Essays on International Private Debt

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

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May, 2021

Acknowledgements

I am grateful to Manuel Amador and Timothy Kehoe, my advisors, and to Javier Bianchi and Marco Bassetto for their invaluable advice and guidance during my graduate study.

My work has greatly benefited from many discussions and comments from Julien Bengui, Anmol Bhandari, V.V. Chari, Dean Corbae, Charles Engel, Terry Fitzgerald, Stelios Fourakis, Benjamin Malin, and Jan Werner. My special thanks go to Monica Tran-Xuan for her invaluable patience and support throughout all stages of my work and, our stimulating conversations helped me develop new ideas and taught me how to communicate them effectively.

I am thankful to the faculty at the Department of Economics at the University of Minnesota, the economists at the Economic Analysis Group, and the people of the Research Department at the Federal Reserve Bank of Minneapolis. I am also grateful to the Minnesota International Macro & Trade Workshop, and Minnesota-Wisconsin Workshop. Finally, I am thankful the University of Minnesota Doctoral Dissertation Fellowship for financial support.

Last but not least, I want to thank my family and friends for their constant support throughout this journey.

Dedication

To my parents for their love and advice, and to France for its intellectual and financial support.

Abstract

This dissertation consists of three chapters. A unifying theme across all chapters is the interaction between international private debt and government's policies.

The first chapter argues that excessive international private debt increases the frequency and severity of sovereign debt crises. I develop a quantitative theory of private and public debt that allows me to measure the level of private overborrowing and its effect on the interest rate spread on public debt. In an environment where private credit is constrained by the market value of income, individually optimal private borrowing decisions are inefficient at the aggregate level. High private debt increases the probability of a financial crisis, characterized by a large deleveraging in private debt and a contraction in consumption. During such crises, the drop in consumption is amplified trough the endogenous decline in the market value of collateral. To counter this reduction, the government responds with fiscal bailouts financed with risky external public debt. This response may cause a sovereign debt crisis, which is characterized by high interest rates spreads, and in some cases, default. I find that the theory is quantitatively consistent with the evolution of international private debt, international public debt, and sovereign spreads in Spain from 1999 to 2015. I estimate that excessive private debt raised the interest rate spread on public bonds by at least 3.8 percentage points at its peak in 2012.

The second chapter, in turn, proposes a theory of foreign reserves as macroprudential policy. This Chapter was written in collaboration with Julien Bengui and Javier Bianchi. We study an open economy model of financial crises, in which pecuniary externalities lead to overborrowing, and show that by accumulating international reserves, the government can achieve the constrained-efficient allocation. The optimal reserve accumulation policy leans against the wind and significantly reduces the exposure to financial crises. The theory can explain the empirical patterns of public and private international capital flows, both in the cross-section and over time.

The third chapter examines the interaction between international debt and inequality. This chapter was written in collaboration with Monica Tran-Xuan. We introduce household heterogeneity in a standard model of sudden stop crises. We find that although there is overborrowing at the aggregate level this result is not true for all types of households. While high income households overborrow, low income households underborrow. In future versions of this paper we would like to study the implications of this result for the redistributional consequences of macroprudential policies.

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Chapter 1

Private Overborrowing under Sovereign Risk

1.1 Introduction

A feature of the 2010-2015 European Debt Crisis is that governments that had previously pursued fiscally frugal policies saw significant increases in their borrowing costs. One of those countries was Spain. From the introduction of the euro in 1999 up to the global financial crisis in 2008, Spain was the largest economy in the Eurozone in uninterrupted compliance with the budgetary and public debt limits set by the Stability and Growth Pact.[1] During this same period, however, Spain accumulated a large stock of international private debt, primarily in its banking sector.[2] As the financial turmoil accelerated, the government responded with multiple rounds of bailouts to highly indebted financial institutions. These interventions led to an abrupt increase in public debt and its interest rate spreads. These events have raised questions about how private crises can lead to public debt crises and how a sovereign with defaultable debt should respond to systemic vulnerabilities in international private credit.[3] Assuming a sovereign with full commitment

[1]Morris, Ongena, and Schuknecht (2006) discuss the reform of the Pact in 2005 and distinguish Spain for its compliance. Schuknecht et al. (2011) describe the evolution of deficits and sovereign debt in the post-reform period and document Spanish compliance up to the 2008 recession.

[2]Lane (2013) and Chen, Milesi-Ferretti, and Tressel (2013) discuss the current account imbalances of periphery European countries. Hale and Obstfeld (2016) and Hobza and Zeugner (2014) analyze capital flows within the Eurozone and document the flow in the form of debt instruments from "core" countries toward financial institutions in the periphery. In't Veld et al. (2014) and Ratto and Roegera (2015) link the increase in capital flows to Spanish banks financing a boom in the construction sector.

[3]This is not the first time that private credit booms have been linked to subsequent sovereign debt crises. An earlier literature analyzing the 1997 currency crises in Thailand, Korea, and Indonesia stresses this link. Burnside, Eichenbaum, and Rebelo (2001) argue that implicit bailout guarantees lead to private credit booms and rise expectations of large fiscal deficits in the future. Schneider and Tornell (2004) show

could lead to policy prescriptions that are not sustainable. Conversely, assuming that bailouts must be financed only with funds raised within period would imply prescribing suboptimal policies that do not incorporate the gains from smoothing costs over time. Thus, to address this issue, a joint analysis of private debt and sovereign risk is necessary in order to provide adequate policy prescriptions.

This paper provides quantitative answers to the following three questions. First, was the Spanish private sector excessively indebted in the lead-up to the crisis and, if so, by how much? Second, by how much did private overborrowing increase the probability of a financial crisis? Third, what was the effect of excessive private debt on the severity of the sovereign debt crisis that followed?

I find that a model of systemic externalities in private credit regulated by a sovereign that can borrow internationally without commitment is quantitatively consistent with the evolution of private debt, public debt, and interest rate spreads in the Spanish data. The calibrated model matches the Spanish environment before the crisis – namely, low public debt, high private debt, and near-zero interest rate spreads. The calibrated model's untargeted business cycle statistics are also consistent with the data. I then verify that the calibrated model can simulate the crisis years, by computing the model dynamics using the productivity and external shocks taken from the Spanish data from 2008 to 2015. As in the data, the government in the model finds it optimal to provide large transfers to the private sector financed with external public debt. This response in turn leads to a sudden decrease in private debt and a rise in the public interest rate spread commensurate with the increase observed in Spain. As a result, the paper contributes by providing a theory that jointly rationalizes the period of near-zero spreads and the subsequent high values observed during the crisis years.

This paper also provides a quantitative estimate of the size of private overborrowing and its effects. I measure the excessive private debt stock from 1999 to 2011 to be 5% of GDP on average. I then define a financial crisis as a contraction of more than one standard deviation below the mean of the current account of the private sector. Under this definition, I estimate that private overborrowing increased the probability of a financial crisis in Spain from 0.1% to 2.5% annually. I also construct counterfactual dynamics of the model around the crisis years if private borrowing had been socially optimal. I show that even when taking public policies as given, the interest spread paid on public debt would have been 3.80 percentage points (p.p.) below the peak observed in 2012.

The paper also finds that private overborrowing increased the annual probability of

that systemic bailout guarantees cause both credit cycles and self-fulfilling crises.

observing a sovereign default from 0.03% to 0.46%. I calculate that the welfare gains of implementing optimal borrowing policies would have been equivalent to an increase of 0.41% in aggregate consumption. Finally, I prove that optimal private borrowing policies could have been implemented by pairing public debt management with macroprudential taxes on international private debt. During the 2008 to 2015 crisis years, I estimate an average tax rate of 5%.

To compute these answers, I combine a dynamic stochastic general equilibrium (DSGE) model of financial crisis caused by collateral debt constraints as developed by Mendoza (2002) and Bianchi (2011) with a sovereign debt structure in the tradition of Eaton and Gersovitz (1981) and Arellano (2008).[4] The model consists of a continuum of competitive identical households, international risk-neutral lenders, and a benevolent government. Households are endowed with tradable and nontradable goods and smooth their consumption by borrowing internationally up to a fraction of the market value of their current income. The government has access to lump-sum transfers and strategically defaultable international public debt. Each period, the government chooses to either default on its debt, tax households to pay back part of its debt, or alternatively increase its debt and transfer resources to the households (bailouts). Both private and public liabilities are priced by competitive, risk-neutral international lenders. I allow for exogenous income shocks to the endowments and financial shocks to the households' borrowing capacity and the default rate on private debt. I then contrast the competitive equilibrium allocations with those chosen by a benevolent social planner. I assume that the planner makes aggregate borrowing decisions on both assets and then transfers the proceeds to the households who make all consumption choices. As a result, the planner and the competitive households are subject to the same market prices, market clearing conditions and credit constraints. Nevertheless, the planner's choice of allocations may be different from that of the competitive equilibrium because it internalizes the general equilibrium effects of the aggregate choices it makes. One main characteristic is that, the planner chooses a lower level of private debt than the competitive equilibrium. These different choices allow the planner to achieve a higher level of welfare. I later show that the planner's allocations can be decentralized by extending the baseline framework to allow the government to impose state-dependent taxes on private borrowing.

There are two main mechanisms at play in the model, the first one is common in the literature and the second one is novel. The first is Irving Fisher's classic debt-deflation

[4] See also Kehoe and Levine (1993) for earlier implementations of collateral debt constraints in a general equilibrium context and Aguiar and Gopinath (2006) for early quantitative adaptations of the Eaton and Gersovitz (1981) model.

effect. This effect is present in Mendoza (2002) and more generally, in models with a collateral constraint that depends on market-determined prices. Consider the situation of a representative household that enters the period with a large amount of private debt and faces an adverse shock in the form of a productivity shock. Without government transfers, the household is unable to roll over its debt without violating the collateral constraint. Under these circumstances, the household reduces both its consumption and its debt. Since all households are assumed to be identical, the reduction in aggregate consumption of tradables induces a decline in the market price of the nontradable goods that reduces the value of collateral. Thus, the credit constraint tightens more, resulting in a greater contraction in consumption. The engine of this feedback loop is a general equilibrium price that competitive households take as given. As a result, their individually optimal borrowing decisions are frequently above the socially optimal level. This exposes them to more frequent and severe credit boom and bust cycles relative to a planner who incorporates the general equilibrium effects in its decision making. This financial amplification mechanism is described in Mendoza (2002) and Bianchi (2011) and is referred to as Fisherian deflation.

The second mechanism of this paper is how this financial amplification interacts with the government's borrowing and default decisions. A benevolent and strategic government will use international public debt to mitigate the costs of Fisherian deflation.

To fix ideas, it is useful to characterize the relation between private and public debt during expansionary states (when the credit constraint is not binding even in the absence of government interventions) and contractionary states (when government inaction implies a binding constraint and a contraction in consumption).In all cases, the government evaluates the benefits of providing a positive transfer to households financed with external public debt against the expected costs of a future with either higher taxes or the deadweight losses of a sovereign default.

In an expansionary environment, households will decrease private borrowing in response to a fiscal transfer financed with international public debt. Private and public debt are substitutes. The classic consumption-smoothing and Ricardian equivalence effects are at play here. Households equate the marginal benefit of an additional unit of consumption with the marginal cost of reducing consumption in the future as a result of higher taxes. Consequently, public borrowing is offset by declines in private borrowing. Fiscal transfers therefore leave aggregate consumption and total indebtedness roughly constant. The composition of the portfolio will be depend on the relative price of the two debt instruments and the trade-off between public debt and the probability of binding credit constraint in the future.[5] Since the probability of a binding credit constraint in the future increases with private debt, the government's incentive to provide these transfers increases with the level of private debt. As a result, these bailouts are commonly seen in the periods preceding a financial crisis and when default costs are high.[6] Since default costs are high and the increase in public debt is offset by a reduction in private debt the interest rate spreads paid on public debt are low in this environment. This mechanism helps the model fit the patterns observed in the data in the years immediately preceding the crisis.

In a contractionary environment, public and private debt behave instead like complements. In these cases, households are facing a binding constraint, and therefore their Euler equation does not hold. In particular, the marginal benefit of current consumption exceeds the marginal cost of lower future consumption. A positive fiscal transfer in this context translates into higher individual private consumption. Moreover, at the aggregate level, the increase in consumption raises the relative price of nontradables and with it the value of private collateral. The increasing valuation of collateral allows for a higher level of private debt, that in turn translates into an additional increase in consumption. However, the cost of the transfer is increasing overall indebtedness, since both private and public debt levels rise in tandem. More indebtedness makes defaulting on public debt more appealing. This increases the risk of default in the future and leads to an increase in the interest rate spread paid on public debt in the current period. Quantitatively, the consumption-smoothing benefits of these interventions are large enough to justify the cost of significant increases in spreads. This is the main channel that allows the model to replicate the patterns observed during the peak years of the crisis.

The benefits of restoring the socially efficient level of private debt in this context are twofold. First, by decreasing the level of private borrowing, the planner decreases the severity and frequency of a private financial crisis. Lower private debt implies fewer episodes with a binding credit constraint and contractions in private credit and output. Second, and because of the first benefit, a lower level of private borrowing reduces the need for government bailouts. Fewer bailouts translate into lower public debt and a lower probability of a sovereign default. The combination of these two factors implies lower interest rate spreads on public debt. I show that macroprudential policies, equated in this paper to taxes on private borrowing, allow the government to decentralize the socially efficient level of private borrowing.

[5] The price of government debt depends on expectations of future default costs. Default costs are usually assumed to be proportional to income in the sovereign default literature. Thus, the persistence of income is passed on to default costs.

[6] In practice, since default costs are assumed be increasing in output this can episodes can coincide with output booms.

The baseline version of the model is calibrated to Spanish data from 1999 to 2011, before the peak of the crisis. The calibration targets the mean and the volatility of the private and total debt as well as the interest rate spreads on public debt. I then use the calibrated parameters to solve the socially planned version of the model. I compare the behavior of this counterfactual socially planned economy and the baseline model at their respective ergodic distributions. Taking differences between them, I measure the level of excessive private debt, the welfare gains, and the change in the probabilities of experiencing a financial crisis.

Finally, I conduct two numerical exercises to evaluate the model's dynamics. Namely, I use the 2008-2015 Spanish data to simulate the crisis in the model. The first exercise is a partial out-of-sample-validation of the modeling approach. I feed into the model the exogenous output and incorporate private default shocks directly from the data. Since financial shocks are unobserved in the data, I use the particle filter approach proposed in Bocola and Dovis (2019) to infer them. I then let the model endogenously generate private and public borrowing and the interest rate spreads. The baseline model replicates the dynamics of private and public debt, bailouts, and spikes on interest rates during the period of interest. Facing the same shocks, the planner completely avoids an increase in the interest rate paid on public debt through a combination of low private and public debt. Since the interest rate spreads are driven by both private and public debt, in the second exercise I impose as an additional restriction that the path of public debt must coincide exactly with the one observed in the data. As a result, the difference between the interest rate spreads measured in the data and in the counterfactual socially planned economy can be directly attributed to excessive private debt. I use this difference as my conservative estimate of the reduction in spreads at the peak of the crisis in 2012, which is 3.80 p.p.

Related Literature: This paper builds upon the literature on sovereign debt as well as the literature on pecuniary externalities and macroprudential policies. It is most closely related to the literature analyzing the relation between sovereign debt and the domestic private financial sector.

Following the theoretical framework of sovereign defaultable debt introduced in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008) developed quantitative models of sovereign debt and business cycles. A growing literature has emerged extending their framework. Chatterjee and Eyigungor (2012) and Hatchondo, Martinez, and Sosa-Padilla (2016) highlight the importance of long-term debt in generating dynamics of the interest rate spread that are consistent with the data[7]. The model presented here incorporates these findings by assuming a long-term structure for public debt while keeping, for simplicity, the short-term maturity in private debt.[8] The paper is closely related to the branch of the sovereign debt literature that focuses on the link between sovereign debt and the private economy. In contrast to Mendoza and Yue (2009) and Arellano, Bai, and Bocola (2017), the analysis presented here assumes that private agents have access to international credit markets even during sovereign default episodes. The article shares this feature with Kaas, Mellert, and Scholl (2020). The main difference with this recent work is that private debt in my model is inefficiently high from a social perspective, and this inefficiency increases the incidence and magnitude of crises. As a result, the frequency of public bailouts, in response to reductions in the borrowing capacity in the private sector, is an endogenous outcome of the model.

The paper is also related to the literature that studies the trade-offs between centralized international public debt and decentralized international private debt. With complete markets, Jeske (2006) finds that a centralized environment, where only the government can issue and default on international debt, allows for more debt and is preferable to a decentralized environment where individual households make the borrowing and default choices. Wright (2006) finds that decentralized private borrowing can lead to underborrowing unless the government can choose to default on behalf of all residents, in which case overborrowing emerges instead. Finally, with incomplete markets, Kim and Zhang (2012) find underborrowing in an environment where decentralized households make the borrowing choices and a centralized government makes the default choice for all agents. My paper assumes incomplete markets and two distinct assets: private and public bonds. Only public debt enjoys sovereign immunity, and the government cannot force private agents to default. In my environment, the decentralization of the private bond leads to overborrowing in both assets relative to a centralized environment where a planner chooses the optimal portfolio.[9] Additionally, I find that both assets are used, even

[7]These papers, along with Aguiar et al. (2019) and Hatchondo, Martinez, and Sosa-Padilla (2016), discuss the issue of "debt dilution", the time-inconsistency problem that emerges when public defaultable debt is long term. Additional papers discussing responses to the trade-offs involved in maturity structures with long-term debt include among many others, Broner, Lorenzoni, and Schmukler (2013), Chatterjee and Eyigungor (2013), and Bianchi, Hatchondo, and Martinez (2018).

[8]The presence of multiple maturities links the paper to literature studying the role of the optimal debt maturity structure, such as Arellano and Ramanarayanan (2012) and Sanchez, Sapriza, and Yurdagul (2018). This paper differentiates itself from this literature by assuming that the government will not be able to fully control the issuances of short-term private debt.

[9]In this context, overborrowing also causes an overvalued real exchange rate in the lead-up to a sovereign default crisis. This is consistent with the Twin D phenomenon, defaults and large devaluations happen in tandem, documented and analyzed in Reinhart (2002) and Na et al. (2018).

in the centralized environment, because they provide insurance in different states. That is, the assets have different spanning.[10]

Furthermore, the paper contributes to the literature on credit frictions, financial crises, and macroprudential policies. In particular, it belongs to the branch on systemic credit risk (see Lorenzoni (2008), Bianchi (2011), and Dávila and Korinek (2018)) and its management with taxes on private borrowing (see Bianchi and Mendoza (2018), Farhi and Werning (2016), and Jeanne and Korinek (2019)).[11] In related work, Arce, Bengui, and Bianchi (2019) show that optimal international reserve accumulation can achieve the same welfare gains as optimal taxes on borrowing.[12] Instead, this paper shows that by themselves, government bailouts financed with external defaultable debt are not a substitute for optimal macroprudential policies. The role of bailouts in the model is similar to the one found in Bianchi (2016), Keister (2016), and Chari and Kehoe (2016). In contrast to those papers, I distinctly assume here that the bailouts can be paid for with long-term strategically defaultable debt. This feature allows the model to create a path from financial crises to sovereign debt crises, a relationship observed in the data.[13]

By analyzing how private credit affects the sovereign spread, the paper also contributes to the growing literature on the feedback loop between sovereigns and the domestic financial sector referred to as "doom loops" or "lethal embrace." Theoretical models of this issue are presented in Korinek (2012), Brunnermeier et al. (2016), and Farhi and Tirole (2018).[14] Acharya, Drechsler, and Schnabl (2014) present empirical evidence documenting a relationship between credit default swaps of European governments and banks for 2007-2011, and propose a close economy model to address this.[15] Another strand of the literature, such as Perez (2015), Bocola (2016), and Sosa-Padilla (2018), has focused instead on developing quantitative models that capture only a part of this loop, the transmission of sovereign risk to private risk. Specifically, the role of sovereign debt in the balance sheet of domestic banks and how the increase in sovereign spreads triggers financial crises. The model presented here is an open economy model, where both private and public

[10]The fact that the two debt instruments have different maturities guarantees different spanning properties even when sovereign default and a biding constraint next period happen with zero probability.

[11]Another recent strand of related literature studies the implications of this pecuniary externality for exchange rate policy, see, for example Fornaro (2015), Ottonello (2015), and Benigno et al. (2016).

[12]Bianchi (2011) also shows the optimal allocations can be implemented with capital reserve requirements.

[13]The literature on bailouts also deals extensively with the issue of moral hazard that the expectation of government bailouts induces. This concern is not addressed in this paper since households take as given that government policies are functions of aggregate states and not their individual actions. Additional research on the issue can be found in Nosal and Ordoñez (2016), Stavrakeva (2020), and Pasten (2020).

[14]Other relevant theoretical papers on this issue include Uhlig (2014) and Cooper and Nikolov (2018).

[15]Another recent closed economy quantitative model that addresses both the doom loop is Hur, Sosa-Padilla, and Yom (2021).

agents have access to international credit markets and is quantitatively consistent with the Spanish data. The paper complements the existing quantitative literature by focusing on the other part the loop, from financial crisis in the private sector to sovereign debt crises. In this paper, excessive private credit will endogenous generate financial crises and increase the incentives for government interventions that increase default risk and spreads.

Finally, methodologically the paper applies recent progresses in dynamic discrete choice methods to solve a sovereign debt model drawing from the contributions of Dvorkin et al. (Forthcoming)[16]. The method is used here to smooth the government policy functions and reduce computational errors. Additionally, to construct a quantitative counterfactual of the Spanish debt crisis, the paper uses the nonlinear particle filter method proposed by Kitagawa (1996). This technique uses likelihood functions to construct a numerical approximation of an unobserved stochastic shock and was first applied to quantitative business cycle models in Bocola (2016) and Bocola and Dovis (2019).

Layout: The paper is organized as follows. Section 1.2 outlines the motivating empirical facts in the Spanish data. Section 1.3 presents the model and the main theoretical result. Section 1.4 details the calibration and discusses the main mechanisms through which private and public debt interact in the model. Section 1.5 provides the quantitative results of the paper. Its first part compares the positive and normative versions of the model at their respective ergodic distributions. Its second part details the two dynamic exercises that simulated the 2008-2015 Spanish debt crisis. It provides the model predictions and counterfactual dynamics for private and official borrowing, and the evolution of interest spreads. Finally, Section 1.6 concludes.

1.2 Motivation: The path of debt and spreads in Spain 1999-2015

This section documents the evolution of international private and public debt in Spain from the creation of the Eurozone in 1999 to the end of the Spanish sovereign debt crisis in 2015. The pattern consists of a period of large accumulation of private debt, with low levels of public debt and spreads, followed by financial and sovereign debt crises. Figure 1.1 shows this pattern for Spain; however, as noted by Reinhart and Rogoff (2011), Lane (2013), and Gennaioli, Martin, and Rossi (2018), similar patterns have been seen in other

[16]Other models using this technique include Mihalache (2020). A review of the method and an alternative can be found in Gordon (2019).

countries and periods.

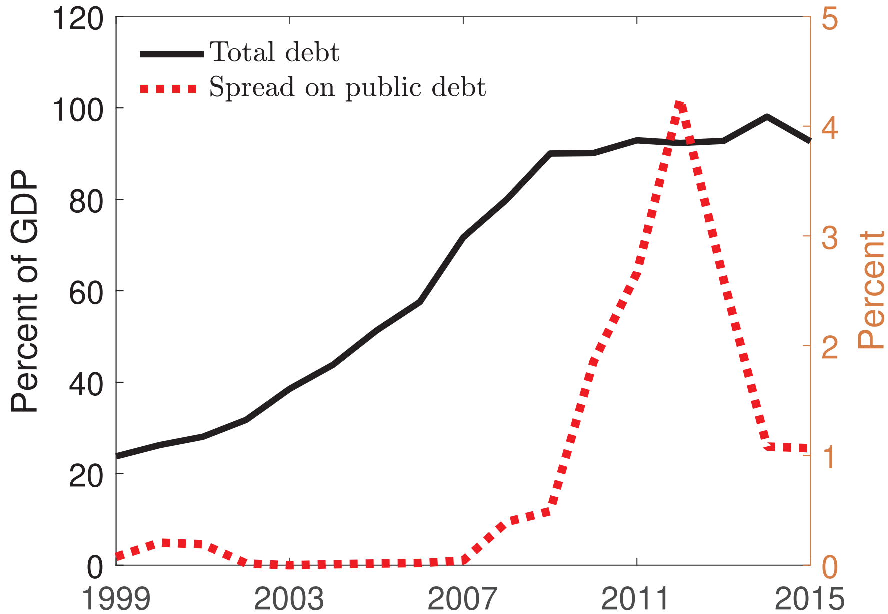

Figure 1.1: Total international debt and sovereign spread

Note: Total debt corresponds to the inverse of the international investment positions. Spreads correspond to the average difference between the interest rate paid on a Spanish six year treasury bill and its German equivalent. The data source for debt is the Bank of Spain, and the interest rate data are from Bloomberg. More details can be found in Appendix A.3.

Figure 1.1 plots the evolution of the Spanish debt crisis. The left axis plots the evolution of the international investment position as a percentage of GDP on an inverted scale; that is, positive numbers represent net liabilities.[17] All types of assets are accounted for in this aggregate. Nevertheless, throughout the paper I refer to this measure of net international liabilities as debt. The right axis plots the sovereign spread (dotted line), calculated as the difference between a six-year Treasury bond issued by Spain and its German counterpart.[18] The figure shows a first period of accumulation of external debt between 1999 and 2008, followed by a period where total debt remained constant at around 92% of GDP. Interest rate spreads remain close to zero up to 2009 and then a spike in 2012. Some observers, like Banco de España (2017), find it hard to reconcile rational financial markets with a period of rapidly increasing debt but low spreads (1999 to 2008) and a period of significant movement in the spread with constant total debt (2009 to 2015). This paper will argue that these two things are not incompatible.

[17] Annualized data are from the Bank of Spain; more details can be found in Appendix A.3.

[18] This maturity is chosen because it corresponds to the average maturity of public debt in Spain during this period. For more details, see Section 1.4 and Appendix A.3.

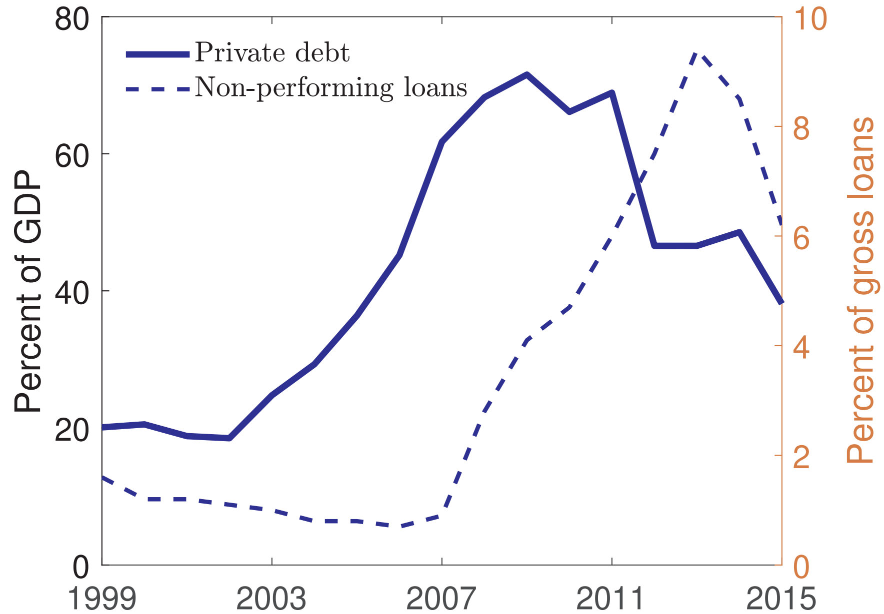

Figure 1.2: Private debt and nonperforming loans

Note: Private debt corresponds to the inverse of the international investment positions of the financial and nonfinancial private sector. Nonperforming loans are computed as a share of total gross loans. The data source for debt is the Bank of Spain, and the loans data are from Bloomberg. More details can be found in Appendix A.3.

Next, I summarize in Figure 1.2 the evolution of the private international liabilities during this time period. The right axis corresponds to the debt position of the private sector as a percentage of GDP (solid line) and the right axis corresponds to nonperforming loans as a percentage of gross loans (dashed line).[19] As before, the evolution of private debt displays two distinct periods. Net liabilities in the private sector grew from 20% of GDP in 1999 to 70% of GDP in 2009. Contemporary observers of this trend, such as International Monetary Fund (2007), classified the growth in private credit as the main risk to Spanish growth but predicted that the imbalances would gradually disappear.[20] After declining slightly for two years, private debt dropped by 22% of GDP in 2012. As noted by International Monetary Fund (2012), International Monetary Fund (2014), and Martin, Moral-Benito, and Schmitz (2019), among others, the buildup of external private debt was primarily driven by a banking sector that was financing a construction boom. When housing prices fell and mortgages started going unpaid, private debt became increasingly more difficult to roll over abroad. For this reason, I use the percentage of nonperforming

[19]To compute the position of the private sector, I subtract from total debt the assets held by the public administration and the Bank of Spain. See details in Appendix A.3.

[20]The empirical literature finds that strong link between international private credit growth and financial crises. See for instance, Schularick and Taylor (2012) and Davis et al. (2016).

loans as a proxy measure of aggregate default risk in the private sector. Figure 1.2 shows that the rapid increase in private debt stopped roughly at the same time as the share of nonperforming loans started increasing. Moreover, the abrupt drop in 2012 coincided with a high mark of the share of private default. On average, 7.5% of gross loans were nonperforming between 2011 and 2015.

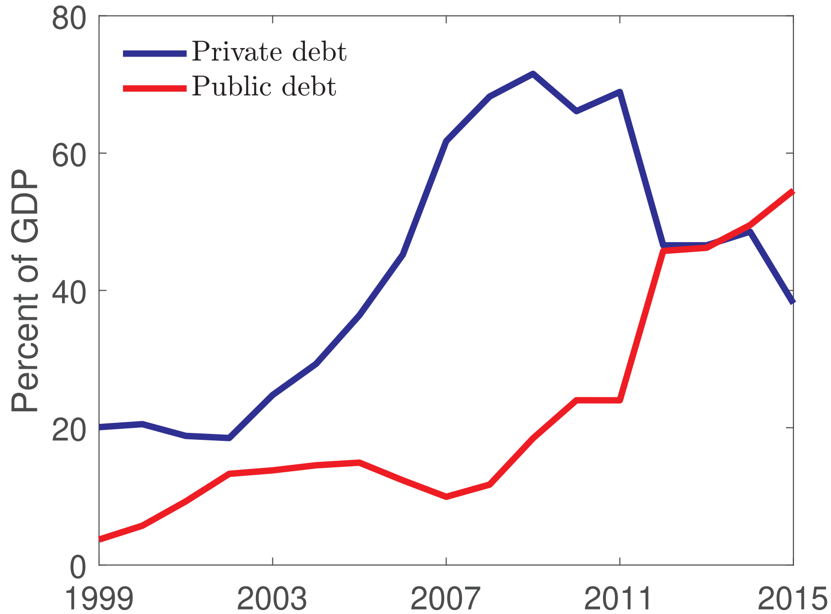

Figure 1.3: Private and public debt

Note: Private debt corresponds to the inverse of the international investment positions of the financial, and non financial private sector. Public debt corresponds to the inverse of the international investment position of the Bank of Spain and other public administrations. The data source is the Bank of Spain. More details can be found in Appendix A.3.

Finally Figure 1.3 complements the analysis by showing the joint evolution of public and private debt. Combined, these two series add up to the total debt presented in Figure 1.1. The symmetry between these two aggregates highlights the importance of the decomposition presented in this section. From 1999 to 2008, public external debt in Spain was below 20% of GDP. In contrast, from 2008 to 2015, public external debt increased from 11% to 55% of GDP. More importantly, the largest yearly increase was also in 2012, when public liabilities increased by 22% of GDP, exactly mirroring the drop in private debt. As noted in Banco de España (2017), this symmetry is not a coincidence. Between 2008 and 2012, the Spanish government funneled financial assistance to its lending institution primarily in the forms of bailouts and transfers of toxic assets. Total direct aid to the Spanish banking sector amounted to 70 billion or around 7% of GDP, with most of these funds being transferred by the newly created Fund for the Orderly Restructuring of the Banking Sector (FROB).[21]

To summarize, in the pre-crisis years, 1999-2007, large buildups of private debt coexist with low public debt and public spreads close to zero. This period was followed by a private financial crisis, corresponding in the data to the years 2008 to 2011. The financial crisis is characterized by an increase in nonperforming loans in the private sector and a moderate private deleveraging. Throughout this period, public debt and spreads increased but remain relatively low. The final period, from 2012 to 2015, corresponds to the sovereign debt crisis. These years are characterized by large public bailouts that reduce net liabilities in the private sector but are financed with issuances of public debt. The symmetric evolution of debt positions coincides with significant increases in the spread paid on public debt.

The next section proposes a theory that sheds light on this interplay between private and public external debt. The goal is to construct a model where both types of debt and their prices are endogenous, and that generates dynamics consistent with the facts presented in this section.

1.3 A model of financial and sovereign debt crises

This section presents a dynamic small open-economy model with one-period international private bonds subject to an occasionally binding borrowing constraint as in Bianchi (2011), and long-term, strategically defaultable, international public bonds, as in Hatchondo and Martinez (2009). The first subsection presents the economy's environment and technologies. The second subsection defines and characterizes the baseline unregulated, competitive equilibrium where the government only has access to public debt and lump-sum transfers. The third shows the optimal policy problem of a social planner (SP) who makes all borrowing decisions in both assets. The fourth subsection demonstrates that the SP's allocations are equivalent to those of a competitive equilibrium where the government gains access to state-contingent taxes on private debt. The last subsection explains the main mechanism of the model.

1.3.1 Environment

Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy is composed of a continuum of identical households of unit measure, a benevolent domestic government, and

[21] Beyond direct transfers, private debt declined following liquidation of private assets while public debt increased to finance unemployment benefits and economic stimulus programs to mitigate the financial crisis. A full overview of the restructuring of the Spanish financial sector is beyond the scope of this paper. More details can be found in International Monetary Fund (2010) and Banco de España (2017).

a continuum of risk-neutral competitive foreign creditors who lend to both domestic agents via two different assets. The focus is on real values as opposed to nominal ones because most Spanish debt was denominated in euros, whose supply is controlled by the European Central Bank[22].

Households

Preferences: The representative household has an infinite life horizon and preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + D_t], \tag{1.1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; and $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada condition. The term D_t is an additive preference shifter that depends entirely on government decisions, and exogenous shocks and the households take it as given. The consumption basket c is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta+1)$ between tradable goods c^T and nontradable goods c^N, given by

$$c = \left[\omega \left(c^T\right)^{-\eta} + (1-\omega)\left(c^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0,1).$$

Endowments: Each period the economy receives a stochastic endowment of tradable goods $y^T \in \mathbb{R}^+$ and nontradable goods $y^N \in \mathbb{R}^+$. Both endowments are drawn from first-order Markov processes independent of each other and of all other stochastic shocks in the model. The numeraire is the tradable good.

Private Debt: Households can borrow using a one-period non-state-contingent debt denominated in units of tradables. Following the standard convention, lowercase b denotes the individual level of private debt, while uppercase B denotes the aggregate level. Each period a stochastic fraction π_t of these bonds is defaulted on. Including these private default shocks allows the model to capture the dynamics of nonperforming loans in Spain and but has otherwise , no major implications in the model. Like the endowment shocks, the fraction of defaulted private bonds is drawn from a first-order Markov process independently from all

[22]The interaction of sovereign default and the inability to inflate away the debt in the context of the European Debt Crisis is studied in Aguiar et al. (2014) and Aguiar et al. (2015). For the specific case of Spain, Bianchi and Mondragon (2018) explore this issue in an environment with nominal rigidities.

the other stochastic shocks in the model. Private debt is issued in international competitive credit markets at price q_t.

In addition, private bonds issuances are subject to a collateral credit constraint, as follows:

$$q_t b_{t+1} \leq \kappa_t \left(y_t^T + p_t^N y^N \right), \tag{1.2}$$

where p_t^N is the equilibrium price of nontradable goods in units of tradables. The market value of private debt issuances $q_t b_{t+1}$ is capped at a fraction $\kappa_t \geq 0$ of the market value of current income.

This credit constraint captures in a parsimonious way the empirical fact that income is critical in determining credit market access.[23] Theoretically, the constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the endowment owned by a defaulting household.[24] Nontradable goods enter the collateral constraint because even though foreign creditors do not value them, I assume they can be seized in the event of default and sold in exchange for tradable goods in the domestic market.[25] Collateral constraints are commonly used in mortgage lending. Consequently, this assumption is particularly suitable in the Spanish context where mortgage loans played an important role in the buildup of private credit. Note that, while private debt is explicitly modeled here as issued internationally by the households, the same constraint arises under a broader set of assumptions. In particular, I could assume instead that credit is provided to households by a competitive domestic financial system with unrestricted access to global capital markets but subject to the same enforcement friction. As noted in Section 1.2, this interpretation is more in line with the events that unfolded in Spain. Commercial and savings banks borrowed internationally and then channeled these funds to households and construction firms. The assumption of short-term maturity is consistent with the empirical literature documenting a reduction in the maturity of private bonds issued in advanced economies during this period.[26]

The fraction of market income required as collateral κ_t is stochastic and drawn from a first-order Markov process. Throughout the paper, I refer to this shock as the financial shock. Stochastic changes in collateral requirements can be viewed as shocks to the creditors'

[23]See Jappelli (1990).

[24]In this context, the punishment is only triggered by private default above the exogenous fraction drawn in each period.

[25]The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks. See Bianchi and Mendoza (2018), for a derivation of a similar constraint.

[26]See, for instance, Gorton, Metrick, and Xie (2020) and Chen et al. (2019).

risk assessment of the borrowers. Financial shocks of this form have been shown to be capable of accounting for the dynamics of private financial crises in advanced economies (see Jermann and Quadrini (2012), and Boz and Mendoza (2014)) as well as balance of payment crises in emerging economies (see Mendoza (2002) and Coulibaly (2018)). From a modeling perspective, these shocks generate fluctuations in private borrowing that are not caused by fluctuations in other domestic fundamentals. This is consistent with recent empirical work by Forbes and Warnock (2020). They document that shocks in international volatility, monetary policy, or sudden-stop crises in similar and/or neighboring countries can cause fluctuations in the lenders' perceptions about the private sector's solvency. In the context of interest, these shocks allow the model to account for a change in investors' behavior toward Eurozone banks in the wake of the Greek sovereign debt crisis.

Finally, note that neither the existence of the financial amplification mechanism nor the government's best responses presented later rely on κ_t or π_t being stochastic.[27] Nevertheless, these shocks will generate fluctuations in private borrowing independently from income fluctuations and as such will have a different impact on government policies.

Households' budget constraint: Each period, individual households face a budget constraint of the form

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t b_{t+1}, + y_t^T + p_t^N y^N + T_t. \tag{1.3}$$

where T_t is a lump-sum transfer from the government. A positive transfer indicates a bailout, while a negative one denotes a lump-sum tax. This transfer is the primary link between the households and the government and will be present in all versions of the model. Access to this instrument allows the government to directly modify the household's cash-in-hand without introducing additional distortions. As a result, the interactions that will arise between private and public debt in this paper are not a consequence of a restrictive set of tax instruments. The last subsection will consider the implications of giving the government an additional tax instrument, a linear tax on private borrowing, τ_t, used for macroprudential purposes.

Government

Public debt: The government borrows by issuing without commitment a long-term bond ($L \geq 0$) on international capital markets *à la* Eaton and Gersovitz (1981). Each period

[27] Models with a constant κ and no private default, such as Mendoza (2010), also generate private crisis dynamics with realistic business cycle features.

the sovereign chooses to either default ($d \in \{0, 1\}$) or to keep its credit market access by paying its obligations and reissuing new ones. As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), I assume that a bond issued in period t promises in case of repayment a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. As such, one unit issued in the current period promises to pay a fraction $(1-\delta)$ of all remaining debt each following period. An advantage of this payment structure is that it condenses all future payment obligations into a one-dimensional state variable proportional to the quantity of long-term coupon obligations that mature in the current period. Hence, the debt dynamics can be summarized by

$$L_{t+1} = (1-\delta)L_t + i_t, \tag{1.4}$$

where L_t is the number of public bonds due at the beginning of period t, and i_t is the bond issuances at t. As in common in the literature, I assume that sovereign debt only takes values in a finite and bounded support with $\mathcal{J}$ points.[28] The grid of potential long-term debt positions can be summarized by a vector Λ, where L_j is the *jth* element, consequently,

$$\Lambda = \Big[L_1, L_2, ..L_{\mathcal{J}}\Big]^T.$$

Default: Default brings immediate financial autarky and an additive utility cost that is an increasing function of tradable output $\phi(y_t^T)$. [29] For simplicity, I assume that the government returns to international credit markets with zero debt after one period of exclusion from markets.[30] Note that sovereign default does not imply default on private debt nor an exclusion of private agents from financial markets. This is in contrast to other papers with both public and private international debt, such as Mendoza and Yue (2009). I make this assumption because empirically, Kalemli-Ozcan, Laeven, and Moreno (2018), Gennaioli, Martin, and Rossi (2018), and Bottero, Lenzu, and Mezzanotti (2020) find that although private borrowing declines during a sovereign default crisis, it still quantitatively significant.

[28]The assumption of a discrete and bounded support is usual in the sovereign default literature with long-term debt; see Chatterjee and Eyigungor (2012).

[29]Utility losses from default in sovereign debt models are also used in Aguiar and Amador (2013), Bianchi and Sosa-Padilla (2020), and Roch and Uhlig (2018), among others. A common alternative is output costs of default. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications.

[30]Assuming an exogenous probability of reentry into financial markets, as in Arellano (2008), would not change the results but would require to keeping track of an additional state.

Government's preferences: The sovereign is benevolent and therefore has the same utility and discount factors as the households. Furthermore, for computational tractability, I follow Dvorkin et al. (Forthcoming) and assume that each period the government draws a random vector ϵ of size $\mathcal{J}+1$ of additive taste shocks. One element of the vector is associated with the choice of default, while the remaining $\mathcal{J}$ elements are associated with each debt choice on Λ in case of repayment. The elements of the vector are labeled

$$\epsilon(L_j) = \epsilon_j,$$
$$\epsilon^{Def} = \epsilon_{\mathcal{J}+1}.$$

The taste shock ϵ is independent and identically distributed (i.i.d.) over time and within Λ. Furthermore, I assume that its distribution is a multivariate generalized extreme value with mean m and variance $v > 0$.[31][32] Combining all this, the government's flow utility at time t is

$$u(\mathcal{C}_t) + d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1}),$$

where d_t is the government default decision, $\mathcal{C}_t$ is private consumption, $\phi(y_t)$ is the utility cost of default, and ϵ_t is the additive taste shock. This equation provides an explicit formulation of the additive preference term in the household preferences (1.1), namely,

$$D_t = d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1}).$$

Government's budget constraint: Each period the government's budget constraint is given by its default decision d_t, the public debt dynamics (1.4), and the lump-sum transfers T_t.[33] The budget constraint is

$$T_t = (1-d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big], \tag{1.5}$$

[31] For additional details regarding the distribution of taste shocks, see Appendix A.1.

[32] Preference shocks affecting the default decisions are now common in the literature; see, for instance, Arellano, Bai, and Bocola (2017), Aguiar et al. (2019), and Aguiar et al. (2020). They are considered an alternative to the i.i.d. income shocks also encountered in the literature (e.g., Chatterjee and Eyigungor (2012)). In this model, the shocks allow the government to break ties between similar portfolio positions. An interpretation of these shocks is that they capture additional costs or benefits of default, such as the perceptions of policy makers of the costs of default. At the same time, as noted by Dvorkin et al. (Forthcoming), provided that the variance of the shocks is small enough, they will have small quantitative consequences in aggregate moments.

[33] The last subsection will modify this constraint by granting the government access to taxes on private debt.

where L_t is the long-term public debt at the beginning of period t, and L_{t+1} is the long-term debt at the end. Finally, Q_t is the price at which lenders purchase these bonds, which in equilibrium depends on the government's and household's portfolio decisions and the exogenous shocks.

International lenders

Private and sovereign bonds are traded with a continuum of risk-neutral, competitive foreign lenders. Lenders have access to a one-period risk-free security paying a net interest rate r. The equilibrium price of private bonds is given by the no-arbitrage condition

$$q_t = \frac{\mathbb{E}_t[1-\pi_{t+1}]}{1+r}.$$

In equilibrium, investors must be indifferent between purchasing a risk-free security and buying a private bond at price q_t. Since private debt is only held for one period, lenders use the exogenous probability of default one period ahead to price it. Similarly, bond prices for sovereign debt in case of repayment are

$$Q_t = \frac{\mathbb{E}_t}{1+r}\Big[(1-d_{t+1})\big(\delta+(1-\delta)Q_{t+1}\big)\Big].$$

As before, the no-arbitrage condition implies that investors will purchase government bonds at a price Q_t that compensates them for the risk of default they bear. In case of default, no public debt is recovered. In case of repayment, the payoff is given by the coupon δ plus the market value Q_{t+1} of the nonmaturing fraction of the bonds next period.

Resource constraints

Since both types of debt are denominated in tradables, the market clearing conditions are

$$c_t^N = y_t^N, \tag{1.6}$$

$$c_t^T + (1-\pi_t)b_t = y_t^T + q_t b_{t+1} + T_t. \tag{1.7}$$

1.3.2 Baseline unregulated competitive equilibrium

This subsection defines and characterizes the baseline problem in recursive form. I first discuss the equilibrium concept and the timing of the events and introduce the notation used throughout the paper. I then present, in order, the problems of the government, the households, and the lenders. I conclude with the formal definition of a competitive equilibrium for this baseline version of the model.

Equilibrium concept: This paper focuses on a Markov perfect equilibrium. Consequently, the current period decisions of all agents will be functions of payoff-relevant state variables and will take all future policies rules as given. The focus on a Markov perfect equilibrium is important. An environment with strategically defaultable long-term bonds with a government that cannot commit to future debt issuances induces a time-inconsistency problem known as debt dilution. The solutions to the recursive, time-consistent problem do not coincide with the solutions to the sequential problem with commitment. Throughout the paper, the focus is on the time-consistent policies.[34] Additionally, government default, borrowing and transfer strategies each period will only depend on current period payoff-relevant states.

One could interpret this environment as a game where the government makes current period decisions while taking as given the best response functions of the other players, households, and foreign lenders, and also the strategies of future governments that decide policies in the future. Thus, the government considers the general equilibrium effects of its policies on the aggregate choices of the private sector, consumption, and private borrowing, and all prices, nontradables, and bonds, but cannot choose those functions.

Recursive notation and timing: In all cases, I denote with a prime symbol the end-of-period levels of private and public debt. The timing of events within the period is as follows:

- The economy enters the period with private debt B and public debt L.
- All shocks are realized. The exogenous state is $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$.
- The state space is now $S = \{s, L, B\}$.
- The government acts first. Facing S, the government makes default d and public debt L' choices.
- The aggregate state of the economy incorporating the government's policies is $S_G = \{S, d, L'\}$.

[34] For a discussion of policies that remedy debt dilution, see Hatchondo, Martinez, and Sosa-Padilla (2016) and Aguiar et al. (2019).

- Households act second. Facing S_G, households choose consumption and private debt, which determine the aggregate consumption C^T and C^N and the aggregate private debt B'.

- The lenders act last. They choose bond schedules Q and q using only the payoff-relevant states.

Policy decisions and best responses: The government's policy decisions are $d(S), \mathcal{L}'(S)$. The private sector's aggregate best responses are $\mathcal{C}^T(S_G)$, $\mathcal{C}^N(S_G)$, and $\mathcal{B}'(S_G)$. The foreign lenders' best responses are the schedules for public bond $Q(s, L', \mathcal{B}'(S_G))$ and for private bond $q(s)$.

Government: Given the best responses of the private sector and foreign lenders, the government chooses $d(S)$ and $\mathcal{L}'(S)$ that maximizes the household's welfare subject to the period budget constraint (1.5) and the resource constraints, (1.6) and (1.7). In detail, the government's problem is

$$W(S) = \max_{d \in \{0,1\}} [1-d]W^R(S) + dW^D(S), \tag{1.8}$$

where $d = 1$ if the government defaults and $d = 0$ otherwise. If the government repays, $S_G = (S, 0, L')$, and the value of repayment is

$$W^R(S) = \max_{L' \in \Lambda} \; u\big(\mathcal{C}^T(S_G), \mathcal{C}^N(S_G)\big) + \epsilon(L') + \beta \mathbb{E}_s[W(s', L', \mathcal{B}'(S_G))] \tag{1.9}$$

subject to

$$\begin{aligned} T(S_G) &= Q(s, L', \mathcal{B}'(S_G))[L' - (1-\delta)L] - \delta L \\ \mathcal{C}^T(S_G) + (1-\pi)B &= y^T + q(s)\mathcal{B}'(S_G) + T(S_G), \\ \mathcal{C}^N(S_G) &= y^N. \end{aligned}$$

Note that in repayment states, the government's public debt decision will affect the value of the transfer directly through issuances and indirectly through the bond schedule. The choice of public debt will then affect the households' decisions on consumption of tradables and private debt via the transfer. The government internalizes that its borrowing decision affects the choices of the households and the price that the lenders will charge for public debt.

In the case of default, $S_G = (S, 1, 0)$, and the government's value is

$$W^D(S) = u\Big(\mathcal{C}^T(S_G), \mathcal{C}^N(S_G)\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\Big[W\big(s', 0, B'(S_G)\big)\Big] \tag{1.10}$$

subject to

$$T = 0,$$

$$\mathcal{C}^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G),$$

$$\mathcal{C}^N(S_G) = y^N,$$

In default, the government loses access to public borrowing. Thus, the transfer is zero. Nevertheless, households still maintain access to financial markets and are still liable for their obligations. Consequently, a sovereign default can still leave the economy highly leveraged, albeit in private bonds.[35]

The solution to the government's problem yields decision rules for default $d(S)$ and public debt $\mathcal{L}'(S)$, which in turn determine the transfers $T(S_G)$ and the preference shift $D(S_G)$ as follows:

$$T(S_G) = (1 - d(S)) \times \Big(Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))[\mathcal{L}'(S) - (1-\delta)L] - \delta L\Big) \tag{1.11}$$

$$D(S_G) = \big(1 - d(S)\big)\epsilon\big(\mathcal{L}'(S)\big) + d(S)\big(\epsilon^{def} - \phi(y^t)\big) \tag{1.12}$$

Households: The households make decisions based on their current level of individual debt b and the aggregate state of the economy when they act, S_G. The aggregate state comprises the exogenous shocks s, the initial level of government debt L, the current level of aggregate private debt B, and the decisions made by the government in the current period regarding default d and public debt L'. Households are competitive, and as such they take all prices and aggregate laws of motion as given: the price of nontradables $p^N(S_G)$, the equilibrium price of private bonds $q(s)$, the government's current and all future borrowing decisions $\mathcal{L}'$ and default decisions d,[36] transfers T, and the preference shock D. Under rational expectations, households predict future states using the perceived law of motion of

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[35]See Mendoza and Yue (2009) for a case where public default also triggers private default.

[36]For concision, I equate in the discussion the solutions to the current government policy functions with the strategies of future governments. This equality holds in a Markov perfect equilibrium. Alternatively, one could impose this equality as an equilibrium condition, as in Bianchi and Mendoza (2018).

aggregate private debt $\mathcal{B}'$. The households' optimization problem in recursive form is

$$V(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V(S'_G, b')] \tag{1.13}$$

subject to

$$c^T + p^N(S_G)c^N + (1-\pi)b = y^T + p^N(S_G)y^N + q(s)b' + T,$$

$$q(s)b' \leq \kappa[p^N(S_G)y^N + y^T],$$

$$T = T(S_G),$$

$$D = D(S_G),$$

$$B' = \mathcal{B}'(S_G),$$

$$L' = \mathcal{L}'(S),$$

$$\text{And } S'_G = (s', L', B', d(s', L', B'), \mathcal{L}'(s', L', B')).$$

In equilibrium, $p^N(S_G)$ is the price of nontradables, and $q(s)$ is the price of private bonds. The solution to the household problem yields decision rules for individual bond holdings $\hat{b}'(S_G, b)$, tradable consumption $\hat{c}^T(S_G, b)$, and non-tradable consumption $\hat{c}^N(S_G, b)$. The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings, $\mathcal{B}'(S_G)$, to an actual law of motion, given the representative agent's choice $\hat{b}'(S_G, B)$. In a rational expectations equilibrium, these two functions must coincide. The same is true for the laws of motion of aggregate consumption in the economy $\{\mathcal{C}^i(s, L, B)\}_{i=T,N}$.

The solutions to the households' problem solve the optimality conditions that include the budget constraint (1.3), the credit constraint (1.2), and the first-order conditions. In particular, the households' intratemporal optimality condition pins down the equilibrium price of nontradables,

$$p^N(S_G) = \frac{1-\omega}{\omega} \left(\frac{\mathcal{C}^T(S_G}{y^N} \right)^{\eta+1}. \tag{1.14}$$

Condition (1.14) is a static optimality condition equating the marginal rate of substitution between tradable and nontradable goods to their relative price. The equation implies that the price of nontradables is an increasing function of c^T.

A pecuniary externality arises in this problem because this equilibrium price affects the value of collateral (1.2) and therefore the level of borrowing in some states. Consequently, a reduction in c^T causes in equilibrium a reduction in the collateral value (1.2). In states where the credit constraint binds, this reduction triggers the financial amplification mechanism, whereby a drop in consumption induces a contraction in private borrowing, which in turn drives consumption further down. Because of standard consumption-smoothing effects, consumption increases with the cash-in hand of the households. Since the government can increase the cash-in-hand of the households via the fiscal transfer, mitigating the amplification mechanism is an important incentive for government bailouts.

Lenders: The competitive risk-neutral foreign lenders use the decision rules of current and future governments and households to price the bonds. The solution to the problem of competitive risk-neutral foreign lenders yields the bond price schedule for private debt,

$$q(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{1.15}$$

and the bond price schedule for public debt

$$\begin{aligned} Q(s,L',B') &= \frac{1}{1+r} \times \mathbb{E}_s\Big[\Big[1-d'\Big] \times \Big[\delta + (1-\delta)Q\Big(s',L'',B''\Big)\Big]\Big], \qquad (1.16)\\ &\quad \text{where:} \\ B'' &= \mathcal{B}'(s',L',B'), \\ L'' &= \mathcal{L}'(s',L',B'), \\ d' &= d(s',L',B'). \end{aligned}$$

The lenders price the debt contracts based on their expectations of future defaults and new issuances of public debt. As a result, when pricing private debt, the only payoff-relevant state is the exogenous shock s. In contrast, when pricing public debt, the payoff-relevant states for the lenders also include the end-of-period levels of private B' and public debt L'. Note that both the levels and composition of debt are important because they affect the future governments' default and public debt issuances decisions.

Definition of equilibrium: The competitive Markov equilibrium combines the problems of the government, households, and lenders, as well as the resource constraints of the economy. Moreover, it also has rational expectations conditions guaranteeing that in equilibrium the households' borrowing and consumption decisions are consistent with the perceived law of motion that all agents are using in their decisions.

Definition 1.1. *A Markov unregulated competitive equilibrium is a set of value functions* $\{V, W, W^R, W^D\}$*, policy functions for the private sector* $\{\hat{b}, \hat{c}^T, \hat{c}^N\}$*, policy functions for the public sector* $\{d, \mathcal{L}'\}$*, a pricing function for nontradable goods* p^N*, pricing functions for* *public debt Q and private debt q, and perceived laws of motion $\{\mathcal{B}', \mathcal{C}^T, \mathcal{C}^N, Q\}$ such that*

1. *Given prices $\{p^N, q\}$, government policies $\{d, \mathcal{L}'\}$, and perceived law of motion $\mathcal{B}'$, the private policy functions $\{\hat{b}', \hat{c}^T, \hat{c}^N\}$ and value function V solve the household's problem (1.13).*

2. *Given bond prices $\{Q, q\}$ and aggregate laws of motion $\{\mathcal{B}', \mathcal{C}^T, \mathcal{C}^N\}$, the public policy functions $\{d, \mathcal{L}'\}$ and value functions W, W^R, and W^D solve the Bellman equations (1.8)–(1.9).*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion $\{\mathcal{B}'(S_G) = \hat{b}'(S_G, B), \mathcal{C}^T(S_G) = \hat{c}^T(S_G, B), \mathcal{C}^N(S_G) = \hat{c}^N(S_G, B)\}$.*

4. *The private bond price function $q(s)$ satisfies (1.15).*

5. *Given public $\{d, \mathcal{L}'\}$ and private $\{\mathcal{B}'\}$ policies the public bond price $Q(s, L', B')$ satisfies (1.16).*

6. *Goods market clear:*

$$
\begin{aligned}
\mathcal{C}^N(S_G) =& y^N, \\
\mathcal{C}^T(S_G) + (1-\pi)B =& y^T + q(s)\mathcal{B}'(S_G) + \\
& \Big\{1 - d(S)\Big\}\Big\{Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))\Big[\mathcal{L}'(S) - (1-\delta)L\Big] - \delta L\Big\}.
\end{aligned}
$$

1.3.3 Recursive social planner's problem

This subsection formulates the problem of a social planner (SP) in the same environment. The formulation is similar to the "primal approach" to optimal policy analysis. The planner chooses aggregate allocations subject to resource, implementability, and collateral constraints. Note that the planner does not set prices and instead takes the pricing functions that solve the lenders problem as given. However, the planner internalizes how its consumption and borrowing decisions affect all general equilibrium prices. As such, the planner behaves like a strategic player and not competitively as the households do in the previous section. Therefore, the equilibrium price of nontradable goods (p^N) and bonds (q, Q) will enter the SP problem as implementability constraints.[37] As before, the focus is on the Markov perfect stationary equilibrium. I assume that the planner cannot commit to

[37] This formulation is equivalent to letting the planner make all borrowing decisions and transfer the proceeds to competitive households who make all consumption decisions taking prices as given.

future policy rules, including future defaulting and borrowing decisions. Consequently, it chooses current period allocations taking as given the strategies of future planners. Equilibrium is characterized by a fixed point of these policy rules.

The social planner's optimization problem consists of maximizing the utility of the households (1.1) subject to the credit constraint (1.2), the resource constraint (1.6), (1.7), and equilibrium prices (1.14),(1.15), and (1.16).The household budget constraint is automatically satisfied by Walras's law. Denote $\{\mathcal{L}^{SP\prime}$ and $\mathcal{B'}^{SP\prime}\}$ as the public and private borrowing decisions, respectively. Let d^{SP} be the default decisions of future planners that the current SP takes as given. The planning problem is[38]

$$W^{SP}(s, L, B) = \max_{d \in \{0,1\}} [1-d] W^{SP,R}(s, L, B) + d W^{SP,D}(s, B), \tag{1.17}$$

where the default value of the planner $W^{SP,D}(s, B)$ is

$$\begin{aligned}
W^{SP,D}\big(s, B\big) &= \max_{c^T, B'} u\big(c^T, y^N\big) - \phi(y^T) + \epsilon_{Def} + \beta \mathbb{E}_s \Big[W^{SP}\big(s', 0, B'\big)\Big], \\
c^T + B(1-\pi) &= y^T + q^{SP}(s) B', \\
q^{SP}(s) B' &\leq \kappa \left(\frac{1-\omega}{\omega} \left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T \right), \qquad (1.18) \\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}.
\end{aligned}$$

and the value of the planner under repayment $W^{SP,R}(s, L, B)$ is

$$\begin{aligned}
W^{SP,R}(s, L, B) &= \max_{c^T, B', L' \in \Lambda} u\big(c^T, y^N\big) + \epsilon(L') + \beta \mathbb{E}_s [W^{SP}(s', L', B')], \\
c^T + B(1-\pi) + \delta L &= y^T + q^{SP}(s) B + Q^{SP}(s, L', B')[L' - (1-\delta) L], \\
q^{SP}(s) B' &\leq \kappa \left(\frac{1-\omega}{\omega} \left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T \right), \\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}.
\end{aligned}$$

[38]For concision, the equilibrium price of nontradables (1.14) and the resource constraint of nontradables (1.6) are already incorporated in this formulation. The price of public bonds Q^{SP} is equated with the equilibrium best response of competitive risk-neutral lenders.

$$Q^{SP}(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s \Big[\Big[1 - d^{SP}(s', L', B') \Big] \times \\ \Big[\delta + (1-\delta) Q^{SP} \Big(s', \mathcal{L}^{SP\prime}(s', L', B'), \mathcal{B}^{SP\prime}(s', L', B') \Big) \Big] \Big].$$

In contrast to the government in the baseline version, the planner directly controls the level of aggregate private borrowing B'. Like the government, the planner chooses aggregates. thus the planner's decisions consider their impact on: the effect of the price of nontradables (1.14) on the private debt limit (1.2) and the equilibrium best response of the foreign lenders.

Definition 1.2. *A Markov stationary socially planned equilibrium is a set of value functions* $\{W^{SP},\ W^{SP,R},\ W^{SP,D}\}$*, policy functions for allocations* $\{\mathcal{C}^{SP,T}, \mathcal{C}^{SP,N}, \mathcal{L}^{SP\prime}, \mathcal{B}^{SP\prime}\}$*, and defaulting* d^{SP}*, and pricing functions for public* Q^{SP} *and private* q^{SP} *debt, that solve (1.17) given conjecture future policies* $\{\mathcal{C}^{SP,T}, \mathcal{C}^{SP,N}, \mathcal{L}^{SP\prime}, d^{SP}\}$

1.3.4 Decentralization with macroprudential policies

In this subsection, I consider another version of the model where the government gains access to state-contingent linear taxes on private borrowing. I show that the Markov competitive equilibrium allocation solves the socially planned problem presented in the previous subsection. The households' budget constraint (1.3) becomes

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t(1-\tau_t)b_{t+1}, + y_t^T + p_t^N y^N + T_t,, \tag{1.19}$$

where τ_t is the tax rate on private borrowing. The introduction of taxes does not modify the credit constraint (1.2). As with all other government policies, taxes on private debt are taken as given by households. At the same time, the government can still tax the households using lump-sum transfers. The budget constraint (1.5) is now

$$T_t = (1-d_t)\Big[Q_t \big[L_{t+1} - (1-\delta)L_t \big] - \delta L_t \Big] + \tau_t q_t B_{t+1}. \tag{1.20}$$

Note that the government can still tax private debt and use lump-sum transfers while in default. Appendix A.1 provides a complete recursive formulation and characterization of the decentralized equilibrium with taxes.

Proposition 1.1. *The socially planned equilibrium allocation can be decentralized with a* *state-contingent tax on debt that satisfies*

$$1-\tau(s,L,B)=\frac{1}{q^{SP}(s)u_T(\mathcal{C}^{SP,T}(s,L,B),y^N)}\times\Big(\mu^{SP}(s,L,B)q^{SP}(s)+ \\ +\beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(\mathcal{C}^{SP,T}(s',L',B'),\mathcal{C}^{SP,N}(s',L',B'))\Big)\Big]\Big), \tag{1.21}$$

where μ^{SP} *corresponds to the Lagrange multiplier associated with the credit constraint in the planner problem* (1.17)*.*

Proof: See Appendix A.2.

The proof is done in two steps. First, I show that the planning problem is equivalent to a relaxed version of the competitive equilibrium with taxes. Second, I show that solutions to the planning problem are sufficient to construct policies that satisfy the additional constraints of the competitive equilibrium problem with taxes.

1.3.5 Difference between the baseline and planned economies

This subsection explains the intuition behind the main difference between the two versions of the model. For this purpose, I compare the intertemporal optimality conditions of the baseline and planner problems presented before. Consider the intertemporal optimality conditions of the households in the baseline problem (1.13),[39]

$$q(s)u_T(\mathcal{C}^T(S_G))=\beta\mathbb{E}_s[(1-\pi')u_T'(\mathcal{C}^{T\prime}(S_G)]+\mu(S_G)q(s), \tag{1.22}$$

$$0\leq\kappa(p^N(S_G)y^N+y^T)-q(s)\mathcal{B}'(S_G)\quad\text{with equality if } \mu(S_G)>0, \tag{1.23}$$

where $u_T(.)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$, the marginal utility of consumption of tradables, and μ is the Lagrange multiplier on the credit constraint. Condition (1.22) is the household's Euler equation for private debt and (1.23) is the complementary slackness condition. If $\mu>0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit of private debt and repaying next period. The main difference between the baseline model and the planning problem is in their private borrowing decisions. Consequently, I compare the Euler equation of private bonds for each problem.[40] Using the same notation as before,

[39] These expressions are obtained by assuming that the policy and value functions are differentiable and then applying the standard envelope theorem to the first-order conditions of the household problem and assuming that rational expectations hold.

[40] The complete characterization of the optimality conditions of the planning problem is discussed in Appendix A.1.

the planner policies (SP) are[41]:

$$q^{SP} + Q^{SP}_{B'}(\mathcal{L}^{SP\prime} - (1-\delta)L) = \frac{1}{u^{SP}_T(\mathcal{C}^{SP,T}) + \mu^{SP}\psi^{SP}} \times \Bigg(\mu^{SP} q^{SP} + \beta \mathbb{E}_s\Big[(1-\pi')\Big(u^{SP}_T(\mathcal{C}^{SP,T\prime}) + \mu^{SP\prime}\psi^{SP\prime}\Big)\Big]\Bigg) \tag{1.24}$$

The prime notation denotes future states and the marginal utility of consumption, and the Lagrange multiplier are u^{SP}_T and μ^{SP}, respectively. In contrast to the baseline's condition (1.22), the planners' Euler equation includes the marginal effect on the collateral value of an additional unit of tradable consumption $\psi^{SP} = \kappa(1+\eta)\frac{(1-\omega)}{\omega}\big(\frac{C^{SP,T}}{y^N}\big)^{\eta}$, public borrowing policies $\mathcal{L}^{SP\prime}$, and the marginal effect on the price of public bonds of an additional unit of tradable consumption $Q^{SP}_{B'}$. These terms capture the additional general equilibrium effects that the planner considers when deciding its level of private borrowing. While the first term is common in the Fisherian debt deflation literature, the latter two are encountered in the sovereign debt maturity management literature. I now briefly discuss the effect of each of them.

The term ψ^{SP} appears in Bianchi (2011). It captures that, relative to the households in the baseline model, the planner considers the marginal benefit of an extra unit of private borrowing on the current and future real exchange rate. First, additional borrowing increases the consumption of tradables and therefore the price of nontradables, which in turn relaxes the credit constraint ($\mu^{SP}\psi^{SP}$). Quantitatively, this effect is generally small, as numerically, it is usually that $\psi^{SP} < 1$. [42] Second, additional private borrowing decreases expected cash-in-hand next period, depressing the expected future price of nontradables ($\mu^{SP\prime}\psi^{SP\prime}$). Thus, additional borrowing increases the probability of facing a binding constraint next period. The planner internalizes this cost; the competitive households in the baseline model do not.[43] Consequently, the planner borrows less. This effect is quantitatively significant and the source of private overborrowing in the baseline model.

The terms $\mathcal{L}^{SP\prime}$ and $Q^{SP}_{B'}$ are seen in Arellano and Ramanarayanan (2012) and Hatchondo, Martinez, and Sosa-Padilla (2016) in models where the government has access

[41] As before, these first-order conditions are obtained by assuming differentiability and the standard envelope conditions. In addition, it also assumed that the equilibrium price of bonds is differentiable.

[42] If $\psi^{SP} > 1$, in some states this can instead lead to underborrowing and/or multiple equilibria. In all quantitative specifications considered in the paper, this case is never encountered. For specifications where this is violated, see Schmitt-Grohé and Uribe (2019). For other models of Fisherian deflation with underborrowing, see Benigno et al. (2013a).

[43] Note that the decision to ignore this effect is rational from the individual household perspective. Each household is small and does not control aggregate borrowing. As a result, its borrowing cannot affect aggregate prices.

to public bonds of different maturities. The private bond discussed here has a short-term maturity and differs from the assets discussed in those papers in two ways. First, it is not directly controlled by the government in the baseline model but by the households. Second, it is not strategically defaultable and is instead subject to the collateral constraint. Nevertheless, some of the trade-offs described in those models apply here. Private borrowing increases the probability of default and also increases the expected issuances of public debt in case of repayment. Keeping all other things equal, an extra unit of private bonds decreases expected wealth next period. Mechanically, this increases the probability of sovereign default. Moreover, even in states of repayment, higher private debt increases the probability of a debt-financed bailout. As a result, in some states an extra unit of private debt is also associated with an increase in expected future public debt. As a consequence of these two effects, increasing private debt increases the premium paid on public debt. Since the planner optimally manages its issuances of both assets, it chooses a lower level of private debt to lower the interest paid on its public debt. Lenders internalize that the government in the baseline problem cannot guarantee this optimal portfolio in either the current or future periods. Consequently, lenders offer a worse price schedule to the government than to the counterfactual social planner. This bond schedule combined with more frequent use of public bailouts will quantitatively explain the difference in average spreads between the baseline and socially planned equilibria.

1.4 Quantitative analysis

In this section, I solve numerically the two versions of the model presented in the previous section. The baseline is solved using time iteration for the private equilibrium and value function iteration for the government problem. The socially planned economy can be solved by value function iteration. More details regarding the numerical solution methods are described in Appendices A.4 and A.5.

1.4.1 Calibration

The baseline version of the model is calibrated using Spanish macroeconomic data from 1999 to 2011. One period in the model corresponds to one year in the data. I assume that Spain was at the ergodic distribution of the baseline version of the model during this period. The calibration consists of selecting a set of parameters so that the ergodic distribution averages coincide with the relevant macroeconomic moments in the data.

The starting year is chosen to coincide with the creation of the Eurozone. Before this, most Spanish public debt was in domestic currency, and therefore its nominal value was subject to government choices. The end year of 2011 is chosen to keep out of sample the significant European policies introduced in 2012. Some of these policies conflict with some of the fundamental assumptions underlying the baseline version of the model. Although Spain had implemented countercyclical prudential policies for its domestic banking sector in 1999, up until 2011 there were no systematic controls on private international borrowing within the European Union.[44] This changed in June of 2012, when European heads of state proposed the creation of the Single Supervisory Mechanism (SSM) to supervise bank debt within the union. By 2014, the Bank of Spain had transferred a substantial portion of its supervisory powers to the SSM. In addition, in June of 2012, European leaders also agreed to allow the European Stability Mechanism to offer direct help to Spanish banks. Finally, one month later, in July 2012, then president of the European Central Bank (ECB) Mario Dragui famously signaled the commitment of the institution to do "whatever it takes to preserve the Euro." That statement was interpreted at the time as a commitment from the ECB to buy Eurozone public bonds from distressed countries.[45]

Given that the baseline version of the model assumes no restrictions in international private debt and that the last two mechanisms of supranational bailouts are not explicitly modeled, I restrict the sample to the year prior to their introduction. As a consequence of this assumption, in the next section, I will use the comparison between the model and the data responses to the large financial shock as an out-of-sample validation.

Functional forms: The utility function is of the constant relative risk aversion (CRRA) form on the composite CES good

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \quad \text{with } \sigma > 0.$$

The default utility cost is parameterized as follows:

$$\phi(y^T) = \max\{0, \phi_0 + \phi_1 \ln y^T\}.$$

As Arellano (2008) and Chatterjee and Eyigungor (2012) discuss, a nonlinear specification of the default costs allows the model to reproduce the mean and standard

[44]Saurina and Trucharte (2017) provide a detailed account of the history of banking regulation in Spain and how it adapted to the adoption of international accounting standards during this period. For an overview of the current provisions, see Mencia and Saurina Salas (2016).

[45]For a discussion of how beliefs can be crucial for sovereign default incentives, see Cole and Kehoe (2000), Conesa and Kehoe (2017), and Aguiar et al. (2020).

deviation of spreads in the data. In particular, I follow Bianchi, Hatchondo, and Martinez (2018) in specifying the default cost function in terms of utility to avoid introducing an direct interaction between sovereign default decisions and private borrowing capacity.

Table 1.1: Parameters estimated outside of the model

Description	Parameter	Value
Risk aversion	σ	2.0
Elasticity of substitution	$1/(1+\eta)$	.83
Share of tradables	ω	.39
Persistence of tradables	ρ^y	.75
Volatility of tradables	σ^y	.010
Mean private default rate	$\bar{\pi}$	.021
Persistence private default rate	ρ^π	.82
Volatility private default rates	σ^π	.33
Risk free interest rate	r	.027
Duration of long-term bonds	δ	.14

Note: The risk aversion and elasticity of substitution between tradables and nontradables are standard in the literature. The share of tradables is the average share of value added of agriculture, manufacturing, and tradable services on GDP. The risk-free rate is the average yield of one-year German Treasury bonds. The duration parameter is chosen to match the average bond duration of six years of Spanish bonds. The tradable income and private default shock parameters are estimated by fitting a first order autoregressive process on the logs of the tradable share of GDP and share of nonperforming gross loans, respectively. All public bonds and yield data are from 1999 to 2011, and the GDP and nonperforming loans process are estimated using the longest available series. The data source for bond yields and nonperforming loans is Bloomberg, and the sectoral GDP series are taken from Eurostat. For details, see data Appendix A.3.

Estimated parameters: Table 1.1 shows the set of parameters that are estimated outside of the model. The risk aversion, σ, and elasticity of substitution between tradables and nontradables, $1/(1+\eta)$, are set at values frequently encountered in the literature.[46] To reduce the state space, the endowment of nontradables, y^N, is set to a constant normalized to one. I assume that the endowment of tradables is drawn from a first-order lognormal autoregressive (AR(1)) process. I estimate this process using the cyclical component of linearly detrended tradable GDP for Spain. Since the focus is on fluctuations around the business cycle, I use the cyclical component of the linearly detrended share of tradable output[47]. The estimated values for persistence and volatility, respectively, are $\rho^y = .75$ and $\sigma^y = .01$. The recursive specification is

$$\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y \quad \text{with } \varepsilon_t^y \sim N(0, \sigma^y).$$

[46]See for instance, Garcia-Cicco, Pancrazi, and Uribe (2010), and Bengui and Bianchi (2018).
[47]Details and sources in Appendix A.3.

The value of ω is chosen to replicate the share of nontradable GDP in the data, which is 60%. [48] To compute the model counterpart of this object at the ergodic distribution, I use the mean value of external private and public liabilities of $\bar{b}$ and $\bar{L}$ at their targeted values.[49] The value of ω is then set so that $\frac{\bar{p}^N y^N}{\bar{p}^N y^N + y^T} = 0.60$ where $\bar{p}^N = \frac{1-\omega}{\omega} \frac{y^T - r\bar{b} - \delta r \bar{L}}{y^N}$. Since the average tradable and nontradable endowments are one, this yields $\omega = 0.39$.

Similarly, I assume that the exogenous share of private bonds defaulted on each period follows a log-normal AR(1) process. The parameters of this process are estimated using the gross share of nonperforming loans as a percentage of total loans.[50] The estimation yields an average private default rate $\bar{\pi} = 2.1\%$, a persistence parameter $\rho^\pi = .82$, and a volatility $\sigma^\pi = .33$. The recursive specification of the process is

$$\ln \pi_t = (1 - \rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon_t^\pi \quad \text{with } \varepsilon_t^\pi \sim N(0, \sigma^\pi).$$

Two parameters affecting interest rates, r and δ, are estimated outside of the model. The risk-free interest rate is set to the average yield of the one-year German Treasury bill over the calibration period, $r = 2.7\%$. One-year bonds are chosen as a benchmark to reproduce the maturity of the short-term private bond in the model. The duration parameter δ is chosen so that average duration in the model corresponds to the average maturity of Spanish bonds in the data. Using Bank of Spain data, I find an average maturity of public debt of six years during the period of interest. This calculation is in line with previous estimates of Spanish maturity, such as Hatchondo, Martinez, and Sosa-Padilla (2016) and Bianchi and Mondragon (2018). The Macaulay definition of duration of a bond given the coupon structure of the model is

$$D = \frac{1 + \bar{i_L}}{\delta + \bar{i_L}},$$

where $\bar{i_L}$ is the constant per-period yield delivered by a long-term bond held to maturity (forever) with no default.[51] The implied duration is then $\delta = .14$.

Calibrated parameters: Six parameters are calibrated to match six aggregate moments from the Spanish data. The calibrated parameters are the two constants in the default cost function ϕ_0 and ϕ_1, the discount factor β, the standard deviation of the taste shocks σ^ϵ,

[48] Tradable GDP is computed using the value-added shares of agriculture, manufacturing, and tradable services. More details can be found in Appendix A.3.

[49] In the baseline calibration described below, $\bar{b} = 0.42$ and $\frac{\delta}{1+\frac{1-\delta}{1+r}} \bar{L} = .14$

[50] Details and sources are in Appendix A.3.

[51] In the baseline calibration, it corresponds to the targeted spread plus the risk-free rate, $\bar{i_L} = 3.1\%$.

Table 1.2: Calibrated parameters

Description	Parameter	Value	Moment	Target	Model
Discount factor	β	.92	Avg. total debt	.56	.56
Vol. taste shock	σ^{ϵ}	.020	Vol. total debt	.048	.050
Avg. financial shock	$\bar{\kappa}$	.45	Avg. private debt	.42	.42
Vol. financial shock	σ^{κ}	.020	Vol. private debt	.071	.058
Default cost	ϕ_0	.31	Avg. spread	.0045	.0045
Default cost	ϕ_1	1.9	Vol. spread	.0061	.0061

Note: Total and private debt are computed using the international investment position presented in Section 1.2. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. All moments are computed using data from 1999 to 2011. For additional details, see Appendix A.3.

and the constants determining the process of the financial shocks $\bar{\kappa}$ and σ^{κ}.[52] Table 1.2 shows a summary of all the targets and their model counterparts.

The parameters associated with the default costs ϕ_0 and ϕ_1 are measured in the data using the difference in returns between the average six-year Spanish bond and the average German bond of the same maturity. The targeted moments are the average and the standard deviation of this spread, and their model counterparts are the average and standard deviation of the spread paid by the long-term bond L_t. To compute the sovereign spread in the model that is implicit in a bond price Q, I use the definition of the constant per-period yield. Given the coupon structure, the yield satisfies

$$Q = \sum_{j=1}^{\infty} \delta \frac{(1-\delta)^{j-1}}{(1+\bar{i_L})^j}$$

The average targeted spread is 0.45% with a standard deviation of 0.47%, which implies values for the default cost parameters of $\phi_0 = .3$ and $\phi_1 = 1.9$. The targets are low when compared to the related literature since they are computed using data from 1999 to 2011. Other quantitative analyses of the sovereign debt crisis in Spain, such as Hatchondo, Martinez, and Sosa-Padilla (2016) and Bianchi and Mondragon (2018), focus on spreads only from a latter period, 2011-2015, and, consequently, target a higher spread. This paper deviates from that by including in the calibration the years 1999-2007 when the interest rate spread of Spanish government debt was very close to zero. Since the aim of the paper is to study the link between the buildup of private debt during those years and the subsequent

[52]The mean of the taste shocks is irrelevant for their quantitative properties and is selected to achieve numerical tractability. More details can be found in Appendix A.4.

sovereign debt crisis, it is important for the model to simultaneously match both the years with zero spreads and the large spikes observed during the crisis.

The discount factor β and the volatility of the taste shocks σ^{ϵ} are selected to match the average and standard deviation of the total debt. To compute the model counterparts of these measures I first calculate the international positions of the public and private sectors. The stock of public debt as a percentage of output at time t in the model is calculated for our coupon structure as the present value of future payment obligations discounted at the risk-free rate, that is, $\frac{\delta}{1+\left(\frac{1-\delta}{1+r}\right)} \times \frac{L_t}{(p_t^N y_t^N + y_t^T)}$. By contrast, the international position of the private sector as a percentage of output at time t is simply $\frac{B_t}{(p_t^N y_t^N + y_t^T)}$. At the calibrated values, $\beta = .92$ and $\sigma^{\epsilon} = .02$.

Finally, since the buildup in private debt in the years leading up to the crisis is a motivating fact of the model, the last two targeted aggregated moments are the average and standard deviations of the private debt. Note that because of the symmetry in the evolution of private and public stocks, the volatility of the private and public positions is higher than the volatility of the total debt. It is therefore important that the model matches not only the aggregate positions but also some of its decomposition. I calibrate the process of financial shocks κ_t to match this. As with the other exogenous shocks in the model, I assume that the financial shock follows a first-order normal AR(1) process of the form

$$\kappa_{t+1} = (1-\rho^{\kappa})\bar{\kappa} + \rho^{\kappa}\kappa_t + \varepsilon_t^{\kappa} \quad \text{with } \varepsilon_t^{\kappa} \sim N(0, \sigma^{\kappa}).$$

For simplicity, I assume that the persistence parameter coincides with the persistence of tradable income $\rho^{\kappa} = \rho$, while the mean ($\bar{\kappa}$) and volatility parameters (σ^{κ}) are estimated within the model. The model successfully replicates the average debt of the private sector and a higher volatility for the private position relative to the aggregate. However, it fits less well the large standard deviation seen in the data. At the baseline calibration, $\bar{\kappa} = .45$ and $\sigma^{\kappa} = .02$.

1.4.2 Untargeted business cycle properties

This subsection evaluates the model's quantitative performance by comparing untargeted moments from the data with moments from the model at the ergodic distribution. I compute the model's moments by simulating the exogenous processes for 10,000 periods and eliminating the first 500 observations. The moments from the data are computed with annual data for the sample period 1999-2017. The longer sample period is chosen to avoid small sample bias. Similar results are obtained when restricting the sample to the period 1999-2011. In Table 1.3, real GDP is equated with output, and consumption corresponds to total final consumption expenditure and is measured in real terms. GDP and consumption data are detrended. The current account and trade balance are computed as a percentage of GDP. All data are from Eurostat, and additional descriptions of the sources can be found in Appendix A.3.

Table 1.3 compares the unconditional second moments in the Spanish data with their baseline model counterparts at the ergodic distribution. The model successfully captures the volatility of consumption, of the current account and of the trade balance, and overestimates the volatility of output. Nevertheless, the model correctly predicts that the volatility of output will exceed the volatility of consumption. This contrasts with traditional sovereign default models where the opposite is true.[53] This suggests that explicitly modelling international private debt is important to simultaneously achieve a volatility of consumption and net capital flows consistent with the Spanish data. Table 1.3 also computes correlations between output and the other business cycle statistics. The model correctly predicts the sign of all the correlations.

Table 1.3: Untargeted business cycle statistics

Statistic	Data	Calibration
Volatility		
Output	.032	.062
Consumption	.031	.037
Current account	.041	.046
Trade balance	.034	.040
Correlations		
Output - Consumption	.97	.99
Output - Current account	-.59	-.91
Output - Trade balance	-.54	-.94
Output - Spread on public debt	-.46	-.10
Public debt - Spread on public debt	.53	.28

Note: Output corresponds to real gross domestic product and consumption to real final consumption expenditure, and both series are detrended. Current account and trade balance are measured as a percentage of output. Public debt corresponds to the international investment position of the public sector. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. For additional details, see Appendix A.3.

[53]Neumeyer and Perri (2005) find that consumption is more volatile than output in emerging economies whereas the opposite is true in advanced economies. Spain is listed by the International Monetary Fund (IMF) as an advanced economy.

1.4.3 Policy functions of private and public debt

To shed light on the workings of the model, this section shows an analysis of the policy functions for public and private debt accumulation. Both variables are functions of the exogenous shocks of the model and of the initial portfolio composition. To fix ideas, this section will first show how the accumulation of private and public debt varies with respect to the two main exogenous shocks, income and financial shocks. Then, I will show how both types of debt issuances vary with the endogenous states, the initial level of total debt and end-of-period public debt. Since the government acts first, the end-of-period private debt is a function of both the beginning of period debt of the country and the newly issued public debt. Considering the best response from the households, the government chooses the issuance of public debt optimally. For simplicity, the initial level of public debt has been set to zero in all the policy function plots, making all initial debt private. Nevertheless, all the implications follow through with a strictly positive level of initial public debt. Unless otherwise specified all debt levels are expressed as a share of mean output at the ergodic distribution.

Policy functions of private debt: Figure 1.4 depicts the optimal private debt accumulation as a function of the income and financial shocks. Panel (a) shows end-of-period private debt as a function of the endowment of tradable shocks, for the mean value of κ and π_t and for two possible values of initial debt. Panel (b) plots end-of-period private debt as a function of the financial shock, for the mean value of y^T, again for two possible values of initial debt.

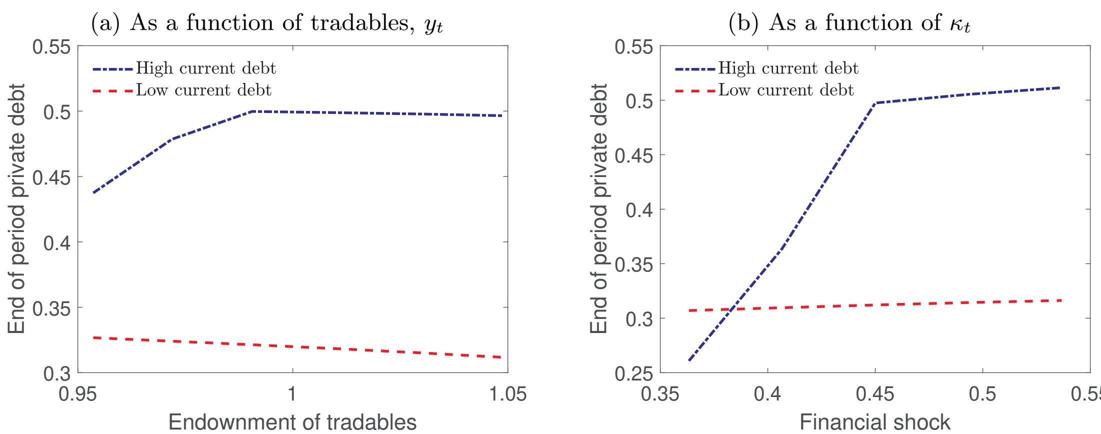

Figure 1.4: Policy function for private debt relative to the exogenous states

The figure shows that households' borrowing choices are most sensitive to the exogenous shocks when the households are facing a binding credit constraint. If the initial level of debt is low, represented by the dashed line in the plot, end-of-period private debt increases only slightly when income is low or the borrowing capacity is larger (smaller y^T or higher κ). However, if the current debt is high enough, households borrow up to their credit constraint. As a result, increases in the endowment of tradables or the value of the financial shock (higher y^T or higher κ) are met with equivalent increases in private borrowing.

Focusing now on the endogenous states, Figure 1.5 plots the law of motion of end-of-period private debt as a function of the initial level of debt, panel (a), and to next period public debt, panel (b). To help visualize the importance of the credit constraint, the total borrowing capacity of the private sector (debt limit) is plotted alongside the policy functions. In both panels, the exogenous shocks are kept constant. In the first panel, the level of end-of-period public debt is set at zero, and in the second panel, the starting level of debt is one standard deviation above the mean.

Panel (a) shows that for low levels of initial debt, the credit constraint does not bind, and end-of-period private debt increases with current total debt. The change in the sign of the slope of the policy function indicates the point at which the credit constraint is satisfied with equality. Beyond this point, higher levels of initial debt imply a lower level of tradable consumption. This in turn lowers the price of nontradables p^N and further restricts the borrowing capacity of the economy. This is therefore an illustration of the Fisherian debt deflation mechanism discussed in the previous section. As a result, similar policy functions can be seen Bianchi (2011) and Bianchi and Mendoza (2018).

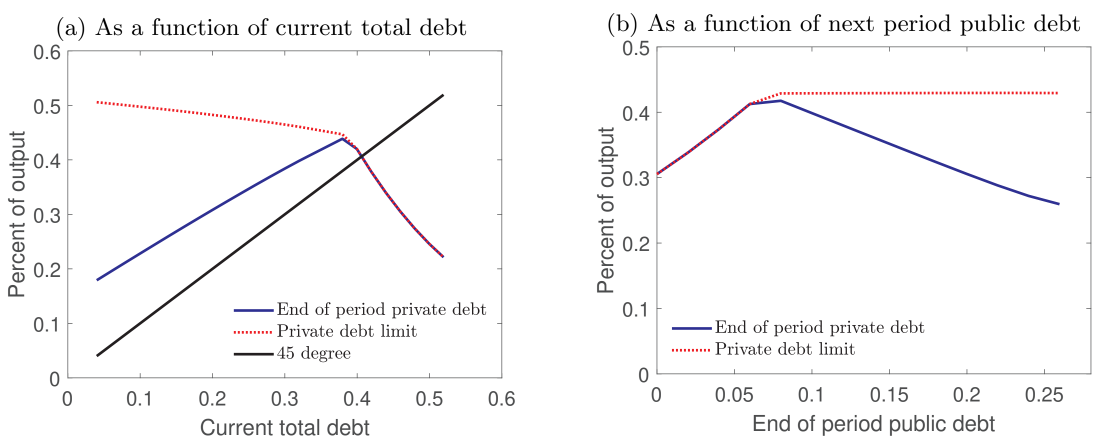

Figure 1.5: Policy function of private debt relative to the endogenous states

In contrast, panel (b) depicts the private sector response to the government's end-of-period debt and is novel to this paper. Low levels of end-of-period public debt imply a reduction in the fiscal transfer received by the household. At the plotted values, without substantial government assistance (above 8% of output), private borrowing will be constrained. Given the financial amplification mechanism described before, in this constrained area, higher government borrowing increases the consumption of tradables, the price of nontradables, the borrowing limit of the private sector, and private borrowing. This process comes to a halt once government assistance is large enough to ensure that the households will not borrow up to their limit. Further government borrowing continues to increase the transfer received by the households, but they now respond by borrowing less. For these states, private and public debt are substitutes.

Figure 1.6 shows the optimal public debt accumulation policy as a function of the income (panel (a)) and financial shocks (panel (b)). When initial debt is low, or when the endowment and the financial capacity κ are high, the optimal end-of-period debt remains mostly constant around a positive value. As in other models with multiple maturity assets, such as Arellano and Ramanarayanan (2012), long-term bonds provide rollover benefits relative to the short-term bonds. Long-term bonds provide more insurance against income fluctuation, which facilitates consumption smoothing. As a result, the government finds it optimal to always have a strictly positive level of public debt, even when the households are unconstrained. Since private and public debt are substitutes in these states, the government can issue debt at low spreads as long as total public debt remains low.

Policy functions of public debt: The government considers the household's best responses when choosing the level of public borrowing. Since the choice of public debt is also affected by the taste shock drawn, I now plot the expected level of next period public debt conditional on repayment. All values are plotted as a share of output. I start by showing public debt as a function of the income and financial shocks and then show how it changes with initial debt.

In contrast, when total debt is high, end-of period public debt varies differently with each type of exogenous shock. A low tradable endowment implies higher default risk and higher spreads, and therefore public borrowing decreases. Instead, an adverse financial shock (low κ) means that private borrowing is more likely constrained. Public debt in these cases has the twofold beneficial effect detailed in the previous section. Public debt allows for higher consumption when the households are constrained. This relaxes the credit constraint by depreciating the real exchange rate and allows for higher private borrowing. Thus, higher end-of-period public debt is desirable.

Finally, Figure 1.7 shows the expected level of end-of-period public debt as a function

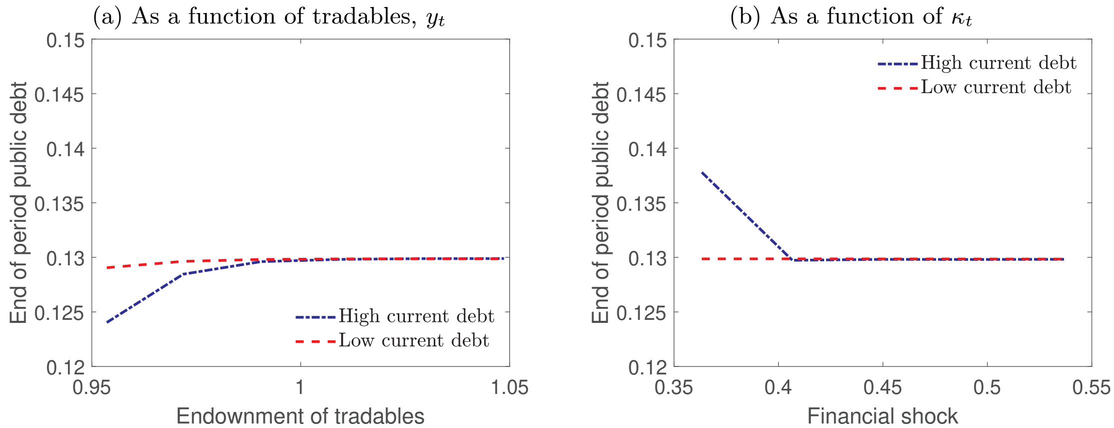

Figure 1.6: Policy function of public debt relative to the exogenous states

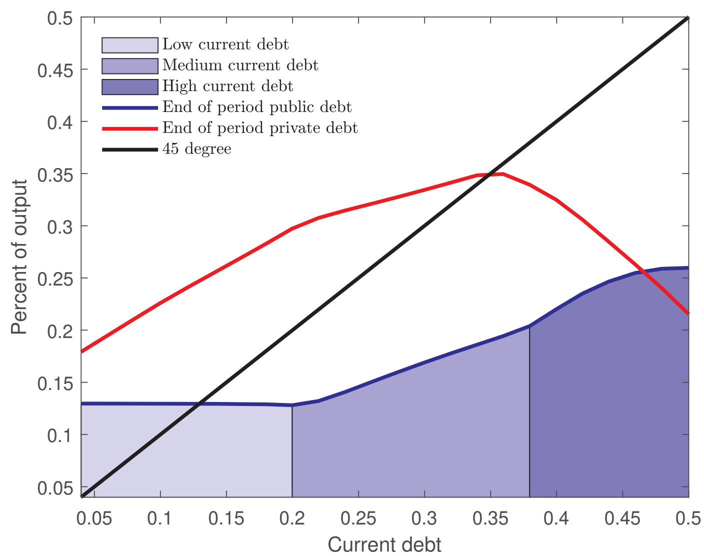

Figure 1.7: Expected end-of-period public and private debt as function of initial debt

of the current level of debt (blue line). To help visualize the situation of the households, the figure also shows the expected end-of-period private debt. All values are plotted as a share of output, and all exogenous shocks and the initial level of public debt are kept at constant values. Depending on the initial level of debt, three types of responses in terms of public debt are possible.

When the initial level of debt is low, issuances of public debt are kept relatively constant and low. Public debt is issued here because of its rollover benefits. Long-term debt allows the government to partially insure the households against transitory fluctuations in all exogenous shocks. Private debt is increasing in initial debt while public debt is almost constant. If the initial debt is large enough, however, the constraint for the private sector will bind if the government end-of-period debt is zero. At these medium levels of initial debt, households are not expected to face a credit constraint on average. The government is expected to transfer enough resources to the household so that the constraint will not bind. Consequently, private and public debt levels are increasing in the initial level of debt. The slope of private debt accumulation is smaller than in the previous case because households will be constrained in some states. Finally, if the initial level of debt is very high, it is never optimal to provide a large enough bailout that would prevent the households from facing a binding constraint. In these cases, issuances of public debt are at their highest. This is because in these states, public debt has a significant positive impact on the private borrowing capacity. The higher the initial level of debt, the more constrained the households are expected to end up, even after receiving transfers, and therefore the lower the level of end-of-period private debt.

Comparison with the socially planned economy: A social planner who controls the issuance of both types of assets would have similar policy functions. In this subsection, we compare those policies to those presented in the baseline model discussed above.

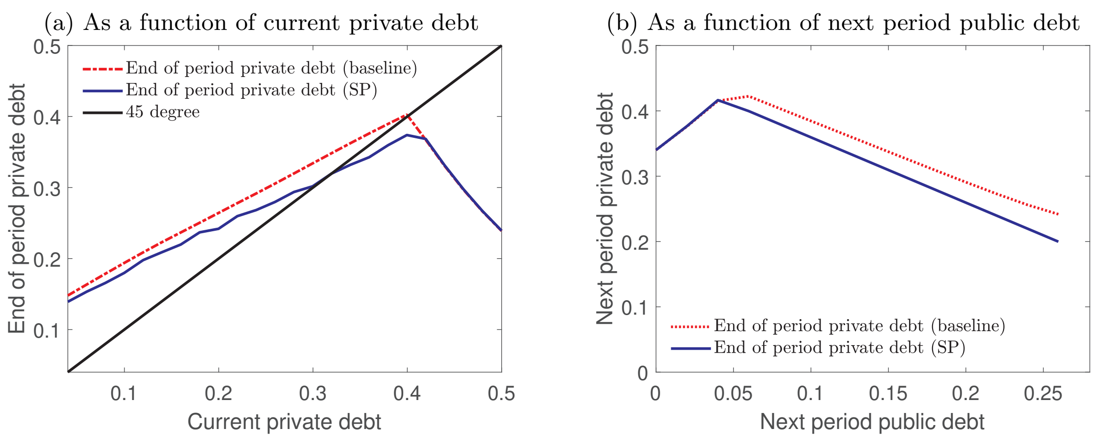

Figure 1.8: Policy function of private debt, baseline and SP

Figure 1.8 compares the evolution of end-of-period private debt in the baseline and socially planned economy as a function of the initial stock of private debt (panel (a)) and end-of-period public debt (panel (b)). In both panels, overborrowing in the baseline economy is present only when the constraint does not bind. When the constraint binds, private borrowing is pinned down by the resource constraints, and therefore there is no room for disagreement between the models. The sources of private overborrowing in both panels, however, are different. In the first panel, households overborrow for low levels of initial private debt because they do not internalize the marginal effect of their debt on the probability of facing a binding constraint next period. This figure is common to models of private overborrowing with a credit constraint that is increasing in the price of nontradables, such as Bianchi (2011) and Bianchi and Mendoza (2018). In contrast, the second panel is novel to this paper. Overborrowing is now caused by a smaller private borrowing response to government issuances of public debt. Unlike the planner, the households do not internalize that higher private debt increases the probability of sovereign default next period. Thus, individual households substitute less private debt for the same increase in public debt relative to the planner.

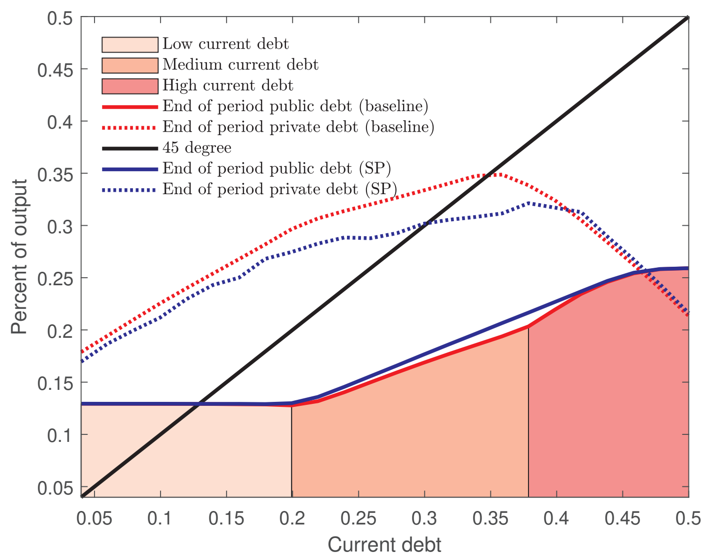

Figure 1.9: Expected end-of-period public and private debt as function of initial debt

Figure 1.9 compares the expected optimal level of public borrowing, conditional on repayment, in the baseline and socially planned economies as a function of the initial debt. As before, the households' private debt responses are plotted alongside the planners'. The figure also shows private overborrowing in the baseline model when the constraint does not bind. Public borrowing is higher in the planned economy when initial debt is small or medium. In these areas, the planner internalizes that it is approaching its borrowing capacity on the private bond and substitutes some of that borrowing with the public bond. The government in the decentralized economy would like to implement the same policy but does not control the issuances of the private bond. Correctly predicting that the household will not reduce private borrowing at the same rate as a planner would, the government decides to issue less public debt. The differences in public borrowing are, however, quantitatively smaller than the differences in private borrowing. As shown in the next section, when we compare the ergodic distributions, the small differences in public borrowing will not compensate for the fact that the baseline economy hits the credit constraint more often than the planned one. Consequently, the government must more frequently relieve the households by issuing public debt. When the constraint is expected to bind, the two economies mostly coincide.[54]

I also compare the evolution of the expected interest rate spreads paid on public debt in both economies conditional on repayment. Figure 1.10 plots the spreads as a function of the initial debt. The figure is computed at the same states as in Figure 1.9. The spreads peak when the debt enters the high debt zone. The shape of this plot shows that the interest rate spreads are mostly driven by the evolution of total end-of-period debt. Default is more likely in a more indebted economy. Up until the moment the constraint binds, both private and public debt are increasing with initial debt. Beyond this point, however, the private sector deleverages at a rate that outpaces the increase in public borrowing. As a result, total indebtedness decreases. This reduces the probability of default and the spread. In all cases, the spreads are higher in the baseline economy. This is the case even though Figure 1.10 shows that for medium or high levels of debt, the planner is expected to issue more public debt. The gap in interest rates exists because total debt is higher in the baseline economy as a result of household overborrowing. Anticipating this, foreign lenders demand a higher spread from the government.

1.5 Results: Quantitative implications of private overborrowing

This section details the main quantitative findings of the paper. The first subsection details the results obtained by comparing the baseline and regulated economies at their ergodic distributions. The second subsection details the quantitative exercises conducted to simulate the model dynamics during the Spanish Debt Crisis and the counterfactual in a socially planned economy.

1.5.1 Social planner and baseline economies at the ergodic

Table 1.4 presents the first set of quantitative results of the paper. The table shows the values first and second moments in the data and at the ergodic distribution of the baseline,

[54]The small amount of underborrowing in the baseline economy in this context is caused by fact that the planner faces a more favorable price schedule and therefore can relax the constraint a little bit more.

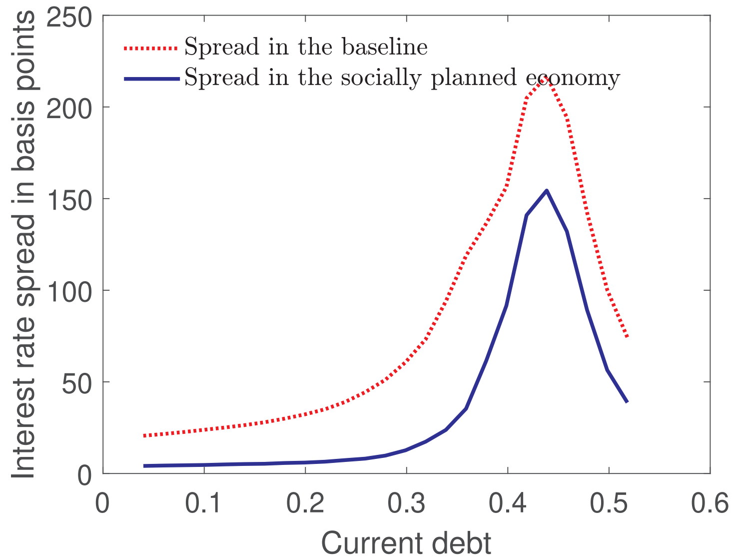

Figure 1.10: Expected spreads on public debt as function of initial debt

and the socially planned economies. The baseline version of the model is calibrated to match the moments from the data; the socially planned economy is not. Instead, I use the calibrated parameters of the baseline to compute the ergodic distribution of the planned problem. The average private debt at the ergodic distribution for the social planner is 36% of output, whereas in the baseline case it is 41%. This difference of 5% of output is the estimate of the total amount of excessive private debt in Spain in the lead-up to the crisis. The table shows that the increase in private debt, in the baseline relative to the planner, is insufficient to explain the increase in overall indebtedness. That is, the baseline economy accumulates on average more public debt, around 2% of output.

The explanation for why there us more public debt in the baseline can be seen in the bottom half of the table. In this part, I compute four measures of aggregate well-being for the baseline and planned economies, namely, the probability of a binding credit constraint, the probability of a financial crisis, the probability of a sovereign default, and a measure of welfare gains. The credit constraint binds more frequently under the baseline. As explained in the previous section, optimal government borrowing is higher when the constraint binds. As a result, average public debt is higher under the baseline because the government must respond more often to crisis. I define a financial crisis as an episode with a binding constraint and a contraction of more than one standard deviation below the mean of the current account of the private sector.[55] Under this definition, I find that excessive private borrowing

[55]Similar definitions are encountered in the related literature; see, for instance, Bianchi (2011) and Bengui

increases the incidence of financial crises by 2.40 percentage points on average.

Table 1.4: Baseline and social planner aggregate moments at the ergodic distribution

Moment	Data	Baseline	Social planner
Total debt	.56	.56	.49
Private debt	.42	.42	.37
Mean spread	.0045	.0045	.00034
Volatility debt	.048	.050	.027
Volatility private debt	.071	.058	.071
Volatility spread	.0061	.0061	.00030
Probability of a binding constraint	-	.099	.024
Probability of a financial crisis	-	.025	.0010
Probability of default	-	.0046	.00030
Welfare gains	-	-	.0041

Note: All calibrated parameters are kept constant in the computation of the socially planned economy. A financial crisis is defined as a episode in which the credit constraint binds and the current account of the private sector contracts by more than one standard deviation below the mean. Welfare gains are calculated as the proportional increase in permanent consumption under the baseline. Debt levels in the data are calculated using the international investment positions. More details are explained in Appendix A.3.

Furthermore, Table 1.4 compares the interest rate spreads paid on public debt relative to the risk-free rate. In the planned economy, spreads are on average an order of magnitude below their baseline counterparts. The reduction in the spread is caused both by the fact that the planner borrows less in general, and that it faces a binding constraint less often. The result is also consistent with the smaller average probability of sovereign default in the regulated economy relative to the baseline.

Finally, Table 1.4 computes the welfare gains of moving from the baseline to the planned economy. The welfare gains are calculated as the proportional increase in consumption for all possible future states that would make the households indifferent between staying in the baseline and moving to the centralized equilibrium. This measure explicitly incorporates the cost of lower consumption in the transition to the ergodic distribution of the planned economy. Taking advantage of the homoscedasticity of the utility function, the expected welfare gains in state (s_0, L_0, B_0) are

$$\theta(s_0, L_0, B_0) = \left(\frac{W^{SP}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1}{W(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1} \right)^{\frac{1}{1-\sigma}} - 1. \quad (1.25)$$

and Bianchi (2018).

On average at the ergodic state, households would need to receive a permanent increase of 0.36% in consumption to be indifferent between the two economies. These welfare gains are larger than the ones encountered in the literature. In Bianchi (2011), the welfare gains from correcting the overborrowing externality are around 0.13%. The welfare gains are larger in my model because optimal private debt management also decreases the probability of experiencing the deadweight losses of sovereign default.

1.5.2 Simulating the 2012 debt crisis

This section uses the data and the calibrated models to provide a model simulation of the events that unfolded in Spain between 2008 and 2015. To shed light on what optimal policies could have achieved, I also plot, alongside the baseline model and the data, the counterfactual dynamics of the socially planned economy. The idea is to feed into the model the exogenous shocks that affected Spain during this period and contrast the endogenous response in terms of debt and spreads of the baseline and socially planned models with their data counterparts. I conduct two exercises. In the first one, I feed into the model the three fundamental exogenous shocks: the income shock, the private default shock, and the financial shock. Public and private debt as well as the spread on public bonds are then allowed to respond endogenously to these shocks. In the second exercise, I impose as an additional restriction the evolution of public debt encountered in the data. I then compute the model-predicted dynamics of private debt and the interest rate spread. This second exercise therefore corresponds to the endogenous response of the baseline and socially planned economies when fixing fiscal policies to the data.

In both exercises, the exogenous income shock, y_t, is taken directly from the Spanish tradable GDP data. Similarly, the share of private bonds defaulted on, π_t matches exactly the data on gross nonperforming loans during this period, but again these latter shocks are not crucial. The taste shocks, ϵ_t, are all set to zero in the first exercise and selected to perfectly match the evolution of the public debt in the second. The financial shock, κ_t, is unobserved in the data. To circumvent this problem, I apply the particle filter method proposed by Bocola and Dovis (2019) to my model. Additional details about the particle filter method can be found in appendix A.6; here I present a summary of the methodology.

The baseline model defines a nonlinear state-space system A

$$Y_t = g(S_t) + e_t,$$
$$S_t = f(S_{t-1}, \varepsilon_t),$$

where $S_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$ is the state vector and ε_t the vector collecting all the innovations in the three structural exogenous shocks. The vector of observables, Y_t, includes average private and public debt as a share of GDP, detrended tradable output, the share of nonperforming loans, and the interest rate spreads on public bonds.[56] The vector e_t represents uncorrelated Gaussian measurement errors and is equal to the difference between the data aggregates Y_t and their model counterparts $g(S_t)$. The functions $g(\cdot)$ and $f(\cdot)$ come from the calibrated numerical solutions of the baseline model. The realizations of the state vector are estimated by applying the particle filter to this system of equations and data from 2008 to 2015. The process yields a path of financial shocks and a set of initial endogenous states. I then feed these shocks into the social planner policy functions $f^{SP}(\cdot)$ to generate the allocations of debts and spreads that would have emerged under counterfactual optimal policies. Note that the social planner functions are not used to estimate the system and are only used expost to generate counterfactuals. Finally, I also construct the implied tax on borrowing, necessary to implement the planner allocations in a competitive equilibrium.

First counterfactual

In the first exercise, I assume that only tradable output and nonperforming private loans are observed with no error. This leaves three observable variables not perfectly fitted in Y_t: public debt, private debt, and spreads. To match them, there are three stochastic variables in S_t, namely, B_t, L_t, and κ_t. By setting the variance of all measurement errors to 1% of their sample variance, I compute the filtered path of these three stochastic variables that is consistent with the data. Figure 1.11 summarizes the results of this exercise.

Positive counterfactual: The baseline model, plotted in dashed red lines, captures the main events of the crisis. In particular, the magnitude of the 2012 public bailout, around 12% of GDP, is financed by an equivalent increase in public debt. This leads to an increase in the interest rate spread on public bonds of around 3%, equivalent to 80% of the increase observed in the data. The baseline model is less successful at tracking the evolution of public debt after 2012, predicting a lower indebtedness than what is observed in the data. Similarly, the interest rate spread increase in the model before 2012 is below its data counterparts. Two observations could partially explain these discrepancies. First, while the model captures some of the fluctuations in the external conditions for borrowing via the financial shock, it may be the case that this shock is not enough to fully replicate the uncertainty around

[56] As in the calibration, I use the linearly detrended cyclical component of tradable output. Public debt is initialized at zero, and initial private debt is adjusted to match the composition of total debt in the data.

government bonds of Eurozone countries during the worst years of the Greek debt crisis. Second, there is no model counterpart to the Mario Draghi speech of June 2012 that can replicate its effect on public interest spreads. Accordingly, the model expects less public debt than the data to replicate the drop in spreads observed in the 2013-2015 period. All things considered, the baseline model predicts a pattern of public debt, private debt, and spreads that is consistent with the data and validates the approach of the paper.

Normative counterfactual: Having validated the positive model, I now turn to the normative counterfactual. In contrast to the baseline case, the socially planned economy is predicting a smooth transition from private liabilities to public debt. Instead of a large bailout in 2012, the planner deleverages in the private bond in three years. The dynamics allow the planner to maintain the interest rate spread near zero throughout the period and halves the size of the 2012 bailout to around 10% of GDP. Note that with the exception of 2012, private debt is lower in the planned economy in all years.

The government could have implemented this with a macroprudential tax on private borrowing that is on average 5% during this period. Similarly, public debt in the socially planned economy is significantly below the levels observed in the data for most of the period, and importantly, even after the bailouts take place.

Second counterfactual

In the first exercise, the 2012 spike in the spread of public debt would have been completely avoided if a planner had managed public and private borrowing optimally. To disentangle how much of the difference is caused by lower public borrowing and how much is caused by excessive private debt, I conduct a second counterfactual exercise. Taking advantage of the probabilistic framework of the model, I can select the taste shocks ϵ_t such that the path of public debt coincides exactly with the one observed in the data in both the baseline and planned economies. With the path of public debt restricted to the data values, the policy functions are used to compute the other endogenous series. The particle filter is then applied to back out the implied financial shock and the filtered endogenous evolution of private debt and the sovereign spread. As before, I then feed into the model this sequence of exogenous shocks to the planner policy functions to compute the counterfactual private debt, and spreads. Finally, I use the planner's policies to compute the optimal taxes on borrowing that could have decentralize these dynamics. The results of the second exercise are presented in Figure 1.12.

Positive counterfactual: The model once again predicts a drop in private debt of 20% of GDP, close in magnitude to the one observed in the data. Overall, private debt is around 5% below what is observed in the data for most of the period. The spread on public debt increases from close to zero in 2008, peaks in 2012, and then falls from 2013 onward. The magnitude of the increase between 2008 and 2012 is not the same in the baseline and the data, however, the model experiences a larger rise in 2012. The small mismatch in private debt and the larger spread are both consequences of the requirement to fit public debt exactly in this exercise. Nevertheless, the baseline model can still replicate the patterns of interest.

Normative counterfactual: Finally, I compare the evolution of the data and the socially planned economy. Private indebtedness in the planned economy is still lower than in the baseline and the data. In this exercise, the data on the evolution of public debt impose that the main bailout takes place in 2012. As a result, the public spread in the planned economy also peaks in 2012. The peak value is .4%, or 3.8 percentage points below the spread observed in the Spanish data. This is the lower bound estimate of the increase in the severity of the sovereign debt crisis caused by excessive private debt. It should be restated here that this estimate is obtained while keeping the paths of public debt at their data values. The reduction in the spread is therefore not a consequence of lower public borrowing but of the only other endogenous factor, private debt. In the planned economy, the lenders internalize that the regulator will pair the increase in high public debt with high taxes on private debt, which is 8% on average during the period. This leads to a reduction in private debt and thus reduces the probability of a sovereign default in the future.

1.6 Conclusions

This paper develops a theory that is quantitatively consistent with the evolution of debt and spreads in Spain that culminated in the 2012 sovereign debt crisis. The theory presented here is also consistent with the business cycle statistics observed in the data during this time period.

The model focuses on the interaction between systemic externalities in private credit and sovereign default. The combination of competitive private households whose borrowing is constrained to a fraction of the market value of their current income and a benevolent government capable of assisting them with public funds creates a pathway from financial to sovereign debt crises. The process begins with a buildup of private debt when financial conditions are favorable. During this time, public debt remains low and the government faces low spreads. As the private sector accumulates more debt, a financial crisis becomes more likely. Eventually an adverse shock materializes, and the households face a tight borrowing limit. In the model, I allow for a crisis to be triggered by the following exogenous factors: slowdowns in output, increases in private default, and shocks to international financial markets. Confronted with an imminent and painful private deleveraging, the government responds with fiscal transfers financed by new issuances of public debt. Bailouts have a multiplicative positive effect in this context. A positive transfer causes an appreciation in the value of collateral, and through this channel increases the borrowing capacity of the private sector. As a result, bailouts allow credit-constrained households to issue more private debt and further increase consumption. Unfortunately, these gains come at the expense of raising the specter of a sovereign default. In all cases, the spreads paid on government debt increase, and in some particularly adverse circumstances, default materializes.

The paper quantifies the level of excessive private borrowing and its impact. I estimate that in the lead-up to the crisis, excessive private debt in Spain was equivalent to 5% of GDP. As a result, the annual probability of experiencing a financial crisis was 2.4 p.p. above the socially desirable level. Finally, I estimate that private overborrowing raised the interest rate spread on public bonds by at least 3.8 p.p. at the peak of the sovereign debt crisis in 2012. As secondary findings, I calculate that private overborrowing raises the annual probability of a sovereign default from 0.03% to 0.46%.

I demonstrate that optimal borrowing policies could have been implemented by pairing public debt management with state-dependent taxes on private borrowing. I estimate an average tax rate of 5% during the crisis years. Finally, I find that the welfare gains of implementing optimal borrowing policies would have been equivalent to an increase of 0.41% in aggregate consumption. Several interesting avenues for future research remain open. It would be fruitful to investigate the quantitative consequences of introducinc moral hazard into the motivations for private overborrowing. Alternatively, one could explore how budgetary covenants or other fiscal limits could simultaneously deal with the incentives for bailouts and with public debt dilution, as in Hatchondo, Martinez, and Sosa-Padilla (2016) and Aguiar and Amador (2018). A final extension would be to investigate how a monetary response to private overborrowing would interact with the fiscal response presented here.

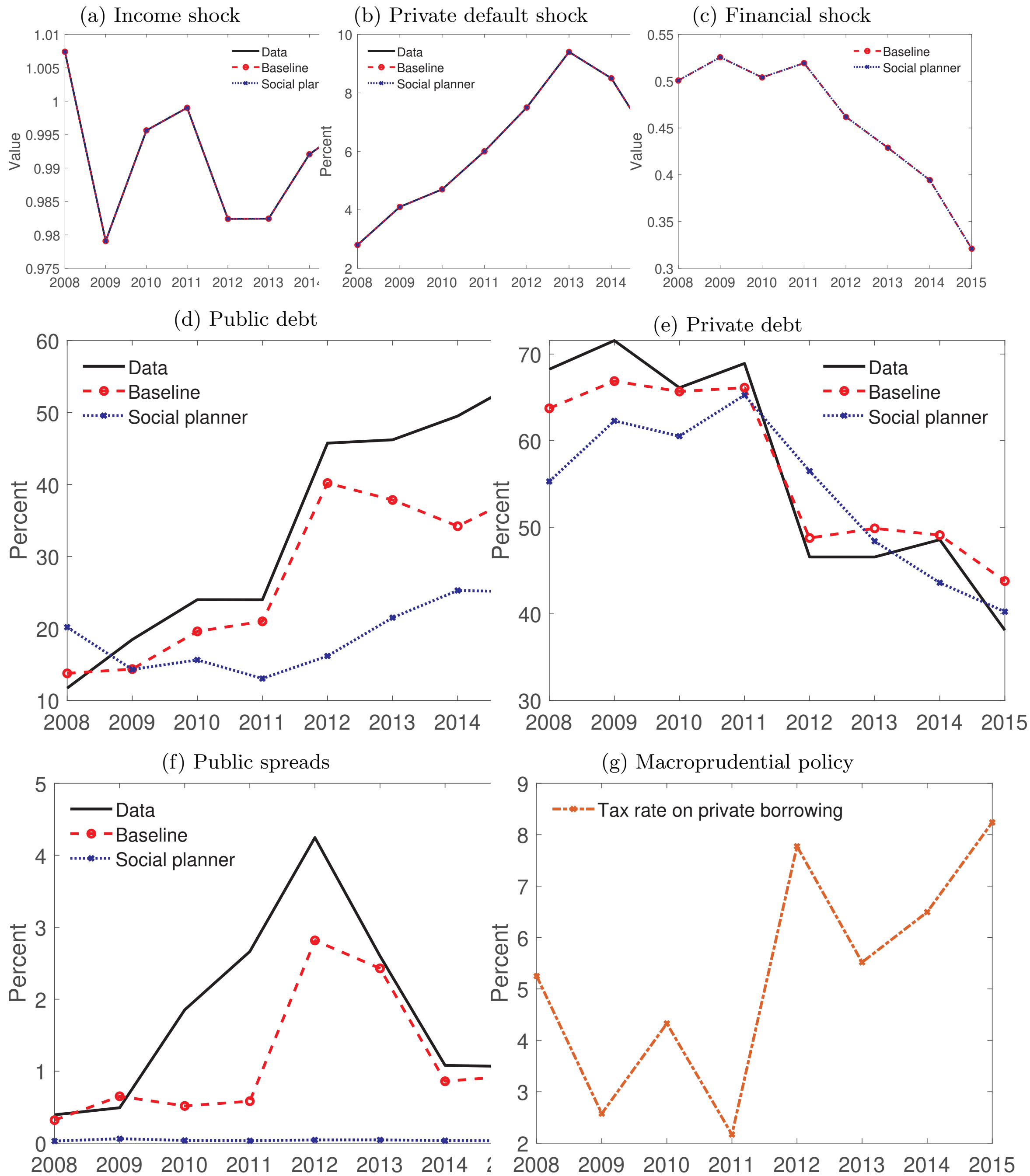

Figure 1.11: Evolution of debt, taxes, spreads, and exogenous shock, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and the most likely path of financial shocks from the particle filter. Public debt, private debt and spreads are the particle-filtered weighted averages. Both debt series are expressed as a percentage of output, while nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix A.3, while details on the particle filter can be found in Appendix A.6.

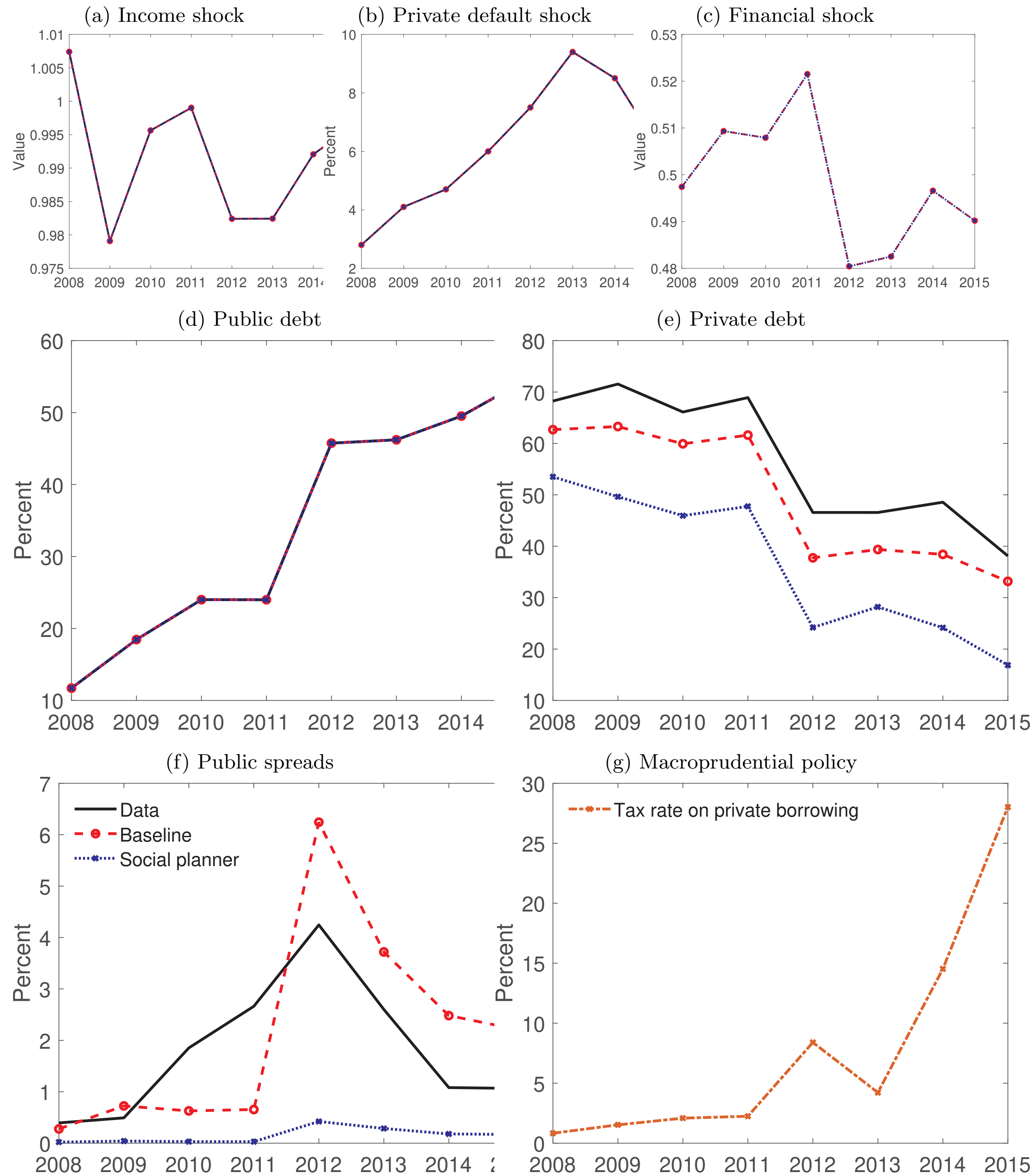

Figure 1.12: Evolution of debt, taxes, spreads, and exogenous shocks, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and taste shocks to match exactly the evolution of public debt. The most likely path of financial shocks is computed using the particle filter. Private debt and spreads are filtered weighted averages. Both debt series are expressed as a percentage of output, while nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix A.3, while details on the particle filter can be found in Appendix A.6.

Chapter 2

A Macroprudential Theory of Foreign Reserve Accumulation

2.1 Introduction

Central banks' holdings of international reserves have nearly quadrupled since the wave of financial globalization of the 1990s. Yet despite an extensive literature, accounting for this surge and the large variation in reserve holdings across countries has remained elusive. In this paper, we propose a theory of foreign reserve accumulation based on a macroprudential motive and show that it can quantitatively account for the buildup of international reserves while being consistent with salient cross-sectional patterns of capital flows.

Our theory is motivated by the intertwined relationship between foreign reserves and private external debt prevalent among middle-income countries. Four facts, which we document in Section 2.2, guide our analysis. First, on the aggregate, the increase in foreign reserves has occurred concomitantly with an increase in private external debt. Second, in the cross-section, the rise in reserves has been high in countries that also experienced an increase in external private debt. Third, reserves and private external debt accumulation appear to be procyclical and positively correlated over time for most countries. Fourth, reserve holdings are larger in economies with a more open capital account.

We argue that these facts point to a hypothesis linking international reserves to the government's prudential management of private capital flows. Few models of international capital flows, however, study explicitly the interaction between private and public capital flows. The literature has focused either on private or public flows, or considered a single borrowing agent without distinguishing between the two categories of flows.[1] A first

[1] A notable early exception is Benigno and Fornaro (2012), which incorporates imperfect substitutability

contribution of our paper is to provide a framework that can quantitatively speak to the evolution of private and public capital flows within a unified setup.

The environment we consider is a two-sector small open economy model with incomplete markets and inefficient private borrowing. Households' borrowing is limited by an occasionally binding credit constraint that depends on income and links the borrowing capacity to the real exchange rate (Mendoza, 2002, Bianchi, 2011). The government accumulates non-state contingent assets, which we refer to as reserves. When an adverse shock hits and the economy is highly leveraged, households hit their credit constraint and become unable to smooth consumption. The contraction in spending leads to a depreciation of the real exchange rate, which further tightens the borrowing constraint and leads to a "sudden stop" in capital inflows. Reserves, in this context, provide a liquidity buffer to mitigate the contraction in capital inflows.

Our second contribution is to show that reserves play a macroprudential role. We demonstrate that the constrained-efficient allocation (i.e., the allocations that would prevail if the government were to make all financial decisions on behalf of private agents) can be implemented via reserve policy. When households deleverage, they fail to internalize how the contraction in spending leads to a real exchange rate depreciation, further tightening economy-wide borrowing constraints. As a result, they do not face proper incentives to accumulate reserves in good times, when the credit constraint is not binding.

In response to the government's accumulation of reserves, households end up borrowing more by a Ricardian equivalence logic, but ultimately, once the government accumulates a large enough stock of reserves, households become borrowing constrained and are unable to offset the government policy. A key insight is that the very same credit constraint that is the source of the overborrowing externality makes reserve accumulation by the government effective. While gross debt increases under this government policy, the net foreign asset position improves, leading to a reduction in the frequency and severity of sudden stops.

A quantitative analysis shows that the macroprudential motive for reserves can go a long way towards accounting for the intertwined relationship between private and public capital flows observed in the data. In particular, model simulations can account for the four aforementioned facts. The model generates the observed upward trend in reserves and debt, the positive correlation between increases in reserves and debt in the cross-section, the procyclicality of debt and reserves over the business cycle, and the positive correlation between the degree of financial liberalization and reserves across countries.

We also show that our results are both theoretically and quantitatively robust to

between private and public capital flows.

several extensions. We first extend the analysis to an economy with production and labor mobility across the two sectors. Although the constrained-efficient allocation may feature *higher* borrowing than the laissez-faire one (Benigno et al., 2013b), we show that reserve accumulation remains an equally effective tool despite inducing a *lower* net level of borrowing in the economy. The insight is that for reserves to be effective, what matters is not whether the laissez-faire economy features over- or underborrowing relative to the constrained-efficient allocation but whether, at the margin, households face a lower or a higher shadow cost of repayment than the planner. Thus, even when ex post stabilizing interventions make higher borrowing more efficient, as long as households face lower shadow costs of repayment than the planner at the margin, reserve accumulation remains effective at achieving the constrained-efficient allocation.

Reserve accumulation remains desirable even when we restrict the government to follow a simple policy rule. Notably, a simple rule reduces the magnitude of the current account reversal in a sudden stop by 5% of GDP relative to the laissez-faire economy. This result is important in light of the quest by central banks for simple implementable rules. In this regard, our work complements the prominent formula for reserves proposed by Jeanne andRanciére (2011).[2] In addition to modelling reserves as a simple non-state contingent asset, a distinctive feature of our analysis is that it considers the strategic interaction between the government and households' financial decisions.

Literature. Our paper is related to several areas of research. First, it belongs to a large literature seeking to explain the demand for international reserves. The idea of a precautionary motive for reserves has a long tradition going back to Kenen and Yudin (1965), Heller (1966), Clower and Lipsey (1968), Clark (1970), and Kelly (1970). More recently, precautionary theories have focused on shocks to income or shocks to countries' access to credit markets. 2009 examine how the risk of sudden stops affects the determination of net foreign asset positions. Caballero and Panageas (2008) and Jeanne andRanciére (2011) model reserves as insurance contracts against the risk of sudden stops. In 2018, reserves are modeled as non-state contingent assets that provide insurance against rollover risk. Bianchi and Sosa-Padilla (2020) examine a macroeconomic stabilization channel and its interaction with a precautionary motive.[3]

Our paper shares with this literature the broad idea that reserve accumulation serve

[2]In addition to the Jeanne and Ranciére rule, the IMF also uses the Greenspan-Guidotti rule for surveillance purposes (see International Monetary Fund, 2011).

[3]Other papers with related approaches include Aizenman and Lee (2007); Bacchetta, Benhima, and Kalantzis (2013); Hur and Kondo (2016); and Jeanne and Sandri (2017), among others.

a precautionary role by reducing the severity of sudden stops. The existing literature, however, considers a single decision maker within a country, which can control all external financial decisions. This single decision-maker is often mapped to the government, but it applies equally to a representative household. While this approach has led to fundamental insights, it is silent on the question of why it is the government that has to accumulate reserves. Our paper tackles this question and underscores the presence of an externality by which private agents do not have incentives to accumulate reserves on their own.

Our paper is closely related to Benigno and Fornaro (2012), who also consider the interaction between private and public capital flows in a dynamic quantitative model in which reserves are motivated by the presence of a learning-by-doing externality in the tradable sector. They show that in the absence of industrial policies, accumulating reserves is desirable to undervalue the real exchange rate and foster export-led growth, therefore providing a rationale for the so-called "Bretton Woods II" international monetary system perspective. Our work is complementary in that it articulates a motive for reserve accumulation based on a macroprudential motive. Moreover, we solve for the optimal intervention and show that the macroprudential motive can go a long way towards accounting for the observed levels of reserves and the interaction between private and public capital flows.[4]

Our paper also relates to the literature that studies foreign exchange interventions in the presence of limits to international arbitrage. Examples include Cavallino (2018), who shows how foreign exchange interventions can deal with dynamic terms of trade externalities and capital account shocks; 2017, who show that reserve accumulation is needed to implement exchange rate policies at the zero lower bound; and Fanelli and Straub (2017), who characterize optimal policies when real exchange rate fluctuations lead to distributional consequences. While a common theme in these papers is that international intermediaries have limited leverage capacity, building on Gabaix and Maggiori (2015), our focus is instead on frictions in domestic financial markets. In addition, a key distinction of our paper is that we study the scope for reserve accumulation due to a financial stability motive. In this respect, our paper is related to the arguments in Calvo (2006) and 2010 and is complementary to Bocola and Lorenzoni (2017), who show that reserves can enhance the credibility of lender of last resort policies by relaxing fiscal constraints.

Our paper also relates to the literature on financial crises and macroprudential policy. This literature has shown how Pigouvian taxes on borrowing and capital requirements can

[4]As Benigno and Fornaro (2012), our paper can account for the empirical finding that countries that accumulate reserves support a more depreciated real exchange rate (Rodrik, 2008), albeit for a different reason.

correct pecuniary externalities that generate excessive systemic risk (e.g., Lorenzoni, 2008; Bianchi, 2011; Bianchi and Mendoza, 2018; Jeanne and Korinek, 2019). We complement this literature by studying the role of international reserves as a macroprudential policy. Moreover, we demonstrate that the accumulation of international reserves constitutes an alternative policy tool that can substitute for the use of financial regulation policies.

Our finding that private borrowing rises as a result of an optimal policy intervention is shared with studies arguing for the possibility of underborrowing. The two most closely related are Benigno et al. (2013b) and Schmitt-Grohé and Uribe (2020).[5] In Benigno et al. (2013b), the government has access to a richer set of tax instruments, enabling it to relax borrowing constraints ex post, which results in more borrowing ex ante than in the laissez-faire economy. In Schmitt-Grohé and Uribe (2020), the government intervention induces more borrowing when there is a possibility of multiple equilibria and the bad equilibrium is selected. In contrast with these studies, our model distinguishes between private and official flows, and we find that the optimal intervention entails higher gross private borrowing but, at the same time, an improved net foreign asset position for the economy as a whole.

Finally, our paper is related to a large empirical literature on capital flows. Particularly relevant is the empirical work on the precautionary motive for reserves (e.g., Edwards, 1983; Frankel and Saravelos, 2012; Bussiere, Cheng, Chinn, and Lisack, 2013; Calvo, Izquierdo, and Loo-Kung, 2013). Our empirical and theoretical analysis emphasizes the interaction between private and public capital flows and the importance of considering gross positions, as stressed in Obstfeld (2012).

The paper is organized as follows. Section 2.2 outlines the motivating facts. Section 2.3 presents the model and the main theoretical result. Section 2.4 contains a quantitative analysis, and Section 2.5 concludes.

2.2 Motivating facts: reserves, debt, and capital mobility

In this section, we present empirical evidence on international reserves and private external debt that illustrates the intertwined relationship between these two variables. We use data for middle-income countries from 1980 to 2015.[6] The data for private external debt are from the International Debt Statistics collected by the World Bank and measure

[5]See also Bianchi (2016), Bornstein and Lorenzoni (2018), and Jeanne and Korinek (2020).

[6]The complete list of countries, based on data availability and other considerations detailed in appendix B.1, is Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Sri Lanka, Thailand, Tunisia, Turkey, and Venezuela

private external debt as non-publicly guaranteed external debt.[7]

We summarize the evidence in four facts:

FACT 1: *Over the past three decades, there has been a concomitant substantial increase in private external debt and international reserves.* Figure 2.1 shows the evolution of the GDP weighted average of private external debt and reserves from 1980 to 2015.[8] Until 1990, both international reserves and private external debt were below 5 percent of total GDP for the average middle-income country. By 2015, reserves and private external debt reached, respectively, 16 percent of GDP and 12 percent of GDP. It is worth noting that the sharp rise in private external debt contrasts with the decline in publicly guaranteed external debt (PGD) in the countries in our sample. Over the same time period, PGD decreased from 27 percent of GDP in 1980 to 14 percent of GDP in 2015.

FACT 2: *Foreign reserve growth has been particularly high in countries that have also experienced high growth of external private debt.* Figure 2.2 shows a scatter plot of the differences between 2015 and 1980 values of the ratios of private external debt to GDP (x-axis) and reserve to GDP (y-axis) for the sample of countries considered, with each dot representing a country. It documents a positive correlation between growth in reserves and growth in private external debt across countries during our sample period.[9]

The cross-sectional association between international reserves and private external debt is also apparent from panel regressions. Table 2.1 reports results from estimations of regressions of logged reserves-to-GDP ratios onto logged private external debt-to-GDP ratios. In the first two columns, we report results of pooled ordinary least squares (OLS) regressions, while in the last two, we report results of regressions with time and country fixed effects. The regressions of columns 1 and 3 include a constant, while those of columns 2 and 4 also control for the logged ratio of the ratio of PGD to GDP, the net foreign assets to GDP, and real GDP growth. In all cases, the coefficient on private external debt is positive and statistically significant at the 1 percent confidence level, indicating a robust statistical association between private debt and reserves.[10]

[7] An advantage of using data from the International Debt Statistics is that it allows us to differentiate PGD and non-PGD. This distinction is important, as some middle-income countries in our sample have large publicly owned companies that issue debt internationally.

[8] This trend also holds when we look at simple averages. Notice that Figure 2.1 excludes China.

[9] 2010 report a positive correlation between *domestic* private debt and reserves. We document the association with external debt instead.

[10] Very similar results are obtained when conducting the regressions in first-differences for reserves and debt, as shown in the appendix.

FACT 3: *The accumulation of reserves and private external debt are correlated with each other over time and are procyclical for most countries.* Figure 2.3 displays correlations of real GDP growth with the growth rates of reserves and of private debt (panels a and b, respectively), and between the growth rates of reserves and private debt (panel c). Similarly to Bianchi, Hatchondo, and Martinez (2018), we find that reserves growth correlates positively with output growth for a wide majority of countries. In addition, we find a positive correlation between the growth of private external debt and output growth for most countries. Finally, in line with Broner et al. (2013), we find that inflows of private debt correlate over time with outflows of international reserves for a majority of countries.

FACT 4: *Reserve holdings are larger in economies with a more open capital account.* Figure 2.4 shows a scatter plot of the Chinn and Ito (2008) index of capital account openness and the average ratio of reserves-to-GDP. It shows a positive correlation between reserves and capital account openness, in line with the evidence presented by Aizenman and Lee (2007) and Bussiere et al. (2013). In other words, emerging countries that impose significant controls on international private flows of capital tend to have relatively smaller ratios of reserves-to-GDP than countries with more liberalized capital accounts.

To summarize the empirical evidence that motivates our theoretical analysis, the data shows that reserves and private external debt are deeply intertwined. The relationship is apparent from time-series and cross-sectional data. In particular, we highlight four facts: (i) there has been a substantial increase in private external debt and international reserves in the aggregate; (ii) there is a positive correlation between reserves and private external debt in the cross-section; (iii) reserve and private external debt accumulation are procyclical and correlate positively with each other; (iv) reserve holdings are larger in economies with a more open capital account.[11]

We next propose a theory that sheds light on the interplay between private external debt and reserves. In our model, both variables are endogenous, and their dynamics are consistent with the four aforementioned facts.

2.3 Model

We consider a dynamic small open-economy model with tradable and non-tradable goods. The economy is populated by a continuum of identical households that borrow

[11] While these observations indicate positive associations, they do not point prima facie to any causality in *either* direction.

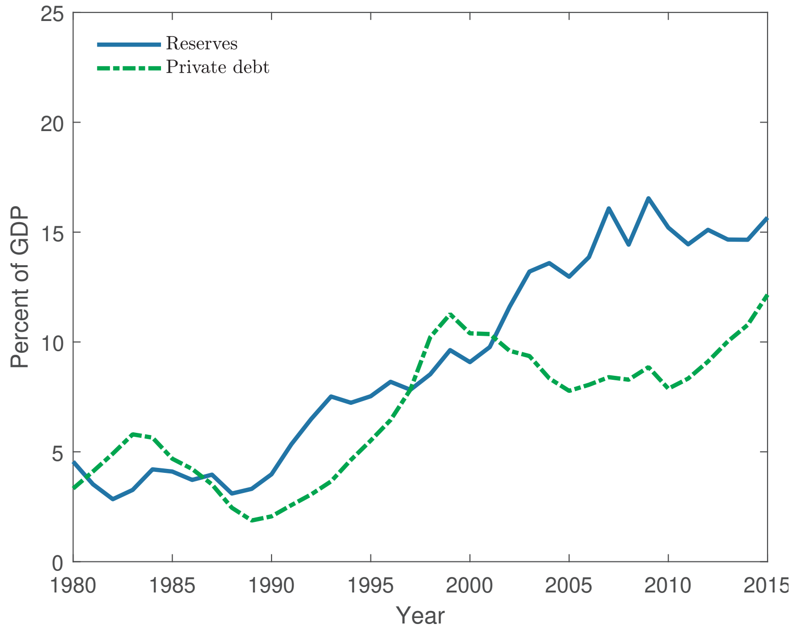

Figure 2.1: Evolution of reserves and private debt (GDP-weighted average)

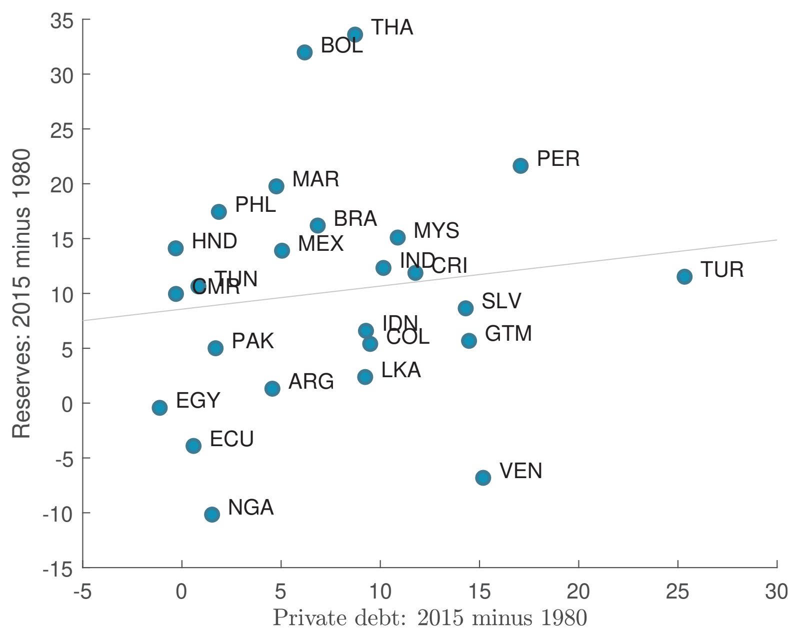

Figure 2.2: Increases in private external debt and reserves in the cross-section

Note: The x-axis denotes the increase in private debt to GDP between 2015 and 1980. The y-axis denotes the increase in reserves to GDP between 2015 and 1980.

externally subject to an occasionally binding borrowing constraint. We describe first the households' problem, and then we analyze the competitive equilibrium and the role of

Table 2.1: Reserves-to-GDP Ratios on Private External Debt-to-GDP Ratios (in logs)

	Reserves	Reserves	Reserves	Reserves
Private External Debt	0.293***	0.267***	0.0915***	0.0881**
	(0.0267)	(0.0273)	(0.0274)	(0.0276)
Net Foreign Assets	0.0145***	0.00829***	0.00703***	0.00870***
as a share of GDP	(0.00126)	(0.00180)	(0.00129)	(0.00165)
Publicly Guaranteed		-0.289***		0.112
External Debt		(0.0649)		(0.0725)
GDP Growth Rate		0.00254		-0.00101
		(0.00194)		(0.00186)
Observations	875	875	875	875
Countries	25	25	25	25
Pooled OLS/ Fixed Effects	pooled	pooled	FE	FE

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

international reserves.

2.3.1 Households' problem

Households' preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2.1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada conditions; and consumption c_t is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta + 1)$ between tradable goods c_t^T and non-tradable goods c_t^N, given by

$$c_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \qquad \text{with} \quad \eta > -1, \omega \in (0,1).$$

In each period, households receive a random endowment of tradable goods y_t^T and a fixed endowment of non-tradable goods y^N. We use the tradable good as the numeraire.

Households can borrow (or save) using a one-period non-state-contingent bond b_{t+1} denominated in units of tradables paying an interest rate R_t, which is exogenously

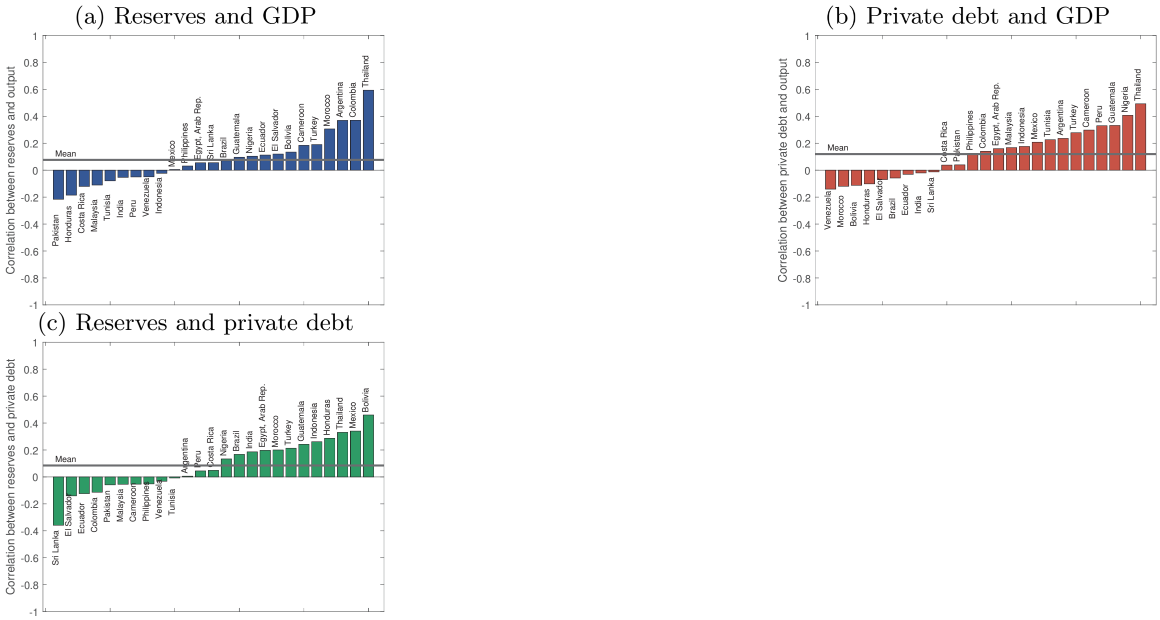

Figure 2.3: Time-series correlations

Note : Correlation between the growth rates of real GDP and growth rate of reserves (panel a), growth rate of real GDP and growth rate of private debt (panel b), and growth rate of reserves and growth rate of private debt (panel c)

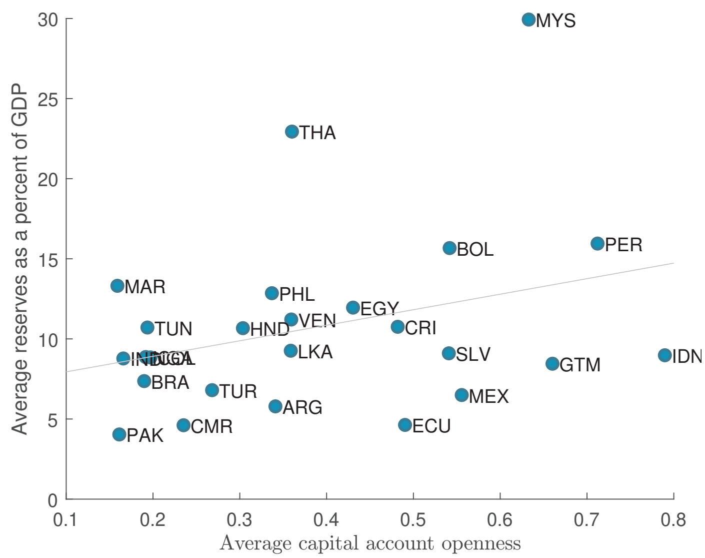

Figure 2.4: Average 1980–2015 reserves and Chinn and Ito (2008) capital account openness

determined in international capital markets and may vary stochastically.[12] Their budget

[12] Assuming no foreign inflation, it is equivalent to denominating the bonds in foreign currency, capturing the liability dollarization phenomenon.

constraint, in units of tradables, is given by

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} = y_t^T + p_t^N y^N - b_t - T_t, \tag{2.2}$$

where p_t^N is the price of non-tradable goods and T_t is a lump-sum tax. In addition, households face a credit constraint given by

$$\frac{b_{t+1}}{R_t} \leq \kappa_t \left(y_t^T + p_t^N y^N \right). \tag{2.3}$$

This credit constraint captures in a parsimonious way the empirical fact that income is critical in determining credit-market access, and it has been shown to be important for accounting for the dynamics of capital flows in emerging markets (e.g., Mendoza, 2002). Non-tradable goods enter the collateral constraint because while foreign creditors do not value these goods directly, they can seize them in the event of default and sell them in exchange for tradable goods on the domestic market.[13] We allow for shocks to the parameter κ_t, which we refer to as a financial shock. One interpretation of this shock is that it captures fluctuations in lenders' perceptions about households' ability to repay or in the country's institutional contract enforcement.

Households choose consumption and borrowing to maximize their utility (2.1) subject to their budget (2.2) and credit constraint (2.3), taking prices and taxes as given. Their optimality conditions are given by

$$p_t^N = \frac{1-\omega}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{\eta+1}, \tag{2.4}$$

$$\lambda_t = u_T(t), \tag{2.5}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t \quad \text{with } \mu_t = 0 \text{ if } b_{t+1}/R_t < \kappa_t \left(y_t^T + p_t^N y^N \right), \tag{2.6}$$

where $u_T(t)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$ and μ_t denotes the non-negative Lagrange multiplier on the borrowing constraint. Condition (2.4) is a static optimality condition equating the marginal rate of substitution between tradable and non-tradable goods to their relative price. Condition (2.5) equates the marginal utility of tradable consumption to the shadow value of current wealth, and Condition (2.6) is the household's Euler equation

[13] The credit constraint can be derived endogenously from a problem of limited enforcement under two assumptions. First, households can default at the end of the current period. Second, upon default, foreign creditors can seize a fraction κ_t of the current income, and households immediately regain access to credit markets. The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks (see Bianchi and Mendoza, 2018, for a derivation of a similar constraint).

for debt. When $\mu_t > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit and repaying next period.

2.3.2 Government

The government accumulates international reserves $A_{t+1} \geq 0$ and finances them with lump-sum taxes and existing holdings of reserves A_t.[14] Its budget constraint is given by

$$\frac{A_{t+1}}{R_t} = T_t + A_t. \tag{2.7}$$

2.3.3 Competitive equilibrium

The market clearing condition for non-tradable goods is

$$c_t^N = y^N. \tag{2.8}$$

We can now define a competitive equilibrium for any government policies. Given initial conditions A_0, b_0, and government policies $\{T_t, A_{t+1}\}_{t\geq 0}$, a *competitive equilibrium* is defined as a stochastic sequence of prices $\{p_t^N\}_{t\geq 0}$ and households' policies $\{c_t^T, c_t^N, b_{t+1}\}_{t\geq 0}$ such that (i) households maximize their utility (2.1) subject to the sequence of budget constraints (2.2) and credit constraints (2.3), taking as given prices and government policies; (ii) the government budget constraint (2.7) is satisfied; and (iii) the market for non-tradable goods clears (2.8).

Combining the household's budget constraint (2.2) with the government's budget constraint (2.7) and the non-tradable goods market clearing (2.8), we obtain the economy's consolidated resource constraint for tradable goods:

$$c_t^T + \frac{A_{t+1} - b_{t+1}}{R_t} = y_t^T + A_t - b_t. \tag{2.9}$$

This constraint illustrates that from the perspective of the resource constraint in the small open economy, official reserves and household-issued bonds are perfect substitutes. Absent the credit constraint (2.3), Ricardian equivalence would hold and the amount of foreign

[14] We could allow the government to finance reserve accumulation with domestic debt, in addition to taxation (motivated by tax-smoothing considerations). In this extended setting, the government would offer a high interest rate on domestic bonds to alter households' intertemporal consumption decision. Provided that foreign investors are excluded from the domestic bond market, the allocations would be isomorphic, with the domestic rate being determined by $\frac{u_T(t)}{\beta u_T(t+1)}$. If foreign investors had access to domestic bonds, they would earn a rent at the expense of the small open economy, generating an extra cost from interventions (see Amador et al., 2017; Fanelli and Straub, 2017).

reserves accumulated by the government would be completely irrelevant. However, as we argue below, the presence of the credit constraint (2.3) makes reserve accumulation both *relevant* and *desirable.*

Remark on households' accumulation of reserves. Notice that we have not explicitly considered the accumulation of the reserve asset by households, but this is without loss of generality. When the borrowing constraint is binding, the return on reserves R_t is lower than the shadow return on borrowing, and so households optimal choice of reserves is zero. When the borrowing constraint is not binding, households are indifferent between reserves and debt because both debt and reserves have the same maturity and deliver a risk-less return R_t in units of tradables.

2.3.4 Constrained efficiency

The competitive equilibrium is constrained inefficient due to a pecuniary externality. Households do not internalize that by borrowing more in the present and consuming less in the future, they put downward pressure on the future price of non-tradables and thereby contribute to tightening other agents' future credit constraint. Following Bianchi (2011), we consider the problem of a constrained social planner who directly chooses the economy's debt subject to the borrowing constraint and allows goods markets to clear competitively. In recursive form, the problem can be written as:

$$V(b, y^T, R, \kappa) = \max_{b', c^T} u(c(c^T, y^N)) + \beta \mathbb{E} V(b', y^{T'}, R', \kappa') \tag{2.10}$$

subject to

$$b + c^T = y^T + \frac{b'}{R}, \tag{2.11}$$

$$\frac{b'}{R} \leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{c^T}{y^N} \right)^{\eta+1} y^N \right]. \tag{2.12}$$

where the substitution of the price of non-tradables expression (2.4) into (2.12) reflects the implementability constraints of the planner.

Using sequential notation, for convenience, the planner's Euler equation for debt is given by

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{2.13}$$

where λ_t and μ_t denote the Lagrange multipliers on (2.11) and (2.12). While equation (2.13) resembles the private Euler equation (2.6), a critical difference is that the shadow value of current wealth differs and is given by

$$\lambda_t = u_T(t) + \mu_t \Psi_t, \tag{2.14}$$

where Ψ_t denotes the equilibrium change in the collateral value associated to a marginal change in tradable consumption, defined as:

$$\Psi_t \equiv \kappa_t (p_t^N c_t^N)/(c_t^T)\,(1+\eta)\,. \tag{2.15}$$

The change in the value of collateral associated to a marginal change in tradable consumption is the product of three terms: the collateral parameter κ_t, the ratio of non-tradable to tradable expenditure, and the inverse of the elasticity of substitution.

The wedge between the planner's and the private marginal value of wealth captures how the planner internalizes that higher demand for consumption relaxes the economy's borrowing constraint. This wedge translates into an "overborrowing" externality whenever the credit constraint does not currently bind but is expected to bind with strictly positive probability in the next period.

2.3.5 Reserve accumulation

In this section, we demonstrate that the constrained-efficient allocations can be implemented using a policy for reserve accumulation. One potential advantage of the implementation with reserves relative to capital controls is the observation that leakages often undermine the effectiveness of the latter (Bengui and Bianchi, 2018). This may make reserve accumulation a more attractive policy to pursue in practice and can, in fact, rationalize why governments seldom resort to the use of capital controls (Fernandez, Rebucci, and Uribe, 2015) and instead use reserves as a primary policy tool.

To establish our result, it is convenient to impose the following assumption.

Assumption 2.1. *Consumption is a Cobb-Douglas aggregator $c = (c^T)^\omega (c^N)^{1-\omega}$, and the credit constraint parameter satisfies $\kappa_t(1-\omega)/\omega < 1$.*

This assumption implies that $\Psi_t < 1$, for Ψ_t defined in (2.15), and guarantees that in any equilibrium, an increase in aggregate consumption by one unit does not relax the credit constraint by more than one unit. We return to the role played by this assumption later in this section, and emphasize that it is by no means necessary for our result to hold.

Our main normative result is summarized in the following proposition.

Proposition 2.1. *Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem* $\{c_t^{T\star}, b_{t+1}^{\star}, p_t^{N\star}\}_{t=0}^{\infty}$*. Then, given initial conditions* (b_0, A_0) *such that* $b_0^{\star} = b_0 - A_0$*, the competitive equilibrium features a tradable consumption allocation* $\{c_t^{T\star}\}_{t=0}^{\infty}$ *if the government follows the reserve policy* $\{A_{t+1}\}$ *given by*

$$\frac{A_{t+1}}{R_t} = \kappa_t \left(y_t^T + p_t^{N\star} y^N\right) - \frac{b_{t+1}^{\star}}{R_t} \quad \textit{for all } t \geq 0. \tag{RP}$$

Proof. The proof is by construction. We will show that, given the sequence of prices $\{p_t^{N\star}\}_{t=0}^{\infty}$ and initial conditions, the sequence of consumption allocations $\{c_t^{T\star}, y^{N\star}\}_{t=0}^{\infty}$ satisfy the households' first-order conditions, which are both necessary and sufficient for optimality.

We start by guessing that given (RP), the households' credit constraint (2.3) holds with equality at all times:

$$\frac{b_{t+1}}{R_t} = \kappa_t \left(y_t^T + p_t^{N\star} y^N\right). \tag{2.16}$$

Combining (2.16) with (RP), we obtain

$$b_{t+1}^{\star} = b_{t+1} - A_{t+1}. \tag{2.17}$$

Substituting (2.17) into the the tradable resource constraint (2.9) yields

$$c_t^T = y_t^T - (b_t - A_t) + \frac{b_{t+1}^{\star}}{R_t}. \tag{2.18}$$

Meanwhile, since $\{c_t^{T\star}, b_{t+1}^{\star}\}$ solve the constrained planning problem, we have

$$c_t^{T\star} = y_t^T - b_t^{\star} + \frac{b_{t+1}^{\star}}{R_t}. \tag{2.19}$$

Given the initial condition $b_0^{\star} = b_0 - A_0$, a comparison of (2.18) and (2.19) reveals that $c_t^T = c_t^{T\star}$ $\forall t \geq 0$. That is, when households' borrowing policy satisfies (2.16) and reserves are set according to (RP), the constrained-efficient sequence of tradable consumption is consistent with the consolidated budget constraints of the household and the government. Notice that the non-negativity of A_{t+1} follows immediately from the reserve policy (RP) and the planner's credit constraint (2.12).

We are left to show that $c_t^T = c_t^{T\star}$, $c_t^N = y^N$ satisfy the optimality conditions of the households. From conditions (2.13)-(2.14) characterizing the constrained-efficient allocation, we have

$$\mu_t^\star = u_T(t) - \beta R_t \mathbb{E}_t u_T(t+1) - \beta R_t \mathbb{E}_t \Psi_{t+1}^\star \mu_{t+1}^\star + \mu_t^\star \Psi_t^\star. \tag{2.20}$$

Rearranging the households' intertemporal Euler equation (2.6), we have that

$$\mu_t = u_T(t) - \beta R_t \mathbb{E}_t u_T(t+1). \tag{2.21}$$

Combining (2.20) and (2.21), we obtain

$$\mu_t = \beta R_t \mathbb{E}_t \Psi_{t+1}^\star \mu_{t+1}^\star + \mu_t^\star (1 - \Psi_t^\star) \geq 0, \tag{2.22}$$

where the the non-negativity of μ_t follows from $\Psi_t^\star = \kappa_t(1-\omega)/\omega < 1$, given Assumption 2.1, and the non-negativity of $\mu_t^\star$. Together, the conjecture (2.16) and the fact that $\mu_t \geq 0$ ensure that the households' intertemporal Euler equation and complementary slackness condition are satisfied. That is, condition (2.6) holds. Finally, notice that the households' intratemporal condition (2.4) follows directly from the definition of the constrained-efficient allocation, implying that $c_t^N = y^N$ is also optimal. □

The proposition establishes that under the reserve accumulation policy (RP), the competitive equilibrium achieves the same level of consumption as in the constrained-efficient allocation—and therefore delivers the same welfare. When the government accumulates reserves, households take on more debt to maintain the same level of consumption, until the credit constraint becomes binding. At that point, further increases in reserves generate a reduction in consumption and an increase in the net foreign asset position. The government then fine-tunes the amount of reserves to deliver the constrained-efficient net foreign asset position.[15]

The reserve policy effectively pushes private agents against their credit constraint whenever consumption in the laissez-faire economy would be above its level in the constrained-efficient allocation. Under the proposed reserve policy, the private credit constraint therefore holds with equality at all times. It strictly binds, however, only when there is a strictly positive probability of a binding credit constraint in the subsequent period under the constrained-efficient allocation. In states in which the credit constraint is not expected to bind next period, the constrained-efficient allocation can also be

[15] We note that the proposition applies generally to an economy that is either a net foreign debtor or a net foreign creditor. What is important is that the economy faces at some point the possibility of a binding borrowing constraint. While this probability is higher when the country is a net debtor, a net creditor can also enter a crisis in the model after a sufficiently large negative shock.

achieved by any alternative reserve policy satisfying $A_{t+1} \leq R_t\kappa_t\left(y_t^T + p_t^{N\star}y^N\right) - b_{t+1}^{\star}$.[16] Intuitively, when the credit constraint does not bind, the anticipation that the constrained-efficient consumption allocation will prevail in the future leads households to pick the constrained-efficient consumption even without a government intervention.

In Appendix B.3, we also provide a dual result, by which the optimal accumulation of reserves that maximize welfare in the competitive equilibrium yields the constrained-efficient allocations. As we show in the appendix, the government's first-order condition for A_{t+1} is given by

$$u_T(t) = \beta R_t \mathbb{E}_t[u_T(t+1) + \underbrace{\hat{\mu}_{t+1}\Psi_{t+1}}_{\text{Externality}} + \underbrace{\hat{\xi}_{t+1}u_{TT}(t+1) - \hat{\xi}_t u_{TT}(t)}_{\text{Incentive}}], \tag{2.23}$$

where ξ is the Lagrange multiplier on the households' first-order condition for borrowing. Equation (2.23) reveals two differences relative to what would be the analogous first-order condition for households.[17] The first difference is the pecuniary externality, which reflects that the planner internalizes that having more reserves in a future state with a binding borrowing constraint has positive general equilibrium effects. The second difference is an incentive term that captures how households respond to government policy. Chiefly important for this effect is that the planner is subject to the households' Euler equation (2.6) as an implementability constraint and that because of the overborrowing externality, the Lagrange multiplier associated with that implementability constraint is non-negative. At the margin, an accumulation of reserves by the government lowers current consumption and increases future consumption. This tightens today's implementability constraint (raises ξ_t) and relaxes tomorrow's implementability constraint (lowers ξ_{t+1}), as reflected in the two components of the "incentive term."

As it turns out, at the optimum ξ_t is zero. Intuitively, once the level of reserves is large enough, the borrowing constraint becomes binding, and thus households cannot offset the government policy. When optimizing, the government fine-tunes its accumulation of reserves so that the borrowing constraint becomes binding exactly at the level of tradable consumption that corresponds to the constrained-efficient allocation. In line with this dual result, in the remainder of the paper, we occasionally refer to the implementation of the constrained-efficient allocation via reserve policy as the "optimal reserve policy" outcome.

[16] This can be seen by noting that if $\mu_{t+1}^{\star} = 0$ in all future states and $\mu_t^{\star} = 0$, (2.22) implies that $\mu_t = 0$—hence the credit constraint does not bind in the competitive equilibrium. In our quantitative analysis, the indeterminacy of the reserve policy arises only in 2.4% of the simulations.

[17] The analogous condition for households would be $u_T(t) \geq \beta R_t \mathbb{E}_t u_T(t+1)$, with equality if reserves are non-negative.

Reserve depletion and liquidity value. As the expression (RP) indicates, when the credit constraint holds with equality in the constrained-efficient allocation, the government depletes its stock of reserves, setting $A_{t+1} = 0$. This result illustrates the liquidity value of reserves for the economy. The government accumulates reserves in good times to be used during crisis times. By rebating reserves to households during a crisis, it stabilizes consumption, raises the price of non-tradables and reduces the amount of deleveraging. Because households do not internalize how a reserve buffer would generate positive general equilibrium effects during crises, it is the government that must accumulate the reserves.

To illustrate the importance of depleting reserves during a crisis, consider an alternative policy by which the government keeps a fraction ϕ of reserves: $A_{t+1} = \phi A_t$. Substituting this reserve policy into the economy's resource constraint (2.9) when the credit constraint (2.12) is binding yields a level of tradable consumption of

$$c_t^T = \frac{(1+\kappa_t)\, y_t^T + \left[1 - \frac{\phi}{R_t}\right] A_t - b_t}{1 - \kappa_t \frac{1-\omega}{\omega}}.$$

Hence, maximizing current consumption—the planner's effective objective when the credit constraint binds—requires a full reserve depletion (i.e., setting $\phi = 0$). The above expression also clarifies why private households undervalue reserves in a crisis. While from an individual perspective, a unit of reserves provides enough resource to consume one additional unit of tradable goods, in equilibrium a unit of reserves raises tradable consumption by $1/[1 - \kappa_t(1-\omega)/\omega] > 1$.

The sharp reduction in reserves when the planner's credit constraint binds is consistent with the evidence that central banks use a large portion of reserves during crises (see, e.g., Broner et al., 2013). In many cases, however, reserves are not entirely depleted (Aizenman and Sun, 2012). A potential explanation for why central banks may choose to keep a positive level of reserves during crises is that policy makers may fear that losing large amounts of international reserves would send a bad signal to market participants. An alternative explanation is pursued by Bocola and Lorenzoni (2017). In their model, holding reserves allows the government to convey to market participants that it has the necessary fiscal resources to intervene if a bad equilibrium occurs. Thus, the sole availability of reserves can implement the good equilibrium without reserves actually being used. By contrast, it is essential in our model that the government actually uses the reserves during crises.

Discussion of Assumption 2.1 and equilibrium multiplicity. Assumption 2.1 is sufficient for our reserve implementation to work, but it is by no means necessary. In

Appendix B.2, we prove a more general version of Proposition 2.1, in which we relax Assumption 2.1—departing in particular from Cobb-Douglas preferences—and show that the reserve accumulation policy (RP) still implements the constrained-efficient allocation under an alternative weaker condition. As we show in the appendix, a necessary condition is

$$\mu_t^\star(\Psi_t^\star - 1) \leq \beta R_t \mathbb{E}_t \mu_{t+1}^\star \Psi_{t+1}^\star \tag{2.24}$$

where stars refer to variables evaluated at the constrained-efficient allocations.

The implications of a violation of condition (2.24) can be more easily understood by assuming that $\mu_{t+1}^\star = 0$ in all successor states. In this case, when $\Psi_t^\star > 1$ and $\mu_t^\star > 0$, the planner is borrowing constrained but chooses a level of consumption which is higher than the unrestricted one (i.e., the level that would prevail at date t absent the date t credit constraint). This occurs because a low elasticity of substitution or a high κ_t (leading to a value of $\Psi_t > 1$) generates a non-convexity in the planner's problem such that that the planner may be forced to choose between very low levels of consumption or very high ones. As can be seen from (2.22), such an allocation cannot be implemented with reserves because it would imply a negative Lagrange multiplier on the borrowing constraint for the household.[18] Intuitively, households would never choose an allocation such that $u_T(t) < \beta R_t \mathbb{E}_t u_T(t+1)$. Achieving this allocation would require a subsidy on borrowing in these states, and reserve accumulation alone would not be enough. Even though our implementation result would not hold in this case, reserves would remain an effective tool to reduce overborrowing ex ante.

Assumption 2.1 is related to the condition for self-fulfilling financial crises identified in Schmitt-Grohé and Uribe, 2020. As they explain, equilibrium multiplicity may occur in this model because even though for an individual agent, an increase in debt tightens the borrowing constraint, in equilibrium, an increase in aggregate debt may actually raise the borrowing capacity more than one-for-one and relax the borrowing constraint. Assumption 2.1 is sufficient to ensure that when aggregate borrowing increases by one unit, the borrowing capacity does not increase by more than one unit. As Schmitt-Grohé and Uribe show, this rules out multiple equilibria by which sharp drops in consumption can become self-fulfilling.

It is worth highlighting that our implementation result would still hold under multiplicity of equilibria, as long as (2.24) holds. However, the reserve policy would not be able to

[18] If the credit constraint is expected to bind in the following period (i.e., $\mu_{t+1}^\star$ in some state at date $t+1$), the necessary condition is weaker. Intuitively, even though $\Psi_t^\star > 1$ indicates that a collective increase in borrowing is feasible, the planner may still choose a level of borrowing below the unconstrained level because it internalizes that more borrowing would tighten the constraint in the next period. As a result, given (2.22), we still have a positive Lagrange multiplier for households evaluated at the constrained-efficient allocations.

uniquely implement the constrained-efficient allocation. To understand why, consider a situation in which, for a given level of debt, reserves, and shocks, the economy features multiple equilibrium levels of current consumption. A planner that can directly choose the level of borrowing would choose the high consumption equilibrium, but it may not be able to implement it using reserve accumulation. If agents were to coordinate on the bad equilibrium, the government would deplete its reserves to raise consumption and support the real exchange rate, but this would not be sufficient to increase the borrowing capacity to reach the good equilibrium unless all households were to further increase their consumption simultaneously. Nevertheless, the macroprudential role for reserves remains intact. In fact, the possibility of being trapped in a bad equilibrium could give rise to an even more significant role for reserves ex ante.[19]

Implementation via a feedback rule

Proposition 2.1 describes a state contingent reserve accumulation policy that implements the constrained-efficient allocations. In this section, we show that the constrained-efficient allocation can also be implemented using a simple *feedback rule* that directly specifies reserve policy as a function of the private sector's borrowing choice. Feedback rules are common in the study of monetary policy (i.e., the "Taylor rule") and under some circumstances can achieve the same outcomes as the state-contingent optimal policy (see e.g. Woodford, 2007). It turns out that a similar equivalence applies in our model.

Proposition 2.2. *Suppose Assumption 2.1 holds and initial conditions* (b_0, A_0) *are such that* $b_0^\star = b_0 - A_0$*. Then, the constrained-efficient allocation is achieved if the reserves are set according to the feedback rule given by*

$$\mathcal{A}_{t+1}(b_{t+1}) = b_{t+1} - b_{t+1}^\star. \tag{2.25}$$

Proof. See appendix. □

The idea behind this result is that by construction of the feedback rule, the NFA position in the competitive equilibrium coincides with its value in the constrained-efficient allocation. This, in turn, leads to the same consumption level as in the constrained-efficient allocation (and achieves the same outcome as the one obtained under the state contingent policy

[19] The parameterization we use for our quantitative analysis violates Assumption 2.1 but delivers a unique equilibrium, as in Bianchi, 2011. Appendix B.5 provides details on how we check numerically for the presence of multiplicity.

(RP)).[20]

The feedback rule provides a simple, yet clear policy insight: the government should save an amount equal to the gap between the private sector's borrowing b_{t+1} and the constrained-efficient level of borrowing $b^{\star}_{t+1}$.

To understand the mechanics of the feedback rule and the strategic interactions between the government and households, consider a scenario in which the constrained-efficient allocation is implemented from date $t+1$ onward and let us focus on the outcome at date t. Denote by $\tilde{c}^T_t$ the level of tradable consumption the household would choose at date t in the absence of any reserve intervention, and by $\tilde{b}_{t+1}$ the associated level of private borrowing. Meanwhile, denote by $c^{T\star}_t$ and $b^{\star}_{t+1}$ the constrained-efficient tradable consumption and net borrowing. Given the overborrowing result, we have $\tilde{c}^T_t \geq c^{T\star}_t$ and $\tilde{b}_{t+1} \geq b^{\star}_{t+1}$.

Let us now examine how households respond to the purchases of reserves A_{t+1} at date t. To stay on their Euler equation (2.6), households adjust their borrowing according to

$$\mathcal{B}_{t+1}(A_{t+1}) = \begin{cases} A_{t+1} + \tilde{b}_{t+1} & \text{for } A_{t+1} < \bar{A}_{t+1} \\ R_t \kappa_t \frac{\frac{1}{\omega} y^T_t - \frac{1-\omega}{\omega}\left(b_t + \frac{A_{t+1}}{R_t}\right)}{1-\kappa_t \frac{1-\omega}{\omega}} & \text{for } A_{t+1} \geq \bar{A}_{t+1}, \end{cases}$$

where $\bar{A}_{t+1} \equiv R_t \kappa_t \left(y^T_t + \frac{1-\omega}{\omega}\tilde{c}^T_t\right) - \tilde{b}_{t+1}$ denotes the threshold of reserve purchases at which point the households' borrowing constraint becomes binding. For $A_{t+1} < \bar{A}_{t+1}$, households react to the lump-sum tax (expected to be offset by a positive future transfer) by a one-to-one increase in debt, following a Ricardian equivalence type of logic. For $A_{t+1} > \bar{A}_{t+1}$, the private debt level required to offset the tax is so large that it violates the household's credit constraint. In fact, above the threshold, more reserves contract the borrowing capacity of the economy and lead to *less* private debt rather than more private debt.

What is the level of reserves that the government needs to accumulate to implement a consumption of $c^{T\star}_t$ and a net borrowing of $b^{\star}_{t+1}$? Figure 2.5 illustrates how the interaction between the government's policy and the households' response determines the necessary level of reserves. The solid blue line represents the private sector's best response $\mathcal{B}_{t+1}(A_{t+1})$, and the dashed red line represents the government's policy $\mathcal{A}_{t+1}(b_{t+1})$ described by (2.25). Notice that the fact that households borrow more in response to the accumulation of reserves leads the government to accumulate even more reserves. Equilibrium is reached when both

[20] In states of natures where the credit constraint neither currently binds nor is expected to bind in any successor states under the constrained-efficient allocation, reserves and private borrowing are not pinned down separately under the feedback rule but they are under the sequential policy (RP). The economy's NFA and consumption allocation, however, always coincide.

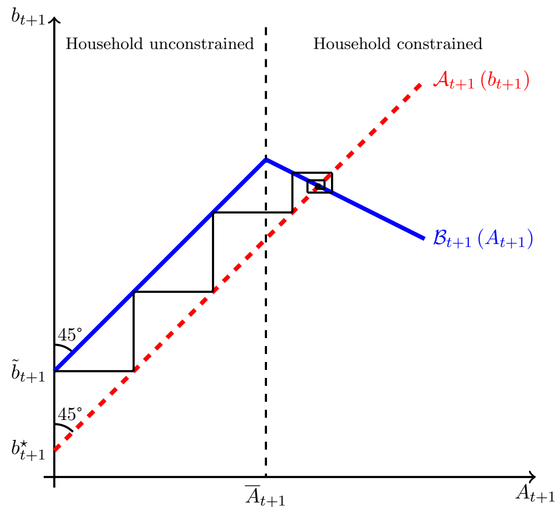

Figure 2.5: Illustration of the implementation of constrained-efficient allocation via feedback rule for reserves when the borrowing constraint is slack for the planner and $c_T^{\star} < \tilde{c}_t$.

the private sector and the government play their best responses, i.e., when $\mathcal{A}_{t+1}(b_{t+1})$ and $\mathcal{B}_{t+1}(A_{t+1})$ intersect. At that point, official reserves are positive and private indebtedness has increased, but the economy's net foreign asset position has improved relative to the level that would have resulted from households' borrowing choice absent any reserve intervention at date t.[21]

This experiment focused on the implementation of the constrained-efficient consumption level via reserves at date t, taking the achievement of the constrained-efficient allocation in future periods as given. A similar logic, however, can be applied to future periods.

2.4 Quantitative analysis

In this section, we present a quantitative analysis.[22] We organize the results as follows. First, we describe the calibration. Second, we present the policy functions to illustrate the workings of the model. Third, we show that the model is able to account for the four empirical facts presented in Section 2.2. Fourth, we present a simple rule for reserve accumulation. Finally, we present various extensions and perform sensitivity analysis.

[21]In a state in which the borrowing constraint binds under the constrained-efficient allocation, we have $\bar{A}_{t+1} = 0$ and hence the households' best response only features a decreasing segment. In that case, the two best responses intersect at $A_{t+1} = 0$.

[22]The competitive equilibrium is solved numerically using time iteration and the optimal policy problem is solved with value function iteration, as in Bianchi (2011).

2.4.1 Calibration

A period in the model represents a year. The preference parameters for risk aversion and the elasticity of substitution are set to standard values from the literature: $\sigma = 2,\ 1/(1+\eta) = 0.83$. The value for the interest rate is set to 4%, also standard in the literature.[23] For the calibration of the remaining parameters, we use data from Mexico, a common choice in studies of reserve accumulation (e.g., Bianchi, Hatchondo, and Martinez, 2018) during the period 1980-2015.

To estimate the tradable endowment stochastic process, we use the value added series in the primary and industrial (net of construction) sectors.[24] We assume a first-order autoregressive process for the cyclical component: $\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y$ with $\varepsilon_t^y \sim N(0, \sigma_y)$, and estimate values of $\rho^y = 0.24$ and $\sigma_y = 0.034$. The value of ω is set to match the share of tradable GDP in the data, which is 33%.[25]

We assume that the process for the financial shock κ_t follows a first-order autoregressive process in logs: $\log(\kappa_t) = (1-\rho^\kappa)\log(\bar{\kappa}) + \rho^\kappa \log(\kappa_{t-1}) + \varepsilon_t^\kappa$ with $\varepsilon_t^\kappa \sim N(0, \sigma_\kappa)$. Unlike income, the financial shock is not directly observable. To discipline the process for κ_t, we exploit the fact that the credit constraint holds with equality under the optimal reserve intervention, and follow the approach proposed by Jermann and Quadrini (2012). Namely, taking (2.12) with equality, we back out a time series for κ_t using the observed sequence of output and debt. Since before the 1994 Tequila crisis, Mexico had very low levels of reserves, we take 1995-2015 as the reference period. We then estimate the aforementioned AR(1) process and obtain $\rho^\kappa = 0.82$, $\bar{\kappa} = 0.46$ and $\sigma_\kappa = 0.11$.

The remaining parameter is the discount factor. We calibrate β so that the average NFA in the economy without government intervention matches the average of Mexicos's NFA position.[26] This calibration yields $\beta = 0.94$. A summary of parameter values is provided in Table 2.2.

[23] In our baseline calibration, we assume a constant interest rate. We consider a stochastic interest rate in Section 2.4.5.

[24] We use value added data in local currency from Mexico's National Institute of Statistics and Geography (INEGI) for 1980-2015, deflated by sector specific prices.

[25] In a non-stochastic version of the model with a level of net foreign asset position equal to $\overline{NFA}$ and tradable and non-tradable output normalized to one, the relative share of non-tradable to tradable output is given by the value of ω such that $1/[1+\frac{1-\omega}{\omega}(1+r\ \overline{NFA})] = 33\%$. Given the mean value of the NFA to be calibrated below, this yields $\omega = 0.325$

[26] Although gross positions have increased quite substantially over time, the average NFA is about the same in the periods 1980-1994 and 1995-2015.

Table 2.2: Parameter Values

	Value	Source/Targets
Interest Rate	$r = 0.04$	Standard value
Risk Aversion	$\sigma = 2$	Standard value
Elasticity of Substitution	$1/(1+\eta) = 0.83$	Standard value
Weight on Tradables in CES	$\omega = 0.33$	Tradable share = 33%
Stochastic structure y^T	$\rho^y = 0.24, \sigma_y = 0.034$	See text
Stochastic structure κ	$\rho^\kappa = 0.82, \bar{\kappa} = 0.46, \sigma_\kappa = 0.11$	See text
Discount Factor	$\beta = 0.94$	Average NFA = −37.0%

2.4.2 Reserves and gross debt

We start by describing the workings of the model through an analysis of the policy functions for reserve accumulation and debt. We highlight how the reserve intervention differs markedly from those based on a Pigouvian tax intervention.

Policy function for reserves. Figure 2.6 presents the optimal reserve accumulation policy as a function of the shocks the economy faces and the current value of debt. In panel (a), the amount of reserves is shown as a fraction of the tradable endowment, for the mean value of κ and for two possible values of beginning-of-period debt. In panel (b), the amount of reserves is shown as a function of the financial shock, for the mean value of y^T, again for two possible values of debt. In both cases, reserves are reported as a function of average GDP. (Unless otherwise noted, we do this for both reserves and debt throughout the section.)

Figure 2.6 shows that the government finds it optimal to hold more reserves in good times, that is, when income is high or when financial conditions are less stringent. The intuition for these results is that when the amount that households can borrow rises (because of either higher y^T or higher κ), the government needs to accumulate more reserves to close the gap between the net amount of borrowing desired by the planner and the borrowing capacity of households. Similarly, when beginning-of-period debt is lower, households are further away from their credit constraint—they want to borrow less and they have more spare borrowing capacity—and the government accumulates more reserves.

Comparison with taxes on debt. An important fact that motivated our analysis was that countries that rely less on capital controls appear to hold larger amounts of reserves (fact 4). In our model, reserves and taxes on debt are substitute policies: a government that uses capital controls has no need for reserve accumulation and conversely, a government that

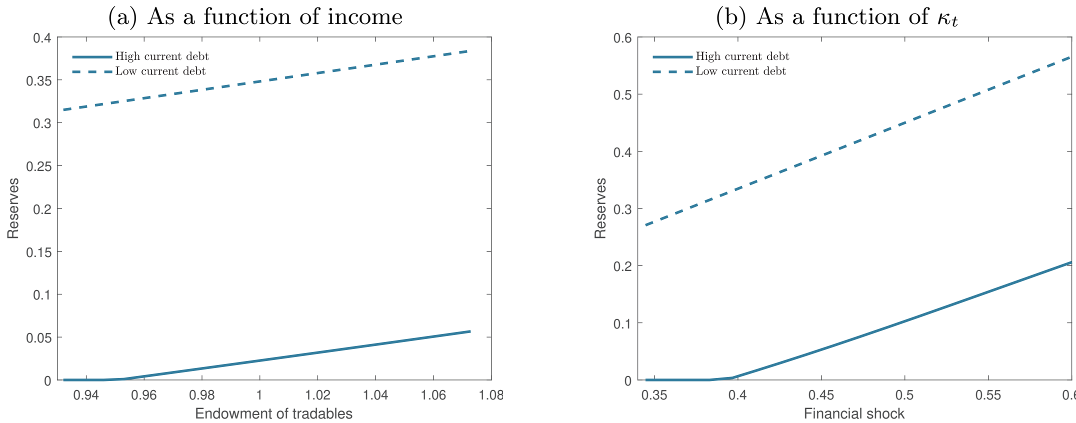

Figure 2.6: Policy function for reserves

accumulates reserves does not need to impose capital controls. It is interesting, however, to contrast the properties of the optimal reserve intervention policy with those of the optimal capital control policy. Figure 2.7 again displays policy functions for reserves, but this time together with policy functions for the optimal tax on debt.[27]

A common feature of the two policies is that they are passive when the constraint is already binding (both taxes on debt and reserve holdings are zero in this case). However, despite both responding to a macroprudential motive when the credit constraint is not binding, they differ markedly in terms of their cyclical properties. While reserves tend to *increase* with output, the tax on debt tends to *decrease* with output. The reason for this different cyclical pattern is as follows. When output is low, agents have stronger incentives to borrow, leading to a higher probability of a binding borrowing constraint in the future—hence, calling for a higher tax on debt.[28] By contrast, as we explained above, when output is low, there is a smaller excess borrowing capacity, which calls for a smaller amount of reserves. A similar contrast applies with respect to the financial shock (see panel (b) of Figure 2.7).

Policy functions for gross private debt. We now show how the profile of private debt depends critically on the government intervention. Figure 2.8 shows the law of motion for b' and its ergodic distribution for three economies: (i) laissez-faire, (ii) constrained-efficient

[27] We apply the optimal borrowing tax formula of Bianchi (2011).

[28] The result that taxes have a negative correlation with output is emphasized in Schmitt-Grohé and Uribe (2017) and Bianchi and Mendoza (2018). 2019 show that persistent shocks to income growth can alter the sign of the cyclicality.

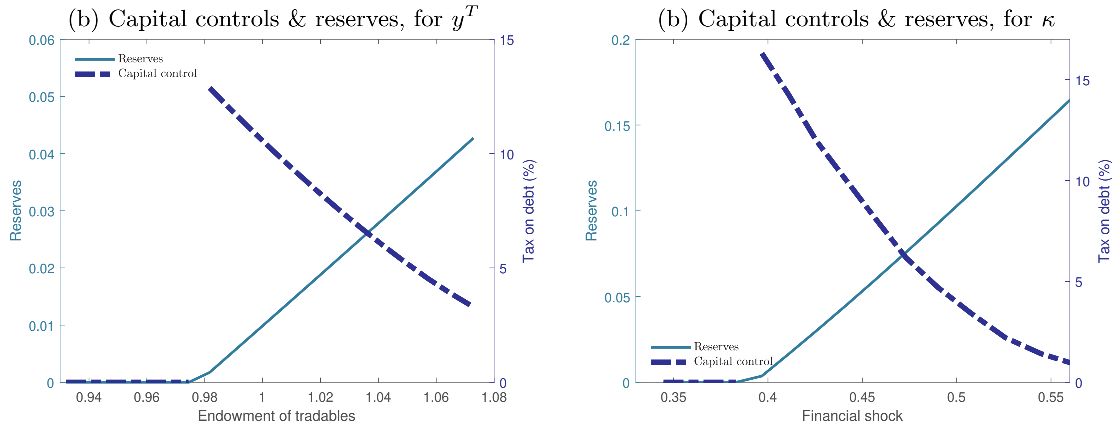

Figure 2.7: Reserve accumulation vs. capital controls

and (iii) foreign reserve intervention.[29] Panel (a) shows that when current debt is high enough, the borrowing constraint binds and all three economies have the same end-of-period debt. For low debt levels, however, private debt choices differ: the constrained-efficient economy is the one in which the least amount of private debt is accumulated, followed by the laissez-faire economy and the economy with foreign reserve intervention. In line with these results, panel (b) shows that the ergodic distribution of gross private indebtedness is located further to the right in the economy with foreign reserve intervention than in the other two economies.

A finding that stands out is that gross private indebtedness is *higher* under the foreign reserve intervention than in the laissez-faire economy.[30] This result emerges even though the laissez-faire economy features overborrowing relative to the constrained-efficient allocation. In other words, the laissez-faire economy displays a lower NFA position than the economy with the optimal reserve intervention but larger gross debt positions. This "underborrowing" result is thus different from the one described by Benigno et al. (2013b). In that paper, the laissez-faire economy also issues too little debt; but critically, it has a higher NFA position relative to an economy in which the government has access to ex post policies.

Figure 2.9 further shows how the optimal reserve intervention changes the cyclical properties of private borrowing: the debt policy functions with respect to income and financial conditions are shown in panels (a) and (b), respectively. When income is low,

[29] By "constrained-efficient," we mean the allocation described in Section 2.3.4, while by "foreign reserve intervention," we mean the implementation of the constrained-efficient allocation presented in Section 2.3.5.

[30] In the state space, this occurs technically for all values of debt except those at which the borrowing constraint is binding under the laissez-faire but not in the constrained-efficient allocation.

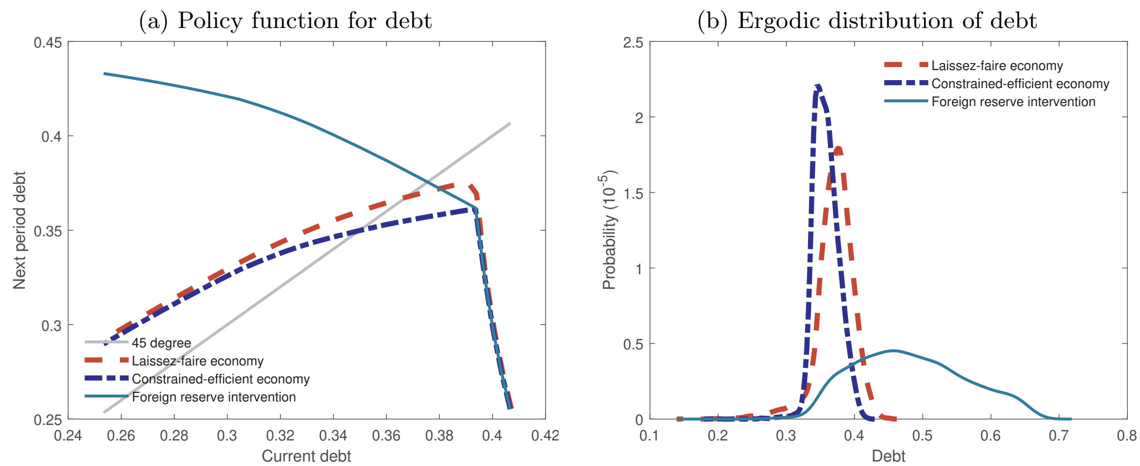

Figure 2.8: Debt: policy functions and ergodic distributions

borrowing is increasing in income for both the laissez-faire economy and the economy with the optimal reserve intervention. The reason is that when income is low, the borrowing constraint is binding and higher income helps relax it. When income is high, however, the two economies differ in the cyclical properties of borrowing: while borrowing is countercyclical under the laissez-faire, it is procyclical under the optimal reserve intervention. Under the laissez-faire, when the credit constraint does not bind, the economy borrows less when income is high, following a permanent income logic. Under the optimal foreign reserve intervention, in contrast, since the excess borrowing capacity is procyclical in the constrained-efficient allocations, the government accumulates more reserves when output is high, inducing households to take on more debt. On the other hand, panel (b) shows that private borrowing is procyclical with respect to financial conditions in both economies.

Our finding that optimal foreign reserve interventions may lead to higher private indebtedness has implications for empirical studies on credit booms and financial crises. In particular, it stresses the importance of taking official reserve dynamics into consideration when determining the role of private credit in predicting financial crises. In our model, the optimal foreign reserve intervention results in higher private indebtedness, yet a lower exposure to financial crises.

Long-run moments. Table 2.3 displays average debt and reserves (as percent of output), together with the probabilities of crisis, according to the ergodic distribution for the different

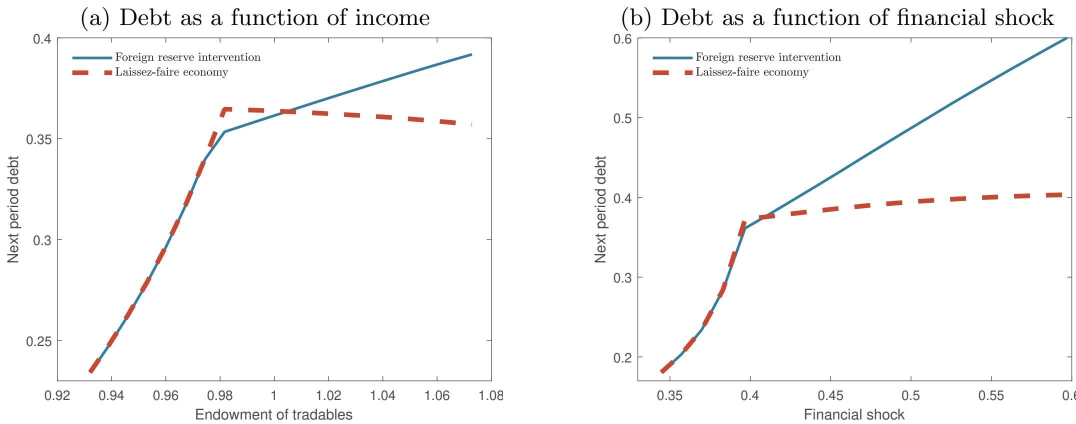

Figure 2.9: Equilibrium policy function for debt

versions of the model. The amount of overborrowing in the laissez-faire economy relative to the constrained-efficient allocations is about 1% of GDP, but the amount of debt under the optimal intervention is about 11% of GDP higher than under the laissez-faire. Meanwhile, the optimal average level of reserves is 12% of output. This value is in the range of the recently observed level of reserves, which is around 15% of GDP for Mexico.[31]

It is worth emphasizing that even though the presence of overborrowing is key to justify the reserve accumulation policy, the scope of the intervention needed is not determined by the difference between the constrained-efficient level of borrowing and the level of borrowing in the laissez-faire. Rather, as indicated by (RP), the amount of intervention is determined by the difference between the constrained-efficient level of borrowing and *the economy's borrowing capacity*. This difference can actually be quite large, especially in states in which financial conditions are loose.[32]

Finally, the optimal intervention is quite successful at reducing the exposure to financial crises. We compute the probability of a financial crisis, defined as an episode where the current account increases by more than two standard deviations above its mean, following the empirical literature. In the laissez-faire economy, the probability of a crisis is 1.8%, which is in the range of the estimated frequency of financial crises for emerging markets (e.g.,

[31] As we explained in Section 1.3, when the credit constraint is currently slack and its probability of being binding in the following period is zero, the reserve policy is indeterminate. While we assume that reserves follows the policy (RP) in our quantitative analysis, assuming that reserves are zero in these circumstances would only change the average level of reserves from 12.2% to 12.0%. The results differ very little because the indeterminacy arises only 2.4% of the time in the simulations.

[32] On the other hand, the difference in the unconditional level of borrowing between the constrained-efficient economy and the laissez-faire is only about 1% of GDP, consistent with Bianchi (2011).

Calvo, Izquierdo, and Talvi, 2006). In the simulations, these events are always characterized by binding credit constraints. The probability that the credit constraint binds is 2.5%, implying that nearly 70% of the times that a shock triggers a binding credit constraint, there is a sharp reversal of capital flows. Following the optimal reserve accumulation policy reduces this probability of financial crises to 0.4%.[33] While the optimal reserve intervention does not fully eliminate the occurrence of crises, it substantially reduces their frequency. As we show in Section 2.4.4, it also reduces their severity.

Table 2.3: Long-run moments

	Laissez-faire	Constrained-efficient	Optimal Reserves
Debt	37.2	35.9	48.1
Reserves	-	-	12.2
Crisis probability	1.8	0.4	0.4

2.4.3 Accounting for the stylized facts

We now assess the model's ability to account for the facts 1–4 outlined in Section 2.2. To do so, we simulate the model to generate artificial data comparable with the data used in our empirical analysis of Section 2.2.

Fact 1. First, we examine whether the model can account for the recent increase in reserves, while being consistent with the simultaneous rise in private external debt observed in the data. We use our calibration for Mexico. Starting in 2000, we fix the initial gross positions from the data and feed the observed path for income. A simulation of the model also requires a path for the financial shock, which is not observable. Given our interest in the joint trend between reserves and debt, we feed the sequence of financial shocks that deliver the sequence of debt observed in the data.[34]

Panel (a) of Figure 2.10 shows that this exercise makes the model predict a significant increase in reserves, of about 10% of GDP, consistent with the increase observed in the data. While the model predicts more volatility in the path of reserves than there is in the data, the model's ability to account for the magnitude of the overall increase is quite remarkable. The model is therefore able to jointly explain the increase in debt and reserves. Notably,

[33] The probability of a binding credit constraint 0.7% in the constrained efficient economy.

[34] The sequence of debt in the data is constructed, analogously to the model, as the sum of the NFA plus reserves.

while the debt path was targeted in our simulation (see panel (b)), the path of reserves was not.[35]

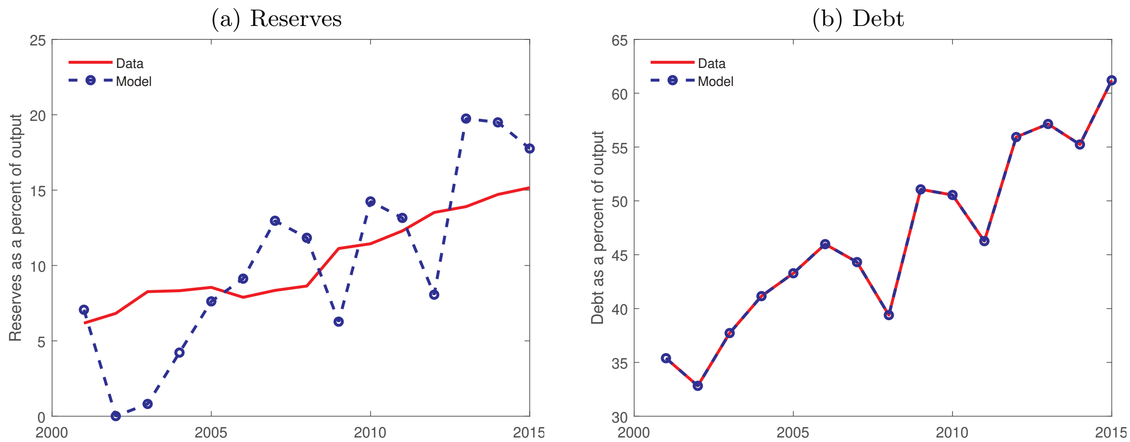

Figure 2.10: Evolution of reserves and debt, 2001–2015: data and model

Note: Model simulation obtained by feeding observed income shocks and calibrating financial shocks to match sequence of NFA (excluding reserves) observed in the data.

Fact 2. Second, we argue that our model is also consistent with the positive cross-sectional association between reserves and private external debt observed in the data. To examine this fact through the lens of our model, we proceed in a way analogous to the way we proceeded in the data: (i) we construct 10,000 samples of simulations of 30 years each; (ii) we compute averages of reserves and private debt over the last four periods of each sample; and (iii) we compute the difference between this end-of-sample average and the beginning-of-period value.

Figure 2.11 shows a scatter plot in which each dot represents a sample, with the x-axis and y-axis respectively measuring changes in private external debt and reserves over the sample. The figure shows that samples displaying significant increases in reserves also display large increases in private external debt, consistent with fact 2. Of course, reserves in the data are also driven by factors other than the macroprudential motive studied in the paper. Hence, the correlation observed in the data in Figure 2.2 is not as tight as the model counterpart in Figure 2.11.

[35] The financial shocks that are reverse-engineered in the procedure are consistent with the log-normal process estimated in the calibration section. In particular, the financial shock is within the 95% confidence interval generated by the estimated process in all but one year from 2001 to 2015 (see Appendix B.6 for details).

Fact 3. Third, we show that our model generates time-series correlations between the changes in reserves, private external debt, and output. For each of our 10,000 samples, we compute the time-series of first differences of reserves, private debt and output.[36] We then calculate the correlation between the reserves and output series, between the private debt and output series, and between the reserve and private debt series for each sample. Finally, we sort these correlations from the lowest to the highest.

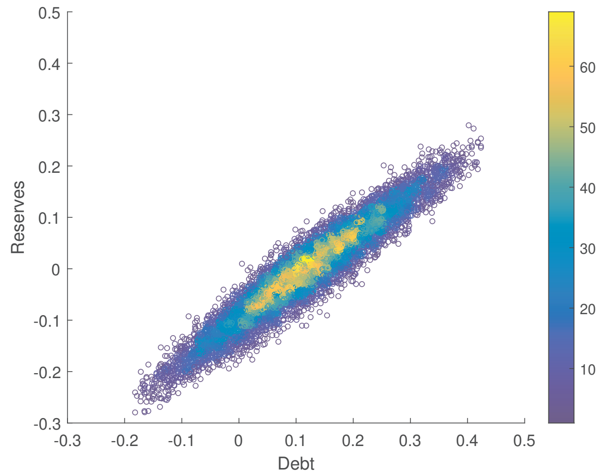

Figure 2.11: Increase in reserves and private debt

Note: Based on 10,000 samples of simulations of 30 years each, with each dot representing a sample. x-axis measures difference between average over last four periods and beginning of period value for private debt. y-axis measures difference between average over last four periods and beginning of period value for reserves.

Figure 2.12 displays these results: panel (a) shows the correlation between the reserve and output series; panel (b) shows the correlation between the private debt and output series; and panel (c) shows the correlation between the reserve and private debt series for all simulated samples. Like in the data, both reserves and private debt are procyclical, while reserves and private debt are positively correlated.

Fact 4. Fourth, we address the correlation between the accumulation of international reserves and capital account openness. So far, we have only considered implementations with either taxes or reserves. However, we can extend our analysis to address fact 4. We postulate that in the background there is a maximum tax rate $\bar{\tau}$ on borrowings that governments can or are willing to impose, either because of a fear of leakages or other

[36]We use the log of the private debt and output series, but not of the reserve series since there are several occurrences of zero reserves in the samples.

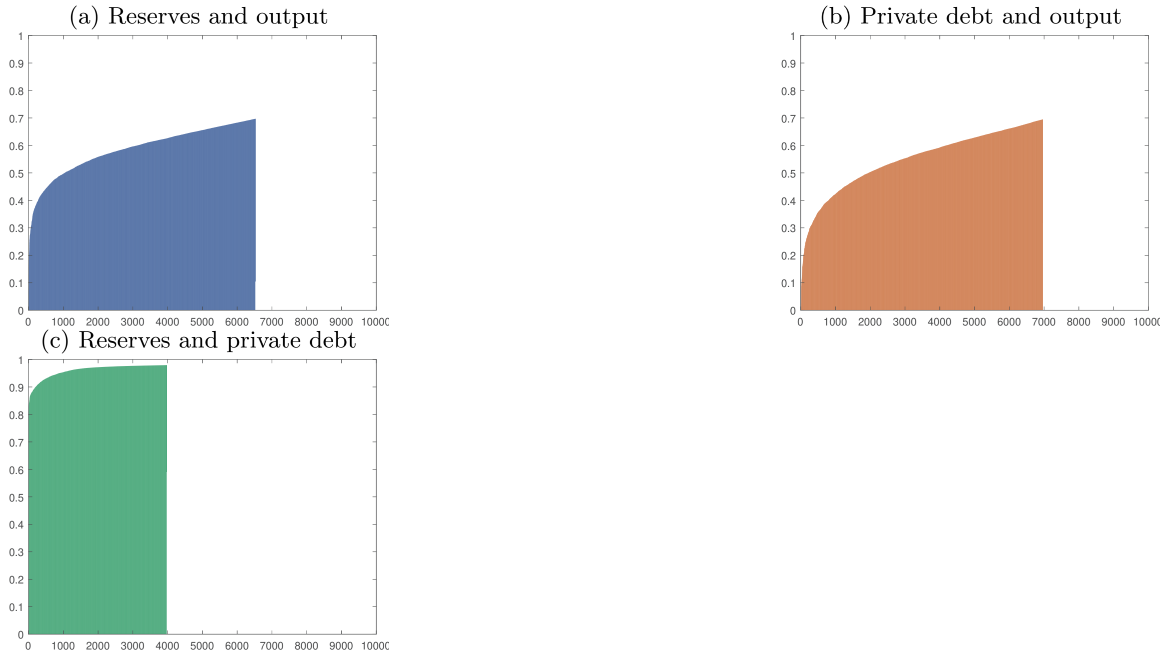

Figure 2.12: Correlations between changes in reserves and output (panel (a)), changes in private debt and output (panel (b)), and changes in reserves and private debt (panel (c))

Note: Based on 10,000 samples of simulations of 30 years each, with each vertical bar measuring the correlation between two variables in a given sample.

unintended consequences. We assume that this maximum tax rate is heterogeneous across countries and draw for each of 10,000 simulations a different $\bar{\tau}$ from a uniform distribution between zero and τ^{max}, where we take τ^{max} to be the largest tax necessary to implement the constrained-efficient allocation in the ergodic distribution. For each of these samples, we consider a government using a mix of capital controls and reserves. We assume that the government implements the constrained-efficient allocation using taxes on borrowing if the optimal tax rate is below the drawn maximum tax rate $\bar{\tau}$ while if the maximum tax is binding, the government sets the maximum tax rate and resorts to reserve accumulation to close the gap.

Figure 2.13 plots the average tax rate and reserves for each sample over 30 years. We find that in samples where average reserves are high, taxes on private borrowing are low. This negative correlation between reserves and traditional capital control policy is consistent with our fact 4.

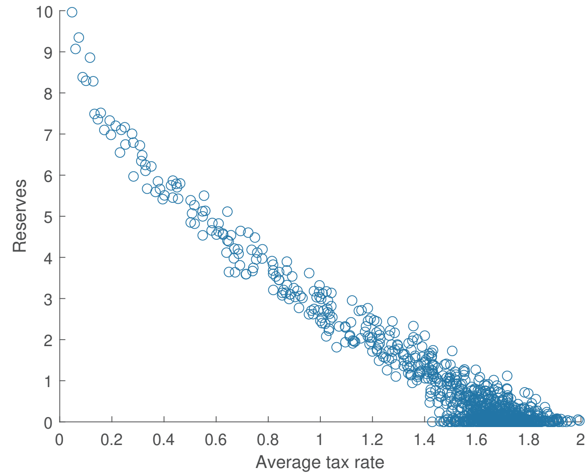

Figure 2.13: Average reserve and taxes in simulated economies

Note: We simulate 10,000 samples for 30 years each, with each dot representing a sample. The x-axis measures the average tax rate over the 30 periods. The y-axis measures the average level of reserves as a percent of output over 30 periods.

2.4.4 A simple rule for reserve accumulation

Motivated by practical policy considerations, we analyze a version of the model in which the government follows a simple policy rule for the accumulation of international reserves.[37]

The simple rule we propose approximates the optimal reserve accumulation by a linear function of state variables, including the NFA position:

$$A_{t+1}^{SR} = \max\{\beta_0 + \beta_1 y_t^T + \beta_2 \kappa_t + \beta_3 (A_t - b_t), 0\},$$

where the $\{\beta_i\}_{i=0}^3$ are constant parameters. We estimate these coefficients by maximizing the unconditional welfare gains from moving from the laissez-faire economy to the economy with the optimal simple rule.[38]

[37]Different from the feedback rule (2.25), the simple rule we consider does not require the government to know the constrained-efficient level of borrowing. It is possible to map this simple rule into a feedback rule where we replace the constrained-efficient level of borrowing in (2.25) by a target that depends on observables.

[38]Numerically, we proceed by first running an OLS regression of the optimal level of reserves at the ergodic distribution on the exogenous state variables and the current NFA position in the economy with the optimal reserve intervention. We restrict the sample to observations where next period's level of international reserves is strictly positive ($A_{t+1} > 0$). Then, we construct a grid for each of the four parameters $\{\beta_i\}_{i=0}^3$ centered around the OLS estimates. Given three values for each parameter and a total four parameters, we have eighty-one possible combinations. We select the combination that gives the highest welfare gains. We repeat the process by centering the new grids on this combination. We iterate until we cannot increase the welfare gains by selecting any other point in the grid different from our initial guess.

The results from the optimization yields the following coefficients:

$$\beta_0 = -0.34,\ \beta_1 = 0.45,\ \beta_2 = 0.68,\ \beta_3 = 0.29.$$

The estimated rule implies a reserve accumulation policy that is increasing in the current NFA position, income and the financial shock. The rule hence inherits the same basic qualitative properties as the optimal state-contingent policy, but differs in that it is a simple linear rule.

The financial stability gains from adopting the simple rule can be illustrated by conducting the following event analysis. First, we simulate the laissez-faire economy for a large number of periods, identify all the financial crisis episodes, and construct 7-year window events centered around the financial crisis episodes. Second, we take the average of key variables across the window period for the laissez-faire economy. Third, we feed the initial state and shock sequence for each event from the laissez-faire economy to the policy functions of the economies with the simple rule and with the optimal state-contingent reserve policy, again taking the average of key variables.

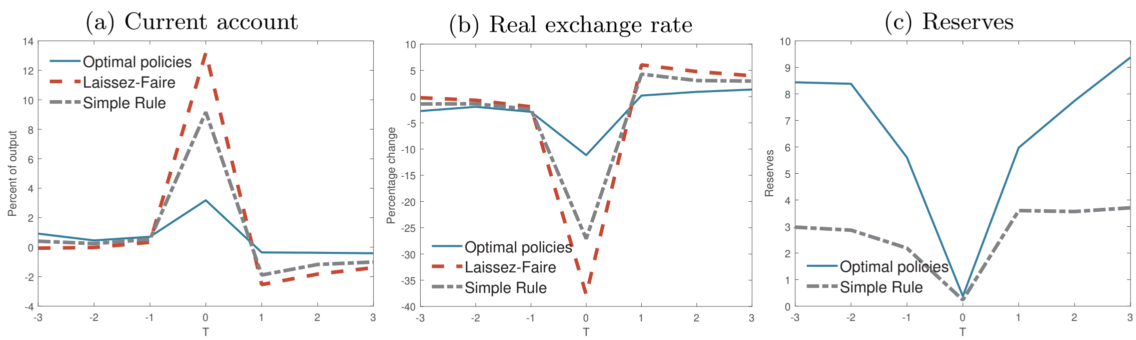

Figure 2.14: Financial crisis event analysis under optimal policies, laissez-faire, and a simple rule

The dynamics of the event analysis are shown in Figure 2.14. The path for the current account and the real exchange rate are shown in panels (a) and (b), respectively, comparing outcomes in the laissez-faire economy, the economy with the optimal state-contingent reserve policy and the economy with the simple rule. The figure shows how, in a crisis, the laissez-faire economy experiences a large current account reversal of about 13% of GDP while the real exchange rate depreciates by close to 40%. These magnitudes are in line with empirical regularities of sudden stops (see, e.g., Calvo, Izquierdo, and Talvi, 2006). The optimal reserve policy is successful at mitigating the severity of sudden stops as reflected in a more modest current account reversal of about 3% of GDP and a real exchange rate depreciation of 10%. The optimal use of reserves, therefore, reduces capital outflows by 10% of GDP and reduces the exchange rate depreciation by 30 percentage points.[39] The simple rule also delivers significant gains, reducing the current account reversal by about 5% of GDP and the real exchange rate depreciation by about 10 percentage points. In terms of welfare, we find that the simple policy rule delivers 12% of the total welfare gains achieved by optimal state-contingent reserve policy.

Panel (c) compares the path for reserves under the optimal policy with its counterpart under the simple rule. As it turns out, the simple rule prescribes less reserve accumulation than the fully optimal policy. The inability to conduct a perfectly state-contingent policy leads the government to err on the side of lower reserves. The intuition for this result is that too large reserve accumulation relative to the optimal may have the effect of excessively tightening households' borrowing constraint. In both cases, reserves fall to a value close to zero around crises, in line with our result on reserve depletion following from Proposition 2.1 and the empirical evidence that reserves fall sharply during crises (Broner et al., 2013).[40].

2.4.5 Extensions and Sensitivity

Production economy. We consider an extension of the model with production. This extension is important in light of the findings that endogenous production may call for ex-post stabilization policies and affect the efficient amount of borrowing (Benigno et al., 2013b, 2020).

Let us assume that households are endowed with a fixed amount of hours $\bar{h}$, perfectly mobile across sectors, and do not value leisure. They receive a competitive wage w_t for their labor, as well as profits from firms in the tradable and nontradable sectors, π_t^T and π_t^N. The household's budget and credit constraint are given by:

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} = w_t \bar{h} + \pi_t^T + \pi_t^N - b_t - T_t, \tag{2.26}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa_t \left(w_t \bar{h} + \pi_t^T + \pi_t^N \right), \tag{2.27}$$

The tradable and nontradable goods are produced by competitive firms that maximize

[39] We note that while the model is purely real, it would be relatively straightforward to extend it to a monetary model where the government defends the *nominal* exchange rate. For example, if the government followed an inflation targeting policy, it would prevent a large nominal exchange depreciation to keep inflation on target during a sudden stop.

[40] The reason why reserves do not exactly fall to zero in the Figure is that for some shock sequences that lead to crises the laissez-faire economy, the ex ante reserve accumulation succeeds at averting a crisis altogether

profits and solve:

$$\max_{h_t^T} z_t^T (h_t^T)^\alpha - w_t h^T \tag{2.28}$$

$$\max_{h_t^N} p_t^N z^N (h_t^N)^\alpha - (1+\tau_t^N) w_t h^N + T_t^N, \tag{2.29}$$

where z_t^T is a stochastic productivity shock, z^N and α are constant parameters, and τ_t^N is a labor tax in the non-tradable sector, to be rebated lump-sum via a transfer T_t^N to non-tradable goods-producing firms.

The competitive equilibrium is given by sequences of consumption, labor, wages and prices of non-tradables such that all optimality conditions are satisfied and market clearing holds for all goods and labor:

$$c_t^T - \frac{b_{t+1}}{R_t} = z_t^T (h_t^T)^\alpha - b_t, \tag{2.30}$$

$$c_t^N = z^N (h_t^N)^\alpha, \tag{2.31}$$

$$\bar{h} = h_t^N + h_t^T. \tag{2.32}$$

A key feature that distinguishes this production economy from our baseline endowment economy is a reallocation of labor across sectors occurring in tandem with movements in the real exchange rate, in line with the empirical regularity documented, for instance, by 2015. During booms, labor moves to the non-tradable sector as the real exchange rate appreciates, while during crises, labor moves to the tradable sector as the real exchange rate depreciates.

From a normative standpoint, the scope for labor reallocation also opens the door to welfare-improving ex-post stabilization policy interventions. In a crisis, reallocating labor from the non-tradable to the tradable goods sector appreciates the real exchange rate and contributes to relaxing agents' credit constraint. Such ex-post interventions, in turn, reduce the need for precautionary savings ex ante, and can result in the constrained-efficient level of borrowing being higher than in the laissez-faire economy, a phenomenon referred to as "underborrowing" by Benigno et al. (2013b).

In Appendix B.4, we show that irrespective of whether the planner has access to an ex-post instrument, such as a labor tax in the non-tradable sector, our main result that the constrained-efficient allocation can be implemented with reserves remains intact.[41] When

[41] Formally, we show that the proof of Proposition 2.1 applies to the production economy considered in this section, whether or not the planner has access to the labor tax.

the planner has access to a labor tax in the non-tradable sector, the optimal tax is positive whenever the credit constraint binds. By reallocating labor toward the tradable sector, this supports the real exchange rate and relaxes the credit constraint. Because the intervention makes financial crises less severe, it reduces the ex ante precautionary savings motive. However, what is key for our results is that households continue to face a low private shadow cost of debt relative to the social one. Hence, reserve accumulation remains desirable as a tool to increase the NFA position, irrespective of whether the planner has access to an ex-post instrument.

A quantitative analysis of the production economy model with an ex-post stabilization policy shows that the scope for reserve accumulation remains broadly in line with those of our baseline endowment economy model. [42] Implementing the constrained-efficient allocation with reserves and a labor tax in the non-tradable sector results in a reduction in the frequency and severity of financial crises comparable to those obtained in the endowment economy model, as indicated in Table 2.4. The average level of reserves, standing at 8.7% of output, is also broadly comparable, albeit somewhat lower.

Liquidity role of reserves. We argued in Section 2.3.5 that in a crisis, it is optimal for the government to fully deplete its reserves. To further highlight the liquidity role of reserves in mitigating the severity of financial crises, we consider a scenario in which the government suboptimally keeps a fraction of reserves. More precisely, we construct an event analysis, as in Section 2.4.4, but assume that at the time of the crisis, the government unexpectedly deviates from the optimal reserve policy for one period and keeps 25% of its current level of international reserves. Figure 2.15 compares the dynamics of crises under this suboptimal policy with that associated with the optimal intervention. The experiment points to a strong liquidity role of reserves, as the depletion of an additional 1 percentage point (of GDP) of reserves reduces the current account reversal by over 3 percentage points (of GDP) and the real exchange rate depreciation by nearly 10 percentage points.

Interest rate shocks Finally, we consider shocks to the risk-free interest rate R_t to capture fluctuations in US monetary policy. Naturally, when interest rates are low, the model predicts an increase in consumption and a reduction in the desired NFA position. However, the effects on gross positions are more subtle. Reserves, in particular, respond

[42] Our calibration assumes that the productivity of tradables follows a log-normal AR(1) of the form: $\ln A_t^T = \rho^A \ln A_{t-1}^T + \varepsilon_t^A$ with $\varepsilon_t^A \sim N(0, \sigma_A)$. The persistence parameter $\rho^A = 0.24$ is equal to the persistence to the endowment of tradables in the original model. The volatility $\sigma_A = 0.017$ is chosen to ensure that the standard deviation of tradable output as a share of output at the ergodic distribution coincides with its counterpart in the endowment economy.

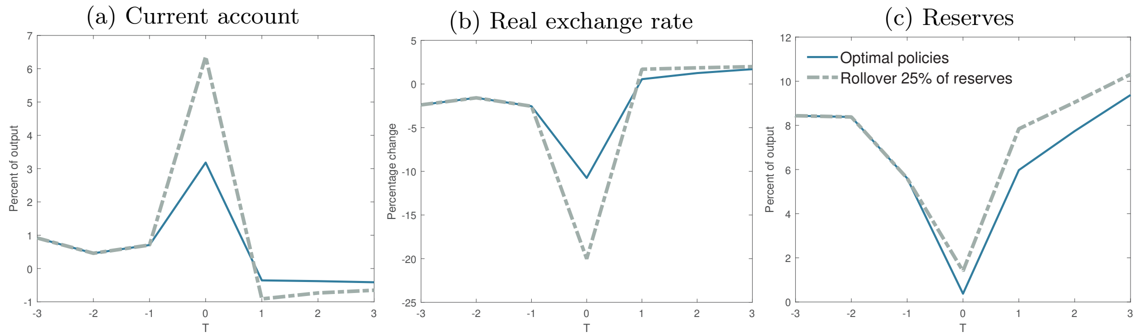

Figure 2.15: Financial crises event analysis when reserves are not fully depleted.

to the gap between the economy's borrowing capacity and the constrained-efficient level of borrowing, as revealed by (RP). Keeping all other parameters at their calibrated values, we obtain quantitative results very similar to those obtained in our baseline model with a constant risk-free rate (see Table 2.4).[43] In particular, the average level of reserves is unchanged at 12.2% of output and the reduction in the frequency and severity of financial crises remain comparable.

Table 2.4: Long-run moments in extended models.

	Baseline		Production economy		Stochastic interest rate	
	LF	Optimal	LF	Optimal	LF	Optimal
Reserves-GDP	-	12.2	-	8.7	-	12.2
Crisis $\mathbb{P}$	1.8	0.4	1.9	0.3	1.8	0.3
CA reversal	13.2	2.8	6.6	2.6	13.0	2.6

2.5 Conclusions

This paper has articulated a macroprudential theory of foreign reserve accumulation in which reserves provide a liquidity value not internalized by private agents. The model is consistent with several aspects of the interaction between private and public capital flows

[43]We follow 2016 in the formulation and calibration of the interest rate shocks. Interest rates follow a regime-switching process with two values to capture the global liquidity phases from Calvo, Leiderman, and Reinhart (1996) and Shin (2014). Bianchi, Liu, and Mendoza estimate a gross interest rate differential between the high and low liquidity regimes of 105 basis points, a probability of staying in the high interest rate regime of $F_{hh} = 0.933$, and the probability of staying in the low interest regime is $F_{ll} = 0.6$. We use their estimates, keeping the same mean for the interest rate and obtain an annual annual gross interest rate for the low interest rate regime of $R^l = 0.9995$ and $R^h = 1.0468$ for the high interest rate regime.

observed in the data. It can account for the increase in reserves and private debt over the last twenty years, the positive association between these variables in the cross-section and the time-series, and the observation that countries with more open financial accounts accumulate more international reserves.

There are several interesting avenues for future research. One would be to apply and further investigate the lessons of our theory for the use of reserve accumulation in models of financial crises that combine aggregate demand externalities and pecuniary externalities. Another would be to extend our theory to allow for frictions in the government's financing of reserve accumulation. We have, in effect, assumed that the central bank can finance reserve accumulation through government transfers. An alternative would be to consider distortionary taxes. A final avenue for future research would be to depart from the assumption of deep-pocket international investors. In this scenario, reserve accumulation would introduce arbitrage losses for the small open economy (Amador et al., 2017; Fanelli and Straub, 2017), and the government would balance these costs against the financial stability benefits uncovered in this paper.

Chapter 3

Optimal Redistribution in a Sudden Stop-Prone Economy

3.1 Introduction

The first two chapters focus on how excessive international private debt can interact with public debt and international reserves management. This chapter instead studies how excessive private debt can affect income inequality and the distributional consequences of macroprudential policies.

We first document the positive relationship between income inequality and sudden stop crises. Using the data from 58 countries over the time period of 1980-2017, we find that increases in pre-tax Gini index are associated with increases in the frequency of sudden stop crisis.

We incorporate heterogeneous agents differentiated by income in a dynamic stochastic general equilibrium model with collateral constraints linking borrowing capacity to the market value of income. Thus, differences in income lead to differences in borrowing capacities across agents.

We study two versions of this model. In the decentralized version, households make their individual borrowing decisions taking all prices and aggregates laws of motion as given. We contrast this version with a constrained efficient version. In this version, a benevolent utilitarian social planner makes borrowing decisions for all households in the economy. The planner is subject to the same budget and credit constraints as the households in the decentralized version, but controls aggregate borrowing and the distribution of debt across households.

As previous authors have showed we find higher indebtedness in the decentralized economy than in the centralized economy. However, we find that this excessive debt burden is more pronounced for lower income households. Moreover, we find that when the credit constraint is biding for lower income households, but not for higher income households, the planner would increase borrowing for this latter group. The intuition for this result is that higher private borrowing by high income households induces an increase in the relative price of nontradables and through this channel, relaxes the credit constraint of the lower income households.

Related Literature. The model builds on the international borrowing model developed by Mendoza (2002) and Bianchi (2011). The contribution is a framework that highlights the interaction between international private debt, financial crises and, and redistributive policies. In contrast to most of the work in this literature, the paper considers the case of heterogeneous agents. Consequently, macroprudential policies will have distributional effects.

This research also draws from the literature that studies the trade-off between debt management and redistribution such as Werning (2007), and Bhandari et al. (2017). By introducing a continuum of households with direct access to international credit markets, the model will highlight the differences between centralized and decentralized international borrowing.

Outline. The chapter is organized as follows. Section 3.2 provides the empirical motivation. Section 3.3 describes a model of international private debt and inequality and describes the competitive equilibrium. Section 3.4 presents the social planner problem. Section 3.5 presents the numerical analysis. Section 3.6 then concludes.

3.2 Empirical motivation

In this section, we document the empirical relationship between the frequency of sudden stop crises and income inequality. We find that high levels of income inequality is associated with high frequency of sudden stop crises in a dataset of 58 countries over the time period 1980-2017. To measure the frequency of sudden stop crises, we use the database on capital flow episodes developed by Forbes and Warnock (2020) with gross flows data. We calculate a sudden stop crisis indicator of values 0-1 that indicating whether there is a capital flow episode in a given year. A capital flow episode is a surge, stop, flight, or retrenchment, defined by Forbes and Warnock (2020). For income inequality, we use pre-tax (market) Gini indices from the Standardized World Income Inequality Database (SWIID) by Solt

(2019). We estimate the following probit model

$$\mathbb{E}(\text{sudden stop crisis}|\text{pre-tax Gini}) = \Phi(\beta_0 + \beta_1 \text{pre-tax Gini}),$$

where Φ is the standard cumulative normal distribution.

Table 3.2 shows the estimation results of the probit model. The coefficient on the pre-tax Gini index is positive and statistically significant both in the cases of no controls and controlling for GDP per capita. Overall, the marginal effect of a 1% increase in the pre-tax Gini index is 0.53% increase in the frequency of sudden stop crisis.

Table 3.1: Probit Regression Result

	Dependent Variable: Sudden stop crisis indicator	
	Time periods: 1980-2017	
	(1)	(2)
Gini index, pre tax (%)	**0.0129*** **	**0.0134*** **
	(0.005)	(0.005)
GDP per capita		4e-6***
		(2e-6)
No. Countries	58	58
No. Observations	2051	1958

Note: This table presents the results from the probit regression. The dependent variable is a sudden stop crisis indicator of values 0-1 indicating whether there is a capital flow episode defined by Forbes and Warnock (2020). The Gini index of pre-tax income is obtained from SWIID database. GDP per capita is from the IMF WEO database. Estimates are obtained using maximum likelihood estimation. *** represents the significant level of 1%.

3.3 Model of international private debt and inequality

This section presents a heterogeneous-agent dynamic model of a small open economy with non-state contingent bonds subject to an occasionally binding borrowing constraint as in Bianchi (2011). We define and characterize the competitive equilibrium.

3.3.1 Environment

Time is discrete and indexed by $t = 0, 1, \ldots, \infty$. There are tradable and nontradable goods sectors. Only tradable goods can be traded internationally, and nontradable goods have to be consumed in the domestic economy. The economy is populated by of a unit-measure continuum of infinitely lived households that are differentiated by endowment shares $\left(s^i\right)_{i\in I}$, where I is finite. The fraction of households with endowment share s^i is π^i. We normalize $\left(\pi^i\right)_{i\in I}$ and $\left(s^i\right)_{i\in I}$ such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i s^i = 1$.

Allocation. Following the standard convention, lowercase denotes the individual level, while uppercase denotes the aggregate level. Individual household i's allocation on consumption and borrowing is $c_t^{T,i}, c_t^{N,i}, c_t^i, b_{t+1}^i$. The aggregate allocation is then $C_t^T = \sum_{i\in I}\pi^i c_t^{T,i}$, $C_t^N = \sum_{i\in I}\pi^i c_t^{N,i}$, $C_t = \sum_{i\in I}\pi^i c_t^i$, $B_t = \sum_{i\in I}\pi^i b_t^i$.

Preference. All households have the same preference that is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \tag{3.1}$$

where $\mathbb{E}_t(\cdot)$ is the time-t expectation operator, and $0 < \beta < 1$ is the discount factor. The consumption basket c is an Armington-type CES aggregator with elasticity of substitution $1/(\eta+1)$ between tradable c^T and nontradable goods c^N,

$$c_t = c(c_t^T, c_t^N) = \left[\alpha\left(c_t^T\right)^{-\eta} + (1-\alpha)\left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \quad \eta > -1, \ \alpha \in (0,1).$$

Endowment. In each period t, household i receives a fraction s^i of endowment of tradable goods Y_t^T and nontradable goods Y_t^N. Both endowments are drawn from first-order Markov processes independent of each other and of all other stochastic shocks in the model. The numeraire is the tradable good.

Assets. Households have access to one-period, non-state contingent international bonds denominated in units of tradables. The bond is issued in international competitive credit markets at price Q. We assume that the discount factor and the international bond price are such that $\beta/Q < 1$. In each period t,individual household i's borrowing b_{t+1}^i is subject to a collateral credit constraint such that the market value of debt issuances cannot exceed a fraction θ of the market value of current income.

3.3.2 Competitive equilibrium

Individual household's problem. Given the price of bond q and the of nontradable goods in units of tradables P_t^N, individual household i chooses allocation $\left\{c_t^{T,i}, c_t^{N,i}, b_{t+1}^i\right\}_{t\geq 0}$ that maximizes utility (3.1) subject to the budget constraint

$$c_t^{T,i} + P_t^N c_t^{N,i} + b_t^i = s^i\left(P_t^N Y_t^N + Y_t^T\right) + Q b_{t+1}^i, \tag{3.2}$$

and the credit constraint

$$Qb^i_{t+1} \leq \theta s^i \left(Y^T_t + P^N_t Y^N\right). \tag{3.3}$$

This credit constraint can be seen as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction θ of the value of current endowment owned by a defaulting household.

Resource constraints. Given the aggregate allocation, the resource constraints in the tradable and nontradable goods sectors are

$$C^T_t + B_t = Y^T_t + QB_{t+1} \tag{3.4}$$

$$C^N_t = Y^N_t \tag{3.5}$$

Recursive Formulation. We consider the optimization problem of individual households in recursive form. Individual household i makes decisions on current consumption and next-period debt based on the current individual debt b, the current exogenous shock on tradables Y^T, and the current aggregate distribution of debt $\boldsymbol{B} = \left(B^i\right)_{i \in I}$. The optimization problem of individual i can be written as

$$V^i(b, Y^T, \boldsymbol{B}) = \max_{c^T, c^N, b'} \quad \frac{c\left(c^T, c^N\right)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{Y^{T'}|Y^T} V^i(b', Y^{T'}, \boldsymbol{B}')$$

subject to

$$c^T + P^N(Y^T, \boldsymbol{B})c^N + b = s^i \left(Y^T + P^N(Y^T, \boldsymbol{B})Y^N\right) + Qb'$$
$$Qb' \leq \theta s^i \left(Y^T + P^N(Y^T, \boldsymbol{B})Y^N\right)$$
$$\boldsymbol{B}' = \Gamma(Y^T, \boldsymbol{B}),$$

where Γ is the law of motion for the distribution of debt. The solution to the household problem gives the individual allocation rule $\left\{c^{T,i}(b, Y^T, \boldsymbol{B}), c^{N,i}(b, Y^T, \boldsymbol{B}), b'^i(b, Y^T, \boldsymbol{B})\right\}$. Then we have the following definition for a recursive competitive equilibrium.

Definition 3.1. *A recursive competitive equilibrium is an individual allocation rule* $\left\{c^{T,i}(b, Y^T, \boldsymbol{B}),\ c^{N,i}(b, Y^T, \boldsymbol{B}),\ b'^i(b, Y^T, \boldsymbol{B})\right\}$ *sand individual value function* $V^i(b, Y^T, \boldsymbol{B})$,*for each* $i \in I$, *aggregate allocation rule* $\left\{C^T(b, Y^T, \boldsymbol{B}),\ C^N(b, Y^T, \boldsymbol{B}),\ B'(b, Y^T, \boldsymbol{B})\right\}$ *, a pricing function* $P^N(Y^T, \boldsymbol{B})$*, and a law of motion* $\Gamma(Y^T, \boldsymbol{B})$ *such that*

- *Household optimization: given* $P^N(Y^T, \boldsymbol{B})$ *and* $\Gamma(Y^T, \boldsymbol{B})$*, for each* $i \in I$,

$\left\{c^{T,i}(b, Y^T, \boldsymbol{B}), \quad c^{N,i}(b, Y^T, \boldsymbol{B}), \quad b'^i(b, Y^T, \boldsymbol{B})\right\}$ *solves household i's problem and* $V^i(b, Y^T, \boldsymbol{B})$ *is the associated value function*

- *Rational expectation:* $\Gamma(Y^T, \boldsymbol{B}) = \left(b'^i(b, Y^T, \boldsymbol{B})\right)_{i \in I}$
- *Aggregation:* $C^T(b, Y^T, \boldsymbol{B}) = \sum_i \pi^i c^{T,i}(b, Y^T, \boldsymbol{B})$, $C^N(b, Y^T, \boldsymbol{B}) = \sum_i \pi^i c^{N,i}(b, Y^T, \boldsymbol{B})$, $B'(b, Y^T, \boldsymbol{B}) = \sum_i \pi^i b'^i(b, Y^T, \boldsymbol{B})$,
- *Market clearance:* $C^N(b, Y^T, \boldsymbol{B}) = Y^N$, $C^T(b, Y^T, \boldsymbol{B}) + \sum_i \pi^i B^i = Y^T + QB'(b, Y^T, \boldsymbol{B})$

3.3.3 Characterizing Competitive Equilibrium

The optimality conditions for individual household i include the budget constraint (3.2), the credit constraint (3.3), and the first-order conditions. In particular, the intratemporal optimality condition implies that

$$P^N(Y^T, \boldsymbol{B}) = \frac{1-\alpha}{\alpha} \left(\frac{c^{T,i}(b, Y^T, \boldsymbol{B})}{c^{N,i}(b, Y^T, \boldsymbol{B})} \right)^{\eta+1}, \ \forall i \in I \tag{3.6}$$

Equation (3.6) is a static optimality condition equating the relative price of tradable to nontradable goods to the marginal rate of substitution between them for any household $i \in I$. This condition implies that the marginal rate of substitution between tradable and nontradable goods are the same across households. Therefore, in equilibrium, it must be that the individual shares of consumption in tradable and nontradable goods are the same, i.e., there exists $\left(\omega^i\right)_{i \in I}$, where $\omega^i \geq 0, \forall i$ and $\sum_i \pi^i \omega^i = 1$, such that

$$\omega^i(b, Y^T, \boldsymbol{B}) = \frac{c^{T,i}(b, Y^T, \boldsymbol{B})}{C^T(Y^T, \boldsymbol{B})} = \frac{c^{N,i}(b, Y^T, \boldsymbol{B})}{Y^N}, \ \forall i \in I. \tag{3.7}$$

Using equation (3.7), we can write the relative price in equation (3.6) as a function of aggregates,

$$P^N(Y^T, \boldsymbol{B}) = \frac{1-\alpha}{\alpha} \left(\frac{C^T(Y^T, \boldsymbol{B})}{Y^N} \right)^{\eta+1}. \tag{3.8}$$

3.4 Efficiency

3.4.1 Social Planner's Problem

We now formulate the problem of a benevolent social planner with restricted planning abilities. Specifically, we consider that the social planner can directly choose the level of borrowing subject to the credit constraints but allows goods markets to clear competitively. In contrast to the competitive equilibrium households that take prices as given, the social planner internalizes the effects of borrowing decisions on the relative price.

The objective of the social planner is

$$\sum_{i\in I}\gamma^i\pi^i\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t u(c_t^i), \tag{3.9}$$

where the social welfare weights are $\boldsymbol{\gamma}=\left(\gamma^i\right)_{i\in I}, \gamma^i\geq 0, \forall i, \sum_i \pi^i\gamma^i=1$. The optimization problem of the social planner is choosing all individual allocation to maximize social welfare (3.9) subject to the resource constraints (3.4)–(3.5), the household's budget constraints (3.2), the credit constraints (3.3), and the equilibrium price condition (3.8).

Recursive Formulation. The social planner's problem in recursive form is

$$\begin{aligned}
V(Y^T,\boldsymbol{B}) = \max_{\left\{c^{T,i},c^{N,i},b'^i\right\}_{i\in I}} & \quad \sum_{i\in I}\gamma^i\pi^i\frac{c\left(c^{T,i},c^{N,i}\right)^{1-\sigma}}{1-\sigma}+\beta\mathbb{E}_{Y^{T'}|Y^T}V(Y^{T'},\boldsymbol{B}') \\
\text{subject to} & \\
\sum_{i\in I}\pi^i c^{T,i}+\sum_{i\in I}\pi^i b^i &= Y^T+Q\sum_{i\in I}\pi^i b'^i \\
\sum_{i\in I}\pi^i c^{N,i} &= Y^N \\
c^{T,i}+P^N c^{N,i}+b^i &= s^i\left(Y^T+P^N Y^N\right)+Qb'^i, \quad \forall i\in I \\
Qb'^i &\leq \theta s^i\left(Y^T+P^N Y^N\right)P^N=\frac{1-\alpha}{\alpha}\left(\frac{\sum_{i\in I}\pi^i c^{T,i}}{Y^N}\right)^{\eta+1}, \quad \forall i\in I
\end{aligned} \tag{3.10}$$

Definition 3.2. *A recursive socially planned equilibrium is the allocation rule* $\left\{c_{SP}^{T,i}(Y^T,\boldsymbol{B}),\ c_{SP}^{N,i}(Y^T,\boldsymbol{B}),\ b_{SP}'^{i}(Y^T,\boldsymbol{B})\right\}_{i\in I}$ *and the value function* $V_{SP}(Y^T,\boldsymbol{B})$ *that solve (3.10) given the welfare weights* $\boldsymbol{\gamma}$.

For each $i\in I$, let φ^i be the share of individual i's borrowing to the aggregate borrowing, and denote $\boldsymbol{\varphi}=\left(\varphi^i\right)_{i\in I}$. Using the property of $\boldsymbol{\omega}$ from equation (3.7), we can rewrite the above problem as one in which the social planner chooses aggregate allocation C^T, B', the consumption distribution $\boldsymbol{\omega}$, and the debt distribution $\boldsymbol{\varphi}$:

$$
\begin{aligned}
V(Y^T, B, \boldsymbol{\varphi}) = \max_{C^T, \boldsymbol{\omega}, B, \boldsymbol{\varphi}'} & \quad \frac{c\left(C^T, Y^N\right)^{1-\sigma}}{1-\sigma} \sum_i \pi^i \gamma^i \left(\omega^i\right)^{1-\sigma} + \beta \mathbb{E}_{Y^{T'}|Y^T} V(Y^{T'}, B', \boldsymbol{\varphi}') \\
\text{subject to} & \\
C^T + B &= Y^T + QB' \\
\omega^i \left(C^T + P^N Y^N\right) + \varphi^i B &= s^i \left(Y^T + P^N Y^N\right) + Q\varphi'^i B', \quad \forall i \in I \\
Q\varphi'^i B' &\leq \theta s^i \left(Y^T + P^N Y^N\right), \quad \forall i \in I \\
P^N &= \frac{1-\alpha}{\alpha} \left(\frac{C^T}{Y^N}\right)^{\eta+1}
\end{aligned}
\tag{3.11}
$$

3.5 Numerical Analysis

In this section, we consider a numerical example of an economy that is populated by two types of households receiving s^H and s^L shares of the endowment in every period, respectively, where $s^H \geq s^L > 0$ and $\pi^H = \pi^L = 0.5$. We normalize $Y^N = 1$ and assume that the endowment of tradables Y^T follows a logged first-order autoregressive process:

$$
\log Y_t^T = \rho_y \log Y_{t-1}^T + \epsilon_t^y,\ \epsilon_t^y \sim \mathcal{N}(0, \sigma_y),
$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. We discretize the tradable endowment process into a Markov chain using Tauchen's method with 20 evenly-spaced nodes.

Table 3.2: Parameters and Values

Parameter	Description	Value	Source/Target
r^*	Risk-free rate	0.02	German 6-year government bond yield
β	Discount factor	0.91	Standard literature value
σ	Risk aversion	2	Standard literature value
$1/(1+\eta)$	Elasticity of substitution T-NT	1.0	Standard literature value
α	Share of tradables	0.39	Spanish data
$\frac{s^H}{s^L}$	Low-type endowment share	2	
ρ_y	Output persistence	0.777	Spanish data
σ_y	Std. dev. of tradeable shock	0.029	Standard literature value

Note: The table describes the parameters and their values.

Table 3.2 reports the parameter values. The risk-free rate r^* is set to 2%, the average annual gross yield on German 6-year government bonds over the period 2000 to 2015. Using the standard values in the literature, we set the discount factor β at 0.91, the risk aversion σ at 2, and the elasticity of substitution between tradables and nontradables $1/\eta$ at 1.0. The persistence of the tradable income shock is set to be 0.91 similar to persistence of Spanish output. The standard deviation of the tradable shock σ_y is set to the standard value of 2.9%.

3.5.1 Preliminary Findings

Using the parameters presented in the previous subsection we solve both versions of the model. This exercise allow us to simulate 10,500 periods of data using the policy rules of each version. We then exclude the first 500 periods of this simulation to focus on the ergodic distribution and not the transitionary dynamics. Figure 3.1 plots the density probabilities of debt issuance by household type at this ergodic distribution. We can see that relative to the decentralized economy, the social planner has a higher probability of observing higher levels of debt for the low type households and lower levels of debt for the high type.

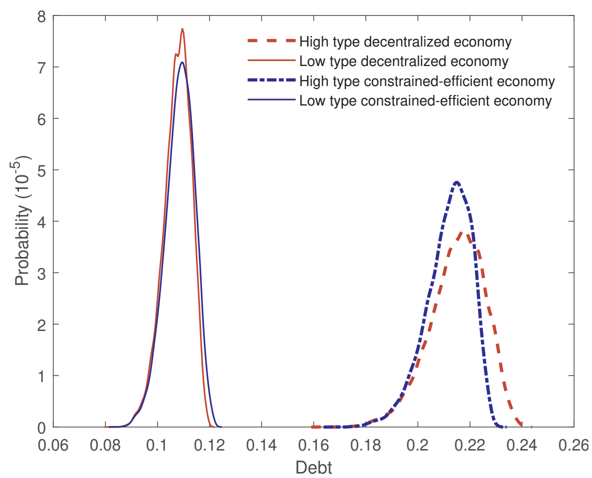

Figure 3.1: Distribution of debt by type at the ergodic distribution

Note: We simulate 10,000 samples and compute the distribution of debt issuance by type for each version of the model.

This and further findings of this numerical simulation are presented in Table 3.3. We find that relative to the decentralized economy the constrained-efficient economy, exhibits higher levels of borrowing by the high type households on average. Conversely, low income households underborrow from a social standpoint. Overall, the first effect dominates the latter, and as a result the economy as a whole overborrow. We define a sudden stop crisis in this context as a period in which the current account of the economy as a whole increases by more than one standard deviation. We find that the probability of experiencing a sudden stop is 11.7% in the decentralized economy as opposed to 3.2% in the constrained-efficient version. Finally, we compare this moments to those obtained in an economy with no inequality. In this representative agent economy there are still welfare gains from moving from a decentralized to a socially planned economy. However, those welfare gains are smaller since the probability of experiencing a sudden stop crisis is smaller in than in the decentralized environment with inequality.

Table 3.3: Decentralized vs. constrained efficient at the ergodic

	Inequality: $s^H/s^L = 2$		No inequality: $s^H/s^L = 1$	
Average (in %)	DE	CE	DE	CE
Debt/income				
Aggregate	32.2	32.1	32.3	32.0
High type	21.5	21.2		
Low type	10.8	10.9		
Prob. of crisis	11.69	3.18	11.63	3.28
Welfare gain	–	0.011	–	0.0099

In Table 3.4 we focus on the sudden stop crises. We compute the average deviations relative to the mean at the ergodic for different aggregates conditional on experiencing a sudden stop. This exercise allows us to measure the magnitude of a sudden stop crisis. We see that the decentralized economy behaves in a homothetically. In other words, the relative impact of a sudden stop crisis is the same for all agents and coincides with the observed magnitude in the economy without inequality. In the constrained-efficient economy the utilitarian preferences of the planner break this hometheticity. During a sudden-stop episode the planer will sustain a higher level of consumption for the high types in order to appreciate the real exchange rate. This will relax the credit constraint of the low income households. The results is that composite good consumption of the high type will increase during a crisis.

3.5.2 Policy functions

In this subsection we provide a brief summary of the policy functions of debt accumulation for both types of households in the numerical simulation. These figures shed some light on the mechanism behind the results presented in the previous subsection. In Figure 3.2 we plot the policy functions of debt issuance for each household type. Concretely, on panel (a) we fix the income shock to its average value and the current level

Table 3.4: Decentralized vs. constrained efficient during a sudden stop

	Inequality: $s^H/s^L = 2$		No inequality: $s^H/s^L = 1$	
Average (in %)	DE	CE	DE	CE
C^T drop during crisis				
High type	2.53	0.04	2.52	0.72
Low type	2.53	1.35	2.52	0.72
C drop during crisis				
High type	1.02	-0.22	1.01	0.31
Low type	1.02	1.09	1.01	0.31

of debt of the low type household to its average at the ergodic distribution. We then plot the evolution of optimal debt issuances of the high type as a function of the initial debt of the high type for each version of our model. Symmetrically, on panel (b) we fix the income shock at the same value and the current level of debt of the high type household to its average at the ergodic distribution. We then plot the evolution of optimal debt issuances of the low type as a function of the initial debt of the low type for both versions of the model.

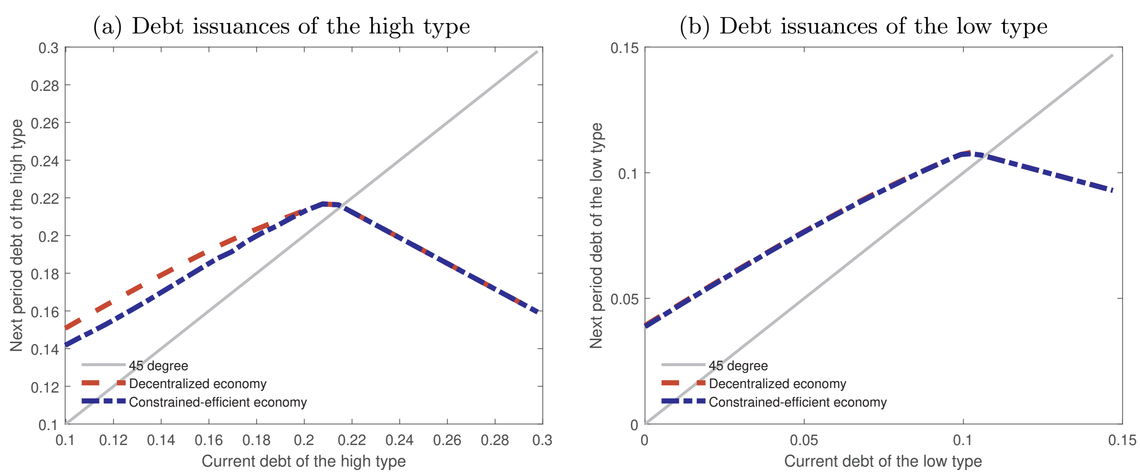

Figure 3.2: Policy functions of debt issuances by type

The results of panel (a) are inline with the standard sudden-stop literature. As in Bianchi (2011). We find that the social planner would like the high type households to issue less debt than in the decentralized economy when the constraint is not binding. Conversely, the results of panel (b) differ from the standard model. This time we do not find any meaningful differences between the decentralized and constrained efficient economy. This is in line with what we found on the previous subsection.

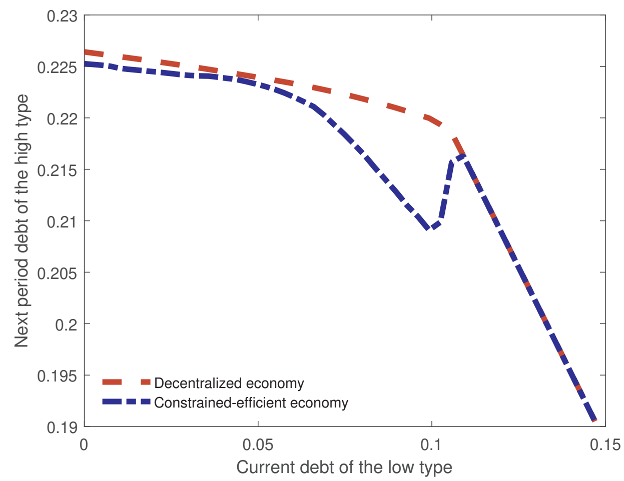

Figure 3.3: Debt issuances of the high type as function of current debt of the low type

Finally, another novelty of our environment is the presence of the interaction between the debt issuances of the low and high types. In figure 3.3, we plot the policy function of the high type as function of the initial debt of the low type when we keep income and the initial debt of the high type fixed at their values from the previous figure. We can see that as the level of initial debt of the low types increases we move from an environment in which both credit constraints are slacking to one where both constraints are binding. We find that the planner has an additional region where the credit constraint of the low type is binding but not the one of the high type. In this area, the planner pushes the high type to issue more debt to relax the credit constraint of the low type creating an additional kink relative to the standard monotonicity exhibited by the decentralized policy function.

3.6 Conclusion

This chapter proposes a theory of how sudden stop crises affect countries with ex-ante income inequality. We find that higher income inequality increases the probability of experiencing a sudden-stop crisis because high income households borrow above the socially optimal level. Once the a sudden-stop crisis materializes, however, high income households underborrow from a social standpoint. This response depresses the real exchange rate and tightens the collateral constraint of the low income households. A social planner who could choose the distribution of international borrowing, while respecting the credit and budget constraints of the households would on average lower borrowing from high income households and increase debt insurances for low income households.

In future versions of this paper, we would like to study how to implement the allocations that solve the constrained efficient problem. In light of the results that we have, however, these optimal policies will have to be type dependent. We, therefore, also want to find ”second best policies” where a planner is constrained to offer a uniform tax to all types of agents and estimate the welfare differences between these policies.

The aim of this paper is ultimately to shed some light on the interaction between income inequality and sudden-stop crises. Both in terms of the link that exists in the absence of government's interventions and on redistributional consequences of macropruential policies.

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Appendix A

Appendix to Chapter 1

A.1 Recursive competitive problem with taxes

For the representative household, the aggregate state of the economy includes the exogenous aggregate shocks denoted by $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$, the initial level of government debt L, the initial level of aggregate private debt B, and the initial level of its own debt b. Following the same notation than in the body of the paper I denote $S = (s, L, B)$ the state space of the economy before government actions. Similarly, let $S_G = (S, d, L', \tau)$ denote the state space after government actions. Note that now that state includes the choice of taxes.

As before, households take as given the price of non-tradables $p^{N\tau}(S_G)$, the equilibrium price of price bonds $q^{\tau}(s)$, and government's current and future decisions regarding default d^{tau}, public debt $\mathcal{L}^{\tau}$, and taxes τ. They also know the functions associates with these choices, the lump-sum transfer $\mathcal{T}^{\tau}$ and the preference shock $\mathcal{D}^{\tau}$. Finally, they also have a perceived law of motion of aggregate private debt $\mathcal{B}^{\tau\prime}$. The household's optimization problem in recursive form is:

$$V^{\tau}(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V^{\tau}(S'_G, b')] \tag{A.1}$$

subject to

$$c^T + p^{N,\tau}(S_G)c^N + (1-\pi)b = y^T + p^{N,\tau}(S_G)y^N + q^{\tau}(s)(1-\tau)b' + T,$$

$$q^{\tau}(s)b' \leq \kappa[p^{N,\tau}(S_G)y^N + y^T],$$

$$T = \mathcal{T}^{\tau}(S_G),$$

$$D = \mathcal{D}^{\tau}(S_G),$$

$$B' = \mathcal{B}^{\tau\prime}(S_G),$$

$$L' = \mathcal{L}^{\tau\prime}(S_G)$$

$$\tau = \tau(S_G),$$

$$\text{And } S'_G = (s', L', B', d^{\tau}(s', L', B'), \mathcal{L}^{\tau\prime}(s', L', B'), \tau(s', L', B')).$$

Using the same notation than in the baseline case for the aggregate laws of motion of the private sector are $\mathcal{B}^{\tau\prime}(S_G)$, and $\{\mathcal{C}^{i,\tau}(S_G)\}_{i=T,N}$, and public bond pricing $\mathcal{Q}^{\tau}(s, L', B')$ function. The government's problem is:

$$W^{\tau}(S) = \max_{d \in \{0,1\}} [1-d]W^{R.\tau}(S) + dW^{D,\tau}(S) \tag{A.2}$$

In case of default, $S_G = (S, 1, 0, \tau)$ and $W^{D,\tau}(S)$ is given by:

$$W^{D,\tau}(S) = \max_{\tau} u\Big(\mathcal{C}^T, \mathcal{C}^N\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\Big[W^{\tau}\big(s', 0, B'(S_G)\big)\Big] \tag{A.3}$$

subject to

$$\mathcal{C}^{T,\tau}(S_G) + (1-\pi)B = y^T + q^{\tau}(s)(1-\tau)B' + T$$

$$\mathcal{C}^{\mathcal{N},\tau}(S_G) = y^N$$

$$T = q^{\tau}(s)\tau B'$$

$$D = \epsilon^{Def} - \phi(y^T)$$

$$B' = \mathcal{B}^{\tau\prime}(S_G)$$

Note that transfers can still be strictly positive in default since the government transfers the proceeds to of the private debt tax to the households. In case of repayment, $S_G = (S, 0, L', \tau)$

and the value is:

$$W^{R,\tau}(S) = \max_{\tau, L' \in \Lambda} u\big(\mathcal{C}^{T,\tau}, \mathcal{C}^{N,\tau}\big) + \epsilon(L') + \beta \mathbb{E}_s[W^{\tau}(s', L', B')] \tag{A.4}$$

subject to

$$\begin{aligned}
\mathcal{C}^{\mathcal{T},\tau}(S_G) + (1-\pi)B &= y^T + q^{\tau}(s)(1-\tau)B' + T, \\
\mathcal{C}^{\mathcal{N},\tau}(S_G) &= y^N, \\
T &= Q^{\tau}(s, L', \tau, B')[L' - (1-\delta)L] - \delta L + q^{\tau}(s)\tau B', \\
D &= \epsilon(L'), \\
B' &= \mathcal{B}^{\tau\prime}(S_G)
\end{aligned}$$

The solution to the government's problem yields decision rules for default $d^{\tau}(S)$, public borrowing $\mathcal{L}^{\tau\prime}(S)$, and taxes $\tau(S)$. The transfers $\mathcal{T}^{\tau}(S_G)$ and preference shifter $D^{\tau}(S_G)$ are also pinned down by these decisions. The solution to the problem of competitive risk neutral foreign lenders yields the bond price schedule for private debt:

$$q^{\tau}(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{A.5}$$

and for public debt:

$$Q^{\tau}(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\Bigg[\Big[1 - d'\Big] \times \Big[\delta + (1-\delta) Q^{\tau}\Big(s', L'', B''\Big)\Big]\Bigg], \tag{A.6}$$

Where:

$$\begin{aligned}
B'' &= \mathcal{B}^{\tau\prime}(s', L', B'), \\
L'' &= \mathcal{L}^{\tau\prime}(s', L', B'), \\
d' &= d^{\tau}(s', L', B')
\end{aligned}$$

Definition A.1. *A Markov regulated competitive equilibrium with taxes is defined by, a set of value functions* $\{V^{\tau}, W^{\tau}, W^{R,\tau}, W^{D,\tau}\}$*, policy functions for the private sector* $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$*, policy functions for the public sector* $\{d^{\tau}, \mathcal{L}^{\tau\prime}, \tau\}$*, a pricing function for nontradable goods* $p^{N,\tau}$*, pricing functions for public debt* Q^{τ} *and private debt* q^{τ}*, and perceived laws of motion* $\{\mathcal{B}^{\tau\prime}, \mathcal{C}^{T,\tau}, \mathcal{C}^{N,\tau}\}$ *such that*

1. *Given prices* $\{p^{N,\tau}, q^{\tau}\}$*, government policies* $\{d^{\tau}, \mathcal{L}^{\tau\prime}, \tau\}$*, and perceived law of motion* $\mathcal{B}^{\tau\prime}$*, the private policy functions* $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$ *and value function* V *solve the household's problem (A.1)*

2. *Given bond prices* $\{Q^{\tau}, q\}$ *and aggregate laws of motion* $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}\}$*, the public policy functions* $\{d^{\tau}, \mathcal{L}^{\tau\prime}, \tau\}$ *and value functions* W^{τ}*,* $W^{R,\tau}$*, and* $W^{D,\tau}$*, solve the Bellman equations (A.2)–(A.4)*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion* $\{\mathcal{B}'(S_G) = \hat{b}^{\tau\prime}(S_G, B), \mathcal{C}^{T,\tau}(S) = \hat{c}^{T,\tau}(S_G, B), \mathcal{C}^{N,\tau}(S_G) = \hat{c}^{N,\tau} S_G, B)\}$

4. *The private bond price function* $q^{\tau}(s)$ *satisfies (A.5)*

5. *Given public* $\{d^{\tau}, \mathcal{L}^{\tau\prime}, \tau\}$*, and private* $\{\mathcal{B}^{\tau\prime}\}$*, policies the public bond price* $Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)')$ *satisfies (A.6)*

6. *Goods market clear:*

$$\mathcal{C}^{N,\tau}(S_G) = y^N$$
$$\mathcal{C}^{T,\tau}(S_G) + (1-\pi)B = y^T + q^{\tau}(s)\mathcal{B}^{\tau\prime}(S_G) + \Big\{1 - d^{\tau}(S)\Big\}\Big\{Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)')\Big[\mathcal{L}^{\tau\prime}(S) - (1-\delta)L\Big] - \delta L\Big\}$$

Similarly to the baseline model the optimality conditions of the households problem are:

$$q^{\tau}(s)(1-\tau(S)u_T(\mathcal{C}^{T,\tau}(S_G)) = \beta \mathbb{E}_s[(1-\pi')u_T(\mathcal{C}^{T,\tau\prime}(S_G))] + \mu^{\tau}(S_G)q^{\tau}(s),$$
$$p^{N,\tau}(S_G) = \frac{1-\omega}{\omega}\left(\frac{\mathcal{C}^{T,\tau}(S_G)}{y^N}\right)^{\eta+1},$$

$$0 \leq \kappa(p^{N,\tau}(S_G)y^N + y^T) - q^{\tau}(s)\mathcal{B}^{\tau\prime}(S_G) \quad \text{with equality if } \mu^{\tau}(S_G) > 0,$$

where μ^{τ} is the Lagrange multiplier associated with the credit constraint.

A.2 Proof of proposition 1

This is a proof by construction. We will show that the recursive equilibrium with taxes can be written as a government problem that coincides with the planning problem (1.17). Start from the recursive competitive equilibrium problem with taxes described in Appendix A.2.

The problem with taxes is equivalent to the recursive problem of a government given that chooses allocations for the current period while taking future policies and prices as given.

Denote these policies $\{d^\tau(S), \mathcal{L}^{\tau\prime}(S), \tau(S), \mathcal{C}^{T,\tau}(S_G), \mathcal{C}^{N,\tau}(S_G), \mathcal{B}^{\tau\prime}(S_G)\}$. This government maximizes utility considering the optimal responses of households and lenders. This is equivalent to let the government choose all policies using the Kuhn-Tucker conditions of households and lenders as constraints. The problem is therefore:

$$W^\tau(S) = \max_{d \in \{0,1\}} [1-d] W^{R,\tau}(S) + d W^{D,\tau}(S),$$

Let $S' = (S', B', L')$ the default value $W^{D,\tau}(S)$ is:

$$\begin{aligned}
W^{D,\tau}\big(S\big) &= \max_{c^T, c^N, B', \tau, \mu} u\big(c^T, c^N\big) - \phi(y^T) + \epsilon_{Def} + \beta \mathbb{E}_s \Big[W^\tau\big(S'\big) \Big] \\
&\text{subject to} \\
c^T + B(1-\pi) &= y^T + q^\tau(s) B', \\
c^N &= y^N, \\
q^\tau(s) B' &\leq \kappa \Bigg(p^{N,\tau} c^N + y^T \Bigg), \\
q^\tau(s)(1-\tau) u_T(c^T, c^N) &= \beta E_s[(1-\pi') u_T(\mathcal{C}^{T,\tau}, \mathcal{C}^{N,\tau}(S', d^\tau(S'), \mathcal{L}^{\tau\prime}(S'), \tau(S'))] + \mu q^\tau(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega} \big(\frac{c^T}{c^N}\big)^{1+\eta} \\
(\kappa(p^{N,\tau} c^N + y^T) - q^\tau(s) B')\mu &= 0 \\
\mu &\geq 0 \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}$$

The value under repayment $W^{R,\tau}(S)$ is:

$$W^{R,\tau}(S) = \max_{c^T, c^N, B', \tau, \mu, L' \in \Lambda} u\big(c^T, c^N\big) + \epsilon(L') + \beta \mathbb{E}_s[W^\tau(S')]$$

$$\text{subject to}$$

$$c^T + B(1-\pi) + \delta L = y^T + q^\tau(s)B + Q^\tau(s, L', B')[L' - (1-\delta)L],$$

$$q^\tau(s)B' \leq \kappa\left(p^{N,\tau}c^N + y^T\right),$$

$$\begin{aligned} q^\tau(s)(1-\tau)u_T(c^T, c^N) =& \beta \mathbb{E}_s[(1-\pi')u_T(\mathcal{C}^{T,\tau}, \mathcal{C}^{N,\tau}(S', d^\tau(S'), \mathcal{L}^{\tau\prime}(S'), \tau(S'))] + \\ & \mu q^\tau(s) \end{aligned}$$

$$p^{N,\tau} = \frac{1-\omega}{\omega}\Big(\frac{c^T}{c^N}\Big)^{1+\eta}$$

$$(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu = 0$$

$$\mu \geq 0$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}$$

$$\begin{aligned} Q^\tau(s, L', B') =& \frac{1}{1+r} \times \mathbb{E}_s\Bigg[\Big[1 - d^\tau(S')\Big] \times \\ & \Big[\delta + (1-\delta)Q^\tau\Big(s', \mathcal{L}^{\tau\prime}(S'), \mathcal{B}^{\tau\prime}(S', d^\tau(S'), \mathcal{L}^{\tau\prime}(S'), \tau(S'))\Big)\Big]\Bigg] \end{aligned}$$

Substituting in the resource constraint for non tradables, and the intratemporal conditions that problem can be simplified to:

$$W^\tau(S) = \max_{d \in \{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S), \tag{A.7}$$

where default value $W^{D,\tau}(S)$ is:

$$
\begin{aligned}
W^{D,\tau}(S) &= \max_{c^T,B',\tau,\mu} u\big(c^T,y^N\big) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\Big[W^\tau\big(S'\big)\Big] \\
c^T + B(1-\pi) &= y^T + q^\tau(s)B', \\
q^\tau(s)B' &\leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r} \\
q^\tau(s)(1-\tau)u_T(c^T,y^N) &= \beta\mathbb{E}_s[(1-\pi')u_T(\mathcal{C}^{T,\tau},\mathcal{C}^{N,\tau})] + \mu q^\tau(s) \\
0 &= \Bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^\tau(s)B'\Bigg]\mu \\
\mu &\geq 0
\end{aligned}
$$

and value under repayment $W^{R,\tau}(S')$ is:

$$
W^{R,\tau}(S') = \max_{c^T,B',\tau,\mu,L'\in\Lambda} u\big(c^T,y^N\big) + \epsilon(L') + \beta\mathbb{E}_s[W^\tau(S')]
$$

$$
c^T + B(1-\pi) + \delta L = y^T + q^\tau(s)B + Q^\tau(s,L',B')[L' - (1-\delta)L] \tag{A.8}
$$

$$
q^\tau(s)B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) \tag{A.9}
$$

$$
q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r} \tag{A.10}
$$

$$
Q^\tau(s,L',B') = \frac{1}{1+r} \times \mathbb{E}_s\bigg[\Big[1-d^\tau\Big] \times \Big[\delta + (1-\delta)Q^\tau\Big(s',\mathcal{L}^{\tau\prime},\mathcal{B}^{\tau\prime}\Big)\Big]\bigg] \tag{A.11}
$$

$$
q^\tau(s)(1-\tau)u_T(c^T,y^N) = \beta\mathbb{E}_s[(1-\pi')u_T(\mathcal{C}^{T,\tau},\mathcal{C}^{N,\tau})] + \mu q^\tau(s) \tag{A.12}
$$

$$
0 = \Bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^\tau(s)B'\Bigg]\mu \tag{A.13}
$$

$$
\mu \geq 0 \tag{A.14}
$$

In this formulation it is apparent that the social planner problem (1.17) is a relaxed version of problem (A.7). In problem (A.7) the government must satisfy three additional constraints (A.12)–(A.14) and has access to two additional instruments μ and τ. Crucially, both μ and τ only appear in problem (A.7) in constraints (A.12)–(A.14). As such, problem (1.17) will be equivalent to problem (A.7) if we can use the solutions of (1.17) to construct two functions $\mu(s,L,B)$ and $\tau(s,L,B)$ that satisfy (A.12)–(A.14).
Let $\{\mathcal{C}^{SP,T}(s,L,B), \mathcal{C}^{SP,N}(s,L,B), \mathcal{L}^{SP\prime}(s,L,B), \mathcal{B}^{SP\prime}(s,L,B), d^{SP}(s,L,B), Q^{SP}, q^{SP}(s)\}$

be a solution of problem (1.17). Additionally let $\mu^{SP}(s, L, B) \geq 0$ be the multiplier on the collateral constraint of the planner problem (1.17). μ^{SP} corresponds to the shadow value of relaxing the collateral constraint from the planner's perspective. This multiplier is different from μ which corresponds to the shadow value of relaxing the collateral constraint for individual households, and is a variable chosen by the government in (A.7). The complementary slackness condition of the social planner problem (1.17) is:

$$0 = \left[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{\mathcal{C}^{SP,T}(s,L,B)}{y^N}\big)^{1+\eta} y^N + y^T\Big) - q^{SP}(s)\mathcal{B}^{SP\prime}(s,L,B),'\right]\mu^{SP}(s,L,B). \quad \text{(A.15)}$$

As such by setting:

$$\mu(s,B,L) = \mu^{SP}(s,L,B)$$
$$1-\tau(s,L,B) = \frac{\beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(\mathcal{C}^{SP,T}(S'),\mathcal{C}^{SP,N}(S'))\Big)\Big] + \mu^{SP}(s,L,B)q^{SP}(s)}{q^{SP}(s)u_T(\mathcal{C}^{SP,T}(s,L,B),y^N)},$$

We can see that (A.12)–(A.14) are satisfied and therefore the two problems are equivalent.

A.3 Data Appendix

Gross Domestic Product (GDP): *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*,-. Corresponds to Total gross value added in all NACE activities. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Non-tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*. Corresponds to the share of total value added produced in the following industries: public administration, wholesale and retail, construction, and real state. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*. Corresponds to the complement of nontradable valued added as a share of total value added. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Private debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España, 2019, table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of Spanish monetary financial institutions (excluding the Bank of Spain) and other resident sectors. The data series used are 3273771 and 3273777. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Public debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España, 2019, table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of the Bank of Spain and all public administrations. The data series used are 2386960 and 3273774. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Total debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España, 2019, table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of Spain and is calculated as the consolidation of private and public positions. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011.

Risk free rate: *Bloomberg ticker GTDEM1Y Govt*, Corresponds to the average interest rate spread paid on 1 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2011.

Spread on public bonds: *Bloomberg tickers GTESP6YR Govt and GTDEM6Y Govt*, Corresponds to the difference between average interest rate paid on 6 year Spanish treasury bonds and 6 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2015. In the calibration we use data only from 1999 to 2011.

Average Maturity: *Table 5 from the Bank of Spain's economic bulletin Alloza et al., 2019, of March 2019*, Average maturity of the stock of public debt for Spain in years.

Annual data from 1999 to 2011.

Nonperforming loans: *Bloomberg ticker BLTLWESP Index*, Nonperforming loans as a share of total gross loans. Annual data from 1999 to 2015.

Consumption: *Eurostat , GDP and main components (output, expenditure and income) nama_10_gdp*. Corresponds to final consumption expenditure. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2017.

Current Account: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to current account as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

Trade Balance: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to the balance of trade on goods and services as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

A.4 Solution Method: The Government's ex-ante problem

Following the approach of Dvorkin et al., Forthcoming, I can re-write the government's Bellman equations before the ϵ shocks are realized. From an ex-ante point of view, the shocks ϵ make the default and borrowing decisions stochastic. By taking expectations over these shocks, the decisions can be viewed as probabilistic. If we view the previously defined equilibrium as a game between the private and public sector each period, the ϵ shocks allow the government to play mixed strategies. This makes the computation of this problem using value function iteration possible. We follow this approach to write (1.8) from a an ex-ante perspective. That is when all the aggregate states have realized except the ϵ. For this we summarize all other exogenous state variables in $z = (y^T, y^N, \kappa, \pi)$. As mentioned in the main text we assume that L' is a finite and bounded grid with $\mathcal{J}$ elements. Denote by $F(\epsilon)$ the joint cumulative density function of the taste shocks and by $f(\epsilon)$ its joint density function. To simplify notation in what follows, the following operator to denotes the expectation of any function $Z(\epsilon)$ with respect to all the elements of ,

$$Z = \mathbb{E}_\epsilon Z(\epsilon) = \int_{\epsilon_1}\int_{\epsilon_2} ... \int_{\epsilon_{\mathcal{J}+1}} Z(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) f(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) d\epsilon_1, ..d\epsilon_{\mathcal{J}+1} \tag{A.16}$$

Given this notation we have that:

$$W(z, L, B) = E_\epsilon[W(s, L, B)]$$

$$W(z, L, B) = E_\epsilon\Big[\max\big\{W^R(s, L, B); W^D(s, B)\big\}\Big]$$

$$W(z, L, B) = E_\epsilon\Big[\max\big\{\max_{L'\in\Lambda}\{u(\mathcal{C}(s, L, B)) + \epsilon(L') + \beta\mathbb{E}_{z'|z}W(z', L', \mathcal{B}'(s, L, B))\};$$

$$u(\mathcal{C}(s, 0, B)) - \phi(y^T) + \epsilon^{Def} + \beta\mathbb{E}_{z'|z}W(z', 0, \mathcal{B}'(s, 0, B))\big\}\Big]$$

Subject to the resource constraints:

$$\mathcal{C}^T(s, L, B) = y^T + q(s)\mathcal{B}'(s, L, B) - (1-\pi)B + Q(s, L', B')[L' - (1-\delta)\mathcal{B}'(s, L, B)] - \delta\mathcal{B}'(s, L, B)$$

$$\mathcal{C}^N(s, L, B) = y^N$$

Furthermore, if its convenient to define the following expected utility objects:

$$\Upsilon_{L,L'}(z, B) = u(\mathcal{C}(s, L, B)) + \beta\mathbb{E}_{z'|z}W(z', L, \mathcal{B}'(s, L, B))$$

$$\Upsilon_{def}(z, B) = u(\mathcal{C}(s, 0, B)) - \phi(y^T) + \beta\mathbb{E}_{z'|z}W(z', 0, \mathcal{B}'(s, 0, B))$$

Suppose that the ϵ shocks follow a multivariate generalized extreme value distribution with parameters $\{m, v, p\}$ and are i.i,d over time. Where v is the scale parameter and p is the shape parameter and is set to 1. m corresponds to the location parameter and is set to $-v\gamma$ where γ is the Euler constant. Suppose that public debt L is on a grid with $\mathcal{J}$ points. Then the ex-ante value function of the government's recursive problem can be re-written as

$$W(z, L, B) = \Upsilon_{def} + v\log\left[1 + \Big(\sum_{L'\in\Lambda}\exp\Big(-\frac{\Upsilon_{def} - \Upsilon_{L,L'}}{pv}\Big)\Big)^p\right] \tag{A.17}$$

Additionally given this distributional assumptions there are closed form solutions for the ex-ante probability of default and borrowing policy functions conditional on repayment.

Proof. Given our distributional assumptions

$$F(\epsilon) = \exp\Bigg[- \Bigg(\sum_{j=1}^{\mathcal{J}} \exp\Big(- \frac{\epsilon_j - m}{v} \Big) \Bigg) - \exp\Big(- \frac{\epsilon_{\mathcal{J}+1} - m}{v} \Big) \Bigg] \tag{A.18}$$

For $j \in 0, \mathcal{J}+1$ we denote by $F_j(\epsilon) = \frac{\partial F(\epsilon)}{\partial \epsilon_j}$, the marginal with respect to element j^{th} element of ϵ.

$$F_j(\epsilon) = \begin{cases} \frac{1}{v} \exp\Bigg[- \Bigg(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \Bigg) \Bigg] & \exp(-\frac{\epsilon_j - m}{v}) \qquad j = 1..\mathcal{J} \\ \frac{1}{v} \exp\Bigg[- \Bigg(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \Bigg) \Bigg] & \exp(-\frac{\epsilon^{def} - m}{v}) \qquad j = \mathcal{J}+1 \end{cases}$$

Using this notation ant the dropping the states (z, B) from the previously defined $\Upsilon_{L,L'}(z, B)$ functions we can compute the ex-ante policy functions of the government in close form solutions. Let the probability of default be $d(z, L, B) = \mathbb{E}_\epsilon d(z, L, B, \epsilon)$. Note that:

$$\begin{aligned} d(z, L, B) &= \int_{-\infty}^{\infty} F_{\mathcal{J}+1}(\Upsilon_{def} + \epsilon^{def} - \Upsilon_1, ..., \Upsilon_{def} + \epsilon^{def} - \Upsilon_{def}) d\epsilon^{def} \qquad \text{(A.19)} \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\Bigg[- \Bigg(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} + \epsilon^{def} - \Upsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \Bigg) \Bigg] \times \\ &\exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\Bigg[- \exp(-\frac{\epsilon^{def} - m}{v}) \Bigg(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \Bigg) \Bigg] \times \\ &\exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \end{aligned}$$

Define $\exp(\phi_{def}) = 1 + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_h}{v})$. We can use this to rewrite (A.19) as:

$$
\begin{aligned}
d(z,L,B) =& \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[-\exp(-\frac{\epsilon^{def}-m}{v}) \exp(\phi_{def}) \right] \exp(-\frac{\epsilon^{def}-m}{v}) d\epsilon^{def} \\
=& \frac{1}{v \exp(\phi_{def})} \times \\
& \underbrace{\int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^{def}-m-v\phi_{def}}{v}) \right] \exp(-\frac{\epsilon^{def}-m-v\phi_{def}}{v}) d\epsilon^{def}}_{=v} \\
=& \frac{1}{1+\left(\sum_{L' \in \Lambda} \exp\left(-\frac{\Upsilon_{def}-\Upsilon_{L,L'}}{v} \right) \right)}
\end{aligned} \tag{A.20}
$$

Where the last equivalence uses the fact that the PDF of the generalized extreme distribution integrates to 1. Similarly, conditional on repayment, the random component ϵ make the public borrowing decisions random from an ex-ante perspective. Given a set of current aggregate states relevant for the government, it is useful to introduce the probability of choosing an amount of public debt L' *conditional on not defaulting* as:

$$
G_{z,L,B}(L') = \mathbb{P}_{\epsilon}\big(L' | d(z,L,B,\epsilon) = 0\big)
$$

Using the same notation as before we have that for the L' that is the j^{th} element of Λ:

$$
\begin{aligned}
G_{z,L,B}(L') =& \frac{1}{1-d(z,L,B)} \int_{-\infty}^{\infty} F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j \\
=& \frac{1}{(1-d(z,L,B))v} \times \\
& \int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m}{v}) \left(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \right) \right] \times \\
& \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j
\end{aligned}
$$

Defining $\exp(\phi_j) = \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v})$, we can simplify:

$$\begin{aligned}
G_{z,L,B}(L') =& \frac{1}{(1-d(z,L,B))v} \int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m}{v}) \exp(\phi_j) \right] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j \\
=& \frac{1}{(1-d(z,L,B))v \exp(\phi_j)} \times \\
& \underbrace{\int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \right] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j}_{=v} \\
=& \frac{1}{(1-d(z,L,B)) \exp(\phi_j)}
\end{aligned}$$

Finally this can be further simplified to:

$$\begin{aligned}
G_{z,L,B}(L') =& \frac{1}{(1-d(z,L,B))} \times \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})}{\sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{1}{\sum_{H \in \Lambda} \exp\left(\frac{\Upsilon_{L,H} - \Upsilon_{L,L'}}{v} \right)}
\end{aligned} \tag{A.21}$$

Finally the value $W(z, L, B)$ is given by:

$$W(z,L,B) = \sum_{j=1}^{\mathcal{J}+1} \int_{-\infty}^{\infty} (\Upsilon_j + \epsilon_j) F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j$$

$$= \sum_{j=1}^{\mathcal{J}} \int_{-\infty}^{\infty} \frac{\Upsilon_j + \epsilon_j}{v} \times$$

$$\exp\left[- \exp(-\frac{\epsilon^j - m}{v}) \Big(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \Big) \right] \times$$

$$\exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j + \int_{-\infty}^{\infty} \frac{\Upsilon_{def} + \epsilon_{def}}{v} \times$$

$$\exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \Big(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \Big) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def}$$

$$= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j) \times \Bigg[\Upsilon_j + m + v\phi_j +$$

$$\underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon_j - m - v\phi_j}{v}) \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Bigg]}_{= v\gamma}$$

$$+ \exp(-\phi_{def}) \times \Bigg[\Upsilon_{def} + m + v\phi_{def} + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \times}$$

$$\underbrace{\exp\big[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \big] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def} \Bigg]}_{= v\gamma}$$

Where in the last equivalence we have used the fact that for all j:

$$\Upsilon_j + m + v\phi_j = \frac{(\Upsilon_j + m + v\phi_j) \int_{-\infty}^{\infty} \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Big]}{v}$$

The last step (underscored in the above equations) uses one of the integral properties of the Euler constant. We now use the fact we assumed the distribution of shocks to be mean zero, that is $m = -\gamma v$. Using the definition of ϕ_{def} one can see that:

$$\exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] = \frac{\Upsilon_{def} + v \log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})}$$

The value of the government is then given by:

$$
\begin{aligned}
W(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j)[\Upsilon_j + v\phi_j] + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
W(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j + v\log(\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v})} + \\
&\quad \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
W(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j - \frac{v\Upsilon_j}{v} + v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j}{v})(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))} + \\
&\quad \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
W(z,L,B) &= \frac{v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
W(z,L,B) &= \frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \\
&\quad \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
W(z,L,B) &= \left[\frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})}\right]\left[\sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + 1\right] \\
W(z,L,B) &= \Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})) \qquad \text{(A.22)}
\end{aligned}
$$

To sum up the distributional assumptions allow us to obtain closed form solutions for the ex-ante value function (A.22), the policy functions for default (A.20), the public borrowing conditional on repayment (A.21), □

Note that the functions $G_{z,L,B}(L')$ and $d(z,L,B)$ are sufficient to express all government decisions. Using the fact that the shocks are i.i.d over time, and assuming a guess $\mathcal{Q}$ of next price schedule functions, we can use $G_{z,L,B}(L')$ and $d(z,L,B)$ to write the pricing equation of public bonds (1.16):

$$
\begin{aligned}
Q(z,L',B') =& q(z)\mathbb{E}_{z'|z}\Bigg[\big[1 - d(z',L',B')\big]\big[\delta + \\
& (1-\delta)\sum_{L''\in\Lambda} \mathcal{Q}\big(z',L'',\mathcal{B}'(z',L',B')\big)G_{z',L',B'}(L'')\big]\Bigg] \qquad \text{(A.23)}
\end{aligned}
$$

In the quantitative section we assume that the shocks are mean zero ($m = -\gamma v$). We also assume that the shape parameter p is one, therefore taste shocks are independent from each other within the period as well. The scale parameter v is calibrated to match the variance of public debt in the data.

A.5 Numerical Solution

In this section we provide more detail about the solution methods we use to solve both the baseline and planner version of the model described in the main text. For both solutions methods we use the closed form ex-ante solutions of the government's problem described in detail in Appendix A.4.

Baseline. This version is solved in three steps. The first step solves the households problem while taking government policies and bond prices as given using time iteration method. The second step uses the implied policy functions of the private sector from the first step and the assumed bond schedules, and computes the closed form solutions that solve the government's ex-ante problem. Finally using private and public policy functions the schedule of private bonds is updated. Iterate until convergence in private en public policies.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (1.15).
- Provide an initial guess of ex-ante policy functions for government default $d(z, L, B)$, and borrowing probabilities conditional on repayment $G(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $Q(z, L', B')$.
- Construct the implied transfer function $T(z, B, L, L')$ using the government budget constraint (1.5).
- Taking all these functions as given find the optimal private borrowing $B'(z, L, B, L')$ and consumption decisions $C'(z, L, B, L')$ using the private sector Euler equation (1.22) to find the binding and non binding states.
- Given households optimal policies $B'(z, L, B, L')$, and $C'(z, L, B, L')$, and the guess schedule of public bonds $Q(z, L', B')$, compute the ex-ante default and borrowing

policy functions of the government using (A.20) and (A.21). Update the government policy functions.

- Compute the government ex-ante value function $W(z, L, B)$ using (A.22).
- Update the schedule of public bonds $Q(z, L', B')$ using (A.23).
- Repeat until convergence in $W(z, L, B)$,$B'(z, L, B, L')$, and $C'(z, L, B, L')$, and $Q(z, L', B')$ is achieved.

Social planner. This version is solved in three steps. The first step finds optimal private borrowing on a grid (*grid search method*) given an initial guess of public for each potential default and public borrowing decisions. The second step uses this optimal private borrowing policy and the assumed bond schedules to computes the closed form solutions for public borrowing and default and the value function. Finally using private and public borrowing policy functions the schedule of private bonds is updated. Iterate until convergence in private borrowing policies and the value function is achieved.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (1.15).
- Construct a grid of potential private borrowing choices B'.
- Provide an initial guess of ex-ante policy functions for government default $d^{SP}(z, L, B)$, and borrowing probabilities conditional on repayment $G^{SP}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $Q^{SP}(z, L', B')$.
- Taking all these functions as given find the optimal private borrowing $B^{SP\prime}(z, L, B, L')$ in the finite grid discarding all choices that violate the credit constraint (1.18) for each potential public borrowing and default decision.
- Given optimal private borrowig policy $B^{SP\prime}(z, L, B, L')$ and the guess schedule of public bonds $Q^{SP}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the planner using (A.20) and (A.21). Update the planner public borrowing and default policy functions.
- Compute the ex-ante value function $W^{SP}(z, L, B)$ using (A.22).

- Update the schedule of public bonds $Q^{SP}(z, L', B')$ using (A.23).
- Repeat until convergence in $W^{SP}(z, L, B)$,$B^{SP\prime}(z, L, B, L')$, and $Q^{SP}(z, L', B')$ is achieved.

A.6 Particle filter method

This appendix details the particle filter method used to conduct the counterfactual exercises of section 1.5. It follows closely the approach presented in Bocola and Dovis, 2019. As noted in the main text, the state space representation of the model is:

$$Y_t = g(S_t) + e_t \tag{A.24}$$

$$S_t = f(S_{t-1}, \varepsilon_t). \tag{A.25}$$

In this formulation, the first equation captures the measurement error e_t, a vector of i.i.d. normally distributed errors with mean zero and a diagonal variance-covariance matrix Σ. The vector of observable, Y_t, includes average private and public debt as share of GDP, detrended tradable output, the share of nonperforming loans, and interest rate spreads on public bonds. The second equation describes the law of motion of the baseline model state variables $S_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$. The vector ε_t corresponds to the innovations in the AR 1 process of the three structural shocks $[y^T_t, \pi_t, \kappa_t]$.

$$\begin{aligned} y^T_t &= \exp(\rho^y \ln y^T_{t-1} + \varepsilon^y_t) \\ \pi^T_t &= \exp((1-\rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon^\pi_t) \\ \kappa_t &= (1-\rho^\kappa)\bar{\kappa} + \rho^\kappa \kappa_t + \varepsilon^\kappa_t \end{aligned}$$

Since we did not observe any defaults in the time periods considered we use the repayment policy functions to compute the transitions. Using the notation of section 1.3 the evolution of private and public debt in the first exercise is then:

$$\begin{aligned} L_{t+1} &= \mathcal{L}'(s_t, L_t, B_t) = \mathcal{L}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t) \\ B_{t+1} &= \mathcal{B}'(s_t, L_t, B_t) = \mathcal{B}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t) \end{aligned}$$

In the first exercise all taste shocks are set to to zero. In the second exercise, we still focus on repayment but this time we select the taste shocks to match public debt exactly to it's data counter part and let private debt the respond endogenously:

$$
\begin{aligned}
L_{t+1} &== L_{t+1}^{data} \\
B_{t+1} &= \tilde{B}'(y_t^T, \pi_t, \kappa_t, L_t, B_t, 0, L_{t+1}^{data}, \tilde{T}(s_t, L_t, L_{t+1}^{data}))
\end{aligned}
$$

These transitions are summarized in function $f(\cdot)$ for each exercise. Similarly we can generate numerical solutions to compute the model counterparts to debt to output ratios and the public spreads and summarize them in $g(\cdot)$.

Let $Y^t = [Y_1, ..Y_t]$, and denote by $p(S_t|Y^t)$ the conditional distribution of the state vector given a history of observations up to period t. In general there is no analytical solution for the density function $p(S_t|Y^t)$. The particle filter method approaches this density by using the fact that the conditional density of Y_t given S_t is Gaussian. It consists of finding a set of pairs of states and weights $\{S_t^i, \tilde{w}_t^i\}_{i=1}^N$ such that for all function $h(\cdot)$:

$$
\frac{1}{N}\sum_{i=1}^{N} h(S_t^i)\tilde{w}_t^i \underset{a.s}{\longrightarrow} \mathbb{E}[h(S_t)|Y^t].
$$

This approximation can then be used to obtain the weighted average path of the state vector over the sample. The states selected S_t^i are called particles and $\tilde{w}_t^i$ corresponds to their weight. To construct this set we follow the algorithm proposed by Kitagawa, 1996.

Step 1: Initialization Set $t = 1$ and $\forall i$ $\tilde{w}_0^i = 1$, draw S_0^i from the ergodic distribution of the baseline model.

Step 2: Transition For each $i = 1..N$ compute the state vector $S_{t|t-1}^i$ given vector S_{t-1}^i by drawing innovations for the fundamental shocks from the calibrated distributions and using the policy functions summa zed in $f(\cdot)$.

Step 3: Filter Assign to each particle $S_{t|t-1}^i$ the weight

$$
w_t^i = p(Y|S_{t|t-1}^i)\tilde{w}_{t-1}^i
$$

where $p(Y|S_{t|t-1}^i)$ is a multivariate Normal density.

Step 4: Rescale & Resample Rescale the weights $\{w_t^i\}$ so that they add up to one, and denote these new weights $\{\tilde{w}_t^i\}$. Sample with replacement N values of the state vector from the set $\{S_{t|t-1}^i\}$ using $\{\tilde{w}_t^i\}$ as sample weights. Denote this draws $\{S_t^i\}$. Set $\tilde{w}_t^i = 1$ $\forall i$.

If $t < T$ set $t = t + 1$ and go to Step 2. Otherwise stop.

In both exercises, it is assumed that measurement error associated with y_t^T and π_t is zero, as such the variance of the measurement error is set to zero for these variables in the measurement equation and the innovations ε_t^y and ε_t^π are set to match the empirical counterparts exactly. Since κ_t has no empirical counterpart, the algorithm help us find the most likely path using its effects on debt aggregates and the spreads. As in Bocola and Dovis, 2019 the filter is tuned with $N = 100,000$.

Equipped with a set of particles and weights $\{S_t^i, \tilde{w}_t^i\}_{i=1}^N$ and the policy functions summarized in $g(\cdot)$ one can approximate the model predictions plotted in figures 1.11 and 1.12. As an example for all $t = [2008..2015]$ the predicted interest rate spread, $spr_t^{Baseline}$ at time t is:

$$spr_t^{Baseline} = \sum_i^N \tilde{w}_t^i [\frac{\delta - \delta Q(S_t^i)}{Q(S_t^i)} - r]$$

Similar weighted averages are computed for the debt to output ratio and the exogenous shocks. When computing objects for the social planner the function $g^{SP}(\cdot)$ is used instead.

Appendix B

Appendix to Chapter 2

B.1 Data Appendix

B.1.1 Data Sources

The sources for the data used in the paper are as follows:

Gross Domestic Product (GDP): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)"*, GDP current USD in current USD converted from domestic currency using the period-average exchange rate, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

International reserves: *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, "FX reserves" in millions of current USD. Does not include gold holdings, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

Net Foreign Assets (NFA): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, NFA in millions of current USD, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

Private debt: *World Bank 2018. International Debt Statistics*, External debt stocks, private nonguaranteed (PNG) (DOD, current USD) from 1980 to 2015.

Public debt: *World Bank 2018. International Debt Statistics*, External debt stocks, public and publicly guaranteed (PPG) (DOD, current USD) from 1980 to 2015.

Real GDP growth: *IMF 2018. The World Economic Outlook (WEO) database*, Gross domestic product, constant prices in Percent change, from 1980 to 2015.

Capital Account openness: *Chinn, Menzie, and Ito (2006).* Index of capital account openness. Annualized using data from 1980Q1-2015Q4.

Current account (CA): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, CA in millions of current USD, for Mexico from 1970 to 2015.

Tradable share of GDP: *National Institute of Statistics and Geography (INEGI))* We first compute the annual sectoral value added in the primary and secondary sector as a share of total value added from 1980 to 2015. Since the secondary sector includes construction we compute the average share in the desegregated series, available from 1993-2015. We estimate the share of the construction sector in total value added, 8% with a standard deviation of 0.5%. We subtract this average share from our first measure of tradable value added.

B.1.2 Sample of Countries

The sample of countries correspond to “Middle-Income Countries”. We arrive to our sample as follows. We start by considering the universe of all countries included in the International Debt Statistics dataset and exclude those listed as “Advanced economies” by the IMF and ”Low income countries” by the World Bank. To have a balanced panel from 1980-2015 we keep only countries that have positive values of private debt in the International Debt Statistics and that do not have missing values for the Chinn-Ito index of capital account openness or in the WEO and Lane and Milesi-Ferretti databases. The requirement of a balanced panel subtracts 40 countries from the sample. Finally, we also exclude countries that record a net foreign assets positions above or below 150% GDP at any point from 1980-2015: Mauritius, Cote d'Ivoire, Jamaica, Papua New Guinea, and Paraguay.

The final list includes the following 25 countries: Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Sri Lanka, Thailand, Tunisia, Turkey, and Venezuela.

B.2 Proofs

In this section, we present the proofs not presented in the main text.

B.2.1 Implementation for more general parameterizations

We provide here a proposition analogous to Proposition 2.1, but without using Cobb-Douglas preferences and Assumption 2.1. Consider the solution to the constrained efficient planning problem $\{c_t^{T\star}, b_{t+1}^\star, p_t^{N\star}\}$. Assume that the solution satisfies the following condition

$$\mu_t^\star(\Psi_t^\star - 1) \leq \beta R_t \mathbb{E}_t \mu_{t+1}^\star \Psi_{t+1}^\star \tag{B.1}$$

for all t where Ψ_t is defined in (2.15) and $\mu_t^\star$ is given by (2.20). Then, given initial conditions (b_0, A_0) such that $b_0^\star = b_0 - A_0$, the decentralized equilibrium with a consumption allocation $\{c_t^{T\star}\}$ can be implemented if the government follows the reserve policy $\{A_{t+1}\}$ given by (RP).

Proof. The proof follows the same steps as in Proposition 2.1, but uses (B.1) to show that given (2.22), we have that $\mu_t \geq 0$ is satisfied in the decentralized equilibrium. Notice that when Assumption 2.1 holds, condition (B.1) is trivially satisfied. □

B.2.2 Proof of Proposition 2.2

Proof. By construction, the feedback rule (2.25) guarantees that the economy's net foreign asset (NFA) position always coincides with its counterpart in the constrained efficient allocation $A_{t+1} - b_{t+1} = -b_{t+1}^\star$. Substituting this equality into economy's resource constraint (2.9) shows that the constrained efficient level of consumption is also achieved. Following the same steps as in the proof of Proposition 2.1, conditions (2.21), (B.17) and (2.22) must hold. When $\mu_t^\star > 0$ or $\mu_{t+1}^\star > 0$ in at least one successor state, (2.22) indicates that $\mu_t > 0$, so that $b_{t+1}/R_t = \kappa_t(y_t^T + p_t^{N\star} y^N)$ and $A_{t+1} = R_t \kappa_t (y_t^T + p_t^{N\star} y^N) - b_{t+1}^\star \geq 0$. Meanwhile, when $\mu_t^\star = 0$ and $\mu_{t+1}^\star = 0$ in every successor state, (2.22) indicates that $\mu_t = 0$, so that $b_{t+1} = b_{t+1}^\star$ is optimal and $A_{t+1} = 0$. Either way, the private borrowing choice is optimal and reserves are non-negative. □

B.3 Optimal reserve accumulation policy

We consider the problem of the government that chooses a state-contingent sequence $\{A_{t+1}\}_{t=0}^{\infty}$ to maximize welfare in the competitive equilibrium. The problem consists of solving:

Problem B.1 (Optimal Policy)**.**

$$\max_{b_{t+1},A_{t+1},c_t^T,\mu_t} \sum_{t=0}^{\infty} \beta^t u(c(c_t^T, y^N)) \tag{B.2}$$

subject to

$$b_t + c_t^T = y^T + A_t + \frac{b_{t+1} - A_{t+1}}{R_t}, \qquad (\hat{\lambda}_t) \tag{B.3}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa \left(y_t^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \qquad (\hat{\mu}_t) \tag{B.4}$$

$$A_{t+1} \geq 0, \qquad (\hat{\zeta}_t) \tag{B.5}$$

$$u_T\left(c(c_t^T, y^N)\right) = \beta R_t \mathbb{E}_t u_T(c(c_{t+1}^T, y^N)) + \mu_t, \qquad (\hat{\xi}_t) \tag{B.6}$$

$$0 = \mu_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \qquad (\hat{\chi}_t) \tag{B.7}$$

$$\mu_t \geq 0. \qquad (\hat{\nu}_t) \tag{B.8}$$

where $\hat{\lambda}_t, \hat{\mu}_t, \hat{\xi}_t, \hat{\zeta}_t, \hat{\chi}_t$ *and* $\hat{\nu}_t$ *are the multipliers associated with the constraints* (B.3)-(B.8).

The government's first-order conditions for b_{t+1}, A_{t+1}, c_t^T and μ_t are given by

$$b_{t+1}: \quad \hat{\lambda}_t + \hat{\chi}_t \mu_t = \beta R_t \mathbb{E}_t \hat{\lambda}_{t+1} + \hat{\mu}_t, \tag{B.9}$$

$$A_{t+1}: \quad \hat{\lambda}_t = \beta R_t \mathbb{E}_t \hat{\lambda}_{t+1} + R_t \hat{\zeta}_t, \tag{B.10}$$

$$c_t: \quad \hat{\lambda}_t = u_T(t) + \hat{\xi}_t u_{TT}(t) + \Psi_t(\hat{\mu}_t - \hat{\chi}_t \mu_t), \tag{B.11}$$

$$\mu_t: \quad \hat{\xi}_t = \hat{\chi}_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right] + \hat{\nu}_t, \tag{B.12}$$

and complementary slackness conditions

$$0 = \hat{\mu}_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \tag{B.13}$$

$$0 = \hat{\zeta}_t A_{t+1}. \tag{B.14}$$

Combining conditions (B.10) and (B.11) reveals that the government faces different marginal benefit from accumulating reserves. In a state in which (B.7)-(B.8) are not binding—as will be indeed the case at the optimum—and assuming that the constraint for reserves and borrowing are currently slack, we have that

$$u_T(t) = \beta R_t \mathbb{E}_t[u_T(t+1) + \underbrace{\hat{\mu}_{t+1}\Psi_{t+1}}_{\text{Externality}} + \underbrace{\hat{\xi}_{t+1}u_{TT}(t+1) - \hat{\xi}_t u_{TT}(t)]}_{\text{Incentive}} \tag{B.15}$$

There are two novel terms in the planner's Euler equation for reserves relative to the household version. First, it is the pecuniary externality, captured by the $\hat{\mu}_{t+1}\Psi_{t+1}$ term, which reflects that the planner internalizes that having more reserves in a future state with a binding borrowing constraint has positive general equilibrium effects. Second, there is an incentive term that captures how households respond to government policy. Chiefly important for this effect is that the planner is subject to households' borrowing Euler equation as an implementability constraint and that because of the overborrowing externality, the Lagrange multiplier is non-negative. When the government accumulates reserves, this lowers at the margin current consumption and increases future consumption. These effects tighten today's implementability constraint and relax next period implementability constraint, as reflected in the two components of the "incentive term."

As it turns out, at the optimum ξ_t becomes zero. Intuitively, once the level of reserves is large enough, the borrowing constraint becomes binding, and thus households cannot offset the government policy. When optimizing, the government exactly fine-tunes the accumulation of reserves so that the borrowing constraint becomes binding exactly at the level of tradable consumption that corresponds to the constrained-efficient allocation. This result is a corollary of the proposition below. Suppose Assumption 2.1 holds. Then, the solution to the optimal reserve accumulation policy presented in Problem B.1 achieves the same utility as the constrained-efficient allocations. Moreover, the optimal policy is time consistent.

Proof. We guess and verify that (B.6)-(B.8) are slack, and so $\hat{\xi}_t = \hat{\chi}_t = \hat{\nu}_t = 0$. Using this conjecture, and combining (B.9) and (B.11), we arrive at

$$u_T\left(c(c_t^T, y^N)\right) = \beta R_t \mathbb{E}_t[u_T(c(c_{t+1}^T, y^N)) + \hat{\mu}_{t+1}\Psi_{t+1}] + \hat{\mu}_t(1-\Psi_t). \tag{B.16}$$

By Assumption 2.1, $\Psi_t < 1$ and since $\hat{\mu}_{t+1} \geq 0$ and $\Psi_{t+1} \geq 0$, we have that:

$$u_T\left(c(c_t^T, y^N)\right) - \beta R_t \mathbb{E}_t u_T(c(c_{t+1}^T, y^N)) \geq \hat{\mu}_{t+1}\Psi_{t+1}. \tag{B.17}$$

Setting $\mu_t = \beta R_t \mathbb{E}_t u_T(c(c_{t+1}^T, y^N)) - u_T\left(c(c_t^T, y^N)\right)$ and using (B.17), it follows that μ and $\hat{\mu}$ have the same sign. Hence (B.8) is satisfied. Moreover, from (B.13), it follows that (B.7) is satisfied as well as conjectured.

Finally, (B.9) and (B.10) imply that (B.5) binds if and only if (B.4) binds, so that (B.4) and (B.5) can be combined to deliver

$$\frac{b_{t+1} - A_{t+1}}{R_t} \leq \kappa_t \left(y_t^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N}\right)^{\eta+1} y^N \right). \tag{B.18}$$

Using $b_{t+1} - A_{t+1} = b_{t+1}^{\star}$, we can see that (B.18) is equivalent to the borrowing constraint in the constrained-efficient problem (2.12). Therefore, Problem B.1 reduces to the same constrained-efficient planning problem of Section 2.3.4. Hence, it follows that the optimal reserve policy achieves the constrained-efficient allocations and is time consistent. □

B.4 Reserve accumulation in production economy

This appendix provides additional details on the production economy model described in Section 2.4.5 and shows that the reserve implementation of the constrained efficient allocation outlined in Proposition 2.1 equally applies to that framework, irrespective of whether an ex post instrument is available.

We consider two constellations in turn. First, in Section B.4.1, we consider a constellation, similar to that considered in our baseline model, where the planner chooses directly the level of non-state contingent debt every period, but lets markets for labor and goods clear competitively. Then, in Section B.4.2, we consider an alternative constellation, where, similar to Benigno et al., 2013b, the planner has in addition also access to a distortionary tax on labor.

In either case, households' optimality conditions are still given by (2.4), (2.5) and (2.6). The tradable and non-tradable goods firm's optimality conditions are respectively given by

$$w_t = z_t^T \alpha \left(h_t^T\right)^{\alpha-1}, \tag{B.19}$$

$$(1+\tau_t^N)w_t = p_t^N z^N \alpha \left(h_t^N\right)^{\alpha-1}. \tag{B.20}$$

Combining (B.19) and (B.20), we obtain

$$1+\tau_t^N = \frac{z^N \left(h_t^N\right)^{\alpha-1}}{z_t^T \left(h_t^T\right)^{\alpha-1}} p_t^N. \tag{B.21}$$

In what follows, we present the planning problems with one and two instruments in Sections B.4.1 and B.4.2, respectively. In each case, we then re-state our main normative result that the constrained efficient allocation can be implemented via reserves for this production economy and show that the proof works similarly to that of Proposition 2.1.

B.4.1 Without availability of ex post instrument

In recursive form, the planner's problem can be written as:

$$V(b, y^T, R, \kappa) = \max_{c^T, c^N, h^T, h^T, b'} u(c(c^T, c^N)) + \beta \mathbb{E} V(b', y^{T\prime}, R', \kappa') \tag{B.22}$$

subject to

$$c^T - \frac{b'}{R} = z^T \left(h^T\right)^{\alpha} - b, \tag{B.23}$$

$$c^N = z^N \left(h^N\right)^\alpha \tag{B.24}$$

$$\bar{h} = h^T + h^N \tag{B.25}$$

$$\frac{b'}{R} \leq \kappa \left[z^T \left(h^T\right)^\alpha + \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} z^N \left(h^N\right)^\alpha \right], \tag{B.26}$$

$$0 = \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} - \frac{z^T}{z^N} \left(\frac{h^T}{h^N}\right)^{\alpha-1}. \tag{B.27}$$

where (B.23) is the resource constraint for tradable goods, (B.24) is the resource constraint for non-tradable goods, (B.25) is the time constraint for labor, (B.26) is the credit constraint, and (B.27) is an implementability constraint associated firms' and households' optimal intratemporal choices when no labor tax is available.

Using sequential notation for convenience, the planner's Euler equation for debt is still given by (2.13), while its remaining optimality conditions for c_t^T, c_t^N, h_t^T and h_t^N are given by

$$\lambda_t = u_T(t) + (\mu_t \kappa_t + \nu_t)(1+\eta) \frac{p_t^N c_t^N}{c_t^T} \tag{B.28}$$

$$\delta_t = u_N(t) - (\mu_t \kappa_t + \nu_t)(1+\eta) p_t^N \tag{B.29}$$

$$\chi_t = (\lambda_t + \mu_t \kappa_t)\, z_t^T \alpha \left(h_t^T\right)^{\alpha-1} - \nu_t \left(\alpha - 1\right) p_t^N \frac{1}{h_t^T} \tag{B.30}$$

$$\chi_t = \left(\delta_t + \mu_t \kappa_t p_t^N\right) z^N \alpha \left(h_t^N\right)^{\alpha-1} + \nu_t \left(\alpha - 1\right) p_t^N \frac{1}{h_t^N} \tag{B.31}$$

where λ_t, δ_t, χ_t, μ_t and ν_t respectively denote the multipliers on constraints (B.23), (B.24), (B.25), (B.26) and (B.27).

Combining (B.28), (B.29), (B.30) and (B.31) to eliminate δ_t, χ_t, and ν_t leads to an expression for the planner's marginal utility of tradable wealth given by

$$\lambda_t = u_T(t) + \mu_t^\star \widetilde{\Psi}_t, \tag{B.32}$$

for $\widetilde{\Psi}_t \equiv \Psi_t \Upsilon_t$, where Ψ_t was defined in (2.15) and

$$\Upsilon_t \equiv \frac{\frac{1-\alpha}{\alpha} \frac{\bar{h}}{c_t^N h_t^T}}{\frac{1-\alpha}{\alpha} \frac{\bar{h}}{c_t^N h_t^T} + (1+\eta) \left[c_t^T / (c_t^T + p_t^N c_t^N)\right]^{-1}}, \tag{B.33}$$

Therefore, a wedge between the planner's and private shadow value of wealth similar to the one present in our baseline endowment economy model is apparent from (B.32). Note that relative to the endowment economy model, the wedge includes an attenuation term $0 < \Upsilon_t < 1$ reflecting the fact that the labor reallocation from the tradable to the non-tradable sector, and the associated rise in non-tradable consumption, mutes some of the increase in the price of non-tradable brought about by a marginally higher tradable goods consumption.

Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}, b_{t+1}^{\star}, p_t^{N\star}\}_{t=0}^{\infty}$. Then, given initial conditions (b_0, A_0) such that $b_0^{\star} = b_0 - A_0$, the decentralized equilibrium features allocations $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}\}_{t=0}^{\infty}$ if the government follows the reserve policy $\{A_{t+1}\}$ given by

$$\frac{A_{t+1}}{R_t} = \kappa_t \left[z_t^T \left(h_t^{T\star}\right)^{\alpha} + p_t^{N\star} z_t^N \left(h_t^{N\star}\right)^{\alpha} \right] - \frac{b_{t+1}^{\star}}{R_t} \quad \text{for all } t \geq 0. \tag{B.34}$$

Proof. The proof follows the same steps as the proof of Proposition 2.1. We will show that, given the sequence of prices $\{p_t^{N\star}\}_{t=0}^{\infty}$ and initial conditions, the sequence of allocations $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}\}_{t=0}^{\infty}$ satisfy the households' and firms' fist-order conditions, which are both necessary and sufficient for optimality.

First, we note that from the firms' first-order conditions (B.19) and (B.20), the labor allocation must satisfy

$$p_t^{N\star} = \frac{z_t^T}{z^N} \left(\frac{h_t^T}{\bar{h} - h_t^T} \right)^{\alpha - 1}. \tag{B.35}$$

It follows that $h_t^T = h_t^{T\star}$ and $h_t^N = h_t^{N\star}$. Next, we guess that given (B.34), the households' credit constraint (2.27) holds with equality at all times:

$$\frac{b_{t+1}}{R_t} = \kappa_t \left[z_t^T \left(h_t^{T\star}\right)^{\alpha} + p_t^{N\star} z_t^N \left(h_t^{N\star}\right)^{\alpha} \right]. \tag{B.36}$$

Combining (B.36) with (B.34), we obtain

$$b_{t+1}^{\star} = b_{t+1} - A_{t+1}. \tag{B.37}$$

Substituting (B.37) into the tradable resource constraint (2.30) yields

$$c_t^T = z_t^T \left(h_t^T\right)^{\alpha} - (b_t - A_t) + \frac{b_{t+1}^{\star}}{R_t}. \tag{B.38}$$

Meanwhile, since $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}, b_{t+1}^\star\}$ solve the constrained planning problem, we have

$$c_t^{T\star} = z_t^T \left(h_t^T\right)^\alpha - b_t^\star + \frac{b_{t+1}^\star}{R_t}. \tag{B.39}$$

Given the initial condition $b_0^\star = b_0 - A_0$, a comparison of (B.38) and (B.39) reveals that $c_t^T = c_t^{T\star}$. That is, when households' borrowing policy satisfies (B.36) and reserves are set according to (B.34), the constrained-efficient sequence of tradable consumption is achieved. Further, the non-negativity of A_{t+1} again follows immediately from the reserve policy (B.34) and the planner's credit constraint (B.26).

We are left to show that $c_t^T = c_t^{T\star}$, $c_t^N = c_t^{N\star}$ satisfy the optimality conditions of the households. From conditions (2.13) and (B.32) characterizing the constrained-efficient allocation, we obtain (2.20) with $\widetilde{\Psi}$ instead of Ψ. Rearranging the households' intertemporal Euler equation (2.6), we have (2.21). Combining (2.20) and (2.21), we obtain (2.22), again with $\widetilde{\Psi}$ instead of Ψ. The non-negativity of μ_t follows from $\widetilde{\Psi}_t = \Upsilon_t \kappa_t (1-\omega)/\omega < 1$, given Assumption 2.1 and $\Upsilon_t^\star < 1$, and the non-negativity of $\mu_t^\star$. Together, the conjecture (B.36) and the fact that $\mu_t \geq 0$ ensure that the households' optimality (2.6) condition is satisfied. Finally, notice that the intratemporal condition (2.4) follows directly from the definition of the constrained-efficient allocation, implying that $c_t^N = c_t^{N\star}$ is also optimal for the household. □

B.4.2 With availability of ex post instrument

In this case, the planner's problem is given by:

$$V(b, y^T, R, \kappa) = \max_{c^T, c^N, h^T, h^T, b'} u(c(c^T, c^N)) + \beta \mathbb{E} V(b', y^{T'}, R', \kappa') \tag{B.40}$$

subject to

$$c^T - \frac{b'}{R} = z^T \left(h^T\right)^\alpha - b, \tag{B.41}$$

$$c^N = z^N \left(h^N\right)^\alpha \tag{B.42}$$

$$\bar{h} = h^T + h^N \tag{B.43}$$

$$\frac{b'}{R} \leq \kappa \left[z^T \left(h^T\right)^\alpha + \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} z^N \left(h^N\right)^\alpha \right], \tag{B.44}$$

where the only difference with (B.22)-(B.27) is that the implementability constraint (B.27) associated with the intratemporal allocation of labor accross sector can now be dropped due to the availability of a labor tax in the non-tradable sector.

Again using sequential notation for convenience, the planner's Euler equation for debt is still given by (2.13), while its remaining optimality conditions for c_t^T, c_t^N, h_t^T and h_t^N are given by

$$\lambda_t = u_T(t) + \mu_t \kappa_t (1+\eta) \frac{p_t^N c_t^N}{c_t^T} \tag{B.45}$$

$$\delta_t = u_N(t) - \mu_t \kappa_t (1+\eta) p_t^N \tag{B.46}$$

$$\chi_t = (\lambda_t + \mu_t \kappa_t)\, z_t^T \alpha \left(h_t^T\right)^{\alpha-1} \tag{B.47}$$

$$\chi_t = \left(\delta_t + \mu_t \kappa_t p_t^N\right) z^N \alpha \left(h_t^N\right)^{\alpha-1} \tag{B.48}$$

where λ_t, δ_t, χ_t and μ_t again respectively denote the multipliers on constraints (B.41), (B.42), (B.43) and (B.44).

Combining (B.45), (B.46), (B.47) and (B.48), together with (B.21) leads to an expression for the optimal labor tax in the non-tradable sector:

$$\tau_t^N = \mu_t \kappa_t c_t^N (1+\eta)\, p_t^N \frac{\left(c_t^T\right)^{-1} + \left(c_t^N\right)^{-1} \frac{z^N \left(h_t^N\right)^{\alpha-1}}{z_t^T \left(h_t^T\right)^{\alpha-1}}}{u_T + \mu_t \kappa_t} \geq 0. \tag{B.49}$$

This optimal tax is positive whenever the credit constraint binds for the planner (i.e., whenever $\mu_t > 0$), indicating that the planner optimally redirects production away from the non-tradable sector so as to support the price of non-tradable goods and relax the credit constraint at the margin.

Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}, b_{t+1}^\star, p_t^{N\star}, \tau_t^{N\star}\}_{t=0}^\infty$. Then, given initial conditions (b_0, A_0) such that $b_0^\star = b_0 - A_0$, the decentralized equilibrium features allocations $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}\}_{t=0}^\infty$ if the government follows the reserve policy $\{A_{t+1}\}$ given by

$$\frac{A_{t+1}}{R_t} = \kappa_t \left[z_t^T \left(h_t^{T\star}\right)^\alpha + p_t^{N\star} z_t^N \left(h_t^{N\star}\right)^\alpha \right] - \frac{b_{t+1}^\star}{R_t} \quad \text{for all } t \geq 0. \tag{B.50}$$

Proof. The proof follows the same steps as those of Proposition B.4.1, but adds the sequence of labor taxes $\{\tau_t^{N\star}\}_{t=0}^\infty$ to the set of variables conditional upon which private agents make their decisions. Accordingly, we invoke the firms' optimality conditions (B.19) and (B.20), together with the optimal tax expression (B.49), to show that the labor allocation satisfies

$h_t^T = h_t^{T\star}$ and $h_t^N = h_t^{N\star}$. Furthermore, (B.45) is used instead of (B.32) to obtain (2.20). Combining (2.20) and (2.21), we obtain (2.22), again concluding that $\mu_t \geq 0$. The remaining steps are analogous those followed in the proof of Proposition B.4.1. □

B.5 Checking for Multiple Equilibria

Open economy models with a collateral constraint like the one we study may feature multiple equilibria, as formally established by Schmitt-Grohé and Uribe, 2020. They provide necessary and sufficient conditions under which a non-stochastic version of the model will feature equilibrium multiplicity. Our parameterization does not fall within the conditions that would allow us to determine unambiguously that there is unique or multiple equilibria. Because of this, we provide a numerical algorithm designed to check whether our equilibrium is unique. For our calibration, we find that there are no other equilibria.

We begin from the solution to the competitive equilibrium absent any intervention, which we solve using time iteration, following Bianchi, 2011. We denote the equilibrium law of motion for debt as $\mathcal{B}(b, s)$. We construct a grid for possible values of debt $B_N = [b_1....b_{max}]$. For the upper bound of the grid, we take a value arbitrarily close to the natural debt limit. The natural debt limit can be obtained as the fixed point of the following problem:

$$b_{\max}(s) = \max_{b'} y^T + \frac{b'}{R} \quad \text{subject to} \tag{B.51}$$

$$b' \leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{y^T + \frac{b'}{R} - b_{\max}}{y^N} \right) y^N \right]$$

Considering that this condition must be satisfied for every possible shock, we obtain $b_{max} = y_{min}(1 + \kappa)$.

For every point in the grid for initial debt and shocks, we then check whether the following conditions are satisfied for every $b' \in B_N$ other than $\mathcal{B}(b, s)$.

$$u_T(y^T - b + b'/R_t, y^N) = \beta R_t E u_T(c_T(\mathcal{B}(b', s), s'), y^N) + \mu \tag{B.52}$$

$$b' \leq \kappa \left[\frac{1-\omega}{\omega} \left(\frac{y^T - b + b'/R_t}{y^T} \right)^{\eta+1} y^N + y^T \right] \tag{B.53}$$

$$\mu \geq 0 \tag{B.54}$$

If there is another $b' \in B_N$ that satisfies these conditions, then there is multiple equilibria.

Following this procedure, we do not find any state with more than one solution to the system of equations. To illustrate this result, we turn to Figure B.1. In the two panels of the figure, the red line computes the excess borrowing capacity, that is the difference between the debt limit and the issued level of debt (B.54), for each value on the grid. The blue line uses (B.53) to calculate the Lagrange multiplier associated with the collateral constraint

(μ) consistent with each potential debt level. In an equilibrium with a binding collateral constraint the excess borrowing capacity is zero and the Lagrange multiplier associated with the collateral constraint (μ) is positive. Conversely, in an equilibrium with a non-binding collateral constraint the excess borrowing capacity is strictly positive and the Lagrange multiplier associated with the collateral constraint is zero.

(a) Binding collateral constraint equilibrium

(b) No-binding collateral constraint equilibrium

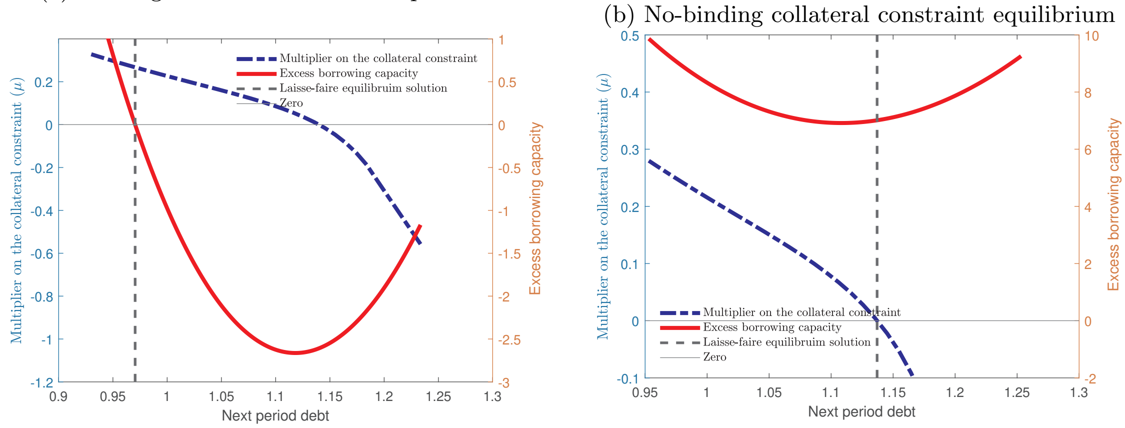

Figure B.1: Equilibrium policy function for debt

Note: Solutions to the optimality conditions for the states with the highest values of the financial shock where the collateral constraint binds and an endowment of tradables two standard deviations below its mean

Panel A, plots the solutions to equations (B.53) and (B.54) for the state with an endowment of tradables two standard deviations below its mean and the highest value of the parameter κ such that the collateral constraint still binds for high enough levels of initial debt. As we can see in panel A, it is possible that a higher level of next period debt that the one that solves the competitive equilibrium will also will also satisfy (B.54) with equality (zero excess borrowing capacity). However, at those higher values of debt, the Lagrange multiplier associated with the constraint is not positive. Similarly, a lower level of end of period debt, will be consistent with a positive excess borrowing capacity, but at those values the Lagrange multiplier μ is strictly positive.

Panel B plots instead an equilibrium at the same exogenous state as panel A but where the initial level of debt is smaller and the competitive equilibrium no longer exhibits a binding collateral constraint. As one can see, it is not possible to find equilibrium with a binding collateral constraints. Solutions with a b Alternative levels of debt are also not a solution with a binding constraint, since the excess borrowing capacity is never zero in this case.

So we have established that under our parameterization, we can only find one equilibrium. The only caveat of this procedure is that we take as given a continuation equilibrium from $t > 0$. However, given a continuation equilibrium for $t > 0$, the procedure can exhaustively determine whether there is multiplicity or not at $t = 0$ for every point in the grid.

B.6 Financial shocks

In the dynamic exercise presented in Figure 2.10 we feed to the regulated economy a sequence of financial shocks (κ_t) that is consistent with our assumption that Mexico was following optimal reserve accumulation policies during those years. As explained in section 2.4.1, the financial shock follows a first-order autoregressive process of the form:

$$\log(\kappa_t) = (1 - \rho^\kappa)\log(\bar{\kappa}) + \rho^\kappa \log(\kappa_{t-1}) + \varepsilon_t^\kappa$$

Figure B.2 plots this sequence of kappa shocks (panel a) as well as the innovations of this autoregressive process (panel b). We also plot the 95% confidence intervals associated with each observation.

(a) Financial shock

(b) Innovations in the financial shock

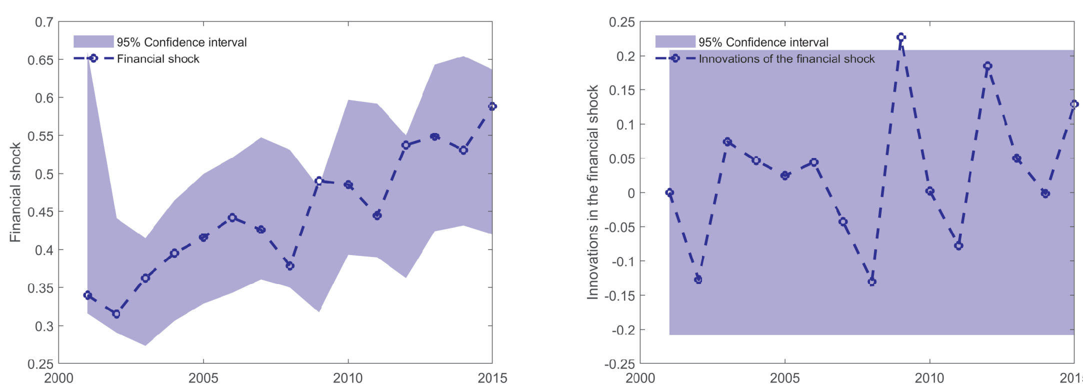

Figure B.2: Financial shock and innovations of the financial shock in the dynamic exercise

Note: For the first observation of the financial the bounds of the 95% confidence intervals are computed using the unconditional mean and standard deviation of the auto-regressive process.

B.7 Additional Tables and Figures

Table B.1: Net changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	Reserves	Reserves	Reserves	Reserves
Private External Debt	0.259***	0.211***	0.261***	0.213***
	(0.0371)	(0.0395)	(0.0370)	(0.0391)
Current Account	0.172***	0.175***	0.252***	0.248***
	(0.0182)	(0.0181)	(0.0222)	(0.0220)
Publicly Guaranteed		0.0501***		0.0663***
External Debt		(0.0135)		(0.0146)
GDP growth rate		-0.00119		0.00502
		(0.00513)		(0.00528)
Observations	850	850	850	850
Countries	25	25	25	25
Pooled OLS/ Fixed Effects	pooled	pooled	FE	FE

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

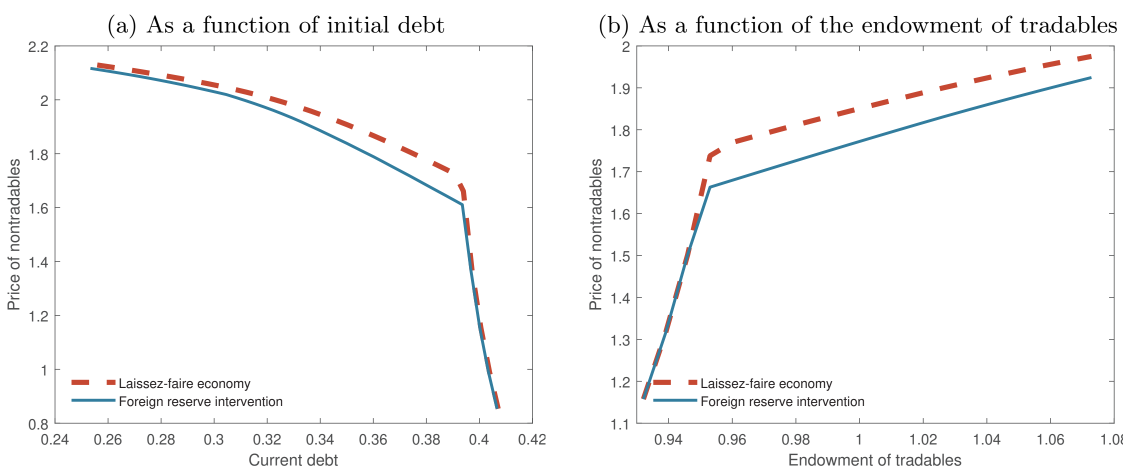

Figure B.3: Price of nontradables