Essays in Dynamic Information Economics

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Jorge David Ramos-Mercado

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Dedication

A mi queridísimo tío Técto Ottaviani por ser la inspiración que me atrajo a la economía. ¡Te recodaré siempre!

Abstract

Dynamic and strategic choices are often made by imperfectly-informed agents observing a groups' past decisions. In such context, past choices inform the tradeoffs that individual agents currently face via two channels. The first channel is strategic in nature i.e., an individual's payoffs from a given action often depend on their peers' choices. For instance, a currency without widespread adoption is worthless and voting for a fringe candidate is mostly futile. Agents, furthermore, learn from their peers actions. Buyers estimate a painting's resale value from past bids; investors impute a firm's long-run profitability from its stock price; et cetera. Although it is well-known that learning over time plays a key role in decision-making, past research ascertained few insights of note. This thesis presents two papers with results in dynamic information economics pertaining auctions and bargaining. I discuss each paper in turn.

In chapter 2, I study the role of learning in sequential auctions under limited commitment. The set up consists of a seller running a sequence of second-price auctions. In each period, the seller first announces a reserve price (the smallest acceptable bid) and then accepts bids. If he receives a bid, he transacts and the game ends; otherwise, he can set a different price in the subsequent auctions.

A seller who can pre-determine all reserve prices from the outset is better off fixing all prices equal to a price that maximizes static revenues. This nets the seller the highest possible (expected) revenues. Sellers, however, are seldom capable of credibly fixing prices after learning that their good failed to sell. Instead, sellers often find it optimal to attract bids by sequentially lowering prices. Doing so, however, forces the seller to (implicitly) compete with his past self.

The Coase conjecture posits that the seller's revenues should converge to running an efficient auction from the outset as market participants (i.e., the seller and buyers) interact increasingly frequently. Note that in an efficient auction, the seller post a

reserve price that elicits bids from all buyers who value the good more than the seller. Such auctions are also known to not maximize revenues.

Liu et al (2019) proves that the Coase conjecture holds (at least) when there are 3 or more buyers who value independent private (IP) valuations. Buyers are said to have IP valuations if and only if each buyer's willingness to pay for the auctioned good is independent of any information held by his peers. This assumption is strong and rarely holds in real-world auctions. Does the Coase conjecture hold when one relaxes the assumption of IP valuations? In general, each buyer's valuations can depend on the private information held by their peers and valuations are said to be interdependent. I find that the Coase conjecture can fail when valuations are interdependent.

The intuition is straightforward. As the item fails to sell, buyers deduce that their peers hold adverse information regarding the auctioned good and hence lower their valuations. The seller then learns that there may be fewer buyers with whom to trade. I find that, in all equilibria, the market unravels after a finite number of re-offerings. In each equilibrium, the seller expects the market to unravel and posts prices that informs when the market unraveling occurs. As a result, initial revenues are bounded above immediately running an efficient and may equal to the maximum revenue attained by fixing prices from the outset.

Next, chapter 3 studies bargaining. Rubinstein (1982) introduced modern, micro-foundations of bargaining.Two impatient players take turns proposing the split of a common "surplus" and deciding whether or not to accept said proposals. The game ends once a proposal is accepted. The paper's unique prediction is that players reach an agreement immediately. This contrasts empirical evidence showing that negotiations concluding with immediate agreement are rare. Backus et al (2020), for instance, finds that only a 32 percent of E-bay buyer-seller transactions concluded with immediate agreement.

To address this discrepancy, Abreu and Gul (2000) extended the model by introducing reputational types. This means that players may only be able to make large, pre-determined demands. In the unique equilibrium, players who are free to make flexible demands and concessions to their opponent find it optimal to behave "as if" they are unable to do so and a war of attrition ensues. This equilibrium rationalizes that negotiations can end with an agreement after multiple rounds of proposals.

I extend the model above by incorporating hidden effort. Such extension allows me to consider bargaining scenarios where players take actions, as they bargain, that can affect the payoff from the negotiation. I particularly focus on wartime, peace negotiations. In such bargaining scenario, combatants negotiate the terms of a peace agreement while they privately manage a war effort. The United Nations and other intergovernmental organizations have observed the destructiveness of war and the fact that most negotiations end without an agreement and have long proposed that combatants pause fighting as they negotiate the terms of a lasting peace. However, the implementation of said policy coincided with an increase in the share of negotiations ending in disagreement from 60 to 87 percent.

The chapter shows that combatants negotiating a peace while they continue to fight. Indeed, it may never be optimal to pause the fighting at any point during the negotiation. This is because learning from the observed military outcomes enables players to quickly deduce their opponent's capacity to make concessions. In general, I find that brief ceasefires (especially at the beginning of the negotiation) may be optimal under some parametric cases.

The insights derived from this model, although applied in the context of wartime negotiations, are relevant to other bargaining scenarios such as mergers and wage renegotiations. By studying the dynamics of hidden actions and the role of ceasefires, the chapter provides valuable insights into the conditions under which such policies are optimal or detrimental in various bargaining contexts.

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Chapter 1

Introduction

Information plays a key role in economics, because most decisions are made by imperfectly informed agents. For example, how much should a buyer bid for a painting when its resale value is unknown? In war, under what conditions should combatants hold a peace negotiation? What kind of information should a seller provide a about his goods unknown common value? Moreover, when should the seller bring the good to market? These scenarios involve imperfectly informed agents making sequential choices, while learning from past outcomes and (sometimes) the choices made by others. In this thesis, I present three papers (chapters 2 and 3) on information economics, providing new insights on the questions posited above.

Chapter 2: Learning and Sequential Rationality in Auctions. In chapter 2, I study learning in sequential auctions when the seller has limited commitment. A seller owns an indivisible good and auctions it to a fixed set of buyers. In each period, the seller announces and runs a second-price auction with a posted reserve price. In such auction, buyers can submit a bid above the reserve price or abstain (from bidding). The highest bid wins but the winning buyer only pays the second highest bid—or the reserve price if his fellow bidders abstained. I assume that the game ends once a buyer submits a bid. The seller is otherwise free to re-auction his good. Crucially, I assume that once the seller announces a reserve price dutifully implements the auction announced, but he cannot pre-determine the reserve prices of subsequent auctions. This assumption is known as sequential rationality or limited commitment.

If the seller could, on the other hand, establish prices at the outset, he cannot improve upon fixing prices equal to a revenue maximizing reserve price. The expected payoff from such strategy is known as the full-commitment revenues. Nevertheless, a seller who has limited commitment is seldom able to guarantee that he will post the aforementioned price schedule and never change his mind. The intuition is clear. If the good failed to sell at a given reserve price, the seller learns that no buyer valued the good enough to place a bid. And the seller deduces that he must lower reserve prices to attract bids. Nevertheless, buyers internalize this behavior, so they wait and revenues fall. The Coase conjecture posits that as the seller interacts with buyers increasingly frequently, revenues converge to running an efficient auction from the outset. In such auction, the seller posts a reserve price eliciting that all buyers who value the good more than himself place a bid.

Previous work by Liu et al (2019) found that the Coase conjecture holds when there are at least 3 buyers and buyers have independent private value (IPV). Buyers are said to have IPV valuations when willingness to pay of any arbitrary pair of potential buyers are statistically independent. This assumption is stark and one (in general) expects valuations to be influence by a "common" value. For example, the IPV setting would imply that wildcatters competing for drilling rights are not in agreement that a rig's oil reserves are key to how much the would bid for the drilling rights in question. Alternatively, it would also imply that not all bankers bidding for a developing nation's sovereign bonds worry about the nation's capacity to repay its recently issued debt. To this end, I revisit the Coase conjecture in auctions when buyer valuations have a common value. My main result is that the Coase conjecture fail in such case.

The intuition is straightforward. At the start of the game, each buyer receives private information about an auctioned good's common value. If the item fails to sell, buyers anticipate that their peers who received unfavorable signals would have refrained from bidding. Consequently, buyers lower their willingness to pay for the item. Similarly, the seller expects that the pool with whom he can trade is smaller than before.

I find that the seller can only re-auction an unsold good a finite number of times until the market fully unravels. This means that there seller is certain that there does not exists a buyer who values the good more than himself. This outcome holds true in every equilibrium and regardless of the frequency at which the seller interacts with buyers. Nevertheless, this certainty that the good can only be re-auctioned a finite number of times provides a lower bound in initial revenues. Indeed, this lower bound on revenues is greater than the expected revenue from running an efficient auction from the outset, does not depend on how often the seller interacts with buyers, and may equal to the maximum possible revenues when the seller has full commitment.

Chapter 3: Bargaining and War In chapter 3, I study bargaining. Like auctions, bargaining is one of the foundations of economics. The modern, non-cooperative foundations of bargaining begin with Rubinstein (1982). The setup consists of two players who take turns proposing how to split a "surplus". For example, how should an employer and a worker split the revenue generated by their interaction?; how much are an acquired firm's stockholders are compensated in a merger be valued?; how do warring nations arrive at a peace treaty? The player who receive a proposal either accept or reject it and once a proposal is accepted, the game end. This model has a unique equilibrium in which players reach an immediate agreement. This is counterfactual. For example, Backus et al (2020) find that only 32 percent of E-bay buyer-seller transactions concludes after the first offer.

Abreu and Gul (2000) were able to extend the aforementioned model and their unique outcome rationalizes negotiations reaching an agreement after multiple rounds of proposals. Their key insight was the introduction of reputational types. This means that, with a positive probability, a player may be obstinate and hence only be able to make predetermined demands. Otherwise, players are able to make concessions and (hence) behave strategically. In the unique equilibrium, strategic players mimic the demands of they would make if they were obstinate and a war of attrition ensues. This means that as long as a strategic player in not certain that he interacts with an obstinate player, he concedes at an exponentially growing rate.

Subsequent research build upon the previously mentioned bargaining framework by analyzing the role of several model features of note. I now provide a brief overview. First, Compte and Jehiel (2002) proved that the unique outcome reaches immediate agreement when players have a sufficiently large outside options. For example, a worker's outside option when negotiating a wage is determined by the value of continued unemployment and the value of meeting another employer at a later date.

Meanwhile, Fanning (2016) introduced a random terminal date. Such addition rationalizes why many negotiations are increasingly likely to reach an agreement near their deadline. Lastly, Fanning (2021) introduce a mediator who proposes mutually acceptable agreements. The findings highlight that intermediaries can enhance bargaining outcomes, but only when there is a possibility that they may fail to effectively communicate that an agreement has been reached. This emphasizes the importance of communication and the potential benefits of involving a third party in the negotiation process.

From a theoretical point of view, my paper accounts for the fact that negotiations may or may not hold in isolation. Should negotiations take place in isolation? For instance, unions negotiate a collective agreement with their employer while they work or during a walking-out strike. Meanwhile, peace negotiations either coincide with active fighting or in a pre-agreed pauses i.e., ceasefires. My main result states that holding a negotiation in isolation may lower welfare. This result holds even when players negotiating in isolation could prevent players from incurring costs and avoid that the negotiation ends in disagreement after multiple rounds of haggling.

In particular, I expand allow players to exert hidden effort. This extension has many real-world applications. For instance, in the context of merging firms, managers invest resources towards talent retention to ensure that the consolidated firm is profitable. Meanwhile, when heads of government negotiate free-trade agreements, they lobby their legislatures to ratify the agreement. Similarly, combatants engaged in a drawn-out conflict negotiate a peace agreement while managing a persisting war effort. By incorporating hidden effort, the model captures the learning dynamics and strategic considerations involved in real-world negotiations.

The model is applied to wartime, peace negotiations. I first note several empirical observation about said negotiations. Using data from Min (2022, 2019) and Fazal (2013), I find that approximately 25 percent of peace negotiations held after 1823 resulted in a lasting peace agreement i.e., they were successful. While this success rate is low, it hides a significant trend. Since 1914, the share of successful negotiations declined from 40 to 13 percent, while the median duration of wars increased from 15 months to 4 years.[1] This prompts the question: Why have peace negotiations become less successful since 1914?

I find that post-1914 peace negotiations are 250 percent more likely to coincide with ceasefires. A ceasefire is a temporary pause in fighting. I estimate that 1 day in which a negotiation coincides with a ceasefire is associated with an additional 10 days of fighting. Is there any reason to expect this relation to be anything other than spurious? In any case, when is it optimal to coordinate ceasefires during armed conflicts?

After the establishment of guidelines for ceasefires during the Hague Convention of 1907, international relations experts promoted pauses in fighting as a precursor to peace negotiations. This approach became further implemented after World War 1, where combatants gradually exited the conflict by first agreeing to a ceasefire before negotiating the terms of their exit. The subsequent establishment of intergovernmental institutions such as the United Nations and the League of Nations reinforced the policy of advocating for ceasefires preceding peace negotiations.

[1]The difference in means is significantly larger.

This coincidence, by itself, does not imply a causal relation. Nevertheless, I provide a rationale for why negotiating during a ceasefire can lead to worse bargaining outcomes relative to negotiating while combatants are actively fighting. In my model, two players negotiate the split of a surplus while they exert effort to manage the war effort. Each player exert two different types of effort. On one hand, players exert defensive effort to prevent that their opponent achieves a decisive military victory and is able to freely impose their terms. The player further exerts effort to prevent the surplus from being destroyed. This is analogous to defending a bridge from enemy attack or a city's telecommunication infrastructure. This is because the value of holding a city depends on the infrastructure and resources that it holds. Lastly and crucially, I assume that obstinate players make pre-determined, hidden effort choices.

In the model, a ceasefire is the case where the rates at which the surplus is destroyed or players reach a decisive military victory are nullified. I find that the decision to hold a ceasefire must take account for the following tradeoff. On one hand, ceasefires preserve the surplus at no cost and avoid a military victory. However, not observing how the fighting proceeds slows down how fast strategic combatants learn that their opponent is unable to make concessions. It also eliminates the incentive to concede as a precaution that there may not be future opportunities to negotiate or that the future negotiation is over a smaller surplus.

When obstinate players exert little effort to prevent the surplus from being destroyed—a natural assumption—I find that holding a ceasefire lowers the welfare of strategic players. Furthermore, it is never optimal to even hold a momentary pause in fighting. Suppose that a benevolent third-party could impose upon the players when they fight or pause the fighting. I find that the aforementioned third-party would never suggest that combatants pause the fighting at any moment in time or after any military occurrence. In general, however, the third-party may find it optimal to hold brief ceasefires at the beginning of the negotiation or after the surplus is partially destroyed.

Chapter 2

Learning to Commit

Summary I study learning in auctions under limited commitment. In each period, the seller sets the terms for an auction selling an indivisible good among multiple buyers; but if the item fails to sell, he cannot pre-commit to the terms of future offerings. I find that, in interdependent value settings, the seller's equilibrium revenues are greater than immediately running an efficient, Vickrey auction. In contrast with private value settings, this result persists regardless of how often agents interact. This is because learning among buyers ensures lowers buyer valuations and ensure that the seller stops re-offering his good in finite time.

2.1 Introduction

The Coase conjecture illustrates how sequential rationality (i.e., limited commitment) reduces profits. A monopolist sells a durable good to buyers with independent, unknwon valuations. At each price, the probability that a buyer purchases the good and exits the market increases with their willingness to pay. This prompts the monopolist to progressively lower prices. In turn, a declining price schedule delays sales and lowers profits. Coase (1972) conjectured that profits vanish as the costs of waiting for future trade opportunities vanish.

The Coase conjecture is robust [1] and extends to other settings like sequential auctions. Nevertheless, the conjecture assumes that buyers have independent and private valuations. A buyer has a private valuation if it is unaffected by others' private information. Such assumption rarely holds. For example, seldom known factors like a painting's resale value, an oil field's deposits, and a bond's default probability inform a buyer's willingness to pay. In such cases, buyers expect their peers to hold private information that would inform their willingness to pay and (thus) buyers are said to have interdependent valuations. Does the Coase conjecture extend to settings where buyers have interdependent valuations? In this paper, I find that the Coase conjecture fails in settings with interdependent valuations.

Previous research showed that the Coase conjecture can fail in durable good markets. The conjecture fails when buyers have an outside option (Board and Pycia 2014) or the monopolist faces capacity constraints (MacAfee and Wiseman 2008). Meanwhile, other papers point out that as the waiting cost vanish, one can approach multiple equilibria e.g., Fundenberg et al (1987) and Ausubel and Deneckere (1989). These papers find that the Coase conjecture is one such limiting equilibria, but there are equilibria in which the seller attains positive profits. In such limiting equilibria, the seller posts a gradually decreasing price schedule and screens buyers.

Liu et al (2019), however, showed that the Coase conjecture is the sole limiting equilibria when there are three or more competing buyers with IP valuations. This means that limiting equilibria in which the seller profitably screen buyers do not exist in the case of first(second)-price or English auctions. Intuitively, the seller faces a trade-off between screening buyers and attracting competing bids. Screening buyers requires posting an initially high reserve price, which is gradually lowered over time. Such prices ensure that the seller extracts a large share of the trade surplus, but screening delays when the good sells. In contrast, the seller attracts bids by starting with a low reserve

[1] see (for example) Kahn 1986, Fuchs and Skrzypacz 2010, Bond and Samuelson 1984, and Sobel 1991.

price from the outset. This strategy immediately prompts the losing buyers to bid up the winning bid. The authors find that screening is sub-optimal, because competition among buyers extracts a large surplus share without delay.

I find that the tradeoff above does not hold when buyer valuations are interdependent valuations. In fact, there exists a maximum, finite number of times in which the seller may offer his good in every equilibria. This implies that equilibria can be derived via backwards induction beginning at the earliest period in which the good may be sold. Consequently, equilibrium revenues are unique and higher than running an efficient auction from the outset, regardless of waiting costs. I even find that commitment considerations may not affect revenues under mild conditions.

The intuition is straightforward. At the start of the game, each buyer receives an independent signal that is informative of the auctioned good's common value. If the item fails to sell, buyers anticipate that their peers who received unfavorable signals would have refrained from bidding. Consequently, buyers lower their willingness to pay for the item. Similarly, the seller expects that the pool of potential buyers contracts as the item remains unsold. These learning dynamics ensure that after a finite number of iteration, the market unravels. This means that the seller does not expect to profit by auctioning his good. Moreover, if the market unravels after a single iteration, then commitment considerations have no effect on revenues.

The paper is organized at follows. Section 2.2 presents the main results in a stylized model. I then introduce the general model in section 2.3 and the main results are in section 2.4. Next, section 2.5 discusses the related literature. Lastly, section 2.6 presents the discussion and concludes.

2.2 Motivation: Selling Art.

This section illustrates the paper's main contribution, i.e. learning among buyers can contravene the Coase conjecture. A seller can auction a painting to a fixed set of patient buyers until the artwork sells. If the seller has full commitment, he is better off immediately running the revenue maximizing, static auction and keeping the painting if it fails to sell. Keeping paintings failing to sell, however, is seldom sequentially rational.

When buyer valuations are independent and the seller has limited commitment (i.e., he is sequentially rational), Liu et al (2019) proves that the seller's revenues converge to immediately running an efficient auction as the seller transacts with buyers increasingly frequently. This is the auction analog of the Coase conjecture. Meanwhile, when buyer valuations are interdependent, I show that the seller can attain strictly higher revenues, regardless of how often the seller transacts with his buyers. In fact, I find conditions for when the seller can immediately run the revenue maximizing, static auction and find it sequentially rational to keep his item when it fails to sell.

2.2.1 Environment

A seller offers a painting to $n \geq 2$ buyers. Each buyer i has a valuation v_i that depends multiplicatively on a private value θ_i and a common value q i.e.,

$$v_i \equiv \theta_i q.$$

The private values (θ_i) are drawn iid uniformly between 0 and 1; meanwhile, q is high (i.e., $q = 1$) with probability $\lambda \in (0, 1)$ or otherwise low ($q = 0$). I assume that q is independent of the private values. Next, each buyer i further observes a signal x_i that is good news (i.e., $x_i = 1$) or bad news ($x_i = 0$). Conditional on q, I assume that the signals are drawn iid, but for each buyer i it holds that $x_i = q$ with probability $\pi \in (1/2, 1)$. This assumption implies that the signals are informative and that the expected common value conditional on observing a good signal is higher than observing a bad signal: $E[q|x_i = 1] > E[q|x_i = 0]$.

Meanwhile, the seller has a commonly known valuation, called θ_s, that does not depend on the common value q. This assumption is important since it simplifies exposition and avoids the issues associated with an informed seller i.e., the seller does not observe a private signal that would inform buyer valuations. Also, the results presented below can be extended to the case when the seller's valuation depends on q: seller's valuation is $\theta_s(q)$. I further assume, for exposition, that the seller's valuation is higher than buyers observing bad news but smaller than some buyers observing good news i.e.,

$$E[q|x_i = 0] < \theta_s < E[q|x_i = 1].$$

Next, the timing of play is as follows. Nature first draws the common value q, private values (θ_i), and the signals (x_i). It then privately informs each buyer i of (θ_i, x_i). In period $t = 0, 1, \ldots$, the seller first posts a reserve price $p_t \in [0, 1]$. Buyers then decide to wait or submit a bid b_{it} such that $b_{it} \geq p_t$. If no buyer bids, the game continues to period $t + 1$. Otherwise, the game ends, the buyer submitting the highest bid wins the auction, and either pays the second highest bid or p_t provided that no other buyer placed a bid. Moreover, if multiple buyers submit the highest bid, then each buyer wins with equal probability. The timing of play is illustrated in figure 2.1. Lastly, if buyer i wins the item in period t and must pay $p_{it} \geq p_t$, payoffs are

i. Buyer i: $\delta^t(\theta_i q - p_{it})$ for common discount factor $\delta \in (0, 1)$,

ii. Buyers $j \neq i$: 0

iii. Seller: $(1 - \delta^t)\theta_s + \delta^t p_{it}$.

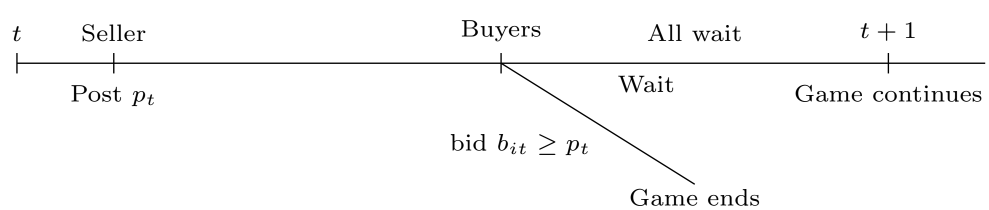

Figure 2.1: Timing of play at each period $t = 0, 1, \ldots$ conditional on the item remaining unsold.

The rest of the section proceeds as follows. First, I illustrate the revenue maximizing equilibrium under full commitment. Next, I provide a sufficient condition ensuring that the strategy profile above remains an equilibrium when one requires the seller to be sequentially rational, i.e. he has limited commitment. I lastly discuss how this result generalizes and extends to more general settings.

2.2.2 Full Commitment Benchmark

Even though I focus on the revenues that a sequentially rational seller can attain, it is useful to first characterize the optimal strategy under full commitment. An optimal strategy for the seller is a price schedule (p_t) that maximizes expected revenues in period 0. Since such price schedules are not required to maximize revenues at each history, the maximum revenues attained by a seller with full commitment cannot be less than the maximum revenue with limited commitment. Hence, the full commitment benchmark is an upper bound on the revenues that a seller can attain.

In lemma 1, I establish that the seller cannot improve posting a constant price schedule i.e., for each period t, $p_t = p \in [0,1]$ almost surely. This is because the seller is impatient and (as shown below) valuations fall as the item remains unsold. Consequently, delaying when buyers submit bids both delays when payments are made and lowers the rents that the seller extracts. It is further immediate that if a price schedule (p_t) does not prompt buyers to delay when they submit a bid, then fixing prices at p_0 nets the seller the same revenues. This implies that the seller cannot improve upon running the optimal, static auction in period 0 and never re-offering his good again. Let $p^* \geq \theta_s$[2] be the reserve price associated with the optimal static auction, then an optimal strategy for the seller is $(p_t = p^*)$.

Next, when the seller fixes prices at p^*, buyers behave myopically. This implies that buyers expect to only submit a bid in period 0 and do so if and only if (iff) their valuation is greater than p^*. But what are buyer valuations? Buyers bid their valuation conditional on winning since buyers face a winner's curse. This means that buyer i is

[2] It is immediate that the optimal reserve price $p^* \geq \theta_s$; otherwise, the seller sells his painting to a buyer for strictly less than he values the painting with a strictly positive probability.

more likely to win if i observed good news (i.e., $x_i = 1$) and most peers j observed bad news ($x_j = 0$). Since $E[q|x_i = 0] < \theta_s$, then a buyer i observing good news expects to outbid each peer j is j observed bad news or he observes good news and a private value that is less than θ_i i.e.,

$$w_i \equiv \theta_t E[q|x_i = 1, \forall j \neq i, x_j = 0 \text{ or } x_j = 1, \theta_j \leq \theta_i].$$

I focus on symmetric equilibria for which this is the unique equilibrium. If one allows the buyers to play asymmetric strategies, revenues are lower but this is an well understood issue that is orthogonal to this paper's contribution. I now states the following lemma, which formalizes the results described above.

Lemma 1 (First Best) *When the seller has full commitment, he posts $p_t = p^*$ in each period t and each buyer i bids w_i in period 0 iff $w_i \geq p^*$.*

The argument behind lemma 1 is standard and I delegated its prove to appendix B.1. What matters is that a buyer submits a bid in period 0 iff they observe $x_i = 1$ and a private value θ_i that is above a cutoff $\theta^* \geq \theta_s$. I further find that the cutoff θ^* is strictly increasing in θ_s, because buyers submitting bids (in equilibrium) must value the good more than the seller.

2.2.3 Limited Commitment

In the previous section, I characterized an optimal strategy under full commitment. This strategy implicitly requires the seller to not elicit bids after after the good fails to sell in period 0. When buyers have independent private values, this strategy is not sequentially rational since gains from trade often remain. Is this still the case in a setting with interdependent values? I find conditions ensuring that the sequentially rational seller can implement the optimal strategy under full commitment and present the argument below.

Suppose that the seller has limited commitment, but players still follow the strategy profile described in lemma 1. I claim that this strategy profile can be sequentially rational. If the seller posts $p_0 = p^*$ and the painting sells, the game is over and commitment considerations are moot. Otherwise, the good failed to sell, which only occurs if no buyer submitted a bid. Each buyer i expects that each alternative buyer $j(\neq i)$ either observed bad new or they observed good news, but they also observed a private value that is below the participation cut-off i.e., $\theta_j \leq \theta^*$. Buyer i then derives a new valuation for the painting: $v_{i1} = \theta_i E_1[q|x_i = 1]$.

Since learning among buyer plays a key role in my result, I characterize how buyers update beliefs given Bayes rule in detail. Considitonal on the common value q, both the signals (x_i) and private values (θ_i) are drawn pairwise independently. This implies that a buyer i expects that the probability that his $n-1$ peers chose to wait (conditional on q) is the probability that a different buyer j waited to the $n-1$ power. The probability that buyer j waits equals to

$$Pr(j\ waits|q) = w(\theta^*, q) \equiv \underbrace{Pr(x_j = 0|q)}_{\text{Bad news}} + \underbrace{Pr(x_j = 1|q)}_{\text{good news}}\underbrace{Pr(\theta_j \leq \theta^*)}_{\text{Low } \theta_j}$$

$$= \begin{cases} (1-\pi) + \pi\theta^* & q = 1 \\ \pi + (1-\pi)\theta^* & q = 0. \end{cases}$$

Since $\pi > \frac{1}{2}$ and $\theta^* < 1$, buyer j is more likely to wait when $q = 0$ than when $q = 1$. Further notice that, conditional on q, signals and private values are drawn iid across buyers. Hence, i expects that, conditional on q, each pair of buyers $j, k \neq i$ decided to wait independently. Thus, the probability that his peers waited was $w(\theta^*, q)^{n-1}$. By Bayes rule, i's period 1 valuation is

$$\begin{aligned} v_{i1} &\equiv \theta_i Pr(q = 1|x_i = 1, no\ trade\ at\ t = 0) \\ &= \frac{\theta_i \pi\lambda w(\theta^*, 1)^{n-1}}{\pi\lambda w(\theta^*, 1)^{n-1} + (1-\pi)(1-\lambda)w(\theta^*, 0)^{n-1}}. \end{aligned} \tag{2.1}$$

In what follows, I provide a condition ensuring that the seller values the good more than all buyers, i.e. $v_{i1} \leq \theta_s$ for each buyer i. This implies that the strategy profile in lemma 1 is sequentially rational.

Sufficient Condition

I now provide a condition for when the seller decides to keep his good after period 1. Intuitively, the seller keeps his painting provided that he does not expect to extract more rents from buyers than his valuation for the good, i.e. θ_s. This is certainly true when no buyer in period 1 values the good more than the seller. In such case, it holds that:

Lemma 2 (Learning to Commit) *A sufficient condition for the seller to optimally posts $p_t = p^*$ in each period t is that*

$$\frac{Pr(q=1|x_i=1, no\ trade\ at\ t=0)}{Pr(q=0|x_i=1, no\ trade\ at\ t=0)} \equiv \left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w(\theta^*,1)}{w(\theta^*,0)}\right]^{n-1} \leq \frac{\theta_s}{\theta^*-\theta_s}. \quad (2.2)$$

This condition states that the seller avoids re-offering the good provided that the event in which all buyers wait is sufficiently informative of q. Informally, it means that the seller avoids re-offering his good, because learning among buyers lowers valuations enough to expunge the seller's gains from trade.

2.2.4 Discussion

I now discuss how and why the Coase conjecture fails in settings with interdependent values. Firstly, negative selection in the demand pool persists, i.e. high value buyers are more likely to bid than their low value peers, and prompts the seller to unprofitably screen his buyers. Secondly, negative selection further prompts buyers to screen each other, and this lowers valuations. When the second effect is sufficiently stark, the seller avoids screening his buyers and implements his optimal strategy.

In general, screening among buyers ensures that the Coase conjecture fails when the seller's valuation lies in the interior of potential buyer valuations. The seller stops re-offering his item in finite time, which this ensures that equilibrium revenues are unique and greater than immediately running an efficient auction. The intuition is threefold. Firstly, the seller still screens his buyers, i.e. if the item fails to sell, agents learn that valuations lie below a falling cutoff. Second, buyer valuations, themselves, fall over time. Lastly, the seller increasingly expects that there are fewer buyers who value the good more than himself.

The rest of the paper proceeds as follows. First, section 2.3 presents the general model. I generalize the distributions of types as well as the payoff function. Furthermore, I ensure that negative selection in the demand pool occurs in equilibrium.[3] Section 2.4 then states the results in the general model. Next, I present the literature review in section 2.5 and discuss the assumptions, results, and extensions in section 2.6. Lastly, appendix B.2 illustrates that the results herein extend to durable goods markets.

2.3 Model

This section presents the model. First, I present the primitives, i.e. types, distributions of types, payoffs, and the seller's valuation. Next, I introduce the timing of the game. Lastly, I define strategies and equilibrium.

2.3.1 Primitives

A seller offers a single, indivisible item to $n \geq 2$ buyers. Each buyer i has a type $\tau_i \in \mathcal{T} \equiv [0,1]^2$. Buyer i's type (i.e., τ_i) consists of a private value $\theta_i \in [0,1]$ and

[3] In order to test whether learning among buyers prevents the Coase conjecture, it must be the case that there is negative selection in the demand pool; otherwise, the conjecture can fail due to non-learning factors.

an interdependent value $x_i \in [0,1]$. All random variables (i.e., $(\theta_i, x_i)_i$) are drawn pairwise independently where each private value is distributed given a CDF K and the interdependent values given a CDF F. I further assume that the CDFs K and F admit PDFs $k \gg 0$ and $f \gg 0$. Next, if buyer i has type τ_i and other buyers have interdependent values $x_{-i} \in [0,1]^{n-1}$, the i's payoff from owning the item is $u(\tau_i, x_{-i})$. I assume that the payoff function $u : [0,1]^{n+1} \to [0,1]$ satisfies the following regularity conditions stated in assumption 1.

Assumptions 1 *The payoff function $u(\cdot)$ is linear i.e., for each tuple $(\theta_i, x_i, x_{-i} \equiv (x_j)_{j \neq i})$*

$$u(\tau_i, x_{-i}) = \theta_i^\alpha + x_i^\beta \prod_{j \neq i} x_j^\eta \tag{2.3}$$

for $\alpha, \beta \in (0,1), \alpha + \beta < 1,$ and $1 = \alpha + \beta + (n-1)\eta$.

Assumption 1 implies that the interdependent values as well as the private value enter payoffs in a linear manner. This assumption imposes a functional form to payoffs, which simplifies the characterization of result. This is because, in every symmetric equilibrium, buyers decisions to participate at an auction ran at any given period follows a threshold strategy: a buyer participates if and only if his valuation is above an equilibrium determined threshold.

Next, I ensure that there are gains from trade; otherwise, the point of the paper is moot. If the seller never expects to gain from auctioning his good, then it will vacuously holds that he never auctions the good in the first place. Now, suppose that the revenue maximizing, static auction has a reserve price of $p^* \in [0,1]$ in which buyers bid their valuation conditional on winning i.e.,

$$w_{i0} \equiv E_0[u(\tau_i, x_{-i}) | \tau_i,\ i \text{ wins}]$$

where for each type τ_i, the expectation is taken with respect to the initial distributions K and F. Each buyer i further expects that each buyer $j (\neq i)$ bids w_{j0} iff

$w_{j0} \geq p^*$; otherwise, buyer j waits and does not place a bid. It is the useful to define the CDF of valuations (w_{i0}) as H_0 and its PDF as h_0. I can now make the assumption ensuring that there are (initially) gains from trade.

Assumptions 2 *Let the seller's valuation for the good be $\theta_s \in (0,1)$, then I assume that the probability that some buyer is willing to bid more than the seller's valuation for the good at the optimal static auction is strictly positive but that some buyers would be willing to bid less than the seller i.e.,*

$$H_0(\theta_s) < 1.$$

2.3.2 Timing and Payoffs

After establishing the setting's primitive, I now state the timing of play. I assume the most standard timing possible in order to ensure that the results herein are not driven by non-standard assumptions. It should further be notice that the timing is similar to the one provided in the example.

The timing of play is as follows. Nature first draws types (τ_i) and privately informs τ_i to player i. The types are drawn as previously discussed. Next, at each period $t = 0, 1, \ldots$, the seller first announces a reserve price $p_t \in [0, 1]$. Each buyer i then decide to wait or submit a bid $b_{it} \geq p_t$. I assume that buyer submit bids at the same time. If buyers wait (i.e., no buyer i submitted some bid b_{it}), the game continues to period $t + 1$. Otherwise, the game ends, the buyer submitting the highest bid wins the auction, and either pays the second highest bid or p_t provided that no other buyer placed a bid. Moreover, if multiple buyers submit the highest bid, then each buyer wins the auction with equal probability. The timing of play is illustrated in figure 2.1. Lastly, if buyer i wins the auction in period $t = 0, 1, \ldots$ and must pay p_{it}, payoffs are

i. Buyer i: $\delta^t[u(\tau_i, x_{-i}) - p_{it}]$ for a common discount factor $\delta \in (0, 1)$

ii. Buyer $j \neq i$: 0

iii. Seller: $\delta^t p_{it} + \sum_{s=0}^{t-1}(1-\delta)\delta^s\theta_s = \delta^t p_{it} + (1-\delta^t)\theta_s$.

Note that if the time never sells, then each buyer i's payoff equals to 0 and the seller nets a payoff of θ_s. Moreover, it is important to note that I assume that types are drawn only once at the beginning of the game.

2.3.3 Strategies and Equilibrium

I now define histories, symmetric strategies, and equilibrium. Note that I focus on symmetric equilibria in a symmetric environment, because this paper is focused on the seller's commitment issue. Informally, a history is a record of all previous reserve prices since any bid would have ensured that the game ended. A strategy for the seller is then a map from histories to reserve prices and a strategy for buyers is a mapping from histories and current reserve prices to a decision to wait or which bid to submit. Meanwhile, an equilibrium imposes that beliefs are derived using Bayes rule and that players are sequentially rational.

I first define histories. In period 0, assume a set of histories $\mathbf{H}_0$ with a single null history. In period $t = 1, 2, \ldots$, however, a history h_t details the preceding reserve prices i.e., $h_t = \{p_\tau\}_{\tau=0}^{t-1}$. The set of period t histories is $\mathbf{H}_t \equiv [0,1]^t$.

Next, I define strategies. A seller strategy is a collection of functions (p_t) $\forall t, p_t : \mathbf{H}_t \to [0,1]$ such that at each period t and history h_t, $p_t(h_t)$ denotes the reserve price that the seller posts. Note that I assume that players follow pure strategies for exposition.[4] It is possible to consider behavioral and mixed strategies, but such extensions would be orthogonal to the point of this paper.

I now define a buyer strategy. A strategy for buyer i consists of a collection of functions (b_{it}), $\forall t, b_t : \mathbf{H}_{t+1} \times \mathcal{T} \to [0,1] \cup \{wait\}$, such that at each period t, history h_t, current reserve price p_t, and type τ_i, it holds that $b_{it}(\tau_i, h_t, p_t)$ denotes a bid choice or

[4]All functions are be assumed Lebesgue measurable. Furthermore, Liu et al (2019), Fundenberg, Levine, and Tirole (1985), and others find that almost surely neither the seller or buyers play a mixed strategy.

a choice to wait. I assume that a buyer that is indifferent between bidding and waiting will submit a bid and in the case that a bidder is indifferent between multiple bids, he submits the highest possible bid. Next, I assume a monotonicity requirement on strategies i.e., for each period t, type τ_i, history h_t, and price p_t if $b_{it}(\tau_i, h_t, p_t) \neq$wait, then for each period $s \geq t$ and history h_s such that $(h_t, p_t) \subset h_s$, it holds that for each price $p_s \in [0,1]$ $b_{is}(\tau_i, h_s) \neq$wait. This condition implies that strategies are consistent with past decisions to participate in the auction. Lastly, beliefs are history dependent joint measures on $(\tau_i)_{i=1}^n$.

Now that strategies and beliefs have been defined, I can define equilibrium. The paper focuses of perfect Bayesian equilibrium (PBE) so as to not deviate from the preceding literature.

Definition 1 (PBE) *A Perfect Bayesian Equilibrium (PBE) is a collection of strategies, $(p_t, (b_{it}))$, and beliefs such that for every period t and history h_t*

i. Given beliefs, strategies are sequentially rational

ii. Beliefs are derived via Bayes rule whenever possible.

2.4 Results

I now state my results. The paper's main result is that equilibrium revenues are unique and greater than immediately running an efficient auction, i.e. setting $p_0 = \theta_s$. I first present three auxiliary results. First, I prove that buyers follow a threshold bidding strategy. This means that buyers bid their valuation conditional on winning the good iff it lies above a time dependent cutoff. Next, this result implies the second result: progressive pessimism. Pessimism implies that in every period, prior beliefs likelihood ratio dominates their Bayes posteriors. This technical result drives all subsequent results in the paper.

I lastly find bounds on equilibrium revenues. First, I prove that an upper bound on equilibrium revenues. In every period, the seller cannot improve upon immediately running the revenue maximizing, static auction given his current beliefs and keeping the good if it fails to sell. Next, I find a revenue floor that is higher than immediately running an efficient auction and does not account δ or the frequency of future re-offerings. In each period, the seller's revenues are greater than running any auction where upon learning that the item failed to sell, buyers lower their valuation enough to justify that the seller keeps his good.

2.4.1 Auxiliary Results

Skimming Property

The first auxiliary result states that buyers play a threshold strategy in every equilibrium i.e., in every equilibrium, each player i participates in an auction iff i's valuation is above a history-dependent cutoff. This result imposes a tractable structure behind equilibrium beliefs. First define expectations given period t beliefs as $E_t[\cdot]$ and each player i's valuations given his type τ_i and the public history until period t as

$$w_{it} \equiv E_t[u(\tau_i, x_{-i})|\tau_i,\ i \text{ wins}].$$

This implies that player i's time t valuation is the expected he expects to net conditional on winning the auction. Note that when it is useful to specify the valuation as a function of the history, I will write it as $w_{it}(h_t) \equiv E[u(\tau_i, x_{-i})|h_t, \tau_i,\ i \text{ wins}]$. Next, I can define a threshold strategy.

Definition 2 (Threshold Strategy) *Buyer i is said to play a threshold strategy iff there exists a collection of functions $(u_{it}), \forall t\ u_{it} : \mathbf{H}_{t+1} \to \Re$ such that i bids his valuation in period t and history h_t if $w_{it}(h_t) \geq u_{it}(h_t)$; otherwise, i waits.*

This definition states that buyer i follows a threshold strategy if he bids his valuation conditional on winning provided that it lies above a cutoff. Otherwise, buyer i waits.

Note that buyers bid their valuation conditional on winning is a standard argument presented Myerson (1981) and Krishna (2004). The following lemma characterizes buyers' equilibrium behavior.

Lemma 3 (Skimming Property) *In every PBE, each buyer i plays a threshold strategy.*

The proofs are in the appendix, but I sketch the argument below. For each buyer i, his type (τ_i) is unverifiable. This implies that when i observes a type τ_i, he can implement the strategy associated with observing type τ_i' without the possibility of being detected. This implies that the difference in payoffs between participating in the auction relative to the payoffs he nets from waiting must be increasing in buyer i's current valuation. This establishes the skimming property.

This result is key for two reasons. First, this result implies that there still exists negative selection in the demand pool. This negative selection ensures that beliefs evolve as described in the next auxiliary result. Secondly, if buyers' decision to participate at auction follows a cut-off rule as described above, then the following auxiliary results proceed regardless of the auction protocols considered. How buyers bid and interact with each other is inconsequential when it comes to characterizing learning from the item failing to sell. The auction format, nonetheless, matters since it determines who participates in the optimal, static auction available to the seller. I delegate this discussion to the appendix to avoid the issue of presenting a general auction format in the main text.

Principle of Progressive Pessimism

The next result characterizes learning. Heuristically, the seller and buyers become increasingly pessimistic regarding other buyers' (IV) components and valuations. This is the key insight to my main result. I first present a standard ordering on distributions and state my result using such ordering. A CDF $H[0,1]$ likelihood ratio dominates another CDF on $G[0,1]$ provided that the CDF G systematically gives higher weight to lower realization of a random variable than H. The formal definition is the following.

Definition 3 (Likelihood Ratio Dominance) *Suppose that $H[a,b], a < b$, and $G[a,b]$ are CDFs admitting pdfs $h \gg 0$ and $g \gg 0$, respectively. Then, H likelihood ratio dominates G, i.e. $H \trianglerighteq G$, if for each pair of $x, x' \in [a,b]$ such that $x \leq x'$, it holds that*

$$\frac{g(x')}{g(x)} \leq \frac{h(x')}{h(x)}. \tag{2.4}$$

This is a strong notion of stochastic dominance. It implies first- and second-order, hazard and inverse hazard rate, stochastic dominance. Indeed, if one has a sequence of random variables (x_n) and for each n, it holds that the CDF of x_n likelihood ratio dominate the CDF of x_{n+1}, then the sequence is stochastically decreasing, i.e. it is increasingly likely that one observes high realizations as n increase and less likely to observe high realizations.

Next, define for each period t the CDFs F_t, K_t, and H_t as the equilibrium path beliefs regarding each buyer's interdependent value, private value, and valuation as of the beginning of the period. Note that $F_t(.)$, for example, refers to the seller or a buyer i's beliefs regarding buyer $j \neq i$'s value x_j. I now present the result.

Theorem 1 (Progressive Pessimism) *In every PBE, the expected valuations and types are stochastically decreasing:*

$$\forall t, \quad F_t \trianglerighteq F_{t+1}, G_t \trianglerighteq G_{t+1}. \tag{2.5}$$

I now state an immediate corollary, i.e. valuations and the expected dispersion of valuations falls.

Corollary 4 (Expected Dispersion in Valuations Falls.) *In every PBE and period t, the dispersion in valuation falls, i.e. for every pair of types τ, τ', it holds that*

$$E_t[|v_t(\tau) - v_t(\tau')|] \geq E_{t+1}[|v_t(\tau) - v_t(\tau')|]. \tag{2.6}$$

Next, for every period t, buyer i, and type τ_i, it holds that $v_t(\tau_i) \geq v_{t+1}(\tau_i)$.

These results allow me to decompose learning in the current IV setting and compare it to a setting with private values. Such comparison clarifies the role. For exposition, I focus on beliefs regarding valuation, i.e. $v_{it} = v_t(\tau_i)$. First, let v_t be the smallest valuation for which $H_t(v_t) = 1$, i.e.

$$v_t \equiv \inf\{x \in [0,1] | H_t(x) = 1\}$$

Note that v_t is non-increasing in t due to the skimming property. In every period t, agents expect that buyers have valuations below v_t and are distributed by the CDF $H_t[0, v_t]$.

If the item fails to sell, agents learn that buyers have a valuations below a cutoff $u_{t+1} \leq v_t$. Note that in a comparable private value setting, the seller's beliefs regarding buyer valuations in period $t+1$ is given by the CDF $\frac{H_t}{H_t(u_{t+1})}[0, u_{t+1}]$. Next, every buyer further lowers their valuation in response to their peers lack of trade and hence the maximum valuation in period $t+1$ is $v_{t+1} \leq u_{t+1}$. Figures 2.2 and 2.3 illustrates the decomposition described above.

The proof of theorem 1's is inductive. Intuitively, when a buyer i observes that his peer $j \neq i$ waited, he learns that $v_{jt} \leq u_t$ and expects that for each pair of values x_j and x'_j where $x_j \leq x'_j$, there exists more potential values θ_j for which the expected valuation of (x_j, θ_j) lies below v_t than for realizations. Consequently, the probabilities ratio is non-increasing is x_j. The stochastic dominance discussed above follows immediately from this observation and argument extends to beliefs regarding private values as well as for valuations.

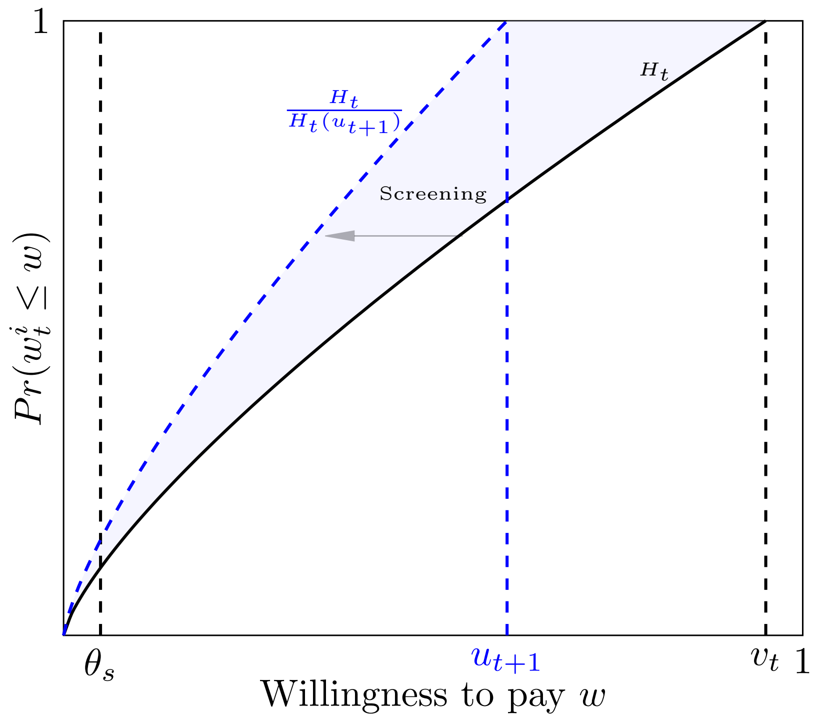

Figure 2.2: Initial distribution of valuations.

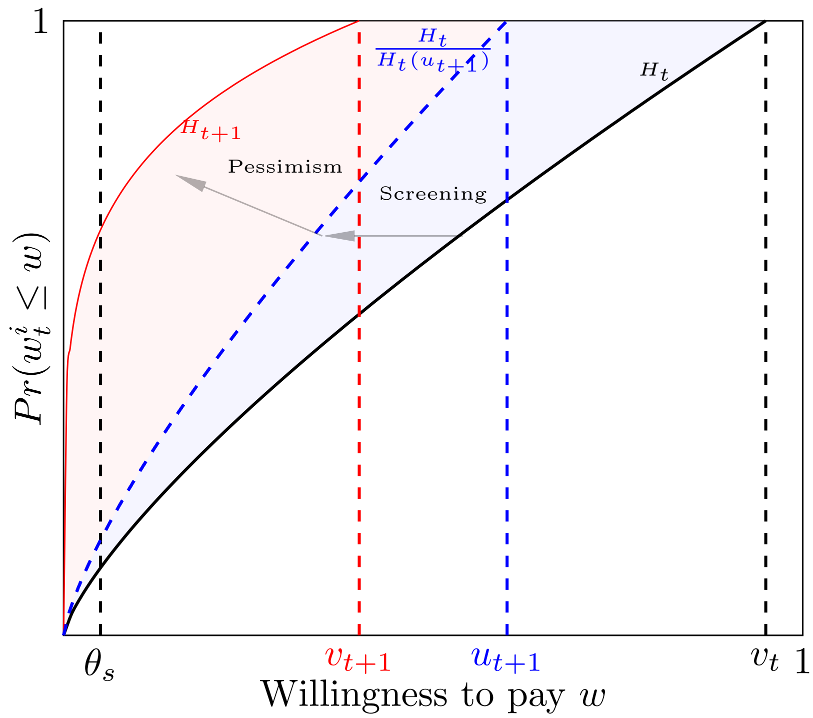

Figure 2.3: Decline in valuations after the good fails to sell

Bound on Revenues

I now derive bounds on equilibrium revenues. First, I define an upper bound on revenues. In each period, I find that the seller cannot improve upon immediately running the revenue maximizing auction given the information at hand and never re-offering his good. Remember that re-offering his good is often sequentially rational. Next, I derive a lower bound. Intuitively, I calculate the revenue maximizing auction after which the seller can commit to keep his good.

First, suppose that after period t, the seller commits to his optimal continuation strategy. What strategy would he pick? I show that the seller cannot improve upon immediately running the static optimal auction given the information he holds in period t and keeping the good if it fails to sell. If p_t^* denotes the optimal, static auction's reserve price, then the seller prevents re-offering his good by setting $p_s = p_t^*$ for every period

$s \geq t$ with probability 1. Next, I derive an expression for the revenues attained by a static auction. A static auction held in period t, when beliefs are $H_t[0, v_t]$, and a reserve price of $p \in [0, 1]$ nets the seller an expected revenue of

$$r(p, H_t) \equiv \theta_s + E_t[\chi(v_t^2 \geq p)\{\bar{\phi}(v_t^2, H_t) - \theta_s\}]$$

where v_t^2 is the second highest valuation among buyers and $\bar{\phi}(x, H_t)$ are the ironed out virtual values as given by the current distribution of valuations conditional on winning the auction. Furthermore, the optimal static auction's revenues are denoted as r_t^* and satisfy

$$r_t^* \equiv \max_{p \in [0,1]} r(p, H_t). \tag{2.7}$$

Next, I find a period 0 revenue floor. The seller may not be able to implement his revenue maximizing auction and never re-auction his good, but there exist some auctions that the seller could run in period 0 whereupon learning that the item failed to sell, buyers end up willing to bid less than the seller's valuation. First define valuations conditional on the seller running a static auction in period 0 with a reserve price of p, buyers bid myopically, i.e. each buyer i bids iff $w_0(\tau_i) \equiv E[u(\tau_i, x_{-i})|i\ wins] \geq p$, and yet the good remains unsold as

$$w_0(\tau_i, p) \equiv E[u(\tau_i, x_{-i})|i\ wins,\ \forall j \neq i, w_0(\tau_j) \leq p].$$

I present the lower bound below.

Definition 4 (Commitment Auctions) *In every PBE, the price $p \in [0, 1]$ defines a commitment auction if for every type τ_i such that $w_0(\tau_i) \leq p$, it holds that $w_0(\tau_i, p) \leq \theta_s$. Next, $\underline{r}_0$ is a revenue floor that satisfies*

$$\underline{r}_0 = \max_{p \in [0,1]} r(p, H_t)\ s.t.\ \forall \tau \in \mathcal{T},\ s.t.\ \ w_0(\tau) \leq p, w_0(\tau, p) \leq \theta_s. \tag{2.8}$$

It should be noted that the set of such auctions is non-empty as $p = \theta_s$ is a commitment auction. Next, denote the expected revenues from running an efficient auction in period 0 as r_0^e and the equilibrium revenues in period t as r_t. Also, a CDF $H[a, b]$, for $a < b$, is regular iff for each x, the function $x - [1 - H(x)]/h(x)$ is increasing. I now state the subsequent theorem.

Theorem 2 (Coase fails) *In every PBE, revenues are below the optimal, static auction revenues, i.e.* $\forall t,\ r_t \leq r_t^*$. *Meanwhile, if* $H_0(\cdot)[0, 1]$ *is a regular distribution, then the seller's equilibrium revenues are strictly higher than immediately running an efficient auction, namely* $r_0^e < \underline{r}_0 \leq r_0$.

I first make some comments before sketching the proof. The revenue floor is independent of how often the seller offer his good and the discount factor. Therefore, the Coase conjecture does not hold in this setting. Lastly, this lower bound need not bind when types are two-dimensional. However, in the accompanying paper, Ramos-Mercado (2022), I prove that when the types are one-dimensional, this revenue floor is binding.

The proof proceeds in three steps. First, suppose that after some history, the seller offers and commits to a dynamic, trade mechanism that only depends on each buyer's current valuation. This mechanism consists of an allocation and payment rules as well as a time when trade and payoffs are realized such that the outcome can be implemented with second-price auctions with reserve prices. I show that the seller might as well focus on mechanisms in which he only trade immediately and if the item fails to sell, the seller keeps the item from henceforth.

The argument follows a replication argument. Fix some individually rational and incentive compatible, dynamic mechanism. I construct an alternative mechanism where the period in which he sells the good is either immediate or never. The proof further shows that such mechanism yields the seller the same revenues as the initial mechanism. Consequently, it is without loss of generality to focus on this restricted class of mechanisms rather than a larger class.

2.4.2 Main Result

The previous results leave three questions unanswered. First, under what conditions are equilibrium revenues with limited commitment equal to revenues with full commitment? Second, are there multiple equilibria? Otherwise, it is possible for learning to mitigate revenue losses in some equilibria and not in others. Lastly, when the seller cannot implement his revenue maximizing auction, what happens?

The answers to these questions are as follows. First, the equilibrium is essentially unique, so multiplicity of equilibria does not undermine revenue predictions. Next, I find sufficient conditions ensuring that the seller implements his optimal strategy. Otherwise, I find that the game effectively ends in finite time. This means that the seller stops eliciting bids almost surely by a finite time horizon and its formal definition is below.

Definition 5 *The game essentially ends in finite time iff there exists a deterministic period $T < \infty$ such that buyers almost surely wait in every period $t \geq T$ and PBE.*

I can now state the paper's main theorem.

Theorem 3 *The PBE is essentially unique and the game essentially ends in finite time. Furthermore, revenues equal the optimal, static auction revenues, i.e. $r_0 = r_0^*$,if the following condition holds:for every type τ_i such that $w_0(\tau_i) \leq p^*$, it holds that*

$$w_0(\tau_i) - v(\tau_i, p^*) \geq \frac{1 - H_0[w_0(\tau_i)]}{h_0[w_0(\tau_i)]}. \tag{2.9}$$

I now discuss theorem 2.9. First, the seller commits to the static, optimal auction iff all excluded buyers from the optimal auction demand smaller information rents than the loss of valuation due to learning. Otherwise, revenues with limited commitment are lower than revenues with full commitment. However, the seller stops eliciting bids in finite time.

The proof has two steps. First, there exists a deterministic period $T < \infty$ such that in all PBE, the seller can fix prices after period t. Intuitively, the seller is sequentially rational, impatient, and can always implement his revenue maximizing commitment auction. But as the item fails to sell, the maximum valuation falls enough by period T such that it is optimal to implement an auction that prevents re-offerings. The second part of the proof characterizes precisely when the seller implements his optimal strategy. I use the virtual value characterization of the optimal reserve price and the observation that no buyer, in period 1, should be willing to bid more than the seller's valuation.

2.5 Related Literature

The Coase Conjecture (1972) illustrates the implications of limited commitment in a stylized manner. Intuitively, patient, high valuation buyers are more likely to buy the good than their low valuation peers. Thus, a sequentially rational monopolist sequentially lowers prices to trade with his remaining buyers. But this pricing behavior persuades some high valuation buyers to delay their purchase decision: thus, lowering profits. Coase conjectured that as the seller interacts with his buyers ever more frequently, then his price converges his marginal cost.

Indeed, Bond and Samuelson (1984), Gul et al (1986), Kahn (1986), and others corroborate the Coase conjecture in stationary equilibria. Subsequent papers, however, illustrate that the conjecture is not robust when either the seller follows a non-stationary strategy or when one slightly perturbs the underlying setting. Ausubel and Denekere (1989) prove that there exist non-stationary equilibria where the seller sustains a gradually declining price schedules and attains profits that are arbitrarily close to the static monopoly rents attainable under full commitment. Intuitively, agents posit a candidate price schedule and if the seller ever posts a price different from the one dictated by the schedule in question, then buyers expect that the subsequent sub-games follow a Markov equilibrium. These Markov equilibria net the seller no profits in the continuous time limit. Thus, the seller prefers sticking to the candidate price schedule if he interacts with buyers sufficiently frequently.

Board and Pycia (2014), meanwhile, added outside options for buyers and found that all equilibria are payoff equivalent to fixing prices equal to the one-shot monopoly price. Heuristically, equilibrium prices are greater than the lowest value among remaining buyers: thus, remaining buyers with relatively low valuations are better off exiting the market than waiting and expecting no consumer surplus in the future. This leads buyers to either immediately purchase the good or exit the market. Thus, the seller *can* fix prices since he has no remaining buyer with whom to trade.

Fundenberg et al (1987) also find that the Coase conjecture fails when seller have an outside option, e.g. consume the good himself or sell it to another buyer. The seller only offers his good until a finite, terminal period since he eventually expects that the rents that he can extract from buyers may not compensate for him forgoing his outside option. This result differs markedly from my result because the seller offers his good to multiple buyers at the same time, via an auction. I too find that the seller eventually becomes sufficiently pessimistic and decides to not re-offer his good but learning among buyers further distorts this intuition twofold. First, learning among buyers further lowers the total surplus above and beyond what the seller learned from his peers. Secondly, social learning allows the seller to extract an increasing share of the surplus among the buyers in question. Further work by McAfee and Wiseman (2008), Madarász (2021), Bagnoli et al (1989), von der Fehr and Kuhn (1995), Montez (2013), Feinberg and Skrypacz (2005), Karp (1993), Ortner (2017) and others similarly find small that environment preventing the Coase conjecture.

Next, the Coase conjecture generalizes to auctions. Vincent and McAfee (1997) first showed that when the seller values the good strictly less than all buyers, there exists an essentially unique equilibrium, where the seller runs a sequence of standard auctions with declining reserve prices until a finite, terminal period in which he ensures that the item is sold. Their paper, however, assumes a "gap" case in a private value setting. This means that the seller who values the good the least values the good more than the seller.

Liu et al (2019) then studied the no gap case and showed that when there are at least 3 buyers and for most independent private value environments with 2 buyers, the seller might as well immediately run an efficient auction in the limit when the time between transactions go to zero. They show that with multiple buyers, one cannot construct a reputational equilibrium—as in Ausubel and Deneckere (1989)—where the seller can profitably screen his buyers and find that the unique level of profit are those attainable by the auction mentioned above.

The intuition behind the Coase conjecture further extends to contracting settings—see Skreta (2006, 2015), Doval and Skreta (2021), and others. When a principle commits only to short term contracts and interacts repeatedly with agents holding private information, he extracts information rents from his agents and changes the contracts offered. This, in general, limits the principal's ability to provide the agents with incentives and restricts implementable outcomes. Doval and Skreta (2021) for instance show that these contracts can be characterized via a generalized version of the Coase Conjecture. Meanwhile, Burzustowski et al (2021) finds that allowing the principal to implement dynamic contracts that he can void at will, however, allows him to avoid the Coase Conjecture.

2.6 Discussion and conclusion

2.6.1 Discussion

In this subsection, I discuss how changes to the primitives affect the paper's results. I also discuss how results extend to more general auction and non-auction environments.

Assumptions on Primitives: Buyers My main results disregarded several issues pertaining preferences and the signal structure. I firstly assume that all random variables are drawn independently from each other, but types can be, for instance, affiliated across buyers. In the motivating example, see section 2.2, this assumption allowed me to describe how each buyer learns deduces information from each one of his peers separately. In general, each buyer learns from the lack of bidding decisions of their peers, as a collective. This means that he updates his beliefs cognizant of the way the signals are jointly drawn. Aside from this distinction, learning among buyers would proceed as before.

The model further assumes symmetric buyers and this does not represent many interesting settings. For example, an art collector's valuation is more responsive to a painting's resale value than the director of a museum since the museum profits from exhibiting the piece rather than reselling it. In equilibrium, however, there still exist negative selection in the demand pool and hence the learning dynamics described herein persist.

Assumptions on Primitives: Seller A less innocuous assumption is that the seller's valuation is constant and in the interior of potential buyer valuations. This assumption avoids three issues. First, if all buyers value the good more than the seller for certain, i.e. $\theta_s = 0$, then the seller never stops re-offering his good in finite time. Indeed, this may seem like an important case, but I claim that it is pathological. First, for every value $\theta_s < 0$, this is a "gap" case and an argument like the one presented in Fundenberg, Levine, and Tirole (1985) shows that the seller stops re-auctioning his good in finite time. Meanwhile, I already established this precise issue when $\theta_s > 0$.

Next, the seller's valuation may depend on a buyers' interdependent components. For example, the seller may care about his artwork's resale value at a different auction. I find that the results herein still hold when the seller's valuation for the good are not as responsive to the resale value as buyers. This too can be understood with the art auction example. The seller may resale his good at the same art markets as his buyers, but he can also run a private sale. Therefore, his valuation for the good is not as responsive to a low resale value at an alternative public offering.

Lastly, the seller could also observe an interdependent value component, making him an informed principal. For example, the seller may be privately informed of the artwork's resale value and his reserve price may signal his private information. I disregard this possibility, because it adds a significant layer of complexity that obfuscates learning among buyers. Such considerations are important, and I plan to study this precise question in my future work.

Assumptions on the auction procedure. The paper further assumes that the seller runs second-price auctions. I make this assumption to simplify exposition and to compare results with the literature, but if the decision to participate in an auction result in negative selection among buyers, the results follow. The auction format only matters in as far as determining who would be excluded from the optimal static auction. Appendix B.1.3 presents a broad collection of auction in which there exist negative selection in the demand pool and hence my result can be extended to those settings.

Is there something special about auctions? The last point of clarification is that the results herein persist beyond auction settings. In appendix B.2, I consider a durable good market with a single seller and a continuum of buyers with a common value. I prove that learning among buyers contravenes the Coase conjecture in stationary settings. This is because buyers observe the mass of consumers who previously purchased the good and this eliminates all dispersion in valuations. Thus, the seller can extract all rents from the remaining buyers.

2.6.2 Conclusion

The Coase conjecture predicts that limited commitment lowers profits since the seller serially auctions his good. In this paper, I proved that this behavior can be rationalized in settings with interdependent IV values. Moreover, it is possible for a sequentially rational seller to attain his maximum expected revenues with full commitment.

Intuitively, learning among buyers serves as an endogenous commitment device. When the seller runs an auction and the good does not trade, the seller and buyers first learn that buyers had lower valuations than expected. Each buyer further expects that their peers' low valuations were informed, at least partially, by interdependent value, private information. Consequently, buyers lower their valuation for the good and this limits, or outright prevents, items from being serially re-offered. When valuations fall by more than buyer's initial information rents, the seller prevents his optimal strategy with full commitment.

I also characterize what happens when the seller cannot commit to his revenue maximizing auction. The seller stops offering his good in finite time, he extracts an increasing share of the trade surplus over time, and his equilibrium revenues are greater than immediately running an efficient auction. I further show that bid shading falls over time. Intuitively, buyers expect it to be increasingly likely that they outbid their peers regardless of their peer's private information. Winning the item at auction, therefore, becomes an increasingly uninformative event and buyers respond by bidding an increasing share of their valuation.

My results, lastly, illustrate that the received Coasian logic is incomplete. The Coase conjecture implies that a sequentially rational seller screens his buyers, which results in delayed market participation and lower profits. In IV settings, however, buyers also screen each other and this that as the seller learns from his buyers, he offers buyers terms of trade that increasingly favor remaining consumers. When types are interdependent, however, *buyers* also lower their willingness to pay, the dispersion in valuations falls, and the seller can extract an increasing share of the remaining trade surplus. Consequently, learning among buyers limits re-offering and the payoff relevance of sequential rationality.

Chapter 3

Reasons for Peace

Summary Why do only 1 in 4 (8 post-1914) wartime, peace talks reach a lasting agreement? In this paper, I rationalize wartime negotiations' declining success and characterize their welfare-maximizing (i.e., optimal) design. I extend the reputational bargaining framework by adding hidden effort to manage the bargained surplus and preclude a military resolution. Wartime negotiations' declining success can be rationalized by a policy-driven increase in ceasefire during peace talks. Intuitively, combatants only opt to negotiate after a prolonged conflict reaches a draw. Learning from active fighting dissuades combatants from posturing and expedites when agreements arrive. This implies that is optimal (if ever) to impose a brief ceasefire at the beginning of the negotiation.

3.1 Introduction

Wars are costly, uncertain, and seldom won in the battlefield. Yet combatants stuck in a bloody, extended deadlock are unlikely to negotiate a peace. And when they decide to negotiate, they are unlikely to reach a lasting agreement i.e., be successful. I estimate that 25 percent of negotiations held since 1823 were successful and this rate fell to 13 percent after 1914. In this paper, I study why wartime negotiations fail; why they are increasingly unsuccessful; and how to improve their design.

My key findings can be summarized as follows. Firstly, moral hazard considerations can rationalize why a negotiation can breakdown after multiple rounds of proposals—a common, real-world occurrence. In my model, combatants privately manage a diminishing surplus as they negotiate. Combatants allow the surplus to be destroyed before an agreement in reached along the unique equilibrium path. This is because each combatant expects their opponent to disproportionately benefit from any agreement and for their efforts to go uncompensated.

Secondly, I find that the declining success of wartime negotiations after 1914 can be rationalized by the increased occurrence of negotiations coinciding with ceasefires. Such occurrence is (crucially) policy-driven. My model illustrates that negotiating during a ceasefire can lower welfare. Nevertheless, the optimal negotiation design may involve an initial, short-lived ceasefire.

It is important to provide some background on wartime negotiations before describing the model and results in detail. Min (2020) finds that negotiations are most likely to reach an agreement when they succeed a decisive victory or after a prolonged draw. Negotiations following a decisive victory succeed because the losing combatant prefers making large concessions to being annihilated. The factors prompting combatants stuck in a draw to reach an agreement, however, are unclear and key for policy. For example, America's 2001 intervention in Afghanistan was only successful in cities. This led to 20 years of draw, 17 years without formal negotiations (USIP 2021), and a 2-year negotiation reaching a soon-abandoned agreement.

This example is representative of empirical trends since 1914. First, the median war duration rose from 15 months to 4 years and the mean duration increased even more. Meanwhile, the share of wars ending with a formal peace treaty declined from 62 to 42 percent over the same period. Wars are also less likely to have a pre-agreed pauses since the share of conflicts with a ceasefire fell from 74 to 55 percent. The share of time spent in a ceasefire also fell from 10 to 7 percent. Nevertheless, ceasefires are 250 percent more likely to coincide with peace negotiations after 1914. I further find that a day spent in both a ceasefire and negotiation is associated with an average of 10 additional days of fighting.

This last relation is stark, but is it real? If not, is there no role for ceasefires during a peace negotiation? To answer these questions, I build a model of wartime negotiations. Two players bargain over a surplus e.g., a territory. Each player is possibly obstinate, meaning that that their actions are exogenously determined as in Abreu and Gul (2000).

While they bargain, players must exert hidden effort to mitigate the risk of losing the war and the risk of destroying the surplus. Each player wins the war at an arrival rate that depends on their opponent's defensive efforts. I assume that obstinate players exert high defensive effort. Such assumption can be relaxed, but doing so only serves to strengthen the results described below.

Next, I assume that the surplus is a continuous-time Markov chain. The surplus is initially high and is (at least partially) destroyed at a rate that depends on the players' collective effort. As players exert more effort, the surplus is destroyed at a slower rate. I assume that obstinate players exert some constant and arbitrary level of effort, but note that assuming low effort is a natural assumption. Doing so, however, leads to a stark result that I discuss later on.

Lastly, the negotiation has two phases: active fighting and ceasefires. I model a ceasefire as the special parametric case, regardless of how much effort players exert, both kinds of risks have an arrival rate of 0. Players take the phases as given, but the last section introduces a benevolent social planner picks when each phase occurs.

Given exogenous negotiation phases, there exists a unique equilibrium. Strategic players mimic the demands of obstinate types and a war of attrition ensues. In private, however, strategic players opt to exert little effort in preserving the surplus and less to avoid a key victory for their opponent. This is because strategic players expect to make significant concessions in any future agreement. Thus, players negotiating during active conflict reach an agreement faster when they negotiate while they fight rather than in a ceasefire.

If obstinate players are expected to exert little effort, it is optimal to never pause the fight. This is not true in general. Nevertheless, I derive an inequality (with a closed-form solution) stating when holding a ceasefire is optimal. Such condition derives 3 insights. I first find that it is *never* optimal to only negotiate during a ceasefire. Heuristically, this means that ceasefires should be brief. Secondly, it is only sensible to hold a ceasefire when players expect that their opponent is unlikely to be obstinate. The last insight is that the only times when a ceasefire may be useful is at the beginning of the negotiation or just after the surplus falls if it occurs early into the negotiation.

The rest of the paper is organized as follows. The following section presents a version of the model without hidden effort, which summarizes how a negotiation proceeds during a ceasefires. Next, I present the model and characterize the unique equilibrium. I then present the social planner's problem and its characterization. Section 3.3 then summarizes the preceding literature and section 3.4 discusses results and concludes. Lastly, I delegate the empirical analysis and technical proofs to the appendix.

3.2 Model

This section presents the basic model and my results therein. I first model the negotiations coinciding with ceasefires as a symmetric version of Abreu and Gul (2000). This is because asymmetric types add do not add new, significant insights to the theory of war negotiations. Next, I extend the setting to allow for an imperfectly and jointly manage surplus. My main result is that the extended setting leads to better bargaining outcomes.

3.2.1 Benchmark: No hidden effort

Two players bargain over a surplus ($s_t = 1$) in continuous time. Each player i either demands a surplus share $\omega_{it} \in [0, 1]$ or accepts j's (i.e., not i) demands. If i concedes to j's time t demands of ω_{jt}, payoffs equal to

$$(u_{jt}, u_{it}) = e^{-rt}(\omega_{jt}, 1 - \omega_{jt}) \tag{3.1}$$

for some common discount factor $r > 0$. I assume that when both players concede at the same time t, they split the surplus equally.

Each player is further obstinate with probability $\mu \in (0, 1)$; otherwise, he is strategic. If a player is obstinate, he demands a constant share $\rho \in (1/2, 1)$ of the surplus and only accepts a share of ω if $\omega \geq \rho$. Strategic players, on the other hand, are free to make any demand or concede.

I now define strategies and beliefs. To be clear, a strategy defines the actions of a strategic player only. A mixed strategy for player i is a CDF H_i such that at each time t, H_{it} denotes the probability that i concedes by time t. Meanwhile, player j's beliefs that i is obstinate at time t are μ_{it}. I further define the functions $F_{it} \equiv (1 - \mu_{it})H_{it}$ and $c_{it} \equiv \dot{F}_{it}$ at each time t whenever they are well defined. Lastly, I focus on Perfect Bayesian Equilibrium (PBE) i.e., at each time t, strategies are sequentially rational given beliefs and beliefs are derived from strategies via Bayes rule whenever possible.

Now that the setting is well defined. I characterize the unique equilibrium. First, I show that c_{it} is well defined and $c_{it} > 0$ if $\mu_{jt} < 1$.

Lemma 5 *At each time t, each player j concedes immediately if $\mu_{it} = 1$; otherwise, c_{it} is well defined and $c_{it} > 0$.*

Abreu and Gul (2000) gave a proof of this statement and I generalize it for my general model below. Next, I characterize c_{it}. Suppose that at time t $\mu_{it} < 1$, then $c_{it}, c_{jt} > 0$ implies that players are indifferent between conceding immediately and waiting until time t+dt (for dt>0) to concede. If j concedes, he nets a payoff of $1 - \rho$ almost surely. If i concedes at time t+dt, then i concedes between time t and t+dt with probability $(1 - \mu_{it}) \int_t^{t+\mathrm{dt}} dH_{is} = c_{it}\mathrm{dt} + o(\mathrm{dt})$ netting him a payoff of ρ. Otherwise, player j (once again) nets a payoff of $1 - \rho$. Indifference implies that

$$1 - \rho = \rho c_{it}\mathrm{dt} + (1-\rho)e^{-r\mathrm{dt}}(1 - c_{it}\mathrm{dt}) + o(\mathrm{dt}).$$

I then reorganize terms, divide by dt, and take the limit of dt to 0 to get that $c_{it} = c$ such that if $\phi \equiv (1-\rho)/[\rho = (1-\rho)]$, then $c = r\phi$. This implies concessions arrive at a constant rate until beliefs converge to 1. By Bayes rule, beliefs initially equal to μ and satisfy $\dot{\mu}_t/\mu_t = c$, so $\mu_t = \mu \min\{1, exp(ct)\}$. The figures C.1 and C.2 illustrates how beliefs and concession rates evolve.

3.2.2 Bargaining with hidden effort

I now extend the model to account for the risks faced by combatants negotiating while fighting a war: they can lose the war outright and destroy the surplus in contention. In response, I assume that players can exert unobserved effort in order to mitigate these risks. First, I model the surplus. The time 0 surplus is normalized to 1 (i.e., $s_0 = 1$) and at each time $t \geq 0$, the surplus transitions from $s_{t^-} \equiv \lim_{s \nearrow t} s_s$ to

$$s_t = s_{t^-} \begin{cases} \epsilon & \text{with probability } \pi \\ 0 & \text{with probability } 1-\pi \end{cases} \tag{3.2}$$

(for parameters $\pi, \epsilon \in (0,1)$) at an arrival rate of $\xi_t \sum_i rf(1-e_{it})$. Each player i exerts effort e_{it}, $\xi_t \in \{0,1\}$ is a function that players take as given, and $f : [0,1] \to [0,1]$ is strictly increasing, strictly convex, continuously differentiable, $f(0) = 0$, and as $x \to 0$, $f'(x) \to \infty$. Note that players do not affect the amount by which the surplus falls.

Next, I assume that ξ_t is continuous almost everywhere. If $\xi_t = 0$, then players are in a ceasefire at time t; otherwise, $\xi_t = 1$ implies that players fight. Also, if i is obstinate, he exerts a constant effort of $e_* \in [0,1]$. Next, I model the possibility of j decisively defeating i in battle and imposes terms. Formally, j defeats i with an arrival rate of $\xi_t rg(1-a_{it})$ for function $g(\cdot)$ satisfying the same regularity conditions as $f(\cdot)$. I assume that obstinate players exert constant, full effort i.e., $a_{it} = 1$.

I now characterize payoffs. If the surplus falls to 0 at time t before players reach an agreement, then player i nets a payoff of $-C_{it}$ such that $C_{it} \equiv \int_s^t r(c + e_{i\tau} + a_{i(\tau-s)})e^{-r(\tau-s)}d\tau$ for a flow cost of fighting $c \geq 0$. Next, j defeats i at time t and $s_t > 0$, then $j's$ payoff is $s_t e^{-r(t-s)} - C_{jst}$, whereas $i's$ payoff equals to C_{it}. Lastly, if i concedes to j's demands of ρ, then i nets a payoff of $(1-\rho)s_t e^{-rt} - C_{jst}$ and j's payoff is $\rho s_t e^{-rt} - C_{jst}$.

I now provide formal definitions. At each time $t \geq 0$, players observe that the surplus fall $n = 0, 1, \ldots$ times at times $h_{nt} \equiv \{\tau_s\}_{s=0}^n$ for $0 = \tau_0 < \tau_1 < \ldots < \tau_n$. The set of h_{nt} histories is $\mathbf{H}_{nt}$. A public player strategy is then a triple (H_{it}, e_{it}, a_{it}) such that $\forall t$, $H_{it}, e_{it}, a_{it} : \mathbf{H}_{nt} \to [0,1]^3$ H_{it} is the probability that i concedes by time t, e_{it} is the effort to preserve the surplus, and a_{it} is the effort to avoid j winning the war. [1] Next, at each time t and history h_{nt}, j's beliefs that i is obstinate are $\mu_{int}(h_{nt})$. Next define $F_{int} \equiv (1-\mu_{int})H_{int}$, $c_{int} \equiv \dot{F}_{int}$, and $s_n = \epsilon^n$ for $n = 0, 1, \ldots$. Lastly, I focus on the set of (public) PBE[2].

Equilibrium with hidden effort

I now characterize the equilibrium with hidden effort. First, I characterize effort decisions. I then derive the concessions behavior. Lastly, I characterize beliefs.

Effort decision I now characterize a strategic player's effort decisions. If i expects that a strategic player j follows the strategy defined in σ, then his nt payoffs are W_{int} satisfying

$$e^{-rt}W_{int} = E_{\sigma t}\left[\omega'_{int'}e^{-r\tau'} - C_{itt'}\right]$$

where $\omega'_{int'}$ is i's expected termination payoff. Observe that the strategies (and thus

[1] Notice that strategies are assumed to only depend on public information. Alternatively, one can extend the definition of strategies to include each players preceding actions. To do this, I considered a sequence of games in which players only make choices at times $t = 0, \mathrm{dt}, \ldots$ for a constant $\mathrm{dt} > 0$ and analyze the limiting equilibria as dt goes to 0. This allows me to define private histories in the standard way. Nevertheless, the limiting equilibria ends up is the same as the equilibrium to the continuous time game reported in this paper.

[2] To simplify exposition, I do not distinguish between PBE and public PBE.

payoffs) are only a function of time, so the Feynman-Kac formula applies and states

$$\begin{aligned} rW_{int} = -c - \overbrace{(e_{int} + a_{int})}^{\text{effort}} + c_{int}[\rho s_n - W_{int}] + \dot{W}_{int} \\ + \underbrace{\xi_t[f(e_{it}) + (1 - \mu_{jnt} f(1 - e_{jnt}))][\pi W_{i(n+1)t} - W_{int}]}_{\text{surplus destruction}} \\ + \underbrace{\gamma g(1 - a_{it})[0 - W_{int}] + \gamma(1 - \mu_{jt}) g(1 - a_{jt})[s_n - W_{int}]}_{\text{military resolution}} \end{aligned} \quad (3.3)$$

Since i expects to mix between conceding and making demands, his expected payoff equals to conceding immediately when the remaining surplus is s_n i.e.,, $W_{int} = (1-\rho)s_n$. Thus, i's effort choices solve

$$\max_{e,a \in [0,1]} -r(e + a) - \xi_t(1 - \rho)s_n[1 - \pi\epsilon] r f(1 - e) - \xi_t r g(1 - a) s_n (1 - \rho) \quad (3.4)$$

This condition holds since effort choices are linearly separable, so j's effort choices does not affect his marginal choice. Also, since feasible effort choices belong to a compact, convex set and f, g are strictly convex and differentiable, the optimal choice is characterized by standard first order conditions (focs)[3]. The optimal level of effort is constant and described below.

Lemma 6 *In every public PBE, time t, and $n = 0, 1, \ldots$, if $\xi_t = 0$, then $e_{int} = a_{int} = 0$; otherwise, $e_{int} = e_{nt}$ and $a_{int} = a_{nt}$ such that*

$$\frac{1}{s_n(1 - \rho)} = (1 - \pi\epsilon) f'[1 - e_{nt}] = g'[1 - a_{nt}]. \quad (3.5)$$

This is just a formal statement of the first order condition. Although standard, it implies that negotiations that always coincide with active fighting can end without an agreement.

[3]The regularity conditions on f and g allow me to abstract away from an moot multiplicity of equilibria

War of Attrition Now that effort choices have been derived, I characterize concessions behavior. I first establish that players gradually concede.

Lemma 7 *In every PBE and for each time $t \geq 0$, player i, and $n = 0, 1, \ldots$, if $\mu_{int} = 1$, then j concedes. Otherwise, c_{int} is well defined and $c_{int} > 0$.*

This lemma implies that F_{int} is differentiable and its derivative (i.e., c_{int}) is strictly positive if $\mu_{jnt} < 1$. Next, define the following terms to simplify exposition: $E_{nt} \equiv c + e_{nt} + a_{nt}$, $\delta_{nt} \equiv \xi_t f[1 - e_{nt}]$, $\gamma_{nt} \equiv \xi_t g[1 - a_{nt}]$, and $\delta_* \equiv \xi_t f(1 - e_*)$. If one plugs in c_{jnt} and the previously defined terms in regards to players effort into equation 3.3, it holds that for each nt

$$\begin{aligned} rW_{int} = -rE_{nt} + c_{jnt}[\rho s_n - W_{int}] + \dot{W}_{int} + [\delta_{nt}(2 - \mu_{jnt}) \\ +\delta_* \mu_{jnt}][\pi W_{i(n+1)t} + (1 - \pi)0 - W_{int}] \\ +\gamma_{nt}[0 - W_{int}] + \gamma_{nt}[s_n - W_{int}]. \end{aligned} \tag{3.6}$$

Note that the players further internalize the effort costs incurred and the payoffs associated with each possible event. Since i is indifferent between conceding at time t or time t+dt, it must be that $W_{int+\mathrm{dt}} = (1 - \rho)s_n$ and c_{jt} satisfies $W_{int} = (1 - \rho)s_n$. Using this observation and re-organizing the statement, equation 3.6 can be re-written as

$$\begin{aligned} r(1 - \rho)s_n = -rE_{nt} + c_{jnt}s_n[2\rho - 1] - [\delta_{nt}(2 - \mu_{jnt}) + \delta_* \mu_{jnt}]s_n(1 - \epsilon\pi) \\ +\gamma_{nt}s_n(2\rho - 1) \end{aligned} \tag{3.7}$$

Define $d \equiv 1/(2\rho - 1) > 1$ and remember that $\phi = (1 - \rho)d$. Given there definitions, the following theorem holds when one takes the limit as dt $\searrow 0$.

Theorem 4 *In every PBE and each time t, $n = 0, 1, \ldots$, and player i if $\mu_{int} < 1$, then $c_{int} = c + c_n(\mu_{int}, \xi_t)$ such that*

$$c_n(\mu_{int}, \xi_t) + \gamma_{nt} \equiv \frac{dE_n}{s_n} + \phi(1 - \pi\epsilon)[\delta_{nt}(2 - \mu_{int}) + \delta_* \mu_{int}].$$

Beliefs with hidden effort I lastly derive equilibrium beliefs. Fix (t, n) and assume that strategic player i expects that j is obstinate with probability $\mu_{jnt} < 1$. Suppose that, for some small dt>0, that by time t+dt the game continues and the surplus remains constant i.e., $s_t = s_{t+\mathrm{dt}}$. If j is obstinate, then he would have not conceded and exerted fix effort. This implies that the probability that the game continues is approximately $\gamma_{jont} \equiv 1-\{r+\delta_{nt}+\gamma_{nt}+\delta_*+h_{jnt}\}\mathrm{dt}+o(\mathrm{dt})$. If j is strategic, however, the probability that the game continued to time t+dt is approximately equal to $\gamma_{jsnt} \equiv 1-\{r+2[\delta_{nt}+\gamma_{nt}]+h_{jnt}\}\mathrm{dt}+o(\mathrm{dt})$. Note that I am stating the linear, Taylor approximation of the probabilities. Bayes rule implies that i's time t+dt beliefs equal to

$$\mu_{jnt+\mathrm{dt}} = \frac{\mu_{jt}\gamma_{jont}}{\mu_{jnt}\gamma_{jont}+(1-\mu_{jont})\gamma_{tsnt}}.$$

Subtracting μ_{jnt} from both sides and then dividing by dt implies that

$$\frac{\mu_{jnt+\mathrm{dt}}-\mu_{jnt}}{\mathrm{dt}} = \mu_{jnt}(1-\mu_{jnt})\left(\frac{\gamma_{jont}-\gamma_{jsnt}}{\mathrm{dt}}\right)+\frac{o(\mathrm{dt})}{\mathrm{dt}}.$$

Lastly, taking the limit as dt $\searrow 0$, beliefs satisfy the following ODE

$$\frac{\dot{\mu}_{jnt}}{\mu_{jnt}} = r\phi + r\mu_n(\mu_{jnt}) \equiv \{c_n(\mu_{jnt},\xi_t)+(1-\mu_{jnt})[\delta_{nt}-\delta_*+\gamma_{nt}]\}\chi(\mu_{jnt}\in(0,1)). \quad (3.8)$$

Next, I characterize beliefs conditional on the surplus falling from s_n to s_{n+1}. Suppose that at time t+dt for am $n+1$ time. If both players are strategic, then a fall in the surplus between time t and t+dt equals to $1-e^{-2\delta_{nt}\mathrm{dt}}$. This is because all processes considered in the paper are jump processes. Conversely, if only i is strategic, then the surplus falls with probability $1-e^{-[\delta_{nt}+\delta_*]\mathrm{dt}}$. Bayes rule implies that $\mu_{j(n+1)t+\mathrm{dt}}$ is

$$\mu_{j(n+1)t+\mathrm{dt}} = \frac{\mu_{jnt}\frac{1-e^{-2\delta_{nt}\mathrm{dt}}}{\mathrm{dt}}}{\mu_{jnt}\frac{1-e^{-2\delta_{nt}\mathrm{dt}}}{\mathrm{dt}}+(1-\mu_{jnt})\frac{1-e^{-[\delta_{nt}+\delta_*]\mathrm{dt}}}{\mathrm{dt}}}. \quad (3.9)$$

Taking the limit as dt goes to 0, then yields that the beliefs immediately after the surplus falls at time t (i.e., t^+) equal to

$$\forall jnt \;\; \mu_{j(n+1)t^+} = \tilde{\mu}_n(\mu_{jnt}) \equiv \frac{\mu_{jnt}[\delta_{nt} + \delta_*]}{\mu_{jnt}\delta_* + (2 - \mu_{jnt})\delta_{nt}} \tag{3.10}$$

The left-hand panel of figures C.4 and C.5 illustrates how beliefs update when the surplus falls as a function of which type (in equilibrium) is expected to exert the most effort. Unsurprisingly, fixing strategic effort, then as obstinate effort e_* increases, the more that the beliefs fall.

As a boundary condition, beliefs converge to 1 by the same terminal period. This implies that beliefs evolve symmetrically both when the surplus remains constant and falls. Since they also converge to 1 at the same time, beliefs are symmetric. Define the belief process (μ_{nt}) such that $\mu_{00} = \mu$ and at each (t, n) such that $\mu_{nt} < 1$, $\dot{\mu}_{nt}/\mu_{nt} = r\phi + r\mu_n(\mu_{jnt})$ if $s_t = s_{t^-}$; otherwise, $\mu_{nt^+} = \tilde{\mu}(\mu_{nt})$. I now state the following result.

Lemma 8 *In every PBE, equilibrium beliefs are symmetric and equal to* (μ_{nt})*.*

Further observe that for fixed beliefs and exogenous process (ξ_t), concession behavior is uniquely pinned down and symmetric. This implies that the equilibrium is unique. Moreover, define $\xi_t = x$ almost surely for $x = 0, 1$ as x. Then, I further find that $\tau_1 < \tau_1$ i.e., negotiations that always coincide with fighting reach a peace agreement earlier than those which always coincide with a ceasefire.

Lemma 9 *For each* (ξ_t)*, there exists a unique, symmetric PBE. Furthermore,* $\tau_1 < \tau_0$*.*

3.2.3 Numerical Example

I now present a numerical example to illustrate how the equilibrium behaves. This exercise provides a graphical intuition to the results presented above. I illustrate two informative quantitative cases and sample paths.

Let us first consider the simple case: $\epsilon = 1, e_* = 0$. The lower line in figure C.3 illustrates beliefs (on the left) as a function of time when the negotiation always coincides with a ceasefire; meanwhile, the top line is when combatants negotiate as they fight. The key insight is that fighting prompts combatants to quickly reach agreements.

Next, I consider a second numerical exercise in which $e_* = 1/2$ and $\epsilon = 1/2$. Figure C.5 plots beliefs as a function of time and the number of times that the surplus falls. When the surplus first falls (change first shift), beliefs fall since a strategic player's initial effort is strictly less than 1/2. However, when they fall twice beliefs are constant in this case (since effort is equal across types) and by the third event of surplus destruction posterior beliefs increase. This result clarifies that the same event (i.e., surplus destruction) can have a time dependent effect.

3.2.4 How to optimally organize a peace negotiation?

Up to this point, I characterized wartime negotiations when the timing of ceasefires is exogenously given. Suppose that a benevolent social planner if free to organize the setting in which a negotiation take place i.e., he picks (ξ_t). When and under what conditions would he choose to hold a ceasefire? I state the problem and provide a succinct characterization in closed form.

I first state the social planner's state variable. The social planner picks a process $\xi = (\xi_{nt})$ such that at each time t, $\xi_{nt} : \mathbf{H}_{nt} \to \{0, 1\}$ such that $\xi_{nt}(h_{nt}) = 1$ denotes that players fight as they negotiate and $\xi_{nt}(h_{nt}) = 0$ implies that combatants negotiate during a ceasefire. Note, however, that given a process ξ, the social planner takes player's strategic actions as given.

Next, I define the planner's payoff i.e., welfare. At time $t \geq 0$, $n = 0, 1, \ldots$, history h_{nt}, and process ξ welfare equals to

$$W_{nh_{nt}}(\mu_{nt}, \xi) = (1 - \mu_{nt}) E_{nh_{nt}\xi} \left[e^{-rt} s_{t^*} - \int_t^{t^*} r E_{ns} e^{-r(s-t)} ds \right]$$

where $\chi(\cdot)$ is the indicator function, and time $t^* \geq t$ is the expected time in which players reach an agreement. Intuitively, welfare at time t is the expected payoff that a strategic player nets in equilibrium. It is without loss of generality to define welfare without mentioning particular weights since the players are symmetric. Of course, in a setting with asymmetric players, welfare weights may matter, but the insights derived from such applications are not significantly different than presented in the symmetric model. I lastly suppress history notation to avoid notation clutter. Thus, the planner's problem is to pick a process ξ to maximize initial welfare:

$$W_n(\mu) \equiv \max_{\xi} W_0(\mu, \xi) \tag{3.11}$$

I present a closed-form characterization of the problem below.

Theorem 5 (Holding Principle) *It is optimal to hold a ceasefire at time $t \geq 0$ when $n = 0, 1, \ldots$ (i.e., $\xi_{nt} = 1$) times iff beliefs μ_{nt} satisfy*

$$\epsilon\left(\frac{1-\hat{\mu}_n(\mu_{nt})^{\frac{1}{1-\rho}}}{1-\mu_{nt}}\right)[(1-\mu_{nt})\delta_{nt}+\mu_{nt}\delta_*]+\rho\mu_n(\mu_{nt})\left(\frac{1-\mu_{nt}^{\frac{1}{1-\rho}}}{1-\mu_{nt}}\right) \leq \frac{E_n}{2s_n}+(\delta_{nt}-\delta_*).$$

The proof of the theorem is a technical exercise and the condition is derived from standard variational condition. Its interpretation, however, is quite natural. It states that a ceasefire may be optimal if beliefs are sufficiently low given the equilibrium costs of fighting and added risk of surplus destruction. This implies that ceasefires can only be implemented at the beginning of the negotiation or quickly after the surplus falls **and** beliefs fall. If $\epsilon = 0$, for instance, this implies that the optimal policy boils down to finding a time $\bar{t} \in [0, \tau_0)$. The social planner urges both sides to negotiate and hold a ceasefire, but (unlike the negotiation) the ceasefire ends for certain at time $\bar{t}$.

I conclude this section by providing a sufficient condition for when holding a ceasefire is **never** useful. Suppose that $e_* = 0$, then the optimal $\bar{t} = 0$. Intuitively, strategic players concede to their opponent's demand at an exponentially increasing rate in response to the risk of surplus destruction associated with an obstinate opponent that exerts no effort. [4] I formally state the result below.

[4]Likewise, it is clear that an obstinate player that does not exert full effort to preserve the surplus only expedites the rate at which agreements are reached at no extra cost. Indeed, if obstinate types did

Corollary 10 *If* $e_* = 0$*, then* $\xi = 1$ *(i.e., never holding a ceasefire) is the optimal setting for a wartime negotiation.*

3.3 Related Literature

This paper contributes to the theory of conflict. The literature, however, focuses on why nations go to war. Fearon (1995), Brito and Intriligator (1985), and others concur that war is the result of asymmetric information. A useful intuition goes as follows. Suppose that two nations have contradicting territorial claims. The nations' leaders can negotiate a border between themselves and resolve the matter without a fight. Negotiating comes at the cost of conceding some claimed territory, which is often unpopular. Conversely, the leaders can declare war. Wars are costly, destructive, and have an uncertain outcome. Nevertheless, the winner imposes their terms. When the expected benefit of fighting or the required concessions are large, nations go to war. This paper differs from the literature by focusing how one can use bargaining to exit unresolved wars.

3.4 Discussion and Conclusion

Unlike the long-standing literature explaining why nations go to war, I study how nations exit armed conflict. In particular, I study the use of ceasefires in wartime negotiations. Ceasefires had been used in wartime negotiations since antiquity, but they became more widespread since the early 20^{th} century. This is because ceasefires avoid further violence and allows for vital, non-combatant tasks to take place e.g., saving the wounded and burying the dead.

Under mild conditions, however, I find that ever holding a ceasefire is ill-advised. Intuitively, negotiations coinciding with active fighting reach agreements quickly, because it is costly to negotiate and combatants quickly learn that posturing is ill-advised.

not exert full effort in order to avoid unilateral defeat, then agreements arrive sooner without additional effort cost. This implies that this assumption makes it harder for fighting to increase welfare.

Ceasefires, for their part, avoid the costs of war but fail also limits learning among combatants: hence, it promotes posturing. This is not to say that ceasefires should never be used in a negotiation, but they should be implemented judiciously. In particular, it may be optimal to hold brief ceasefires at the beginning of a negotiations.

Future work should go in two directions. First, it may be very fruitful to consider how to implement additional policy tools. This would provide a robust theoretical framework for fine-tuning diplomacy. Second, future research should consider be applied to other arenas in which negotiations are often contested as in collective bargaining agreements as well as mergers and acquisitions.

Chapter 4

Conclusion and Discussion

This thesis presents 3 papers on dynamic, information economics. These are economic scenarios in which an agent makes choices while receiving imperfect, payoff relevant information over time. Such information may be acquired (e.g., chapter 4) or deduced by observing the choices of others e.g., chapters 2 and 3. In both cases, these scenarios are central to the study of economics. I summarize each paper each paper and their results.

In chapter 2, I studied the Coase conjecture in sequential auctions. I find that such result seldom exists when buyer valuations are interdependent. In other words, a seller who auctions an item with an unknown, common value to multiple buyers attains more than the efficient level of revenue provided. Crucially, this statement holds regardless of how frequently the seller interacts with his buyers. This result is further in contrast to the case in which buyers have independent private values (IPV). In IPV settings, revenues must converge to their efficient level when the seller transacts with at least 3 buyers arbitrarily frequently.

The intuition behind my result is relatively simple. As the item fails to sell, buyers respond by lowering their willingness to pay for the good being auctions and the seller expects that fewer buyers would ever choose to place a bid. Hence, the market unravels after a few rounds of re-auctioning an item that fails to sell. Ex-ante, the seller internalizes these unraveling dynamics, posits that the future prices will be deduced by backwards induction, and thus ensures a revenue floor above running an efficient auction. My result implies that learning can restore the seller's pricing power by (implicitly) preventing the seller to perpetually compete with his future selves.

In chapter 3, I study wartime negotiations.These negotiations are ineffective i.e., even in the time period in which negotiations were most successful only 8 in 20 negotiations reach a lasting, peace agreement. Moreover, negotiations held in the last century have been abnormally ineffective since less than 3 in 20 negotiations reached a lasting agreement. What explains their declining effectiveness and how can one improve their performance? I find that a key factor behind their recent decline in effectiveness has been a policy-driven coordination of ceasefires during peace negotiations. I show that it is (in general) ill-advised to hold a ceasefire that coincides with a negotiation as it lowers the chances of the negotiation faltering. This too is driven by learning dynamics which allow combatants to quickly reach a peace agreement. I further find, nevertheless, that the optimal way to organize a peace negotiation may begin with combatants holding a brief ceasefire. Lastly, this result (in general) contributes to the theory of reputational bargaining.

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Appendix A

Proofs

A.1 Learning to Commit Proofs

This section of the paper presents the proofs. First, I present the proof pertaining the motivating example. I then present the proofs to the auxiliary results. Lastly, I present the proof of the main theorem.

Proof of lemma 2.

Proof

The proof proceeds as follows. I will posit that the seller posts the optimal strategy and that buyers expect that the good will not be re-offered. Next, I rationalize the conjecture posited.

Suppose that the seller posts a price schedule $p_t = p^*$ for each period t and the item fails to sell in period 0. Then, every buyer who observed good news has a valuation equal to $v_{i1} \equiv \theta_i q(\theta^*) \leq \theta^* q(\theta^*)$ and

$$\theta^* E[q|x_i = 1, agent\ w/\ \theta_i = \theta^*,\ wins] = \frac{\theta_* \pi\lambda w(\theta^*, 1)^{n-1}}{\pi\lambda w(\theta^*, 1)^{n-1} + (1-\pi)(1-\lambda)w(\theta^*, 0)^{n-1}}$$
$$= \frac{\theta^*\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}}{\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1} + 1}$$
$$\leq \frac{\frac{\theta^*\theta_s}{\theta^*-\theta_s}}{\frac{\theta_s}{\theta^*-\theta_s} + 1}$$
$$= \frac{\theta_s\theta^*}{\theta_s + (\theta^* - \theta_s)} = \theta_s. \tag{A.1}$$

Observe that the second line uses the condition stated in the prompt. Next, notice that no buyer is willing to bid more than θ_s in period 0, so the seller prefers keeping the item and when he posts $p_1 = p^*$ no buyer bids with probability 1.

Assume that by period $t \geq 1$, the seller maintains the prices at $p_s = p^*$ for $s \leq t$. Then in period t, no buyer is expected to submit a bid at any period $s \geq 1$ and thus beliefs do not change from period 1. Consequently, all buyers still value the good less than θ_s, the seller still prefers keeping the good, and he might as well fix prices at $p_t = p^*$.

The opposite direction is immediate. Suppose that $\theta^* q(\theta^*) \leq \theta_s$, then

$$\theta_s \geq \theta^* q(\theta^*) = \frac{\theta_* \pi\lambda w(\theta^*, 1)^{n-1}}{\pi\lambda w(\theta^*, 1)^{n-1} + (1-\pi)(1-\lambda)w(\theta^*, 0)^{n-1}}$$
$$= \frac{\theta^*\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}}{\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1} + 1}. \tag{A.2}$$

If one re-arranges the inequality at hand, it holds

$$\theta_s\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*, 1]}{w[\theta^*, 0]}\right]^{n-1} + \theta_s \geq \theta^*\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*, 1]}{w[\theta^*, 0]}\right]^{n-1} \tag{A.3}$$

and collecting terms leads to equation 2.2. □

A.1.1 Proof of Lemma 3, Theorem 1, and Theorem 2.

I now prove the auxiliary results. First, I prove that buyers follow a threshold strategy in every equilibrium. Next, I establish of progressive pessimism. The last prove provides bounds on the equilibrium revenues that the seller attains.

Buyers follow a threshold strategy, lemma 3.

Proof

I first prove that buyers follow a threshold strategy (u_t). This argument has two steps. First, I show that if buyer i has a greater valuation at some period t when his type is τ than when his type if τ', then his valuation given type τ remains greater than given type τ' for every period $s \geq t$. The next result establishes that the net payoff between bidding right away and waiting is non-decreasing in the current valuation.

Single Crossing Property Holds The first section of the proof first establishes a non-crossing difference in payoffs.

Proposition 11 *For every pair $\tau, \tau' \in [0,1]^2$ and $x_{-i}, \in [0,1]^{n-1}$, $\Delta(\tau, \tau', x_{-i}) \equiv u(\tau, x_{-i}) - u(\tau', x_{-i}) \geq 0$ iff $\Delta(\tau, \tau', 0) \geq 0$.*

Proof

Fix some pair $\tau, \tau' \in [0,1]^2$ and a $x_{-i} \in [0,1]^{n-1}$. First, if $\Delta(\tau, \tau', 0) \geq 0$, then the non-decreasing differences condition implies that $\Delta(\tau, \tau', x_{-i}) \geq 0$ for each $x_{-i} \in [0,1]^{n-1}$. This establishes the inverse direction. Next, suppose for contradiction that $\Delta(\tau,\ tau', x_{-i}) \geq 0$ but $\Delta(\tau, \tau', 0) < 0$. Since $u(.)$ is continuous, then $\Delta(\tau, \tau', .)$ is also continuous. Since $[0,1]^{n-1}$ is connected, then the intermediate value theorem implies that $\Delta(\tau, \tau', [0,1]^{n-1})$ is also connected. Hence, there exists some $x'_{-i} \in [0,1]^{n-1}$ such that $\Delta(\tau, \tau', x'_{-i}) = 0$ and by non-decreasing difference, it holds that

$$0 < |\Delta(\tau, \tau', 0)| \leq |\Delta(\tau, \tau', x'_{-i})| \leq \Delta(\tau, \tau', x'_{-i}) = 0. \tag{A.4}$$

This is a contradiction and concludes the proof. □

Belief Independence Ordering in Valuations. Next, I establish that if buyer i values the good more when he observes τ than when he observes τ' for some given beliefs regarding x_{-i}. Then he would still value the good more when his type is τ rather τ' given any other alternative belief regarding x_{-i}.

Corollary 12 *Fix some PBE, history h, and suppose that for some pair of types τ, τ' it holds that $E[u(\tau, x_{-i})|h] \geq E[u(\tau', x_{-i})|h]$, then for every h', it holds that $E[u(\tau, x_{-i})|h'] \geq E[u(\tau', x_{-i})|h']$.*

Proof Fix some PBE, history h, pair of types τ, τ', and assume that $E[u(\tau, x_{-i})|h] \geq E[u(\tau', x_{-i})|h]$. This equivalently implies that $E[\Delta(\tau, \tau', x_{-i})|h] \geq 0$ and hence there exists some $x_{-i} \in [0,1]^{n-1}$ such that $\Delta(\tau, \tau', x_{-i}) \geq 0$. By proposition 11, it follows that for each x'_{-i} ,$\Delta(\tau, \tau', x'_{-i}) \geq 0$. Consequently, for each history h'

$$0 = E[0|h'] \leq E[\Delta(\tau, \tau', x'_{-i})|h']. \tag{A.5}$$

This concludes the proof. □

Payoffs from participating in an auction increases with valuations I now prove that the payoff that a buyer receives from participating in an auction is non-decreasing in his valuation conditional on winning.

Proposition 13 *For every feasible terms of trade $m = (p, x, t)$ and belief on valuations H, buyer payoffs from participating in the terms of trade is non-decreasing in his valuation v. Furthermore if some buyer with a valuations v wins the item with a strictly positive probability, then the*

Proof Fix some reserve price p, a belief H, and a pair of valuations conditional on winning v, v' such that $p \leq v \leq v' \leq 1$. Then, the payoff that a buyer nets equals to his valuation minus the second highest valuation

$$\begin{aligned} V(v',H) &= \int_0^{v'} (v' - \max\{y,p\})dH_2(y) \\ &= \int_0^{v} (v' - \max\{y,p\})dH_2(y) + \int_v^{v'} (v' - \max\{y,p\})dH_2(y) \\ &\geq \int_0^{v} (v - \max\{y,p\})dH_2(y) + \int_v^{v'} (v' - \max\{y,p\})dH_2(y) \\ &= V(v',H) + \int_v^{v'} (v' - \max\{y,p\})dH_2(y) \\ &\geq V(v',H) \end{aligned} \tag{A.6}$$

where $H_2(\cdot)$ refers to the distribution of the second highest valuation given belief CDF H and $V(x,H)$ is the valuation that a buyer with a valuation conditional on winning the good when beliefs are given by H. This concludes the proof. □

Buyers follow a Threshold strategy I now prove that the decision to participate in the auction must be characterized by a threshold strategy. The argument presented here is standard.

Fix some PBE, period t, history h_t, and a buyer i. Assume that buyer i has a type τ and he weakly prefers to participate in the current terms of trade m_t. Then it must be the case that his payoff from participating in the current terms of trade is weakly greater than waiting: formally, let the buyer's valuation be $v(\tau,h_t) \equiv E[u(\tau,x_{-i})|h_t]$

$$V(v(\tau,h_t),H(.|h_\tau)) \geq W(\tau,h_t) \equiv E\Big[\sum_{s=1}^{\infty} \delta^\tau b_{t+s}(\tau,h_{t+s})s_{t+s}(h_{t+s})V(v(\tau,h_{t+s}),H(.|h_{t+s}))|h_t\Big] \geq 0 \tag{A.7}$$

for $b_{t+s}(x,h_t)$ is the probability that a buyer participates in an auction conditional on the good remaining unsold by period $t+s$, the buyer choosing not to bid right away, and history h_t, $s_{t+s}(h_{t+s})$ denotes the probabilities that the item has failed to sell and $W(\tau,h_t)$ his option value from waiting. Now suppose that the buyer had a valuation τ' such that $V(v(\tau',h_t),H(.|h_t)) > V(v(\tau,h_t),H(.|h_t))$. I claim that if buyer had type τ' rather than type τ, the seller still prefers bidding. First, observe that the type is unverifiable, so buyer i observing τ' can replicate the equilibrium strategy he is supposed to pick when his type is τ instead, and vice versa, and cannot receive strictly higher payoff from deviating. Furthermore, as $v(\tau', h_t) \geq v(\tau, h_t)$ then corollary 12 implies that his valuation in any subsequent history h_{t+s} satisfies that $v(\tau', h_{t+s}) \geq v(\tau, h_{t+s}$; moreover, proposition 13 implies that

$$V(v(\tau, h_{t+s}), H(.|h_{t+s})) \leq V(v(t, h_{s+\tau}), H(.|h_{s+\tau})). \tag{A.8}$$

Therefore, it holds that the PBE equilibrium payoffs satisfy

$$U(\tau', h_t) \geq E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau}) V(v(t', h_{s+\tau}), H(.|h_{s+\tau}))|h_t\Big] \geq W(t, h_t) \tag{A.9}$$

as the buyer with type τ must henceforth fix his participation to 1 and

$$U(\tau, h_t) \geq E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau}) b_{s+\tau}(t', h_{s+\tau}) V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))|h_t\Big]. \tag{A.10}$$

Consequently, the difference in equilibrium payoffs must satisfy

$$\begin{aligned} U(\tau', h_t) - U(\tau, h_t) \geq E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) \\ -V(v(t, h_{s+\tau}), H(.|h_{s+\tau})]|h_t\Big] \geq 0. \end{aligned} \tag{A.11}$$

However, since types are unverifiable, then the difference in options values from delay must be greater than the payoff that a buyer observing type τ could get if he plays the strategy of a buyer observing τ', i.e.

$$\begin{aligned} W(t', h_t) - W(t, h_t) \leq \\ E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau}) b_{s+\tau}(t', h_{s+\tau}) \Big\{ V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau})) \Big\} |h_t\Big] \end{aligned} \tag{A.12}$$

This implies that that

$$\begin{aligned} E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[b_{s+\tau}(t', h_{s+\tau}) - 1] \\ \Big\{ V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau})) \Big\} |h_t\Big] \geq 0. \end{aligned}$$

But since for each history $h_{t+\tau}$, $V(v(\tau', h_{t+\tau}), H(.|h_{t+\tau})) \geq V(v(\tau, h_{t+\tau}), H(.|h_{t+\tau}))$ and the choices $1 \geq b_{t+s}(., \tau)$ and non-decreasing in the history set order, then it must hold that

$$E\Big[\sum_{\tau=1}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[b_{s+\tau}(t', h_{s+\tau})-1]\Big\{V(v(t', h_{s+\tau}), H(.|h_{s+\tau}))-V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))\Big\}|h_t\Big] \leq 0 \quad \text{(A.13)}$$

and

$$\begin{aligned}[b_s(t', h_s) - 1]\{V(v(t', h_s), H(.|h_{s+})) - V(v(t, h_s), H(.|h_{s+\tau}))\} \\ +E\Big[\sum_{\tau=1}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[b_{s+\tau}(t', h_{s+\tau}) - 1] \\ \times\Big\{V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))\Big\}|h_t\Big] = 0\end{aligned} \quad \text{(A.14)}$$

This concludes the proof since for the only way both equations hold is for the buyer observing history τ' to also bids immediately. □

Progressive Pessimism, Theorem 1.

Proof

This proof establishes that in every period t and history h_t, prior beliefs $F_t(\cdot|h_t)$ likelihood ratio dominate posterior beliefs $F_t^{'}(\cdot|h_t)$. I present the argument for interdependent values since the argument for valuations and private values is identical to the one in question. It is worth noting that these proofs go by induction and that I present the induction step first and the initial condition.

Now that this lemma is established, it will be immediate that if $H_t[0, v_t]$ likelihood ratio dominated $H_{t+1}[0, u_{t+1}]$, for $u_{t+1} \leq v_t$, then $H_t/H_t(u_{t+1})[0, v_t]$ also likelihood ratio dominates $H_{t+1}[0, u_{t+1}]$.

Learning and Induction Step Fix some PBE whose threshold strategy is (u_t), period t, history h_{t+1}, and assume that the prior belief are given by a CDF $F(.|h_t)$ denote each buyer i's belief regarding each buyer $j \neq i$'s interdependent value x_j with pdf $f_t(.|h_t)$, and the item remains unsold. Since the item remains unsold it holds that for every buyer i $v(\tau_i, h_t) \leq u_t(h_{t+1})$. Therefore, for each interdependent value $x_j \in [0,1]$ Bayes' rule implies that the posterior beliefs in period t, i.e. beliefs once the good fails to sell, satisfiy

$$\begin{aligned} f_{t+1}(x_j|h_{t+1}) &= \frac{Pr(v(x_j, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})|h_t) f(x_j|h_t)}{\int_0^1 Pr[v(y, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})|h_t] f(y|h_t) dy} \\ &= \frac{f(x_j|h_t) \int_{v(x_j, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})} K_t(\tilde{\theta}|h_t) d\tilde{\theta}}{\int_0^1 f(y|h_t) \int_{v(x'_j, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})} K_t(\tilde{\theta}'|h_t) d\tilde{\theta}'_j dy} \end{aligned} \tag{A.15}$$

where $K_t(\cdot|h_t)$ are the equilibrium beliefs regarding private values, i.e. each agent i in period t and history h_t expects that the private value of each peer j is distributed given $\theta_j \sim K_t(\cdot|h_t)[0,1]$.

Now, pick some pair $x_j, x'_j \in [0,1]$ such that $x_j \leq x'_j$, then as $u(.)$ is strictly increasing in (θ, x), then so is $v(., h_t) = E[u(., x_{-i})|h_t, (\cdot), i \text{ wins}]$ for each history h_t given that the expectation operator preserves monotonicity. By proposition 13, it further holds that if $(\theta, x_j) < (\theta, x'_j)$, then $v(\theta, x_j, h_t) \leq v(\theta, x'_j, h_t)$. This is because the payoff that each buyer would net from participating in an auction as well as the equilibrium payoff must be non-decreasing in each buyer's valuation. Consequently, for each pair of interdependent values x_j, x'_j such that $x_j \geq x'_j$ the set of private values θ_j for which buyer j does not participate in the auction when he observes x_j is smaller than comparable set given x'_j:

$$\{\theta \in [0,1] | w(\theta, x'_j, h_t) \leq u_t(h_{t+1})\} \subset \{\theta \in [0,1] | w(\theta, x_j, h_t) \leq u_t(h_{t+1})\}. \tag{A.16}$$

Since beliefs denote a measure on the set of private values and probability measures are monotone, then

$$Pr_t(V(x'_j, \tilde{\theta}_j, h_t) \leq \theta_s(h_{t+1})|h_t) \leq Pr_t(V(x_j, \tilde{\theta}_j, h_t) \leq \theta_s(h_{t+1})|h_t). \tag{A.17}$$

Informally, this argument states that since payoffs are strictly increasing and $x'_j \leq x_j$, then there exists a larger collection of private values that buyer j could have observed and his valuation to lie below any given cutoff. Next, if one divides $f_{t+1}(x'_j|h_{t+1})$ by $f_{t+1}(x_j|h_{t+1})$, then it holds that

$$\frac{f_{t+1}(x'_j|h_{t+1})}{f_{t+1}(x_j|h_{t+1})} = \left[\frac{f_t(x'_j|h_t)}{f_t(x_j|h_t)}\right]\left[\frac{Pr_t(v(x'_j,\tilde{\theta}_j,h_t) \leq u_t(h_{t+1})|h_t)}{Pr_t(v(x_j,\tilde{\theta}_j,h_t) \leq u_t(h_{t+1})|h_t)}\right] \leq \frac{f_t(x'_j|h_t)}{f_t(x_j|h_t)}. \tag{A.18}$$

Since x_j, x'_j were arbitrarily chosen, one can conclude that $F_t(.|h_t)$ likelihood ratio dominates $F_{t+1}(.|h_{t+1})$.

Initial Step In this part of the proof, I establish the initial step in the inductive argument. Fix some price $p_0 \in [0,1]$ and let $h_1 = \{p_0\}$. Then suppose that the item failed to sell, the each buyer i updates his belief that his peer $j \neq i$ observes x_j via Bayes rule as follows

$$f_0(x_j|h_1) = \frac{Pr(v_0(x_j,\tilde{\theta}_j) \leq u_0(h_1))f(x_j)}{\int_0^1 Pr(v_0(y,\tilde{\theta}_j) \leq u_0(h_1))f(y)dy} = \frac{f(x_j)\int_{v_0(x_j,\tilde{\theta}_j)\leq u_0(h_1)} k(\theta)d\theta}{\int_0^1 f(y)\int_{v_0(x'_j,\tilde{\theta}_j)\leq u_0(h_1)} K(\theta',y)d\theta' dy}. \tag{A.19}$$

Now, pick some pair of interdependent values x_j, x'_j such that $x_j < x'_j$, then the same argument in period t and h_t implies that

$$\{\theta \in [0,1]|v_0(\theta,x'_j) \leq u_0(h_1)\} \subset \{\theta \in [0,1]|v_0(\theta,x_j) \leq u_0(h_1)\}. \tag{A.20}$$

Consequently, by the monotonicity of probability measures, it holds that

$$Pr(v_0(x'_j,\tilde{\theta}_j) \leq u_0(h_1)|h_0) \leq Pr(v_0(x_j,\tilde{\theta}_j) \leq u_0(h_1)|h_0). \tag{A.21}$$

Lastly, if one divides $f'_0(x'_j|h_1)$ by $f'_0(x_j|h_1)$, it holds that

$$\frac{f_0(x'_j|h_1)}{f_0(x_j|h_1)} = \left[\frac{f(x'_j)}{f(x_j)}\right]\left[\frac{Pr(v_0(x'_j,\tilde{\theta}_j) \leq u_0(h_1))}{Pr(v_0(x_j,\tilde{\theta}_j) \leq u_0(h_1))}\right] \leq \frac{f(x'_j)}{f(x_j)}. \tag{A.22}$$

Since the choice of $x_j, x'_j \in [0,1]$ and p_0 are arbitrary, then $F(\cdot)$ likelihood ratio dominates the posterior $F_0(\cdot|h_1)$ for each price p_0 such that $h_1 = \{p_0\}$. $\square$

Proof of Corollary 4. This subsection characterizes how the dispersion in valuations falls over time.

Proof

I will first prove that valuations fall. Fix some PBE, period t, history h_t, and price p_t. Define $h_{t+1} = (h_t, p_t)$, then progressive pessimism, i.e. the previous theorem, states that $F_t(\cdot|h_t)$ likelihood ratio dominates $F_{t+1}(\cdot|h_{t+1})$. Next, progressive pessimism implies First order stochastic dominance. Consequently, since $u(\cdot)$ is strictly increasing, then for each type τ_i, it holds that

$$v_t(\tau_i, h_t) = E_t[u(\tau_i, x_{-i})|h_t, \tau_i, i \text{ wins}] \geq E_{t+1}[u(\tau_i, x_{-i})|h_{t+1}, \tau_i, i \text{ wins}] = v_{t+1}(\tau_i, h_{t+1}). \tag{A.23}$$

Next, I show that the dispersion in valuations falls. Fix some PBE, some period t, a history h_t, price p_t, and a pair of types τ, τ'. Then let us suppress conditioning on winning the auction and it holds that

$$\begin{aligned} E_t[|v_t(\tau) - v_t(\tau')||h_t] &= E_t[|E_t[u(\tau, x_{-i}) - u(\tau, x_{-i})|h_t]||h_t] \\ &= E_t[|E_t[\Delta(\tau, \tau', x_{-i})|h_t]||h_t] \\ &= E_t[E_t[\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \geq 0) - [\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \leq 0)|h_t] \\ &\geq E_{t+1}[E_t[\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \geq 0) - [\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \leq 0)|(h_t, p_t)] \\ &= E_{t+1}[|E_t[u(\tau, x_{-i}) - u(\tau, x_{-i})|h_t]||(h_t, p_t)] \\ &= E_{t+1}[|v_t(\tau) - v_t(\tau')||(h_t, p_t)] \end{aligned} \tag{A.24}$$

where the third line just implements the definition of the absolute value. Meanwhile, the fourth line uses the fact that when the indicators are active both functions are increasing in their arguments and the beliefs in period t likelihood ratio dominate their posterior beliefs. □

Revenue Bounds, Theorem 2.

Proof

In this proof, I provide bounds on equilibrium revenues. First, I provide an upper bound. Suppose that after a given period, the seller can commit to a direct, dynamic mechanism (see Myerson 1986) where buyers who have identical, initial valuations expect to have the same equilibrium payoff. Then, I find that for any dynamic mechanism, there exists a comparable mechanism that either elicits trade immediately or never trades with buyers. I show that the payoff from both mechanism are the same; meanwhile, the restriction in mechanisms ensures that it can be implemented in the current environment. The second part of the proof characterizes a lower bound. I find that there exists a collection of auctions that the seller can run in period 0 after which buyer valuations conditional on winning the good fall below the seller's valuation of θ_s and the seller might as well keep his good.

Revenue Ceiling. Fix some PBE, period t, and history h_t. Define a contract under full commitment as a tuple $m = (x, r, T) : \mathcal{T}^n \to \Delta^{n-1} \times \Re^n \times \mathbb{Z}_+ \cup \{\infty\}$ such that for each tuple $\tilde{\tau} = (\tau_i)$ and buyer i,

i. Probability that buyer i wins the good is $x_i(\tilde{\tau})$

ii. Expected payment made by buyer i is $r_i(\tilde{\tau})$

iii. The expected period in which agents expect the good to trade $t + T(\tilde{\tau})$.

I assume that the seller can only consider contracts in which buyers who have the same valuation for the good win the good with equal odds and make the same expected payment, i.e. for every pair of buyer i, j and types τ_i, τ_j such that $v(\tau_i, h_t) = v(\tau_j, h_t)$, it holds that

$$x_i(\tau_i, \tilde{\tau}_{-i}) = x_j(\tau_j, \tilde{\tau}_{-j}) \text{ and } r_i(\tau_i, \tilde{\tau}_{-i}) = r_j(\tau_j, \tilde{\tau}_{-j}) \tag{A.25}$$

Next, I define standard functions that will allow me to state which terms of trade are feasible in a compact manner. Fix some buyer i and types τ_i, τ_i', then define the following functions

i. Discounted, payoff from buyer i winning the good when he observes types τ_i but reports type τ_i':

$$q_i(\tau_i, \tau_i') \equiv E[\delta^{T(\tilde{\tau}_{-i}, \tau_i')} u(\tau_i, x_{-i} \in \tilde{\tau}_{-i}) x_i(\tau_i', \tilde{\tau}_{-i}) | h_t, i \; wins] \tag{A.26}$$

ii. Discounted, expected payment made by buyer i when he reports type τ_i' but he is actually of type τ_i:

$$p_i(\tau_i', \tau_i, B) \equiv E[\delta^{\tau(\tilde{\tau}_{-i}, \tau_i')} r_i(\tilde{\tau}_{-i}, \tau_i') | h_t, i \; wins]. \tag{A.27}$$

Next, the contract m is feasible iff it is incentive compatible and individually rational, i.e.

$$\forall i, \tau_i \;\; E[q_i(\tau_i, \tau_i) - p_i(\tau_i, \tau_i)] \geq 0 \qquad \text{(Individual Rationality)}$$

and

$$\forall i, \tau_i, \tau_i', \;\; q_i(\tau_i, \tau_i) - p_i(\tau_i, \tau_i) \geq q_i(\tau_i, \tau_i') - p_i(\tau_i, \tau_i'). \qquad \text{(Incentive Compatability)}$$

Replication Argument Now, pick some feasible contract m. Define an alternative terms of trade m' as follows. For every $\tilde{\tau}, i$ let

i. $\tau'(\tilde{\tau}) = 0$,

ii. $r_i'(\tilde{\tau}) = \delta^{\tau(\tilde{\tau})} r_i(\tilde{\tau})$,

iii. and $x_i'(\tilde{\tau}) = \delta^{\tau(\tilde{\tau})} x_i(\tilde{\tau})$.

Define for each buyer i and types τ_i, τ_i', the functions

$$q_i'(\tau_i, \tau_i') = E[x_i'(\tilde{\tau}_{-i}, \tau_i') u(\tau_i, \tilde{\tau}_{-i}) | h_t, i \; wins]$$

and

$$p_i'(\tau_i, \tau_i') = E[r_i'(\tilde{\tau}_{-i}, \tau_i') | h_t, i \; wins].$$

By construction, for every buyer i and pair of types $\forall \tau_i, \tau_i'$, it holds that $q_i(\tau_i, \tau_i') = q_i'(\tau_i, \tau_i')$ and $p_i(\tau_i, \tau_i') = p_i'(\tau_i, \tau_i')$: thus as m is feasible, then m' is also feasible. Next, terms of trade m's expected revenues equal to

$$r(m) = \mathbb{E}\left[\sum_{i \in I} \delta^{t(s)} r_i(s)\right] = \mathbb{E}\left[\sum_{i \in I} r_i'(s)\right] = r(m'), \tag{A.28}$$

so the seller expects to gain the same returns in terms of trades m and m'. This implies that the seller, with full commitment, might as well only consider terms of trade where he either trades in the initial period or does not trade. This establishes the proof that $r_s \leq r_s^*$ for every history almost surely.

Revenue Floor I now prove that the seller's revenue is greater than immediately running the revenue maximizing auction after which the seller avoids re-auctioning his good, i.e. $r_0^e < \underline{r}_0 \leq r_0$. I first establish the following technical result:

Proposition 14 *There exists some value $\underline{p} \in (\theta_s, v_0(1,1))$ such that for each type τ such that $w_0(\tau)$, $E[u(\tau, x_{-i})|\forall j \neq i, w_0(\tau_j) \leq \underline{p}] \leq \theta_s$.*

Before proving this result, it is important to describe why it matters. Suppose that one dealt with a private value setting. Then, if the item fails to sell, buyers do not lower their valuation and thus there is no such $\underline{p}$ for which the seller nets higher revenues than by just running an efficient auction and this result is moot.

Proof This proof has several steps. First, I derive the posterior distribution on interdependent values if the seller runs an auction with a reserve price p, buyers decide to bid if their valuation is above p, but the good failed to sell. Next, I show that the initial beliefs likelihood ratio dominate these beliefs and I order posterior beliefs by p. I then show that valuations in period 1 are strictly increasing in the reserve price p. Lastly, I prove the proposition.

Bayes rule. Pick some pair of values $p, p' \in (v_0(0,0), v_0(1,1))$ where $p < p'$ and for each type τ $v_0(\tau) = E[u(\tau, x_{-i}|\tau_i, i \text{ wins}]$. Then for each interdependent value realization

x_i, it holds that for each $P \in \{p, p'\}$ one can define $f(x|P) = Pr(x|m(\theta_j, x_j) \leq P)$ and it satisfies,

$$f(x|P) = \frac{Pr(v_0(\theta, x) \leq P|x)f(x)}{\int_0^1 Pr(v_0(\theta, y) \leq P|x)f(y)dy} \tag{A.29}$$

Note that since $p < p'$, then for each (x, θ) such that $v_0(\theta, x) \leq p < p'$, so $Pr(v_0(\theta, x) \leq p) \leq Pr(v_0(\theta, x) \leq p')$. Furthermore, if there exists some value $\theta \in (0, 1)$ such that $v_0(\theta, x) > p$, then the inequality is strict. Further notice that since $p, p' < v_1(1, 1)$, then it holds that

$$\int_0^1 Pr(v_0(\theta, y) \leq p|y)f(y)dy < \int_0^1 Pr(v_0(\theta, y) \leq p'|y)f(y)dy. \tag{A.30}$$

Ordering posterior beliefs by p. I claim that this fact allows me to prove that $F(\cdot|p')$, i.e. the CDF associated with $f(\cdot|p')$, first order stochastically dominates $F(\cdot|p)$.

Suppose for contradiction that there exists some value $y \in (0, 1)$ such that $F(y|p) \leq F(y|p')$, then this implies that

$$\begin{aligned}
0 \leq F(y|p') - F(y|p) &= \int_0^y f(x|p') - f(x|p)dx \\
= \int_0^y f(x)\Big[\frac{Pr(v_0(\theta, x) \leq p'|x)}{\int_0^1 Pr(v_0(\theta, y) \leq p'|y)f(y)dy} &- \frac{Pr(v_0(\theta, x) \leq p|x)}{\int_0^1 Pr(v_0(\theta, y) \leq p|y)f(y)dy}\Big]dx \\
\leq \int_0^y f(x)Pr(v_0(\theta, x) \leq p'|x)dx\Big[\frac{1}{\int_0^1 Pr(v_0(\theta, y) \leq p'|y)f(y)dy} &- \frac{1}{\int_0^1 Pr(v_0(\theta, y) \leq p|y)f(y)dy}\Big] \\
&< 0\int_0^y f(x)Pr(v_0(\theta, x) \leq p'|x)dx \\
&= 0
\end{aligned} \tag{A.31}$$

This is a contradiction. Note that the third line follows from the fact that $Pr(v_0(\theta, x) \leq p) \leq Pr(v_0(\theta, x) \leq p')$ and the fourth line from equation A.30. This implies that $F(\cdot|p')$ first order stochastically dominates $F(\cdot|p)$.

Initial Beliefs Dominate Posteriors Next, I claim that F likelihood ratio dominates $F(\cdot|p)$ for $P \in (v_0(0, 0), v_0(1, 1))$. Fix P, then note that for each pair $0 < x < x' < 1$, it holds that $Pr(v_0(\theta, x') \leq P|x') \leq Pr(v_0(\theta, x) \leq P|x')$ and $v_0(\cdot, \cdot)$ is a strictly increasing function. Furthermore, if $v_0(q, x) > P$, then the inequality is strict.

Now, pick some pair $z, y \in [0,1]$ such that $z < y$, then the aforementioned inequality implies that

$$\frac{f(z|P)}{f(y|P)} = \left[\frac{Pr(v_0(\theta,z) \leq P|z)}{Pr(v_0(\theta,y) \leq P|y)}\right]\frac{f(z)}{f(y)} \leq \frac{f(z)}{f(y)} \tag{A.32}$$

where the inequality is strict if $v_0(y,1) > P$. Therefore, F likelihood ratio dominates $F(\cdot|p)$ for $P \in (v_0(0,0), v_0(1,1))$ and hence F first order stochastically dominates $F(\cdot|p)$ for $P \in (v_0(0,0), v_0(1,1))$.

Beliefs are continuous in p. Pick some $p \in (v_0(0,0), v_0(1,1))$, then observe that for each value $x \in [0,1]$

$$Pr[v_0(x,\theta) \leq p|x] \equiv t(p,x) = \begin{cases} 1 & if\ v(x,1) \leq p \\ K[v_0^{-1}(x,p)] & if\ v(x,1) > p \end{cases} \tag{A.33}$$

where $v_0^{-1}(x,\cdot)$ is well defined for each x and differentiable since $u(\cdot)$ is a strictly increasing and continuously differentiable function. This implies that in all but a Lebesgue-measure zero set, the partial derivative $\partial_p t(x,p)$ is well defined for each x. Moreover, beliefs can be re-written as

$$f(x|p) = \frac{t(x,p)f(x)}{\int_0^1 t(y,p)f(y)dy}, \tag{A.34}$$

so the partial derivative $\partial_p f(x|p)$ is well defined almost surely. Next, I define valuations given a reserve price $p \in (v_0(0,0), v_0(1,1))$ for each type τ as

$$v(\tau,p) = E[u(\tau,x_{-i})|\forall j \neq i, v_0(\tau_j) \leq p] = \int_{[0,1]^{n-1}} u[\tau, x_{-i} = (x_j)_{j\neq i}] \prod_{j\neq i} f(x_j|p)d(x_j)_{j\neq i}. \tag{A.35}$$

Observe that the partial derivative of $\partial_p v(\tau,p)$ is well defined and hence for each type τ, $v(\tau,\cdot)$ is a continuous function since it is continuously differentiable.

Beliefs are increasing in p. I now continue with the function $v(\tau,\cdot)$ for a fixed τ. I claim that for each type τ, $v(\tau,\cdot)$ is strictly increasing.

Fix some $\tau \in \mathcal{T}$ and a pair of reserve prices $p, p' \in (v_0(0,0), v_0(1,1))$ such that $p < p'$. Then as $u(\cdot)$ is a strictly increasing function of x_{-i} and $F(\cdot|p')$ first order stochastically dominates $F(\cdot|p)$, then

$$\begin{aligned} v(\tau, p) = \int_{[0,1]^{n-1}} u[\tau, x_{-i} = (x_j)_{j\neq i}] \prod_{j\neq i} f(x_j|p) d(x_j)_{j\neq i} \\ < \int_{[0,1]^{n-1}} u[\tau, x_{-i} = (x_j)_{j\neq i}] \prod_{j\neq i} f(x_j|p') d(x_j)_{j\neq i} = v(\tau, p'). \end{aligned} \tag{A.36}$$

This proves that for each τ, $v(\tau, \cdot)$ is strictly increasing. Also, note that as F likelihood ratio dominate $F(\cdot|p)$ for each $p \in (v_0(0,0), v_0(1,1))$, then it first order stochastically dominates it. Hence,

$$\forall \tau \in \mathcal{T}, p \in (v_0(0,0), v_0(1,1)), v(\tau, p) < v_0(\tau). \tag{A.37}$$

Proof Conclusion. I now prove that $\underline{\text{p}} > \theta_s$ exists. Fix some τ such that $v_0(\tau) \leq \theta_s$, then for each the previous result implies that $v_0(\tau, \theta_s) < v_0(\tau) \leq \theta_s$. This implies that if the seller runs an efficient auction and the item fails to sell, then valuations fall enough such that the seller keeps his good. Next, suppose that the seller picks as his reserve price $v_0(1,1)$, then with probability 1 for every type $\tau \in \mathcal{T}$ do not place a bid, i.e. $v_0(\tau) \leq v_0(1,1)$. Since almost surely no buyer bids, then beliefs do not update and $v_0(\tau) = v(\tau, v_0(1,1))$ for each type τ and it holds that there exists types such that $v(\tau, v_0(1,1)) > \theta_s$; for instance, $\tau = (1,1)$. This implies that if the seller announces a price that is large enough such that all buyers wait, then beliefs do not update and the higher valuation among buyers remains higher than θ_s.

Next, let us consider the types $\tau = \epsilon(1,1)$ for some $\epsilon \in [0,1]$, then the image of $v_0[\epsilon(1,1)]$ is $[v_0(0,0), v_0(1,1)]$. Hence, for each type τ, there exist an $\epsilon(\tau)$ such that

$$v_0[\epsilon(\tau)(1,1)] = v_0(\tau)$$

. Moreover, by the assumption of monotone differences, then for each belief regarding

x_{-i} it holds that

$$E[u(\tau, x_{-i}) - u(\epsilon(\tau)(1,1), x_{-i})] = 0. \tag{A.38}$$

This allows me to finish the proof by only considering the ray $\{\epsilon(1,1) | \epsilon \in [0,1]\}$. Define for each reserve price $p \in (v_0(0,0), v_0(1,1))$ the value $\epsilon(p)$ which solves

$$v_0[\epsilon(p)(1,1)] = p. \tag{A.39}$$

Notice that as $u(\cdot)$ is strictly increasing, then $\epsilon(\cdot)$ is strictly increasing. Next, notice that $\epsilon(1) = v_0(1,1)$ and that for $v_0[\epsilon(\theta_s)] = \theta_s$ it holds that $\epsilon(\theta_s) \in (0,1)$.

I conclude the proof by defining one last function. Let let the maximum valuation when the item fails to sell when the reserve price is p as $m(p) = v[\epsilon(p)(1,1), p]$ and note that $m(\cdot)$ is a strictly increasing and continuous function. As previously shown, $m(1) = v_0(1,1) > \theta_s$ and $m(\theta_s) < \theta_s$. By the intermediate value theorem, there exists value $\underline{\mathrm{p}} \in (\theta_s, 1)$ such that $m(\underline{\mathrm{p}}) = \theta_s$. This concludes the proof.

□

The next result characterizes the revenue associated with a one-shot auction and compares revenues. First, define $H_{j0}(.)$ to be the distribution of the j^{th} highest valuation in period 0 and given a minimal valuation participating $p \in [\theta_s, \underline{\mathrm{p}}]$, the revenues are

$$r_0(p, H_0) = \theta_s + \int_p^{w_0(1,1)} x - \theta_s - \frac{1 - H_0(x)}{h_0(x)} dH_0^2(x) = \theta_s + \int_p^{w_0(1,1)} \phi(x) - \theta_s dH_0^1(x) \tag{A.40}$$

This equation just defines the payoff to an auction in terms of the virtual value of the highest valuation, i.e. in terms of $\phi(x) = x - \frac{1-H_0(x)}{h_0(x)}$. Next, observe that in an efficient auction $p = \theta_s$, so $r_0^e = r_0(\theta_s, H_0$. Further note that $\phi(x) \leq x$ and in the case where H_0 is regular, the optimal auction with full commitment rest p^* to solve $\phi(p^*) = \theta_s$. Hence, the efficient auction is suboptimal and increasing the reserve price in $p \in (\theta_s, p^*]$ will increase payoffs. If $p^* \leq \underline{\mathrm{p}}$, then the seller can implement his static, optimal auction and the commitment issue is moot; otherwise, it holds that $p^* > \underline{\text{p}}$ and the revenue maximizing, feasible auction has a reserve price of $\underline{\text{p}} > \theta_s$ and

$$r_0(p, H_0) - r_0^e = \int_{\theta_s}^{p} \phi(x) - \theta_s dH_0^1(x) > 0. \tag{A.41}$$

Since for each $r_0(p, H_0) \leq \underline{\text{r}}_0$, then this establishes the revenue floor assumption. $\square$

A.1.2 Proof of Theorem 3.

Proof

I now establish that the equilibrium is essentially unique and provide a condition for when the revenues with full commitment equal to those without it. This proof has three parts. First, I prove that the game essentially ends in finite time. Since actions are observable, this implies that the equilibrium is essentially unique. Lastly, I characterize conditions for which the seller implements his revenue maximizing terms of trade.

Game effectively ends in finite time.

Lemma 15 *There exists a deterministic period $\hat{T} < \infty$ such that for every period $t \geq \hat{T}$, $u_t(h_t) \leq \theta_s$ for every history h_t and PBE.*

Proof

Suppose for contradiction that there exists some PBE, period t, and history h_{t+1} such that

$$v_t(h_{t+1}) \equiv \inf\{x \in [0, 1] : H_t(x|h_t) = 1\} > \theta_s$$

and that after a large period $s \in \{1, 2, \ldots\}$ and small $\epsilon > 0$, it holds that

$$E_t[1 - H_t[v_{t+s}(h_{t+s})|h_{t+1}]^n|h_t] < \epsilon.$$

Then the seller's revenues are bounded above by the probability that the seller trades the good in the following s periods and he extracts the maximum possible rents from the winner plus the static optimal period 0 rents in period s times the remaining probability that a buyer remains who values the good more than the seller, i.e.

$$r_t(h_{t+1}) < v_t(h_{t+1})\epsilon + \delta^s r_0^*(1 - H_t(\theta_s|h_{t+1})^n) - \epsilon). \tag{A.42}$$

Meanwhile, if the seller runs an efficient auction, i.e. he posts $p_t = \theta_s$, his revenues are greater than netting his private value θ_s times the probability that a buyer values the good more that the seller and otherwise keeping his good:

$$r_t^e \geq \theta_s[1 - H_t(\theta_s|h_{t+1})^n] + \delta\theta_s H_t[\theta_s|h_{t+1}]^n \tag{A.43}$$

This implies that the difference between the expected equilibrium revenues and those attainable by immediately running the efficient auction are bounded above as

$$\begin{aligned} r_t(h_{t+1}) - r_t^e &< v_t(h_{t+1})\epsilon + \delta^s r_0^*(1 - H_t(\theta_s|h_{t+1})^n) - \epsilon) - \theta_s[1 - H_t(\theta_s|h_t)^n] - \delta\theta_s H_t[\theta_s|h_t]^n \\ &= (v_t(h_{t+1}) - \delta^s r_0^*)\epsilon - \delta\theta_s H_t[\theta_s|h_t]^n + (\delta^s r_0^* - \theta_s)[1 - H_t(\theta_s|h_t)^n]. \end{aligned} \tag{A.44}$$

For sufficiently large s, it holds that $\delta^s r_0^* < \theta_s$ and for such values of s, it further holds that

$$\begin{aligned} r_t(h_t) - r_t^e &< (v_t(h_{t+1}) - \theta_s)\epsilon - \delta\theta_s H[\theta_s|h_t] \\ &\leq (v_t(h_{t+1}) - \theta_s)\epsilon - \delta\theta_s(1 - \epsilon) \\ &\leq [1 - \theta_s(1 - \delta)]\epsilon - \delta\theta_s. \end{aligned} \tag{A.45}$$

For $\epsilon \in [0, \theta_s]$, it holds that $r_t(h_t) < r_t^e$. This is a contradiction, because it implies that the seller strictly prefers running an efficient auction rather than continuing with candidate equilibrium strategy. This implies that for s solving $\delta^s = \theta_s$ and $\epsilon = \delta\theta_s$, it holds that by period

$$\hat{T} = \frac{1}{\delta\theta_s}\left[\frac{\ln\theta_s}{\ln\delta}\right] \tag{A.46}$$

the item must either sells or the seller keeps his item. This concludes the proof □

Since the game essentially ends by some finite period $\hat{T}$, then all PBE can be characterized via backwards induction. As actions are further perfectly observable, then the equilibrium is essentially unique.

Before moving on, I make a quick remark. Suppose that agents interact in period $t = \Delta_t, 2\Delta_t, \ldots$ for some $\Delta_t > 0$. Then $\delta = e^{-r\Delta_t}$ for some discount factor $r > 0$ and one can construct a value $s\Delta_t$ instead of s and it holds that by period $\hat{T}$, the game must end. In other words, as Δ_t goes to zero, the number of times an item is re-auctioned may diverge. But the period after which the game ends does not.

Characterizing Full Mitigation.

I lastly prove that the condition 2.9 is sufficient for the seller to implement his optimal auction under full commitment. For every type τ_i excluded from the optimal static auction, it holds that $\theta_s \geq \bar{\phi}[w_0(\tau_i)]$. Therefore, if inequality 2.9 holds, then

$$w(\tau_i, p^*) \leq \phi(\tau_i, p^*) \leq \bar{\phi}(\tau_i, p^*) \leq \theta_s; \tag{A.47}$$

where $v(\tau_i, p^*)$ is the buyer i's valuation conditional on the good failing to sell when the seller posted $p_0 = p^* = p_0^*$, buyers expected to good to not re-auction the good and yet no buyer submitted a valid bid. Therefore, inequality A.47 implies that the seller values the good more than his buyers and he cannot gain from re-auctioning his good as he must accept a payment below his valuation.

□

A.2 Reasons for Peace Proofs.

Proof of Lemma 7. **Proof**

I now characterize concession behavior i.e., the probability that a player concedes to his opponent's demands. If i is certain that j is obstinate, i immediately concedes since he is impatient and cannot extract more than $1 - \rho$ surplus from j. This is an important observation to be noted throughout the paper. Define the cumulative probability that player i concedes by time t when the surplus fell $n = 0, 1, \ldots$ times as $F_{int} \equiv (1 - \mu_{int})H_{int}$. Fix i, n, t. I claim that if i is not certain that j is obstinate (i.e., $\mu_{jnt} < 1$), however, i mixes between conceding and demanding ρ and that is strictly increasing and F_{int} admits a PDF i.e., at each time t such that $\mu_{jt} < 1, F_{int^-} \equiv \lim_{s \nearrow t} F_{is} = F_{int}$.

First, I establish that F_{int} is strictly increasing for each i if $\mu_{jnt} < 1$. Suppose, for contradiction, that after time $t \geq 0$ and for a given n, it holds that $t' = \sup\{\tau | F_{in\tau} = F_{int}\} > t$, so for a maximal interval $[t, t']$ player i does not concede provided that the surplus remains intact. Since j is impatient, he is strictly better off conceding at time t+dt than at t'+dt for each dt$\in (0, t' - t)$. In turn, this implies that i strictly prefers to concede at time t' rather than at any time between $s \in (t', t' + (t' - t)]$ and since $F_{it} = F_{it'}$, it must be the case that $F_{int} = F_{ins}$. This is a contradiction, because $s > t'$ and I establishes that F_{int} is strictly increasing when $\mu_{jnt} < 1$.

Next, I establish that if $\mu_{jnt} < 1$ at time $t > 0$, $F_{it^-} = F_{it}$. Suppose for contradiction that there exist some time $t > 0$ such that $F_{it^-} < F_{it}$[1]. Then, for sufficiently small dt> 0, j is strictly better off demanding ρ, because he nets a time t payoff weakly greater than

$$\rho(F_{it} - F_{it^-}) + [1 - (F_{it} - F_{it^-})]e^{-r\mathrm{dt}}(1 - \rho) > 1 - \rho.$$

This implies that j strictly prefers to wait until time t+dt rather than to concede immediately, so $F_{jt} = F_{jt+\mathrm{dt}}$. Nevertheless, this contradicts the result that F_{jnt} is strictly increasing.

as j strictly prefers to wait: thus, contradicting that F_{it} is strictly increasing. This concludes the proof.

□

Proof of Lemma 9.

Proof

The only result that was not directly established in the text is that for each $x > 0$, it holds that $\tau_x < \tau_0$. Informally, I prove that conditional on the surplus never falling, players reach beliefs $\mu' \in [\mu, 1]$ at an earlier date when $x > 0$ rather than equal to 0.

[1]The function $F_{it}/(1 - \mu)$ is a CDF, so it is non-decreasing.

Fix some time $t > 0$, such that $\mu_{0xt} \in [\mu, 1]$, then for $\gamma_{0xt} \equiv \ln \mu_{0xt}$ it holds that

$$\gamma_{0xt} = \gamma_{0x0} + \int_0^t \dot{\gamma}_{0xs} ds = \ln \mu + \int_0^d \frac{\dot{\mu}_{0xs} ds}{\mu_{0xs}} = \ln \mu + \int_0^d [c_0(e^{\gamma_{0xs}}) + (1 - e^{\gamma_{0xs}})(f_0(\delta) - \delta_*)] ds.$$

If $c \geq \delta(2\rho - 1)$, then $[c_0(e^{\gamma_{0xs}}) + (1 - e^{\gamma_{0xs}})(f_0(\delta) - \delta_*)] > r\phi$. This implies that

$$\gamma_{0xt} > \ln \mu + r\phi t = \gamma_{00t}$$

or that $\mu_{0xt} > \mu_{00t}$ for each $t > 0$ such that $\mu_{0xt} \leq 1$. This concludes the proof.

□

A.2.1 *Proof of Theorem 5*

Proof

I first derive the welfare as function of a player's beliefs that their opponent in obstinate $x \in [0, 1]$ and the number of times that the surplus fell $n = 0, 1, \ldots$. I use x to avoid confusion with the primitive μ and welfare can be reduced to a function of $(x, n, \xi) \in [0, 1] \times \{0, 1, \ldots\} \times \{0, 1\}$ since player strategies are Markov in (x, n, ξ).

The first step is to derive welfare assuming that players are in a ceasefire i.e., $\xi = 1$. I do this, because the argument for the more involved setting is a simple extension of this argument. Fix some small time interval dt> 0 and suppose that the current state is $(x, n, 1)$. If $x = 1$ or $n > N$, then welfare equals to 0 (i.e., $W_{n0}(x) = 0$) since players never reach an agreement and/or the game is over. Otherwise, $x < 1$ and $n \in \{0, 1, \ldots, N\}$ and two events can happen: players reach an agreement or fail to do so. Players reach an agreement if either player concedes and a concession arrives in the time interval dt with a probability of $(1 - x)[1 - exp(-r\phi/(1 - x)\mathrm{dt})] = r\phi\mathrm{dt} + o(\mathrm{dt})$ such that $\lim_{z \searrow 0} o(z)/z = 0$. If players do not reach an agreement after waiting dt, then players update their beliefs via Bayes rule to $\hat{\mu}_{0n}(x)$, which satisfies

$$x\hat{\mu}_{0n\mathrm{dt}}(x) - x \approx x \left[\frac{(1-0)}{+x(1-0) + (1-x)(1 - r\phi/(1-x)\mathrm{dt})} \right] - x \approx xr\phi\mathrm{dt}.$$

This equation states that if a player is obstinate he does not concede with probability 1, but if he is strategic, the does not concede with probability of approximately $1-r\phi/(1-x)\mathrm{dt}$. Therefore, the posterior belief is $x(1+\hat{\mu}_{0n\mathrm{dt}}(x))$ and the continuation welfare is $W_{n0}[x(1+\hat{\mu}_{0n\mathrm{dt}}(x))]$. Since current payoff is s_n, continuation valuation satisfies

$$\begin{aligned} W_{n0}(x) &= 2r\phi\mathrm{dt}s_n + (1-r\phi\mathrm{dt})(1-2r\phi\mathrm{dt})W_{n0}[x(1+\hat{\mu}_{0n\mathrm{dt}}(x))] + o(\mathrm{dt}) \\ &= 2r\phi\mathrm{dt}s_n + [1-r(1+2\phi)\mathrm{dt}]W_{n0}[x(1+\hat{\mu}_{0n\mathrm{dt}}(x))] + o(\mathrm{dt}). \end{aligned}$$

Note that (since player randomize independently from each other) the probability of an agreement is precisely twice the probability that any given player concedes. I also discount the continuation welfare by $r>0$. Next, I subtract $W_{n0}(x)$ from both sides of the equation above and divide by dt, yielding

$$0 = 2r\phi[s_n - W_{n0}(x)] - rW_{n0}[x(1+\hat{\mu}_{0n\mathrm{dt}}(x))] + \frac{W_{n0}[x(1+\hat{\mu}_{0n\mathrm{dt}}(x))] - W_{n0}(x)}{\mathrm{dt}} + \frac{o(\mathrm{dt})}{\mathrm{dt}}$$

Taking the limit as dt goes to 0, then yields the ordinary differential equation (ODE)

$$\begin{aligned} 0 &= 2r\phi[s_n - W_{n0}(x)] - rW_{n0}(x) + W'_{n0}(x)x\hat{\mu}_{0n}(x) \\ &= 2r\phi[s_n - W_{n0}(x)] - rW_{n0}(x) + W'_{n0}(x)xr\phi \\ &= 2\phi s_n - (1+2\phi)W_{n0}(x) + W'_{n0}(x)x\phi \end{aligned}$$

The ODE above, combined with the fact that $W_{n0}(1)=0$, implies that welfare is uniquely defined and equal to

$$W_{n0}(x) = s_n\frac{2\phi\big(1-x^{2+\frac{1}{\phi}}\big)}{2\phi+1} = s_n\frac{2(1-\rho)\big(1-x^{2+\frac{2\rho-1}{1-\rho}}\big)}{2(1-\rho)+2\rho-1} = 2s_n(1-\rho)\big(1-x^{\frac{1}{1-\rho}}\big) \quad \text{(A.48)}$$

Now that the ODE has been derived for the case in which players negotiate during a ceasefire, I study the more general case. Note that the derivation is a modest generalization of the proof above. It still remains the case that I discretely approximate the HJB and take limits. Fix some small dt>0. At each (x,n) such that $x<1$ and

$n < N+1$, the social planner expects that an agreement is reached in one of two ways. First, either players may attain a decisive military victory. If player i is obstinate, he concedes a military defeat with a Poisson rate of 0, but if i is strategic, he concedes at a Poisson rate of γ_{nt}. This implies that the probability that i concedes a military defeat in a time interval of length dt can be linearly approximated to be $rd_n(x)\mathrm{dt} + o(\mathrm{dt})$ where

$$d_n(x) \equiv (1-x)\gamma_{nt}.$$

On the other hand, the negotiation could end due to a player i conceding. If i is obstinate, he never concedes; otherwise, he concedes given a time-dependent rate $h_n(x)$. This implies (once again) that the probability that a player i concedes in a time interval of length dt (given n, x) is linearly approximated to be $r\phi + rc_n(x,1)\mathrm{dt} + o(\mathrm{dt})$ where

$$\phi + c_n(x,1) \equiv (1-x)rh_n(x).$$

Next, if the players do not reach an agreement, then the surplus is either destroyed or left intact. The ϵ surplus is destroyed with an expected probability of $2rf_n(x)\mathrm{dt} + o(\mathrm{dt})$ where

$$f_n(x) \equiv x\delta_* + (1-x)\delta_{nt}.$$

Next, continuation beliefs now depend on whether or not the surplus was destroyed. If the surplus is destroyed more than N times, then no agreement is ever reached and welfare is precisely 0. Otherwise, when current state is (x, n) and the surplus is destroyed, updated beliefs conditional on the $n+1$ fall in surplus is $\tilde{\mu}_n(x)$. Notice that this equation was derived in the main body of the text. If (in contrast), the surplus is not destroyed further, then the beliefs update to approximately $x[1 + r(\phi + \mu_n(x))\mathrm{dt}]$. Combining the approximations given above, it holds that

$$\begin{aligned} W_n(x) = {} & r[2s_n\phi + 2s_nc_n(x,1) + 2s_nd_n(x) - E_n(1-x)]\text{dt} \\ & + [1 - 2r(\phi + c_n(x,1) + d_n(x))\text{dt}](1 - r\text{dt})\Big[W_{(n+1)y}[\tilde{\mu}_n(x)]2rf_n(x)\text{dt} \\ & + (1 - 2rf_n(x)\text{dt})W_n\{x[1 + r(\phi + \mu_n(x))\text{dt}]\}\Big] + o(\text{dt}). \end{aligned}$$

If one now uses the approximation of $\text{dt}^2 \approx 0$, then the equation above can be re-written as

$$\begin{aligned} W_n(x) = {} & r[2s_n\phi + 2s_nc_n(x,1) + 2s_nd_n(x) - E_n(1-x)]\text{dt} + W_{(n+1)y}[\tilde{\mu}_n(x)]2rf_n(x)\text{dt} \\ & + [1 - (r + 2r[\phi + c_n(x,1) + d_n(x)) + f_n(x)]\text{dt}]W_n\{x[1 + (r\phi + \mu_n(x))\text{dt}]\} + o(\text{dt}). \end{aligned}$$

Next, I subtract $W_n(x)$ from both sides and divide by dt, it implies that

$$\begin{aligned} 0 = {} & r[2s_n\phi + 2s_nc_n(x,1) + 2s_nd_n(x) - E_n(1-x)] + \pi W_{(n+1)y}[\tilde{\mu}_n(x)]2rf_n(x) \\ & + \frac{W_n\{x[1 + r(\phi + \mu_n(x))\text{dt}]\} - W_n(x)}{\text{dt}} - (r + 2r[\phi + c_n(x,1) + d_n(x)) \\ & + f_n(x)]W_n\{x[1 + r(\phi + \mu_n(x))\text{dt}]\} + \frac{o(\text{dt})}{\text{dt}}. \end{aligned}$$

Taking the limit as dt goes to 0, it holds that

$$\begin{aligned} 0 = {} & r[2s_n\phi + 2s_nc_n(x,1) + 2s_nd_n(x) - E_n(1-x)] + \pi W_{(n+1)y}[\tilde{\mu}_n(x)]2f_n(x) \\ & + W_n'(x)x[r\phi + \mu_n(x)] - (r + 2r[\phi + c_n(x,1) + d_n(x)) + f_n(x)]W_n(x). \end{aligned}$$

Further notice that for each $(x, n, 1)$, it holds that

$$r\mu_n(x) = rc_n(x,1) + (1-x)r[\delta_{nt} - \delta_* + \gamma_{nt}]$$

If one then adds $r(\phi + \delta_*)$ to both sides, it holds that

$$\begin{aligned} r\phi + r\mu_n(x) + r\delta_* & = r\phi + rc_n(x,1) + (1-x)r\delta_{nt} + xr\delta_* + (1-x)r\gamma_{nt} \\ & = r\phi + rc_n(x,1) + rd_n(x) + rf_n(x). \end{aligned}$$

This condition lastly implies that $\mu_n(x) = c_n(x,1) + d_n(x) + f_n(x) - \delta_*$ and that the value function satisfies

$$W_n(x) = 2s_n\phi + 2s_n[\mu_n(x) + \delta_* - f_n(x)] - E_n(1-x)$$
$$+ \pi W_{(n+1)y}[\tilde{\mu}_n(x)]2f_n(x) + W_n'(x)x[\phi + \mu_n(x)] - 2[\phi + \mu_n(x)]W_n(x)$$

The boundary condition is that if $n > N$ or $x = 1$, then $W_n(x) = 0$.

Next, I characterize the social planner's problem. Suppose that the social planner could impose when players hold a ceasefire while they negotiate. The social planner, however, must take the strategic behavior of players as given i.e., he determines a policy $\xi : \{0, 1, \ldots, N\} \times [0,1] \to \{0,1\}$ where $\xi_n(x) = 1$ implies that at state (n, x), combatants negotiate while they fight; otherwise, $\xi_n(x) = 0$ implies that combatants negotiate during a ceasefire. Given a policy $\xi(\cdot)$, welfare satisfies

$$W_n(x) = [2s_n\phi + 2\xi_n(x)\{s_n[\mu_n(x) + \delta_* - f_n(x)] - E_n(1-x)\}]$$
$$+ \pi W_{(n+1)y}[\tilde{\mu}_n(x)]2\xi_n(X)f_n(x) + W_n'(x)x[\phi + \xi_n(x)\mu_n(x)] - 2[\phi + \xi_n(x)\mu_n(x)]W_n(x).$$

This implies that the welfare maximizing policy $\xi^*(\cdot)$ solves the HJB equation

$$(1+2\phi)W_n(x) = 2s_n\phi + W_n'(x)x\phi +$$
$$[2s_n[\mu_n(x) + \delta_* - f_n(x)] - E_n(1-x) + \pi W_{(n+1)y}[\tilde{\mu}_n(x)]2f_n(x) + W_n'(x)x\mu_n(x) - 2\mu_n(x)W_n(x)]^+ \tag{A.49}$$

Next, observe that $\mu_n(x) = c_n(x,1) + d_n(x) + f_n(x)$, so the equation above satisfies

$$(1+2\phi)W_n(x) = 2s_n\phi + W_n'(x)x\phi +$$
$$[2s_n[\mu_n(x) - f_n(x)] - E_n(1-x) + \pi W_{(n+1)y}[\tilde{\mu}_n(x)]2f_n(x) + W_n'(x)x\mu_n(x) - 2\mu_n(x)W_n(x)]^+$$

This implies that if holding a ceasefire is optimal, then welfare equals to $P_n(\cdot)$ and that this function must satisfy

$$2s_n\mu_n(x)+\{\pi W_{(n+1)y}[\tilde{\mu}_n(x)]-s_n\}2f_n(x)+W_n'(x)x\mu_n(x)-2\mu_n(x)W_n(x)+2s_n\delta_* \leq E_n(1-x)$$

Or equivalently that

$$\{\pi P_{n+1}[\tilde{\mu}_n(x)]2f_n(x)+2s_n(\delta_*-\delta_{nt})(1-x)+\mu_n(x)[2s_n+P_n'(x)x-2P_n(x)] \leq E_n(1-x)$$

Since $P_n(x)=s_n2(1-\rho)[1-x^{\frac{1}{1-\rho}}]$, then $xP_n'(x)=-2s_nx^{\frac{1}{1-\rho}}$ and the equation above can be re-written as

$$\{\frac{s_{n+1}}{s_n}2\pi(1-\rho)(1-\tilde{\mu}_n(x)^{\frac{1}{1-\rho}})\}f_n(x)+\rho\mu_n(x)(1-x^{\frac{1}{1-\rho}}) \leq \left(\frac{E_n}{2s_n}+\delta_{nt}-\delta_*\right)(1-x).$$

Next, if one divides both sides by $1-x$, it holds that

$$2\epsilon\pi(1-\rho)\left(\frac{1-\tilde{\mu}_n(x)^{\frac{1}{1-\rho}}}{1-x}\right)f_n(x)+\rho\mu_n(x)\left(\frac{1-x^{\frac{1}{1-\rho}}}{1-x}\right) \leq \frac{E_n}{2s_n}+(\delta_{nt}-\delta_*).$$

This concludes the proof.

□

Proof of lemma 10.

Proof

Suppose (for contradiction) that $e_*=0, \hat{\delta}=0, \epsilon$, and it is optimal to hold a ceasefire when beliefs are $x\in[0,1)$ and the surplus fell n times, then the inequality derived in lemma 5 implies that for each x

$$\rho\mu_n(x)\left(\frac{1-x^{\frac{1}{1-\rho}}}{1-x}\right) \leq \frac{E_n}{2s_n}+\delta_{nt}-\delta=\frac{E_n}{2s_n}-(\delta-\delta_{nt}). \tag{A.50}$$

This inequality implies simply forces $\epsilon=0$. If $\epsilon\in(0,1]$, then one adds a positive term on the left-hand side of the inequality and it is easier to arrive at a contradiction.

Given that $\rho\in(1/2,1)$, $e_*=0$, and $\hat{\delta}=0$, then

$$\mu_n(x)=\frac{E_nd}{s_n}+\phi[\delta+(1-x)(\delta_{nt}-\delta)]+(1-x)(\delta_{nt}-\delta)=\frac{E_nd}{s_n}+\phi\delta-\rho d(1-x)(\delta-\delta_{nt})$$

where $d = [\rho - (1-\rho)]^{-1}$ and $\phi = (1-\rho)d$. Since $1 - x^{\frac{1}{1-\rho}} > 1 - x$ and I assume that the inequality A.50 hold, then it must be the case that $\frac{E_n}{2s_n} - (\delta - \delta_{nt}) > \rho\mu_n(x)$ and plugging in the functional form of $\mu_n(x)$ and re-organizing implies that

$$
\begin{aligned}
0 > \frac{E_n(d\rho - 1/2)}{s_n} + \phi\rho\delta + [1 - \rho^2 d(1-x)](\delta - \delta_{nt}) \\
> \phi\rho\delta + [1 - \rho^2 d(1-x)]\delta \\
= \delta\left[\frac{\rho(1-\rho) + 1 - \rho^2(1-x)}{\rho - (1-\rho)}\right] \\
> \delta\left[\frac{\rho(1-\rho) + 1 - \rho^2}{\rho - (1-\rho)}\right] > \delta\left[\frac{(1+\rho)(1-\rho)}{\rho - (1-\rho)}\right] > 0.
\end{aligned}
$$

The last line implies that there is a contradiction, however, the argument crucially depends on the fact that $\rho^2 d > 1$. This inequality holds since it can be equivalently written as $\rho^2 > \rho - (1-\rho)$ and this equation implies that $1 - \rho > \rho(1-\rho)$. This concludes the proof and establishes the argument presented.

□

Appendix B

Non-proof appendices for Learning to Commit.

B.1 One-shot Benchmark.

I return to the setting in section 2.2 and characterize the revenue maximizing, static auction. First, I derive the optimal, symmetric bidding strategy. Next, I state the seller's revenue as a function of his chosen reserve price and derive the optimal reserve price p^*.

B.1.1 Symmetric Bidding Strategy.

In this subsection, I prove that buyer participating in the auction bid their valuation conditional on winning the good in the unique, symmetric equilibrium. Note that the seller picks price $p \geq \theta_s$; otherwise, he accepts payments for his good that are below his valuation with a strictly positive probability. This implies that only buyers observing good news bid.

I first derive the valuation of a winning buyer. Suppose that the seller posted a price of p, buyer i bid b, and i wins. Then, if buyer i expects that only his peers $j \neq i$

observing good news submit bids $b(\theta_j)$, for some strictly increasing function b, then the initial probability that i wins conditional on q is

$$w(b^{-1}(b), \theta^*, q) = \begin{cases} \pi b^{-1}(b) + (1-\pi) & if\ q = 1 \\ (1-\pi) b^{-1}(b) + \pi & if\ q = 0. \end{cases}$$

Since conditional on q, (θ_i, x_i) are drawn iid, it holds that the odds that buyer i wins is $w(b^{-1}(b), \theta^*, q)^{n-1}$. By Bayes rule, it then implies that buyer i's valuation, conditional on winning is

$$w_i = \theta_i \left[\frac{\lambda \pi w(b^{-1}(b), \theta^*, 1)}{\lambda \pi w(b^{-1}(b), \theta^*, 1) + (1-\lambda)(1-\pi) w(b^{-1}(b), \theta^*, 0)} \right].$$

Note that for a strategy $b(\cdot)$ to form in equilibrium, it must be the case that the optimal bid is $b(\theta_i)$ and hence his valuation conditional on winning is

$$w_i = w(\theta_i) = \theta_i \left[\frac{\lambda \pi w(\theta_i, \theta^*, 1)}{\lambda \pi w(\theta_i, \theta^*, 1) + (1-\lambda)(1-\pi) w(\theta_i, \theta^*, 0)} \right].$$

Next, let $\Phi(\cdot)$ denote the distribution of the second highest valuation private value conditional on receiving good news, then each buyer i's payoff from bidding $b \in [p, b(1)]$ is

$$r(b) = \int_{w^{-1}(p)}^{b^{-1}(b)} [w(\theta_i) - \max\{p, b(y)\}] d\Phi(y).$$

This means that the optimal bid must satisfy the first order condition, which implies that

$$0 = r'(b) = \frac{w(\theta_i) - b(b^{-1}(b))]}{b'(b)]} = \frac{w(\theta_i) - b}{b'(b)]}. \tag{B.1}$$

and in equilibrium $B = b(\theta_i)$. Hence, the equilibrium condition implies, as desired, that

$$b(\theta_i) = w(\theta_i).$$

This implies that the only strictly increasing bidding function that can be sustained in a symmetric, monotone equilibrium is to bid one's valuation conditional on winning. In the following section, I use this observation to derive the seller's problem.

B.1.2 Optimal Reserve Price.

I now state and solve the seller's problem. Rather than picking p^* the seller might as well pick θ^* such that $p^* = w(\theta^*)$.

Suppose that the seller picks a value θ and $m \in \{0, 1, \cdots, n\}$ buyers observe good news. Then the seller expects to keep his item with odds θ^m and to net a payoff of

$$r(\theta, m) = m\theta(1-\theta)\theta^{n-1} + \int_\theta^1 m(m-1)x^{m-1}(1-x)dx + \theta^m\theta_s$$

Next, the odds of m buyers receiving good news are $\tilde{\lambda}(m) = \binom{n}{m}[\lambda\pi^m(1-\pi)^{n-m} + (1-\lambda)(1-\pi)^m\pi^{n-m}]$. This implies that the seller's expected revenue from the choice of θ is the expected revenue of $r(\theta) = \tilde{\lambda}(m)r(\theta, m)$. Lastly, the seller's optimal choice solves $\max_{\theta\in[0,1]} r(\theta)$.

B.1.3 Contracts, For Online Publication

In this subsection, I present the terms of trade that the seller may offer buyers. The auction format is fixed, i.e. the protocol determining how buyers who participate in the auction interact with each other. The seller, however, picks who wins the good and how much the winner pays subject to the allocation rule being implementable by an equilibrium.

The resulting extensive form game satisfies the following conditions. First, if a buyer decides to not participate in the auction, then he neither wins the good nor makes a payment. On the other hand, if at least one player decides to participate in the auction, the item sells to whichever buyers holds the highest valuation at a price that is incentive compatible. Lastly, I assume that buyers are treated symmetrically, i.e. if two buyers have the same valuation, then they expect to win the auction with equal probabilities and to make the same expected payment.

Auction Format

I assumed that the seller runs second-price auctions; is this necessary? My result holds if there exist negative selection among buyers choosing to participate in the auction. I now present a general auction setting in which my results persists. First, I model the auction procedure as a particular type of extensive form game that I call an "auction format". Next, I assume that the seller gets to pick a particular kind of indirect contract allowing that must form an equilibrium in the auction format.

I first define the set of outcomes. An outcome is either a buyer who wins the good and the payment he makes or a failure to trade, i.e. let the set of buyers be $\mathcal{I}$, then the set of outcomes is

$$\mathcal{X} \equiv \{(i, p_i) | i \in \mathcal{I}, p_i \in \Re\} \cup \{\text{failure}\}. \qquad \text{(Set of Outcomes)}$$

Note, the outcome $x =$ failure means that no buyer won the good. Next, I define the auction format.

Definition 6 (Auction Format, is an extensive form game) *An auction format is a tuple* $\Gamma \equiv \{\mathcal{H}, \succeq, \rho, A, \mathcal{A}, (\mathfrak{I}_i)_{i \in \mathcal{I}}\}$ *consisting of*

i. A game tree $(\mathcal{H}, \succeq)$ *with initial history* $\hat{h}_o$*,*

ii. The set of terminal histories is $\mathcal{Z} (\subset \mathcal{H})$ *such that* $\#\mathcal{X} = \#\mathcal{Z}$*,*

iii. A function assigning buyers to non-terminal histories: $\rho : \mathcal{H} - \mathcal{Z} \to 2^{\mathcal{I}} - \{\emptyset\}$*. This function denotes who get to take an action when.*

iv. A set of acts A *and I assume that* $[0, 1] \subset A$*,*

v. A map from non-terminal histories to acts feasible at the history in question: $\mathcal{A} : \mathcal{H} - \mathcal{Z} \to 2^A - \{\emptyset\}$*,* *vi.* *In the initial period, buyers have a participation decision: i.e.* $\rho(\hat{h}_o) = \mathcal{I}$ *and for each buyer* i*, it holds that* $\mathcal{A}(\hat{h}_o) = \{0, 1\}$ *such that if a buyer* i *plays* 1*, then they succeeding outcomes is not in* $\{i\} \cup \Re$ *and* i *does not play in a subsequent non-terminal history.*

vii. *For each buyer* i*, there exists a collection of measurable information sets* $\mathfrak{I}_i$ *that are well defined, i.e. for each set* $B_i \in \mathfrak{I}_i$ *and pair of histories* $h, h' \in B_i$*, it holds that* $\mathcal{A}(h) = \mathcal{A}(h')$.

viii. *I assume that the game has perfect recall and a PBE.*

This definition allows for the second and first price as well as English auctions. Next, I define a strategy to the game above.

Definition 7 (Behavioral Strategy) *A (behavioral) strategy for buyer i is a measurable function* $\sigma_i : \mathcal{T} \times \mathfrak{I} \to \Delta(A)$ *such that for every set* $B_i \in \mathfrak{I}_i$ *and type* $\tau \in \mathcal{T}$*, it holds that* $supp\sigma_i(\tau, B_i) \subset \mathcal{A}(h_i)$ *for some history* $h_i \in B_i$*. Note that from henceforth, I denote the set of actions directly as a function of his information set.*

I can now define a PBE. Note that this is important since the seller picks an equilibrium of the game.

Definition 8 (PBE) *Given a function* $g \equiv (q, p) : \mathcal{Z} \to \mathcal{X}$*, a PBE is a collection of behavioral functions* (σ_i) *and beliefs such that for every set buyer* i*, set* $B_i \in \mathfrak{I}_i$ *and type* $\tau \in \mathcal{T}$*, it holds that* $\sigma_i(\tau_i, B_i)$ *solves*

$$u_i(\tau_i, B_i) = \max_{a \in \mathcal{B}_\rangle} E[u(\tau, x_{-i})|B_i,\ i\ wins] E[q[a, \sigma_{-i}]|B_i] - E[p[a, \sigma_{-i}]|B_i] \tag{B.2}$$

where $q(\cdot)$ *denotes the expected probability that buyer* i *wins the good,* $p(\cdot)$ *the payment he makes, and the expectations are made given beliefs.*

The next subsection, I allow the seller to pick an outcome function $g(\cdot)$ given a fixed set of constraints. The seller implicitly picks an indirect mechanism that must form an equilibrium in the extensive form game in question.

Terms of Trade

I now define a set of contracts that the seller can offer buyers. The seller announces a rule $g(\cdot)$ denoting who wins the good and how much the winner pays. Meanwhile, each buyer i either reports a type τ_i or decides to not participate by reporting $\emptyset$. For what follows, assume that beliefs are given by $\beta = \{(F_i, K_i)\}$ where F_i is CDF denoting each buyer $j \neq i$'s beliefs of x_i; whereas K_i denote beliefs regarding the private value θ_i. Next, define a buyer i's valuation given beliefs β and a type τ_i as

$$v(\tau_i, \beta) = E_\beta[u(\tau, x_{-i})|\tau_i].$$

I now define the terms of trade.

Definition 9 (Terms of Trade) *Given beliefs $\beta = \{(F_i, K_i)\}$, the terms of trade are a triple $m \equiv (q, \pi, p)$ consisting of a minimal trading valuation $p \in [0, 1]$ as well as an allocation and transfer rule pair $(q, \pi) : (\mathcal{T} \cup \{\emptyset\})^n \to [0, 1]^n \times \Re^n$ such that for every collection of reports $\tilde{r} \equiv (r_j)$, it holds that*

i. $q_i(\tilde{r}) \in [0, 1]$ *is the probability i wins and $\pi_i(\tilde{r}) \in [p, \infty)$ i's payment,*

ii. Buyers who abstain or have valuations below p neither win or make transfers*: for every buyer i report $r_i = \emptyset$ or $r_i = \tau_i$ such that $v(\tau_i, \beta) < p$, it holds that* $q_i(\emptyset, \tilde{r}_{-i}) = \pi_i(\emptyset, \tilde{r}_{-i}) = 0,\ \forall\ \tilde{r}_{-i}$,

iii. If a buyer reports a valuation above p, the good sells*: For every collection of reports $\tilde{r}$, if some buyer i reports $r_i = \tau_i \in \tilde{r}$ such that $v(\tau_i, \beta) \geq p$, then $\sum_{i \in \mathcal{I}} q_i(\tilde{r}) = 1$,*

iv. The buyer with the highest valuation wins the good*: Suppose that there exists a report $r_i = \tau_i$ such that $v(\tau_i, \beta) \geq p$, then define the set*

$$W(\tilde{r}) = \{i \in \mathcal{I} : v(\tau_i, \beta) \geq v(\tau_j, \beta),\ \text{or}\ r_j = \emptyset \forall\ j \in \mathcal{I}\} \tag{B.3}$$

and for each buyer $i \in W(\tilde{r})$, let $q_i(\tilde{r}) = 1/\#W(\tilde{r})$, *v.* The mechanism can be implemented*: there exists a strategy profile* (σ_i) *and a function* $g(\cdot)$ *such that given* $g(\cdot)$*,* (σ_i) *is a PBE and the composition of* (σ_i) *and* g *implements* m*,*

vi. The seller implements the mechanism in the revenue maximizing PBE*: There does not exists an alternative PBE* (σ_i') *such that the expected revenue to the seller from implementing* m *via* (σ_i') *is strictly higher than implementing* m *via PBE* (σ_i)*.*

I now explain this definition. Terms of trade are an indirect mechanism denoting who wins the good, how much each buyer pays, and who is excluded. They also satisfy the following conditions. First, If a buyer reports a type associate with a valuation above a cutoff p, the item sells. Second, the agent who values the good the most wins the auction. Note that this assumption, in general, implies that an auction like Myerson (1981) is not feasible. Next, I assume that the mechanism can be implemented via a mechanism in equilibrium and it maximizes the seller's revenues.

Note that the only lever controlled by the seller is the cutoff value p. The revenues as a function of p when beliefs are $\beta = \{(F_i, K_i)\}$ can be expressed as a function of the virtual values of valuations, i.e. a distribution over $v(\tau, \beta)$ that is expected to have a CDF H_i for each buyer i. Define the ironed out virtual value when beliefs over valuations are H_i as $\bar{\phi}(\cdot, H_i)$ and the expected revenues is

$$r(p, (H_i)) = E_H[\max_i \chi(v_i \geq p)\bar{\phi}(v_i, H_i)\}] + \theta_s \prod_{i \in \mathcal{I}} H_i(p)$$

for $\chi(\cdot)$ is an indicator function.

I conclude this section noting that since the seller only manages the choice p and incentives are increasing in valuation. Then, one can define a game where the seller still picks reserve prices p_t and buyers pick rules to participate in the auction. The resulting games will have negative selection among buyers, so the results in this paper follow.

B.2 Durable Goods Market, For Online Publication

Up to this point, I showed that interdependence precludes the Coase conjecture in auction settings, but doe this insight persists in non-auction settings? The answer is yes. I present a durable goods monopoly example in which interdependence allows the seller to contravene the Coase conjecture in a stationary equilibrium.

A monopolist offers a durable good to a unit mass of consumers. Nature first draws a common quality q such that $\ln q \sim N(\mu, \sigma)$ for $(\mu, \bar{\sigma}) \gg 0$. Then nature privately informs each buyer i a private signal $x_i = q + \epsilon_i$ for $\epsilon_i \sim N(0, \hat{\sigma}), \hat{\sigma} > 0$, such that for each pair of distinct buyers i, j, it holds that ϵ_i is pairwise independent of ϵ_j. At each period $t = 0, 1, \ldots$, the seller first announces a price $p_t \geq 0$. Buyers then decide the either purchase the good (and exit the market) or wait. If a buyer purchases the good at a period t, at a price of p_t, then his payoff is $\delta^t(q - p_t)$ for some common $\delta \in (0, 1)$.

I now define histories, strategies, and equilibrium. At each period t a history consists of the set of past prices and share of buyers who purchased the good: i.e. $h_t = \{p_s, m_s\}_{s=0}^{t-\Delta} \in H_t \equiv \Re_+^t \times [0,1]^t$. Next, a seller strategy is a collection of functions $(p_t), \forall t, p_t : H_t \to \Re_+$ denoting the current price; meanwhile, I an anonymous buyer strategy is a collection of functions $(c_t), \forall t, c_t : H_t \times \Re \times \Re_+ \to [0,1]$ where for each tuple (h_t, x_i, p_t), $c_t(h_t, x_i, p_t)$ denotes the probability that the buyer purchases the good conditional on not previously purchasing the item. A PBE is then a pair $\beta = \{(p_t), (c_t)\}$ coupled with beliefs such that given beliefs, the strategies are sequentially rational, beliefs are derived from β via Bayes rule whenever possible.

I claim that there exists an equilibrium where the seller fixes an initial price at $p_0 > 0$ and for every period $t > 0$ he fixes prices at $p_t = q$. First, buyers expecting this seller strategy profile to be played in an equilibrium expect that there are no gains from trade to be had by delaying their purchases decide to buy the good at a price of p_0 provided that $p_0 \leq E[q|x_i] = e^{x_i}$. Hence, for each quality q, the share of buyers purchasing the good in period 0 if $D(p_0, q) = 1 - \Phi\left[\frac{1}{\hat{\sigma}} \ln\left(\frac{p_0}{q}\right)\right]$ for $\Phi(.)$ being the CDF of the standard normal distribution. Observe that the corresponding p.d.f. of a normal distribution with mean μ ans variance σ will be defined as $\phi(x,\mu,\sigma)$.

The seller also conjectures that he would fix prices after period Δ and expects that for each price p_0 he picks, his revenues are

$$\begin{aligned} r(p_0) = \int_0^\infty p_0\left\{1-\Phi\left[\frac{1}{\hat{\sigma}}\ln\left(\frac{p_0}{q}\right)\right]\right\} + \delta q\Phi\left[\frac{1}{\hat{\sigma}}\ln\left(\frac{p_0}{q}\right)\right]d\phi(\ln q,\mu,\sigma) \\ = p_0 + \int_0^\infty \phi(\ln q,\mu,\sigma)\Phi\left[\frac{1}{\hat{\sigma}}\ln\left(\frac{p_0}{q}\right)\right][\delta q - p_0]dq. \end{aligned}$$

The optimal price p_0 then maximizes $r(p_0)$ among all prices $p_0 \geq 0$ and it satisfies the first order condition

$$\hat{\sigma} + \int_0^\infty \phi(\ln q,\mu,\sigma)\phi\left[\ln\left(\frac{p_0}{q}\right),0,\hat{\sigma}\right]\left[\delta - \frac{p_0}{q}\right]dq = \hat{\sigma}\int_0^\infty \phi(\ln q,\mu,\sigma)\Phi\left[\frac{1}{\hat{\sigma}}\ln\left(\frac{p_0}{q}\right)\right]dq. \tag{B.4}$$

Now, once buyers purchase the good in period 0, they all observe a share of buyers purchasing the good $m_0 \in (0,q)$ and this quantity can only be associated with a unique quality q satisfying $D(p_0,q) = m_0$. Hence, buyers learn that the common value equals to q. The seller then fixes the price at q and buyers are indifferent between buying and waiting, so it is an equilibrium for them to buy the good.

Appendix C

Non-proof appendices for Reasons for Peace.

C.0.1 Extended Literature review for Reasons for Peace

In their review of bargaining models of war, Baliga and Sjöström (2013) argue that nations going to war is puzzling. This is because of the Coase Theorem. The theorem states that strategic agents who can freely negotiate a mutually acceptable agreement will reach a Pareto efficient outcome. Wars are destructive and (by their very nature) hence a source of inefficiency.

Brito and Intriligator (1985) first showed that incomplete information can rationalize why nations go to war. They argue that potential combatants can either negotiate a peaceful agreement or go to war at cost that is not known by their opponent. They find a semi-separating equilibrium where combatants facing low costs of war go to war when their demands are not met; meanwhile, combatants facing high costs of fighting mimic the demands of combatants facing low costs. In response, the opponent challenges the combatant in question and a war may ensure with a strictly positive probability. Incomplete information, manifested in other ways, also rationalizes why nations go to war. For instance, Fearon (2007) further argues that combatants may not know how long their opponents can sustain a conflict and establishes that this may prompt nations to fight protracted wars as they "screen" each other's capacity to sustain an armed conflict.

Fearon (1995) further showed that, in general, bargaining failures rationalize why strategic combatants go to war. A different source of bargaining failure noted to play a role in the decision to go to war is a limited capacity to bargain. For example, the peaceful resolution to a war may include a transfer from the wealthier combatant to their opponent, but if the transfers required to appease the poorer combatant are too high, then it may be optimal to fight rather than appease. Lastly, another factor that can prompt strategic players to go to war are "strategic moves" (Baliga and Sjöström 2020). Such decisions allow a party to extract high rents from their opponent by forcing them to risk a major conflict right away or major concessions. In equilibrium, strategic moves stave off major conflicts in the short run but may promote war later. For example, a major (military) power may annex a disputed territory and threaten a costly war against the opposing powers that object. Nevertheless, the major power is likely to face local insurgents in the newly annexed territory.

C.0.2 Literature of Bargaining

Bargaining has long been a central question in economics, but a natural point to begin the relevant literature is with Nash (1950). He considered a cooperative approach where two players partition on perfectly divisible item among themselves and he studied efficient allocations satisfying further properties. Nash found that the way in which the surplus is partitioned can be understood by a notion known as "bargaining power". The notion of bargaining power, although useful, leaves unanswered the features of a bargaining setting that influence how surplus is divided. In response, Rubinstein (1982) considered a dynamic game where alternate proposing ways to partition the good in question. He further assumed that all the pertinent features of the game were common knowledge. This assumption results in a unique subgame-perfect Nash Equilibrium were the player makes an offer which ensures that his opponent immediately accepts the first offered made and features like each player's outside option and discount factors endogenize the notion of bargaining power.

Rubinstein's model, although successful in rationalizing bargaining power, makes the following counterfactual prediction: when there are gains from trade, players come to an agreement without delay. First, real-world negotiations are notoriously time-consuming. The literature primarily understood that part of the reason for which negotiations can be time-consuming is that players have incomplete information regarding the bargaining setting in question, see for instance Ausubel and Deneckere (1989), Fundenberg, Levine, and Tirole (1985, 1989), Cho (1990), Myerson (1990), Cramton (1998), Compte and Jehiel (2004). A particularly fruitful approach was promoted by Myerson (1990) and Abreu and Gul (2000). They considered a dynamic bargaining game where players can either be strategic and bargain as in Rubinstein (1982) or obstinate. When players are obstinate, they make the same demands and only accept terms which gives them a higher payoff than the demands they make.

This type of approach is called reputational bargaining. It implies that at most one player concedes immediately or the negotiation is from henceforth characterized by a War of attrition in which strategic players come to an agreement in finite time. This approach has two benefits. First, it rationalizes why negotiations are protracted since strategic players have an incentive to posture. Indeed, Min (2020) finds direct evidence of posturing behavior in wartime peace negotiations. The second significant contribution is that the model predictions do not significantly depend on the way players bargain, better known are the bargaining protocol. This is important since negotiations proceed in many differing ways and models focusing on a particular bargaining protocol often make predictions that crucially depend on the order of play. Such dependence on the bargaining protocol both limits the predictive power of a model and makes it difficult to study many empirical applications where fine details determining how negotiations proceeded are often not observed by the econometrician.

Reputational bargaining has become a leading approach with an extensive literature. I detail some of the main results therein which have focused on the role of model features on bargaining. Since the predictions of these models do not greatly depend on the protocol, the role of model features on the outcome is particularly robust and generalizable. Comte and Jehiel (2002) show that the presence of outside options reverses the role of posturing in bargaining i.e., if players have outside options that are weakly better than conceding to an opponent's demands, then strategic players reveal their types immediately and come to an agreement as soon as possible. Meanwhile, Abreu and Pearce (2007) study the role of more involved types and bargaining with contracts in which players can also make **observable** actions. Their main result is that reputational considerations limit the set of model repeated equilibria that can be sustained. Wolitsky (2012), for his part, shows that one can relax the assumption of what is common knowledge and the maximum payoff that players can sustain. Fanning (2016) then considered the role of exogenous deadlines on bargaining resolution rationalizes "deadline effects": agreements often arrive close to the deadline, even when a similar agreement could have been agreed to much earlier. Lastly, Fanning (2021) considers the benefit of arbitrators in a negotiation. He finds that an arbitrator can expedite the arrival of an agreement when he imperfectly communicates the possibility of a consensus agreement. Otherwise, arbitration is futile. Other papers

It is also the case that negotiations often end in a protracted disagreement. Cramton (1992) rationalize this behavior in a game of imperfect information, without reputational types, in which players do not know their opponent's outside option. He finds that an agreement may arrive gradually, but it be possible for players to learn that there were no gains from trade to begin with. Note that this model differs markedly from my own in that there is no initial uncertainty in whether strategic player can initially come to a mutually beneficial agreement. I generate disagreement because posturing prompts players to not concede to each other's demands.

My model further contributes to the bargaining literature in three ways. First, this model allows for players to arrive at a protracted impasse on the equilibrium path even when players are strategic and there were initial gains from trade. This is because equilibrium incentives to shirk endogenize the risk that the surplus is destroyed before players come to a agreement. Next, I consider a bargaining setting in which the players' actions are not perfectly observable and this allows strategic players to behave strategically without fully revealing their types. This is important as it generalizes additional information that affects the way that the negotiation proceeds. Lastly, I rationalize the empirical trends in wartime, peace negotiations. The literature pertaining this application is detailed below.

Next, my model significantly departs from the hold-up problem literature (see Rogerson 1992, Grossman and Hart 1986, Hart and Moore 1988, Hölmstrom and Roberts 1998, among others). The hold-up problem occurs in bargaining when a party to the negotiation must make a costly and non-contractible investment before a negotiation. Parties exerting effort expect their peer to extract a share of the return from investments, but to not assume its costs. Therefore, players under-invest. My model differs from the hold-up problem because I assume that investment decisions occur as players bargains. Such difference in timing qualitatively affects bargaining outcomes as described below.

The political science literature has studied bargaining in war from several points of view. First, since Schelling (1967), political scientist has found it useful to view war, itself, as negotiation among opposing parties and aggression as a tool through which belligerents pressure each other to capitulate. Filson and Werner (2002) rationalize, for example, why nations go to war in a similar way that Cramton justifies protracted negotiations: costly and time-consuming negotiations allow players to extract information rents from their opponent. Meanwhile, Powell (2004), as well as Reiter (2010), models how coming to an agreement and fighting is overlayed in a conflict; therefore, the progression of the conflict crucially depends on negotiations among parties and the negotiations depend on the war. Fearon (2007), however, further illustrates that fighting serves as a form of screening that may preclude bargaining from occurring in earnest.

My model can be understood as a model of war where the diplomats must exert effort in preserving the potential gains for a potential peace.

Political scientists have also studied the outcome of wartime negotiations: how often peace talks lead to peace treaties; how do peace negotiations proceed; et cetera. Pillar (1983), for example, analyzed the bargaining dynamics within wartime negotiations. First, Pillar (25) finds that from a sample of wars occurring from the later part of the 18^{th} century to the mid 20^{th} century, 68 percent of all wars fought among differing states ended with a peace treaty, whereas the rate among all wars is roughly 48 percent. Note that this observation does not contradict my findings since he only considers wars and no single negotiations themselves. He further recognizes that these talks are bargaining scenarios where players have imperfect information and (like in my model) the parties to a negotiation often influence hostilities which occur in tandem with the negotiation at hand. Next, Min (2020) finds that since 1945, the likelihood that a war ends after a negotiation further fell. He argues that this predominantly occurs because of interventionists policies from third parties who force belligerents to negotiate before either party is willing to come to an earnest agreement. It should be noted, however, that Walter (2002) and Page-Fortna (2004) provide evidence that third party interventions can be crucial in driving peace negotiations as they can enforce the guarantees signed in a peace treaty. I recognize this dimension, but the current paper does not study issues of commitment occurring after the players come to an agreement.

On the other hand, Fazal (2013) postulates that jus in bello could have been a significant factor determining why diplomacy often fail to end conflicts since 1945. Jus in bello are the international laws determining what are war crimes. She argues that with the advent of the International Court of Justice and the United Nations, belligerents are less willing to sign a formal document in which they account for their war crimes as the previously mentioned institutions could prosecute and sanction the nations and actors responsible. My model and empirical work suggests a more mundane justification for why wartime, peace negotiations have become less effective over time. I argue that the fall in share of talks resulting in a peace treaty is associated with the fact that it is 150 percent more likely that, at least part, of peace negotiations occurring since 1914 to take place during a ceasefire relative to previous conflicts. In my model, I show that observing the surplus i.e., additional information that is informative of the opponent's reputation, expedites agreements. Therefore, if one increases the share of conflicts occurring in a peace treaty, then one expects that players are more likely to arrive at an impasse since posturing is more time consuming then: thus, gains from negotiations can fall before strategic belligerents come to an agreement. The empirical section provides evidence of this composition effect in detail.

C.1 Figures

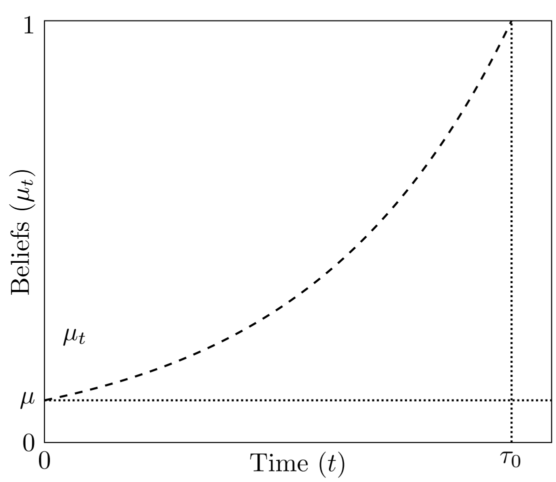

Figure C.1: Equilibrium beliefs

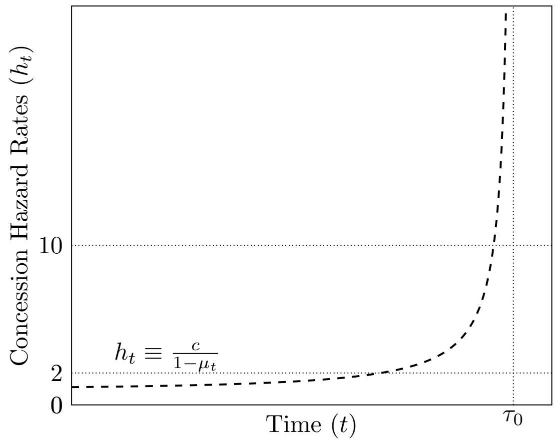

Figure C.2: Equilibrium concession rate.

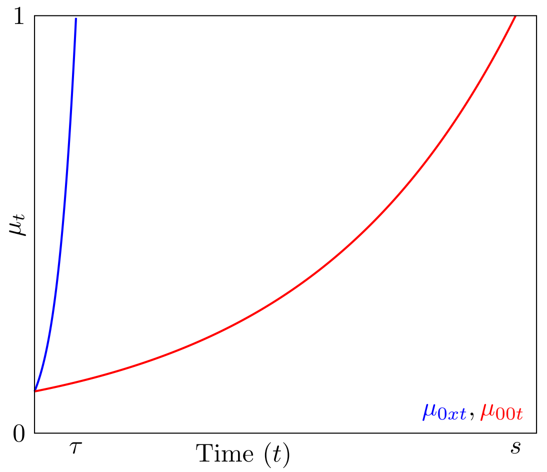

Figure C.3: Equilibrium beliefs when negotiations coincide with a ceasefire versus active fighting when $\epsilon = 1$.

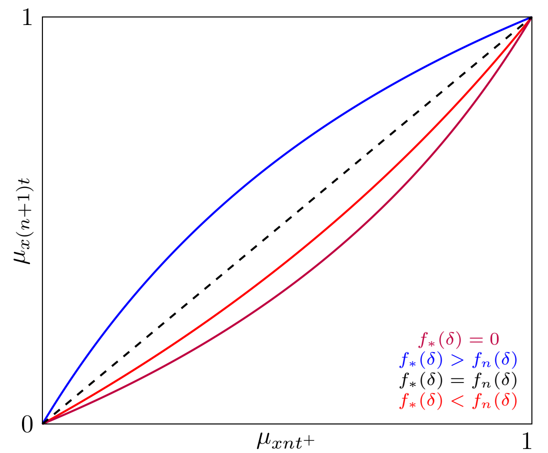

Figure C.4: Equilibrium Bayesian update rule.

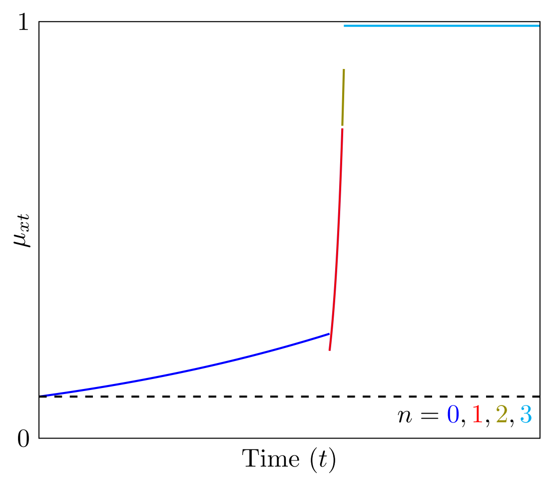

Figure C.5: Belief sample path.

C.2 Empirical Evidence

In this section, I study the patterns of wartime diplomacy from a panel of 92 wars fought between 1823-2003. The data combines the daily data from Min 2020 and 2021 alongside with data from Fazal (2013) in order to study the dynamics of diplomacy. I present and discuss the policy changes associated with decline in the share of wartime negotiations leading to an end of hostilities. My main result is that after the Hague convention of 1907, ceasefires were codified and after WWI they were used as an intermediary step in negotiating a peace. The section also considers the role of jus in bello in this trend (i.e., how combatants expose themselves to sanctions when recognize war crimes while signing a treaty) and clarifies the trend. With regards to these policies, I find that they do not paint a clear and uniform picture.

C.2.1 Descriptive Statistics

I first describe several summary results. Table C.1 first describes summary facts regarding wartime negotiations at the conflict level. Almost 80 percent of all wars in the panel had peace negotiations at some point in the talk, but less than half of wars ended after a formal peace treaty. It should be noted that this does not mean that others wars ended with a unilateral victory. The table also shows that roughly 16 percent of all wars ending after negotiations did so after multiple attempts.

	1823-2003	1823-1913	1914-2003
Share of Conflicts with (wartime) negotiations	79 %	82 %	78 %
Share of Conflicts ending after the first negotiations	41	50	35
Share of Conflicts ending after multiple negotiations	8	11	6
Share of negotiations leading to an end of a conflict	24	40	13

Table C.1: Summary of Peace Negotiation outcomes of conflicts fought from 1823 to 2003.

Next, I split the sample into two periods: pre-1914 (1823-1913) and post-1914 (1914-2003). I find that roughly the same share of wars had peace negotiations in both period. However, roughly 60 percent of wars in the pre-1914 period ended with a peace treaty in comparison to 41 percent post-1914. This implies that a significantly smaller share of conflicts end in a negotiated conflict. Looking at each period at the negotiation level, I further find that roughly 25 percent of negotiations led to an end of fighting but that this rate has been falling over time. Pre-1914 40 percent of negotiations led to an end of wars, but post-1914 this rate fell to 13 percent: a 27 percentage point decline.

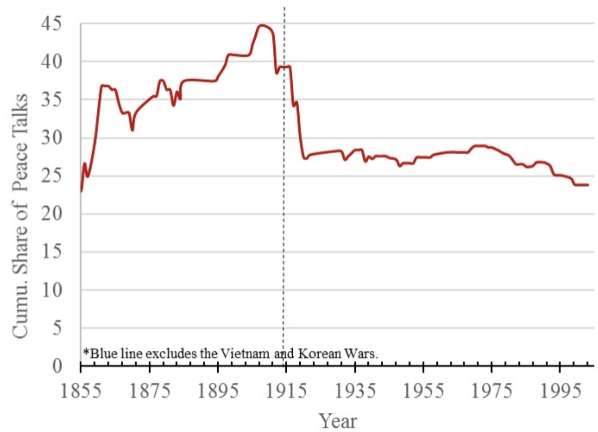

Figure C.6: Summary of Peace Negotiation outcomes of conflicts fought from 1823 to 2003.

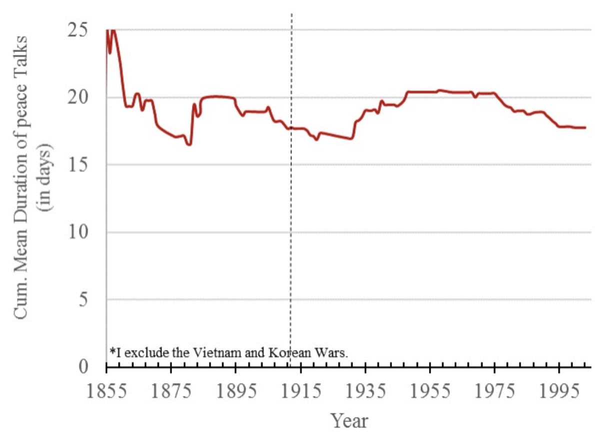

Figure C.7: Mean duration of negotiations.

Next, rather than looking at the pattern of formal peace talks, one may study the pattern of multi-week pauses between major battles. This is how I quantitatively define ceasefires. Figure C.7 illustrates the distribution of ceasefires duration for wars fought from 1823 to 2003. Around 30 percent of ceasefires lasted less than two weeks, but roughly 39 percent of ceasefires lasted between 1 to 9 months. On the other hand, figure C.9 illustrates the distribution of the number of ceasefires per war during the same conflict. I find that roughly 23 wars had no ceasefires, but that a similar proportion had more than 6 ceasefires. These facts imply that ceasefires are commonplace and often lengthy, which gives belligerents the opportunity to informally negotiate with each other.

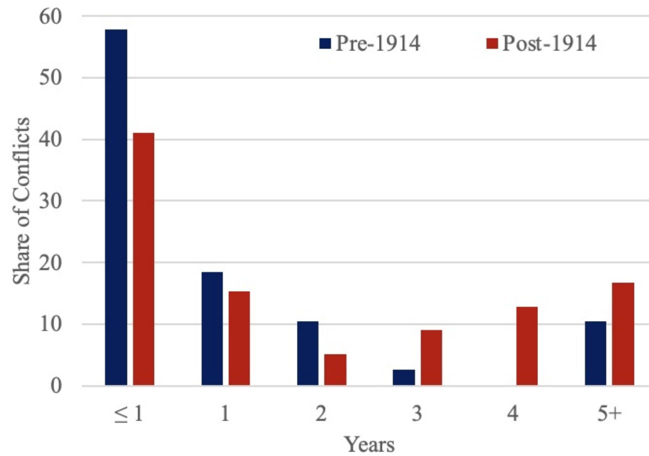

Figure C.8: Distribution of War duration.

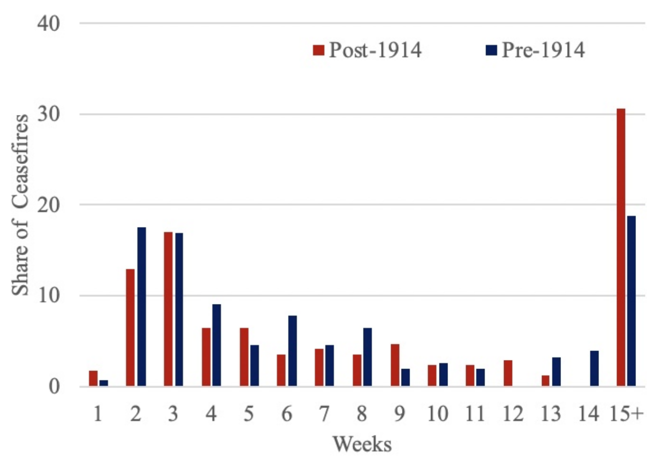

Figure C.9: Distribution of Ceasefire duration.

Correlates	Duration
In negotiation (yes/no)	-0.120 (0.401)
Ceasefire (yes/no)	-0.222 (0.130)
Ceasefire X In negotiation + Controls	0.373** (0.155)
Sample size (war X days)	36,849
R-Square	0.495

Clustered Standard Errors, **(p<0.05)

Figure C.10: Regression of select correlates on the duration of wars (in years).

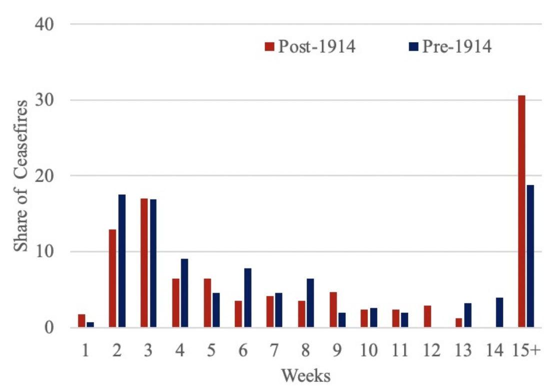

Figure C.11: Histogram of Ceasefire duration for Wars fought from 1823 to 2003.

Table C.2 considers how the Ceasefires differ between the periods 1823-1913 versus 1914-2003. A war belongs to the period 1823-1913 if it ends before 1913. I find that after 1914 was spend a smaller share of their duration in a ceasefire and a higher share of conflicts had no ceasefire lasting more than 2 weeks. Note that I also consider *effective* ceasefires. These are ceasefires that culminate with an end to the conflict. I find that the fact wars after 1914 spend a smaller share of the time in such ceasefires. Moreover, roughly 25 to 30 percent of all time spend in a ceasefire coincides with the end of a war.

This implies that most of the time spent in a ceasefires is not associated with an end to hostilities. Next, I look at several correlations of note. First, I find that the correlation between a war ending, ceasefires, and negotiation are tenuous at best. However, the correlation between a ceasefire and a diplomatic negotiation more than doubled after 1914. This implies that a larger share of the time in which player negotiates occurs when belligerents do not fight after 1914 relative to before 1913. Secondly, the correlation between negotiating and a war ending almost halved from the period 1823-1913 to 1914-2003. Therefore, the fact that parties partake in peace talks is less associated with an end to the conflict after 1914. Lastly, the correlation between a ceasefire and the war ending is negative and small.

	1823-1913		1914-2003	
Share of time in a Ceasefire				
Average	30.1	%	28.0	%
Median	27.2		24.3	
Share of time in an Effective Ceasefire				
Average	10.8	%	7.0	%
Median	2.4		0.0	
No Ceasefires	25.6		41.4	
Correlation between				
Ceasefire and Peace negotiations	2.8	%	7.1	%
War ends and Peace negotiations	14.1		7.9	
War ends and Ceasefire	-4.9		-5.0	

Table C.2: Summary Description of Ceasefire Patterns.

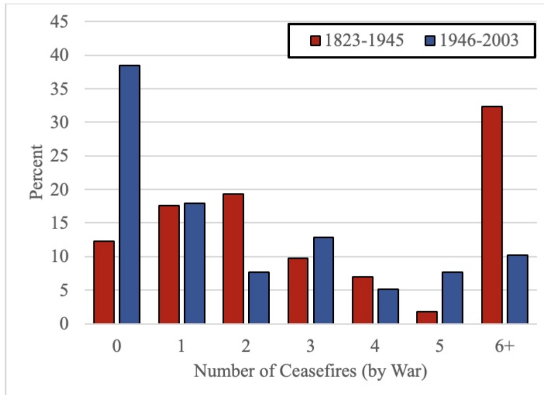

Figure C.12: Mean number of Ceasefires per war and period

It is further important to analyze the distribution of the number of ceasefires by the periods pre- and post-1945. Figure C.8 and C.9 shows that before 1945 the distribution of ceasefire duration approximates an exponential distribution but after 1945 the distribution has the shape of a log-normal distribution. This implies that the average duration of ceasefires after 1945 is longer than before 1945. Meanwhile, figure C.9 illustrates that wars occuring before 1945 had more ceasefires than wars after 1945.

Figure C.13, however, does show that the cumulative share of negotiations resulting in a peace treaty *has* fallen over time. The issue is that the fall in the cumulative share of conflicts with a negotiation that did not end the conflict markedly falls since 1914, i.e. much earlier than predominantly noted in the literature. Indeed, if one were to remove the Vietnam and Korean wards, this pattern persists. Consequently, over time peace talks are shorter and less effective; meanwhile, ceasefires are longer than before.

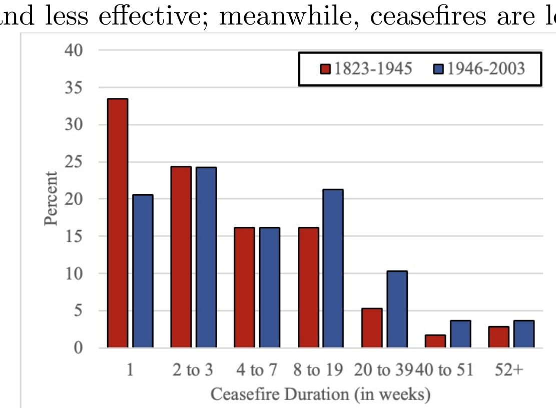

Figure C.13: Share of negotiations leading to a lasting peace agreement.

C.2.2 Jus in Bello

The descriptive statistics show that after 1914, formal negotiations have become shorter and less likely to lead to a formal peace treaty. However, wars after 1914 have longer and fewer ceasefires. Such pauses in the conflict allow the parties involved to informally negotiate an end to the conflict without having to write a formal theory. These observations bring forth a natural questions: why don't nations negotiate formal, peace treaties?, have informal agreements taken the place of treaties? I argue that ceasefires play a significant role in this process, but previous authors also note the role of jus in bello (i.e. laws of war) in accounting for these trends.

Fazal 2013 postulates a potential justification for why belligerent states do not sign peace treaties. She argues that peace treaties became disfavored by belligerent nations after the establishments of the laws of war, i.e. jus in bello. Informally, once the heads of states write and sign a treaty, they account the actions each side made during the conflict as well as the accord that brings an end to hostilities. The issue lies that since 1899, there are formal laws and legal entities that can penalize the heads of states and government officials of committing "war crimes". Moreover, states found responsible of these acts open themselves to sanctions. This implies that laws of war impose costs to signing formal treaties that are not fully captured by the concessions made in the negotiation.

In order to test this hypothesis, it is important to note which rules of war are associated with the decline in the efficacy of wartime diplomacy. Before doing so, it is important to distinguish between terms that may be confused. Stahn (2006) clarifies that since the late 19^{th} century, laws regarding wars have been divided into jus ad bello and jus in bello. Jus and bello refers to the laws determining when belligerent states are legally justified to enter a military conflict. It states when a nation can decide to attack another nation. Meanwhile, just in bello focuses on the behavior of military actors during a conflict. For instance, jus in bello regulates which weapons should not be used in a conflict as well as the treatment of prisoners of war.

Jus in bello violations hence describe acts deemed inappropriate during a conflict. I study the effect associated with four particular treaties. The first treaty pertains the Hague treaty of 1899 that was promoted by Czar Nicholas. According to Best (1980), this is one of the foundational treaties that that codifies both just in and ad bello. Unlike other treaties, starting since 1856, this Hague convention established a permanent Court of Arbitration, promoted the less frequented Geneva conference of 1864, and explicitly forbid several practices as the use of poisons and expanding bullets in open battle.

The second treaty considered is the Hague Convention of 1907. This convention expands upon the previous convention, explicitly defines neutral parties, the treatment of enemy merchant (i.e. non-war) ships, and other protocols. Both Hague conventions are seen as the first codification of jus in bello, but it failed to establish a judicial system that was capable of carrying out sanctions and prosecuting individual actors of war crimes. The treaty establishing the League of Nations (1920), however, filled this gap by formally establishing the first iteration of the International Court of Justice (ICJ). Lastly, the modern iteration of the ICJ and the United Nations as an entity that can organize sanctions against a state occurs in 1946.

C.2.3 Regressions

Now that both the empirical facts have been established and the treaties in consideration, I study the associated effect of such treaties, presence of negotiations, and ceasefires on the duration of conflicts, that probability that belligerent nations set up a formal treaty, and the probability that the war ends. For each convention discussed above, I make an indicator for whether the date of the conflict was before or after each treaty. The subsequent regressions further control for conflict specific, geographic, and year fixed effects.

Table C.3 first measures each treaty's marginal effect on the duration of a conflict. The results are mixed. First, the first Hague Convention is associated with a 3 year decline in the duration of a conflict; whereas the second Hague conference is associated with an extension of a conflict of more than a year and a half. Later conventions, for their part, are not associated with a statistically significant change in the duration of the conflict and the magnitude of the effect is half or an order of magnitude less than the previous conventions. Meanwhile, when one runs a probit and estimate the probability that the war ends on each treaty, then they are all statistically significant and all but one is associated with an increase in the likelihood that a war ends quickly.

Next, do not find evidence to support that ceasefires or holding negotiations (by themselves) has a statistically significant correlation with the duration of wars. The interaction between the two factors, however, is associated with an increase in the duration of conflicts. Indeed, spending 1 day negotiating a peace treaty during a ceasefire is associated with 10 more days of fighting. I further find that holding negotiations and holding ceasefires are uncorrelated with each other.

The last regression of note, i.e. the central column, details the probability that belligerents partook in formal peace talks during the conflict. I find that the evidence appears mixed. First, the Hague conventions are associated with significant declines in the probability that parties negotiate formally, but the formation of the UN is associated with more, albeit less effective, talks. It should be noted, as expected, that the probability that nations held negotiations after the Hague Convention of 1907 with 57 percent more likelihood.

These regressions, lastly, present that jus in bello have an ambiguous, but important, association with the duration of a conflict and the probability that parties involved in a conflict negotiate. However, any future analysis must consider a longer historical scope and recognize that there exist a distinction between a legalistic process like a negotiation being effective and an era where conflicts of short lived.

VARIABLES	Duration of Conflict	Pr (Negotiation)
	(in years)	
Hague Peace Conference (1899)	5.541	-0.807***
	(4.080)	(0.304)
Second Hague Peace Conference (1907)	0.984	-0.052
	(0.422)	(0.152)
League of Nations (ICJ) 1920	-0.270	0.158
	(0.557)	0.158***
International Court of Justice (1946)	9.904	(0.055)
	(6.200)	0.990***
		(0.299)
Parties in Negotiation	-0.120	
	(0.401)	
Ceasefire in Place	-0.222	-0.004
	(0.130)	(0.017)
Ceasefire X Negotiation	0.373**	
	(0.155)	
Conflict, geography, and year fixed effects		
Observations	36,849	36.849
R-squared	0.495	0.224

War clustered standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table C.3: Associate effect of the Hague Conventions and other Jus in Bello treaties on the duration of wars.

It is indeed possible for diplomacy to still be effective without an explicit contract that compromises the parties involved. I now study whether there is any evidence that jus in bello forced negotiations to be informal; i.e. belligerents do not sign formal treaties expecting that the liability associated with the formal declaration, but still negotiate a lasting truce without a formal document. Is there any evidence that this is the case? I find little evidence supporting this hypothesis. First, table C.4 runs a probit with the probability of a ceasefire, i.e. a pause in a conflict lasting more than 2 weeks, on the same correlates as before. I find that all conventions, but the second Hague conference, are associated with a decline in the probability of a ceasefire. Meanwhile, the left-hand side panel calculates the probability of a ceasefire which terminates the conflict. I find that not convention is associated with a statistically significant change in the probability of such ceasefires.

These regressions make the following clarifications. Firstly, conventions and treaties establishing the laws of war and their implementation may be significantly correlated with the prevalence of peace talks, ceasefires, and the end of conflict. It is, however, secondly the case that the direction of the association depends on the convention in question. It is thirdly not clear that the conventions are associated with multi-week ceasefires that end with an end in the war. Thus, the degree to which laws of war conventions are associated with a fall in the prevalence in the formal peace treaty, they are not, in its place, associated with an increase in ceasefires which (informally) take their place.

Variables	Pr (Ceasefire)		Pr (Final Ceasefire)
Hague Peace Conference (1899)	-0.880	***	-0.279
	(0.325)		(0.325)
2nd Hague Peace Conference (1907)	0.547	***	-0.057
	(0.165)		(0.010)
League of Nations (ICJ) (1920)	-0.064		-0.039
	(0.072)		(0.051)
International Court of Justice (1946)	-1.234	**	-0.308
	(0.488)		(0.322)
+ Conflict, geography, and year fixed effects			
Observations	31,985		28,985
Pseudo R-squared	0.19		0.26

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table C.4: Associate effect of the Hague Conventions and other Jus in Bello treaties on the duration of wars.