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Extraction of pavement and soil thermal diffusivity from measured temperature times series

by

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Abstract

This report describes several methods to extract pavement thermal diffusivity from pavement temperature measurements at two or more depths. Two methods use analytic solutions for heat transfer in an infinite solid to relate the attenuation of diurnal temperature change with depth to thermal diffusivity. The first approach considers the surface temperature forcing to be a simple sinusoidal function with a period of one day. The second method considers the surface temperature forcing to be a general periodic signal that can be decomposed with a Fourier series. The accuracy of these two methods are limited by non-homogeneous nature of pavement/subgrade/soil systems. The third method uses a one-dimensional finite difference heat transfer model to extract thermal diffusivity from measured pavement temperature. This method requires more computational effort, but can take into account the variation in thermal diffusivity between the pavement and underlying layers.

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1. Introduction

Pavement temperature, along with traffic loading, is a primary factor in determining the service life of pavement. Pavement temperature, in turn, is determined by surface heat fluxes and the thermal properties of the pavement and underlying base materials. In general, laboratory testing has been used to measure thermal conductivity and specific heat of pavement materials, using steady state and/or transient tests (Luca and Mrawira, 2005). In this report, several methods are described to extract pavement thermal diffusivity from pavement temperature measurements at two or more depths.

The first approach considers the surface temperature forcing to be a simple sinusoidal function with a period of one day. The second method considers the surface temperature forcing to be a general periodic signal that can be decomposed with a Fourier series. The third method uses a one-dimensional finite difference heat transfer model to extract thermal diffusivity from measured pavement temperature. In all cases, the analyses were applied to 15 minute temperature data from MnROAD test section 33, with an asphalt thickness of 10.1 cm (4 inches). The test section also includes measured temperatures at five other depths in the sub-grade and base layers (Figure 1).

2. Exponential decay of a sinusoidal surface temperature variation

2.1 Methodology

For a semi-infinite, uniform slab with no internal heat generation and a specified surface temperature or heat flux periodic in time, analytic solutions exist for the amplitude and phase of the temperature profile in the slab (Eckert and Drake 1972). For a surface temperature in the form of a simple sinusoid, $T(t) = T_o \,\mathrm{Cos}(2\pi\, t/\tau)$, the temperature as a function of depth and time is given by:

$$T(z,t) = T_o \,\mathrm{Exp}\left(-\sqrt{\frac{\pi}{\alpha\tau}}\, z\right)\mathrm{Cos}\left(\frac{2\pi t}{\tau} - \sqrt{\frac{\pi}{\alpha\tau}}\, z\right) \qquad (1)$$

where T is temperature, T_o is the surface temperature amplitude, t is time, z is depth normal from the surface, a is thermal diffusivity, and τ is the period. Thermal diffusivity determines the decrease in amplitude with depth and the phase shift with respect to the surface temperature. Given measured pavement temperature time series at two depths with periodic variation, the thermal diffusivity can be extracted from the measured variation of amplitude and phase with depth.

The first method assumes that the surface or near-surface temperature of the pavement can be represented as a pure sinusoid with a period of one day. Temperature time series from MnROAD cell 33 were analyzed as follows:

1) The diurnal variation was separated from temperature variations at longer time scales (detrended) by subtracted the 24 hour running average from the raw temperature data for each depth (Figure 2).
2) The phase relationship between the diurnal variation at 2.4 cm and 7.6 cm was then examined by calculating the correlation coefficient between the two time series for varying phase shift (Figure 3).
3) The optimum phase shift was then applied to 7.6 cm temperature data, and the detrended, phase-shifted temperature at 7.6 cm was plotted against the detrended 2.4 cm data (Figure 4).

The slope of the relationship, S, gives the decrease in diurnal temperature amplitude from 2.4 cm to 7.6 cm depth, and the thermal diffusivity was calculated as:

$$\alpha_1 = \frac{\pi}{\tau}\left(\frac{\Delta z}{-\ln(S)}\right)^2 \qquad (2)$$

where Δz is the vertical distance between the two measurements. The optimum phase angle, ϕ_o, was then used to calculate a second value of the pavement thermal diffusivity:

$$\alpha_2 = \frac{\tau}{\pi}\left(\frac{\Delta z}{2\phi_o}\right)^2 \qquad (3)$$

2.2 Results

The above procedure was performed on segments of temperature data varying from 1 to 14 days. Segment lengths of 7 to 10 days gave the best relationships, i.e. the highest r^2 in determining the slope S (Figure 4). A full year of data (Jan 1 – Dec 31, 2002) from cell 33 was used to calculate α_1 and α_2 using non-overlapping 7 day segments, giving 52 separate values of α_1 and α_2 for the year (Figure 5). While each value of diffusivity is reasonably constant from April through October, α_2 is consistently about 30% higher than α_1. Much more variation in diffusivity was seen in the winter months, with up to an order of magnitude disagreement between α_1 and α_2, and lower r^2 in the relationship between the two temperatures. Figure 6 gives the variation in diffusivity over one year, where the values of α_1 and α_2 were calculated using two day blocks of temperature data. The results are very similar to those obtained using 7 day blocks (Figure 5), but with a slightly lower overall r^2; 0.92 for 2 day blacks versus 0.94 for 7 day blocks.

The same procedure was used to extract the thermal diffusivity of the cell 33 sand/gravel base layer using temperature measurements at 13 and 38 cm below the surface, and for the underlying clay layer using temperature measurements at 43 and 61 cm below the surface. The resulting thermal diffusivity values are given in Figures 7. Compared to the pavement, there is more uncertainty in the extracted diffusivity for the base layers due to smaller diurnal temperature variations. As with the asphalt layer, there is no evidence of a systematic variation in diffusivity with season.

Table 1 gives the average and standard deviation of each diffusivity value for each layer, for the period May 1 to September 1, 2002. While the standard deviation is similarly small for diffusivity extracted from amplitude or phase, the diffusivity values extracted from phase (α_2) are significantly higher compared to those obtained from amplitude (α_1). These values are somewhat lower than literature values of asphalt thermal diffusivity summarized by Luca and Mrawira (2005), ranging from 3.5E-07 to 14.4E-07 m^2/s, with the authors own measurements giving values close to 5.0E-07 m^2/s.

An advantage of using in-situ temperature data to extract material properties is the ability to examine long term trends, e.g. aging effects. Table 2 gives the extracted diffusivity for the asphalt test section for six years, 2000 – 2005. α_1 and α_2 exhibit slight trends over six years, but in opposite direction (Figure 8), so that no meaningful trends can be extracted from this analysis.

Table 1. Extracted diffusivity mean ($\bar{x}$) and standard deviation (σ) for asphalt, sand/gravel, and clay for the period May 1 to September 1, 2002.

Material	α_1 (m^2/sec)		α_2 (m^2/sec)	
	$\bar{x}$	σ	$\bar{x}$	σ
Asphalt (4" thick)	2.25E-07	1.94E-08	3.00E-07	3.22E-08
Sand/Gravel (12" thick)	1.11E-06	3.13E-08	1.26E-06	5.40E-08
Clay	4.31E-06	3.93E-07	2.82E-06	3.69E-07

Table 2. Extracted diffusivity mean ($\bar{x}$) and standard deviation (σ) for asphalt for the period May 1 to September 1, 2000 - 2005.

Year	α_1 (m^2/sec)		α_2 (m^2/sec)	
	$\bar{x}$	σ	$\bar{x}$	σ
2000	2.29E-07	1.47E-08	2.77E-07	1.74E-08
2001	2.17E-07	1.36E-08	2.93E-07	3.28E-08
2002	2.25E-07	1.94E-08	3.00E-07	3.22E-08
2003	2.23E-07	1.15E-08	2.92E-07	2.43E-08
2004	2.10E-07	6.05E-09	2.97E-07	2.96E-08
2005	2.09E-07	8.41E-09	3.17E-07	3.85E-08

3. Exponential decay of a periodic surface temperature variation with multiple frequency components

3.1 Methodology

The diurnal variation in pavement temperature is not a single frequency sinusoid, even for cloudless days (Figure 2), mainly due to the non-sinusoidal nature of solar radiation. However, non-sinusoidal periodic functions can be decomposed into the sum of sinusoidal functions using a Fourier series, e.g as in Equation 4:

$$T(t) = a_0 + a_1 \cos(\omega t) + a_2 \cos(2\omega t) + \ldots \qquad (4)$$

where ω is frequency, t is time, and the coefficients a_i are complex numbers that represent both a magnitude and a phase shift for each frequency component.

For a varying surface temperature composed of multiple frequency components, each component will decay with depth with a distinct decay rate and phase shift, as given by Equation 1. For a linear heat conduction problem, i.e. constant thermal properties, the temperature variation at some depth can be found as the sum of the response to each frequency component of the surface temperature. This method was applied to the MnROAD cell 33 temperatures using Matlab, as follows:

1) A Fourier transform was applied to a temperature time series at 2.5 cm depth to find the complex Fourier coefficients, A_i.
2) For each frequency component, the attenuation coefficient and phase shift was calculated as in Equation 1, based on a starting value of the thermal diffusivity.
3) The attenuation and phase shift was applied to each complex Fourier coefficient, a_i, and the corresponding time series was calculated with an inverse Fourier transform.
4) The calculated and measured time series at 7.5 cm depth were then compared, and the root-mean-square (RMSE) error was calculated
5) Steps 2 – 5 were repeated for different values of thermal diffusivity, and an optimum value of thermal diffusivity was found by minimizing the RMSE error.

3.2 Results

Method 2 was applied to temperature time series from test cell 33 in one week time increments. Figure 9 gives an example of a Fourier transform of a temperature time series from cell 33. Most of the frequency content lies at frequencies of 1 cycle/day and 2 cycles/day, with much smaller peaks at higher harmonics. Figure 10 gives an example of a temperature time series at 7.5 cm depth synthesized from the inverse-FFT, i.e. step 3 in the procedure. The synthesized time series for May, 2004 matches the measured temperature very closely using an optimized value of α = 2.1 E-07 m^2/sec. Figure 11 gives the calculated thermal diffusivity for each week of 2004 using the FFT method. The diffusivity value is in the range of 2.1 to 2.5 m^2/sec for April through October, with RMSE less than 0.05 C. There is a modest variation in diffusivity over this period, with lower values in the mid-year. Figure 12 gives a relationship between extracted diffusivity and mean weekly temperature for April through October, 2002.

The same analysis was performed for the other five years of pavement temperature data from cell 33, with very similar results (Figure 13). For November through March, the RMSE of the synthetic temperature fit is much higher, and the extracted diffusivity values vary greatly. This variation is likely due to freeze/thaw processes in the pavement, base and subgrade layers, which introduce non-linear thermal behavior to the linear heat conduction problem.

Overall, method 2 gave more consistent results than method 1. The mean thermal diffusivity for May-September for each year (2000-2005) calculated using the two methods is given in Table 3.

4. Extraction of thermal diffusivity from calibration of a finite difference model

4.1 Methodology

The third method used to extract thermal diffusivity used a one-dimensional finite difference model for the pavement, subgrade and soil layers. The model is described in the report, "Simulation and Characterization of Asphalt Pavement Temperatures", Herb et al., 2006. The model is typically used to simulate pavement and soil temperature time series based on surface heat transfer calculated from measured climate data. For the purposes of the present study, the measured pavement temperature at 2.5 cm depth was used as the surface boundary condition. The model was then used to simulate sub-surface temperature time series, and the thermal diffusivity values of the pavement, subgrade and soil were adjusted to best match measured temperatures.

4.2 Results

Optimization results using the finite difference model are given in Figure 14. Two cases were examined:

Case 1: Independent values of pavement, subgrade and soil diffusivity

Setting the subgrade and soil thermal diffusivity equal to a constant, 1.0E-06 m^2/s, the optimum pavement thermal diffusivity was 3.0 E-07 m^2/s, based on minimizing the RMSE of the simulated pavement temperature from June to September, 2004.

Case 2: Constant diffusivity for all layers

Setting the subgrade and soil thermal diffusivity equal to that of the pavement, the optimum pavement thermal diffusivity was significantly lower, 2.25 E-07 m^2/s. This is close to the values of diffusivity obtained in methods 1 and 2, which assume uniform thermal properties.

The difference in diffusivity between case 1 and case 2 may be considered to be the error associated with the assumption of a uniform slab used for methods 1 and 2, i.e. the application of heat transfer equations derived for a uniform slab to a non-uniform series of layers.

Table 3. Extracted diffusivity for test cell 33 asphalt for the period May 1 to September 1, 2000 - 2005.

Year	Single Frequency, Amplitude	Single Frequency, Phase	Multiple Frequency	Finite Difference
2000	2.29E-07	2.77E-07	2.27E-07	
2001	2.17E-07	2.93E-07	2.25E-07	
2002	2.25E-07	3.00E-07	2.29E-07	3.0E-07
2003	2.23E-07	2.92E-07	2.30E-07	
2004	2.10E-07	2.97E-07	2.25E-07	
2005	2.09E-07	3.17E-07	2.27E-07	

5. Conclusions

The single frequency thermal diffusivity extraction method (method 1) works best for large, smoothly periodic temperature variations. Days with cloud cover and/or snow cover give a non-sinusoidal variation in temperature so that equations (1) – (3) do not accurately capture the variation in temperature with depth and time. Temperature data that includes freeze/thaw cycles in the pavement or underlying soil result in substantial errors in diffusivity, because the freeze/thaw processes introduce non-linear behavior to the heat conduction through the soil. Finally, the variation in thermal properties between asphalt, sub-grade, and base layers also introduces errors in the extracted diffusivity. The best relationships were obtained for the near surface data, e.g. in the asphalt layer, while measurements in the base layers gave less certain diffusivity values. There is no clear evidence of a systematic variation in diffusivity with age, but there is evidence of a systematic variation with mean temperature at seasonal time scales.

The multiple frequency method addresses the problem of non-sinusoidal temperature functions, and produced more consistent results than the single frequency analysis for days with varying cloud cover, etc. The multiple frequency method is still susceptible to freeze thaw cycles and to non-constant material properties over depth. Extracting material properties with the finite difference model allows for both non-sinusoidal temperature forcing and non-uniform material properties over depth, and likely results in the most representative values for thermal diffusivity of the 3 methods. With further work, the effects of freeze-thaw cycles could also be included in the finite difference model.

References

Eckert, E.R.G. and Drake, R.M., Jr. (1972). *Analysis of Heat and Mass Transfer*. McGraw-Hill, New York.

Herb, W.R., Marasteanu, M., and H.G. Stefan, 2006. Simulation and Characterization of Asphalt Pavement Temperatures, Saint Anthony Falls Laboratory Technical Report #480.

Luca, J. and Mrawira, D., 2005. New Measurement of Thermal Properties of Superpave Asphalt Concrete, ASCE Journal of Materials in Civil Engineering, 17(1):72-79.

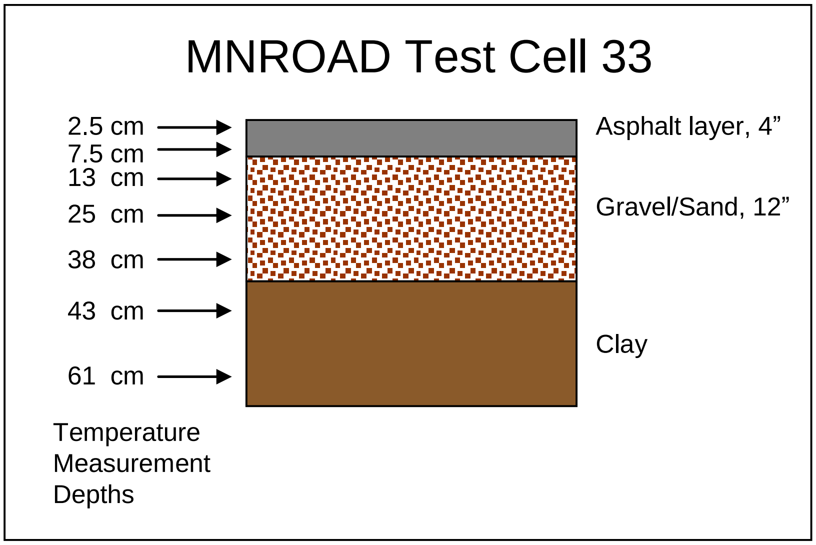

Figure 1. Schematic cross section of MnROAD test cell 33, showing the relative position of the seven thermocouples with respect to the pavement, sub-grade, and base layers.

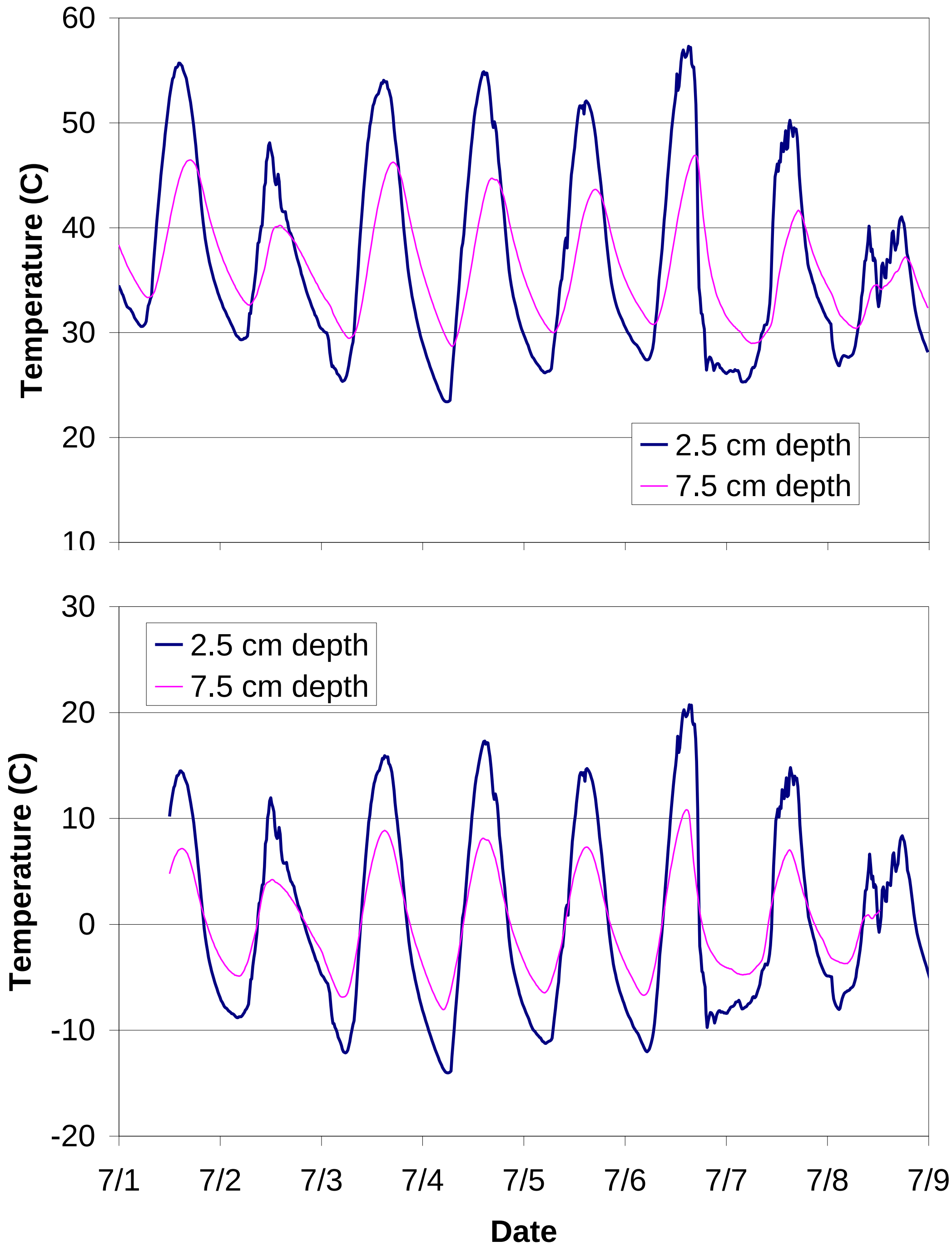

Figure 2. Time series of measured temperature at 2.4 and 7.6 cm depth for an asphalt test section (MNROAD cell 33) for July 1 – July 8, 2002. The upper panel gives the raw 15 minute temperature data, while the lower panel gives the detrended data after the 24 hour running average has been subtracted from each raw data set, and a phase shift of 2.25 hours (lag) has been applied to the 7.6 cm depth data.

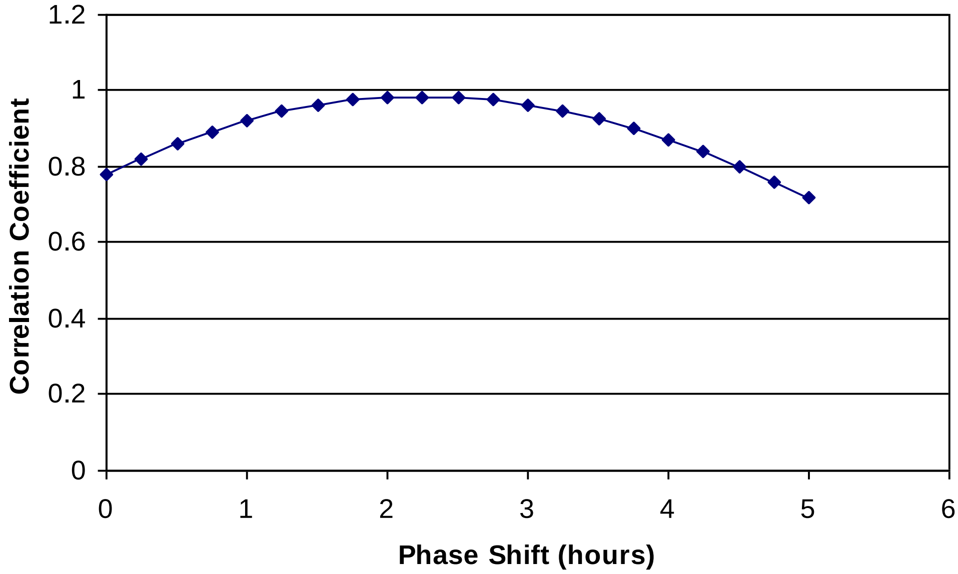

Figure 3. Correlation coefficient versus time phase shift for measured temperature at 2.4 and 7.6 cm depth for an asphalt test section (MNROAD cell 33) for July 1 – July 8, 2002. The phase shift values given were applied to the 7.6 cm depth temperature data.

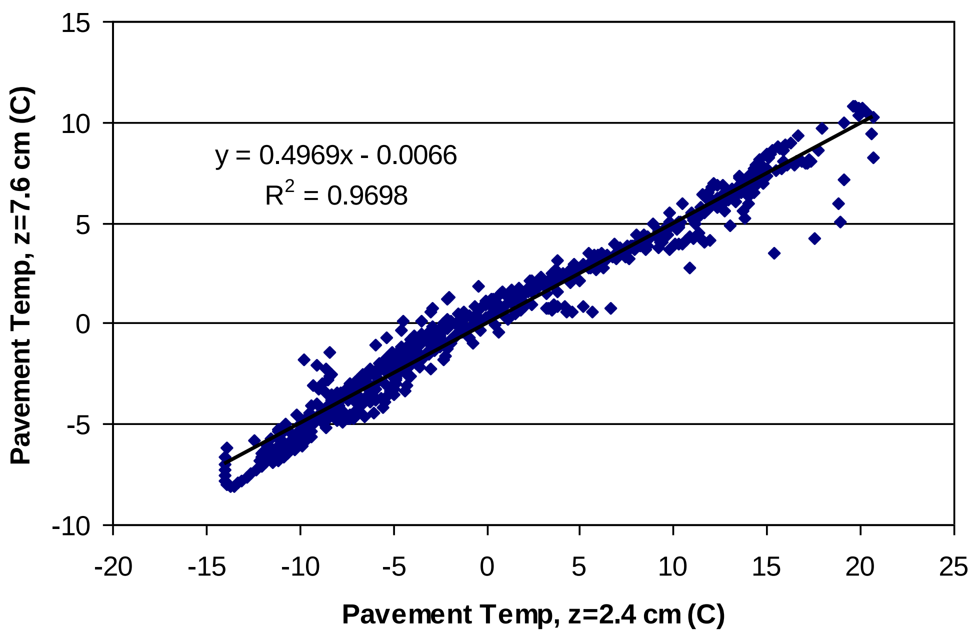

Figure 4. Measured temperature at 7.6 cm depth versus 2.4 cm depth for an asphalt test section (MNROAD cell 33) for July 1 – July 8, 2002. A phase shift of 2.25 hours (lag) was applied to the temperature data for 7.6 cm depth to achieve the best linear fit.

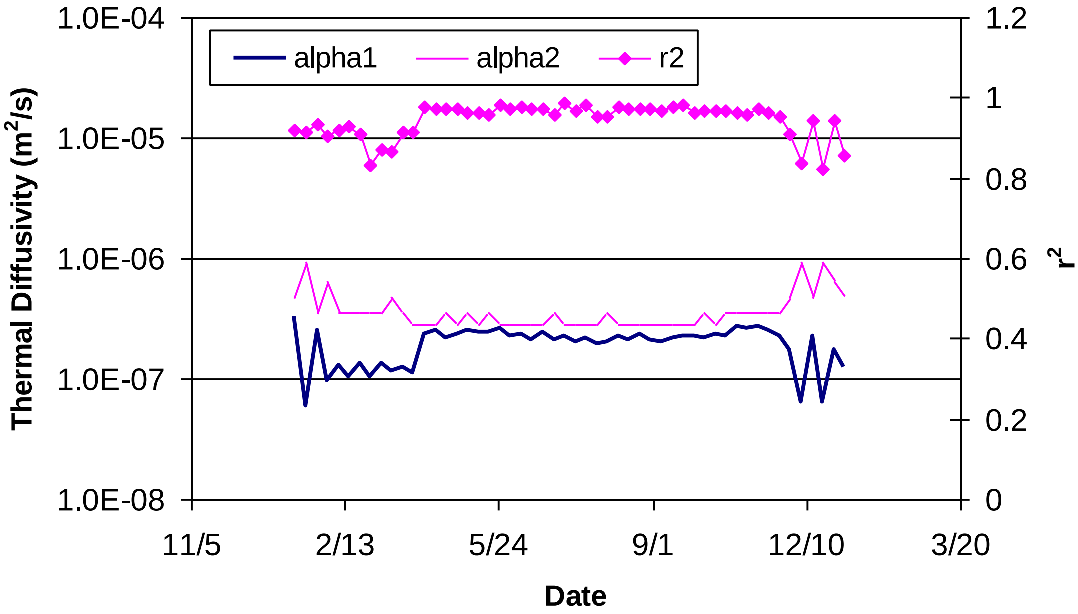

Figure 5. Thermal diffusivity in asphalt pavement layer (MNROAD cell 33) extracted from measured temperatures at 2.4 cm and 7.6 cm depth using amplitude (α_1) and phase (α_2) information in 7 day blocks of 15 minute data (January 1 – December 31, 2002). The fit coefficient for the amplitude relationship (Figure 4) is also given.

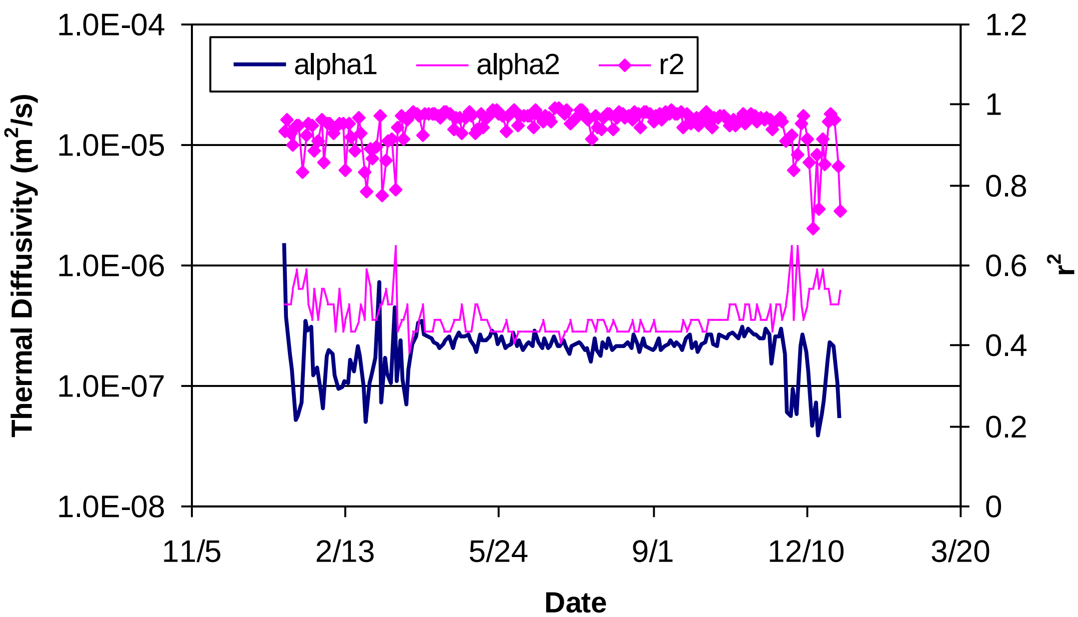

Figure 6. Thermal diffusivity in asphalt pavement layer (MNROAD cell 33) extracted from measured temperatures at 2.4 cm and 7.6 cm depth using amplitude (α_1) and phase (α_2) information in 2 day blocks of 15 minute data (January 1 – December 31, 2002).

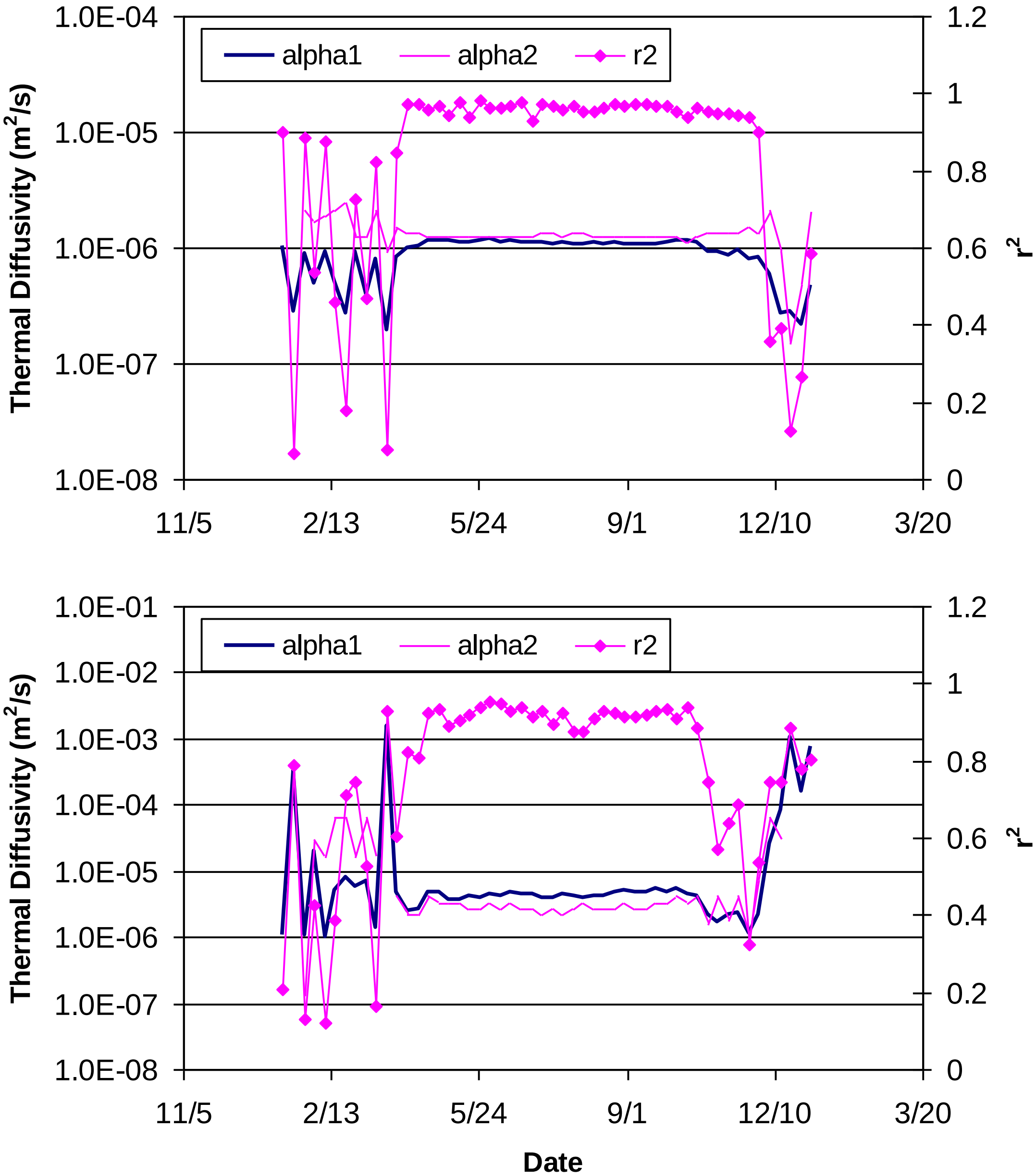

Figure 7. Thermal diffusivity in sand/gravel subgrade layer (upper panel) and the clay base layer (lower panel) for MnROAD cell 33, January 1 – December 31, 2002.

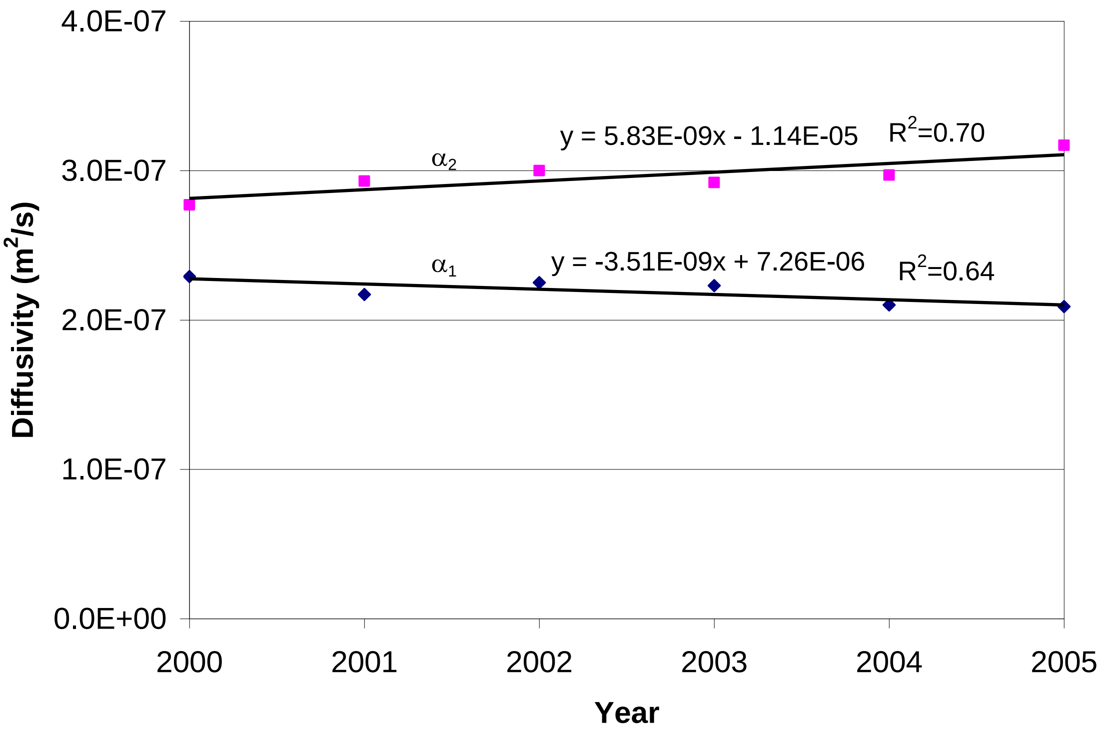

Figure 8. Thermal diffusivity in sand/gravel subgrade layer (upper panel) and the clay base layer (lower panel) for MnROAD cell 33, January 1 – December 31, 2002.

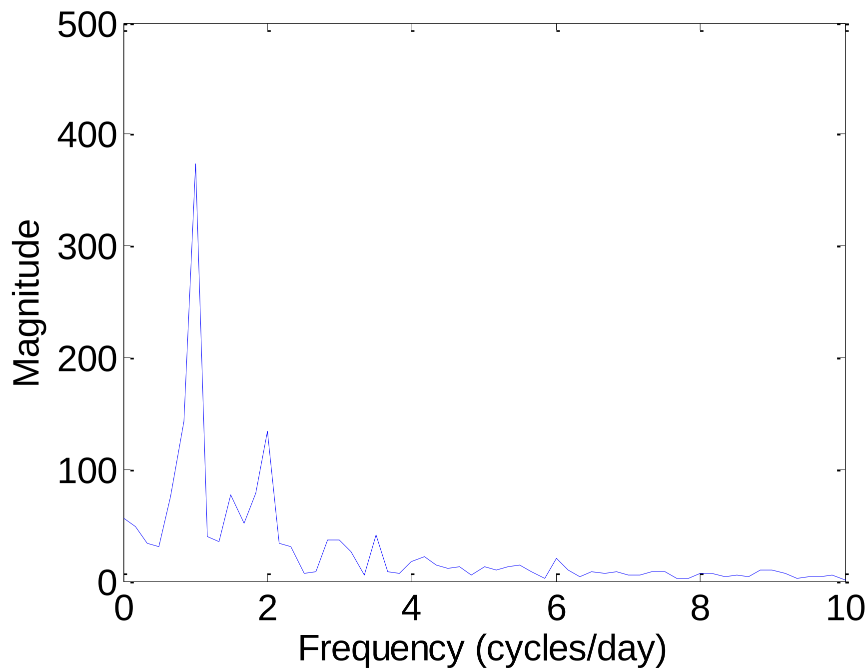

Figure 9. Fourier transform magnitude versus frequency for 7 day pavement temperature time series (June, 2004) at 2.5 cm depth.

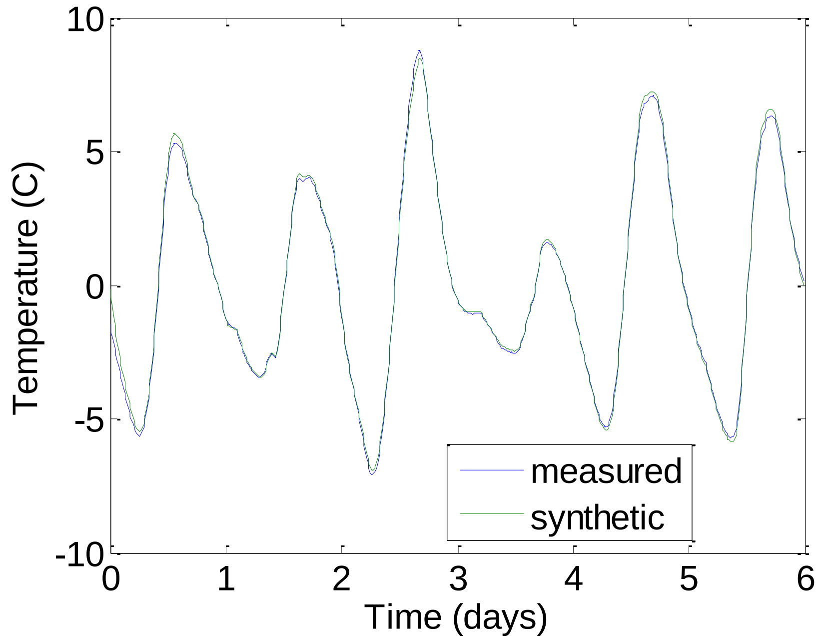

Figure 10. Measured temperature and synthesized temperature time series at 7.5 cm depth. The RMSE of the synthetic time series is 0.04 °C. The temperature shown is the detrended temperature, i.e. the fluctuation from the mean for the time period.

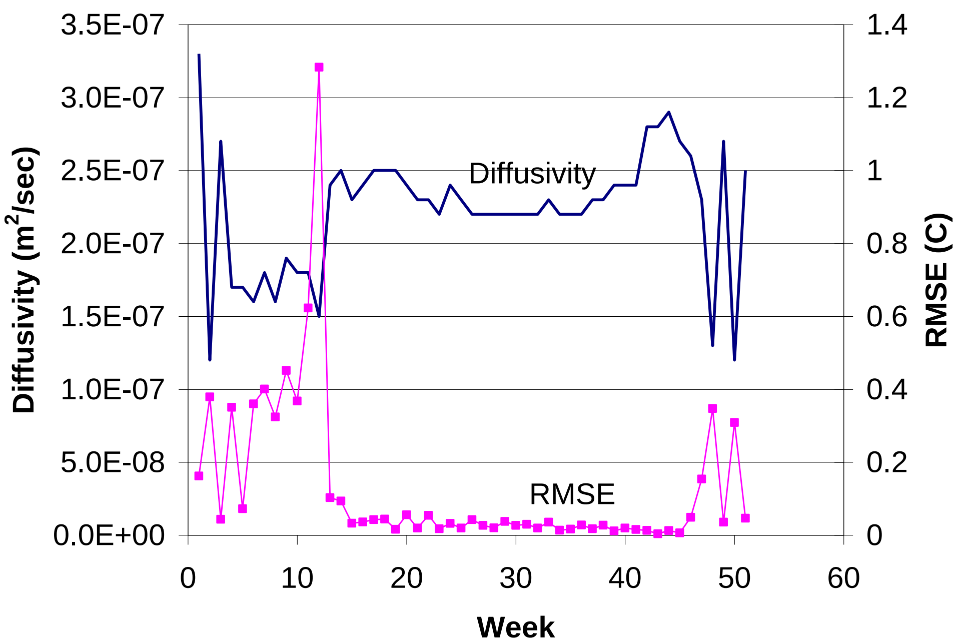

Figure 11. Thermal diffusivity and RMSE of fitted temperature versus week number for asphalt pavement layer (MnROAD cell 33), 2002. Thermal diffusivity extracted from measured temperatures at 2.4 cm and 7.6 cm depth using the Fourier transform method in 7 day blocks of 15 minute data.

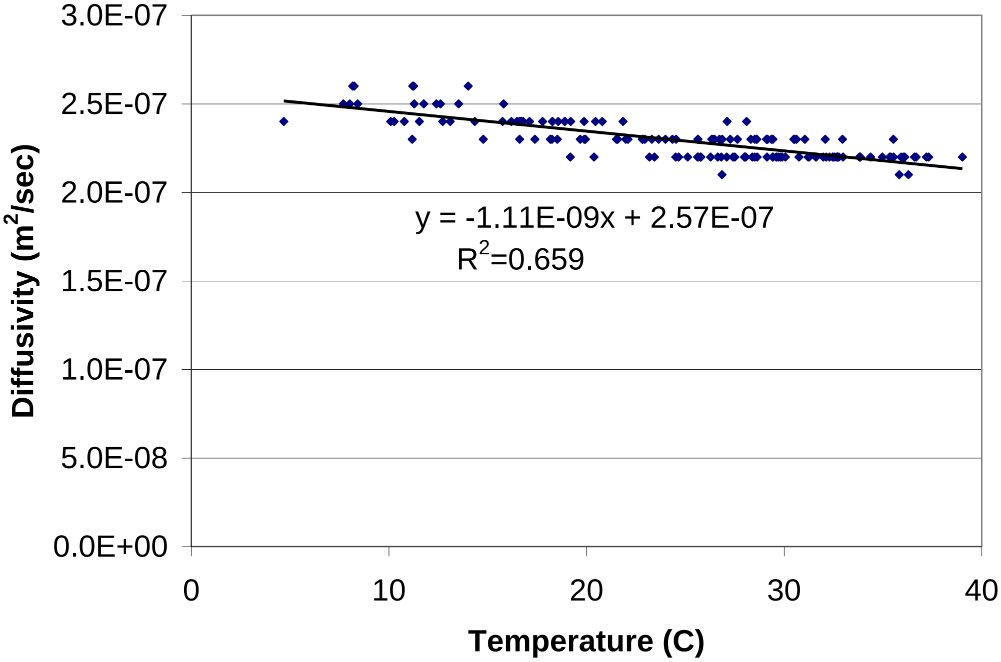

Figure 12. Thermal diffusivity versus weekly mean pavement temperature for asphalt pavement layer (MnROAD cell 33), week 11 through week 41, 2000-2005. Thermal diffusivity extracted from measured temperatures at 2.4 cm and 7.6 cm depth using the Fourier transform method in 7 day blocks of 15 minute data. Data points with an RMSE greater than 0.1 °C were not included.

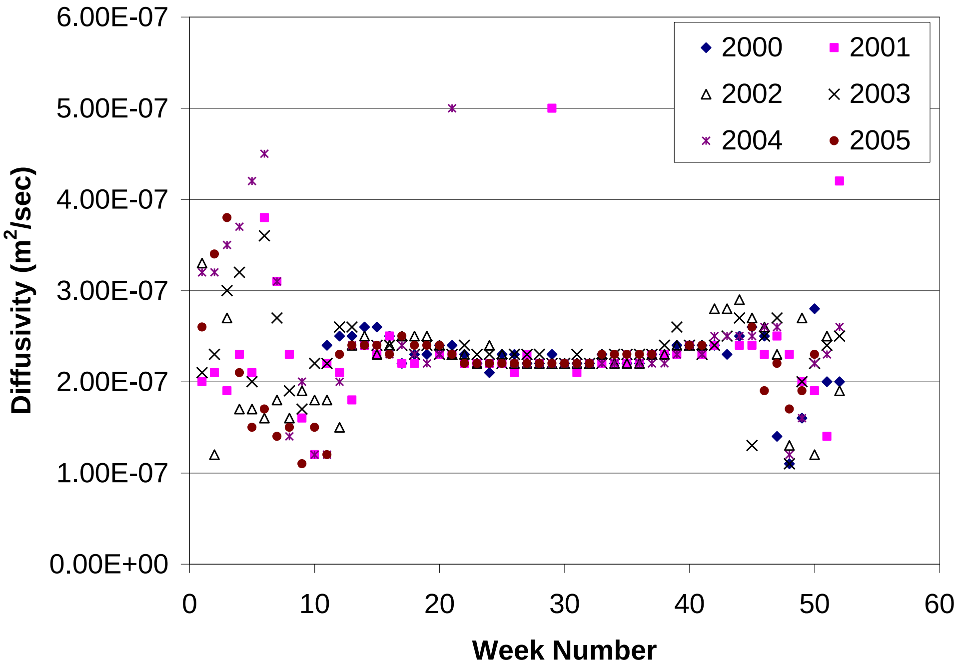

Figure 13. Thermal diffusivity versus week number for asphalt pavement layer (MnROAD cell 33). Thermal diffusivity extracted from measured temperatures at 2.4 cm and 7.6 cm depth using the Fourier transform method in 7 day blocks of 15 minute data.

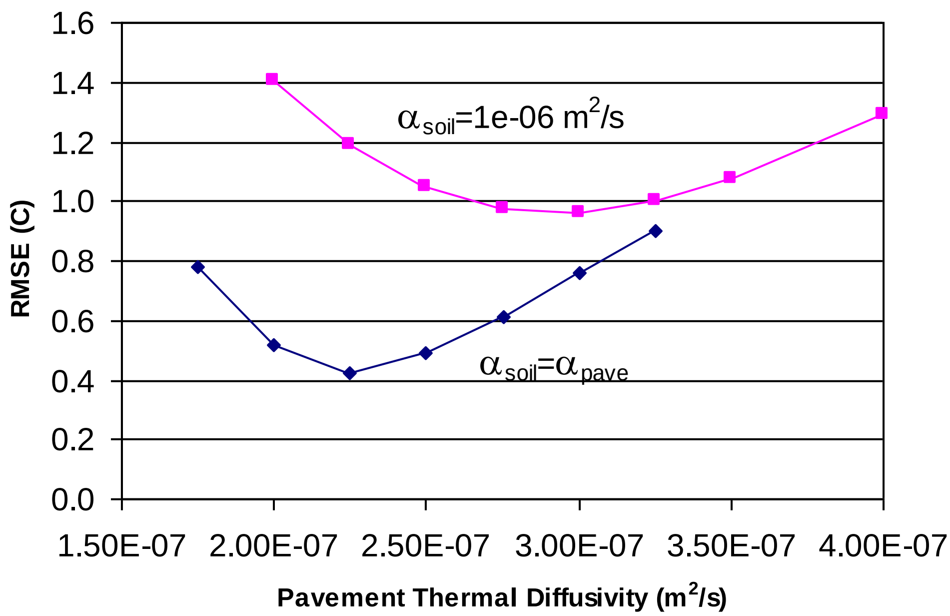

Figure 14. Simulated pavement temperature RMSE versus pavement thermal diffusivity for fixed and varying values of subgrade/soil diffusivity.