THE EFFECT OF STRATEGIC BEHAVIOR ON RICARDIAN EQUIVALENCE

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Abstract

The presence of strategic behavior is often believed to be sufficient to negate the neutrality assertions of the Ricardian Equivalence Theorem. I present a two-period, two-consumer (parent and child) model with one-sided altruism. The child behaves strategically in the sense that he seeks to manipulate the size of the parent's second period transfer. The parent behaves strategically as he seeks to minimize this manipulation. I show that, for general utility functions, this form of strategic behavior does not alter the effects of a change in the timing and incidence of a lump-sum tax. The intuition for this result derives from the fact that the child's utility is a public good. Under certain conditions (present in this model) wealth redistributions have no effect on total voluntary contributions to a public good.

*Preliminary and incomplete. All comments welcome.

1 Introduction

The Ricardian Equivalence Theorem provides us with a powerful result: no real redistributive effects result from deficit financing by government. Unfortunately, the conditions required for complete debt neutrality are numerous and restrictive.[1] One condition believed necessary for Ricardian equivalence, but little explored in the literature, is the absence of strategic interactions between members of different generations.[2] This paper investigates the impact of allowing intrafamily strategic behavior on the debt neutrality assertions of the Ricardian equivalence theorem. I consider a two-period model with one-sided altruism and show that strategic behavior does not negate these assertions in this framework. This result stems in part from the public good character of a child's welfare, as is discussed further below.

The Ricardian Equivalence Theorem is one of the most examined theorems in macroeconomics. Robert Barro's (1974) reintroduction of this theorem to economics provided the impetus for a substantial body of economic literature. This is not surprising given the theorem's implications for public policy analysis. In addition to numerous theoretical studies, empirical analysis explores both macro and micro data in search of evidence for or against Ricardian equivalence.[3]

Formally, Ricardian equivalence asserts that substituting government debt for current taxation has no real effect on consumption, interest rates or output. The limitations required for its application are believed to include perfect foresight, no uncertainty, perfect capital markets, only nondistortionary taxes and an absence of strategic behavior.[4] Barro's

[1]See Barro (1989), Seater (1993) and Leiderman and Blejer (1988) for surveys of the required conditions and their respective significance. Barro (1989) and Seater (1993) also provide reviews of the micro and macroeconomic studies which test for evidence of Ricardian equivalence.

[2]Bruce and Waldman (1990) use a two-period model with altruism and manipulation to demonstrate that government transfers may not be neutralized by private transfers. Kotlikoff, Razin and Rosenthal (1990) present a stylized model of parent-child strategic behavior with altruism. They conclude with "We suspect that for the most part such models will not, however, satisfy Ricardian equivalence..." (p. 1267) They also suggest that further research of strategic-altruism models may be fruitful. Some of the literature follows Bernheim, Shleifer and Summers (1985) who also conclude that Ricardian equivalence does not hold when strategic behavior is allowed. In their model bequests are part of an exchange with children. This is not the path taken by this paper.

[3]For example, see Bernheim, Shleifer and Summers (1985), Poterba and Summers (1987) and Evans (1993).

[4]Two comments about this list of restrictions are in order. First, this list is culled from a number of different works. Each work focuses on the significance of one, or at most two, restrictions at a time. Second, this list is not intended to be exhaustive. The scrutiny given Ricardian equivalence means that new

(1974) contribution was to show that infinite lifetimes are not required when consumers are intertemporally linked, as they are with intergenerational altruism.

A major problem with this result is uncertainty about the true nature of parent-child interactions. Bernheim, Shleifer and Summers (1985) assert that the true nature is not altruistic, but rather one of exchange.[5] Even those who stick to the altruistic formulation encounter difficulties expanding the analysis to multi-period regimes. Laitner (1993) describes the problem succinctly when he writes:

> Given multi-period overlaps between parents and their adult children ... 'single-sided altruism' would potentially complicate the analysis significantly ... a T-period Stackelberg game would emerge – with parents the 'leaders' and their children the 'followers.' Children might behave strategically – fully consuming their earnings early in marriage, for example, in order to extract large transfers when the arrival of their own children raised their marginal utility of wealth. (p. 70)

In an earlier writing, James Buchanan (1975) described the problem an altruist faces in this context as the Samaritan's Dilemma. Briefly stated, when two consumers, with different objective functions, interact over multiple periods, altruists are open to possible manipulation of their choices. Selfish individuals, knowing they will receive an altruistic transfer in the near future, may impoverish themselves today to appear needier at the time of the transfer.

This paper studies this type of manipulation within a family. As inclusion of strategic behavior is the main focus of this paper, it seems judicious to clarify the nature of the behaviors to be studied.

Consider two individuals, both alive for two periods.[6] One individual (denoted A) is altruistic. He gains utility from the utility (or consumption) of the other individual (denoted

information is regularly revealed.

[5]This theme has expanded to include parents providing wealth or human capital investments in exchange for personal 'services' or old-age financial support. See Cox (1987), Chakrabarti, Lord and Rangazas (1993) or Cremer, Kessler and Pestieau (1992) for example.

[6]This is an application of the Samaritan's Dilemma as presented by Buchanan (1975), and expounded on by Bruce and Waldman (1990). For a similar approach see Lindbeck and Weibull (1988) who allow

B) who is selfish. Each makes a choice in the first period (e.g. savings or investment) that determines their respective wealth levels in the second period. In the second period individual A may make a nonnegative transfer to B. The question is: when does A determine how much to give to B? If A and B make their decisions simultaneously in the first period, then A will condition his choice on B's initial endowment. Individual A expects B to rationally distribute his own wealth and the proceeds of the transfer between first and second period consumption. In this case B cannot manipulate the transfer amount. This scenario is called *precommitment* and provides an efficient allocation of resources.

But what if choices are made sequentially, as may be more realistic? Now individual A chooses the transfer amount in the second period. His transfer choice is contingent on B's state – most likely on B's wealth – in period 2. Individual B, knowing of A's altruism, may make a first period choice that leaves B a lower second period wealth than under precommitment. This strategy is likely to elicit a larger transfer from A and can easily lead to a higher overall utility for B at A's expense. This scenario is called *manipulative*, and is the focus of this paper.

Individual A, the good Samaritan, when making his first period choice, knows of B's likely manipulation. A must balance the competing goals of personal consumption and altruism, knowing that B will impoverish himself to elicit a large transfer. Conceivably, individual A could choose an action that provides him less second period wealth than under precommitment. He may do this to minimize the excess transfer B's impoverishment might otherwise elicit. The end result is an inefficient allocation of resources. B overconsumes in the first period and underconsumes in the second. A likely underconsumes in both periods, having transferred more than an efficient amount to B.

In this manipulative environment we might expect that intrafamily transfers would not negate the transfer effects of deficit financing. Kotlikoff, Razin and Rosenthal (1990) and Bruce and Waldman (1990) conclude that Ricardian equivalence fails in these environments. The problem is that allowing strategic behavior alters the margins at which choices are made. Since B over-consumes in the first period (relative to a Pareto efficient consumption amount), it seems likely that A would not increase his transfer by the entire tax amount out

manipulation by either individual; or Coate (1995) who studies inefficiencies that can arise from altruistic public transfers.

of concern that B would squander even more of his wealth in the first period. An alternative possibility is that B, faced with the second period tax, will squander less of his wealth in the first period so as to be better able to pay the tax in the second period. This also would lead A to increase his transfer amount by less than the amount of the tax. In this paper, I present a proof that Ricardian equivalence is preserved in spite of the strategic behavior, assuming only separability of the utility functions.

This result is not surprising given the literature on public goods. In a model with non-paternalistic one-sided altruism, the utility of the nonaltruist is a pure public good. Bergstrom, Blume and Varian (1986) and Warr (1983) demonstrate that voluntary contributions to a public good are unaffected by a relatively small income redistribution amongst contributors.[7] In effect, a change in the timing of lump sum taxes is identical to an income redistribution across generations.

The remainder of the paper is organized as follows. The model is described in section 2. Section 3 presents a proof of debt neutrality. Conclusions and directions for ongoing and future research are presented in section 4.

[7]Specifically, the requirement is that the resources taken from an individual cannot exceed his initial contribution to the public good.

2 The Model

This section describes a two-consumer two-period model with one-sided altruism. The two consumers (denoted Parent and Kid) engage in a two-stage game.

The goal is to demonstrate, assuming only additive separability of the utility functions, that Ricardian equivalence always holds in this model. To accomplish this, I present proofs for two theorems:

1. That a unique Nash equilibrium always exists.
2. That changing the timing and statutory incidence of a lump sum tax has no real effects.

A change in the timing and incidence of lump sum taxes is used to evaluate whether or not Ricardian equivalence holds in this model. Thus the second theorem presents this paper's main result.

A specific example may help illustrate the interactions modeled here. Suppose a parent has a child in his early years of high school. Both parent and child want the child to attend college after high school. The question both are thinking about is how to finance the cost of attending college. The parent is already saving for this expense and hopes the child will also contribute some funds.

The child must decide whether or not to work after school to help accumulate funds for college. The child knows the parent is already accumulating resources for this expense. The child also values his after-school leisure time. What should he do? By working he could save for college, but at the cost of his leisure time. The parent then will contribute his saved funds. By not working the child can enjoy his leisure time and still go to college. However, this choice means that he cannot afford as good a college, or as nice a place to live and food to eat, or in some other way will have a lower quality experience.

The child recognizes the value the parent places on having the child obtain a good education. If the child doesn't work, the parent may be willing to pay more than he originally intended in order for the child to obtain a quality education. If so, not working today may be the child's optimal choice.

The child's ability to be manipulative depends on the parent's affinity for the child and on the parent's wealth and income levels. The child's interest in being manipulative depends primarily on his substitution rate between current leisure time and an additional unit of college education (beyond what the parent's savings will pay for.) Finally, the parent may anticipate the child's manipulation and prepare for it by saving less – or perhaps even more – than he otherwise might have.

2.1 Details of the Model

There are two consumers who are alive for both of two time periods. One consumer (denoted P for parent) is altruistic towards the other consumer (denoted K for kid).[8] Each individual j is endowed with initial income w^j. This income can either be consumed or put into savings. For each unit put into savings in the first period, $(1+r)$ units are returned in the second period. The net return on savings (r) is exogenously specified.

Let c_t^j denote individual j's $(j = P, K)$ consumption in period $t(= 1, 2)$. The kid's utility function is $U^K(c_1^K, c_2^K)$. The parent's utility function is $U^P(c_1^P, c_2^P, U^K)$. To allow separate descriptions for the utility maximization problem each consumer faces in each time period, I assume the utility functions U^P and U^K are separable in their arguments. Then we have

$$U^K(c_1^K, c_2^K) \equiv u_1^K(c_1^K) + u_2^K(c_2^K)$$

and

$$U^P(c_1^P, c_2^P, U^K(c_1^K, c_2^K)) \equiv u_1^P(c_1^P, u_1^K(c_1^K)) + u_2^P(c_2^P, u_2^K(c_2^K))$$

where u_t^j are continuously differentiable, strictly concave and increasing for $j = P, K; t = 1, 2$. Thus U^j are continuously differentiable and strictly concave for $j = P, K$. We assume all goods are normal for each consumer and that $0 < \frac{\partial U^P}{\partial U^K} < 1$.

The parent can transfer any nonnegative amount of wealth to the kid in the second

[8]Identifying the consumers as parent and kid is not meant to restrict application of the results to intrafamily interactions. Other applications, such as Coate (1995), also exist. He uses a similar framework to evaluate the efficiency of public transfers from rich to poor individuals.

period. There is perfect foresight and no individual or aggregate uncertainty.

We now turn our attention to the timing of the model. Two possible approaches are identified in the literature. First, the parent may precommit to a transfer amount by choosing this amount in the first period. This approach has three advantages. It prevents manipulation by the kid; obtains an efficient resource distribution; and, presumably satisfies Ricardian equivalence.[9] We expect Ricardian equivalence is satisfied because the parent bases his bequest decision on their combined endowments rather than on their combined second period wealth. Changing the timing of taxes may alter the distribution of second period wealth, but does not alter the combined endowments. The primary disadvantage of this approach is the lack of time consistency on the part of the parent in the second period. The kid may choose an action in the first period that makes the parent's previously chosen transfer amount sub-optimal in the second period. This problem makes precommitment a difficult assumption to defend in practice.

The second approach allows the parent to choose the transfer amount in the second period. While providing a time consistent solution, this approach leaves the parent vulnerable to manipulation of his transfer choice by the kid's first period choices. It is believed that the potential for manipulation leads to a failure of Ricardian equivalence.[10] This work uses this second approach.

We formalize the model by constructing a two-stage game. The first stage consists of the consumption and savings decisions of the first period. The second stage consists of the consumption and transfer decisions of the second period. In the first period, I assume both individuals choose their consumption and savings amounts simultaneously.[11] In the second period the parent chooses his consumption and transfer amounts first. The kid then receives the transfer and chooses his consumption amount last. This structure produces a unique subgame perfect equilibrium.

Let s^j denote the amount consumer j puts into savings. Let T denote the amount the parent transfers to the kid in the second period. Then the period budget constraints for the

[9] Another attraction of this method is its relative ease of computation.

[10] As in Kotlikoff, Razin and Rosenthal (1990) and Bruce and Waldman (1990).

[11] Altering the timing of choices in the first period offers an avenue for further exploration.

parent are

$$c_1^P + s^P \leq w^P \tag{1}$$

$$c_2^P + T \leq s^P(1+r). \tag{2}$$

The period budget constraints for the kid are

$$c_1^K + s^K \leq w^K \tag{3}$$

$$c_2^K \leq s^K(1+r) + T. \tag{4}$$

The sequential nature of the game allows us to use backwards induction to develop a solution. With backwards induction we address the second stage of the game first. The kid chooses last in the second period. His problem at that time is

$$\max_{c_2^K} u_2^K(c_2^K)$$

subject to (4) and $c_2^K \geq 0$.

Since u^K is strictly increasing (and T and s^K are chosen earlier), the solution to this problem is simply

$$c_2^K = s^K(1+r) + T. \tag{5}$$

In fact, the strictly positive marginal utilities for both consumers implies that, in equilibrium, each budget constraint is satisfied with equality. In what follows I also assume the equilibrium transfer amount (T) is strictly positive. This stems from wanting to study only the effects of strategic behavior and not the effects of possible corner solutions – which are known to negate Ricardian equivalence.

Continuing with the backwards induction we next look at the parent's second period problem:

$$\max_{c_2^P, T} u_2^P(c_2^P, u_2^K(c_2^K))$$

subject to (2) and $c_2^P, T \geq 0$, given (5).

Substituting all pertinent constraints into the parent's second period problem allows us to write down the following function:

$$T(s^P, s^K) \equiv \underset{T}{\text{argmax}} \left[u_2^P(s^P(1+r) - T, u_2^K(s^K(1+r) + T))\right] \tag{6}$$

$$\text{such that } T \geq 0.$$

Remember that s^P and s^K are known when T is chosen. Implicit in this function is the fact that there is a unique T which maximizes the parent's second period problem. While not explicitly demonstrated here, this is not difficult to show given the strict concavity of u_2^P and u_2^K. In addition, since u_2^P and u_2^K are strictly concave and continuous, $T(s^P, s^K)$ is continuous. This function will be useful to have when determining the optimal first period choices.

The following paragraph illustrates how all final allocations of this two-stage game are completely specified by the first period choices of s^P and s^K.

Clearly, given w^j, the choices of s^P and s^K uniquely determine c_1^j for $j = P, K$ from equations (1) and (3) respectively. To see that s^P and s^K also uniquely determine c_2^j for $j = P, K$ we first note that knowing s^P and s^K allows unique determination of T using equation (6). Then, recalling that r is exogenous, c_2^P and c_2^K are uniquely determined from equations (2) and (4) respectively.

Definition: Let $\mathcal{S}^j = [0, w^j]$ be the space of possible savings amounts for individual $j(= P, K)$. These are the strategy spaces for the parent and kid respectively.

Payoffs are given by the respective utility functions.

2.2 Equilibrium

We now define a Nash equilibrium of this model.

Definition: A Nash equilibrium of this model is a savings pair $(\tilde{s}^P, \tilde{s}^K) \in \mathcal{S}^\mathcal{P} \times \mathcal{S}^\mathcal{K}$ such that:

1. $\tilde{s}^K$ solves the kid's first period problem given $\tilde{s}^P$:

$$\max_{s^K}\Big[U^K\Big(w^K - s^K, s^K(1+r) + T(\tilde{s}^P, s^K)\Big)\Big],$$

2. $\tilde{s}^P$ solves the parent's first period problem given $\tilde{s}^K$:

$$\max_{s^P}\Big[U^P\Big(w^P - s^P, s^P(1+r) - T(s^P,\tilde{s}^K), U^K\Big(w^K - \tilde{s}^K, \tilde{s}^K(1+r) + T(s^P,\tilde{s}^K)\Big)\Big)\Big],$$

3. $T(s^P, s^K)$ is as defined in equation (6) for all $(s^P, s^K) \in \mathcal{S}^{\mathcal{P}} \times \mathcal{S}^{\mathcal{K}}$.

In addition, we define the best response functions for the parent and kid respectively as follows:

$$f^P(s^K) \equiv \underset{s^P}{\text{argmax}}\Big[U^P\Big(w^P - s^P, s^P(1+r) - T(s^P, s^K), U^K\Big(w^K - s^K, s^K(1+r) + T(s^P, s^K)\Big)\Big)\Big] \tag{7}$$

and

$$f^K(s^P) \equiv \underset{s^K}{\text{argmax}}\Big[U^K\Big(w^K - s^K, s^K(1+r) + T(s^P, s^K)\Big)\Big]. \tag{8}$$

The continuity and strict concavity of the utility functions means the best response functions are continuous and single-valued.

Theorem 1: *A unique Nash equilibrium (in pure strategies) exists.*

Proof: This proof relies on the contraction mapping theorem. The key is to demonstrate that the best response functions comprise a contraction mapping.

First let $\mathcal{S} \equiv \mathcal{S}^{\mathcal{P}} \times \mathcal{S}^{\mathcal{K}}$ and note that $\mathcal{S}$ is a closed and bounded subset of $\mathcal{R}^2$. Thus $\mathcal{S}$ is compact and convex. For a metric I use the standard Euclidian norm,

$$\rho(x,y) = \| x - y \| = \Big(\sum_{j=1}^{2}(x_j - y_j)^2\Big)^{1/2} \quad \forall x, y \in \mathcal{S}.$$

To show that a function $F(\cdot) : \mathcal{S} \to \mathcal{S}$ is a contraction we must show that for some $\beta \in (0,1)$,

$$\rho(F(x), F(y)) \leq \beta\rho(x,y) \quad \forall x, y \in \mathcal{S}. \tag{9}$$

For any $s_1^P, s_2^P \in \mathcal{S}^{\mathcal{P}}$ and any $s_1^K, s_2^K \in \mathcal{S}^{\mathcal{K}}$, let $x = \begin{bmatrix} s_1^P \\ s_1^K \end{bmatrix}$ and $y = \begin{bmatrix} s_2^P \\ s_2^K \end{bmatrix}$.

Define $F(\cdot) = \begin{bmatrix} f^P(\cdot) \\ f^K(\cdot) \end{bmatrix}$. Then $F(x) = \begin{bmatrix} f^P(s_1^K) \\ f^K(s_1^P) \end{bmatrix}$ and $F(y) = \begin{bmatrix} f^P(s_2^K) \\ f^K(s_2^P) \end{bmatrix}$.

We first show that

$$\rho(F(x), F(y)) < \rho(x, y). \tag{10}$$

We later show $\exists \beta \in (0, 1)$ satisfying (9).

Expanding (10) gives

$$\left[(f^P(s_1^K) - f^P(s_2^K))^2 + (f^K(s_1^P) - f^K(s_2^P))^2\right]^{1/2} < \left[(s_1^P - s_2^P)^2 + (s_1^K - s_2^K)^2\right]^{1/2}. \tag{11}$$

That (11) is satisfied can be demonstrated by proving that

$$|f^K(s_1^P) - f^K(s_2^P)| \leq |s_1^P - s_2^P| \tag{12}$$

and

$$|f^P(s_1^K) - f^P(s_2^K)| \leq |s_1^K - s_2^K| \tag{13}$$

$\forall (s_1^P, s_1^K), (s_2^P, s_2^K) \in \mathcal{S}$, with at least one equation satisfied with strict inequality.

Without loss of generality, assume $s_2^P > s_1^P$ and $s_2^K > s_1^K$. Then define $s_2^P = s_1^P + \delta^P$ and $s_2^K = s_1^K + \delta^K$. We proceed with proofs of (12) and (13) separately.

1. Given the above definition, equation (12) can be rewritten as

$$|f^K(s_1^P) - f^K(s_1^P + \delta^P)| \leq \delta^P.$$

The following arguments evaluate how the kid's optimal savings choice changes when the parent's savings increases by δ^P. Therefore, s^K is a variable in the following equations.

Rewrite equation (6) as follows:

$$T(s_1^P, s^K) = \underset{T}{\text{argmax}} \left[u_2^P(s_1^P(1+r) - T, u_2^K(s^K(1+r) + T))\right], \tag{14}$$

and

$$T(s_1^P + \delta^P, s^K) = \underset{T}{\text{argmax}} \left[u_2^P((s_1^P + \delta^P)(1+r) - T, u_2^K(s^K(1+r) + T))\right]. \quad (15)$$

Let $\Delta T = T(s_1^P + \delta^P, s^K) - T(s_1^P, s^K)$.

In equation (15) the parent's second period wealth is $\delta^P(1+r)$ units larger than in equation (14). The parent consumes some of this additional wealth and passes some of it on to the kid. Because of the strict concavity of u_2^P, we know he neither consumes all of it himself nor passes all of it on to his kid. Thus we have $0 < \Delta T < \delta^P(1+r)$.

Then rewrite equation (8):

$$f^K(s_1^P) = \underset{s^K}{\text{argmax}} \left[U^K(w^K - s^K, s^K(1+r) + T(s_1^P, s^K))\right]. \quad (16)$$

Similarly,

$$\begin{aligned} f^K(s_1^P + \delta^P) &= \underset{s^K}{\text{argmax}} \left[U^K(w^K - s^K, s^K(1+r) + T(s_1^P + \delta^P, s^K))\right] \\ &= \underset{s^K}{\text{argmax}} \left[U^K(w^K - s^K, s^K(1+r) + T(s_1^P, s^K) + \Delta T)\right]. \quad (17) \end{aligned}$$

Define $\Delta s^K = f^K(s_1^P) - f^K(s_1^P + \delta^P)$ (i.e., the left-hand side of equation (12).)

Compare the solutions to equations (16) and (17). In the latter the kid receives an additional transfer amount ΔT. He consumes some of this in the second period, but also saves less and consumes more in the first period. Because of the strict concavity of U^K, the decrease in savings is less than the additional transfer amount. More specifically, the decrease in the amount returned from savings in the second period is less than the additional transfer amount. Therefore,

$$\Delta s^K(1+r) < \Delta T < \delta^P(1+r)$$

$$\Longrightarrow \Delta s^K < \delta^P.$$

Thus equation (12) holds with strict inequality.

2. We now turn our attention to equation (13), which can be rewritten as

$$|f^P(s_1^K) - f^P(s_1^K + \delta^K)| \leq \delta^K.$$

This section evaluates how the parent's optimal savings choice changes when the kid's savings increases by δ^K. Therefore, s^P is a variable in the following equations.

Again rewrite equation (6):

$$T(s^P, s_1^K) = \underset{T}{\text{argmax}} \left[u_2^P(s^P(1+r) - T, u_2^K(s_1^K(1+r) + T))\right], \tag{18}$$

and

$$T(s^P, s_1^K + \delta^K) = \underset{T}{\text{argmax}} \left[u_2^P(s^P(1+r) - T, u_2^K((s_1^K + \delta^K)(1+r) + T))\right]. \tag{19}$$

Here we define $\Delta T = T(s^P, s_1^K) - T(s^P, s_1^K + \delta^K)$.

In equation (19) the kid's second period wealth increases by $\delta^K(1+r)$. Thus the parent reduces his transfer amount from the amount given in equation (18). Because of the strict concavity of u_2^P and the strict concavity of u_2^K, the parent decreases his transfer amount by less than the kid's wealth increase. Thus we again have

$$0 < \Delta T < \delta^K(1+r).$$

Next rewrite equation (7):

$$\begin{aligned} f^P(s_1^K) = \underset{s^P}{\text{argmax}} \Big[&U^P(w^P - s^P, s^P(1+r) - T(s^P, s_1^K), \\ &U^K(w^K - s_1^K, s_1^K(1+r) + T(s^P, s_1^K)))\Big]. \end{aligned} \tag{20}$$

Similarly,

$$\begin{aligned} f^P(s_1^K + \delta^K) = \underset{s^P}{\text{argmax}} \Big[&U^P(w^P - s^P, s^P(1+r) - T(s^P, s_1^K + \delta^K), \\ &U^K(w^K - s_1^K - \delta^K, (s_1^K + \delta^K)(1+r) + T(s^P, s_1^K + \delta^K)))\Big] \\ = \underset{s^P}{\text{argmax}} \Big[&U^P(w^P - s^P, s^P(1+r) - (T(s^P, s_1^K) - \Delta T), \\ &U^K(w^K - s_1^K - \delta^K, (s_1^K + \delta^K)(1+r) + T(s^P, s_1^K) - \Delta T))\Big]. \end{aligned} \tag{21}$$

Define $\Delta s^P = f^P(s_1^K) - f^P(s_1^K + \delta^K)$ (the left-hand side of equation (13)) and compare the solutions to equations (20) and (21). We only need to consider two of the three differences. These two are the increase (of ΔT) in the parent's second period wealth and the increase (of $\delta^K(1+r) - \Delta T$) in the kid's second period wealth.[12]

The *aggregate* increase in second period wealth is $\delta^K(1+r)$. The parent, via his transfer and savings decisions chooses how this additional wealth will be distributed between himself and the kid. Since U^P is strictly concave, he distributes this wealth amongst his own first and second period consumption and the kid's second period consumption. Greater first period consumption implies less first period savings. We quantify the savings decrease by looking at the effect a savings decrease has on second period wealth. In the second period, the decreased return caused by a savings decrease ($\Delta s^P(1+r)$) must be less than the aggregate increase in second period wealth ($\delta^K(1+r)$).

Therefore,

$$\Delta s^P(1+r) < \delta^K(1+r)$$

$$\Longrightarrow \Delta s^P < \delta^K.$$

Thus equation (13) holds with strict inequality.

Given that (12) and (13) are satisfied with strict inequality we know (10) is satisfied.

We now turn our attention to showing $\exists \beta \in (0,1)$ satisfying equation (9).

Rewriting (10) gives

$$\frac{\rho(F(x), F(y))}{\rho(x,y)} < 1. \tag{22}$$

Since F and ρ are continuous, the left-hand side of (22) defines a continuous function from $\mathcal{S} \times \mathcal{S}$ to $[0,1]$.

Define β as follows:

$$\beta = \sup_{x,y \in \mathcal{S}} \frac{\rho(F(x), F(y))}{\rho(x,y)}.$$

[12]The third difference is the decrease (of δ^K) in the kid's first period consumption. Since the parent's transfer only directly affects the kid's second period consumption, the kid's first period consumption amount is not relevant to the parent's savings choice when taking the kid's savings amount as given.

Note that $0 \leq \beta \leq 1$.

It is known that a continuous function on a compact set achieves its supremum. That is, $\exists(\bar{x}, \bar{y}) \in \mathcal{S} \times \mathcal{S}$ such that

$$\beta = \frac{\rho(F(\bar{x}), F(\bar{y}))}{\rho(\bar{x}, \bar{y})}.$$

Suppose $\beta = 1$. Then $\rho(F(\bar{x}), F(\bar{y})) = \rho(\bar{x}, \bar{y})$ which contradicts equation (10).

Therefore it must be that $\beta < 1$.

Thus β satisfies (9).

The above arguments demonstrate that F is a contraction on $\mathcal{S}$. Then, by the contraction mapping theorem, we know F has a unique fixed point in $\mathcal{S}$.

Q.E.D.

3 Ricardian Equivalence

3.1 Introduction

The Ricardian equivalence theorem asserts that changing the timing of taxes has no real effect on the distribution of resources. To examine this theorem in the two-period two-agent setting I contrast the effects of two possible tax policies. The first imposes a lump-sum tax of τ on the parent in the first period. The second imposes a lump-sum tax of $\tau(1+r)$ on the kid in the second period.[13] The larger second period tax reflects the interest that accumulates when governments use deficit financing. Instead of thinking about imposing one policy or the other, we view this as substitution of the latter policy for the former and ask what effect this substitution has on the distribution of resources.

Ricardian equivalence predicts that, when faced with a reduction of his own taxes and

[13] For completeness we point out that government expenditures remain the same under each policy. Under the second policy, the government finances these expenditures by issuing a one period bond, at rate r, to some external agent.

a corresponding increase in his kid's taxes, the parent increases the size of his transfer to help the kid with his tax burden. As mentioned earlier, this assertion has been questioned in models which allow the parent and kid to behave strategically.

3.2 The Neutrality of Changing Statutory Tax Incidence

Theorem 2: *Assume we are initially in a Nash equilibrium. Consider a change in policy from a lump-sum tax of τ on the parent in the first period to a lump-sum tax of $\tau(1+r)$ on the kid in the second period. After the policy change there exists a new Nash equilibrium in which both individuals consume the same amounts as they did before the change.*

Proof: The proof of this theorem is presented in two parts.[14] First I show that the kid's optimal consumption choices remain unchanged *given a specific action by the parent.* Next I show that this parental action is optimal given the kid's unchanged choices. Thus we have a new Nash equilibrium, which we have already shown will be unique.

1. We start with the kid's problem. First, consider the transfer function of equation (6). The kid considers how his savings choice affects the transfer amount, given the parent's savings choice. Let $\tilde{s}^P$ be the parent's savings choice. Rewrite equation (6) to describe the transfer as a function of the kid's savings choice:

$$T(s^K) = \underset{T}{\text{argmax}} \left[u_2^P(\tilde{s}^P(1+r) - T, u_2^K(s^K(1+r)+T))\right]. \tag{23}$$

   Differentiating with respect to T gives the following first order condition:

$$u_{21}^P\Big(\tilde{s}^P(1+r) - T, u_2^K(s^K(1+r)+T)\Big) = u_{22}^P\Big(\tilde{s}^P(1+r) - T, u_2^K(s^K(1+r)+T)\Big) \cdot u^{K'}(s^K(1+r)+T) \tag{24}$$

   where $u_{2i}^P(x_1, x_2) = \partial u_2^P / \partial x_i$ for $i = 1, 2$.

   Next assume the policy change causes the parent to increase his savings amount by τ. Remember the second part of this proof demonstrates this is an optimal strategy for

[14]This proof draws on a similar result presented by Bergstrom, Blume and Varian (1986).

the parent. After the policy change the kid's period budget constraints become

$$\hat{c}_1^K \leq w^K - \hat{s}^K$$
$$\hat{c}_2^K \leq \hat{s}^K(1+r) + \hat{T} - \tau(1+r)$$

where the hat-ed terms represent post-policy change amounts. The last term is the tax collected from the kid in the second period.

Rewriting equation (24) with the post-policy change budget constraints gives

$$u_{21}^P\Big((\tilde{s}^P+\tau)(1+r)-\hat{T}, u_2^K((\hat{s}^K-\tau)(1+r)+\hat{T})\Big) = u_{22}^P\Big((\tilde{s}^P+\tau)(1+r)-\hat{T}, u_2^K((\hat{s}^K-\tau)(1+r)+\hat{T})\Big)\cdot u_2^{K'}\Big((\hat{s}^K-\tau)(1+r)+\hat{T}\Big). \tag{25}$$

Let $\hat{T}(\hat{s}^K)$ denote the solution to equation (25).

Comparison of equations (24) and (25) reveals the following:

$$\hat{T}(s^K) = T(s^K) + \tau(1+r). \tag{26}$$

It therefore follows that $\hat{T}'(s^K) = T'(s^K)$.

Now consider the kid's first period problem. Before the policy change, taking derivatives and combining first order conditions gives the following result:

$$U_1^K\Big(w^K - s^K, s^K(1+r)+T(s^K)\Big) = U_2^K\Big(w^K - s^K, s^K(1+r)+T(s^K)\Big)((1+r)+T'(s^K)) \tag{27}$$

where $U_t^K = \partial U^K/\partial c_t^K$ for $t = 1, 2$.

Equation (27) equates the first period utility decrease of an additional unit of savings and the second period utility increase of additional unit of savings. The last term $(T'(s^K))$ arises because the kid considers the effect his additional saving has on the parent's transfer choice.

Inserting the post-policy change budget constraints into equation (27) gives

$$U_1^K\Big(w^K - \hat{s}^K, (\hat{s}^K-\tau)(1+r)+\hat{T}(\hat{s}^K)\Big) = U_2^K\Big(w^K - \hat{s}^K, (\hat{s}^K-\tau)(1+r)+\hat{T}(\hat{s}^K)\Big)((1+r)+\hat{T}'(\hat{s}^K)). \tag{28}$$

Use equation (26) to rewrite (28) as follows:

$$U_1^K\Big(w^K - \hat{s}^K, (\hat{s}^K-\tau)(1+r)+T(\hat{s}^K)+\tau(1+r)\Big) = U_2^K\Big(w^K - \hat{s}^K, (\hat{s}^K-\tau)(1+r)+T(\hat{s}^K)+\tau(1+r)\Big)((1+r)+T'(\hat{s}^K)). \tag{29}$$

Clearly, the choice of $\hat{s}^K$ which satisfies (29) is identical to the choice of s^K which satisfies (27). Thus the kid's first period consumption choice is unchanged (i.e., $\hat{c}_1^K = c_1^K$). Combining equation (26) with the kid's post-policy change second period budget constraint gives $\hat{c}_2^K = c_2^K$. Thus the kid's post-policy change consumption choices are identical to his initial choices when the parent increases his savings choice by τ.

2. Now show the parent's optimal choice is increasing his savings amount by τ. Take as given the kid's unchanged first period consumption and savings amounts.

Again start with the transfer function. The parent anticipates how his savings choice will affect his later transfer choice, given the kid's savings choice. Let $\tilde{s}^K$ be the kid's savings choice. Rewrite equation (6) to describe the transfer as a function of the parent's savings choice as follows:

$$T(s^P) = \underset{T}{\text{argmax}} \left[u_2^P(s^P(1+r) - T, u_2^K(\tilde{s}^K(1+r) + T))\right]. \tag{30}$$

Differentiating with respect to T gives

$$u_{21}^P\left(s^P(1+r) - T, u_2^K(\tilde{s}^K(1+r) + T)\right) = u_{22}^P\left(s^P(1+r) - T, u_2^K(\tilde{s}^K(1+r) + T)\right) \cdot u_2^{K'}(\tilde{s}^K(1+r) + T). \tag{31}$$

Again, $u_{2i}^P(x_1, x_2) = \partial u_2^P / \partial x_i$ for $i = 1, 2$.

Under the initial policy the parent's period budget constraints are

$$c_1^P + s^P \leq w^P - \tau$$

$$c_2^P + T \leq s^P(1+r).$$

After the policy change the parent's period budget constraints become

$$\hat{c}_1^P + \hat{s}^P \leq w^P$$

$$\hat{c}_2^P + \hat{T} \leq \hat{s}^P(1+r)$$

where hat-ed terms represent the new choices.

Rewrite (31) with the post-policy change budget constraints:

$$u_{21}^P\left(\hat{s}^P(1+r) - \hat{T}, u_2^K(\tilde{s}^K(1+r) + \hat{T} - \tau(1+r))\right) = u_{22}^P\left(\hat{s}^P(1+r) - \hat{T}, u_2^K(\tilde{s}^K(1+r) + \hat{T} - \tau(1+r))\right) \cdot u_2^{K'}(\tilde{s}^K(1+r) + \hat{T} - \tau(1+r)). \tag{32}$$

Let $\hat{T}(\hat{s}^P)$ denote the solution to equation (32).

Comparison of equations (31) and (32) reveals that

$$\hat{T}(s^P + \tau) = T(s^P) + \tau(1+r). \tag{33}$$

Now consider the parent's first period problem. Before the policy change, taking derivatives and combining first order conditions gives the following results:

$$\begin{aligned} &U_1^P\Big((w^P - s^P - \tau), (s^P(1+r) - T(s^P)), U^K(w^K - \tilde{s}^K, \tilde{s}^K(1+r) + T(s^P))\Big) = \\ &\quad U_2^P\Big((w^P - s^P - \tau), (s^P(1+r) - T(s^P)), U^K(w^K - \tilde{s}^K, \tilde{s}^K(1+r) + T(s^P))\Big)(1+r), \end{aligned} \tag{34}$$

and

$$\begin{aligned} &U_2^P\Big((w^P - s^P - \tau), (s^P(1+r) - T(s^P)), U^K(w^K - \tilde{s}^K, \tilde{s}^K(1+r) + T(s^P))\Big) = \\ &\quad U_3^P\Big((w^P - s^P - \tau), (s^P(1+r) - T(s^P)), U^K(w^K - \tilde{s}^K, \tilde{s}^K(1+r) + T(s^P))\Big) \cdot \\ &\qquad U_2^K\Big(w^K - \tilde{s}^K, \tilde{s}^K(1+r) + T(s^P)\Big), \end{aligned} \tag{35}$$

where $U_j^P = \partial U^P(x_1, x_2, x_3)/\partial x_j$ for $j = 1, 2, 3$.

Equation (34) gives the parent's first order condition for the amount of wealth put into savings. The first period utility decrease from saving an additional unit must be equal to the second period utility increase from getting $(1+r)$ units back from savings. Equation (35) gives the parent's first order condition for the transfer amount. The utility decrease caused by foregoing one additional unit of consumption must equal the marginal utility of increasing the kid's utility times the kid's utility increase from an additional unit of consumption.

After the policy change, the parent's first order conditions are

$$\begin{aligned} &U_1^P\Big(w^P - \hat{s}^P, \hat{s}^P(1+r) - \hat{T}(\hat{s}^P), U^K(w^K - \tilde{s}^K, (\tilde{s}^K - \tau)(1+r) + \hat{T}(\hat{s}^P))\Big) = \\ &U_2^P\Big(w^P - \hat{s}^P, \hat{s}^P(1+r) - \hat{T}(\hat{s}^P), U^K(w^K - \tilde{s}^K, (\tilde{s}^K - \tau)(1+r) + \hat{T}(\hat{s}^P))\Big)(1+r), \end{aligned} \tag{36}$$

and

$$\begin{aligned} &U_2^P\Big(w^P - \hat{s}^P, (\hat{s}^P(1+r) - \hat{T}(\hat{s}^P)), U^K(w^K - \tilde{s}^K, (\tilde{s}^K - \tau)(1+r) + \hat{T}(\hat{s}^P))\Big) = \\ &U_3^P\Big(w^P - \hat{s}^P, (\hat{s}^P(1+r) - \hat{T}(\hat{s}^P)), U^K(w^K - \tilde{s}^K, (\tilde{s}^K - \tau)(1+r) + \hat{T}(\hat{s}^P))\Big) \cdot \\ &\qquad U_2^K\Big(w^K - \tilde{s}^K, ((\tilde{s}^K - \tau)(1+r) + \hat{T}(\hat{s}^P))\Big). \end{aligned} \tag{37}$$

Use equation (33) and compare the pre- and post-policy change first order conditions. When s^P satisfies equations (34) and (35), then $\hat{s}^P = s^P + \tau$ satisfies equations (36) and (37).

Using the parent's new budget constraints to determine new consumption amounts gives

$$\hat{c}_1^P = c_1^P \text{ and } \hat{c}_2^P = c_2^P.$$

Thus the new Nash equilibrium is $\hat{s}^K = s^K$ and $\hat{s}^P = s^P + \tau$. This equilibrium provides the same consumption amounts as did the initial equilibrium.

Q.E.D.

3.3 Discussion

First consider the intuition underlying this result. The policy change is analogous to a redistribution of wealth from kid to parent. When choosing savings amounts both parent and kid know how second period wealth will be divided via the transfer function. The kid faces a new tax and realizes the parent's second period wealth has increased by the amount of the tax. Thus aggregate second period wealth is unchanged. Significant here is that the redistribution causes the parent to increase his transfer by the amount of the redistribution. Then, since the kid's first period endowment is unchanged, he effectively perceives the same resource constraint as before the policy change. By the axiom of revealed preference, his initial optimal choice continues to be his optimal choice. The argument is similar for the parent. He effectively maximizes the family's utility subject to a family budget constraint. He considers the kid's wealth, as well as his own, when choosing his savings and transfer amounts. The redistribution of wealth from kid to parent does not change the family's total wealth, so he too effectively perceives an unchanged budget constraint. Again, the axiom of revealed preference dictates that the initial choice is still the optimal choice.

We can extend this result to a policy shift from a second period tax on the kid to a first period tax on the parent – a public transfer from parent to kid. One stipulation we need to add is that the new tax on the parent cannot exceed his initial savings amount. These two results together can be used to demonstrate the neutrality of a range of policy options including deficit financing and social security programs.

To repeat, this result is not surprising given the literature on voluntary contributions to a public good. Bergstrom, Blume and Varian (1986) and Warr (1983) demonstrate that a wealth redistribution amongst contributors to a public good has no effect of the provision of the public good. In our model, the public good is the kid's utility. The parent and kid both enjoy the kid's utility non-rivalrously and without possibility of exclusion. Thus we could expect that changing the timing of taxes has no effect on the final consumption amounts.

The result is also consistent with Varian's (1994) study of private provision of public goods. He compares public good provision when contribution choices are simultaneous or sequential. He shows underprovision of a public good results when the individual valuing it most chooses his contribution first. This underprovision is relative to the amount provided under simultaneous choices.

To apply this result, we distinguish between the kid's first and second period utilities by recalling that the parent only directly affects the kid's second period consumption. Then the kid's second period utility is the public good to which both individuals make voluntary contributions – the parent via transfer and the kid via savings. The kid's first period utility is considered just a positive externality for the parent.

In our model the kid values the public good most. In the manipulative framework he makes his contribution choice (i.e., his savings amount) first. In precommitment both individuals choose their contributions simultaneously. The numerical example of the following section shows that the kid's second period consumption is lower under the manipulative regime than under precommitment.

3.4 A Numerical Example

The following example is presented as an illustration of the foregoing arguments. This example uses a common CES period utility function with U^P linear in U^K. That is:

$$U^K(c_1^K, c_2^K) = \frac{(c_1^K)^\gamma}{\gamma} + \beta\frac{(c_2^K)^\gamma}{\gamma}$$

and

$$U^P(c_1^P, c_2^P, U^K) = \frac{(c_1^P)^\gamma}{\gamma} + \beta\frac{(c_2^P)^\gamma}{\gamma} + \rho U^K,$$

where $\rho \in (0,1)$ is the degree of intergenerational altruism. $\beta \in (0,1)$ is an individual's intertemporal discount rate.

With this parameterization the best response functions are linear and have different slopes. The transfer function T is also linear in both arguments.

Table 1 presents the results of a specific paramaterization of this example. These results are representative of the variety of different parameterizations evaluated. The specific parameter values of this example are as follows: $\gamma = -2; \beta = 0.7; \rho = 0.6; w^P = 12; w^K = 8; \tau = 2$ and $r = 1/\beta - 1$.

The first two columns of Table 1 present results for the precommitment and manipulative regimes respectively. A tax of τ is collected from the parent in the first period in each case. Notice that when manipulation is allowed, the kid's first period consumption increases while second period consumption decreases. Correspondingly, the parent chooses a larger transfer and the kid experiences a net utility increase. This illustrates the manipulative potential of the kid as discussed earlier.

Table 1: Precommitment vs. Manipulative Specifications

Precommitment - Tax on Parent	Manipulative - Tax on Parent	Manipulative - Tax on Kid
$C_1^P = 5.746$	$C_1^P = 5.318$	$C_1^P = 5.318$
$C_2^P = 5.746$	$C_2^P = 5.318$	$C_2^P = 5.318$
$C_1^K = 4.843$	$C_1^K = 5.821$	$C_1^K = 5.821$
$C_2^K = 4.843$	$C_2^K = 4.485$	$C_2^K = 4.485$
$Beq = 0.332$	$Beq = 1.371$	$Beq = 4.448$
$U^P = -0.04749$	$U^P = -0.04935$	$U^P = -0.04935$
$U^K = -0.03624$	$U^K = -0.03216$	$U^K = -0.03216$

The second and third columns present typical results from comparison of the two tax policies. As predicted, the only difference is an increase of $\tau(1+r)$ in the transfer amount. All consumption amounts and utilities are identical. This illustrates that Ricardian equivalence holds in the manipulative environment.

4 Concluding Remarks

The Ricardian Equivalence Theorem presents a powerful result: there are no real redistributive effects resulting from deficit financing by governments. Parents merely increase their intergenerational transfers to help children pay increased future taxes. Other works have demonstrated the theorem's limited scope of application and that the data does not fully support the formulation used to develop this result.

This paper presents a two-period model of strategic parent-child interactions and asks whether Ricardian equivalence holds in this framework. I present a proof, for fairly general utility functions, that Ricardian equivalence does hold in this framework. The literature on private provision of public goods provides a foundation for this result.

Why does this result differ from that of similar studies? To answer this we briefly consider two related studies. Bruce and Waldman (1990) also study a two-period model with one-sided altruism. They allow private transfers in both periods and study public transfers from parent to kid. Each consumer chooses an action, which influences both consumers' second period incomes. Bruce and Waldman discuss the possibility of allowing the parent to make a first period transfer which leaves him impoverished in the second period. Thus he chooses not to give a second period transfer. Without a second period transfer the kid acts selfishly. He chooses an action which maximizes his own income, potentially at the expense of the parent's income. Thus the public transfer is not offset by private action. The failure of Ricardian equivalence stems largely from allowing the parent to impoverish himself, a strategy unlikely to be pursued in practice. While not explicitly discussed, my conjecture is that public transfers from kid to parent would be neutral in their model.

Kotlikoff, Razin and Rosenthal (1990) study a two-stage game with two-sided altruism. The two-sided altruism can lead to a scenario of "competing transfers". Negotiations about the direction and size of transfers can lead to a range of asset distributions in which no private transfers occur. When government redistributions begin or end within this range, private transfers do not fully offset public transfers. It is the "competing transfer" scenario, a result of two-sided altruism, which negates Ricardian equivalence.

The final question: where to go from here? Already underway is the work of incorporating this form of strategic behavior into the general equilibrium framework of an overlapping generations model. By using three-period lived consumers I obtain a sequence of the two-period games described in this paper. Today's potentially manipulative kid becomes tomorrows potentially manipulated parent. The goal is to explore the possibility that the dynamics of an infinite horizon model affect the debt neutrality results of the static model. Given that each two-period game has a unique subgame perfect equilibrium, extending to an overlapping generations framework should not affect the result. I expect to find that Ricardian equivalence holds in this model as well.

One interesting line of research lies in studying other types of strategic behavior. Bernheim, Shleifer and Summers (1985) suggest bequests are part of an exchange with children. They use a one-period model and show Ricardian equivalence fails in this environment. Two questions arise here. First, does this result extend to a dynamic environment? Second, how closely is Ricardian equivalence satisfied in their model?[15] Another promising line would be an empirical evaluation of the ability of different models of intrafamily bargaining to explain observed transfer behaviors.

This work is a step towards developing a better understanding of the implications of including intrafamily strategic behavior in analysis of government financing decisions. The exact nature of parent-child interactions is a subject of continuing debate amongst economists. This work demonstrates that one possible specification of these interactions presents no implications for government financing decisions. Perhaps the main value of this work lies in resurrecting hope for the Ricardian equivalence theorem. Many works find conditions which negate debt neutrality. It may be refreshing to proponents to have one affirm it.

[15]This question can be asked of other possible failures of Ricardian equivalence as well.

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