Some issues about infrared divergences during inflation

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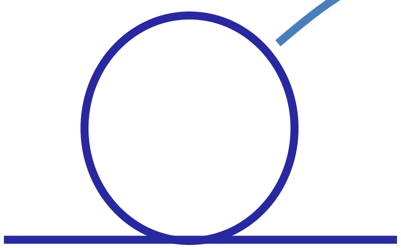
§ IR divergence in single field inflation

ζ : curvature perturbation in co-moving gauge.

$$\begin{cases} h_{ij} = e^{2N+2\zeta}\left(\delta_{ij} + h_{ij}\right) \\ \delta\phi = 0 \end{cases}$$

Transverse traceless



Factor coming from this loop:

$$\langle \zeta(y)\zeta(y) \rangle \approx \int d^3k \, \underline{P(k)} \approx \log\left(aH / k_{\min}\right)$$

$\propto 1/k^3$

for scale invariant spectrum

− no typical mass scale

Special property of single field inflation

Yuko Urakawa and T.T., PTP122: 779 arXiv:0902.3209

– In conventional cosmological perturbation theory, gauge is not completely fixed.

Time slicing can be uniquely specified: $\delta\phi = 0$ OK!

but spatial coordinates are not.

$$h_j^{\,j} = 0 = h_{i,j}^{\,j}$$

Residual gauge d.o.f.

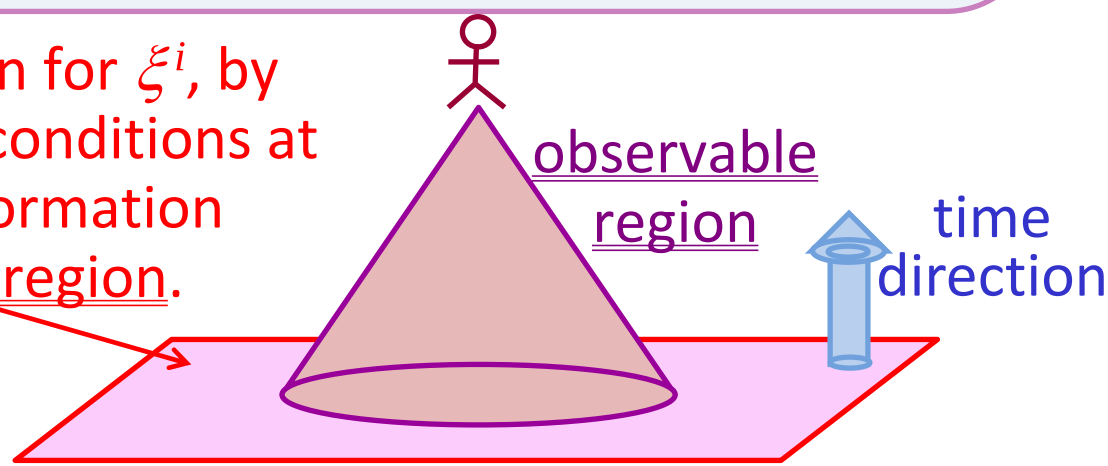
$$\delta_g h_{ij} = \xi_{i,j} + \xi_{j,i}$$

Elliptic-type differential equation for ξ^i.

$$\Delta \xi^i = \cdots$$

Not unique locally!

- To solve the equation for ξ^i, by imposing boundary conditions at infinity, we need information about unobservable region.



Basic idea of the proof of IR finiteness in single field inflation

- The local spatial average of ζ can be set to 0 identically by an appropriate gauge choice.
- Even if we choose such a local gauge, the evolution equation for ζ formally does not change, and it is hyperbolic. So only the interaction vertices inside the past light cone are relevant.
- Therefore, IR effect is completely suppressed as long as we compute ζ in this local gauge.

However, we later noticed that the above argument is true only when correlation functions of ζ are free from divergence at the initial time, which is not in general guaranteed.

Genuine gauge-invariant quantities

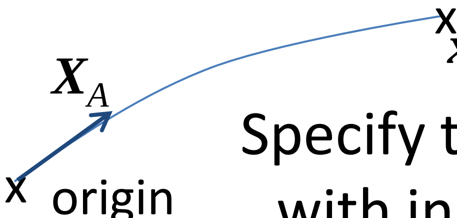
- If we evaluate genuine gauge-invariant quantities, we should obtain finite results whatever gauge we may use.

A genuine gauge-invariant quantity:
Correlation functions for 3-d scalar curvature on ϕ =constant slice.

$$\langle R(\boldsymbol{x}_1)\, R(\boldsymbol{x}_2)\rangle$$

But coordinate values do not have gauge invariant meaning.

(Giddings & Sloth 1005.1056)
(Byrnes et al. 1005.33307)



$$\boldsymbol{x}(\boldsymbol{X}_A, \lambda=1) = \boldsymbol{X}_A + \delta \boldsymbol{x}_A$$

Specify the position by solving geodesic Eq. $D^2 x^i / d\lambda^2 = 0$
with initial condition $Dx^i / d\lambda \big|_{\lambda=0} = X^i$.

Then, use X^i to specify the position.

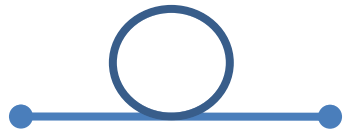
$${}^{g}R(\boldsymbol{X}_A) := R(\boldsymbol{x}(\boldsymbol{X}_A, \lambda=1)) = R(\boldsymbol{X}_A) + \delta \boldsymbol{x}_A \nabla R(\boldsymbol{X}_A) + \ldots$$

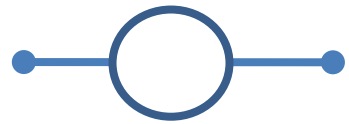
$\langle {}^{g}R(\boldsymbol{X}_1)\, {}^{g}R(\boldsymbol{X}_2)\rangle$ should be genuine gauge invariant.

Translation invariance of the vacuum state takes care of the ambiguity in the choice of the origin.

One-loop 2-point function at the leading slow-roll exp.

- No interaction term in the evolution equation at $O(\varepsilon^0)$ in flat gauge.





◎flat gauge → $\delta\phi$=const. gauge ◎$R(X_A) \sim e^{-2\zeta}\Delta\zeta$ ◎$R \rightarrow {}^gR$

$$\langle {}^gR(X_1)\,{}^gR(X_2)\rangle^{(4)} = \langle {}^gR^{(3)}(X_1)\,{}^gR^{(1)}(X_2)\rangle + \langle {}^gR^{(2)}(X_1)\,{}^gR^{(2)}(X_2)\rangle + \langle {}^gR^{(1)}(X_1)\,{}^gR^{(3)}(X_2)\rangle$$

$$\propto \langle \zeta_I^{\,2}\rangle \int d(\log k) k^3 \Big[\Delta(D^2 u_k(X_1))\Delta(u^*_k(X_2)) + 2\Delta(D u_k(X_1))\Delta(D u^*_k(X_2)) + \Delta(u_k(X_1))\Delta(D^2 u^*_k(X_2))\Big] + \text{c.c.}$$

+ (manifestly finite pieces)

where $\zeta_I = \int d^3k\,(u_k a_k + u^*_k a^\dagger_k)$ $\quad D := \partial_{\log a} - (x\cdot\nabla)$

- ◆ IR divergence from $\langle \zeta_I^{\,2}\rangle$ exists in general.

However, the integral vanishes for the Bunch-Davies vacuum state.

$u_k = k^{-3/2}(1\text{-}ik/aH)e^{ik/aH + ikx}$ ⟹ $D u_k = k^{-3/2}\,\partial_{\log k}(k^{3/2}u_k)$

Then $\langle {}^gR(X_1)\,{}^gR(X_2)\rangle^{(4)} \propto \langle \zeta_I^{\,2}\rangle \times \int d(\log k)\, \partial_{\log k}^{\,2}[\Delta(k^{3/2}u_k(X_1))\Delta(k^{3/2}u^*_k(X_2))] + \text{c.c.}$

- ◆ To remove IR divergence, the positive frequency function corresponding to the vacuum state is required to satisfy $D u_k = k^{-3/2}\,\partial_{\log k}(k^{3/2}u_k)$.

IR regularity requests scale invariance!

Summary of what we found in our previous work

1) To avoid IR divergence, the initial quantum state must be "scale invariant/Bunch Davies" in the slow roll limit.

"Wave function must be homogeneous in the residual gauge direction"

2) To the second order of slow roll, a generalized condition of "scale invariance" to avoid IR divergence was obtained, and found to be consistent with the EOM and normalization.

However, we later noticed …