Primordial Black Holes and the QCD Phase Transition

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Outline

- Formation of PBH's
- Description of the QCD phase transition
- Impact on PBH formation
- Cosmological implications
  - Mass spectrum
  - Present day abundance

Formation – Qualitative Description

- Adapted from an approach by Carr. We also closely follow Cardall and Fuller [astro-ph/9801103].
- Over-dense region crosses the horizon
- Expansion
- Region stops expanding: Turnaround
- Depending on conditions, region will then either collapse, or disperse

Classifying Over-dense Regions

- Regions are specified by
  - When they enter the horizon
  - Their over-density $\dfrac{\delta\rho}{\rho} \equiv \delta$
  - At horizon crossing:

$$\rho = \bar{\rho}_h$$
$$T = \bar{T}_h$$

$$\rho = \bar{\rho}_h(1+\delta)$$
$$T = T_h$$

Evolution of Region

- Model the region as spherical and homogeneous
- Apply metric for a closed FRW universe
- Match inner/outer regions at horizon crossing

$$\left(\frac{dS(\tau)}{d\tau}\right)^2 = \frac{8\pi G}{3}\left[\rho(\tau)S(\tau)^2 - \bar{\rho}_h R_h{}^2 \delta\right]$$

$\rho =$ Energy density

$S =$ Scale factor for region

$\tau =$ Time coordinate for region

$\kappa =$ Constant (from matching)

- Expansion stops when:

$$\rho_* S_*{}^2 = \bar{\rho}_h R_h{}^2 \delta$$

Collapse Condition

- Density fluctuation satisfies equations of General Relativity
- Size of region must exceed relativistic Jeans Length at turnaround

$$d_* > R_J = \sqrt{\frac{\pi \cdot v_s^2}{4G(1+3v_s^2)(\rho + P)}}$$

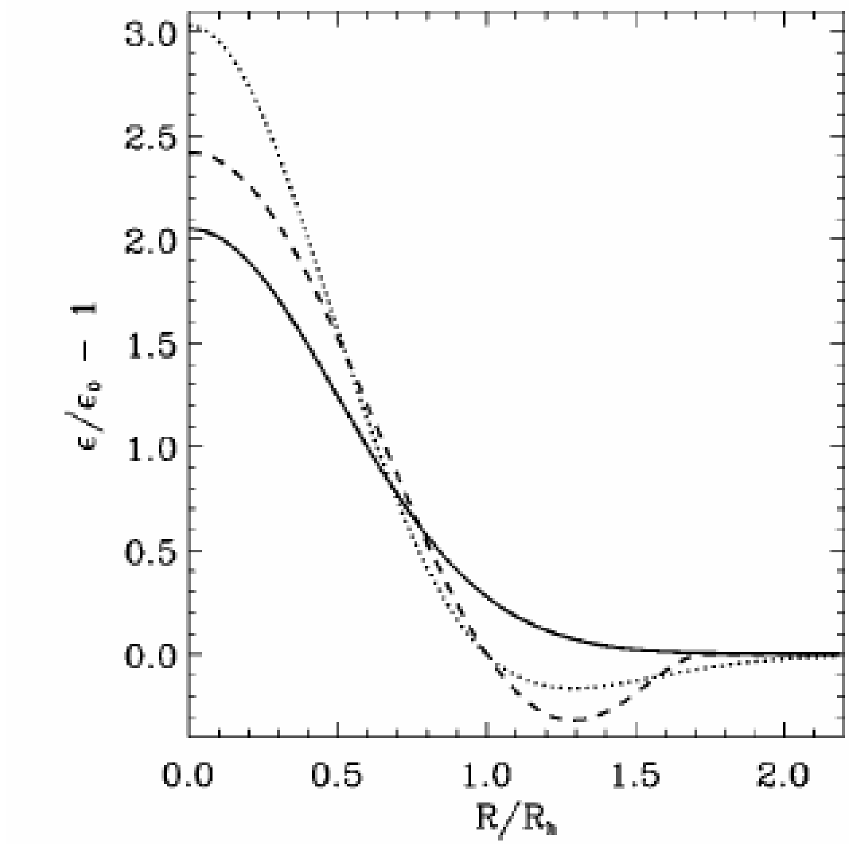
v_s = Speed of sound
ρ = Energy density
P = Pressure

For a constant sound speed:

$$\delta_c(v_s^2) = \frac{8\pi^2}{3} \frac{v_s^2}{(1+v_s^2)(1+3v_s^2)^3} \approx 0.822 \quad \text{for } v_s^2 = \tfrac{1}{3}$$

Determining Critical Over-density

- This is roughly consistent with Niemeyer and Jedamzik (1999) who found $\delta_c \approx 0.70 \pm 0.01$ in a numerical approach.



Determining Critical Over-density

- This is roughly consistent with Niemeyer and Jedamzik (1999) who found $\delta_c \approx 0.70 \pm 0.01$ in a numerical approach.
- Green et al. (2004) and Musco et al. (2005) found $\delta_c \approx 0.45 \pm 0.02$ (considering growing mode only)

$$\delta_+ \sim t \text{ (growing mode)} \quad \delta_- \sim t^{-1} \text{ (decaying mode)}$$

$$\delta(\mathrm{t}) = \mathrm{A}\left(\frac{\mathrm{t}}{\mathrm{t}_0}\right) + \mathrm{B}\left(\frac{\mathrm{t}_0}{\mathrm{t}}\right)$$

$$\frac{d\delta}{dt}(t_0) = 0 \;\Rightarrow\; A = B \;\text{ (50\% mixture)}$$

- Perturbations which are “pure growing modes” would require only ½ the initial over-density

Models of Degrees of Freedom

- Bag Model

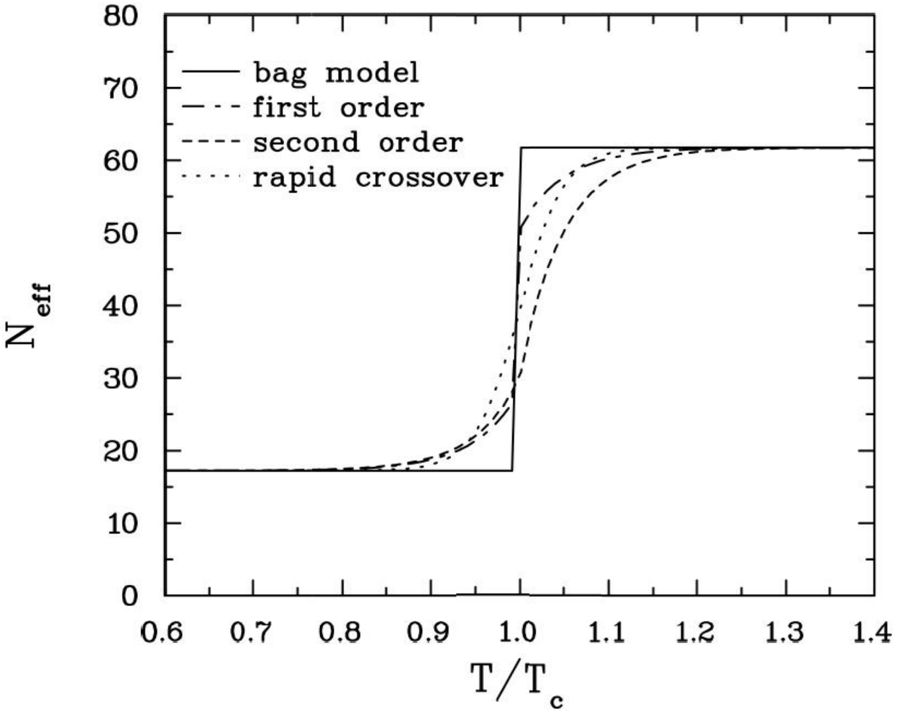
$$N(T)=\begin{cases} N_q & (T>T_c) \\ N_h & (T<T_c) \end{cases}$$

- Softened First Order/ Second Order

$$N(T)=\begin{cases} N_q-\beta\exp\left(\frac{T_c-T}{\Delta}\right) & (T>T_c) \\ N_h+\alpha\exp\left(\frac{T-T_c}{\Delta}\right) & (T<T_c) \end{cases}$$

- Rapid Crossover

$$N(T)=\frac{1}{2}\left[N_q+N_h+\left(N_q-N_h\right)\tanh\left(\frac{T-T_c}{\Delta}\right)\right]$$



$N_q = 61.75$ (u, d, s quarks, gluons)

$N_h = 17.25$ (leptons, pions)

$\Delta = 0.05\,T_c$

$\alpha = 11.125\,(1st)\;\; 13\,(2nd)$

$\beta = 11.125\,(1st)\;\; 33.375\,(2nd)$

“Softening” of the Equation of State

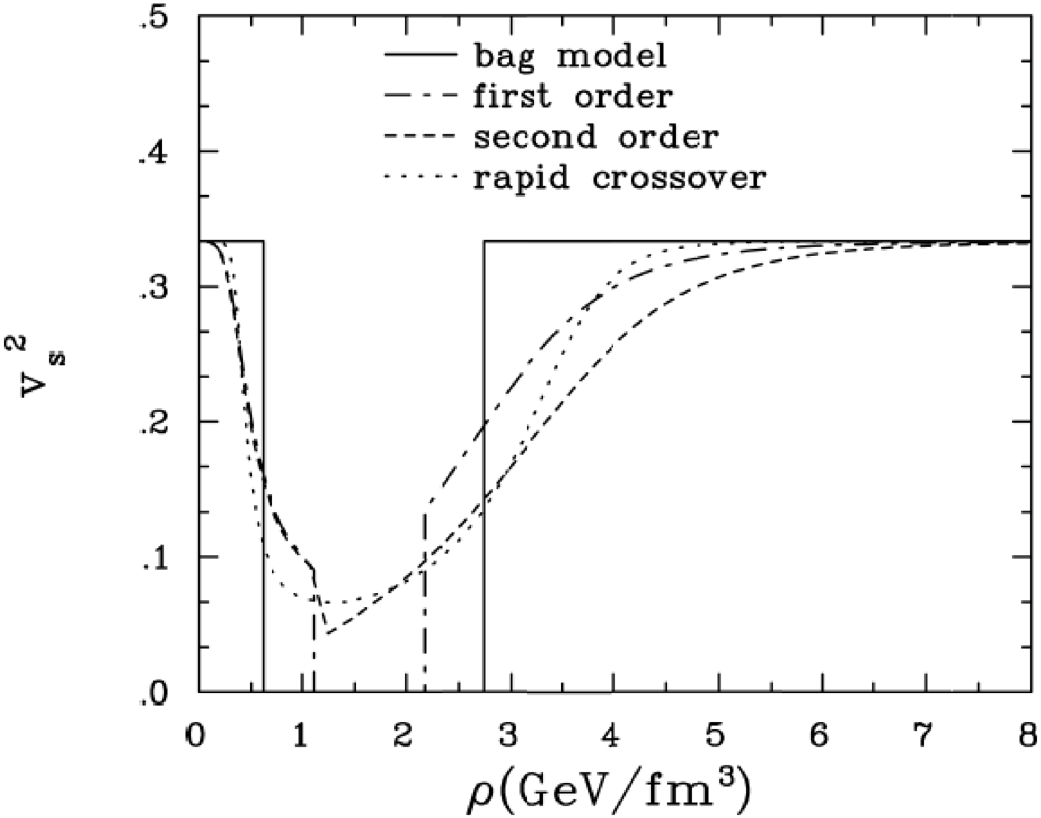
Reduction in the speed of sound

$$s(T) = \frac{4\pi^2}{90} T^3 N(T)$$

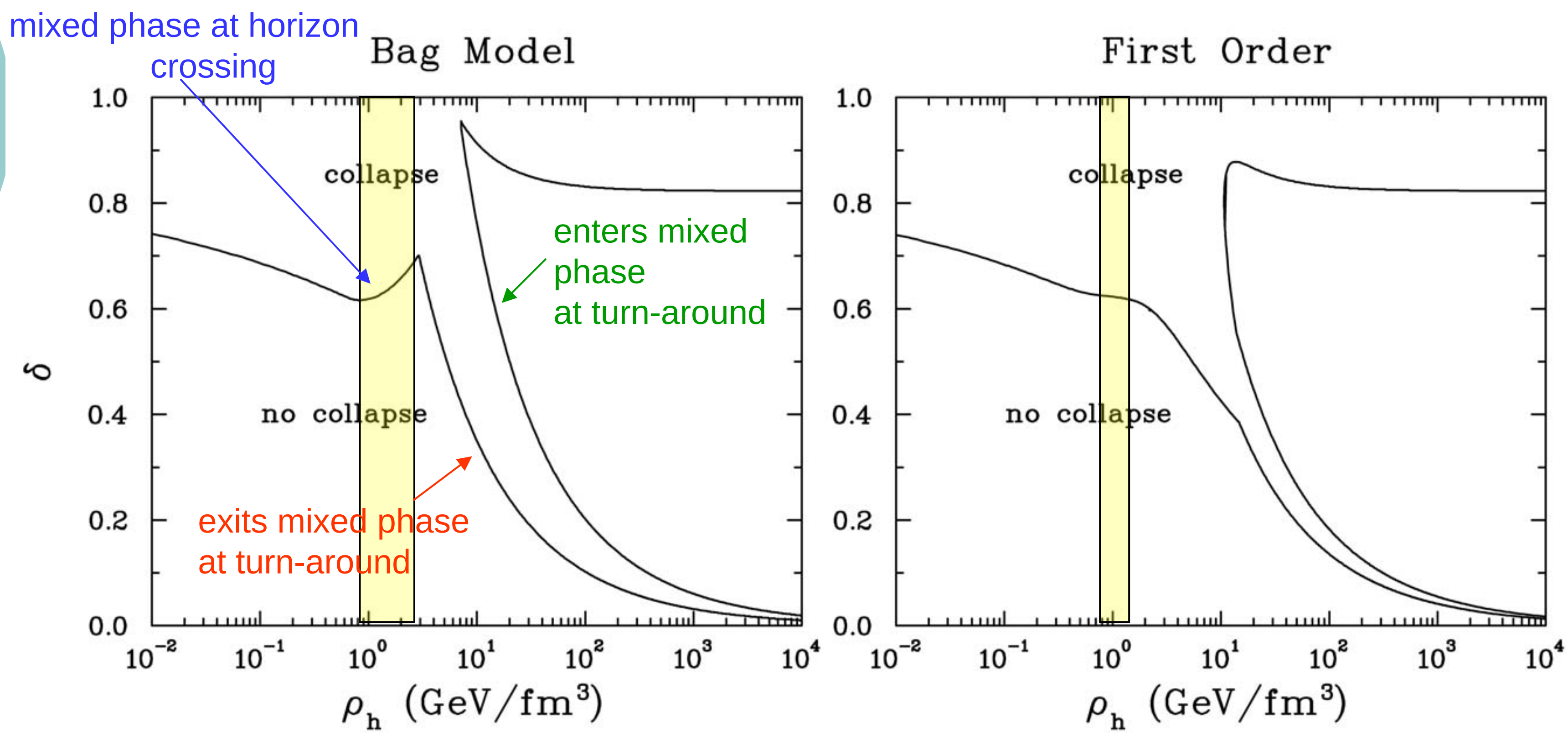
$$P(T) = \int_0^T s(T')dT'$$

$$\rho(T) = -P + Ts$$

$$v_s^2 = \frac{dP}{d\rho} = \frac{dP/dT}{d\rho/dT}$$

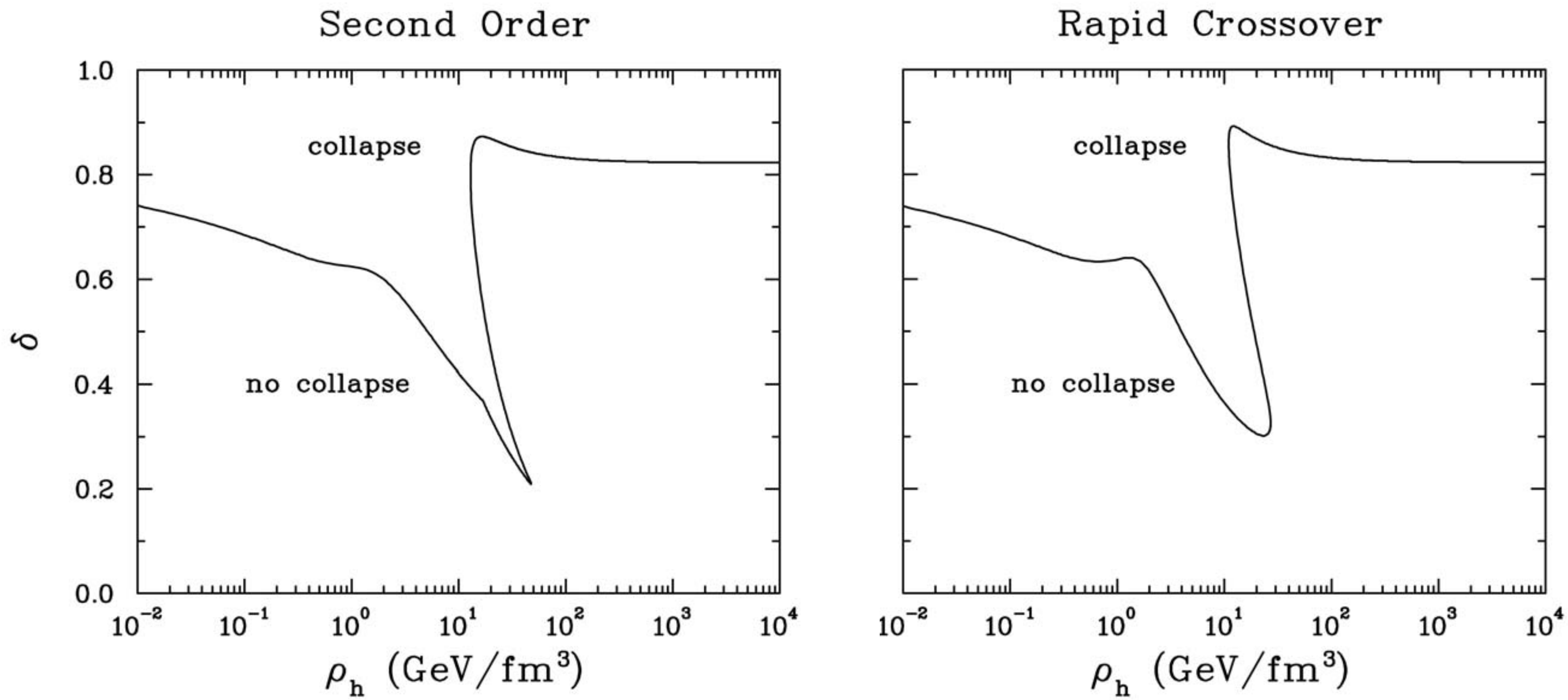


Critical δ – First Order Transitions



- ρ_h is the density at horizon crossing
- Yellow region denotes the mixed phase

Critical δ – Higher Order Transitions



- ρ_h is the density at horizon crossing

Spectrum of Perturbations

- Assume Gaussian distribution

$$P(\delta, M_h) = \frac{1}{\sqrt{2\pi}\sigma(M_h)} \exp[-\frac{\delta^2}{2\sigma^2(M_h)}] \qquad \sigma = 9.5\times10^{-5}\left(\frac{M_h}{10^{22}M_\odot}\right)^{\frac{1-n}{4}}$$

- σ is COBE normalized variance which depends on the spectral index n.
  [Green and Liddle: astro-ph/9704251, 1997]

$$\text{Power spectrum of fluctuations } P(k) \sim k^n$$

$$\text{WMAP suggests } n \approx 1 \text{ but}$$
$$\text{this pertains to larger length scales.}$$

Cumulative Number Density

$$\frac{dn}{dt} = \left| \frac{d}{dt}\left(\frac{1}{V_h(t)} \right) \right| \varepsilon(t)$$

Number density formation rate

Horizon crossing rate

Probability that over-density at horizon crossing leads to collapse

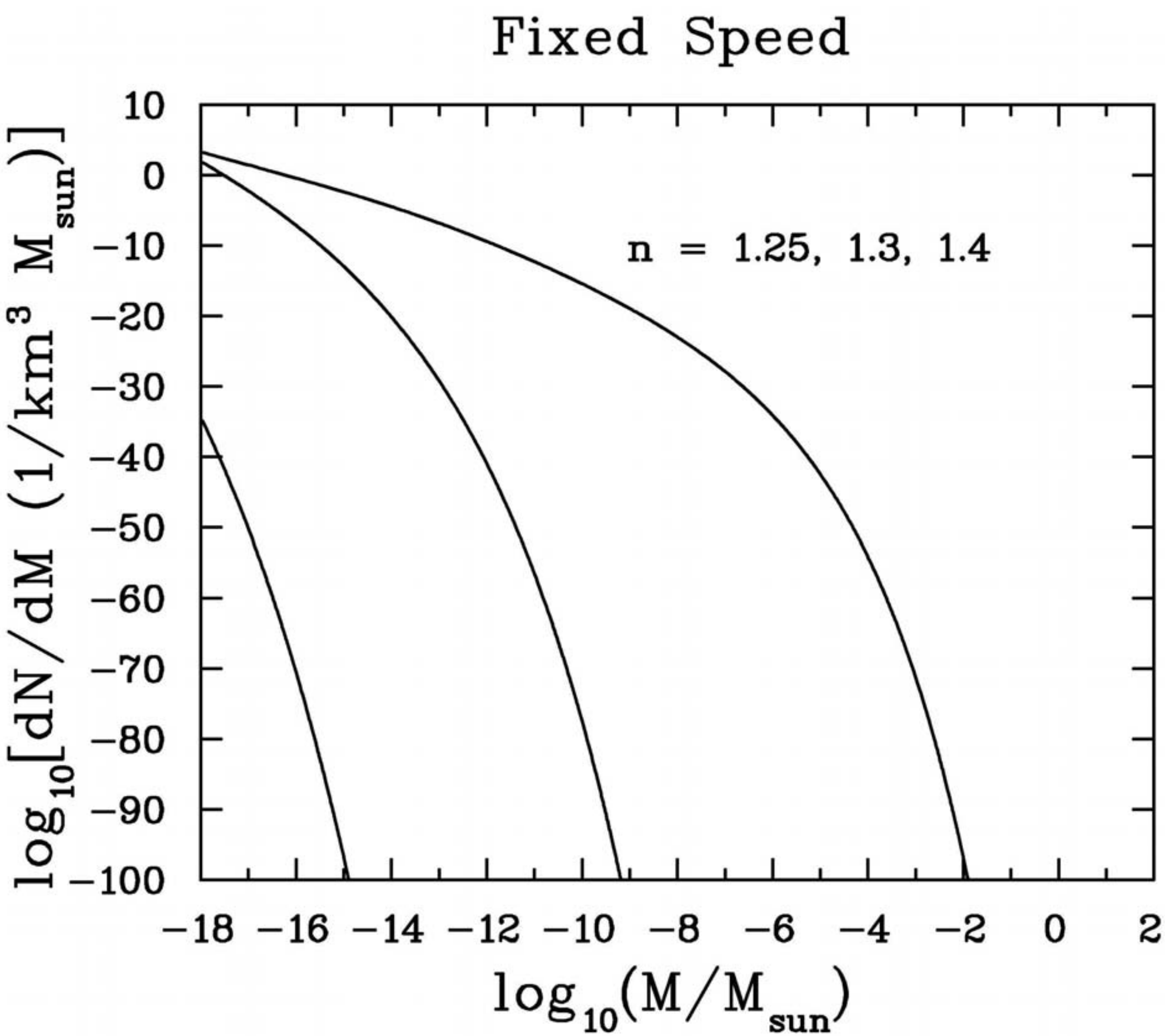
$$n_{today}(m > m_c) = \int_{t_c(m_c)}^{t_0} \frac{dn}{dt}\left(\frac{R(t)}{R(t_0)} \right)^3 dt$$

Dilution from formation to present time

t_c is the horizon crossing time which first results in PBHs of mass m_c

$$\frac{dn_{today}}{dm}(m_c) = -\frac{d}{dm_c}\left(n_{today}(m > m_c)\right)$$

PBH Mass Spectrum Results



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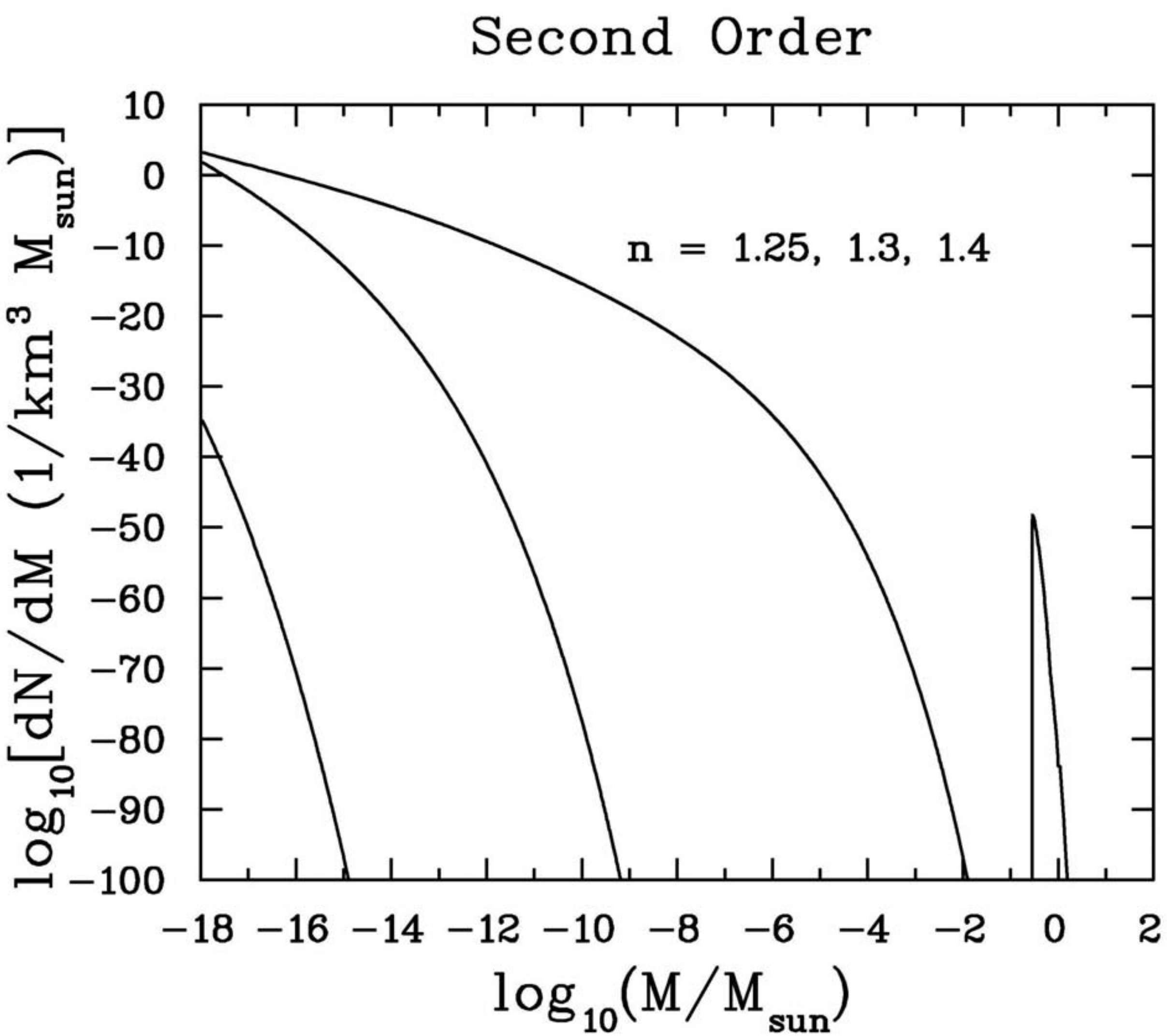

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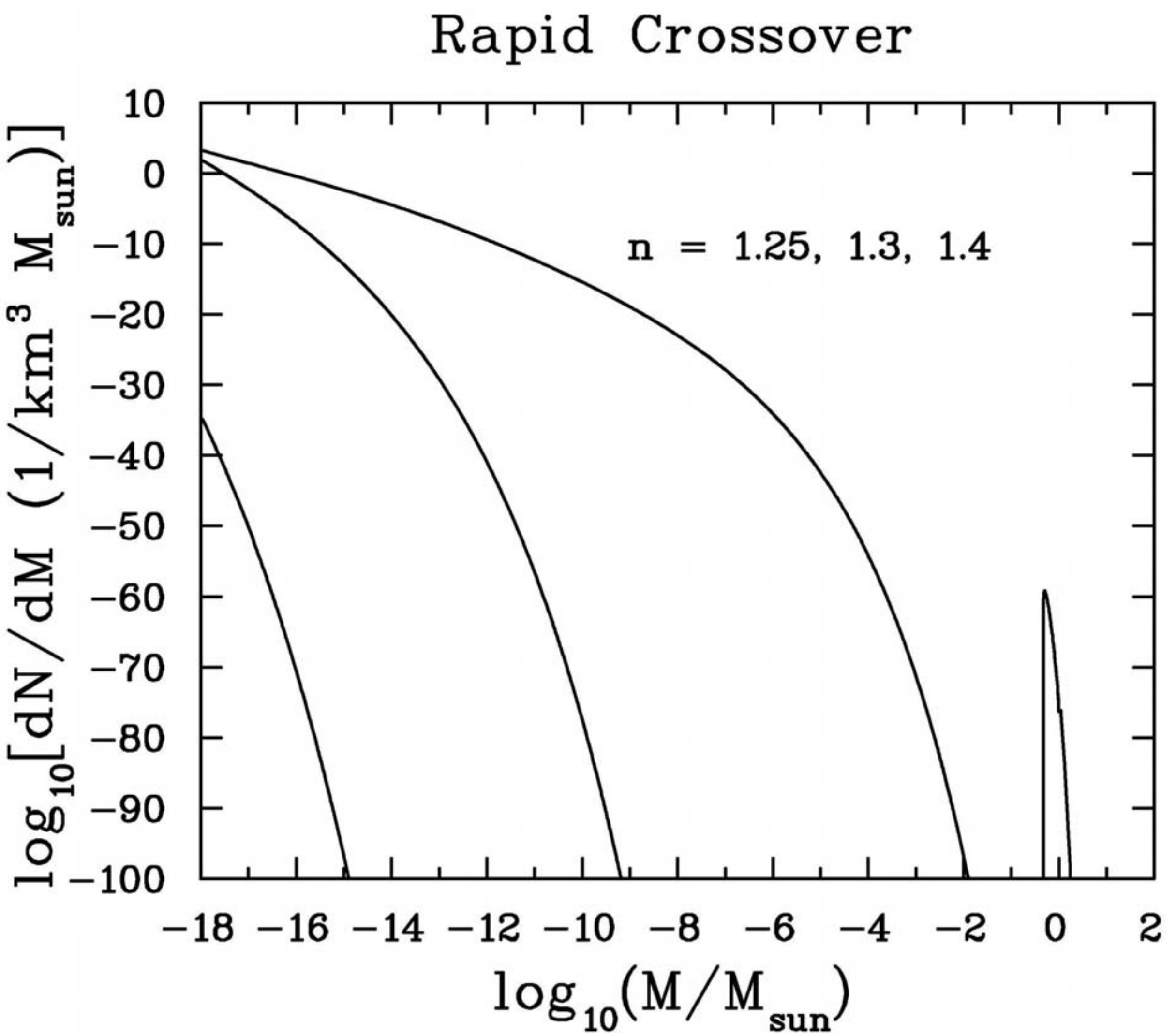

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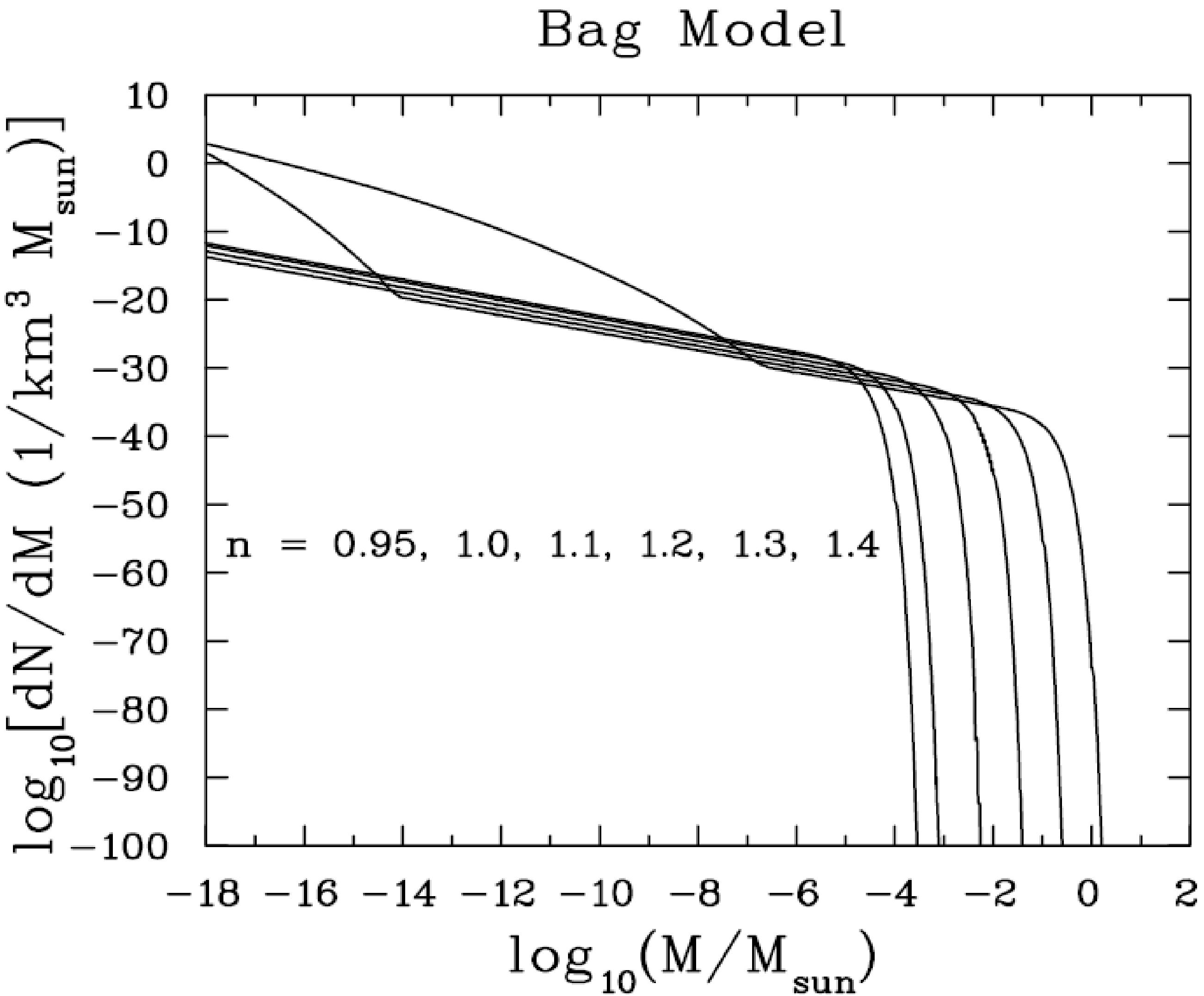
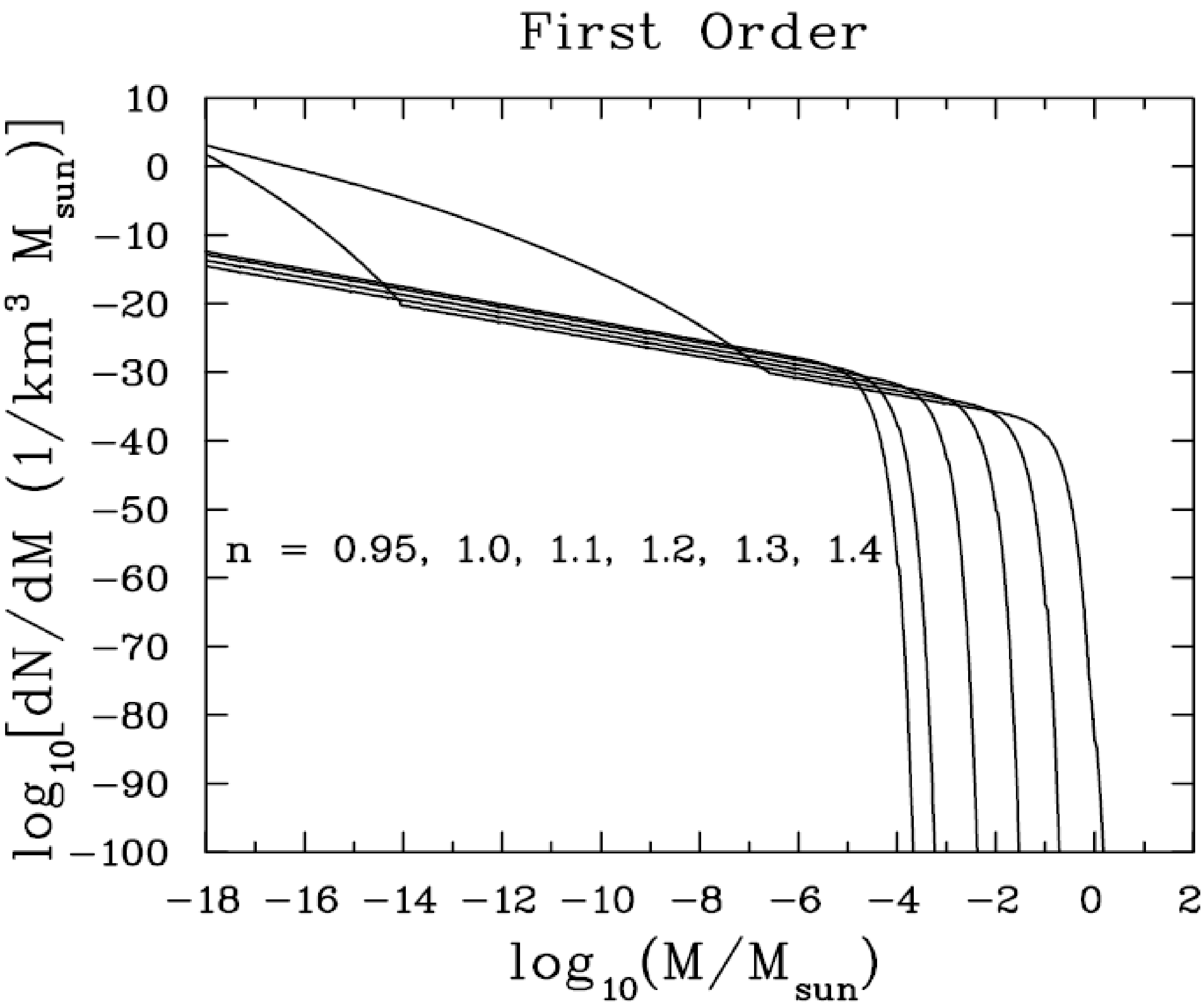


PBH Mass Spectrum Results



PBH Abundance (QCD Transition)

Ω_{PBH}	n=1.4	n=1.3	n=1.25	n=1.2	n=1.1	n=1.0	n=0.95
Bag Model	7.8×10^{14}	7.8×10^{11}	4.4×10^{7}	4.5×10^{7}	4.6×10^{7}	4.7×10^{7}	4.7×10^{7}
First Order	1.3×10^{15}	1.3×10^{12}	2.3×10^{7}	2.3×10^{7}	2.3×10^{7}	2.3×10^{7}	2.2×10^{7}
2nd Order	1.8×10^{15}	1.9×10^{12}	9.7×10^{-26}	~ 0	~ 0	~ 0	~ 0
Crossover	1.8×10^{15}	1.9×10^{12}	9.7×10^{-26}	~ 0	~ 0	~ 0	~ 0
Fixed Speed	1.8×10^{15}	1.9×10^{12}	9.7×10^{-26}	~ 0	~ 0	~ 0	~ 0

Number / pc^3	n=1.4	n=1.3	n=1.25	n=1.2	n=1.1	n=1.0	n=0.95
Bag Model	6.4×10^{24}	7.5×10^{22}	1.5×10^{8}	4.0×10^{8}	2.6×10^{9}	1.7×10^{10}	4.1×10^{10}
First Order	1.0×10^{25}	1.2×10^{23}	3.1×10^{7}	8.1×10^{7}	5.4×10^{8}	3.6×10^{9}	9.3×10^{9}
2nd Order	1.5×10^{25}	1.8×10^{23}	1.3×10^{-14}	~ 0	~ 0	~ 0	~ 0
Crossover	1.5×10^{25}	1.8×10^{23}	1.3×10^{-14}	~ 0	~ 0	~ 0	~ 0
Fixed Speed	1.5×10^{25}	1.8×10^{23}	1.3×10^{-14}	~ 0	~ 0	~ 0	~ 0

PBH Abundance (EW Transition)

Ω_{PBH}	n=1.4	n=1.3	n=1.25	n=1.2	n=1.1	n=1.0	n=0.95
$L=1.5T_c^4$	1.7×10^{15}	2.7×10^{12}	4.8×10^{8}	4.8×10^{8}	5.0×10^{8}	5.0×10^{8}	5.1×10^{8}
$L=1.0T_c^4$	1.7×10^{15}	2.6×10^{12}	9.6×10^{8}	9.5×10^{8}	9.9×10^{8}	1.0×10^{9}	1.0×10^{9}
$L=0.5T_c^4$	1.7×10^{15}	2.6×10^{12}	1.4×10^{9}	1.4×10^{9}	1.4×10^{9}	1.5×10^{9}	1.5×10^{9}

Analysis also carried out for a Bag Model type Electroweak phase transition with:

$$T_c = 100\ \mathrm{GeV}$$

$$\mathrm{L} = 0.5T_c^{\ 4},\ 1.0T_c^{\ 4},\ 1.5T_c^{\ 4}\ \text{(latent heat density)}$$

Summary

- Enhanced black hole formation for all models of a QCD phase transition.

- QCD equation of state, spectrum of perturbations, PBH abundance, and PBH mass distribution are directly related.

- Experimental data on one or more of these quantities can help determine the others.

- In the context of our model, first order phase transitions in the early universe may be incompatible with the observed lack of primordial black holes.