsewhere.

This research in part has been supported by the NSF grant DMS-8803669 to Texas A&M University.

REFERENCES

[1] Slimane Adjerid, and Joseph E. Flaherty, *A moving finite element method with error estimation and refinement for one-dimensional time dependent partial differential equations,* SIAM. J. NUMER. ANAL.,23 (1986), pp. 778-796.

[2] I. Babuska, J. Chandra, and J. E. Flaherty, eds., *Adaptive ComputationalMethods for Partial Differential Equations,* Society for Industrial and Applied Mathematics, Philadelphia, PA, 1983.

[3] M. Bieterman, and I. Babuska, *An Adaptive Method of Lines with Error Control for Parabolic Equations of the Reaction-Diffusion Type,* J. Comput. Phys.,63 (1986), pp. 33-66.

[4] J. G. Blom, J. M. Sanz-Serna, and J. G. Verwer, *On Simple Moving Grid Methods for One-Dimensional Evolutionary Partial Differential Equations,* J. Comput. Phys.,74 (1988), pp. 191-213.

[5] P. Daripa, To appear.

[6] E. A. Dorfi, and L. Drury, *Simple Adaptive Grids for 1-D initial Value Problems,* J. Comput. Phys.,69 (1987), pp. 175-195.

[7] H. A. Dwyer, *Grid Adaption for Problems in Fluid Dynamics,* AIAA JOURNAL.,22

(1984), pp. 1705-1712. SIAM. J. NUMER. ANAL.,23 (1986), pp. 778-796.

[8] G. W. Hedstrom, and G. H. Rodrigue, *Adaptive-grid methods for time-dependent partial differential equations,* Proc. Conference on Multigrid Methods, Cologne, November 1981, pp. 474-484.

[9] B. Larrouturou, *A Conservative Adaptive Method For Flame Propagation,* SIAM. J. SCI. STAT. COMPUT.,10 (1989), pp. 742-755.

[10] K. Matsuno, and H. A. Dwyer, *Adaptive Methods for Elliptic Grid Generation,* J. Comput. Phys.,77 (1988), pp. 40-52.

[11] Keith Miller, *Moving finite elements. II,* SIAM. J. NUMER. ANAL.,18 (1981), pp. 1033-1057.

[12] Mehtab M. Pervaiz, and Judson R. Baron, *Temporal and spatial adaptive algorithm for reacting flows,* Communications in Applied Numerical Methods.,4 (1988), pp. 97-111.

[13] J. D. Ramshaw, *Conservative rezoning algorithm for generalized two dimensional meshes,* J. Comput. Phys.,59 (1985), pp. 193-199.

[14] J. D. Ramshaw, *Simplified second order rezoning algorithm for generalized two dimensional meshes,* J. Comput. Phys.,67 (1986), pp. 214-222.

[15] J. G. Verwer, J. G. Blom, and J. M. Sanz-Serna, *An Adaptive Moving Grid Method for One-Dimensional Systems of Partial Differential Equations,* J. Comput. Phys.,82 (1989), pp. 454-486.

Recent IMA Preprints

#	Author/s	Title
547	**M. Slemrod**	Dynamics of Measured Valued Solutions to a Backward-Forward Heat Equation
548	**Avner Friedman and Jürgen Sprekels**	Steady States of Austenitic–Martensitic–Domains in the Ginzburg–Landau Theory of Shape Memory Alloys
549	**Avner Friedman and Bei Hu**	Degenerate Hamilton–Jacobi–Bellman Equations in a Bounded Domain
550	**E.G. Kalnins, Willard Miller, Jr., and M.V. Tratnik**	Families of Orthogonal and Biorthogonal Polynomials on the N-Sphere
551	**Heinrich Freistühler**	On Compact Linear Degeneracy
552	**Matthew Witten**	Quantifying the Concepts of Rate and Acceleration/Deceleration of Aging
553	**J.P. Albert and J.L. Bona**	Total Positivity and the Stability of Internal Waves in Stratified Fluids of Finite Depth
554	**Brian Coomes and Victor Zurkowski**	Linearization of Polynomial Flows and Spectra of Derivations
555	**Yuriko Renardy**	A Couette-Poiseuille Flow of Two Fluids in a Channel
556	**Michael Renardy**	Short wave instabilities resulting from memory slip
557	**Daniel D. Joseph and Michael Renardy**	Stokes' first problem for linear viscoelastic fluids with finite memory
558	**Xiaxi Ding**	Superlinear Conservation Law with Viscosity
559	**J.L. Ericksen**	Liquid Crystals with Variable Degree of Orientation
560	**F. Robert Ore, Jr. and Xinfu Chen**	Electro-Optic Modulation in an Arbitrary Cross-Section Waveguide
561	**M.V. Tratnik**	Multivariable biorthogonal continuous-discrete Wilson and Racah polynomials
562	**Yisong Yang**	Existence of Solutions for a Generalized Yang-Mills Theory
563	**Peter Gritzmann, Laurent Habsieger and Victor Klee**	Good and Bad Radii of Convex Polygons
564	**Martin Golubitsky, Martin Krupa and Chjan. C. Lim**	Time-Reversibility and Particle Sedimentation
565	**G. Yin**	Recent Progress in Parallel Stochastic Approximations
566	**G. Yin**	On H-Valued SA: Finite Dimensional Approximations
567	**Chien-Cheng Chang**	Accurate Evaluation of the Effect of Diffusion and Conductivity in Certain Equations
568	**Chien-Cheng Chang and Ruey-Ling Chern**	The Effect of Viscous Diffusion in Discrete Vortex Dynamics for Slightly Viscous Flows
569	**Li Ta-Tsien (Li Da-qian) and Zhao Yan-Chun**	Global Existence of Classical Solutions to the Typical Free Boundary Problem for General Quasilinear Hyperbolic Systems and its Applications
570	**Thierry Cazenave and Fred B. Weissler**	The Structure of Solutions to the Pseudo-Conformally Invariant Nonlinear Schrödinger Equation
571	**Marshall Slemrod and Athanasios E. Tzavaras**	A Limiting Viscosity Approach for the Riemann Problem in Isentropic Gas Dynamics
572	**Richard D. James and Scott J. Spector**	The Formation of Filamentary Voids in Solids
573	**P.J. Vassiliou**	On the Geometry of Semi-Linear Hyperbolic Partial Differential Equations in the Plane Integrable by the Method of Darboux
574	**Jerome V. Moloney and Alan C. Newell**	Nonlinear Optics
575	**Keti Tenenblat**	A Note on Solutions for the Intrinsic Generalized Wave and Sine-Gordon Equations
576	**P. Szmolyan**	Heteroclinic Orbits in Singularly Perturbed Differential Equations
577	**Wenxiong Liu**	A Parabolic System Arising In Film Development
578	**Daniel B. Dix**	Temporal Asymptotic Behavior of Solutions of the Benjamin-Ono-Burgers Equation
579	**Michael Renardy and Yuriko Renardy**	On the nature of boundary conditions for flows with moving free surfaces
580	**Werner A. Stahel**	Robust Statistics: From an Intellectual Game to a Consumer Product
581	**Avner Friedman and Fernando Reitich**	The Stefan Problem with Small Surface Tension
582	**E.G. Kalnins and W. Miller, Jr.**	Separation of Variables Methods for Systems of Differential Equations in Mathematical Physics
583	**Mitchell Luskin and George R. Sell**	The Construction of Inertial Manifolds for Reaction-Diffusion Equations by Elliptic Regularization
584	**Konstantin Mischaikow**	Dynamic Phase Transitions: A Connection Matrix Approach
585	**Philippe Le Floch and Li Tatsien**	A Global Asymptotic Expansion for the Solution to the Generalized Riemann Problem
586	**Matthew Witten, Ph.D.**	Computational Biology: An Overview
587	**Matthew Witten, Ph.D.**	Peering Inside Living Systems: Physiology in a Supercomputer

#	Author/s	Title

588 **Michael Renardy**, An existence theorem for model equations resulting from kinetic theories of polymer solutions

589 **Daniel D. Joseph and Luigi Preziosi**, Reviews of Modern Physics: Addendum to the Paper "Heat Waves"

590 **Luigi Preziosi**, An Invariance Property for the Propagation of Heat and Shear Waves

591 **Gregory M. Constantine and John Bryant**, Sequencing of Experiments for Linear and Quadratic Time Effects

592 **Prabir Daripa**, On the Computation of the Beltrami Equation in the Complex Plane

593 **Philippe Le Floch**, Shock Waves for Nonlinear Hyperbolic Systems in Nonconservative Form

594 **A.L. Gorin, D.B. Roe and A.G. Greenberg**, On the Complexity of Pattern Recognition Algorithms On a Tree-Structured Parallel Computer

595 **Mark J. Friedman and Eusebius J. Doedel**, Numerical computation and continuation of invariant manifolds connecting fixed points

596 **Scott J. Spector**, Linear Deformations as Global Minimizers in Nonlinear Elasticity

597 **Denis Serre**, Richness and the classification of quasilinear hyperbolic systems

598 **L. Preziosi and F. Rosso**, On the stability of the shearing flow between pipes

599 **Avner Friedman and Wenxiong Liu**, A system of partial differential equations arising in electrophotography

600 **Jonathan Bell, Avner Friedman, and Andrew A. Lacey**, On solutions to a quasilinear diffusion problem from the study of soft tissue

601 **David G. Schaeffer and Michael Shearer**, Loss of hyperbolicity in yield vertex plasticity models under nonproportional loading

602 **Herbert C. Kranzer and Barbara Lee Keyfitz**, A strictly hyperbolic system of conservation laws admitting singular shocks

603 **S. Laederich and M. Levi**, Qualitative dynamics of planar chains

604 **Milan Miklavčič**, A sharp condition for existence of an inertial manifold

605 **Charles Collins, David Kinderlehrer, and Mitchell Luskin**, Numerical approximation of the solution of a variational problem with a double well potential

606 **Todd Arbogast**, Two-phase incompressible flow in a porous medium with various nonhomogeneous boundary conditions

607 **Peter Poláčik**, Complicated dynamics in scalar semilinear parabolic equations in higher space dimension

608 **Bei Hu**, Diffusion of penetrant in a polymer: a free boundary problem

609 **Mohamed Sami ElBialy**, On the smoothness of the linearization of vector fields near resonant hyperbolic rest points

610 **Max Jodeit, Jr. and Peter J. Olver**, On the equation grad $f = M$ grad g

611 **Shui-Nee Chow, Kening Lu, and Yun-Qiu Shen**, Normal form and linearization for quasiperiodic systems

612 **Prabir Daripa**, Theory of one dimensional adaptive grid generation

613 **Michael C. Mackey and John G. Milton**, Feedback, delays and the origin of blood cell dynamics

614 **D.G. Aronson and S. Kamin**, Disappearance of phase in the Stefan problem: one space dimension

615 **Martin Krupa**, Bifurcations of relative equilibria

616 **D.D. Joseph, P. Singh, and K. Chen**, Couette flows, rollers, emulsions, tall Taylor cells, phase separation and inversion, and a chaotic bubble in Taylor-Couette flow of two immiscible liquids

617 **Artemio González-López, Niky Kamran, and Peter J. Olver**, Lie algebras of differential operators in two complex variables

618 **L.E. Fraenkel**, On a linear, partly hyperbolic model of viscoelastic flow past a plate

619 **Stephen Schecter and Michael Shearer**, Undercompressive shocks for nonstrictly hyperbolic conservation laws

620 **Xinfu Chen**, Axially symmetric jets of compressible fluid

621 **J. David Logan**, Wave propagation in a qualitative model of combustion under equilibrium conditions

622 **M.L. Zeeman**, Hopf bifurcations in competitive three-dimensional Lotka-Volterra Systems

623 **Allan P. Fordy**, Isospectral flows: their Hamiltonian structures, Miura maps and master symmetries

624 **Daniel D. Joseph, John Nelson, Michael Renardy, and Yuriko Renardy**, Two-Dimensional cusped interfaces