EXTINCTION AND POSITIVITY FOR A SYSTEM OF SEMILINEAR PARABOLIC VARIATIONAL INEQUALITIES

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EXTINCTION AND POSITIVITY FOR A SYSTEM OF SEMILINEAR PARABOLIC VARIATIONAL INEQUALITIES*

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Abstract. A simple model of chemical kinetics with two concentrations u and v can be formulated as a system of two parabolic variational inequalities with reaction rates v^p and u^q for the diffusion processes of u and v respectively. It is shown that if $pq < 1$ and the initial values of u and v are "comparable" then at least one of the concentrations becomes extinct in finite time. On the other hand for any $p = q > 0$ there are initial values for which both concentrations do not become extinct in any finite time.

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§**1. Introduction.** It is well known that nonnegative solutions of various initial and boundary value problems associated to the semilinear heat equation

$$u_t - u_{xx} + u^p = 0 \qquad \text{with} \qquad 0 < p < 1 \tag{1.1}$$

vanish identically in finite time; see [7] [2] [3] [1]. This phenomenon is termed *extinction*, and is clearly illustrated by the explicit solution

$$u = ((1-p)(T-t)_+)^{\frac{1}{p-1}} \quad , \ T > 0 \tag{1.2}$$

where $s_+ = \max\{s, 0\}$. This particular solution plays an important role in describing the asymptotic behavior of the extinction process; cf. [5] [6].

In this paper we consider a semilinear parabolic system which may be thought of as a toy model in chemical kinetics. Let $u(x,t)$ and $v(x,t)$ denote the concentrations of two species which diffuse and react in a one-dimensional domain $L = \{-1 < x < 1\}$ according to

$$u_t - u_{xx} + v^p = 0 \quad , \quad v_t - v_{xx} + u^q = 0 \qquad (p > 0, q > 0).$$

Since the concentrations must be nonnegative, we are led to the following variational inequality formulation:

$$u \geq 0, \quad v \geq 0 \quad \text{in} \quad Q = (-1,1) \times (0,\infty), \tag{1.3}$$

$$u_t - u_{xx} + v^p \geq 0 \ , \quad v_t - v_{xx} + u^q \geq 0 \quad \text{a.e. in } Q, \tag{1.4}$$

$$u(u_t - u_{xx} + v^p) = 0, \ v(v_t - v_{xx} + u^q) = 0 \quad \text{a.e. in } Q. \tag{1.5}$$

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We also impose initial conditions

$$u(x,0) = u_0(x)\ ,\ v(x,0) = v_0(x) \quad \text{in} \quad [-1,1] \tag{1.6}$$

where $u_0 \geq 0,\ v_0 \geq 0$, and boundary conditions which are either Dirichlet

$$u(\pm 1,t) = v(\pm 1,t) = 0 \quad \text{for} \quad t > 0\ , \tag{1.7}$$

or Neumann

$$\frac{\partial u}{\partial x}(\pm 1,t) = \frac{\partial v}{\partial x}(\pm 1,t) = 0 \quad \text{for} \quad t > 0\ . \tag{1.8}$$

We shall briefly refer to the problem (1.3)–(1.7) as (DP) and to (1.3)–(1.6),(1.8) as (NP).

For simplicity we assume that

$$\begin{aligned} &u_0''(x), v_0''(x) \quad \text{belong to} \quad L^\infty[-1,1]\ , \\ &u_0(\pm 1) = v_0(\pm 1) = 0 \text{ for } (DP) \text{ and } u_0'(\pm 1) = v_0'(\pm 1) = 0 \text{ for } (NP)\ , \end{aligned} \tag{1.9}$$

although our results easily extend to the case where u_0, v_0 are just continuous in $[-1,1]$. We define a solution of (DP) to a pair of functions (u,v) be such that (1.3)–(1.7) hold and

$$\iint\limits_{Q_T} (|w_x|^r + |w_{xx}|^r + |w_t|^r) < \infty \quad \text{for } w = u, v \text{ and for all } r > 1, T > 0. \tag{1.10}$$

For (NP) we replace (1.7) by (1.8) and require, in addition to (1.10), that

$$u_x, v_x \quad \text{are continuous in } \overline{Q}. \tag{1.11}$$

We observe that, if $pq < 1$, then for any $T > 0$ (NP) has a solution

$$u = c_1(T-t)_+^{\frac{p+1}{1-pq}} \quad , \quad v = c_2(T-t)_+^{\frac{q+1}{1-pq}} \tag{1.12}$$

with

$$c_1 = \left(\frac{(1-pq)^{1+p}}{(p+1)(q+1)^p}\right)^{\frac{1}{1-pq}} \quad , \ c_2 = \left(\frac{(1-pq)^{1+q}}{(p+1)^q(q+1)}\right)^{\frac{1}{1-pq}} \quad . \tag{1.13}$$

This example and the extinction results for the single equation (1.1) suggest that one may expect the extinction phenomenon to hold for the system (1.3)–(1.5) whenever $pq < 1$. As it will turn out, however, this is not always the case. But before going any further we need to define the concept of extinction more carefully. If

$$u(x,0) \equiv 0 \ , \ v(x,0) \not\equiv 0$$

then there exists a solution (u,v) such that $u(x,t) \equiv 0$ if $t > 0$, whereas $v(x,t) > 0$ for all $-1 < x < 1, \ t > 0$. Also, if

$$u(x,0) = \text{const} = c_1 \ , \quad v(x,0) = \text{const} = c_2 \ , \quad pq < 1 \ ,$$

then, for (NP), there exists a solution (u,v) which is a function of t only and, for general c_1, c_2, only one component becomes zero in finite time (the case (1.13) in exceptional).

These examples show that in order to capture the phenomenon of extinction one should define:

A solution (u,n) has finite extinction time T if T is the smallest positive number such that either $u(x,t) \equiv 0$ for all $t > T$, or $v(x,t) \equiv 0$ for all $t > T$.

In §3 we give a sufficient condition on the data u_0, v_0 which ensures extinction, and in §4 we give an example (with $p = q$) where there is no extinction. (The same initial data as in that example also leads to a positivity result of one component in case $p \geq q$.) In §2 we briefly establish existence of solutions to (DP) and (NP).

§2. Existence.

THEOREM 2.1. *Let u_0, v_0 be nonnegative functions satisfying (1.9). Then there exists a solution (u,v) to (DP) (respectively (NP)).*

Proof. We consider only (DP); the case (NP) is similar. For any $0 < \varepsilon < 1$ let $\beta_\varepsilon(s)$ be a C^∞ function such that

$$\beta'_\varepsilon(s) \geq 0 \ , \quad \beta_\varepsilon(s) = 0 \quad \text{if } s \geq 0, \ \lim_{\varepsilon \to 0} \beta_\varepsilon(s) = -\infty \text{ if } s < 0.$$

Let $f_{\varepsilon,p}, \ f_{\varepsilon,q}$ be smooth, nonnegative, monotone nondecreasing and bounded functions satisfying:

$$\lim_{\varepsilon \to 0} f_{\varepsilon,p}(s) = s_+^p \ , \lim_{\varepsilon \to 0} f_{\varepsilon,q}(s) = s_+^q \ .$$

Consider the system of penalized equations:

$$\begin{aligned} &u_t - u_{xx} + \beta_\varepsilon(u) + f_{\varepsilon,p}(v) = 0 \quad \text{in} \quad Q \ , \\ (2.1) \qquad & \\ &v_t - v_{xx} + \beta_\varepsilon(v) + f_{\varepsilon,q}(v) = 0 \quad \text{in} \quad Q \end{aligned}$$

with the same data as for (DP). One can easily prove (as in [4: Chap. 1]) that this problem has a solution $(u_\varepsilon, v_\varepsilon)$ and

$$u_\varepsilon \le \|u_0\|_{L^\infty} \;, \quad v_\varepsilon \le \|v_0\|_{L^\infty} \;;$$

a standard energy inequality can be used to establish uniqueness. It follows that $f_{\varepsilon,p}(v_\varepsilon)$ and $f_{\varepsilon,q}(u_\varepsilon)$ are bounded uniformly in ε and then (as in [4; p. 25])

$$\beta_\varepsilon(u_\varepsilon) \quad , \quad \beta_\varepsilon(v_\varepsilon)$$

are bounded uniformly in ε. We can then deduce that for any sequence $\varepsilon \to 0$ there is a subsequence which converges to a solution of (DP).

We note that the question of uniqueness of the solutions is open.

§3. Extinction result.

THEOREM 3.1. *Suppose that $pq < 1$ and*

(3.1) $$u_0(x) \ge \left(\frac{q+1}{p+1}\right)^{\frac{1}{q+1}} v_0(x)^{\frac{p+1}{q+1}} \;, \quad p \ge q \;.$$

Then there exist a solution (u, v) of (DP) (respectively (NP)) such that $v(x,t) \equiv 0$ for $t \ge T$, for some $T > 0$.

Proof. Consider the auxiliary functions

$$\theta = cv^\beta, \; \theta_\lambda = c(v+\lambda)^\beta \quad \text{where} \quad c = \left(\frac{q+1}{p+1}\right)^{\frac{1}{q+1}} \;, \; \beta = \frac{p+1}{q+1} \;, \; \lambda > 0 \;.$$

Notice that θ_λ satisfies

$$\theta_{\lambda,t} - \theta_{\lambda,xx} = -c\beta(v+\lambda)^{\beta-1}(v_t - v_{xx}) - c\beta(\beta-1)(v+\lambda)^{\beta-2}v_x^2 \qquad (\beta \ge 1) \;.$$

Dropping the last term and letting $\lambda \to 0$ we get

(3.2) $$\theta_t - \theta_{xx} \le -c\beta v^{\beta-1} u^q$$

in some weak sense. Recalling (1.4) we deduce that

(3.3) $$(u-\theta)_t - (u-\theta)_{xx} \ge c\beta v^{\beta-1} u^q - v^p \qquad \text{in} \quad Q \;.$$

We now replace $u_0(x)$ by $u_0(x)+\tau \quad (\tau > 0)$ and in the case of (DP) replace the conditions $u(\pm 1, t) = 0$ by $u(\pm 1, t) = \tau$. We continue to denote by (u, v) the corresponding solution of (1.3)–(1.5). Observe that

$$\text{if } u \geq cv^\beta \text{ then } c\beta v^{\beta-1} u^q \geq v^p ,$$

so that the right-hand side of (3.3) is ≥ 0. Using this fact, and the strong maximum principle (which holds for our solution u, in view of the regularity (1.10)) we can deduce that if $u(x,t) \geq cv(x,t)^\beta$ for $-1 \leq x \leq 1$, $0 \leq t \leq s$ then also $u(x,s) > cv(x,s)^\beta$ for $-1 \leq x \leq 1$; here we needed the modification of the Dirichlet conditions at $x = \pm 1$. Since $u(x, 0) = u_0(x) + \tau > cv_0(x)^q$ for $-1 \leq x \leq 1$ (by (3.1)), it follows that

$$u(x,t) > cv(x,t)^\beta \quad , \quad -1 \leq x \leq 1$$

for small t and then also for all $t > 0$.

Letting $\tau \to 0$ we obtain $u \geq cv^\beta$ for the solution (u, v) of (DP) or (NP). Substituting this into the differential equation for v (on the set $\{v = 0\}, v_t = 0$ and $v_{xx} = 0$ a.e.), we get

$$v_t - v_{xx} + kv^\alpha \leq 0 \quad \text{with} \quad \alpha = \frac{q(p+1)}{q+1} \ , \ k = c^q \ .$$

The assumption $pq < 1$ implies that $\alpha < 1$, and therefore there exists a $T > 0$ such that $v(x,t) \equiv 0$ for $t \geq T$.

§4. Non-extinction and positivity.

We begin with a result on non-extinction.

THEOREM 4.1. *Assume that* $p = q$ *and*

$$u_0(x) = v_0(-x) \quad \textit{for} \quad -1 \leq x \leq 1 , \tag{4.1}$$

$$u_0(x) \geq u_0(-x) , \quad u_0(x) \not\equiv u_0(-x) \quad \textit{for} \quad -1 \leq x \leq 0 . \tag{4.2}$$

Then there exists a solution of (DP) *(respectively* (NP)*) such that*

$$u(x,t) > 0 \quad \textit{in} \quad Q^- \equiv (-1,0) \times (0,\infty) , \tag{4.3}$$

$$v(x,t) > 0 \quad \textit{in} \quad Q^+ \equiv (0,1) \times (0,\infty) . \tag{4.4}$$

Thus the solution does not have finite extinction time.

Proof. We consider the system

$$u_t(x,t) - u_{xx}(x,t) + \beta_\varepsilon(u(x,t)) + f_{\varepsilon,p}(u(-x,t)) = 0 , \tag{4.5}$$

$$v_t(x,t) - v_{xx}(x,t) + \beta_\varepsilon(v(x,t)) + f_{\varepsilon,p}(u(x,t)) = 0 . \tag{4.6}$$

with the same initial and boundary data as before. It is easy to show that (4.5) has a solution with the required data. If we set $v(x,t) = u(-x,t)$ then v satisfies (4.6) and the required data (here we used (4.1)). Finally, since

$$f_{\varepsilon,p}(u(-x,t)) = f_{\varepsilon,p}(v(x,t)\ ,$$

the pair (u,v) is a solution of the penalized problem (2.1).

Denoting this solution by $(u_\varepsilon, v_\varepsilon)$ we thus have

$$v_\varepsilon(x,t) = u_\varepsilon(-x,t) \tag{4.7}$$

and

$$u_\varepsilon \to u\ ,\ v_\varepsilon \to v \quad \text{as} \quad \varepsilon \to 0\ ,$$

where (u,v) is a solution of (DP) (respectively (NP)).

The function

$$z_\varepsilon(x,t) = u_\varepsilon(x,t) - v_\varepsilon(x,t)$$

satisfies:

$$z_t - z_{xx} + c_1 z + c_2 z = 0$$

where

$$c_1 = \frac{\beta_\varepsilon(u_\varepsilon(x,t)) - \beta_\varepsilon(v_\varepsilon(x,t))}{u_\varepsilon(x,t) - v_\varepsilon(x,t)}\ ,\ c_2 = \frac{f_{\varepsilon,p}(v_\varepsilon(x,t)) - f_{\varepsilon,p}(u_\varepsilon(x,t))}{u_\varepsilon(x,t) - v_\varepsilon(x,t)}$$

if $u_\varepsilon(x,t) \neq v_\varepsilon(x,t)$ and $c_1 = c_2 = 0$ otherwise. Notice that c_1, c_2 are bounded functions for any fixed ε. Since

$$z_\varepsilon(x,0) = u(x) - u_0(-x) \geq 0\ ,\ \not\equiv 0 \quad \text{in} \quad (-1,0)\ ,$$

and z_ε satisfies homogeneous boundary conditions at $x = -1, 0$, the strong maximum principle yields $z_\varepsilon > 0$ in Q^-. Letting $\varepsilon \to 0$ we obtain a solution (u,v) satisfying:

$$u(x,t) = v(-x,t) \quad \text{in} \quad Q\ , \tag{4.8}$$

$$u(x,t) \geq u(-x,t) \quad \text{in} \quad Q^-\ . \tag{4.9}$$

For any $T > 0$ we consider the function

$$z(x,t) = u(x,t) - u(-x,t) \quad \text{in} \quad Q_T^- = (-1,0) \times (0,T)\ .$$

In the subset where $u(x,t) \geq u(-x,t) > 0$, we have

$$z_t - z_{xx} = u(x,t)^p - u(-x,t)^p \geq 0 \quad \text{by (4.9).}$$

In the subset where $u(x,t) > u(-x,t) = 0$, $u_t(-x,t) = 0, u_{xx}(-x,t) = 0$ a.e. so that

$$z_t - z_{xx} = u_t - u_{xx} = -u(-x,t)^p = 0 \quad \text{a.e.}$$

Finally in the subset where $u(x,t) = u(-x,t) = 0$ we have a.e.

$$z_t = z_{xx} = u_t - u_{xx} = 0 \ , \quad \text{since} \quad u_t = 0 \ , \ u_{xx} = 0 \quad \text{a.e.}$$

We conclude that

$$z_t - z_{xx} \geq 0 \quad \text{in} \quad Q_T^- \ .$$

Since also

$$z(x,0) \geq 0 \ , \not\equiv 0 \quad \text{in} \quad (-1,0)$$

and z satisfies homogeneous boundary conditions,

$$z(x,T) > 0 \quad \text{if} \quad -1 < x < 0 \ .$$

This implies that $u(x,T) > 0$ if $-1 < x < 0$, and since T is arbitrary, (4.3) is satisfied. Recalling (4.8), the assertion (4.4) also follows.

Theorem 4.1 can be used to derive a positivity result in case $p \neq q$:

THEOREM 4.2. *Assume that $p \geq q$, (4.1), (4.2) are satisfied, and*

$$\max\{\|u_0\|_{L^\infty} \ , \ \|v_0\|_{L^\infty}\} \leq 1 \ . \tag{4.10}$$

Then there exists a solution of (DP) (respectively (NP)) such that (4.3) holds.

Proof. We choose $f_{\varepsilon,p}$, $f_{\varepsilon,q}$ such that

$$f_{\varepsilon,p}(s) \leq f_{\varepsilon,q}(s) \quad \text{if} \quad 0 \leq s \leq 1 \ . \tag{4.11}$$

Denote by $u_\varepsilon, v_\varepsilon$ the solution of the penalized problem as defined in §2 and by $v_{\varepsilon,1}$ the solution of

$$v_t - v_{xx} + \beta_\varepsilon(v) + f_{\varepsilon,p}(u_\varepsilon) = 0 \quad \text{in} \quad Q$$

with the same data as v_ε. Since $\|u_\varepsilon\|_{L^\infty} \leq 1$, $\|v_\varepsilon\|_{L^\infty} \leq 1$ (here we use (4.10)), it follows from (4.11) that

$$v_{\varepsilon,1} \geq v_\varepsilon \qquad \text{in} \qquad Q \ . \tag{4.12}$$

Next we define $u_{\varepsilon,1}$ to be the solution of

$$u_t - u_{xx} + \beta_\varepsilon(u) + f_{\varepsilon,p}(v_{\varepsilon,1}) = 0 \qquad \text{in} \quad Q$$

with the same data as u. Recalling (4.12) and the fact that the $f_{\varepsilon,p}$ are monotone nondecreasing, we deduce that

$$u_{\varepsilon,1} \le u_\varepsilon \quad \text{in} \quad Q \; .$$

Iterating this procedure we obtain sequences of nonnegative functions, bounded above by 1, $\{u_{\varepsilon,j}\}$ and $\{v_{\varepsilon,j}\}$ such that

$$v_\varepsilon \le v_{\varepsilon,1} \le v_{\varepsilon,2} \le \cdots \quad ,$$

$$u_\varepsilon \ge u_{\varepsilon,1} \ge u_{\varepsilon,2} \ge \cdots \quad ,$$

and $u_{\varepsilon,j}$ are uniformly bounded from below by a negative constant independent of ε, j. The limits $U_\varepsilon = \lim_{j\to\infty} u_{\varepsilon,j}$, $V_\varepsilon = \lim_{j\to\infty} v_{\varepsilon,j}$ satisfy

$$v_\varepsilon \le V_\varepsilon \ , \ u_\varepsilon \ge U_\varepsilon \ ; \tag{4.13}$$

further, $(U_\varepsilon, V_\varepsilon)$ form the solution of the penalized problem with $p = q$. By uniqueness for the penalized problem,

$$U_\varepsilon(x,t) = V_\varepsilon(-x,t) \; .$$

By the proof of Theorem 4.1, for any limits

$$U = \lim_{\varepsilon\downarrow 0} U_\varepsilon \ , \quad V = \lim_{\varepsilon\downarrow 0} V_\varepsilon$$

there holds

$$U(x,t) > 0 \qquad \text{if} \quad -1 < x < 0, \;\; t > 0 \; .$$

Since, by (4.13), the limit $u = \lim u_\varepsilon$ satisfies $u \ge U$, the assertion of the theorem follows.

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