

TESTING FOR INDEPENDENCE WITH
ADDITIONAL INFORMATION

by

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$$\begin{aligned}
(2.30) \quad \pi(\phi, \beta) &= \int \phi \, dP_{\beta}^T = \int \phi R dP_0^T = \mathcal{E}_0 \phi R \\
&= \mathcal{E}_0 \phi \left[1 + \frac{n(n+m)}{2p_1 p_2} \operatorname{tr} \bar{g}_1 x' \bar{g}_2' \bar{g}_2 x \bar{g}_1' \operatorname{tr} \beta \beta' \right. \\
&\quad \left. - \frac{(n+m)}{2p_2} \operatorname{tr} \bar{g}_2 u' u \bar{g}_2 \operatorname{tr} \beta \beta' + o(\operatorname{tr} \beta \beta') \right] \\
&= \alpha + (\mathcal{E}_0 B(\phi)) \operatorname{tr} \beta \beta' + o(\operatorname{tr} \beta \beta')
\end{aligned}$$

where the remainder term is uniform in ϕ , and $B(\phi)$ is given by (2.4).

The last equality in (2.30) follows by noting that

$$\begin{aligned}
\operatorname{tr} \bar{g}_2 u' u \bar{g}_2' &= \operatorname{tr} S_{22} (S_{22} + V)^{-1} \quad \text{and} \\
\operatorname{tr} \bar{g}_1 x' \bar{g}_2' \bar{g}_2 x \bar{g}_1' &= \operatorname{tr} S_{11}^{-1} S_{12} (S_{22} + V)^{-1} S_{21}.
\end{aligned}$$

This completes the proof of Theorem 2.1.

Este condicele de la funcția de distribuție S(t).

$$\text{cu } \frac{dS(t)}{dt} = \lambda e^{-\lambda t} (e^{-\lambda t} + A) e^{-\lambda t}$$

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Această funcție de distribuție (S(t)) reprezintă numărul de...

Acesta este rezultatul pentru funcția de distribuție ϕ sau $P(\phi)$ în cazul de (S(t)).

$$= \lambda + (\lambda^0 P(\phi)) \text{ cu } \phi, + \phi(\text{cu } \phi)$$

$$- \frac{\lambda^0 \phi}{(1+\lambda)} \text{ cu } \frac{\lambda^0 \phi}{(1+\lambda)} \text{ cu } \phi, + \phi(\text{cu } \phi)$$

$$= \lambda^0 \phi + \frac{\lambda^0 \phi^2}{2(1+\lambda)} \text{ cu } \frac{\lambda^0 \phi^2}{2(1+\lambda)} \text{ cu } \phi$$

$$(S:0) \quad \mu(S) = \int_0^\infty \phi \lambda e^{-\lambda t} dt = \int_0^\infty \phi \lambda e^{-\lambda t} dt = \lambda^0 \phi$$

$$(2.22) \quad IV = \int_{\mathcal{O}(p_2)} \left[\sum_{j=2}^{\infty} \frac{(-F)^j}{j!} + \sum_{j=2}^{\infty} \frac{E^{2j}}{(2j)!} + \sum_{\ell=1}^{\infty} \sum_{j=1}^{\infty} \frac{E^{2j} (-F)^\ell}{(2j)! \ell!} \right] \mu_2^{(2)}(dk_2)$$

$$\equiv V + VI + VII .$$

Recall that for matrices $a: r_1 \times r_2$ and $b: r_2 \times r_1$,

$|\operatorname{tr} ab| \leq (\operatorname{tr} aa')^{\frac{1}{2}} (\operatorname{tr} bb')^{\frac{1}{2}}$ and $\operatorname{tr}(aa')^2 \leq (\operatorname{tr} aa')^2$. Using these inequalities and arguing as in Schwartz (1967) yields

$$(2.23) \quad \begin{cases} |-F|^j & \leq (2p \operatorname{tr} \beta \beta')^j \left(\frac{1}{4} \operatorname{tr} h_2 h_2' \right)^j \\ E^{2j} & \leq (4p \operatorname{tr} \beta \beta')^j \left(\frac{\operatorname{tr} h_1 h_1' + \operatorname{tr} h_2 h_2'}{4} \right)^{2j} \end{cases}$$

where we have used the inequalities $\operatorname{tr} \bar{g}_2 u u' \bar{g}_2' \leq p$ and $\operatorname{tr} \bar{g}_1 x' \bar{g}_2' \bar{g}_2 x \bar{g}_1' \leq p$.

Thus for $\operatorname{tr} \beta \beta' < \frac{1}{4p}$,

$$(2.24) \quad |V| \leq \sum_{j=2}^{\infty} (j!)^{-1} [2p \operatorname{tr} \beta \beta']^2 \left[\frac{1}{4} \operatorname{tr} h_2 h_2' \right]^j \\ \leq [2p \operatorname{tr} \beta \beta']^2 \exp \left[\frac{1}{4} \operatorname{tr} h_2 h_2' \right] ,$$

$$(2.25) \quad |VI| \leq \sum_{j=2}^{\infty} [(2j)!]^{-1} (4p \operatorname{tr} \beta \beta')^j \left[\frac{\operatorname{tr} h_1 h_1' + \operatorname{tr} h_2 h_2'}{4} \right]^{2j} \\ \leq (4p \operatorname{tr} \beta \beta')^2 \exp \left[\frac{\operatorname{tr} h_1 h_1' + \operatorname{tr} h_2 h_2'}{4} \right]$$

and

$$(2.26) \quad |VII| \leq \sum_{j=1}^{\infty} \sum_{\ell=1}^{\infty} [(2j)! \ell!]^{-1} [4p \operatorname{tr} \beta \beta']^j [4p \operatorname{tr} \beta \beta']^\ell \\ \left[\frac{1}{4} (\operatorname{tr} h_1 h_1' + \operatorname{tr} h_2 h_2') \right]^{2j} \left[\frac{1}{8} \operatorname{tr} h_2 h_2' \right]^\ell \\ \leq [4p \operatorname{tr} \beta \beta']^2 \exp \left[\frac{1}{4} \operatorname{tr} h_1 h_1' + \frac{3}{8} \operatorname{tr} h_2 h_2' \right] .$$

write $g_i = h_i k_i$ where $h_i \in G_T^+(p_i)$, $k_i \in \mathcal{O}(p_i)$ and let $\mu_1^{(i)}$ and $\mu_2^{(i)}$ be left invariant measures on $G_T^+(p_i)$ and $\mathcal{O}(p_i)$ respectively for $i = 1, 2$. (For convenience we take invariant measure on $\mathcal{O}(r)$ to be invariant probability measure.) Then N in (2.12) can be written as

$$(2.13) \quad N = \int \int \int \int_{G^*} \exp(E-F) \left[\prod_{i=1}^2 |h_i h_i'| \right]^{\frac{n(i)}{2}} \exp\left(-\frac{1}{2} \text{tr } h_i h_i'\right) \prod_{i=1}^2 \mu_1^{(i)}(dh_i) \mu_2^{(i)}(dk_i)$$

where $G^* = G_T^+(p_1) \times G_T^+(p_2) \times \mathcal{O}(p_1) \times \mathcal{O}(p_2)$ and

$$(2.14) \quad \begin{cases} E = \text{tr } \beta' h_2 k_2 \bar{g}_2 \times \bar{g}_1' k_1 h_1' \\ F = \frac{1}{2} \text{tr } \beta \beta' h_2 k_2 \bar{g}_2 (u'u) \bar{g}_2' k_1 h_1' . \end{cases}$$

Similarly, the denominator in (2.10) can be written as $D = D_1 D_2$ where

$$(2.15) \quad D_i = \int |h_i h_i'| \left[\prod_{i=1}^2 \exp\left[-\frac{1}{2} \text{tr } h_i h_i'\right] \mu_1^{(i)}(dh_i) \mu_2^{(i)}(dk_i) \right]$$

for $i = 1, 2$. Defining $H_i(h_i)$, $i = 1, 2$, by

$$(2.16) \quad H_i(h_i) = \frac{1}{D_i} |h_i h_i'| \left[\prod_{i=1}^2 \exp\left[-\frac{1}{2} \text{tr } h_i h_i'\right] \right],$$

the ratio R becomes

$$(2.17) \quad R = \int \int \int \int_{G^*} \exp[E-F] \left[\prod_{i=1}^2 H_i(h_i) \right] \prod_{i=1}^2 \mu_1^{(i)}(dh_i) \mu_2^{(i)}(dk_i) .$$

To evaluate (2.17), we first evaluate

10. DEFINITION (S.11). Let \mathcal{A} be a \mathcal{C}^* -algebra.

$$(S.11) \quad \mathcal{A} = \overline{\text{span} \{ \sum_{i=1}^n a_i^* a_i \mid a_i \in \mathcal{A} \}}.$$

PROOF. It is clear that \mathcal{A} is a \mathcal{C}^* -subalgebra of \mathcal{A} .

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PROPOSITION (S.12). Let \mathcal{A} be a \mathcal{C}^* -algebra. Then $\mathcal{A} = \overline{\text{span} \{ \sum_{i=1}^n a_i^* a_i \mid a_i \in \mathcal{A} \}}$.

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with (x,u) , y , and v independent, $x:p_2 \times p_1$, $y:n_1 \times p_1$, $u:n \times p_2$ and $v:m \times p_2$. Then, the data in (2.7) corresponds to the data in (2.2) in terms of the new coordinates (x,y,u,v) . Consider the group $\tilde{G} \equiv GL(p_1) \times GL(p_2) \times \mathcal{O}(n_1) \times \mathcal{O}(n) \times \mathcal{O}(m)$ acting on (x,y,u,v) by

$$(2.8) \quad (g_1, g_2, g_3, g_4, g_5)(x, y, u, v) = (g_2 x g_1', g_3 y g_1', g_4 u g_2', g_5 v g_2')$$

with $g_1 \in GL(p_1)$, $g_2 \in GL(p_2)$, $g_3 \in \mathcal{O}(n_1)$, $g_4 \in \mathcal{O}(n)$, and $g_5 \in \mathcal{O}(m)$. Clearly, the group action of \tilde{G} on (x,y,u,v) corresponds to the group action of G on $(S_{11.2}, S_{21}, S_{22}, V)$ defined by (1.5).

Now, let $\tilde{\mathcal{D}}_\alpha$ be the class of all level α test functions [functions of (x,y,u,v)] for $H_0:\beta = 0$ versus $H_1:\beta \neq 0$ which are invariant under \tilde{G} . The mapping $S_{11.2} = y'y$, $S_{21} = x$, $S_{22} = u'u$, $V = v'v$ determines a 1-1 correspondence between test functions in \mathcal{D}_α and test functions in $\tilde{\mathcal{D}}_\alpha$. Thus, it is sufficient to verify (2.3) for test functions in $\tilde{\mathcal{D}}_\alpha$.

The joint density of (x,y,u,v) , with respect to Lebesgue measure, is

$$(2.9) \quad f(x,y,u,v|\beta) = c |u'u|^{-\frac{p_1}{2}} \exp[-\frac{1}{2} \text{tr}[x'(u'u)^{-1}x + y'y] + \text{tr}\beta'x - \frac{1}{2} \text{tr}\beta'u'u\beta - \frac{1}{2} \text{tr}(u'u + v'v)]$$

where c is a constant. Now, let P_β^T denote the probability measure of a maximal invariant, T , under the action of \tilde{G} on (x,y,u,v) .

From a result due to Wijsman (1966), the Radon-Nikodym derivative

$$R = dP_\beta^T / dP_0^T \text{ can be written as}$$

$E = \sum_{\alpha} \sum_{\beta} \delta_{\alpha\beta}^0$ and so we have

Let us assume that the system (1.2) is a homogeneous system of linear equations in the variables x_{α} on $(\mathbb{R}^n, \mathbb{R})$. Then, it is clear that the system (1.2) is homogeneous if and only if $\sum_{\alpha} \delta_{\alpha\beta}^0 = 0$ for all β .

$$+ \sum_{\alpha} \delta_{\alpha\beta}^0 = [\sum_{\alpha} \delta_{\alpha\beta}^0 - \sum_{\alpha} (\delta_{\alpha\beta}^0 + \delta_{\alpha\beta}^0)]$$

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$$(2.2) \quad \begin{cases} S_{11.2} \sim W(I_{p_1}, p_1, n_1) \\ S_{21} | S_{22} \sim N(S_{22} \beta, S_{22} \times I_{p_1}) \\ S_{22} \sim W(I_{p_2}, p_2, n) \\ V \sim W(I_{p_2}, p_2, m) . \end{cases}$$

Theorem 2.1: For $\phi \in \mathcal{D}_\alpha$, the power function of ϕ , say $\pi(\phi, \beta) \equiv \pi(\phi; \delta_1, \dots, \delta_t)$, is given by

$$(2.3) \quad \pi(\phi; \delta_1, \dots, \delta_t) = \alpha + \left(\sum_{i=1}^t \delta_i \right) B(\phi) + o\left(\sum_{i=1}^t \delta_i \right)$$

where

$$(2.4) \quad B(\phi) = a_1 \mathcal{E}_0 [\phi(a_0 \text{tr} S_{11}^{-1} S_{12} (S_{22} + V)^{-1} S_{21} - \text{tr} S_{22} (S_{22} + V)^{-1})]$$

with $a_0 = \frac{n}{p_1}$ and $a_1 = \frac{m+n}{2p_2}$. \mathcal{E}_0 denotes expectation when β in (2.2) is zero. The remainder term $o\left(\sum_{i=1}^t \delta_i\right)$ is uniform in ϕ , i.e.,

$$(2.5) \quad \lim_{\sum \delta_i \rightarrow 0} \sup_{\phi \in \mathcal{D}_\alpha} \frac{o\left(\sum_{i=1}^t \delta_i\right)}{\sum_{i=1}^t \delta_i} = 0 .$$

The proof of Theorem 2.1 is rather lengthy and is deferred to the end of this section. An immediate consequence of Theorem 2.1 is

Theorem 2.2: Define the level α -test function $\phi^* \in \mathcal{D}_\alpha$ by

$$(2.6) \quad \phi^* = \begin{cases} 1 & \text{if } a_0 \text{tr} S_{11}^{-1} S_{12} (S_{22} + V)^{-1} S_{21} - \text{tr} S_{22} (S_{22} + V)^{-1} > k \\ 0 & \text{otherwise} \end{cases}$$

(S.1) $\varphi_* = \dots$

In case S.S: ...

... of this region: ...

The area of ...

(S.2) $\frac{\dots}{\dots} = \dots$

(S.3) ...

$\varphi^0 = \frac{\dots}{\dots}$...

(S.4) $\varphi(\varphi) = \dots$

... ..

(S.5) $\varphi(\varphi: \varphi^1, \dots, \varphi^F) = \dots$

$\varphi(\varphi) = \dots$

In case S.S: ...

(S.6) \dots

(S.7) \dots

\dots

\dots

$$(1.19) \quad R(u, w, \delta_0) \equiv \frac{g(u, w; \delta_0)}{g(u, w; 0)} .$$

But, it is not hard to see that such tests depend on the particular value δ_0 . Hence there can be no uniformly most powerful invariant test of H_0 versus H_1 .

conditionally independent (given S_{22}), the joint conditional density is $S_{22} h_m(S_{22} w) q(u; S_{22} \delta)$. Multiplying this by the density of S_{22} (with $B_{22} = 1$) and integrating out S_{22} yields

$$(1.15) \quad g(u, w; \delta) = \sum_{j=0}^{\infty} \frac{\Gamma(\frac{m+n}{2} + j)}{\Gamma(\frac{m}{2}) \Gamma(\frac{n}{2})} \frac{\delta^j}{j!} \frac{w^{\frac{m}{2} - 1}}{(1 + \delta + w)^{\frac{m+n}{2} + j}} g_{p-1+2j, n-p+1}(u); \quad u > 0,$$

as the joint density of U and W . Since the marginal density of W is $g_{m, n}(w)$, the conditional density of U given W is

$$(1.16) \quad g(u|w, \delta) = \sum_{j=0}^{\infty} \frac{\Gamma(\frac{m+n}{2} + j)}{\Gamma(\frac{m+n}{2})} \frac{(\frac{\delta}{1+w})^j}{j!} \frac{1}{(1 + \frac{\delta}{1+w})^{\frac{m+n}{2} + j}} q_{p-1+2j, n-p+1}(u), \quad u > 0.$$

Proposition 1.2: For each fixed value of w , $q(u|w, \delta)$ has a monotone likelihood ratio (MLR) in u and δ .

Proof: For w fixed, the function $(\frac{\delta}{1+w})^j / (1 + \frac{\delta}{1+w})^{\frac{m+n}{2} + j}$ has a MLR in δ and j . Also $q_{p-1+2j, n-p+1}(u)$ has a MLR in j and u . By a result due to Karlin (1956), $q(u|w, \delta)$ has a MLR in u and δ . This completes the proof.

Now let $C_{\alpha}(W)$ be the class of tests of H_0 versus H_1 which are invariant under G and which have conditional level α for each fixed value of W . Since all tests which are invariant under G can be written as functions of (U, W) , we restrict our attention to the density of (U, W) when discussing the power of tests in $C_{\alpha}(W)$.

(1.1) $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$ is the inverse of $f^{-1}(x) = \frac{1+x}{1-x}$.
 The function $f(x)$ is defined for $x \in (-1, 1)$. The function $f^{-1}(x)$ is defined for $x \in (-1, 1)$.
 The function $f(x)$ is an odd function. The function $f^{-1}(x)$ is an odd function.
 The function $f(x)$ is concave down. The function $f^{-1}(x)$ is concave up.

(1.2) $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$ is the inverse of $f^{-1}(x) = \frac{1+x}{1-x}$.
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(1.3) $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$ is the inverse of $f^{-1}(x) = \frac{1+x}{1-x}$.

$$= \frac{\frac{1}{2} \ln \frac{1+x}{1-x}}{\frac{1}{2} \ln \frac{1+x}{1-x}} = \frac{\frac{1}{2} \ln \frac{1+x}{1-x}}{\frac{1}{2} \ln \frac{1+x}{1-x}} = 1$$

(1.4) $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$

(1.5) $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$ is the inverse of $f^{-1}(x) = \frac{1+x}{1-x}$.

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(1.7) $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$ is the inverse of $f^{-1}(x) = \frac{1+x}{1-x}$.
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 The function $f(x)$ is an odd function. The function $f^{-1}(x)$ is an odd function.
 The function $f(x)$ is concave down. The function $f^{-1}(x)$ is concave up.

When $p_2 = 1$, then $|I - S_{11}^{-1} S_{12} S_{22}^{-1} S_{21}| = 1 - R^2$ where $R^2 = S_{22} S_{11}^{-1} S_{12} / S_{22}$ is the multiple correlation coefficient.

For the remainder of this section we treat only the case $p_2 = 1$.

Proposition 1.1: Under the action of G defined by (1.5) a maximal invariant in the sample space is

$$(1.7) \quad (U, W) \equiv \left(\frac{S_{21} S_{11.2}^{-1} S'_{21}}{S_{22}}, \frac{V}{S_{22}} \right)$$

and a maximal invariant in the parameter space is

$$(1.8) \quad \delta \equiv B_{22}^{-1} B_{21} B_{11}^{-1} B'_{21} \geq 0.$$

Proof: The proof of this proposition is routine and is omitted.

To study invariant tests of H_0 versus H_1 , we now need to derive the joint distribution of U and W . The conditional distribution of (U, W) given S_{22} is first calculated and then S_{22} is integrated out. Since (U, W) is a maximal invariant under G , it suffices to find the distribution of (U, W) under the assumption that $B_{11} = I_{p-1}$ and $B_{22} = 1$ which we now assume. Conditional on S_{22} , $V/S_{22} \sim \frac{1}{S_{22}} \cdot \chi_m^2$ where " χ_m^2 " denotes a chi-square random variable with m degrees of freedom. Since $B_{11} = I_{p-1}$ and $B_{22} = 1$,

$$(1.9) \quad \frac{S_{21}}{\sqrt{S_{22}}} | S_{22} \sim N(\sqrt{S_{22}} \sum_{21}, I_{p-1})$$

$$S_{11.2} \sim W(I_{p-1}, p-1, n-1).$$

Thus, conditional on S_{22} ,

$$U = \frac{S_{21}}{\sqrt{S_{22}}} S_{11.2}^{-1} \frac{S'_{21}}{\sqrt{S_{22}}} \sim \frac{\chi_{p-1}^2 (S_{22} \delta)}{\chi_{n-p+1}^2}$$

§ 1: Invariant Tests and the LRT for $p_2 = 1$.

In reduced form, the testing problem under consideration is the following. Given independent S and V ,

$$(1.1) \quad \begin{cases} S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \sim W(\Sigma, p, n) \\ V \sim W(\Sigma_{22}, p, n) \end{cases}$$

we want to test $H_0: \Sigma_{12} = 0$ versus $H_1: \Sigma_{12} \neq 0$. Here, S_{11} is $p_1 \times p_1$, S_{22} is $p_2 \times p_2$ and $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ is partitioned in the same way as S .

Let G be the group of transformations defined by

$$(1.2) \quad G = \left\{ A \left| \begin{array}{l} A = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix} : p \times p, \quad A_{11} : p_1 \times p_1 \\ A_{22} : p_2 \times p_2, \quad \det A_{ii} \neq 0, \quad i = 1, 2 \end{array} \right. \right\},$$

with the group action given by

$$(1.3) \quad \begin{cases} S \rightarrow ASA' \\ V \rightarrow A_{22} VA'_{22} \\ \Sigma \rightarrow A \Sigma A' \\ \Sigma_{22} \rightarrow A_{22} \Sigma_{22} A'_{22} \end{cases} .$$

The testing problem is clearly invariant under G .

It is convenient to transform the observations S and V to other variables. Let $S_{11.2} = S_{11} - S_{12} S_{22}^{-1} S_{21}$ and $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$. Recall that (Eaton (1972))

Умножив (1.5) на $(I - S)$

$$\text{получим: } (I - S) \cdot (I + S) = (I - S) \cdot (I + S) \cdot S^{-1} \cdot S \quad \text{или} \quad (I - S) = (I - S) \cdot (I + S) \cdot S^{-1} \cdot S$$

Из последнего равенства вытекают следующие соотношения для матрицы A и вектора e :

где e — вектор единичности, т.е. $e = (1, 1, \dots, 1)$.

$$S \cdot S^{-1} = I$$

$$(1.6) \quad \begin{cases} A \rightarrow I \\ A \rightarrow S \cdot S^{-1} \end{cases}$$

$$(1.7) \quad e \rightarrow S \cdot e$$

Таким образом, матрица A имеет вид:

$$(1.8) \quad e = \begin{pmatrix} S \cdot e \\ \vdots \\ S \cdot e \end{pmatrix} \quad \text{где } S^{-1} = 0 \quad \text{или} \quad S^{-1} = I$$

Для e по формуле (1.8) получаем следующие соотношения:

$$(1.9) \quad \text{где } S \cdot S^{-1} = I \quad \text{или} \quad S^{-1} = 0 \quad \text{или} \quad S^{-1} = I$$

$$\text{или} \quad A = S \cdot (S^{-1} \cdot S)$$

$$(1.10) \quad e = \begin{pmatrix} S \cdot e \\ \vdots \\ S \cdot e \end{pmatrix} \quad \text{или} \quad (S \cdot e)$$

Таким образом, матрица A имеет вид:

Из последнего равенства вытекают следующие соотношения для матрицы A и вектора e :

$$1. \quad \text{где } S \cdot S^{-1} = I \quad \text{или} \quad S^{-1} = 0 \quad \text{или} \quad S^{-1} = I$$

Reducing the data first by sufficiency and then by translations (under which the testing problem is invariant), we have the statistics

$$(0.1) \quad \begin{cases} S = \sum_1^N (X_i - \bar{X})(X_i - \bar{X})' \\ V = \sum_1^M (Y_i - \bar{Y})(Y_i - \bar{Y})' \end{cases}$$

where S and V are independent, $S \sim W(\Sigma, p, n)$ and $V \sim W(\Sigma_{22}, p_2, m)$ with $n = N - 1$ and $m = M - 1$. " $S \sim W(\Sigma, p, n)$ " means that S has a p -dimensional Wishart distribution with n -degrees of freedom and expectation $n\Sigma$. Our problem is to test $H_0: \Sigma_{12} = 0$ versus $H_1: \Sigma_{12} \neq 0$. This problem has been considered previously by Lee and Geisser (1972) in the context of testing for "Rao simple structure" in the linear growth curve model. Lee and Geisser derived the likelihood ratio test and, surprisingly, the test is the same as if the additional data, V , was not present.

A slightly different testing problem which has the same structure as the above problem, after a reduction by sufficiency and translation, is the following. Consider a random sample X_1, \dots, X_N from a $N_p(\mu, \Sigma)$ distribution and partition Σ as above and let $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ with $\mu_1: p_1 \times 1$ and $\mu_2: p_2 \times 1$. Also assume that Y_1, \dots, Y_M is a random sample (independent of the X 's) from a $N_{p_2}(\mu_2, \Sigma_{22})$ distribution. Again it is desired to test $H_0: \Sigma_{12} = 0$ versus $H_1: \Sigma_{12} \neq 0$. Reducing the data by sufficiency we have $\bar{X} \sim N(\mu, \frac{1}{N}\Sigma)$,

$$S = \sum_1^n (X_i - \bar{X})(X_i - \bar{X})' \sim W(\Sigma, p, N-1),$$

$\bar{Y} \sim N(\mu_2, \frac{1}{m}\Sigma_{22})$ and $\tilde{V} = \sum_1^m (Y_i - \bar{Y})(Y_i - \bar{Y})' \sim W(\Sigma_{22}, p_2, m-1)$. The problem

which the least squares problem is invariant under the transformation

$$\begin{aligned} \text{and } \sum_{i=1}^n (\bar{y}_i - \bar{y})(\bar{x}_i - \bar{x}) &= S_{xy} \\ \sum_{i=1}^n (\bar{y}_i - \bar{y})(\bar{y}_i - \bar{y}) &= S_{yy} \end{aligned} \quad (1.1)$$

where \bar{x} and \bar{y} are the means of X and Y respectively, S_{xy} is the covariance of X and Y , and S_{yy} is the variance of Y .

Let β_0 and β_1 be the parameters of the regression line $\hat{y} = \beta_0 + \beta_1 x$. The least squares problem is to find β_0 and β_1 such that

$$\sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$$

is minimized. This is a standard least squares problem. The normal equations are

$$\begin{aligned} n\beta_0 + \beta_1 \sum_{i=1}^n x_i &= \sum_{i=1}^n y_i \\ \beta_0 \sum_{i=1}^n x_i + \beta_1 \sum_{i=1}^n x_i^2 &= \sum_{i=1}^n x_i y_i \end{aligned}$$

which can be written in matrix form as

$$\begin{pmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{pmatrix}$$

The matrix on the left is symmetric and positive definite, so it is invertible. The solution is

$$\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{pmatrix}$$

Using the formulas for the inverse of a 2x2 matrix, we can write the solution as

$$\beta_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

and

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101 PROCESSES OF THE INDUSTRIAL AND COMMERCIAL SECTORS
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110 PROCESSES OF THE INDUSTRIAL AND COMMERCIAL SECTORS
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Thus, for $\text{tr } \beta\beta' < \frac{1}{4p}$,

$$\begin{aligned}
(2.27) \quad |IV| &\leq |V| + |VI| + |VII| \leq [2p \text{tr } \beta\beta']^2 \exp\left[\frac{1}{4} \text{tr } h_2 h_2'\right] \\
&\quad + [4p \text{tr } \beta\beta']^2 \exp\left[\frac{1}{4} \text{tr } h_1 h_1' + \frac{1}{4} \text{tr } h_2 h_2'\right] \\
&\quad + [4p \text{tr } \beta\beta']^2 \exp\left[\frac{1}{4} \text{tr } h_1 h_1' + \frac{3}{8} \text{tr } h_2 h_2'\right] \\
&\equiv [\text{tr } \beta\beta']^2 K(h_1, h_2) .
\end{aligned}$$

Integrating (2.21) and (2.27) with respect to $\mu_2^{(1)}(dk_1)$ yields

$$(2.28) \quad \begin{cases} \int_{II} \mu_2^{(1)}(dk_1) = \frac{1}{2p_2} \text{tr } \bar{g}_2 u' u \bar{g}_2' \text{tr } h_2 h_2' \beta\beta' \\ \int_{III} \mu_2^{(1)}(dk_1) = \frac{1}{2p_1 p_2} \text{tr } \bar{g}_1 x' \bar{g}_2' x \bar{g}_1' \text{tr } h_1' \beta' h_2 h_2' \beta h_1 \\ \int |IV| \mu_2^{(1)}(dk_1) \leq [\text{tr } \beta\beta']^2 K(h_1, h_2) \text{ for } \text{tr } \beta\beta' < \frac{1}{4p} . \end{cases}$$

Now, integrating the three members of (2.28) with respect to

$\prod_{i=1}^2 H_i(h_i) \mu_1^{(i)}(dh_i)$, and thereby completing the integration in (2.17),

we have, for $\text{tr } \beta\beta'$ small,

$$\begin{aligned}
(2.29) \quad R &= 1 + \frac{n(n+m)}{2p_1 p_2} \text{tr } \bar{g}_1 x' \bar{g}_2' x \bar{g}_1' \text{tr } \beta\beta' \\
&\quad - \frac{(n+m)}{2p_2} \text{tr } \bar{g}_2 u' u \bar{g}_2' \text{tr } \beta\beta' + o(\text{tr } \beta\beta')
\end{aligned}$$

where the remainder term is uniform in (x, y, u, v) .

Since $\phi \in \mathfrak{D}_\alpha$ is invariant, it is a function of any maximal invariant.

Thus, for $\sum_1^t \delta_i = \text{tr } \beta\beta'$ small, we have

where for $\frac{1}{\epsilon} \delta^2 = \epsilon \delta \delta$, where δ is small

where δ is small, we have $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small.

where δ is small, we have $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small.

$$= \frac{\delta \delta \delta}{(\delta \delta \delta)} \approx \frac{\delta \delta \delta}{\delta \delta \delta} \approx \delta \delta \delta + o(\delta \delta \delta)$$

$$(S \cdot S) \cdot E = I + \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \delta \delta \delta \delta$$

where for $\frac{1}{\epsilon} \delta^2 = \epsilon \delta \delta$, where δ is small

where δ is small, we have $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small.

$$|II| \approx \frac{\delta \delta \delta}{\delta \delta \delta} \approx \frac{\delta \delta \delta}{\delta \delta \delta} \approx \delta \delta \delta \approx \frac{\delta \delta \delta}{\delta \delta \delta}$$

$$(S \cdot S) \cdot III \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \delta \delta \delta \delta$$

$$|II| \approx \frac{\delta \delta \delta}{\delta \delta \delta} \approx \frac{\delta \delta \delta}{\delta \delta \delta} \approx \delta \delta \delta$$

where for $\frac{1}{\epsilon} \delta^2 = \epsilon \delta \delta$, where δ is small, we have $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small, and $\delta \delta \delta$ is small.

$$= \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \delta \delta \delta \delta$$

$$+ \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \delta \delta \delta \delta$$

$$+ \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \delta \delta \delta \delta$$

$$(S \cdot S) \cdot |II| \approx |II| + |II| + |II| \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta} \approx \frac{\delta \delta \delta \delta}{\delta \delta \delta \delta}$$

where for $\frac{1}{\epsilon} \delta^2 = \epsilon \delta \delta$, where δ is small

$$(2.18) \quad A \equiv \int_{\mathcal{O}(p_2)} \exp[E-F] \mu_2^{(2)}(dk_2) = \int_{\mathcal{O}(p_2)} \sum_{j=0}^{\infty} \frac{(E-F)^j}{j!} \mu_2^{(2)}(dk_2) .$$

Since the terms in odd powers of E vanish, we have

$$(2.19) \quad A = \int_{\mathcal{O}(p_2)} [1 - F + \frac{1}{2}E^2 + \frac{1}{2}F^2 + \sum_{j=3}^{\infty} \frac{(E-F)^j}{j!}] \mu_2^{(2)}(dk_2)$$

$$= 1 - \int_{\mathcal{O}(p_2)} F \mu_2^{(2)}(dk_2) + \frac{1}{2} \int_{\mathcal{O}(p_2)} E^2 \mu_2^{(2)}(dk_2)$$

$$+ \int_{\mathcal{O}(p_2)} [\frac{1}{2} F^2 + \sum_{j=3}^{\infty} \frac{(E-F)^j}{j!}] \mu_2^{(2)}(dk_2)$$

$$\equiv 1 - II + III + IV .$$

For any $q \times q$ matrices B_1 and B_2 , recall that

$$(2.20) \quad \begin{cases} \int_{\mathcal{O}(q)} \text{tr } B_1 \Gamma B_2 \Gamma' \lambda(d\Gamma) = \frac{1}{q} \text{tr } B_1 \text{tr } B_2 \\ \int_{\mathcal{O}(q)} (\text{tr } B_1 \Gamma)^2 \lambda(d\Gamma) = \frac{1}{q} \text{tr } B_1 B_1' \end{cases}$$

where λ is invariant probability measure on $\mathcal{O}(q)$ (see James (1961, 1964)).

Using (2.20), we have

$$(2.21) \quad \begin{cases} II = \frac{1}{2p_2} \text{tr } \bar{g}_2 u' u \bar{g}_2' \text{tr } h_2 h_2' \beta \beta' \\ III = \frac{1}{2p_2} \text{tr } \bar{g}_1 x' \bar{g}_2' \bar{g}_2 x \bar{g}_1' k_1' h_1' \beta' h_2 h_2' \beta h_1 k_1 . \end{cases}$$

Further, a bit of manipulation shows that

$$(S^{(b)}(s))^{-1} \frac{(s-1)}{s} = (S^{(b)}(s))^{-1} [s-1] \text{ and } (S^{(b)}(s))^{-1} = A \quad (21.2)$$

and so, taking the inverse Laplace transform we have

$$(S^{(b)}(s))^{-1} \left[\frac{(s-1)}{s} + \frac{s}{s^2} + \frac{s}{s^3} + \dots \right] = A \quad (21.3)$$

$$(S^{(b)}(s))^{-1} \frac{s}{s^2} + (S^{(b)}(s))^{-1} \frac{s}{s^3} + \dots =$$

$$(S^{(b)}(s))^{-1} \left[\frac{(s-1)}{s} + \frac{s}{s^2} + \dots \right] +$$

$$= I + III + IV + V$$

For each of the terms \$I, II, III, IV, V\$ we have

$$\frac{1}{s} = (1/s) \text{ and } (1/s) = \int_0^\infty e^{-st} dt \quad (22.2)$$

$$\frac{1}{s^2} = (1/s^2) \text{ and } (1/s^2) = \int_0^\infty t e^{-st} dt$$

where \$A\$ is a constant depending on \$s\$ and \$t\$ (see Table 19.4).

Using (22.2) we have

$$I = \int_0^\infty e^{-st} dt = \frac{1}{s} \quad (23.2)$$

$$II = \int_0^\infty t e^{-st} dt = \frac{1}{s^2}$$

where \$A\$ is a constant depending on \$s\$ and \$t\$.

$$(2.10) \quad R = \frac{\int_{\tilde{G}} f(g(x,y,u,v)|\beta)|J|^{-1}v(dg)}{\int_{\tilde{G}} f(g(x,y,u,v)|0)|J|^{-1}v(dg)}$$

where v is a left invariant measure on \tilde{G} and $|J|$ is the Jacobian of the transformation defined by (2.8). We note here that the reason for transforming to the new variables (x,y,u,v) is so that the conditions necessary to use the representation (2.10) could be easily verified.

The remainder of the proof is primarily concerned with the evaluation of (2.10) for $\sum \delta_i$ small. For $g = (g_1, g_2, g_3, g_4, g_5) \in \tilde{G}$ as in (2.8), the Jacobian of the transformation (2.8) is

$$(2.11) \quad |J| = |g'_1 g_1|^{\frac{n}{2}} |g'_2 g_2|^{\frac{p_1+n+m}{2}}.$$

Now, select $\bar{g}_1 \in G_T^+(p_1)$ such that $\bar{g}_1(x'(u'u)^{-1}_x + y'y)\bar{g}'_1 = I_{p_1}$ and select $\bar{g}_2 \in G_T^+(p_2)$ such that $\bar{g}_2(u'u + v'v)\bar{g}'_2 = I_{p_2}$. Substituting $(g_1 \bar{g}_1, g_2 \bar{g}_2)$ for (g_1, g_2) without changing the value of R , the numerator in (2.10) becomes, after some cancellation,

$$(2.12) \quad N = \int_{G\ell(p_1)} \int_{G\ell(p_2)} |J_1| |J_2| \exp[-\frac{1}{2}\text{tr } g'_1 g_1 - \frac{1}{2}\text{tr } g'_2 g_2] \\ \exp[\text{tr } \beta' g_2 \bar{g}_2 x \bar{g}'_1 g'_1 - \frac{1}{2}\text{tr } \beta' g_2 \bar{g}_2 (u'u) \bar{g}'_2 g'_2 \beta] v_1(dg_1) v_2(dg_2),$$

where $|J_1| = |g'_1 g_1|^{\frac{n(1)}{2}}$, $|J_2| = |g'_2 g_2|^{\frac{n(2)}{2}}$, v_i is left invariant measure on $G\ell(p_i)$, $i = 1, 2$, and $n(1) = n$, $n(2) = m+n$. For $g_i \in G\ell(p_i)$,

where k is chosen to make ϕ^* level α . Then ϕ^* is the unique locally best invariant test of H_0 . Further, ϕ^* is admissible in the class \mathcal{D}_α .

Proof: The assertions follow from the generalized Neyman-Pearson Lemma given in Lehmann (1959).

Since the test ϕ^* is locally most powerful in \mathcal{D}_α , it is natural to ask about the local minimaxity of ϕ^* within the class of all level α tests. (See Giri-Kiefer (1964) for the definition and sufficient conditions under which a given test is locally minimax.) When $p_1 = p_2 = 1$, it is clear that ϕ^* is locally minimax. However, we have been unable to show that ϕ^* is locally minimax when either p_1 or p_2 is bigger than 1.

Proof of Theorem 2.1:

Throughout this proof, $G_T(r)$ will denote the group of $r \times r$ lower triangular matrices with positive diagonal elements, $\mathcal{O}(r)$ will denote the group of $r \times r$ orthogonal matrices, and $Gl(r)$ will denote the group of $r \times r$ non-singular matrices. Recall that if $\eta \sim W(\Sigma, r, k)$ then $\eta \sim \xi' \xi$ where ξ is $k \times r$ and $\xi \sim N(0, I_k \times \Sigma)$. Using this decomposition of the Wishart distribution, let $S_{11.2} = y'y$, $S_{21} = x$, $S_{22} = u'u$ and $V = v'v$ where the joint distribution of (x, y, u, v) is given by

$$(2.7) \quad \begin{cases} x|u \sim N(u'u\beta, (u'u) \times I_{p_1}) \\ y \sim N(0, I_{n_1} \times I_{p_1}) \\ u \sim N(0, I_n \times I_{p_2}) \\ v \sim N(0, I_m \times I_{p_2}) \end{cases}$$

(S.1)

$$\begin{aligned}
 A &= \sum_{\alpha} (O I^{\alpha} \times I^{\alpha \sigma}) \\
 B &= \sum_{\alpha} (O I^{\alpha} \times I^{\alpha \sigma}) \\
 C &= \sum_{\alpha} (O I^{\alpha} \times I^{\alpha}) \\
 D &= \sum_{\alpha} (O I^{\alpha} \times I^{\alpha \sigma})
 \end{aligned}$$

is given by

$S^{\sigma} = \sum_{\alpha} A_{\alpha} A_{\alpha}$ where the elements A_{α} are given by (S.1) and the decomposition of the algebra S^{σ} is given by $S^{\sigma} = A + B + C + D$. The elements A_{α} are given by $A_{\alpha} = \sum_{\beta} (O I^{\beta} \times I^{\beta \sigma})$. The elements B_{α} are given by $B_{\alpha} = \sum_{\beta} (O I^{\beta} \times I^{\beta \sigma})$. The elements C_{α} are given by $C_{\alpha} = \sum_{\beta} (O I^{\beta} \times I^{\beta})$. The elements D_{α} are given by $D_{\alpha} = \sum_{\beta} (O I^{\beta} \times I^{\beta \sigma})$.

Proof of Theorem S.1:

Let ϕ be a linear map from A to S^{σ} . We show that ϕ is injective. Suppose $\phi(x) = 0$ for some $x \in A$. Then $\phi(x) = \sum_{\alpha} (O I^{\alpha} \times I^{\alpha \sigma}) = 0$. This implies $\sum_{\alpha} O I^{\alpha} \times I^{\alpha \sigma} = 0$. Since $O I^{\alpha} \times I^{\alpha \sigma} = 0$ implies $O I^{\alpha} = 0$, we have $x = 0$. This shows that ϕ is injective. (See also (S.1) for the definition of ϕ and the definition of S^{σ} .)

Proof: The case where ϕ is not injective follows immediately from the above.

Let ϕ be a linear map from A to S^{σ} . We show that ϕ is surjective. Let $y \in S^{\sigma}$. Then $y = \sum_{\alpha} (O I^{\alpha} \times I^{\alpha \sigma})$. We show that $y \in \text{Im } \phi$. Let $x = \sum_{\alpha} (O I^{\alpha} \times I^{\alpha \sigma}) \in A$. Then $\phi(x) = y$. This shows that ϕ is surjective. (See also (S.1) for the definition of ϕ and the definition of S^{σ} .)

§ 2: A Locally Most Powerful Invariant Test.

In the notation of the previous section, the data for the testing problem under consideration is $(S_{11.2}, S_{21}, S_{22}, V)$ where

$$(2.1) \quad \begin{cases} S_{11.2} \sim W(\Sigma_{11.2}, p_1, n_1); & n_1 = n - p_2 \\ S_{21} | S_{22} \sim N(S_{22} \Sigma_{22}^{-1} \Sigma_{21}, S_{22} \times \Sigma_{11.2}) \\ S_{22} \sim W(\Sigma_{22}, p_2, n) \\ V \sim W(\Sigma_{22}, p_2, m) \end{cases}$$

and $S_{11.2}$, (S_{21}, S_{22}) and V are mutually independent. The problem is to test $H_0: \Sigma_{21} = 0$ versus $H_1: \Sigma_{21} \neq 0$. This problem is invariant under the group G defined by (1.2) with the group action being defined by (1.3) (or in terms of the new variables by (1.5)). Since p_1 and p_2 are now arbitrary, an analytically tractable maximal invariant seems difficult to obtain. A maximal invariant parameter is the set of ordered characteristic roots of $\Sigma_{22}^{-1} \Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12}$ -- say, δ_i , $i = 1, \dots, t$ where $t = \min\{p_1, p_2\}$, with $\delta_1 \geq \delta_2 \geq \dots \geq \delta_t \geq 0$.

Let \mathfrak{D}_α be the set of all G -invariant level α test functions for H_0 versus H_1 . Only test functions in \mathfrak{D}_α will be considered in this section. If $\phi \in \mathfrak{D}_\alpha$, it is well known (Lehmann (1959)) that the power function of ϕ will be a function of a maximal invariant parameter. Let $\beta: p_2 \times p_1$ be such that $\beta_{ii} = \sqrt{\delta_i}$ for $i = 1, \dots, t$ with the remaining elements of β being zero. Since our concern is the computation of power functions for invariant tests $\phi \in \mathfrak{D}_\alpha$, we can, without loss of generality, assume that the data is

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$$\begin{aligned}
 & \dots \\
 & \dots \\
 & \dots \\
 & \dots \\
 & \dots
 \end{aligned}$$

... of the ... system ...

S: ...

Theorem 1: Let $\phi^*(U)$ be the test defined by

$$(1.17) \quad \phi^*(U) = \begin{cases} 1 & \text{if } U > c \\ 0 & \text{if } U \leq c \end{cases}$$

where c is chosen to make ϕ^* level α . Then ϕ^* is uniformly most powerful in the class $C_\alpha(W)$.

Proof: Let $\varphi(U,W) \in C_\alpha(W)$. Then $\mathcal{E}_{\delta=0}(\varphi(U,W)|W) = \alpha$ a.e. (W).

Since the conditional density of U given W has a MLR, we have that

$$(1.18) \quad \mathcal{E}_\delta(\phi^*(U)|W) \geq \mathcal{E}_\delta(\varphi(U,W)|W) .$$

Integrating both sides of (1.18) with respect to the distribution of W yields the result.

A simple calculation shows that $U = R^2/(1-R^2)$ where $R^2 = S_{21}S_{11}^{-1}S_{12}/S_{22}$ so the test ϕ^* is equivalent to the likelihood ratio test. Since W is an ancillary statistic, one might argue, using the Principle of Conditionality, that after a reduction by invariance, testing should be done conditionally on W .

In the next section, we show that the test ϕ^* can be dominated locally (in terms of power). More specifically, a locally most powerful invariant test is constructed for general p_1 and p_2 . As will be seen, this test depends on both U and W and so is not equivalent to ϕ^* .

Let C_α be the class of all level α invariant tests. To show that a uniformly most powerful test in C_α does not exist we argue as follows. Fix $\delta_0 > 0$. For testing $H_0: \delta = 0$ versus $H_1: \delta = \delta_0$, the most powerful invariant test rejects for large values of

Let $\epsilon > 0$ be arbitrary. For $\epsilon > 0$ choose $\delta > 0$ such that $\epsilon > \delta$ and $\delta > 0$ such that $\epsilon > \delta$ and $\delta > 0$ such that $\epsilon > \delta$.

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$$\epsilon = \frac{\delta}{2} \quad \text{where } \delta = \frac{\epsilon}{2}$$

$$(T, T) = \epsilon^2 \phi_*(a) \approx \epsilon^2 \phi_*(a^2)$$

Let $\epsilon > 0$ be arbitrary. For $\epsilon > 0$ choose $\delta > 0$ such that $\epsilon > \delta$ and $\delta > 0$ such that $\epsilon > \delta$ and $\delta > 0$ such that $\epsilon > \delta$.

$$(T, T) = \begin{cases} 0 & \text{if } T \leq 0 \\ \epsilon & \text{if } T > 0 \end{cases}$$

Let $\epsilon > 0$ be arbitrary. For $\epsilon > 0$ choose $\delta > 0$ such that $\epsilon > \delta$ and $\delta > 0$ such that $\epsilon > \delta$ and $\delta > 0$ such that $\epsilon > \delta$.

where χ_{n-p+1}^2 and $\chi_{p-1}^2(S_{22}\delta)$ are independent and $\chi_{p-1}^2(S_{22}\delta)$ denotes a non-central chi-square random variable with non-centrality parameter $S_{22}\delta$.

The density of $\chi_k^2(\tau)$ is given by

$$(1.10) \quad f(x; \tau) = \sum_{j=0}^{\infty} \frac{e^{-\frac{\tau}{2}}}{j!} \left(\frac{\tau}{2}\right)^j h_{k+2j}(x)$$

where

$$(1.11) \quad h_{\alpha}(x) = \frac{x^{\frac{\alpha}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{\alpha}{2}} \Gamma(\alpha/2)}, \quad x > 0$$

is the density of a χ_{α}^2 random variable. Also, the density of $F_{\alpha, \beta} \equiv \frac{\chi_{\alpha}^2}{\chi_{\beta}^2}$, where χ_{α}^2 and χ_{β}^2 are independent, is

$$(1.12) \quad g_{\alpha, \beta}(z) = \frac{\Gamma(\frac{\alpha + \beta}{2})}{\Gamma(\alpha/2)\Gamma(\beta/2)} \frac{y^{\alpha/2-1}}{(1+y)^{\frac{\alpha + \beta}{2}}}, \quad y > 0.$$

From (1.10) and (1.12) it follows that the density of $\chi_{\alpha}^2(\tau)/\chi_{\beta}^2$ is

$$(1.13) \quad g(z; \tau) = \sum_{j=0}^{\infty} \frac{e^{-\frac{\tau}{2}}}{j!} \left(\frac{\tau}{2}\right)^j g_{\alpha+2j, \beta}(z); \quad z > 0.$$

Thus, the conditional density of U , given S_{22} , is

$$(1.14) \quad q(u; S_{22}\delta) = \sum_{j=0}^{\infty} \frac{e^{-\frac{S_{22}\delta}{2}}}{j!} \left(\frac{S_{22}\delta}{2}\right)^j g_{p-1+2j, n-p+1}(u); \quad u > 0.$$

Further, the conditional density of $W \equiv V/S_{22}$, given S_{22} is

$S_{22} h_m(S_{22} w)$ where h_m is given by (1.11). Since U and W are

(1.1) shows that the function $f(x)$ is positive in the interval $(0, 1)$ and that $f(0) = 0$ and $f(1) = 0$.

$$(1.2) \quad f(x) = \frac{1}{\pi} \int_0^1 \frac{1-t}{1-tx} dt \quad (0 < x < 1)$$

From the definition of $f(x)$ it follows that

$$(1.3) \quad f(x) = \frac{1}{\pi} \int_0^1 \frac{1-t}{1-tx} dt \quad (0 < x < 1)$$

From (1.2) and (1.3) it follows that the function $f(x)$ is

$$(1.4) \quad f(x) = \frac{1}{\pi} \frac{(1-x) \ln(1-x)}{-x} \quad (0 < x < 1)$$

It is easy to see that the function $f(x)$ is positive in the interval $(0, 1)$ and that $f(0) = 0$ and $f(1) = 0$.

$$(1.5) \quad f(x) = \frac{1}{\pi} \frac{(1-x) \ln(1-x)}{-x} \quad (0 < x < 1)$$

$$(1.6) \quad f(x) = \frac{1}{\pi} \int_0^1 \frac{1-t}{1-tx} dt \quad (0 < x < 1)$$

It is easy to see that the function $f(x)$ is

It is easy to see that the function $f(x)$ is positive in the interval $(0, 1)$ and that $f(0) = 0$ and $f(1) = 0$.

From (1.2) and (1.3) it follows that the function $f(x)$ is

$$(1.4) \quad \begin{cases} S_{11.2} \sim W(\Sigma_{11.2}, p_1, n-p_2) \\ S_{21} | S_{22} \sim N(S_{22} \Sigma_{22}^{-1} \Sigma_{21}, S_{22} \times \Sigma_{11.2}) \\ S_{22} \sim W(\Sigma_{22}, p_2, n) . \end{cases}$$

Also, $S_{11.2}$ is independent of the pair (S_{21}, S_{22}) . The second line in (1.4) means that the conditional distribution of S_{21} given S_{22} is multivariate normal with mean matrix $S_{22} \Sigma_{22}^{-1} \Sigma_{21}$ and covariance operator $S_{22} \times \Sigma_{11.2}$ - the Kronecker product of S_{22} and $\Sigma_{11.2}$.

Let $B_{11} = \Sigma_{11.2}$, $B_{21} = \Sigma_{22}^{-1} \Sigma_{21}$ and $B_{22} = \Sigma_{22}$, so our testing problem is now $H_0: B_{21} = 0$ versus $H_1: B_{21} \neq 0$. The group action on the new variables induced by (1.3) is

$$(1.5) \quad \begin{aligned} S_{11.2} &\rightarrow A_{11} S_{11.2} A_{11}' \\ S_{21} &\rightarrow A_{22} S_{21} A_{11}' \\ S_{22} &\rightarrow A_{22} S_{22} A_{22}' \\ V &\rightarrow A_{22} V A_{22}' \\ B_{11} &\rightarrow A_{11} B_{11} A_{11}' \\ B_{21} &\rightarrow (A_{22}')^{-1} B_{21} A_{11}' \\ B_{22} &\rightarrow A_{22} B_{22} A_{22}' . \end{aligned}$$

With the observations given by (1.4) plus V , it is easy to calculate the maximum likelihood estimators for the parameters B_{11} , B_{21} and B_{22} under H_0 and H_1 , and thus compute the likelihood ratio test. The likelihood ratio test rejects H_0 for small values of the statistic

$$(1.6) \quad |I - S_{11}^{-1} S_{12} S_{22}^{-1} S_{21}| .$$

(11) \dots

... and to ... of the ...
... the ...
... the ...
... of the ...

$$SS \rightarrow SS, SS$$

$$SI \rightarrow (SS) SI$$

$$SI \rightarrow SI, SI$$

(12) \dots

$$SS \rightarrow SS, SS$$

$$SI \rightarrow SS, SI$$

$$SI \rightarrow SI, SI$$

... (13) ...
... $SI = 0$... $SI = 0$...
... $SI = SI$... $SI = SS$...
... SI ... SI ...
... SI ... SI ...
... (14) ... SI ... SI ...
... (15) ... SI ... SI ...

$$SI \rightarrow (SS, SI)$$

(16) \dots

$$SI \rightarrow (SI, SI)$$

is invariant under $\bar{X} \rightarrow \bar{X} + a$, $S \rightarrow S$, $\bar{Y} \rightarrow \bar{Y} + a_2$, $\tilde{V} \rightarrow \tilde{V}$ where $a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $a_1 = p_1 \times 1$, $a_2 = p_2 \times 1$. A maximal invariant under this group of translations is $(S, V, \bar{Y} - \bar{X}_{(2)})$ where $\bar{X} = \begin{pmatrix} \bar{X}_{(1)} \\ \bar{X}_{(2)} \end{pmatrix}$ with $\bar{X}_{(1)}: p_1 \times 1$ and $\bar{X}_{(2)}: p_2 \times 1$. But

$$\bar{Y} - \bar{X}_{(2)} \sim N_{p_2} \left(0, \left(\frac{1}{N} + \frac{1}{m} \right) \Sigma_{22} \right) \text{ so}$$

$$Z = \left(\frac{1}{N} + \frac{1}{m} \right)^{-\frac{1}{2}} (\bar{Y} - \bar{X}_{(2)}) \sim N(0, \Sigma_{22}) .$$

Reducing the data $(S, V, \bar{Y} - \bar{X}_{(2)})$ by sufficiency, we have the sufficient statistic $(S, \tilde{V} + ZZ')$ $\equiv (S, V)$. Here $S \sim W(\Sigma, p, N-1)$ and is independent of $V \sim W(\Sigma_{22}, p_2, m+1)$. In this reduced form, the problem for testing $H_0: \Sigma_{12} = 0$ versus $H_1: \Sigma_{12} \neq 0$ is identical to the problem when the extra observations have a mean which is unrelated to the mean of the original sample.

In Section 1, we show that for $p_2 = 1$ the likelihood ratio test is the uniformly most powerful invariant conditional level- α test. The conditioning is with respect to a natural ancillary statistic. In addition, it is argued that a uniformly most powerful invariant (under the appropriate group of linear transformations leaving the problem invariant) test does not exist.

A locally most powerful invariant test for H_0 versus H_1 is derived in Section 2. This test is seen to depend on an ancillary statistic and the test is different from the likelihood ratio test. A representation theorem for the probability ratio of the density of a maximal invariant due to Wijsman (1967) is used to derive the locally most powerful invariant test.

1948.

que se obtiene (0,0) se debe a que las dos rectas, una horizontal y otra vertical, se cortan en el origen. La ecuación de la recta horizontal es $y = 0$ y la ecuación de la recta vertical es $x = 0$. El sistema de ecuaciones es:

$$\begin{cases} y = 0 \\ x = 0 \end{cases}$$

El sistema tiene una única solución: $(0, 0)$. Esto se debe a que las rectas no son paralelas y se intersectan en un solo punto.

El sistema de ecuaciones lineales $ax + by = c$ tiene una única solución si el determinante de la matriz de coeficientes no es cero. En este caso, el determinante es $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. Si $ad - bc \neq 0$, el sistema tiene una única solución.

El sistema de ecuaciones lineales $ax + by = c$ tiene infinitas soluciones si el determinante de la matriz de coeficientes es cero y el determinante de la matriz ampliada es cero. En este caso, las rectas son coincidentes.

El sistema de ecuaciones lineales $ax + by = c$ tiene ninguna solución si el determinante de la matriz de coeficientes es cero y el determinante de la matriz ampliada no es cero. En este caso, las rectas son paralelas y no se intersectan.

$$x = \frac{\begin{vmatrix} c & b \\ d & a \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{ca - bd}{ad - bc}$$

$$y = \frac{\begin{vmatrix} a & c \\ c & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{ad - bc}{ad - bc} = 1$$

El sistema de ecuaciones lineales $ax + by = c$ tiene una única solución si el determinante de la matriz de coeficientes no es cero. En este caso, el determinante es $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. Si $ad - bc \neq 0$, el sistema tiene una única solución.

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§ 0: Introduction and Notation.

Let X_1, \dots, X_N be a random sample from a p -dimensional multivariate normal distribution $N_p(\mu, \Sigma)$ and partition Σ as

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}; \quad \Sigma_{11}: p_1 \times p_1, \quad \Sigma_{22}: p_2 \times p_2$$

with $p = p_1 + p_2$. For testing $H_0: \Sigma_{12} = 0$ versus $H_1: \Sigma_{12} \neq 0$, when $p_2 = 1$, it is well known that the best invariant test (which is the likelihood ratio test) rejects H_0 for large values of the sample multiple correlation coefficient between the first $p - 1$ coordinates and the last coordinate. For general p_1 and p_2 , a variety of invariant tests have been proposed -- all of which are functions of the sample eigenvalues of $S_{11}^{-1} S_{12} S_{22}^{-1} S_{21}$ where S is the sample covariance matrix based on X_1, \dots, X_n and

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \quad S_{11}: p_1 \times p_1, \quad S_{22}: p_2 \times p_2 .$$

In particular, Schwartz (1967) has shown that the test which rejects H_0 for large values of $\text{tr} S_{11}^{-1} S_{12} S_{22}^{-1} S_{21}$ (the Pillai trace test) is locally most powerful invariant and locally minimax in some special cases.

The purpose of the present paper is to consider the above testing problem when one has additional observations on the last p_2 coordinates. In addition to X_1, \dots, X_N assume that Y_1, \dots, Y_M is a random sample (independent of the X 's) from an p_2 -dimensional multivariate normal distribution - $N_{p_2}(\tau, \Sigma_{22})$. Presumably, the Y 's can be used to help estimate Σ_{22} and thus be used to construct a more powerful test of H_0 .

\mathbb{S}^S and also be used to construct a new boundary set of \mathbb{H}^0 .
 Construction - \mathbb{H}^1 (\mathbb{S}^S). In addition, the \mathbb{H}^1 set is used to form
 (in addition to the \mathbb{H}^0) and the \mathbb{S}^S -determined boundary set
 in equation $\mathbb{H}^1 \dots \mathbb{H}^N$ where $\mathbb{H}^1 \dots \mathbb{H}^N$ is a unique set of
 boundary set for the construction of the new \mathbb{S}^S construction.

A number of the above set to be covered the same set of
 boundary set and the result is that in some cases, cases.

For each of the \mathbb{S}^S , \mathbb{S}^S , \mathbb{S}^S , \mathbb{S}^S (\mathbb{S}^S is the same as \mathbb{H}^0) to \mathbb{H}^1 set
 is constructed (100) as shown in the case of \mathbb{H}^0

$$\mathbb{H}^1 = \begin{pmatrix} \mathbb{S}^S & \mathbb{S}^S \\ \mathbb{S}^S & \mathbb{S}^S \end{pmatrix} \quad \mathbb{H}^2 = \begin{pmatrix} \mathbb{S}^S & \mathbb{S}^S \\ \mathbb{S}^S & \mathbb{S}^S \end{pmatrix}$$

The set of boundary set is $\mathbb{H}^1 \dots \mathbb{H}^N$ and
 construction of the set of boundary set of $\mathbb{H}^1 \dots \mathbb{H}^N$ is the
 same as the case of \mathbb{H}^0 -- set of \mathbb{H}^1 and
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 set of boundary set is $\mathbb{H}^1 \dots \mathbb{H}^N$ and \mathbb{S}^S
 is the same as the case of \mathbb{H}^0 . For \mathbb{H}^1 and \mathbb{S}^S
 set of \mathbb{H}^1 is the same as the case of \mathbb{H}^0 (where
 and $\mathbb{H}^1 = \mathbb{H}^0 + \mathbb{S}^S$. For \mathbb{H}^1 and \mathbb{S}^S is $\mathbb{H}^1 = \mathbb{H}^0$ and $\mathbb{S}^S = \mathbb{H}^0$

$$\mathbb{H}^1 = \begin{pmatrix} \mathbb{S}^S & \mathbb{S}^S \\ \mathbb{S}^S & \mathbb{S}^S \end{pmatrix} \quad \mathbb{H}^2 = \begin{pmatrix} \mathbb{S}^S & \mathbb{S}^S \\ \mathbb{S}^S & \mathbb{S}^S \end{pmatrix}$$

and the set of boundary set \mathbb{H}^1 and \mathbb{S}^S is
 the same as the case of \mathbb{H}^0 and \mathbb{S}^S is the same as the case of \mathbb{H}^0

0: In addition to the above.

ABSTRACT

Suppose $S : p \times p$ has a Wishart distribution with n degrees of freedom and expectation $n \Sigma$, $n \geq p$. Write

$$S = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \text{ where}$$

$\Sigma_{11} : p_1 \times p_1$ and $\Sigma_{22} : p_2 \times p_2$, $p_1 + p_2 = p$. Also suppose $V : p_2 \times p_2$ has a Wishart distribution with m -degrees of freedom and expectation $m \Sigma_{22}$. For testing $H_0 : \Sigma_{12} = 0$ versus $H_1 : \Sigma_{12} \neq 0$, we derive a locally most powerful invariant test and show that this test is different from the likelihood ratio test.

1. The first part of the text

Let \mathcal{H}^0 and \mathcal{H}^1 be Hilbert spaces. Let $\mathcal{H} = \mathcal{H}^0 \oplus \mathcal{H}^1$ be the direct sum.

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$$\mathcal{H} = \begin{pmatrix} \mathcal{H}^0 & \mathcal{H}^1 \\ \mathcal{H}^1 & \mathcal{H}^0 \end{pmatrix} \text{ and } \mathcal{H} = \begin{pmatrix} \mathcal{H}^0 & \mathcal{H}^1 \\ \mathcal{H}^1 & \mathcal{H}^0 \end{pmatrix} \text{ where}$$

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