A CURRICULUM FOR THE SIXTH GRADE MATHEMATICS ENRICHMENT PROGRAM OF HIBBING, MINNESOTA

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Problems in
Curriculum Construction
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Under the Direction of
William C. Gemeinhardt

A Requirement for the Degree

Master of Arts (Plan B)

by
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University of Minnesota
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To:

Dr. William R. McEwen who taught me to understand and love mathematics,

Dr. William C. Gemeinhardt who encouraged and guided my writings,

My father, William, who instilled in me the quality of perseverance,

I dedicate this paper.

Thank you, William.

[redacted]

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

How can you capture the imagination of the mathematically talented elementary school students, and provide them with an education which reflects the recent advances in mathematics and provides for each of them the fullest personal achievement? This paper in the hands of a teacher who knows and likes mathematics may help provide an answer for the students of Hibbing, Minnesota.

I. THE PROBLEM

Statement of the Problem

It was the purpose of this paper to create a curriculum for the Hibbing Sixth Grade Mathematics Enrichment Program utilizing the guided discovery method of teaching. The material was developed to enrich the existing basic mathematics program of the Hibbing public and parochial schools.

Importance of the Paper

Since the origin of the Hibbing Mathematics Enrichment Program, there had been no definite curriculum for the program. The author of this paper, also the instructor for the program, was well aware of this need and so developed the curriculum which was put into use in the fall of 1967.

The author spent the academic year 1966-67 participating in an Experienced Teacher Fellowship Program in Science and Mathematics at the University of Minnesota, Duluth. The objectives of this institute were to upgrade the teaching of mathematics and science in the elementary schools, and to prepare the participants to teach science and mathematics using the discovery approach. The curriculum in this paper was designed with this philosophy in mind.

II. DEFINITIONS OF TERMS USED

Enrichment

Enrichment was interpreted as meaning an addition to the basic school mathematics program. It consisted of topics that were completely new but related to the basic program, and basic mathematics topics to be studied in greater depth.

Guided Discovery

Throughout this paper the term guided discovery referred to the teaching-learning process which encouraged the student to explore situations and discover, while the teacher guided him by carefully selected experiences and skillfully formulated questions.

CHAPTER II

REVIEW OF THE LITERATURE

"The gifted is both an asset and a responsibility," stated Henry Nelson in the _Forty-ninth Yearbook of the National Society for the Study of Education_. He continued further stating that:

> He can be an asset of incalculable value to society. His potentialities for good or bad are difficult to overestimate. Our socio-economic structure, both national and international, demands leadership of the highest quality and keenest intelligence. Where else should we look for this type of leadership except among those of intellectual superiority?[1]

With this as a guiding statement, it behoved educators and laymen alike to focus attention on the problem of utilizing to the fullest the talents of all students.

Figures obtained from elementary school teachers indicated there were ten million people in this country with definite mathematical talent or who had shown a definite liking for mathematics at an early age. What

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[1]Henry Nelson, _The Education of Exceptional Children_, Forty-ninth Yearbook of the National Society for the Study of Education, Part II (Chicago, Illinois: University of Chicago Press, 1950), p. 260.

happened to these students in later life? Somehow in the educational process much of the talent was discouraged, bored, or otherwise lost.[2]

Paul Rosenbloom, a pioneer in the School Mathematics Study Group, suggested that:

> One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum--one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge and at the same time one which reflects recent advances in mathematics itself.[3]

In The Process of Education by Jerome S. Bruner the third chapter is devoted to "Readiness for Learning." "It seems clear from the experimental work of David Page, Robert Davis, Max Beberman, Patrick Suppes, and many others that we have grossly underestimated the mathematical power of students and their capacity to learn mathematics."[4]

Vincent J. Glennon in the Twenty-seventh Yearbook of the National Council of Teachers of Mathematics

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[2]National Council of Teachers of Mathematics, The Revolution in School Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1961), p. 9.

[3]Paul C. Rosenbloom, "Mathematics K-14," Educational Leadership, (Cambridge, Massachusetts: Harvard University Press, 1960), p. 64.

[4]Jerome S. Bruner, The Process of Education (Cambridge, Massachusetts: Harvard University Press, 1960), p. 64.

reported, "Enrichment is the most widely accepted practice for providing learning experiences for the talented child."[5]

Many writers defined enrichment, and though they differed in words their concept was the same. Howard Fehr in his latest book Teaching Modern Mathematics in the Elementary School said, "Enrichment means giving children material that will prepare them for later study and that will give them a deeper insight into the content of the regular program of study."[6]

Inherent in the idea of enrichment are the concepts of breadth and depth. The concept of breadth is concerned with the introduction of new but related topics and the concept of depth is concerned with developing new insights into what is presently taught. A number of articles have been written about the two aspects of enrichment with no conclusions as to the superiority of one over the other.

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[5]Vincent J. Glennon, "Some Perspectives in Education," Enrichment Mathematics for the Grades, Twenty-seventh Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1963), p. 26.

[6]Howard F. Fehr and Jo McKeeby Phillips, Teaching Modern Mathematics in the Elementary School (Reading: Addison Wesley Publishing Company, 1966), p. 393.

In regard to an enrichment curriculum for the academically talented, writers agreed that the material must be forward looking, comprehensive, and challenging. The Department of Public Instruction of Pennsylvania presented the following guidelines to consider in the development of enrichment materials, consider that talented pupils:

1. Need less repetition of material.
2. Need fewer concrete illustrations: they have more ability to visualize.
3. Have extensive vocabularies.
4. Enjoy discovering their own answers to questions and problems.
5. Have a long attention span.
6. Need materials to develop their abilities for independent study and research.[7]

"A theory of instruction is concerned with how what one wishes to teach can best be learned,"[8] defined Jerome S. Bruner.

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[7]Pennsylvania Department of Public Instruction, Guide to Planning for Able Pupils, A Report Prepared by the Department of Public Instruction, Commonwealth of Pennsylvania (Harrisburg: Commonwealth of Pennsylvania, 1962), p. 19.

[8]Jerome S. Bruner, Toward a Theory of Instruction (Cambridge, Massachusetts: Harvard University Press, 1966), p. 40.

Much has been written in recent years concerning the discovery method of instruction, and the new mathematics textbooks all emphasized this method to some degree.

Typical of the many writers who strongly advocated the discovery approach was Dr. Lola J. May, Mathematics Consultant for the Winnetka Public Schools, who stated:

> The best method of teaching modern mathematics is the discovery method. If you encourage pupils to find relationships and solutions for themselves rather than to look to you for the answers, you are not only teaching mathematics, but a way to think about mathematics. This is the basis of learning. This searching attitude produces enjoyment and a feeling of satisfaction.[9]

The Cambridge Conference on School Mathematics supported the discovery method but added this note of caution, ". . . the teacher should be prepared to introduce required ideas when they are not forthcoming from the class; that he should bring attention to misleading statements in the way of discussion, then summarize results clearly as they come forward. He should not allow the 'moments of triumph' to pass by unnoticed."[10]

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[9]Lola J. May, "How to Teach the New Mathematics," Grade Teacher, LXXXII (September, 1964), p. 50.

[10]Cambridge Conference on School Mathematics, Goals for School Mathematics (Boston: Houghton Mifflin Company published for Educational Services Incorporated, 1963), p. 17.

Another viewpoint was expressed by Dr. Robert Kane who suggested that substantial numbers of teachers and students will not use discovery techniques as their basic operational mode. Only some are able to succeed in the wholesale use of discovery techniques. ". . . it is possible to take some comfort in what I believe to be true: that while discovery may be a sufficient condition for understanding, it does not seem to be a necessary one."[11]

Analyzing the literature of the past five years showed that attention had shifted from the discovery approach to a guided discovery approach.

One exponent of guided discovery was Irving Adler. He urged educators not to pursue the mirage of every student making all his own discoveries in splendid isolation but rather to provide the student with guided discovery and allow for the learning which comes from an exchange of ideas between a student and his classmates.[12]

School systems throughout the United States have made provisions for the talented student but the literature describing these programs was limited.

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[11] Robert B. Kane, "School Mathematics--Where to Now?" The Arithmetic Teacher, XIV (February, 1967), p. 130.

[12] Irving Adler, "Mental Growth and the Art of Teaching," The Mathematics Teacher, LIX (December, 1966), p. 711.

California provided the Saturday class plan as a means of enriching the educational opportunities offered gifted pupils by the rural schools they attended. Each Saturday morning the pupils studied two subjects, mathematics was one of the choices, each for one period with a play period between the classes. The classes were taught in groups of ten to fifteen pupils. The teachers chosen to work with the mathematics groups had special competence in this area. Close cooperation was maintained between those responsible for the Saturday classes and the teachers of the pupils' regular classes.[13]

A high achievers program segregated the gifted elementary school students at Bloomington, Minnesota. Beginning at the fifth grade and continuing through the sixth grade the seven top students in each building were clustered in one high achievers class. Four schools provided students for one class. Selection was based upon high intellectual ability and achievement consistant with ability in academic areas. The curriculum, defined as an expanded curriculum, emphasized both acceleration and enrichment in mathematics. Evaluation of the program by

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[13]Ruth A. Martinson, Special Programs for Gifted Pupils, A Bulletin of the California State Department of Education (Sacramento: California State Department of Education, 1962), pp. 77-78.

teachers and parents indicated that mathematics ranked first according to the subjects showing the most student improvement and second according to the interest developed in the subject areas.[14]

The elementary school at Illinois State Normal University had a special mathematics teacher who worked for two fifty minute sessions each week with children in grades five and six who had been selected as very able in mathematics. The teacher did not duplicate the regular classroom instruction, but helped the pupils to be creative with arithmetic and elementary algebra in topics they had not studied. Rather than giving formal instruction, the teacher served as an adviser, guided the pupils, and tried to help them explore and discover for themselves.[15]

Pittsburg, Pennsylvania was one of the many schools which offered a summer school mathematics enrichment program for elementary students judged to be mathematically

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[14]High Achievers Program, A Report Prepared by the Bloomington Public Schools (Bloomington, Minnesota: Bloomington School System, 1962), pp. 11-25. (Mimeographed)

[15]Veryl Schult, Present Practices in Mathematics Instruction and Supervision, A Report Prepared by the United States Department of Health, Education, and Welfare--Bureau of Research. (Washington, D.C.: United States Government Printing Office, 1966), p. 11.

talented. Classes were held daily for a period of six weeks.[16]

Milwaukee schools provided for the mathematically talented by the use of television. Twice a week selected students watched "Patterns in Arithmetic."[17]

In San Francisco, the local chapter of the Association for Childhood Education sponsored a Mathematics Fair for elementary school students. Interest was widespread and the fair was attended by one thousand teachers, administrators and parents.[18]

For years Winnetka, Illinois has been known for individualized instruction. In 1965 Winnetka introduced the learning laboratory. The laboratory was a large L-shaped room that had carpeting on the floor, drapes on the windows, individual mobile carrels, tape booths, and the usual variety of visual aid materials. Some talented students were ahead of their class and were released from class a few periods a week. A few of the students were so advanced that they did all of their mathematics in the laboratory. The students were under the guidance of

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[16]Ibid., p. 51

[17]Ibid., p. 52

[18]Ibid., p. 23

Dr. Lola May, Winnetka mathematics consultant, who planned an individual mathematics program for each student.[19]

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[19] Lola J. May, "Individualized Instruction in a Learning Laboratory Setting," The Arithmetic Teacher, XIII (February, 1966), pp. 110-111.

CHAPTER III

THE HIBBING MATHEMATICS ENRICHMENT PROGRAM

Origin of the Program

In 1960, Dr. Rachel Bodoh, then Director of Elementary Education in Hibbing, Minnesota, brought to the attention of the community the availability of enrichment activities in art, music, and athletics; and the lack of any program, outside the regular classroom, in any academic area. Dr. Bodoh called a meeting of the elementary administrators and the sixth grade teachers to study this need. After a thorough study, a decision was reached to establish enrichment programs in the areas of mathematics, science, and creative writing for the sixth grade students.

The Hibbing Mathematics Coordinating Committee, a group of mathematics teachers representing all grade levels, kindergarten through twelve, offered suggestions for the mathematics program.

The program was presented to the Hibbing Board of Education. They gave the program enthusiastic support and agreed to provide funds for the instructor's salary and the purchase of any desired materials. A new program had joined the offerings of the Hibbing elementary schools.

Objectives of the Program

The objectives of the Hibbing Mathematics Enrichment Program are:

1. To provide additional learning experiences for the mathematically talented student.
2. To help the students develop a better understanding of the logical structure of mathematics.
3. To teach the student the precise vocabulary of mathematics.
4. To teach the student that mathematics is exciting, interesting, and challenging.

Selection of Participants

Participants in the program are selected from the sixth grade classes in both the public and parochial schools of Hibbing. The selections are made by the classroom teachers using the following guidelines:

1. Analysis of intelligence test scores.
2. Analysis of achievement in mathematics on The Iowa Test of Basic Skills.
3. Analysis of fifth grade scholastic achievement in mathematics.
4. Comments of the previous fifth grade teacher.
5. Observations of the student and analysis of his

work. (Special emphasis is given to the student's interest in mathematics and his ability to think and work abstractly.)

Parents of the students selected receive a letter from the Director of Elementary Education informing them of their child's selection and of the program itself. Enclosed with the letter is a form which is completed by the parents if they desire their child to participate in the program.

The Program in Operation

The mathematics enrichment class begins to meet shortly after the opening of school in September. The class meets every Monday, from four to five o'clock, at the Washington Elementary School. There is no conflict with other student interests as the boys' athletic program and the Girl Scouts meet on Tuesday and Thursday, and Wednesday is reserved for church activities. Class sessions are held until the middle of April. The class size varies from fifteen to twenty-five students depending on the number of students recommended that year.

Various topics in mathematics are studied by the students. There are usually no out-of-class assignments, but students often work on topics which have interested them and bring their information or questions to the next class session. No formal tests or grades are given. The progress of the class is evaluated by the instructor on the basis of worksheet results, students' classroom responses, and the interest exhibited by the students.

Participation certificates are presented at the final class session. This session may include the student's parents and classroom teacher. A statement indicating that the student participated in the program is placed in his cummulative folder.

CHAPTER IV

THE CURRICULUM

NUMERATION SYSTEMS WITH BASES OTHER THAN TEN

Objectives

Upon completion of these activities the student should be able:

1. to explain what is meant by a numeration system.
2. to realize that using a base of ten was an arbitrary choice made by early peoples probably because of their ten fingers.
3. to demonstrate his understanding of the principles of place value by the reading and writing of numerals in other bases.
4. to perform the operations of addition and multiplication in a non-decimal numeration system.
5. to solve a multiplication problem by the use of Napier's bones.

Suggested Time Allotment

Four class sessions

Vocabulary

Number, numeral, numeration, place value, base

Materials

Flannel board

Flannel cut-outs including 23 arrowheads

Six strips of cardboard (1 inch by 5 inches) for use by each student

Demonstration set of Napier's bones

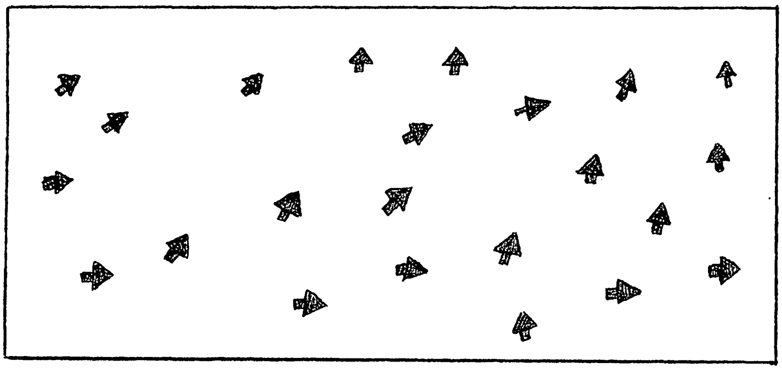
String of Christmas tree lights

Styrofoam, wood, or other base for mounting the set of lights

Abacus

Originating the Problem

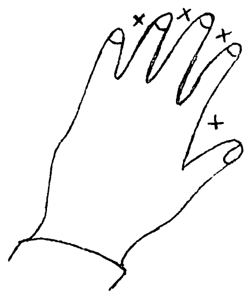
Randomly place 23 flannel cut-outs of arrowheads on the flannel board.



An Indian of early Minnesota had this number of arrowheads. Quickly tell me, how many did he have? (Student's answers will probably vary from 21 to 25.) We had some difficulty in obtaining our answer. How could we arrange the arrowheads so that we could tell quickly how many there are? (Various methods of grouping will be suggested. Two groups of ten and three arrowheads remaining. Four groups of five and three arrowheads remaining.)

Civilizations have not always grouped by tens. The Mayan Indians of Central America grouped by twenties. Can you guess why? (They lived in a warm climate and didn't wear shoes so they counted on both their fingers and toes.)

There was an Indian tribe in California that grouped by fours. Would anyone like to guess why? (The correct answer will probably not be given. It is believed that they used the space between the digits on one hand as a group.)



We group things by tens. Why do you suppose ten was the choice made by most early people? (man's ten fingers)

In the activities that follow it is assumed that the students fully understand grouping by ten and the Base Ten Numeration System.

Instructional Procedure

Activity One

We use some groupings of five in our everyday life. Let us look at our system of money. If I have 14 cents, I might have a dime and four pennies. This is grouping by tens. What does 14 mean? (1 ten + 4 ones) I might have two nickels and 4 pennies. This is grouping by fives. A group of five pennies is equivalent to? (a nickel) A group of five nickels is equivalent to? (a quarter) Grouping by fives, 14 would be written 24_{five}. The subscript five tells us we are grouping by fives. We say the numeral has been written in base five--our groupings are based on 5 objects in a group. What does 24_{five} mean? (2 fives + 4 ones)

In your previous work with base ten you have made place value charts. Ask the students to fill in the place values in a base ten chart.

Ten x Ten or Hundreds	Tens	Ones

In grouping by fives, we also make a place value chart. Ask the students to fill in the values.

Five x Five or Quarters	Fives or Nickels	Ones or Pennies

Be sure the students understand that a numeral is a symbol used to stand for a number.

Using the smallest number of coins, separate the following amounts of money into quarters, nickels, and pennies. Also express in base five notation.

How much money?	How many quarters?	How many nickels?	How many pennies?	Base five notation
1) 23 cents	0	4	3	43_{five}
2) 26 cents				
3) 29 cents				
4) 33 cents				
5) 42 cents				
6) 57 cents				
7) 73 cents				
8) 97 cents				
9) 124 cents				

One way to see that we are using different numerals to represent the same number is to write in one column the first fifty counting numbers and then in a parallel column the same number in base five notation.

Base Ten	Base Five
1	1
2	2
3	3
4	4
5	10
6	11
7	12
8	13
9	14
10	20
⋮	⋮
32	112
⋮	⋮
50	200

Let us agree that whenever we write "13", base ten will automatically be understood, and when we want base five to be understood instead, we shall write "13_{five}", read "one five and three ones or one three, base five."

32 = 112 (read "one twenty-five, one five, and two ones or one one two, base five.")

50 = 200 (read "two twenty fives or two zero zero, base five.")

If the students have difficulty in understanding the groupings, use an abacus or use groups of concrete objects.

On the chalkboard place a few numbers written in base five notation and ask the students to determine how to rewrite them in base ten notation.

$24_{five} = (2 \times 5) + 4 = 14$

$111_{five} = (1 \times 25) + (1 \times 5) + 1 = 31$

Ask the students to determine a way of changing a number written in base ten notation to base five notation.

$89 = \underline{\qquad}_{five}$

$89 \div 25 = 3r\ 14$ — This division shows there are three twenty-fives and fourteen units left over.

$14 \div 5 = 2r\ 4$ — This division shows there are two groups of five and 4 units left over.

Thus $89 = 324_{five}$

Activity Two

Ask the students to work out a base five addition table which they may refer to for the basic facts. The teacher may then show a completed table on the overhead projector so the students may verify their answers. Encourage the students to refer to this table.

Addition Table - Base Five

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

Renaming numbers is a very important process in addition. Rename the following in their simplest form.

2 fives + 4 ones (24_{five})

3 twenty-fives + 2 ones (302_{five})

1 five + 2 ones (12_{five})

Given a numeral in base five, express it in expanded notation.

21_{five} (2 fives + 1 one)

314_{five} (3 twenty-fives + 1 five + 4 ones)

Using expanded notation, develop a method of solving the following problem: $22_{five} + 2_{five} = n$. The student will be able to develop one of the following forms:

$$\begin{array}{ll} 22_{five} & = 2 \text{ fives} + 2 \text{ ones} \\ \underline{+2_{five}} & \quad\quad\quad\;\; \underline{+ 2 \text{ ones}} \\ & \;\; 2 \text{ fives} + 4 \text{ ones or } 24_{five} \end{array}$$

$$\begin{array}{ll} 22_{five} & = 20_{five} + 2_{five} \\ \underline{+2_{five}} & \quad\quad\quad\; \underline{+ 2_{five}} \\ & \;\; 20_{five} + 4_{five} \text{ or } 24_{five} \end{array}$$

$$\begin{aligned} 22_{five} + 2_{five} &= (20_{five} + 2_{five}) + 2_{five} \\ &= 20_{five} + (2_{five} + 2_{five}) \\ &= 20_{five} + 4_{five} \\ &= 24_{five} \end{aligned}$$

Ask the students to solve a more difficult problem requiring regrouping.

$$\begin{array}{ll} 23_{five} & = 2 \text{ fives} + 3 \text{ ones} \\ +\underline{13}_{five} & = \underline{1 \text{ five} \;\; + 3 \text{ ones}} \\ & \;\; 3 \text{ fives} + 11 \text{ ones} \\ & \;\; 4 \text{ fives} + 1 \text{ one} \;\; \text{or } 41_{five} \end{array}$$

Use the abacus if students are having difficulty regrouping.

Ask the students to work out a few examples to ascertain if they have mastered the idea of regrouping.

$$\begin{array}{l} 144_{five} \\ +\underline{204}_{five} \end{array} \qquad \begin{array}{l} 34_{five} \\ +\underline{22}_{five} \end{array} \qquad \begin{array}{l} 21_{five} \\ +\underline{13}_{five} \end{array}$$

Solve for n in each of the following equations:

$4_{five} + 3_{five} = n_{five}$

$n_{five} + 2_{five} = 10_{five}$

$n_{five} + 11_{five} = 23_{five}$

If you really understand the ideas of regrouping you will be able to discover, on your own, how to solve these subtraction problems.

$$\begin{array}{r} 44_{five} \\ -23_{five} \\ \hline \end{array} \qquad \begin{array}{r} 320_{five} \\ -42_{five} \\ \hline \end{array} \qquad \begin{array}{r} 34_{five} \\ -12_{five} \\ \hline \end{array}$$

Activity Three

Ask the students to work out a base five multiplication table. After they have finished, present a completed table so they may check their answers.

Multiplication Table - Base Five

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

To help us with our multiplication in base five, we can make a set of Napier's bones. This multiplication device was introduced by John Napier, a Scottish mathematician, in 1617.

Provide each student with six strips of cardboard, each one inch by five inches. Mark five of the strips into squares with diagonals as shown in figure a, and label these five strips with the base five products as shown in figure b. The remaining strip is marked only in squares, as in figure c, and will be used as the index.

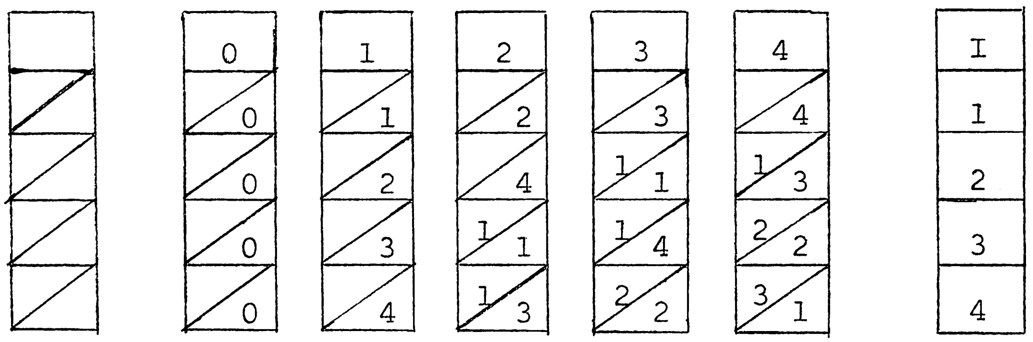
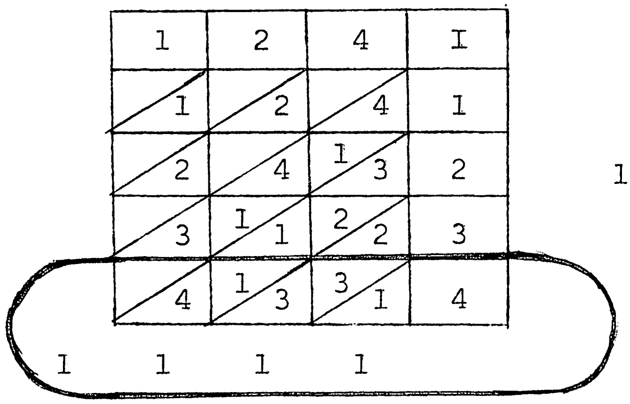


Fig. a. Fig. b. Fig. c.

Given the problem, $124_{five} \times 4 = n$, to solve -- use your Napier's bones as follows:

Select the strips or "bones" for 1, 2, and 4 and place them in place value order next to the index. Since we are multiplying by four, we will obtain our answer by adding along the diagonal columns in this row, regrouping whenever necessary.



$124_{five} \times 4 = 1111_{five}$

Work out a number of examples until the students are sure they understand the use of the "bones".

Ask the students to solve the following problems by developing their own multiplication operation. Answers may be checked by using Napier's bones.

$$\begin{array}{r} 22_{five} \\ \underline{\times 2} \end{array} \qquad \begin{array}{r} 243_{five} \\ \underline{\times 2} \end{array} \qquad \begin{array}{r} 32_{five} \\ \underline{\times 3} \end{array}$$

Challenge the students to find a solution to the following problem:

$$\begin{array}{r} 103_{five} \\ \underline{\times 42}_{five} \end{array}$$

Activity Four

You may have read about computers that can solve difficult mathematical problems very quickly. The base two numeration system is used in many computers. A German mathematician, Leibniz advocated this system many years ago. He was a very religious man and considered one as representing God and zero as representing the void. With these two symbols he could write any numeral just as he felt God could make anything from the void.

How many symbols would be needed for a base two system? (two, 1 and 0) Ask the students to write the base two numerals for the numbers from one to thirty-two. Ask helpful questions whenever you find a student having difficulty.

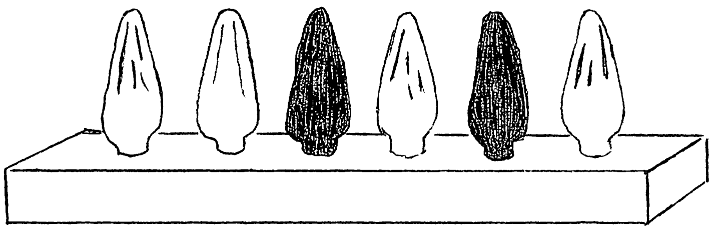
1_{two}	1001_{two}	10001_{two}	11001_{two}
10_{two}	1010_{two}	10010_{two}	11010_{two}
11_{two}	1011_{two}	10011_{two}	11011_{two}
100_{two}	1100_{two}	10100_{two}	11100_{two}
101_{two}	1101_{two}	10101_{two}	11101_{two}
110_{two}	1110_{two}	10110_{two}	11110_{two}
111_{two}	1111_{two}	10111_{two}	11111_{two}
1000_{two}	10000_{two}	11000_{two}	100000_{two}

What are the place values for base two? (ones, twos, fours, eights, sixteens, thirty-twos, and so on)

Since base two has only two digits, a computer can represent any number by having the electric current on or off. When a light is on, it shows the digit 1; when it is off, it shows the digit 0.

Have a group of students mount a string of Christmas tree lights and visually demonstrate binary numerals.

1010_{two} would look like this:



Can the four basic operations be performed using base two numerals?

$$\begin{array}{r} 11001_{two} \\ +101_{two} \\ \hline \end{array} \qquad \begin{array}{r} 1111_{two} \\ -11_{two} \\ \hline \end{array}$$

$$10_{two} \overline{)\ 1011}_{two} \qquad \begin{array}{r} 11_{two} \\ \times 11_{two} \\ \hline \end{array}$$

Discuss the advantages and disadvantages of adopting a base two numeration system.

Activity Five

Many people are interested in a base twelve numeration system. Can you suggest any reasons why 12 might be a good base for a numeration system? (12 has many even divisors; 2, 3, 4, and 6. Many of our units of measures are based on 12 or multiples of 12, such as: twelve inches equal a foot, twelve hours on the clock face, and twelve eggs in a dozen.)

Divide the class into groups of four or five and ask each group to write base twelve numerals for the numbers from zero to twenty-five. Discuss the results and the various ways the groups handled the writing of 10 and 11. New symbols must be introduced for example, ten = T and eleven = E. Discuss the place value involved in a base twelve system.

If your mother is 33 years old, what is her age written in base 12? (29) This would make her happy.

Let me pose a more difficult problem. Can you express the year 1967 in base twelve numeration? (118E)

Ask the students to suggest problems involving base twelve numeration.

Activity Six

I have developed a new system of numeration. Can you discover it and solve the exercises?

The symbols and names for numbers are defined as follows:

	[ ]	[x]	[xx]	[xxx]	[xxxx]
Symbol	△	—	=	≡	— △
Name	fe	fi	fo	fum	fi fe

Example

[xxxxxxx] means fi fum or — ≡

Exercises

a. Write the symbol and name of each number shown.

[xxxxx xxxxx] [xxxxx xxx]

b. Write the next consecutive number after each of the following:

— =,

≡ ≡,

— △,

c. Add: ≡ and _ =

d. Subtract: = △ from ≡ _

e. What picture would you draw to represent fi fe fe? (_ △△)

Activity Seven

Ask interested students to develop numeration systems of their own.

Suggested Supplementary Readings

Ellison, Alfred. "That Backward Yllis Math," *The Arithmetic Teacher*, X-5, (May, 1963), pp. 259-261.

Hensley, C. A. E. "New Mathematics With the Pioneers' Dial Tally," *School Science and Mathematics*, LXVII (January, 1967), pp. 51-59.

Johnson, Donovan A. and William H. Glenn. *Understanding Numeration Systems*. St. Louis: Webster Publishing Company, 1960. 56 pp.

National Council of Teachers of Mathematics. *Enrichment Mathematics for the Grades*. Twenty-seventh Yearbook. Washington, D.C.: The National Council of Teachers of Mathematics, 1963. pp. 41-64 and 234-235.

Olsen, Elizabeth P. "There's Sense in Nonsense Arithmetic," The Arithmetic Teacher, XII (May, 1965), pp. 341-342.

Rahmlow, Harold F. "Understanding Different Number Bases," The Arithmetic Teacher, XII (May, 1965), pp. 339-340.

Schupback, Sister Joseph M. "Does Base Four Bewilder You?" The Arithmetic Teacher, XIV (April, 1967), pp. 208-210.

Weyer, Virginia. "Base Popsicle," The Arithmetic Teacher, XIV (April, 1967), pp. 312-313.

EXPONENTIAL AND SCIENTIFIC NOTATION

Objectives

Upon completion of these activities the student should be able:

1. to express a number in exponential notation.
2. to express a number, given in exponential notation, as a product.
3. to give an explanation of why any number (except zero) raised to the zero power is one.
4. to multiply and divide numbers written in exponential notation.
5. to use expanded notation and exponents to express a numeral written in any base.
6. to write large numbers in scientific notation.

Suggested Time Allotment

Three class sessions

Vocabulary

Base, exponent, power, exponential notation, scientific notation, input, output

Materials

Instruction cards for the duplicating machine in

Martian and English. (Place on an overhead transparency or make a large sample card to display.)

Originating the Problem

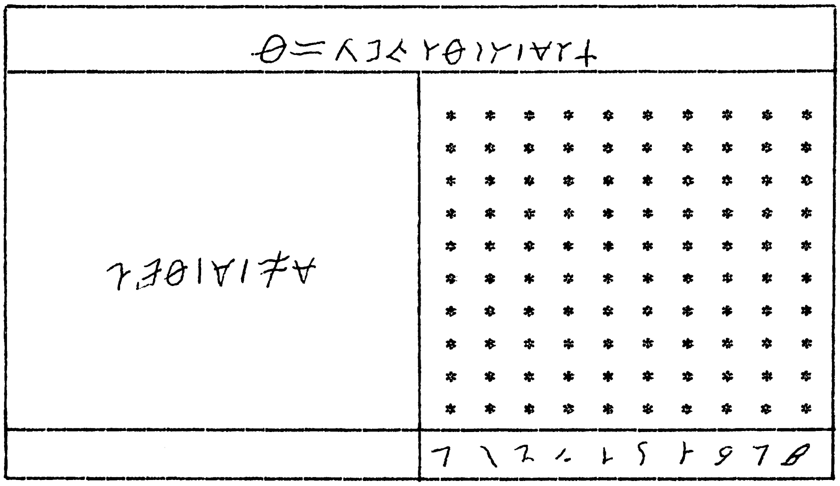
In the sciences, you will deal with measurements of distance, surface, and space which are often represented by extremely large numbers. Our next group of activities will prepare you mathematically to work with these large numbers.

Instructional Procedure

Activity One

Instruction is to begin with the following narration being given by the teacher. The newspapers have been full of news of the recent landing of astronaut Revocsid on the planet Mars. Telling of his visit, he reported that the most interesting thing he had seen was a marvelous duplicating machine. If a Martian had one candy bar and wanted five, he merely placed the one candy bar and a set of instructions in one slot called the input slot and out the other side would come five candy bars. Revocsid would have liked to bring this machine to earth, but of course, this was impossible. He did, however, bring back an instruction card and the great scientists on earth discovered how the machine works. Maybe some day we will have the machine, too.

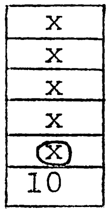
Today I am going to see what good scientists you can be and if you can discover how the Martian duplicating machine works. Show a picture of the Martian instruction card.



Are you having trouble reading the card? That's natural, it's written in Martian. Let's look at it after it was translated into English.

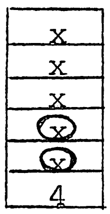
Instruction Card										
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
Duplication	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
	x	x	x	x	x	x	x	x	x	x
Output Number	1	2	3	4	5	6	7	8	9	10

The numbers along the bottom of the instruction card represent the number of objects desired in the output. If I want to get ten balloons from the machine, the instruction card will have one x circled above the ten, like this:



Ask the students to explain how the instruction card would be marked to obtain an output of 3, 4, 5, 6, 7, 8, and 9 objects.

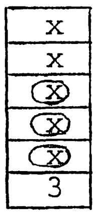
Suppose someone wanted to make 16 baseballs, how do you think the instruction card would be marked? The students will give many responses, such as; circle two x's above the eight (8 + 8 = 16) or circle one x above two and one x above eight (2 x 8 = 16). Continue the discussion until the correct response is reached. Circle two x's above the four. (4 x 4 = 16)



Note to the teacher: The machine operates only on instructions to multiply a number by itself, in other words to raise a number to a power.

Ask the students to now determine how the instruction card would be marked to produce outputs of 25, 36, 49, and 81. At this point see if the students have discovered how the machine operates. Someone will probably suggest it multiplies and the numbers used (factors) must be the same.

The students should now be asked to determine how the instruction card should be marked to obtain an output of 27. (Circle three x's above the three.) (3 x 3 x 3 = 27)



Ask the students to discuss various ways of marking the instruction card to obtain outputs of 8, 81, 16, 32, 100, and 64. By this time, the students should understand how the machine operates.

Another way of writing our instructions without using our instruction card, would be to write x^{x}_{9}. What do you suppose this means? (Circle two x's above the nine or 9 x 9 = 81.) Have the students explain other examples.

x							
x							
x	x		x				x
x	x	x	x		x	x	x
x	x	x	x	x	x	x	x
2	2	4	5	4	5	6	4

Mathematicians have a way of expressing how the duplicating machine works. They call the operation raising the number to a power.

x
x
x
2 would be written a 2^3 and read two to the third power.

x
x
5 would be written as 5^2 and read five to the second power.

At this point take time to develop with the class the correct vocabulary:

in 5^3, 5 names the <u>base</u>.

in 5^3, 3 names the <u>exponent</u>.

5^3 names the <u>power</u> - five to the third power.

5^3 is <u>exponential</u> <u>notation</u> for the number 5 x 5 x 5 or 125.

Activity Two

Present the students with the following problems to solve.

1. Express each of the following in exponential notation.

   a. 3 x 3 x 3   d. 5 x 5

   b. 5 x 5 x 5 x 5   e. 17 x 17

   c. 7 x 7 x 7 x 7 x 7   f. 2 x 2 x 2 x 2

2. Complete the chart below. The first exercise is done for you.

	Power	Base	Exponent
a.	4^5	4	5
b.	2^3		
c.	6^2		
d.	10^3		
e.	2^{10}		

3. Express each of the following as a product.

   a. 6^3

   b. 2^2

   c. 10^3

   d. 30^2

4. How would you write five to the seventh power?

5. Express each of the following numbers in exponential notation.

   a. 36

   b. 1000

   c. 144

6. Braintwister -- Express the following numbers in exponential notation.

   a. 1/9

   b. 1/4

Activity Three

Mathematicians and other scientists save a great deal of time by using exponents. They can use exponents when they are multiplying or dividing quantities that have the same base number. See if you can discover an approach that uses exponents to solve the problems.

Study the problem $2^3 \times 2^5$

a. Express 2^3 as a product $(2 \times 2 \times 2)$

b. Express 2^5 as a product $(2 \times 2 \times 2 \times 2 \times 2)$

c. Express $2^3 \times 2^5$ as a product consisting of the factor 2 repeated the correct number of times. $(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$

d. Complete the exponential notation $2^3 \times 2^5 =$ /__/ (2^8)

Consider other problems like this until the students discover the pattern and can formulate the general rule: $x^m \circ x^n = x^{m+n}$

Next study the problem $3^6 \div 3^4$ or $3^6 / 3^4$

a. Express 3^6 as a product $(3 \times 3 \times 3 \times 3 \times 3 \times 3)$

b. Express 3^4 as a product $(3 \times 3 \times 3 \times 3)$

c. Simplify the expression

$$\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} \qquad (3 \times 3)$$

d. Complete in exponential notation $3^6/3^4 = \square$ (3^2)

Consider other problems like this until the students discover the pattern and can formulate the general rule: $x^m/x^n = x^{m-n}$

The purpose of our next activity is to see if we can develop some meaning for 3^0, 7^0, etc.

Work out the following pattern with the students.

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$ divide by 3

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$ divide by 3

$3^4 = 3 \times 3 \times 3 \times 3 = 81$ divide by 3

$3^3 = 3 \times 3 \times 3 = 27$ divide by 3

$3^2 = 3 \times 3 = 9$ divide by 3

$3^1 = 3$ divide by 3

$3^0 = ?$ $(3^0 = 1)$

Consider many more examples of a similar nature until the students can formulate the general statement $x^0 = 1$. (except when $x = 0$)

Activity Four

Present the following problems to provide practice in working with exponents.

1. Write each product in exponential notation.
   a. $2^2 \times 2^3$
   b. $3^5 \times 3^7$
   c. $5^3 \times 5^3$
   d. $5^3 \times 5^0$
   e. $(1/2)^3 \times (1/2)^2$
2. Write each quotient in exponential notation.
   a. $\frac{3^4}{3^2}$ b. $\frac{2^5}{2^2}$ c. $\frac{5^6}{5^3}$ d. $\frac{7^8}{7^3}$
3. Write "true" in the space at the right of each exercise if the statement is true. Write "false" if the statement is false.
   a. $2^3 \cdot 2^4 = 2^7$ ______ e. $2^3 \cdot 2^0 = 1$ ______
   b. $3^4 \cdot 3^7 = 3^{28}$ ______ f. $(1/2)^2 = 1/4$ ______
   c. $9^3 \cdot 9^1 = 9^3$ ______ g. $10^3 = 100$ ______
   d. $\frac{3^7}{3^4} = 3^3$ ______ h. $\frac{5^2}{5^2} = 5$ ______

Activity Five

Express the number 2345 in expanded notation and then in expanded notation using exponents.

$$2345 = 2 \times 1000 + 3 \times 100 + 4 \times 10 + 5 \times 1$$
$$2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$$

You can express a number written in any base in expanded notation. Try a few examples.

$$3342_{five} = 3 \times 125 + 3 \times 25 + 4 \times 5 + 2 \times 1$$
$$= 3 \times 5^3 + 3 \times 5^2 + 4 \times 5^1 + 2 \times 5^0$$

$$136_{seven} = 1 \times 49 + 3 \times 7 + 6 \times 1$$
$$= 1 \times 7^2 + 3 \times 7^1 + 6 \times 7^0$$

Activity Six

On the chalkboard write 93000000. Ask the students to read the number. Scientists have developed a way to shorten the writing of very large numbers by using the power of ten. Ask the students to tell you what to write as you fill in the following table.

ten	$= 10 = 10^1$
one hundred	$= 10 \times 10 = 10^2$
one thousand	$= 10 \times 10 \times 10 = 10^3$
ten thousand	$= 10 \times 10 \times 10 \times 10 = 10^4$
one hundred thousand	$= 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

How could we express our original number 93 million using the powers of ten? (93×10^6) Are there other ways the number could be expressed? (Yes)

$$9300000 \times 10^1$$

$$930000 \times 10^2$$

$$93000 \times 10^3$$

$$9300 \times 10^4$$

$$930 \times 10^5$$

$$9.3 \times 10^7$$

In order to avoid any confusion, scientists all agreed to follow the same rules and to call this way of writing a number scientific notation. A number is said to be expressed in scientific notation if it is named as the product of two factors, one factor a number between one and ten, and the other factor the correct power of ten.

Work out a few examples.

$$80{,}000 = 8 \times 10^4$$

$$100 = 1 \times 10^2$$

$$350 = 3.5 \times 10^2$$

$$275 = 2.75 \times 10^2$$

$$1{,}500 = 1.5 \times 10^3$$

$$2{,}135 = 2.135 \times 10^3$$

Activity Seven

Ask each student to complete the following table:

Decimal	Scientific Notation
1,000,000	
	2.3×10^3
350	
	7.63×10^7
	2×10^9
25,000	

Express the numbers in each of the following statements in scientific notation.

1. The radius of the earth is about 4,000 miles.
2. The average distance from Pluto to the sun is 3,600,000,000 miles.
3. The earth's orbit around the sun is 595,000,000 miles.
4. The moon is about 238,000 miles away from the earth.

Activity Eight (Optional)

How many subsets can be formed from a set of one element? (2)

Set $\{a\}$

Subsets $\{a\}$ $\{\ \}$

How many subsets can be formed from a set of two elements? (4)

Set $\{a, b\}$

Subsets $\{a, b\}$ $\{a\}$ $\{b\}$ $\{\ \}$

How many subsets can be formed from a set of three elements? (8)

Set $\{a, b, c\}$

Subsets $\{a, b, c\}$ $\{a, b\}$ $\{a, c\}$ $\{b, c\}$ $\{a\}$ $\{b\}$ $\{c\}$ $\{\ \}$

How many subsets can be formed from a set of four elements? (16)

Set $\{a, b, c, d\}$

Subsets $\{a, b, c, d\}$ $\{a, b, c\}$ $\{a, c, d\}$ $\{a, b, d\}$ $\{b, c, d\}$ $\{a, b\}$ $\{a, c\}$ $\{a, d\}$ $\{b, c\}$ $\{b, d\}$ $\{c, d\}$ $\{a\}$ $\{b\}$ $\{c\}$ $\{d\}$ $\{\ \}$

How many subsets can be formed from a set of five elements? (32) six elements? (64) seven elements? (128)

Analyze your results and see if there is a pattern.

1 element	2 subsets
2 elements	4 subsets
3 elements	8 subsets
4 elements	16 subsets
5 elements	32 subsets

Ask the students to try and develop a general expression for the number of subsets.

The number of subsets of a given set is equal to 2^n where n = the number of elements in the given set.

Suggested Supplementary Readings

Mallory, Curtis. "Intuitive Approach to $X^0 = 1$," The Mathematics Teacher, LX (January, 1967), pp. 41.

Marks, John L., James Smart, and Irene Sauble. Extending Mathematical Ideas - Book Six. Boston: Ginn and Company, 1961. pp. 70-76.

Nichols, Eugene D., and others. Elementary Mathematics Accelerated Sequence - 7. New York: Holt, Rinehart, and Winston, Inc., 1966. pp. 135-154.

MATHEMATICAL SYSTEMS

Objectives

Upon completion of these activities the student should be able:

1. to look upon a mathematical table as a storehouse of data, a reference source.
2. to read, understand, and effectively use a mathematical table.
3. to explain what is meant by a mathematical system.
4. to understand and use the properties associated with mathematical systems, namely: commutative, associative, closure, distributive, identity element, and inverse element.

Suggested Time Allotment

Three class sessions

Vocabulary

Closure, commutative, associative, distributive, operation, identity element, inverse element, modular, mod, mathematical system

Materials

Overhead transparencies of the tables used in the activities

12 hour clock face

5 minute clock face

Rectangular card (file card) for each student

Originating the Problem

Using the overhead projector the teacher shall present the familiar addition and multiplication tables.

+	0	1	2	3	4	...
0	0	1	2	3	4	...
1	1	2	3	4	5	...
2	2	3	4	5	6	...
3	3	4	5	6	7	...
4	4	5	6	7	8	...
⋮	⋮	⋮	⋮	⋮	⋮	

X	0	1	2	3	4	...
0	0	0	0	0	0	...
1	0	1	2	3	4	...
2	0	2	4	6	8	...
3	0	3	6	9	12	...
4	0	4	8	12	16	...
⋮	⋮	⋮	⋮	⋮	⋮	

Can you read and use these tables?

Ask each student to make a list of information which he can obtain from the tables. Discuss the student's answers. Hopefully the students will list:

1. The operations of addition and multiplication.
2. The inverse operations of subtraction and division.
3. Closure property of addition and multiplication.
4. Commutative property of addition and multiplication.
5. Associative property of addition and multiplication.
6. Identity elements.

<u>Instructional Procedure</u>

Activity One

Ask the students to demonstrate their understanding of the use of tables by solving the following problems using the given tables.

1. ◇ + ○ = ______
2. ○ x ⌂ = ______
3. // + △ = △ + ______
4. ⌂ + ______ = | △
5. △ x | =
6. ◇ x ⌂ =

7. △ x ______ = △ 0

8. //0 = 1 0 + ______

9. △ + // + ◇ = ______

+/−	0	1	//	△	◇	⌂	10
0	0	1	//	△	◇	⌂	10
1	1	//	△	◇	⌂	10	11
//	//	△	◇	⌂	10	11	1//
△	△	◇	⌂	10	11	1//	1△
◇	◇	⌂	10	11	1//	1△	1◇
⌂	⌂	10	11	1//	1△	1◇	1⌂
10	10	11	1//	1△	1◇	1⌂	//0

x/÷	0	1	//	△	◇	⌂	10
0	0	0	0	0	0	0	0
1	0	1	//	△	◇	⌂	10
//	0	//	◇	10	1//	1◇	//0
△	0	△	10	1△	//0	//△	△0
◇	0	◇	1//	//0	//◇	△//	◇0
⌂	0	⌂	1◇	//△	△//	◇1	⌂0
10	0	10	//0	△0	◇0	⌂0	100

Activity Two

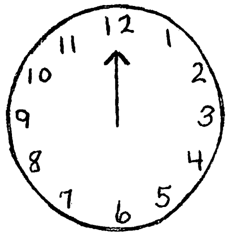
Ask the students to complete the given table and then use it to solve problems.

+/-	2^0	2^1	2^2	2^3	2^4
2^0	2	3	5	9	17
2^1		4			
2^2			8		
2^3				16	
2^4					32

1. $2^2 + 2^3 =$ ________
2. $2^4 + 2^4 =$ ________
3. ________ $+ 2^3 = 24$
4. $2^2 +$ ________ $= 5$
5. $12 - 2^3 =$ ________
6. $20 - 2^2 =$ ________
7. $32 =$ ________ $+ 2^4$
8. ________ $= 2^0 + 2^1$
9. $2^0 + 2^0 + 2^0 =$ ________
10. $16 = 2^3 +$ ________

Activity Three

Show the students a clock face.



If it is 2 o'clock and I work for two hours, at what time will I finish? (4 o'clock) How did you arrive at your answer? (by counting or adding 2 + 2 = 4) What time is it three hours after 6 o'clock? (9 o'clock) How did you arrive at this answer? (6 + 3 = 9) If it is 6 o'clock and I work for six hours, what time will it be when I finish? (12 o'clock, 6 + 6 = 12) What if I start at 8 o'clock and work six hours, what time will it be when I finish? (2 o'clock) Could you say 8 + 6 = 2 is a correct addition fact when dealing with clocks? Continue the discussion as long as necessary and then ask each student to prepare a clock arithmetic table for addition.

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4	5	6	7	8	9	10	11	12	1	2
3	4	5	6	7	8	9	10	11	12	1	2	3
4	5	6	7	8	9	10	11	12	1	2	3	4
5	6	7	8	9	10	11	12	1	2	3	4	5
6	7	8	9	10	11	12	1	2	3	4	5	6
7	8	9	10	11	12	1	2	3	4	5	6	7
8	9	10	11	12	1	2	3	4	5	6	7	8
9	10	11	12	1	2	3	4	5	6	7	8	9
10	11	12	1	2	3	4	5	6	7	8	9	10
11	12	1	2	3	4	5	6	7	8	9	10	11
12	1	2	3	4	5	6	7	8	9	10	11	12

Encourage the students to seek out patterns. They will discover patterns such as:

1. The numbers across each row and down each column are in sequence up to 12.
2. Reading along each diagonal, the numbers increase by twos.
3. No column or row has a sum repeated.
4. No zero appears in the table.
5. There are no addends or sums greater than 12.

Discuss with the students the use of 0 in the place of 12. Ask the students to replace the 12's in their table with 0's and check the validity of this substitution.

We have been working with a set of elements, the numerals 0 - 11, and the binary operation of addition. Mathematicians would say we are working with a mathematical system.

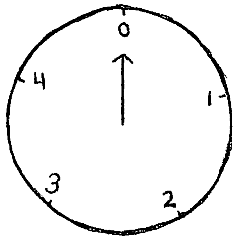
Definition for the teacher: A mathematical system is a set of elements together with one or more binary operations defined on the set.

It is interesting to find out how an operation behaves. Is it commutative? associative? Is there an identity element? Ask the students to examine their clock arithmetic addition table and discuss the properties which apply. (Associative, commutative, identity element is 0, inverse elements, closure) Provide practice problems if they are needed by the students.

The mathematical system we have been working with and referring to as clock arithmetic is a modular system. Since the system consists of only 12 elements, we say the system has a modulus of 12. The system may be referred to as modular 12, modulo 12, or mod 12. Accordingly, a modulo 4 system would contain how many elements? (four $\{0, 1, 2, 3\}$) a molulo 5 system? (five $\{0, 1, 2, 3, 4\}$) a molulo 2 system? (two $\{0, 1\}$).

Activity Four

Let us extend our knowledge to a five minute clock.



We shall call the five-number system a mod 5 system. Ask the students to work out the following exercises in mod 5 and discuss the results.

Exercises - Part One

1. Complete the mod 5 addition table.

+	0	1	2	3	4
0				3	
1					0
2		3			
3				1	
4	4				

2. What is the additive identity in mod 5? (0)

3. Fill in the frames in each of the following and answer each question.

a. 0 + /__/ = 0 (0)

What is the additive inverse of 0? (0)

b. 1 + /__/ = 0 (4)

What is the additive inverse of 1? (4)

c. 2 + /__/ = 0 (3)

What is the additive inverse of 2? (3)

d. 3 + /__/ = 0 (2)

What is the additive inverse of 3? (2)

e. 4 + /__/ = 0 (1)

What is the additive inverse of 4? (1)

4. Does addition in the mod 5 system have the commutative property? (yes)

5. Does addition in the mod 5 system have the associative property? (yes)

6. Use the table to find answers to the following:

a. 3 - 1 (2)
b. 4 - 2 (2)
c. 0 - 4 (1)
d. 1 - 3 (3)
e. 1 - 4 (2)
f. 3 - 2 (1)
g. 2 - 3 (4)
h. 0 - 1 (4)
i. 3 - 4 (4)
j. 0 - 2 (3)

Do you think it is possible to multiply in our mod 5 system? (yes) Why? (Multiplication is repeated addition.) What does 4 x 2 mean? (Four 2's)

2 + 2 + 2 + 2 = ? (2 + 2) + 2 + 2 = (4 + 2) + 2 = 1 + 2 = 3 or 4 x 2 = 3 Work out a few more examples with the students and then present the following exercises.

Exercises - Part Two

1. Complete the following multiplication table for mod 5.

X	0	1	2	3	4
0			0		
1					4
2				1	
3	0				
4			3		

2. For every number n in mod 5, n • 0 = /__/ (0)

3. Using the table fill in the frames and answer each question.

   a. 1 x /__/ = 1 (1)

      What is the multiplicative inverse of 1? (1)

   b. 2 x /__/ = 1 (3)

      What is the multiplicative inverse of 2? (3)

   c. 3 x /__/ = 1 (2)

      What is the multiplicative inverse of 3? (2)

   d. 4 x /__/ = 1 (4)

      What is the multiplicative inverse of 4? (4)

4. Does multiplication in mod 5 have the cummutative property? (yes)
5. Does multiplication in mod 5 have the associative property? (yes)
6. Compute the answers to each pair of exercises in a and b.
   a. 3 x (0 + 1) ; (3 x 0) + (3 x 1) (3;3)
   b. 4 x (4 + 0) ; (4 x 4) + (4 x 0) (1;1)
   c. Is multiplication distributive over addition in mod 5? (yes)
7. What is the multiplicative identity element in mod 5? (1)
8. Fill in the frames.
   a. 2 x (3 - 1) = /__/ (4)
   b. (3 x 4) x 4 = /__/ (3)

The elements in a mathematical system do not have to be numbers. They may be any objects whatever. There are other operations besides the four we use every day in mathematics class. Can anyone give an example? (Raising to a power, rotating through a number of degrees, moves in a checker game.)

Exercises - Part Three

Suppose you are given a system consisting of the elements a, b, c, d, and e and two operations * and #. The operations are defined by the tables below.

*	a	b	c	d	e
a	a	b	c	d	e
b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	c
e	e	a	b	c	d

#	a	b	c	d	e
a	a	a	a	a	a
b	a	b	c	d	e
c	a	c	e	b	d
d	a	d	b	e	c
e	a	e	d	c	b

1. Is there an identity element for * in this system? (yes, a)
2. What is the identity element for # in this system? (b)
3. Are the operations * and # commutative? (yes)
4. Are the operations * and # associative? (yes)

5. Does the system have closure under * ? (yes)

   Under # ? (yes)

6. Do you think # is distributive over * ? (yes)

   $c \# (b * e) \stackrel{?}{=} (c \# b) * (c \# e)$

   $c \# (a) \stackrel{?}{=} (c) * (d)$

   $a = a$

7. Do you think * is distributive over # ? (no)

   $d * (c \# e) \stackrel{?}{=} (d * c) \# (d * e)$

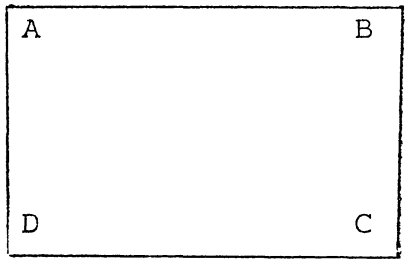
   $d * (d) \stackrel{?}{=} (a) \# (c)$

   $b \neq a$

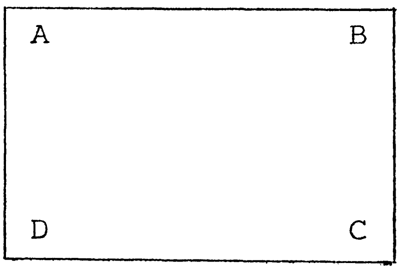
Activity Five

In our last activity we were working with a mathematical system without numbers. Suppose we want to invent a system of our own. What do we need? (A set of elements and some kind of a binary operation.) It would be nice if our system could have some of the properties we have been studying; closure, commutativity, etc. Let's see what we can develop.

Provide each student with a rectangular shaped card. Ask the students to label the corners of the card as shown. Label both sides of the card being sure the two letters "A" are back-to-back so they are labels for the same corner of the card.

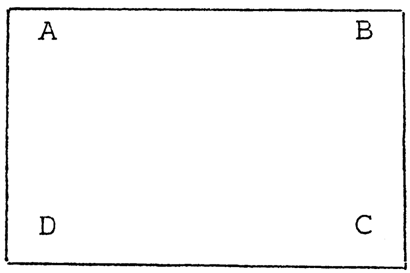


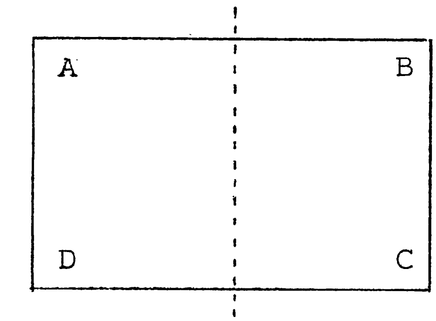
We now need a set of elements. Let us take elements which have something to do with the card. Place your card in front of you on your desk with the long side parallel to the front of your desk. Now move the card, pick it up, turn it over, turn it in any way just so its final position has the long sides parallel to the front of your desk. The card will look just the same as it did before except the corners may be labeled differently. We will take these changes of position as our elements. How many of these changes can you discover? After experimenting, the students will come up with the following changes:



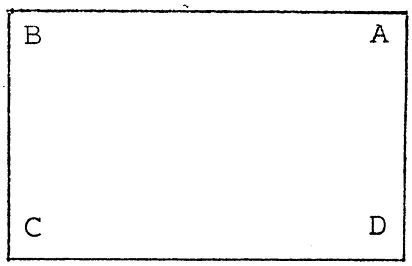
Leave the card in place

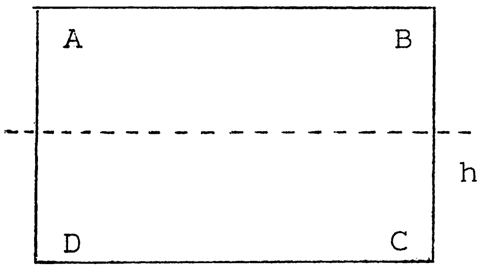
Element I



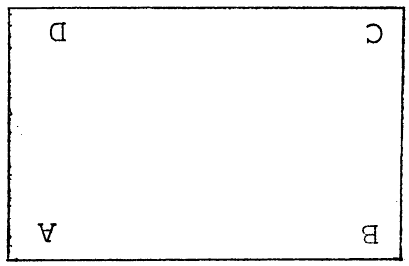


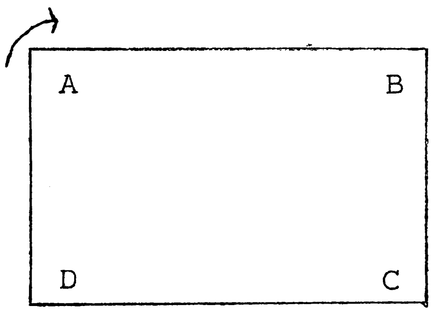
Flip the card over the vertical axis. Element V



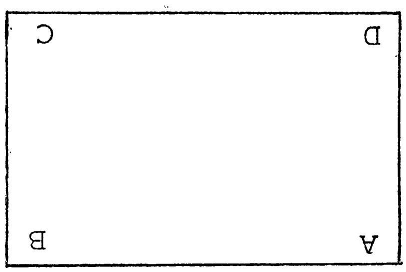


Flip the card over the horizontal axis. Element H



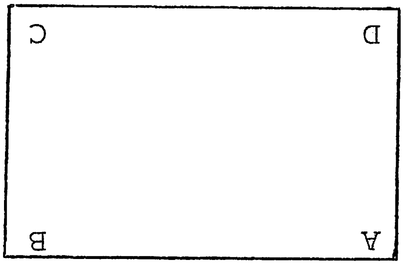


Rotate the card halfway around. Element R



What is our set of elements? ({I, V, H, R}) We now need an operation. Here is a way of combining any two elements of our set. We will do one of our changes followed by another change of position. We can use the symbol " (FB) " for this operation. Thus H (FB) V means a flip of the card over the horizontal axis followed by a flip of the card over the vertical axis. Start with the card in standard position and do these changes to it.

What is the final position of the card?



The result of these two changes is the same as the result of what single change? (R) Thus we can say H (FB) V = R. Using your card, and actually making the moves, complete the table for the operation (FB).

Second Change

First Change	FB	I	V	H	R
	I				
	V				
	H		R		
	R				

Ask the students to work out the following exercises using their tables.

1. Fill in each frame.

   a. R (FB) H = /__/ (V)

   b. R (FB) /__/ = H (V)

   c. /__/ (FB) R = H (V)

   d. /__/ (FB) H = R (V)

e. (R (FB) H) (FB) V = /‾‾/ (I)

f. (/‾‾/ (FB) H) (FB) V = R (I)

2. Is the set {I, V, H, R} closed for the operation (FB)? (yes)

3. Is the operation commutative? (yes)

4. Is there an identity element for the operation? (yes - I)

5. Does each element of the set have an inverse under the operation (FB)? (yes) List them.

Element	Inverse
I	I
V	V
H	H
R	R

Activity Six (Optional)

The students may develop other mathematical systems by changing the position of various geometric shapes. Some suggestions are:

1. A system of 2 elements using a 0° rotation and a 180° rotation of a line.

2. A system of 3 elements using a 0° rotation, a 120° rotation, and a 240° rotation of an equilateral triangle.

3. A system of 4 elements using a 0° rotation, a 90° rotation, a 180° rotation, and a 270° rotation of a square.
4. A system of 5 elements by rotating a pentagon.
5. A system of 6 elements by rotating a hexagon or by rotating an equilateral triangle and flipping it over its three axes.

Suggested Supplementary Reading

Fehr, Howard F. and Jo McKeeby Phillips. Teaching Modern Mathematics in the Elementary School. Reading: Addison - Wesley Publishing Company, 1967. pp. 396-402.

National Council of Teachers of Mathematics. Enrichment Mathematics for the Grades. Twenty-seventh Yearbook. Washington, D.C.: The National Council of Teachers of Mathematics, 1963. pp. 73-91 and 273-281.

Nichols, Eugene D., and others. Elementary Mathematics Accelerated Sequence - 7. New York: Holt, Rinehart, and Winston, Inc., 1966. pp. 45-76.

Norton, M. Scott. Finite Mathematical Systems. St. Louis: Webster Division, McGraw-Hill Book Company, 1963. 64 pp.

GRAPHING

Objectives

Upon completion of these activities the student should be able:

1. to describe the location of a point on a line.
2. to graph the solution set of a simple number sentence.
3. to describe the location of a point in a plane by the use of a rectangular coordinate system.
4. to define a function and realize that a function may be expressed as a set of ordered pairs, as an equation, or pictured by a graph.
5. to use a given rule, English sentence or mathematical sentence, to make a table and graph the results.

Suggested Time Allotment

Three class sessions

Vocabulary

Horizontal axis, vertical axis, axes, origin, ordered pair, coordinates, function, graph

Materials

Turtle (plastic toy or felt cut-out)

Demonstration number line

Blackboard compass

Large sheet of graph paper (1 inch squares) for teacher demonstration in activity five

Overhead transparency of number line

Overhead transparency of graph paper

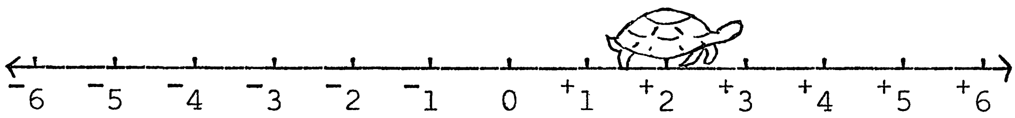
Graph paper for each student

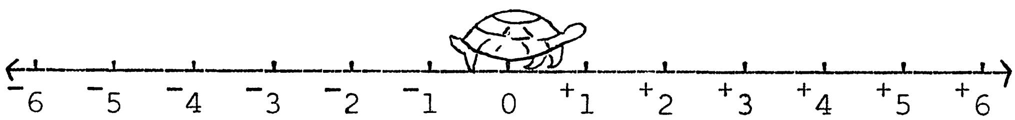
Ruler for each student

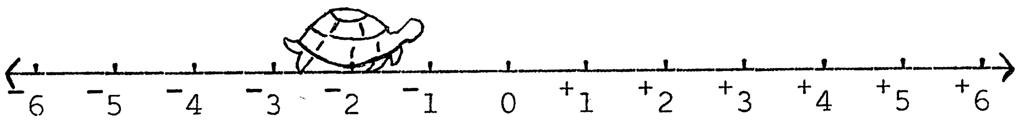
<u>Originating the Problem</u>

A turtle is sitting on a line as shown below. How can you describe its location using arithmetic?

The students will realize that they must establish a point of reference (the origin) and then divide the line into units of equal length. Various solutions will be offered by the students such as:







We have described the location of the turtle by making use of a number line. A number that tells both the distance and direction of a point on a line from the 0-point (origin) is called the coordinate of the point.

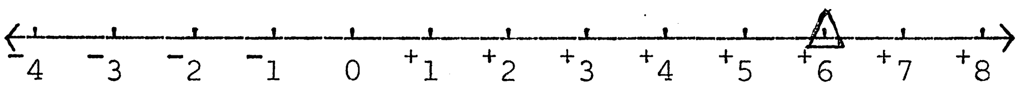
<u>Instructional Procedure</u>

Activity One

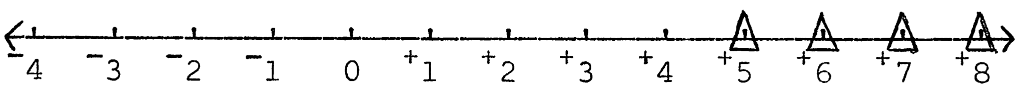
Sometimes mathematicians picture their solutions to equations on a number line. This is called the graph of the solution set.

Ask the students to graph the solution set of each of the following, if we assume our universe is the set of integers. (positive and negative whole numbers and zero) Mark the points with a $\triangle$.

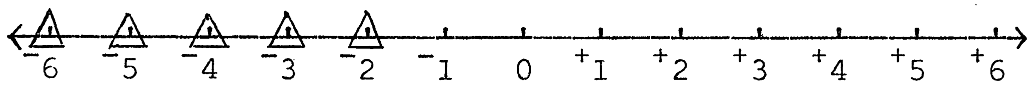
1. $x = 6$



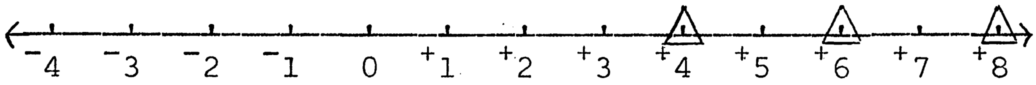
2. $x > 4$



3. $x < {}^{-}1$

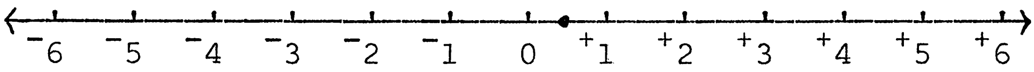


4. $x > 3$ and x is an even number.

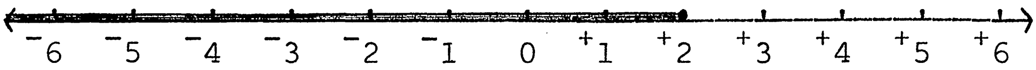


Let us now assume that our universal set is the set of real numbers. (all rational and irrational numbers) Mark the points with a •.

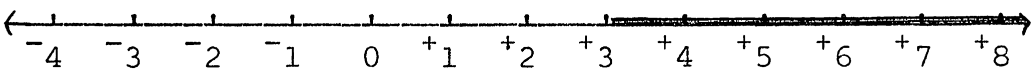
I. $x = 1/2$



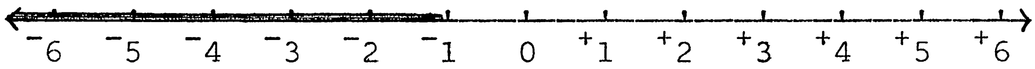
2. $x \leq 2$



3. $x > 3$

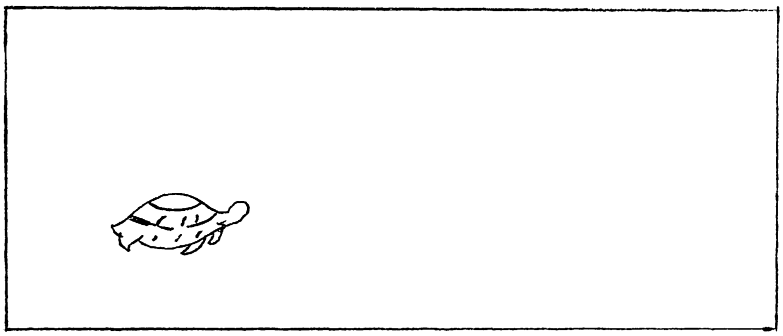


4. $x < {}^{-}1$



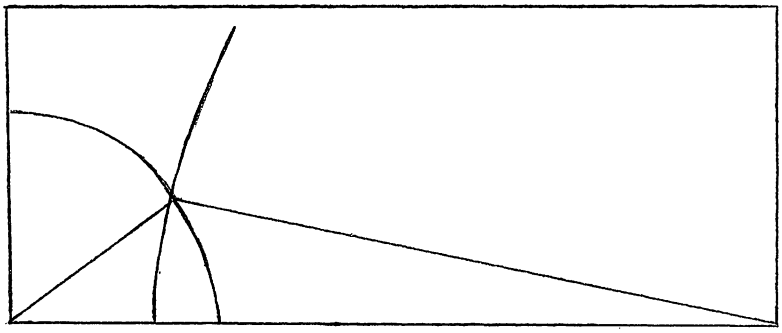
Activity Two

Now suppose our turtle is located on a spot on this table where there are no lines.

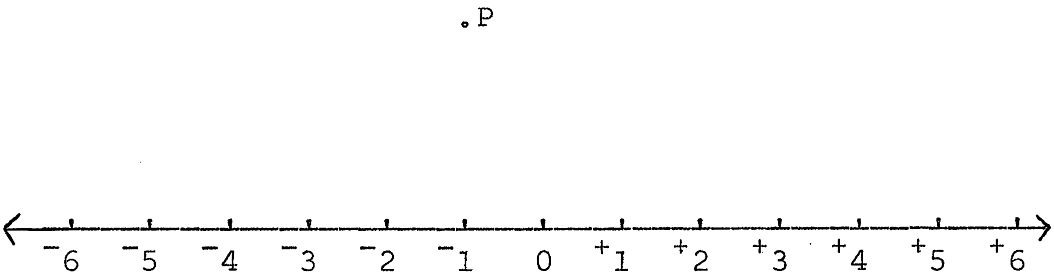


How can its location now be pin-pointed? Give the students ample opportunity to discover and discuss ways of giving the turtle's location. They may suggest the following:

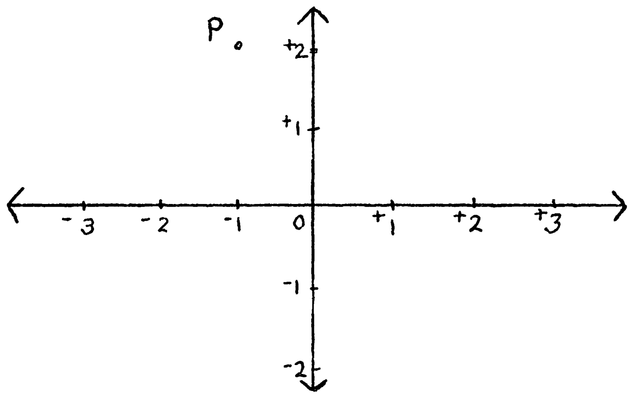
1. The turtle is a given number of inches from the lower left hand corner and a given number of inches from the lower right-hand corner. This method is one of triangulation. The point of intersection of the circles drawn from each corner would mark the location of the point.



2. The turtle is a given number of inches from the left-hand edge and a given number of inches from the lower edge. This is basically the use of rectangular coordinates.



Since P is not on the number line, we cannot state its position by naming its coordinate. It appears to be directly above ($^{-}1$) ? We need a way to tell how far above the line it is. We can do this by adding a second number line which is perpendicular to the first and has the same zero point.



We can now describe the position of P by using the two numbers, $^-1$ and $^+2$. The numbers $^-1$ and $^+2$ are both co-ordinates of the point P. The first number tells how far right or left of the origin the point is and the second number tells how far up or down it is. The order in which the numbers are named is important, so ($^-1$, $^+2$) is called an ordered pair.

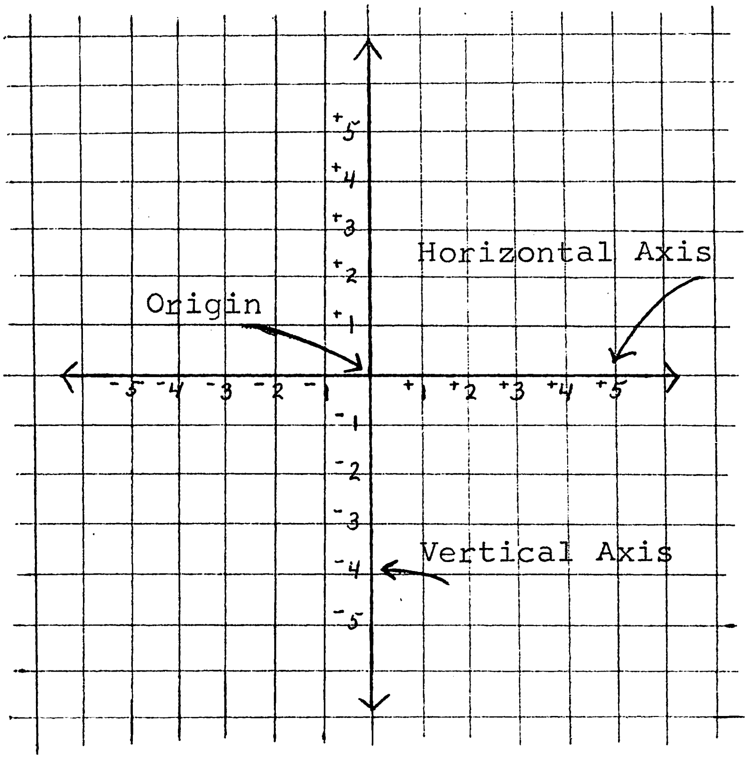
To assist in describing the location of points it is customary to use graph paper. Using the overhead projector, show the students a sheet of graph paper and draw in a horizontal and vertical number line intersecting at zero. Let us now locate some points.

To locate point A ($^-3$, $^-1$), start at (0, 0) and count (three) units to the (left) and then (one) unit (down).

To locate point B ($^-1$, $^+3$) start at (0, 0) and count (one) unit to the (left) and then (three) units (up).

To locate point C ($^{+}2$, $^{-}1$), start at (0, 0) and count (two) units to the (right) and then (one) unit down.

When number lines are used in this way, we call each number line an axis. The horizontal number line is called the x-axis and the vertical number line is called the y-axis. ("axes" is the plural of axis) The point of intersection of the x axis and the y axis is called the origin.

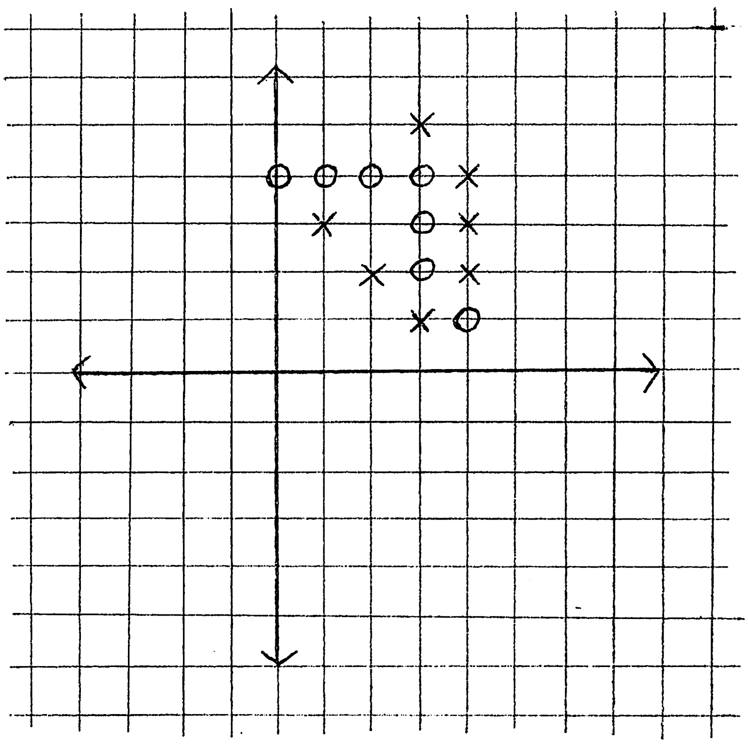


Activity Three

To provide the opportunity for practice in locating coordinates, allow the students to play some of the following games:

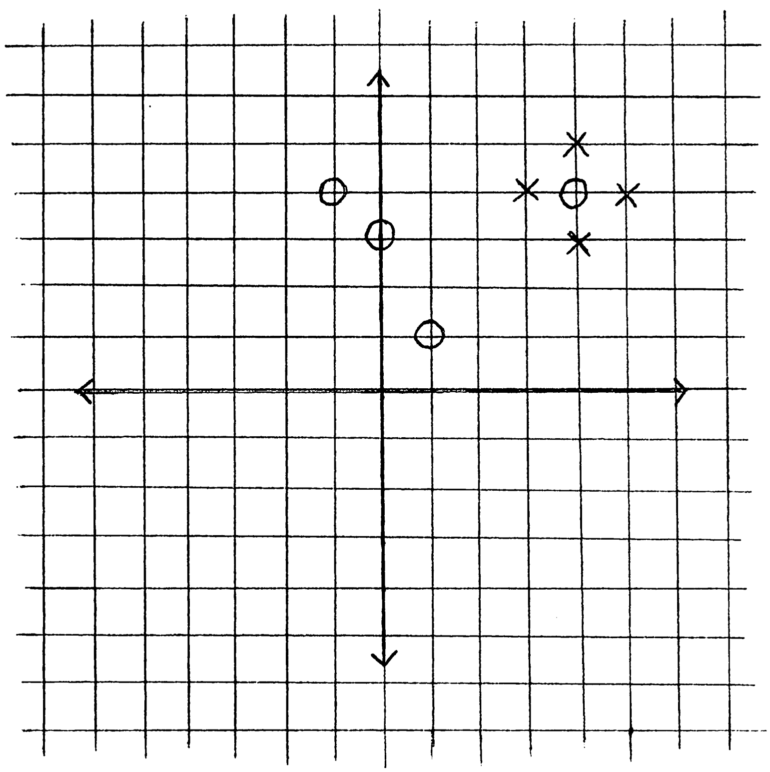
Tic-Tac-Toe

Divide the students into two teams. The teams take turns giving pairs of numbers which are the coordinates of points on the graph. The teacher or a student records the points given by team A using an "O" and by team B using an "X". The goal of the game is to get four consecutive points on any vertical or horizontal line. In the example below team A won since they got four consecutive "O's" in the fourth horizontal line.



Go

Divide the students into two teams. When the teacher says go the teams take turns giving pairs of numbers which are the coordinates of points on the graph. The teacher or a student plots the points given by team A using an "O" and by team B using an "X". The object of the game is to surround the enemies' soldiers and thus eliminate them. For example, when an "O" is surrounded by four "X's" as shown below, the "O" is taken off the board. Any number of soldiers can be completely encircled and thus eliminated.



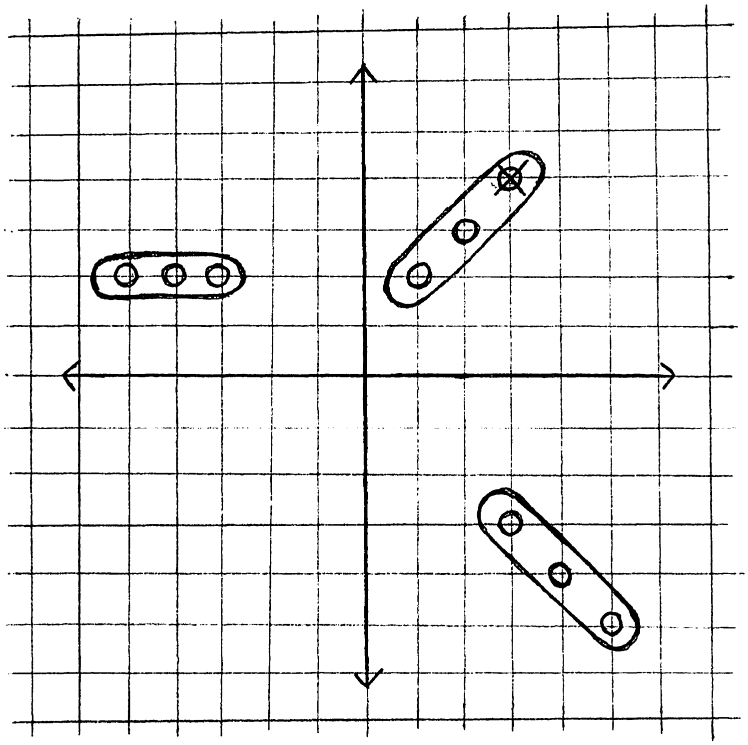
The team which has the most soldiers on the board at the end of the specified time wins the game.

The Naval Battle Game

Each student chooses a partner. Each player has a piece of graph paper. (to start with it might be easier to use a 10 by 10 grid) Each player secretly marks the location of his three ships on his graph paper by covering three points in a line with "O's". To start the game, the first player calls out a number pair. For example, (3, 4). If his number pair names a point on the second player's ship, the second player must say, "Hit!" and record it as ⊠. If the number pair does not name a point on his ship, the second player says, "Miss!" If the first player gets a hit he gets another turn. If he misses it is the second player's turn. To sink a ship, all three points of the ship must be hit. When a ship is sunk, the owner must announce the sinking.

Sometimes players may discover that they have placed ships on the same coordinates. If a player hits his own ship, he must announce the fact and so mark it. At times a player must decide if he wants to fire upon himself in order to sink his enemy.

The winner of the game is the player to first sink his opponent's ships.



⊠ = a hit at (3, 4)

Activity Four

Give each student a piece of graph paper and ask him to plot the following coordinates. If the points are now joined in order by straight line segments a familiar figure will be formed. (A five pointed star)

A. (5, 0) F. ($^-$2, 0)

B. (1 3/4, 1) G. ($^-$4, $^-$3)

C. (1 3/4, 4 1/2)

D. ($^{-}$1/2, 2)

E. ($^{-}$4, 3)

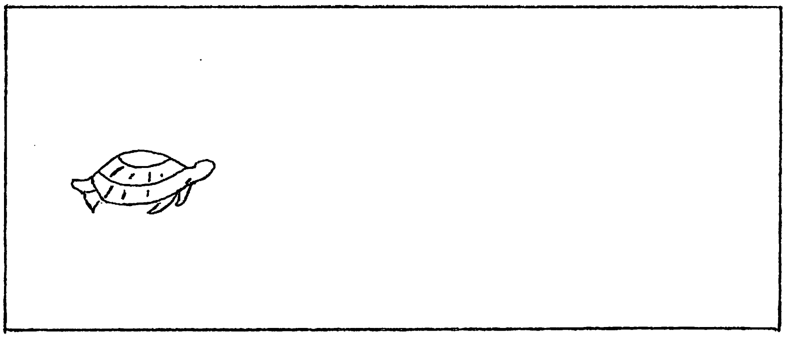
H. ($^{-}$1/2, $^{-}$2)

I. (1 1/2, $^{-}$4 1/2)

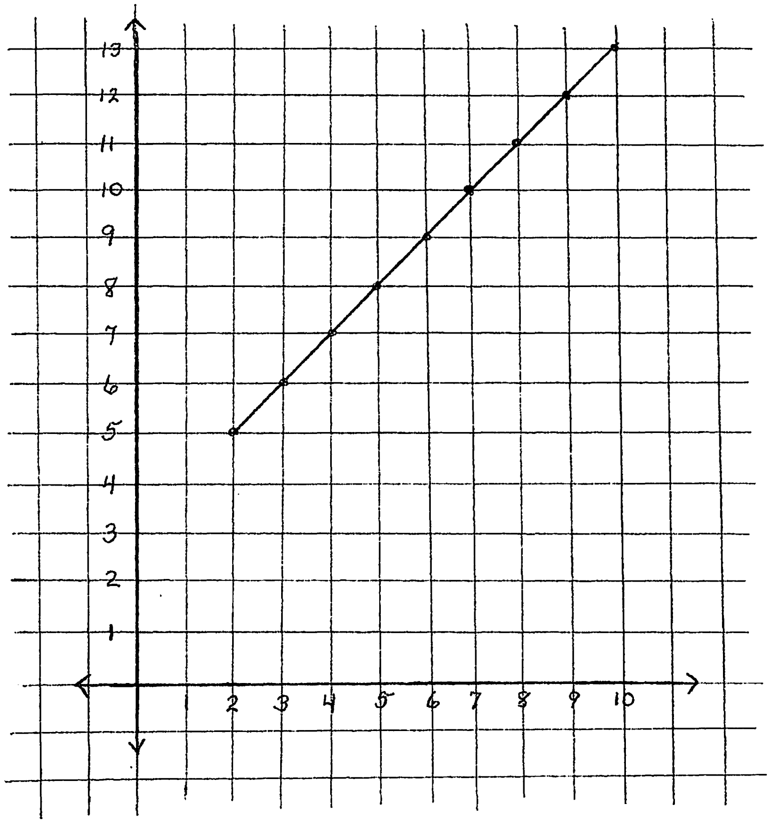
J. (1 1/2, $^{-}$1)

Activity Five

Suppose our turtle decided to take a little walk on the table top.



Do you think we could mathematically describe his walk? The teacher shall place on the table a large sheet of graph paper. As the teacher moves the turtle forward in a straight line, have a student draw his path on the graph paper.



Ask the students to list the coordinates through which the line passes. (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (8,11), (9,12), (10,13). Instead of listing the points as ordered pairs we can write the coordinates in a table.

x	2	3	4	5	6	7	8	9	10
y	5	6	7	8	9	10	11	12	13

Row x gives the horizontal coordinates.

Row y gives the vertical coordinates.

Ask the students to study the table. What was done to each value of x to get the value of y named below? (3 was added)

Move the turtle along another straight line path and ask the students to make a table of the points passed through.

A set of ordered pairs such as we have been working with, where the first number in each pair can be matched with one and only one second number, is called a function. Sometimes a rule can be written to show the function. Rules may be written in English sentences or mathematical sentences. The rule for our first example was; add 3 to each number or $y = x + 3$.

Ask the students to write the following English sentences as mathematical sentences.

1. Multiply each number by 3. ($y = 3 \times n$)
2. Subtract 4 from each number. ($y = x - 4$)
3. Multiply each number by 2 and add 1. ($y = 2x + 1$)

Provide each student with a ruler and graph paper. Ask the students to make a table for each of the given rules and then graph each rule. Suggest that the students choose six numbers between $^{-}6$ and $^{+}6$ for the horizontal coordinates.

$y = x + 2$	$y = x - 1$
$y = 2x$	$y = 2x + 1$

Activity Six (Optional)

Ask the students to graph each of the following rules on the same grid using the same coordinate axes.

$y = x + 3$	$y = x - 3$
$y = x + 1$	$y = x$

Ask the students to describe their results. (A set of parallel lines)

Ask the students to graph these rules on the same grid using the same coordinate axes. (A different grid than the one used in the previous problem will be easier for the students to work with.)

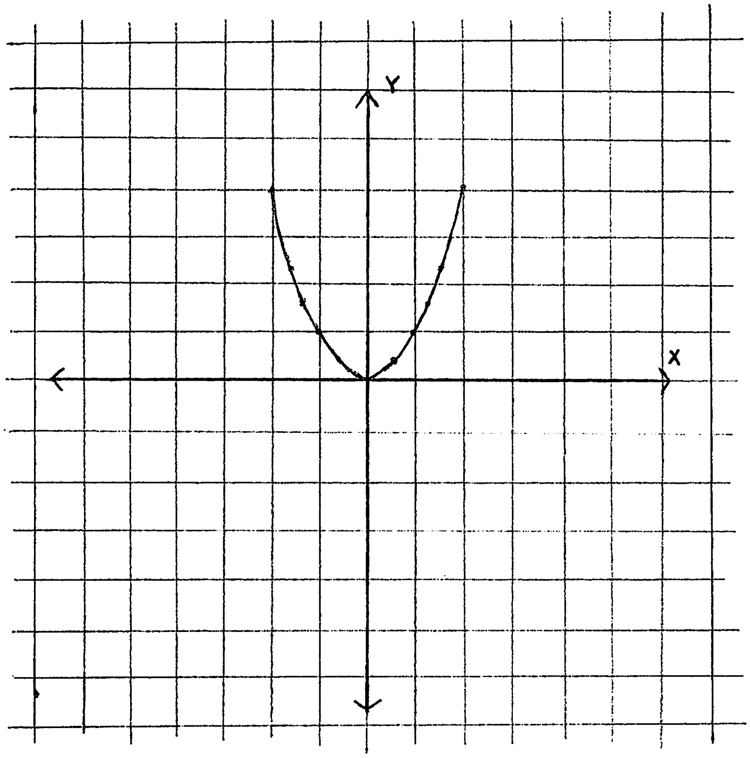
$y = x$

$y = 2x$

$y = 3x$

Again, ask the students to describe their results. (A set of lines all passing through the origin.)

Graph the function or rule $y = x^2$. Try to draw a line segment to connect the points for the function. Can you always draw a line segment that includes all the points for a function? (no) The figure you obtained is called a parabola.



Suggested Supplementary Readings

Anderson, Rosemary C. "Let's Consider The Function," The Arithmetic Teacher, XIV-4 (April, 1967), pp. 280-285.

Fehr, Howard F., and Jo McKeeby Phillips. Teaching Modern Mathematics in the Elementary School. Reading, Massachusetts: Addison-Wesley Publishing Company, 1967. pp. 369-372.

Glenn, William H., and Donovan A. Johnson. Adventures in Graphing. St. Louis: Webster Publishing Company, 1961. 64 pp.

Gold, Sheldon. "Graphing Linear Equations--A Discovery Lesson," The Arithmetic Teacher, XIII-5 (May, 1966), pp. 406-407.

Hajek, Roy D. "New Learning and Subverbal Knowledge," The Mathematics Teacher, LX-5 (May, 1967), pp. 444-447.

May, Lola J. Elementary Mathematics - Enrichment 5. New York: Harcourt, Brace, and World, Inc., 1966. pp. 42-44.

Minnesota Mathematics and Science Teaching Project. Unit XIX - Multiplication in Squareville. University of Minnesota, 1966. 172 pp.

Minnesota Mathematics and Science Teaching Project. Unit XVI - Squareville. University of Minnesota, 1965. 95 pp.

MATHEMATICS-THE INDISPENSABLE TOOL OF ALL THE SCIENCES

EARTH SCIENCE

Objectives

Upon completion of these activities the student should be able:

1. to cite an example of how mathematics is used in the field of earth science.
2. to use a scale drawing or model to represent large distances.
3. to explain why it would be necessary, in making a scale drawing of the solar system, to use one scale for the distances and another for the sizes of the planets.

Suggested Time Allotment

One class session

Vocabulary

Diameter, scale drawing

Materials

Modeling clay

Ruler

Originating the Problem

Look out the window at the school garage. Now close your eyes and imagine the same scene. Did you have any trouble doing this? Now try to visualize the distance from Hibbing to Chisholm. Was this more difficult? Why? Can you now visualize the distance from the earth to a planet? Not really, because neither you nor anyone else has yet made the trip. Today you may learn to appreciate some of the problems that face the astronomer and the astronaut as we study the solar system.

Activity One

Distances and Sizes of the Planets

Planets	Average Distance from Sun (miles)	Scale Distance (inches)	Diameter of Planet (miles)	Scale Diameter (inches)
Mercury	36,000,000		2,900	
Venus	67,000,000		7,600	
Earth	93,000,000		7,900	
Mars	142,000,000		4,200	
Jupiter	483,000,000		86,800	
Saturn	886,000,000		71,500	
Uranus	1,783,000,000		29,400	
Neptune	2,794,000,000		28,000	
Pluto	3,670,000,000		3,600 (?)	

Have the class make a model of the solar system using the information given. Begin by discussing the selection of a convenient scale. (1 inch = 20,000,000 miles would be suitable for the classroom. On this scale the earth would be a little less than 5 inches from the sun, and Pluto about 15 feet from the sun.) Discuss the possibility of using the same scale for the planets and lead the students to realize the planets would be too small to be seen. Select a new scale for the planets. (1 inch = 16,000 miles would be suitable.) Do not attempt to make the sun to scale.

After agreeing upon the scales, have each student compute the scale distances and diameters. The planets may now be made from modeling clay and the scale model of the solar system set up.

After everything is in position, discuss the sizes and distances. Is the solar system crowded? The earth is very important to us, but does it stand out as something special in the solar system?

PHYSICAL SCIENCE

Objectives

Upon completion of these activities the student should be able:

1. to cite examples showing the close relationship between mathematics and the physical sciences.
2. to state why measurements are made more than once in gathering experimental data and why an average of these measurements is used.
3. to systematically collect data from an experiment and analyze the results.
4. to state a scientific principle in mathematical terms.

Suggested Time Allotment

One class session

Vocabulary

Data, average, directly proportional, force, acceleration

Materials

Roller skate or small cart

Rubber bands

Paper clips

String

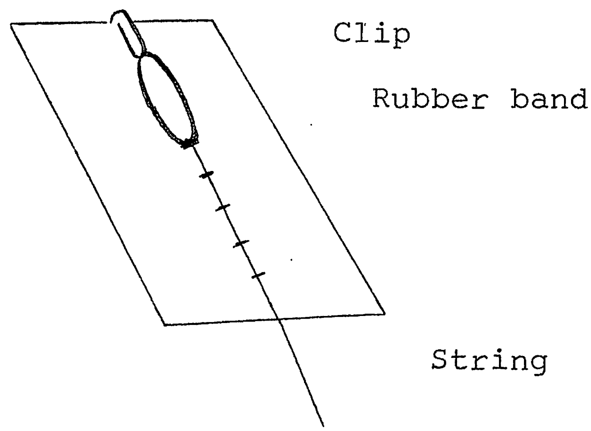
Cardboard

Chalk for marking the floor

Watch with easily read second hand

Originating the Problem

Make a rubber band scale to measure force. At the edge of a piece of cardboard fasten a paper clip. Attach a rubber band to the clip. Attach a five-foot piece of string to the other end of the rubber band. Allowing your scale to hang loosely, mark the cardboard at the bottom of the rubber band. This will be your starting point. Add more marks each 1/2 inch apart.



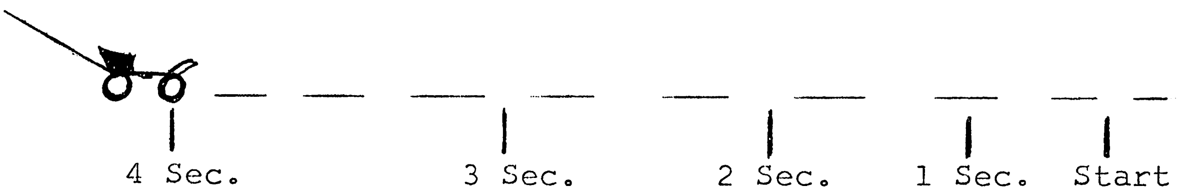
Tie the string to the skate. Ask a student to pull the skate, with the rubber band at the first mark, down the hallway. What happens to the speed of the skate? (gains speed) Students will have difficulty keeping the force constant with the rubber band scale. Try the activity many times with different students and select the best ones to do the pulling in the following activities.

Instructional Procedure

Activity One

In the last activity you applied a constant force to the skate and it continued to gain speed. What is this gain in speed called? (acceleration) Let's see if we can measure this acceleration or change of speed.

Place a marker on the floor next to the front wheels of the roller skate to show the starting point. Assign one student to be the timer and another student to be the marker. Keeping the force scale at the first mark, pull the roller skate down the hall. The timer will call out the seconds and the marker will mark the position of the front wheels at that instant.



Do this activity several times to find the average position after each second of the skate's motion.

Marking the skate's position as it passes is very difficult. The results may be better if one student marks the position at the end of the first second, and a different student is ready to mark each of the other positions. Begin with a few trial runs. The timekeeper should give a short count down to make sure the student pulling begins at the right time.

Using the distance from start to the position at the end of the first second as one unit of length measure the other distances and fill in the following table. Round off all fractional parts of the unit length to the nearest whole number of units.

Time from Start (Seconds)	Distance Traveled from Start (Units)	Speed (Units/sec.)	Acceleration (Units/sec. each sec.)
0	0		
1			
2			
3			
4			

What do you notice about the speed of the skate from second to second? (increases) What did you notice about the acceleration of the skate? (A constant force results in a constant acceleration.)

Activity Two

What do you think will happen to the speed and the rate of acceleration if we increase the force pulling the skate? List the suggestions given by the various students.

Perform the experiment again using more force, stretch the rubber band two units this time. Make your measurements and compile a table as you did in the previous activity.

Compare your data with that obtained in the first experiment. How do the distances, speeds, and acceleration compare? Can you make a statement which will explain how force affects acceleration? (You have just stated one of Newton's laws that the acceleration of an object depends directly on the force exerted on it.)

Does anyone know how to express this statement mathematically? The acceleration of an object is directly proportional to the force exerted on the object.

$$a \propto F$$

BIOLOGY

Objectives

Upon completion of these activities the student should be able:

1. to understand the difficulty of measurement of biological species because of their great variability.
2. to construct a frequency distribution table and graph.
3. to state the median, mean, mode, and the range of a set of data.

Suggested Time Allotment

One class session

Vocabulary

Median, mode, mean, range, frequency distribution, variation

Materials

Whole green beans or fresh pea pods

Graph paper

Originating the Problem

How tall is a sixth grade boy? How much does a dog weigh? How many seeds are there in this bean? Why can't you answer these questions with a single numerical value? (There is variation among individuals.) How would you attempt to answer the question about the height of a sixth grade boy? The students will provide many answers.

Instructional Procedure

Activity One

Divide the class into groups of four or five students. Provide each group with a supply of beans and ask them to make a frequency distribution table of the number of seeds per pod. The resulting table may look somewhat like this:

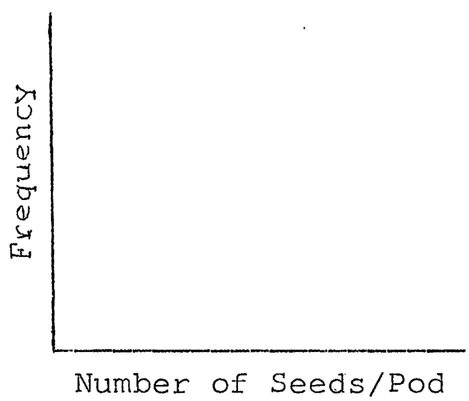
Seeds/Pod	Tally Marks	Frequency
0		
1		
2		
3	~~////~~	5
4	///	3
5	~~////~~ ~~////~~ /	11
6	~~////~~ ///	8
7	~~////~~	5
8	/	1
9		

A discussion should now be held as to the methods of communicating your information to others. The students will probably give some of the following suggestions:

1. The fewest seeds in any pod and the most seeds in any pod. (range)
2. The average number of seeds per pod. (median or mean)
3. The most frequent number of seeds in a pod. (mode)
4. Present the data table.
5. Show the data on a graph.

Activity Two

Ask each student to construct a frequency distribution graph of the data collected in activity one. The controlled variable is, by convention, plotted along the horizontal axis. The graph may be either a bar or line graph.



Suggested Supplementary Readings

American Association for the Advancement of Science. Science-A Process Approach. AAAS Miscellaneous Publication, 1965. Books One, Two, Three, Four, Five, and Six.

Elementary School Science Project. Astronomy. Urbana: University of Illinois, 1963. Series of six books.

National Council of Teachers of Mathematics, Enrichment Mathematics for the Grades. Twenty-seventh Yearbook. Washington, D.C.: The National Council of Teachers of Mathematics, 1963. pp. 165-172.

MATHEMATICS AND THE SCIENCE FAIR

Objectives

Upon completion of these activities the student should be able:

1. to exhibit at the Hibbing Science Fair a complete and accurate mathematics project, to explain the project, and to answer questions about it.
2. to locate mathematics materials in a library.
3. to carry on independent research into a mathematical topic that interests him and report on his findings.

Suggested Time Allotment

Four class sessions and independent work outside of class

Vocabulary

Each project will have its own vocabulary

Materials

All available mathematics reference materials

Originating the Problem

Have you ever entered a project in the Hibbing Science Fair? Allow time for the students to tell about their projects and their experiences at the fair. Discuss the plans for this year's fair, values of participating, areas of exhibit, and the criteria for judging.

Information for the Teacher

The Hibbing Science Fair is held each year for the students of the public and parochial schools in Hibbing, grades four through twelve. The High School Science Club sponsors the event. It is held on a Friday evening and the following Saturday. Students set up their exhibits Friday after school and the exhibits are open for public viewing Friday evening. Judging is held Saturday morning and a student must be present to explain and answer questions about his project. Saturday afternoon an awards ceremony is held. In addition to the awards listed below, every exhibitor receives a participation certificate. At the conclusion of the ceremony, each student is responsible for taking down his own exhibit.

Science Fair Awards

Grand Award - $100.00 for best exhibit at the fair.

I. Senior High

Medals 1st and 2nd in this division

	1st place	2nd place
1) Chemistry	(blue ribbon)	(red ribbon)
2) Biology	"	"

3) Mathematics	"	"
4) Physics	"	"
5) Miscellaneous	"	"

II. Junior High

Medals 1st and 2nd in this division

	1st place (blue ribbon)	2nd place (red ribbon)
1) Chemistry		
2) Biology	"	"
3) Mathematics	"	"
4) Physics	"	"
5) Miscellaneous	"	"

III. Grades 4-6

Medals 1st and 2nd in this division

	1st place (blue ribbon)	2nd place (red ribbon)
1) Chemistry		
2) Biology	"	"
3) Mathematics	"	"
4) Physics	"	"
5) Miscellaneous	"	"

Judging Criteria

Clarity and Dramatic Value . . . . 20 points

The exhibit should be attractive and command attention of visitors, whether laymen or scientists. Is the exhibit more attractive than others in the same field beyond the point of making loud noises and "gadgetry"?

Remember, this is not an art contest. Projects which are masterpieces of art should not win on this basis alone. Workmanship will be evaluated as well as the skill shown in handling, preparation and mounting of the material in the exhibit.

Creative Ability 30 points

Does the exhibit show originality in plan and execution? Consider ingenious uses of material. Think of collections as being creative if they serve a scientific purpose for the exhibitor's grade level.

Scientific Thought and Thoroughness 50 points

The exhibit should indicate evidence of the application of the scientific method. Has a planned system been followed? Does the exhibit represent real study and effort? The exhibit should tell a clear, full but concise story about the project, with proper emphasis on the important items. Judging will be based on the completeness and accuracy with which the exhibit is presented.

Instructional Procedure

Activity One

Ask each student to select a topic for research and development into a mathematics project. Students may work individually or in small groups. The teacher should help by suggesting some possible topics, providing reference material for browsing, and relating student interests to mathematics.

Suggested Topics for Student Research

1. The History of Measurement
   a. History of common measures such as foot, inch, etc.
   b. Development of the English system of measurements
   c. Greek and Roman measures
   d. Egyptian measures
   e. Metric system
2. Map Making
   a. Mapping land
   b. Mapping the sea
   c. Mapping the sky
3. Time
   a. History and development of devices for telling time
   b. Different kinds of calendars
4. Measurement and Science
   a. Gravity
   b. Atomic weight

c. Electricity

d. Temperature

e. Heat

f. Sound

g. Force, work, and energy

5. Mathematics in Nature

a. Patterns in nature - snowflakes, spiral shells, crystals, branch distribution, etc.

b. Mathematics of the honeycomb

c. Geometric and symmetric shapes

6. Topology - The Rubber-Sheet Geometry

a. Topological curves and regions

b. Moebius strip

c. Traveling networks

d. Euler's formula for networks

e. Topological puzzles

7. Number Curiosities

a. What are:

Prime numbers

Perfect numbers

Amicable numbers

Irrational numbers

b. Shapes of numbers

c. Infinity

d. Rapid growth by doubling - Start by saving a penny the first day, and double the amount saved each day following. How much is saved in a month's time?

8. How Early People Wrote Numerals
    a. Egyptians
    b. Sumerians
    c. Babylonians
    d. Greeks
    e. Romans
    f. Chinese
    g. Hebrews
    h. Mayas
9. Sets
    a. Basic concepts of sets
    b. Set operations
    c. Venn diagrams
    d. Using set concepts to solve problems
10. Logic
    a. Truth value of statements
    b. The connective "and"
    c. The connective "or"
    d. The connective "if,then"
    e. Negation and negating combinations

11. The Logarithm Story

    a. What are logarithms?

    b. Reading a simple logarithmic table

    c. Simple computation with logs

12. Geometric Constructions

    a. Tools of construction - compass and straight-edge

    b. Line constructions

        (1) Bisect a line

        (2) Construct a perpendicular to a point on a line

        (3) Construct a perpendicular to a line from a point not on the line

        (4) Copy an angle

        (5) Construct two parallel lines

        (6) Divide a line segment into any number of equal parts

    c. Working with angles

        (1) Bisect an angle

        (2) Construct an angle equal to the sum of two given angles

        (3) Construct the complement and supplement of a given angle

    d. Construction of triangles

    e. Geometric designs

13. Vectors
    a. What are vectors?
    b. Adding vectors
    c. Solving problems by the use of vectors
14. Right Triangles
    a. Proportions and similar right triangles
    b. The trigonometric functions - sine, cosine, tangent
    c. Using a table of trigonometric functions
    d. Computing measures of inaccessible objects or distances by the use of trigonometric functions
15. Business and the Use of Per Cent
    a. Discounts
    b. Commissions
    c. Bank interest
    d. Buying on credit
    e. Borrowing money
16. Conducting a Survey
    a. Plan and carry out a survey on some topic of interest to students
    b. Analyze and report the results
17. Finite Mathematical Systems (Modular Arithmetic)
    a. Mod 12 system
    b. Basic operations in the mod 12 system

c. Properties of the mod 12 system

(1) Commutative

(2) Associative

(3) Distributive

(4) Identities

(5) Inverses

(6) Closure

d. Other modular systems

18. Computing Devices

a. Abacus

b. Napier's bones

c. Nomograph

d. Slide rule

19. Probability

a. Meaning of probability and chance

b. Plan and carry out experiments where predicted results can be matched with actual results.

20. Front-End Arithmetic

a. Addition

b. Multiplication

c. Subtraction

d. Division

Activity Two

Provide the students with time for independent research and study of their topics. Each student should find as much information about his topic as possible. The teacher should be available to answer questions and to guide the student's progress.

Activity Three

Each student should prepare and organize his material into a concise, interesting report. Include drawings, pictures, and examples that will get the readers' attention and add meaning.

The student should build an exhibit that tells the story of his topic. Use models, applications, and charts that lend variety. Give the exhibit a catchy title. Make the display simple but at the same time attractive. Be sure the labels are large, neat and easy to read.

Activity Four

The student shall enter his project at the science fair. He should be well informed about his topic so that he can answer questions about it.

Suggested Supplementary Reading

Kemp, Grace K. and Jean Graham. "The Birth of an Idea: The Science Fair," Education, LXXXV - 1 (September, 1964), pp. 51-53.

PROBABILITY

Objectives

Upon completion of these activities the student should be able:

1. to identify all possible outcomes of an event (assuming a finite number of outcomes), the sample space.
2. to state the probability of a certain outcome (E) using the definition:

$$P\ (E) = \frac{\text{favorable outcomes}}{\text{all possible outcomes}}$$

3. to demonstrate an experimental procedure to approximate the probability of an event.
4. to demonstrate a procedure for determining the probability of two events connected by "or".
5. to explain the difference between theoretical probability and experimental probability.

Suggested Time Allotment

Four class sessions

Vocabulary

Probability, sample space, favorable outcomes, possible outcomes, "or", theoretical probability, experimental probability

Materials

Coins - 2 nickels, 1 penny, and 1 dime for each student

Dice - 1 red and 1 white for each student

1 toothpaste cap for each student

Originating the Problem

We are living in a world of danger as well as opportunity. Almost every day there is the probability of an accident or the chance of a new success. When we talk about these events we make statements such as:

"I will probably be on the honor roll next year."

"It isn't likely that my family will move from Hibbing."

"My chances of being a millionaire are not very good."

We use the words chances, likely, or probably to describe expected occurrences.

The teacher now holds up a message from the principal's office which is for John. (Choose the name which occurs most frequently in the class.) Tell the students you are not sure whether the message is for John Brown, John Smith, or John Wilson. Ask the students what the chances are that the message is for John Brown. Students will give reasons for thinking the message is or is not for John Brown. Suppose there is an equally good chance that the message is for any one of the Johns. What is the chance that the message is for John Brown? (one out of three or 1/3) If the students do not come forth with an answer, ask the students how many Johns there are. (three) How many of these are John Brown? (one) John Smith? (one) John Wilson? (one) Ask the students why one-third would be a good answer.

We say the probability of the message being for John Brown is 1/3. What is the probability of the message being for John Smith? (1/3) What is the probability of the message being for John Wilson? (1/3)

<u>Instructional Procedure</u>

Activity One

We are now going to perform some experiments with coins. Suppose I toss this penny in the air, how will it land? (head or tail) There are only two possible outcomes, either a head or a tail. We give a name to the set of all possible outcomes. It is called the sample space.

Supply each student with a penny. What is the probability of getting a head on any one toss of the penny? (1/2) There are two possible outcomes but since we are interested in getting a head we say there is one favorable outcome. The probability can then be defined as:

$$\frac{\text{favorable outcomes}}{\text{possible outcomes}} \text{ or } 1/2.$$

Ask each student to toss the penny 50 times and keep a record of the number of heads and tails that come up. Combine the results of all the students and find the ratio of the number of heads to the total number of tosses. Is the ratio very close to the mathematical probability of 1/2?

Activity Two

Supply each student with a nickel and a penny. If these two coins are tossed together, list all the possible outcomes, the sample space.

Nickel	Penny		
H	H	(H,H)	both heads
H	T	(H,T)	head on nickel, tail on penny
T	H	(T,H)	tail on nickel, head on penny
T	T	(T,T)	both tails

How many possible ways are there for the two coins to land? (4) What is the probability of getting two heads on one toss of the coins? (1/4) a head on the nickel and a tail on the penny? (1/4) a tail on the nickel and a head on the penny? (1/4) two tails? (1/4)

If you toss the coins twenty times, how many times do you think they will land HH? (5) TT? (5) either HT or TH? (10) Try the experiment and record your results. Did they turn out as you expected? Compare your results with classmates.

Probability sentences can be written as mathematical sentences.

P (HH) = 1/4

We read the various parts of this sentence in this way:

1. The symbol P is read "the probability of".
2. The symbol (HH) is read "two heads occuring".
3. The symbol = is read "is" or "equals".
4. The symbol 1/4 is read "one-fourth".

"The probability of two heads occuring is one-fourth."

Activity Three

Supply each student with two nickels. Make a table of the sample space for tossing the two nickels.

Nickel	Nickel	
H	H	(H,H)
H	T	(H,T)
T	H	(T,H)
T	T	(T,T)

Ask the students to toss the coins a few times and record the results. What difficulty did you encounter? (distinguishing between the outcomes HT and TH) Let us consider this as one outcome, a head and a tail. (H,T) What is the probability of two heads? (1/4) two tails? (1/4) a head and a tail? (2/4 or 1/2)

If I hold one nickel in my hand and I offer to give you a coin from my hand, what is the probability that I will give you a nickel? (1) a dime? (0)

If E is certain to occur, then P (E) = 1

If E is certain not to occur, then P (E) = 0

All other probabilities vary between 0 and 1

Activity Four

Supply each student with a dime, nickel, and penny. Tossing the coins at the same time list the outcomes.

Dime	Nickel	Penny
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

What is the probability of obtaining all heads? (1/8) all tails? (1/8) two heads and one tail? (3/8) two tails and one head? (3/8) What is the sum of the probabilities? (1)

Activity Five (Optional)

As we increase the number of coins which we toss at one time it becomes increasingly difficult to list the outcomes. There is a famous table of numbers, called Pascal's triangle, which can help us out.

Row	
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

Ask the students if they can discover a pattern which would help them to make a triangle. (Every number in the triangle is the sum of the two numbers nearest to it in the preceding line. The circle shows 10 = 6 + 4)

Row 3 would be used for giving the results of tossing three coins. The sum of the row will tell you the number of possible outcomes. (8) The probability of three heads is 1/8, two heads and one tail is 3/8, one head and two tails is 3/8, and three tails is 1/8.

Row 6 would be used to give the results for tossing 6 coins. What is the probability of 6 heads? (1/64) 5 heads and 1 tail? (6/64) 4 heads and 2 tails? (15/64) 3 heads and 3 tails? (20/64) 2 heads and 4 tails? (15/64) 1 head and 5 tails? (6/64) and 6 tails? (1/64)

The table can be used in other problems where only 2 alternatives exist. If a family has four children the probability of four girls is 1/16. What is the probability of three girls and one boy? (4/16) two girls and two boys? (6/16) one girl and three boys? (4/16) four boys? (1/16)

Activity Six

Supply each student with a die. (singular of dice) In rolling a die, how many possible outcomes are there? (6) What are the elements of the sample space? (1, 2, 3, 4, 5, 6) What is the probability of rolling a 1? (1/6) 2? (1/6) 3? (1/6) 4? (1/6) 5? (1/6) 7? (0)

What is the probability of rolling a 2 or 3? (2/6 or 1/3) The students may arrive at their answer by thinking of two favorable outcomes out of six possible outcomes, or as a sum of the probabilities of each occurrence, 1/6 + 1/6 = 2/6 or 1/3. Ask the students to individually work out examples of the following types:

P (4)

P (1 or 4)

P (1, 3, or 6)

P (odd number)

Activity Seven

Supply each student with two dice of different colors, for example white and red. When two dice are rolled we add the numbers together and give the sum. Let us first consider all the ways that the dice may land. Ask the students to make a chart showing all the possible outcomes. Students will come up with a variety of ways of recording their outcomes.

Red	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3
White	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Red	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	6
White	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6

How many possible outcomes are there? (36) We are interested in sums. How many ways are there of obtaining a sum of 7? (6) What is the probability of rolling a 7? (6/36 or 1/6)

Ask the students to list all the possible sums obtainable by one throw of the dice and to find the probability of each occuring.

P (2) = 1/36	P (8) = 5/36
P (3) = 2/36	P (9) = 4/36
P (4) = 3/36	P (10) = 3/36
P (5) = 4/36	P (11) = 2/36
P (6) = 5/36	P (12) = 1/36
P (7) = 6/36	

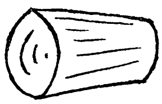
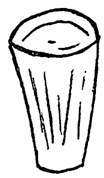
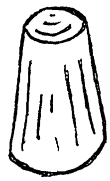
What is the probability of rolling a 7 or an 11? (8/36 or 2/9) What is the probability that a 2, 3, or 12 is thrown? (4/36 or 1/9) What is the probability that any total except 7 is thrown? (30/36 or 5/6) What is the probability that the sum of the two numbers will be even? (18/36 or 1/2)

The probabilities we have been listing are theoretical probabilities. Experimental results would compare quite favorably if data from a large number of trials was used.

Activity Eight

In tossing coins, the theoretical probability for the different outcomes could be found. Many events are not this predictable. For example, what is the probability that a tornado will hit a certain community, or you will live to the age of 100? The only way probabilities of this kind can be found is by evidence or data from past events. Experimentation may provide the basis for establishing the probability for each possible event.

Give each student a toothpaste cap. It can come to rest in three different positions when it is dropped on a table. Let the letter S represent the case where it comes to rest on its side, the letter T if it comes to rest on its top, and the letter B if it comes to rest on its bottom.

S  T  B 

Can the theoretical probability of each outcome be determined? (no) Why? (Some outcomes will occur more frequently than others.)

Ask the students to predict whether the probability of T or B is greater, and whether that of S or B is greater, and then arrange them in order of their probability. The numerical probability for each outcome could also be predicted.

Ask the students to design and carry out an experiment to check the predicted probabilities.

Activity Nine

Susan has five skirts: blue, white, red, green, and yellow. She also has three sweaters: red, white, and blue. Susan chooses a skirt and a sweater at random. Work out the sample space for choosing a skirt and sweater. (Skirts are listed first.)

(BR)	(WR)	(RR)	(GR)	(YR)
(BW)	(WW)	(RW)	(GW)	(YW)
(BB)	(WB)	(RB)	(GB)	(YB)

If Susan chooses an outfit at random, what is the probability that her skirt will be green? (1/5)

What is the probability that her sweater will be white? (1/3)

What is the probability that either her sweater or skirt will be blue? (7/15)

What is the probability that both her sweater and her skirt will be blue? (1/15)

What is the probability that the color of her sweater and skirt will be the same? (1/5)

What is the probability that her skirt and sweater do not match in color? (4/5)

If Susan's school colors are green and white, what is the probability that she will select an outfit with her school colors? (1/15)

Activity Ten (Optional)

Have you heard about T.V. ratings and the Gallup Poll of public opinion? These are illustrations of the work of groups who obtain their information by sampling and base their predictions on the information.

Do you think we can predict the favorite television program of the sixth graders in Hibbing? We will need some information on which to base our predictions. Suggest that the students survey the class and list the five most popular television shows. On the basis of this information, ask each student to write on a slip of paper his prediction of the most popular television program. The teacher shall collect the predictions and save them until the next class session.

During the week, class members from each school shall survey all the sixth graders attending their school and bring their results to class. The students can now compile their results and determine what really is the favorite television program of the Hibbing sixth graders.

Compare the actual results with the various predicted results. Do you think our class was a good random sample? Students may suggest the sample was too small, our class has more boys than girls, we are a special group interested in mathematics and science, etc. Lead the students to realize that a good sample group is one that you pick by chance.

Suggested Supplementary Readings

Adler, Irving. Probability and Statistics for Everyman. New York: Signet Science Library, 1963. entire book.

American Association for the Advancement of Science. Science - A Process Approach, Part Seven A. AAAS Miscellaneous Publications, 1965. pp. 29-35.

Bergamini, David, and the Editors of Life. Mathematics. New York: Time, Inc., 1963. pp. 126-147.

Johnson, Donovan A. Probability and Chance. St. Louis: Webster Division, McGraw-Hill Book Company, 1963. 64 pp.

May, Lola J. Elementary Mathematics Enrichment-4. New York: Harcourt, Brace, and World, Inc., 1966. pp. 30-33.

May, Lola J. Elementary Mathematics Enrichment-5. New York: Harcourt, Brace, and World, Inc., 1966. pp. 6-10.

Wilkinson, Jack D. and Owen Nelson. "Probability and Statistics - Trial Teaching in Sixth Grade," The Arithmetic Teacher, XIII (February, 1966), pp. 100-106.

BIBLIOGRAPHY

BIBLIOGRAPHY

A. BOOKS

Bruner, Jerome S. The Process of Education. Cambridge, Massachusetts: Harvard University Press, 1960.

Bruner, Jerome S. Toward a Theory of Instruction. Cambridge, Massachusetts: Harvard University Press, 1966.

De Vault, M Vere. Improving Mathematics Programs. Columbus, Ohio: Charles E. Merrill Books, Inc., 1961.

Fehr, Howard F. and Jo McKeeby Phillips. Teaching Modern Mathematics in the Elementary School. Reading: Addison Wesley Publishing Company, 1966.

Mager, Robert F. Preparing Instructional Objectives. Palo Alto: Fearon Publishers Inc., 1962.

Riedesel, C. Allan. Guiding Discovery in Elementary School Mathematics. New York: Appleton-Century-Crofts, Division of Meredith Publishing Company, 1967.

Spitzer, Herbert F. Enrichment of Arithmetic. St. Louis: Webster Division, McGraw-Hill Book Company, 1964.

B. PUBLICATIONS OF THE GOVERNMENT LEARNED SOCIETIES, AND OTHER ORGANIZATIONS

Cambridge Conference on School Mathematics. Goals for School Mathematics. Boston: Houghton Mifflin Company published for Educational Services Incorporated, 1963.

Martinson, Ruth A. Special Programs for Gifted Pupils. A Bulletin of the California State Department of Education. Sacramento: California State Department of Education, 1962.

National Council of Teachers of Mathematics. Enrichment Mathematics for the Grades. Twenty-seventh Yearbook of the Council. Washington: The National Council of Teachers of Mathematics, 1963.

National Council of Teachers of Mathematics. The Revolution in School Mathematics. Washington: The National Council of Teachers of Mathematics, 1961.

Nelson, Henry. The Education of Exceptional Children. Forty-ninth Yearbook of the National Society for the Study of Education, Part II. Chicago, Illinois: University of Chicago Press, 1950.

Pennsylvania Department of Public Instruction. Guide to Planning for Able Pupils. A Report Prepared for the Department of Public Instruction. Harrisburg: Commonwealth of Pennsylvania, 1962.

Schult, Veryl. Present Practices in Mathematics Instruction and Supervision. A Report Prepared by the United States Department of Health, Education, and Welfare--Bureau of Research. Washington: United States Government Printing Office, 1966.

C. PERIODICALS

Adler, Irving. "Mental Growth and the Art of Teaching." The Mathematics Teacher, LIX (December, 1966), pp. 706-715.

Adler, Irving. "The Cambridge Conference Report: Blueprint or Fantasy?" The Arithmetic Teacher XIII (March, 1966), pp. 179-186.

Bolding, James. "A Look at Discovery," The Mathematics Teacher LVII (February, 1964), pp. 105-107.

Carlow, Chester D. "Possible Misunderstandings Arising Out of Kersh's Article on Learning by Discovery," The Arithmetic Teacher, XIII (April, 1966), pp. 275-277.

Grade Teacher, LXXXIV (January, 1967), Entire Issue.

Johnson, Harry C. "What Do We Mean by Discovery?" The Arithmetic Teacher, XI (December, 1964), pp. 538-539.

Kane, Robert B. "School Mathematics--Where to Now?" The Arithmetic Teacher, XIV (February, 1967), pp. 126-131.

Kersh, Bert Y. "Learning by Discovery: Instructional Strategies," *The Arithmetic Teacher*, XII (October, 1965), pp. 414-417.

May, Lola J. "How to Teach the New Mathematics," *Grade Teacher*, LXXXII (September, 1964), p. 50.

May, Lola J. "Individualized Instruction in a Learning Laboratory Setting," *The Arithmetic Teacher*, XIII (February, 1966), pp. 110-112.

Ranucci, Ernest R. "Discovery in Mathematics," *The Arithmetic Teacher*, XII (January, 1965), pp. 14-18.

Rosenbloom, Paul C. "Mathematics K-14," *Educational Leadership*, XIX (March, 1962), p. 359.

D. UNPUBLISHED MATERIALS

High Achievers Program, A Report Prepared by the Bloomington Public Schools. Bloomington, Minnesota: Bloomington School System, 1962. (Mimeographed.)

The Elementary Gifted Program, A Guide Prepared by the San Diego Public Schools. San Diego, California: San Diego School System, 1965. (Mimeographed.)