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Time in General Relativity

Introduction

In this essay I shall discuss two intimately related properties of time, or of the temporal aspects of phenomena, which, having at first been developed through philosophical arguments, have now come to play central roles in general relativistic physics.

The first property is the *directionality* (or *anisotropy*) of time, a phrase which I use loosely to refer to the fact that the peculiarities of the events around us on Earth can consistently be used to distinguish between two possible ways of ordering time. In other words, no one is for long in doubt about which way to run a film through a projector. This is one of the most basic physical facts about the world we live in.

If this directionality is assumed to exist at all points of a general relativistic model of the universe, then the possible structure of the space-time is accordingly restricted: it is called *time-orientable*, meaning that its metric structure is such that at each point one can designate one temporal direction as "positive" and the other as "negative," with the designations at neighboring points agreeing. The assignment of positive and negative can be made arbitrarily at one point, but it is then fixed at all other points by this requirement of neighboring agreement (continuity). If the physicist goes on to say that one direction is *future* and the other *past*, then through the wide connotations of these words he tacitly assumes that by far the most proper form of physical argument is the prediction of the future from the past.

The second property I shall discuss is again drawn from experience and then generalized to a mathematical principle: time is *strictly linear*. My history, and, as far as I know, the history of any object, is describable as a (finite or infinite) linear extension and is not like a circle. Nor, in our normal experience, can two distinct instants[1] of a person's or an object's history be physically contemporaneous; indeed, there is much in our experience that weighs against the mere possibility of, for example, my

use of a time-machine to make me contemporary with an event I have already experienced. This is reflected in our language, which so assumes the linearity of time that any contrary statement (such as that in the previous sentence) is liable to be internally inconsistent unless one restricts the usual implications of the words used.

The mathematical correlative of such a time-trip, or of a circular history, is a curve in space-time which is closed,[2] in that it returns to its starting point, and yet is timelike, locally describing a possible history for an object. It is almost invariably assumed in general relativity that space-time is *causal*, meaning that such closed timelike curves do not occur. While time-orientability asserted that an absolute distinction between past and future could be made locally near every point, causality implies that this distinction is meaningful on the entire history of any object, in which an event once past can never be regained.

The discussion of these properties in general relativity differs from the parallel arguments in philosophy. For instance, many philosophical writers claim that the "normal" properties of time are logically necessary if temporal language is to have anything like its everyday meaning, or if we are speaking of a world in which human discourse and action as we know it is possible. But relativity uses a language where words have new technical meanings to discuss cosmological models in parts of which human action, or even existence, is undoubtedly not possible! Yet the distinction is all too often ignored, and the conclusions of philosophical arguments are uncritically used, mainly by physicists, to justify various mathematical restrictions on space-time. I hope to exemplify here how such restrictions should be sought only through a discussion which is consistently within the general relativistic context.

Directionality

While few would deny that on Earth time is directional in the sense described above, controversy centers on whether the fact is a physical law in its own right or a consequence of laws and contingent circumstances together. I shall first examine the standard example of work in support of the latter case, showing not only the greater explanatory power of this approach but also the deep consequences of this controversy for general relativity, extending far beyond the classification of types of scientific explanation. In the example to be described, the laws used are the laws of electrodynamics in the form of partial differential equations that are indifferent with respect to any distinction between past and future; they are combined with the contingent fact that the universe (or at least a very large part of it surrounding us) is in that kinetic state which, with our usual assignment of time direction, we call expansion.

The effect of this circumstance was examined by Sciama[3] in the context of the usual idealized cosmological model (the homogeneous isotropic Robertson-Walker solutions of the Einstein equations for a perfect fluid). In models like this[4] it is possible to express the electromagnetic field at any point p in one of two extreme forms, or as any mixture of the two: (1) a sum of contributions from all the particles lying in a region to the past of p, each radiating in the usual way so that the radiation recedes from the particle as one progresses to the future, together with a contribution from radiation already present at the past boundary of the region; and (2) a sum of contributions from particles in a future region, each radiating "backward" (the radiation receding from the particle as one progresses toward the past), together with a contribution from radiation at the future boundary.

Both descriptions are mathematically admissible for any solution of Maxwell's equations. Why, then, is it customary always to use (1)? The answer is clear in the idealized model used by Sciama, in which only (1) has the property that, as the contributing region used is extended progressively further into the past, the contribution from radiation entering the region can be taken to become less and less, while the contribution from the radiating particles tends to a finite value. In the limit, the field at p is then represented as due simply to the sum of contributions from all the particles to its past. If, however, we try to perform a similar limiting procedure with (2), we find that both the contribution from the particles and the contribution from radiation at the boundary increase without limit (on the simplest analysis)[5] as the region is extended to the future. Thus the only representation of the electromagnetic field which could describe it as being produced solely by particles is that in which the particles radiate in the usual time-sense, relative to the time-sense[6] defined by the kinetics of the universe. (One might note in passing that this example illustrates the process of transition from time-symmetric *laws*, expressing only neutral connections between temporally neighboring events, to a system in which the motions of particles *cause* the field. On the present analysis, the effect must follow its cause because 'causation' is linked to the kinetic structure of the universe.)

This argument is not merely a specimen instance of one way in which contingent facts can impart one sort of directionality to temporal phenomena. As is well known, there are good reasons for believing that it describes in simplified form the central process that determines the directionality of time in any reasonable model of the universe, all the various possible arrows of time being dependent on the electromagnetic arrow. A full proof of this belief has yet to be given, but the lines which it would take are fairly clear. The kinetic state of the universe, through electromagnetic phenomena, determines a direction of time with respect to which matter loses heat by radiating it away into space. This condition of thermal disequilibrium then gives a thermodynamic directionality to physical processes; from this the anisotropy of time as expressed through recording processes and the second law of thermodynamics could plausibly arise from Reichenbach's "branch system" argument.[7]

My aim in this section is to explore the implications for general relativity, if it be accepted that the viewpoint I have just described is generally valid—the view, that is, that all processes characteristic of the directionality of time have their sense determined by the large-scale kinetic structure of the universe. I shall suggest that if such a determination takes place, then the direction of time is not some metaphysical absolute that must be related to a relativistic model by an interpretative convention: rather, it is grounded in the kinetic structure of the model itself.

This proposition has drastic consequences. For if time is thus kinetically determined, then there is no reason to expect it to have all the properties which it would possess as a primary absolute. For instance, even if a time-coordinate can be defined in the model, the interpretation of the sense of this coordinate—whether it measures time "forward" or "backward"—must be determined by the intrinsic physical properties of the model; in particular it may vary from place to place[8] and be in some places undefinable.

An example illustrates the physical importance of this and reveals the difficulties that arise. Consider a homogeneous and spherically symmetric star collapsing into a black hole. The appropriate solution[9] is the Schwarzchild metric outside the star, joined onto the "interior Schwarzchild metric" inside. Now this latter is identical with the Robertson-Walker cosmological solution discussed by Sciama *with the time direction reversed*, which suggests that the time-sense as determined intrinsically might be anomalous. And, indeed, when Sciama's argument is applied to points inside the black hole, it is found that one can realize the representation (2) of the field as being caused by future motions of charges. Whether or not (1) is also allowed depends on the state of motion of the matter in the universe before the black hole forms.

In this situation one reaches quite different physical predictions according to the attitude adopted toward time. Usually a certain direction of time, with respect to which the star is collapsing, is taken as an a priori datum for physical reasoning. Causation and explanation are strictly unidirectional; the task of physics is to explain or predict later stages of the system in terms of the earlier stages that give rise to them. Typically, these earlier stages might be regarded as "initial conditions" from which the system evolves. In the case of the black hole one can find a spacelike hypersurface (a Cauchy surface) whose physical condition determines uniquely the conditions everywhere in the space-time. Then one can, for instance, argue that a small departure from exact symmetry on this "initial" hypersurface does not hinder the formation of a black hole.[10]

But suppose that we apply to the region inside the black hole arguments based on the "reversed" time direction kinetically determined there. Such arguments will be qualitatively like those usually developed in Robertson-Walker cosmologies, but time-reversed. In cosmology, for example, galaxies are usually regarded as having been formed by the gravitational amplification of small fluctuations of density present at a very early time in an otherwise homogeneous universe. The origin of these fluctuations is often sought in quantum processes which, very early on, introduce a random element into a cosmos which initially was quite homogeneous. In this way it is hoped to provide an explanation of the occurrence of the galaxies which, if successful, should account for their observed distribution of sizes and angular momenta. The direction of time enters twice: once in designating the conditions at the $t = 0$ boundary of space-time as *initial* conditions which can be postulated a priori; then again in giving a directionality to the *growth* of quantum fluctuations, which are regarded as being statistically independent in accordance with Penrose and Percival's[11] analysis of the directionality of time.

If the time-sense in the black hole were determined intrinsically, then we could postulate a *time-reversed "growth" of perturbations*, in the same way as in the cosmological case. The physical consequences are then dramatic: it turns out that such perturbations grow indefinitely large near the boundary (horizon) of the black hole. In this analysis the conventional

black hole model is an unstable configuration which cannot physically exist!

Such is the confidence placed in reasoning from an a priori time-sense that virtually no physicist would draw the conclusion that black holes do not exist. They would rather say either that the model in question is grossly unrealistic in its description of the final singularity (which may be equally true in the cosmological case) or that one must restrict one's reasoning to the domain in which everything is normal, outside the black hole, trusting that this domain will remain unaffected by processes in the interior, however extreme. In either case, they are then able to fall back on an a priori time-sense, which is the "real" direction of time—with respect to which an anomalous part may perhaps appear to be running backward. But if an *intrinsically* determined time-sense is so rejected, then we must recognize and justify the alternative: a mode of explanation which is unsymmetric with respect to time and which applies the time-sense determined by processes near the earth to the entirety of the universe, irrespective of the nature of the processes elsewhere; or which seeks to establish an absolute time *extrinsic* to the physical universe to govern its evolution. This is the dilemma to which I shall return in the final section, after examining the second conventional property of time.

Cyclic versus linear histories

Perhaps the most important result in modern relativity theory has been the prediction by Hawking and Penrose of the necessary occurrence of singularities in general relativistic models of the universe. The theorems they prove use various assumptions about the reasonableness of matter, together with a condition (strong causality) intermediate between the nonexistence of closed timelike curves (causality) and the existence of a global time coordinate (stable causality).[12] These theorems are applied to regions at the center of a collapsing star or to the early stages of the universe—regimes which are totally unlike any of which we have experience. Yet the philosophical arguments on which the assumption of causality rests[13] collapse if only the slightest departures from everyday experience are contemplated. The danger of transplanting the conclusions of philosophical arguments to an alien relativistic context could hardly be better illustrated.

A selection of some recent arguments[14] against "cyclic time" will bear the point out.

(a) Cyclic time is postulated only when one first considers the possibility of events recurring periodically, and then joins time up on itself to form a circle. (Thus, if event B, a recurrence of A, is in all respects similar to A, then it should be regarded as being numerically the same event.) But, the argument runs, this is fallacious because it overlooks the essential role of time: the provision of a framework which enables us to speak of qualitatively similar but numerically distinct events. Time determines the identity of events, not vice versa.

(b) In any case, the reason for postulating cyclic time in (a) is not even self consistent; for, in saying that an event recurs we are implying that there must be *two* events, one of which is a *re*currence of the other.

(c) One of time's definitive features, part of the essence of the concept, is its linearity: an extension without linearity could therefore not be called 'time' without gross abuse of language. This arises because 'time,' through its basic definition, is linked to our consciousness and the ideas of before and after, which require linearity.

(d) Physical processes are essentially directional (enabling us to think of time as directional). But in a cyclic time all processes are periodic and so cannot have any unidirectional trend.

(e) Cyclic time is inconsistent with our undoubted participation in the world as agents. For, suppose I travel backward in time and meet my former self at an earlier age. As a free agent, what is to prevent me from drawing a gun and shooting my former self, which is a logical impossibility?

The first four of these can be dealt with summarily. The objections (a) and (b) are irrelevant to relativity because the construction of cyclic time which is countered in (a) is *not* the reason for postulating closed timelike curves in general relativity. These curves are postulated only when they are forced by the dynamics of the universe; they do not arise from making identifications in a periodic universe and, in general, no periodic universe exists from which such a causality-violating universe could be derived. For example, in the Taub-NUT or Kerr solutions, the progressive development of the universe causes the null-cones to tip so that closed curves which in one region are spacelike become timelike in another region. In neither of these models is it possible to "unwind" the time so as to remove the anomaly.

Argument (c) may be quite proper, but it is directed against our use of the word 'timelike' to describe these curves, not against their existence.

The case (d) is interesting, as it rests on a confusion between the directionality of *time* and the directionality of *systems in time*, a distinction which I have discussed at length in the previous section. We can in fact quite well have the physical processes in all relevant systems directional while time as a whole is cyclic. An example of this is provided by the steady-state model with the time coordinate "rolled up" to become periodic; this is possible because the *metric* is static, although the physical processes within that metric are directional (the universe expands). But in any case we are not interested in situations in which the entire universe has a cyclic time coordinate, but in those where there may be just one curve[15] which violates causality.

Thus we are left with (e), seemingly the most powerful argument as it rests on a clear logical contradiction. One could point out that in the realms of astrophysics under consideration (the very earliest phases of the universe or the final stages of a collapse) one cannot conceive of the presence of human beings, whether free agents or not. But this consideration alone will defeat (e) only if one is prepared to accept the position that the existence or non-existence of closed timelike curves is to be determined by the ability of human beings to withstand the climate. A factual and physical matter such as causality should not depend on such a criterion, which is not only physically arbitrary, but is also dependent on the level of technology at our disposal. Therefore I shall argue against (e) directly, showing that closed timelike curves can occur even in regions of the universe occupied by normally functioning human beings. I shall include the idea of free will, not only because I hold it to be an important fact of our experience that cannot yet be discussed satisfactorily in other terms, but also because free will produces the most powerful form of (e): my arguments will hold a fortiori if free will is not referred to.

While a will which could never be exercised would be nonsense, it is likewise unreasonable to demand that will should always achieve its ends: possession of free will does not imply omnipotence in its execution. Herein lies the solution to the apparent logical paradoxes of acausality: it turns out, from purely physical reasoning, that in a universe with closed timelike curves the laws of physics manifest themselves in an abnormal manner ("normality" being established by the behavior of physics in a universe without such curves). This abnormality is precisely such as to frustrate the execution of any wish whose accomplishment would create a logical antinomy. One's acts of will still count in the world as partial causes of what occurs; the totality of events which transpire if a certain will is exercised is different from that which would obtain were that will not exercised. The only consequence of the acausality is that the result of an act of will is not always what would be expected on a naïve analysis based on "normal" experience.

To see how this is so, consider the case already cited of a person who meets his former self in circumstances in which, if physics were normal, he would be able to shoot him. Then, as a preliminary step in the analysis, let us replace the complex human being by a simple automaton which nonetheless exhibits the abnormal physics referred to. This apparatus [16] is to consist of a gun, a target, and a shutter so arranged that the impact of a bullet on the target will trigger the shutter so as to move in front of the gun. It pursues a causality-violating curve in space-time in such a way that two points on the object's world line A and B, with B later in the object's history than A, are physically contemporaneous and disposed as in Figure 1 so that the gun at B is aimed at the target at A and the shutter is initially up at A.

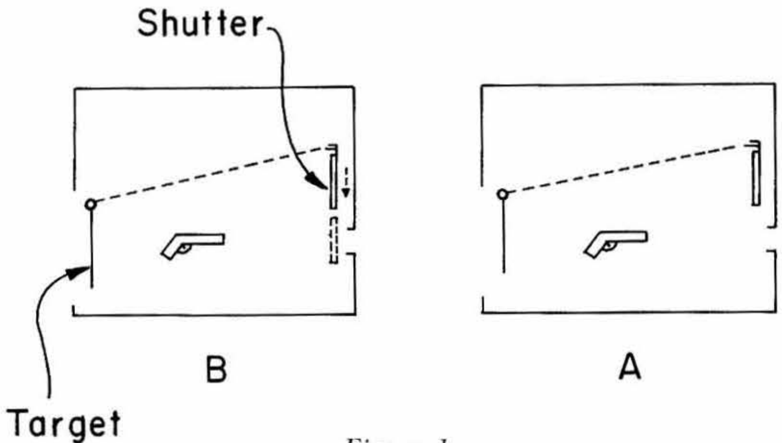

Figure 1

Suppose now that the machine "shoots its former self": the gun at B is fired, either by an automatic timing mechanism or by the intervention of a human being making a conscious decision. If the shutter in B were still up, the bullet would strike the target at A, which would cause the shutter in B to be down, a contradiction. But if the shutter were down in B, then the bullet would be stopped, the target in A would not be hit, and the

shutter in B should still be up: the shutter is up if, and only if, it is down; the situation is logically impossible.

First I shall make a classical analysis, allowing for quantum effects later. Classically, as Feynman and Wheeler [17] pointed out, all the equations involved are *continuous*, and the argument to a contradiction just given is fallacious because it assumes that it is possible to set up a discontinuous situation in which the shutter is either up or down, with no intermediate state. The position of the shutter at B, x, say, is a continuous variable on which depends continuously the angle by which the bullet is deflected; this is in turn continuously related to the force of impact on the target and to the speed y with which the shutter in A is triggered to start descending. Thus $y = f(x)$, where f is a continuous function which is large for $x = 0$ (shutter right up) and zero for $x = 1$ (shutter right down). Suppose that the proper time in the apparatus between A and B is T, so that $x = Ty$; then the physical processes we have described, each a normal classical process, give rise to the equation $x = Tf(x)$ for x. This will always have at least one solution corresponding to the shutter just grazing the bullet so that it is deflected and gives the target a glancing impact, marginally triggering the shutter. Paradox is thereby avoided.

The general features of this situation are applicable to all such paradoxical arrangements. At a local level the ordinary equations of physics can be written down. They must then be solved in a global context which is abnormal. Consequently, the solution is abnormal, in that it corresponds to a type of behavior which in a causal universe would have only an infinitesimal chance of occurring. In an acausal universe miracles can occur quite often, and one must set aside one's normal judgment as to what is likely and what unlikely.

It might seem that quantum processes are peculiarly discrete and so might produce a real discontinuity. (Actually this is open to doubt: Schrödinger's cat [18] is indeed either alive or dead; but is this a property of atomic decays, or of Geiger counters, cyanide capsules, and cats?) But in any case, if we work in the quantum domain, then we must recognize that such discreteness, if it occurs, is accompanied by indeterminacy. The continuous deterministic evolution that characterizes both the classical equations that we have just examined and the Schroedinger equation becomes (in a way still highly disputed) a *probabilistic* evolution of discrete possibilities when translated into observed outcomes. Hence it is of no avail to replace the mechanical gun and target by a quantum mechanical decaying atom and Geiger counter, for example. What one might gain in discreteness one loses in indeterminacy: even if the shutter were to close fully, there would still be a finite probability of the emitted particle tunelling through it and so triggering the counter.

When a closed timelike curve brings about a coincidence of events—such as the shutter just grazing the bullet—which would be grotesquely implausible under normal circumstances, then, and only then, normal concepts of causation and likelihood are completely disrupted. But if we are concerned only with a few timelike curves, and not with a completely cyclic time, then such coincidences will be seen as the exceptions to the normal behavior in which the concepts of causality, free will, and so on are grounded. In particular, the occasional closed timelike curve[19] will not alter the psychology of taking a free decision: it will merely alter the consequences of that decision.

It is now not difficult to imagine a way in which these factors might operate in the fully human example with which I started. The gun-toting protagonist is free to choose whether or not to shoot. If he decides against it, then there is no paradox. But if he decides to shoot, then his hand will waver and he will only graze his former self, his unexpected weakness being caused, not by divine intervention but by the flesh wound which he thereby inflicts on his former self!

Causes in general relativity

I have argued first that a general relativistic model of the universe may have no globally valid way of assigning a local sense of time, so that it may not even be time-orientable; and second, that even where a time sense can be defined, there is no reason to suppose that space-time need be causal. The most cogent objection to these ideas, in my estimation, has yet to be examined: my argument has been within the context of a physical theory whose aim is to explain the structure of the universe; but, if there is no conventional pattern of cause and effect, can any account be offered which is in any sense an explanation, and not a mere description?

This objection is usually based on too narrow an attitude to explanation—the attitude of assuming that the only possible explanation is of the "Cauchy problem" type in which a system is explained entirely in terms of physical laws and its *initial* conditions. In these last few paragraphs I shall briefly give my reasons for believing that this approach is

neither necessary nor sufficient for achieving a reasonable explanatory scheme.

There are certainly some cases in which the Cauchy prediction approach might be reasonable. For instance, if we were concerned with a system of which we ourselves were external observers, then predictive arguments could be part of the prediction-test-hypothesis cycle of Popperian methodology. The scientist could gather information about, or experimentally create, the initial situation of the system. Then its state would be examined after a few seconds, days, or years to see if the theory had been supported or refuted. Clearly in this case the time which enters the theory when a prediction is made is totally linked to the laboratory time in which the physicist operates, and so must be directionally uniform and topologically linear. Yet even here, in a laboratory system, the scheme of Cauchy data may not be relevant. In studying gas contained in a cylinder in which a piston is moving, any explanation must include the externally imposed motion of the piston as part of the data.[20] Such an explanation, though not Cauchy, would be regarded as proper and scientifically illuminating.

In general relativity we are ourselves within the system, and any predictive arguments we may use about the universe are not in "real time": the mathematical process of explaining the present state in terms of an earlier state is separate from the historical process of testing the theory (a situation which is in practice almost always the case). Thus, if we wish, we can be Popperian in our methodology without using strict prediction in the mathematical models of the universe. And it may well be that we cannot use Cauchy data arguments in these models, since the work of Yodzis, Müller zum Hagen, and Seifert[21] has recently shown that "naked singularities" can occur in the universe. Data have to be given on these, just as on the surface of the piston in the cylinder. If this is so, then we must abandon any hope of being able to explain the universe in terms of initial conditions only.

If we allow types of explanation other than those based on Cauchy data, then to me there seems to be no pressing reason for retaining the sort of temporal causation that is required for the "initial condition" sort of explanation. Instead we may have to try to understand the universe in terms of laws of physics acting within a context which may be highly noncausal, having as data the conditions on all the boundaries[22] of the space-time which are not future relative to a locally determined time-sense. Such data can then be assigned according to the same criteria as are at present used in cosmology: they may be, for example, "simple" or "chaotic." As in normal predictive cosmology the data may not be given arbitrarily, but are subject to constraints which in some cases can be thought of as arising from interactions between different datum-points through the space-time.

According to the conventional view the universe can be seen as evolving from its initial condition like a watch wound up at the moment of creation.[23] This is not the case with the view I am proposing. According to my view it is not even correct to say that the data can be "prescribed," as in the piston example, since the structure of the boundaries on which the data reside is itself determined by those data. There is a web of interconnecting causation, proceeding in all temporal directions, which explains the universe as a spatio-temporal whole. By conventional standards the explanation thus achieved may appear post hoc, in that one cannot directly state acceptable data and then decide what universe results; rather, one must examine any proposed model as a whole to decide whether its data are acceptable at an explanatory level.

If this novel position is forced on us by, for example, the observation of naked singularities, or of the time-reversed instability of a black hole, then one can imagine two possible lines of development. In the first, it may prove possible to accommodate all our observations in a model in which the boundaries and the data on them are very simple: a special case of this is the standard homogeneous cosmological model usually used at present. Then all the complexity which we observe in the universe is a consequence of the interplay of physical laws within space-time. If this proves workable then a real explanation of this complexity could indeed be achieved, irrespective of causality.

The second possibility would be that the complexity of the universe could only be swept onto the boundary of space-time, not disposed of. This could happen with conventional cosmology as well as with an unconventional causality structure: adopting the wound-up-watch model does not in itself guarantee explanatory power. If this were to happen, then, in the absence of any wider theory which in turn explained the boundary data, we should have to admit that cosmology was more descriptive and less explanatory than its recent practitioners have hoped.

Notes

1. By an "instant of a history" I refer to what is usually idealized as a point on a world-line: a part of the world-tube of a person or object which has a small enough temporal extent to be regarded as being within the "now" of some relevant observer. Because this instant is a localized concept, the statement here is not tautologous.

2. 'Closed' is used in the sense of 'compact without boundary' (as in 'closed universe,' etc.).

3. There is a full presentation of this argument, followed by a critical discussion, in D. W. Sciama's chapter "Retarded Potentials and the Expansion of the Universe," in T. Gold and D. L. Schumacher, eds., *Symposium on the Nature of Time* (Ithaca: Cornell University Press, 1967), pp. 55–67.

4. This is possible in any globally hyperbolic universe. See, for example, F. Friedlander's *The Wave Equation in Curved Space-time*, Cambridge Monographs on Mathematical Physics 2 (Cambridge: Cambridge University Press, 1975).

5. In fact a high limit is eventually reached because of the onset of correlations between the movements of charges at different places. See note 3 above. This does not affect the validity of the conclusion, however.

6. I use 'sense' to mean one of the two possible orientations of a line or curve.

7. H. Reichenbach, *The Direction of Time* (Berkeley: University of California Press, 1971 reprinting). From the physicist's point of view the weakest link in his argument (and in later versions of it by other authors) is the lack of a general account of *how* branch systems in a relatively low entropy state should be formed actively in such a disequilibrium situation. The work of Prigogine and his collaborators has now clarified this to some extent.

8. This might suggest the possibility of two intercommunicating worlds whose time senses were opposite, a situation whose possibility has often been opposed by philosophers. In fact this cannot happen since it is a consequence of Sciama's argument that two worlds in mutual intercommunication must have the same time-sense, if they have any at all.

9. See, for example, B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, *Gravitation Theory and Gravitational Collapse* (Chicago: University of Chicago Press, 1965).

10. R. H. Price, "Nonspherical Perturbations of Relativistic Gravitational Collapse: I Scalar and Gravitational Perturbations" and "II Integer-Spin, Zero-Rest-Mass Fields", *Physical Review* D5 (1972): 2419–2438 and 2438–2454.

11. O. Penrose and I. C. Percival, "The Direction of Time", *Proceedings of the Physical Society* 79 (1962): 605–616.

12. A space-time M with metric g is called stably causal if the space-times (M, g') are causal for all g' sufficiently near g (in the fine topology on metrics). This is the case if and only if the space-time has a global time-coordinate. See S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-time* (Cambridge: Cambridge University Press, 1973).

13. The dependence of singularity theorems on causality assumptions has been greatly lessened by the work of F. J. Tipler, *Causality Violation in General Relativity*, University of Maryland Ph.D. thesis, (1976).

14. These arguments are derived, with heavy paraphrase, from R. Lucas, *A Treatise on Time and Space* (London: Methuen, 1973); but I am responsible for the form which they take here.

15. Metaphorically speaking! If there is one closed timelike curve, then there is an infinity of them, but they may still occupy only a small volume of space-time.

16. This example, and its resolution, comes from J. A. Wheeler and R. P. Feynman, "Classical Electrodynamics in Terms of Direct Interparticle Action," *Reviews of Modern Physics* 21 (1949): 425–434.

17. See note 16, and also A. Peres and L. S. Schulman, "Signals from the Future," *International Journal of Theoretical Physics* 6 (1972): 377–382.

18. For a critical discussion of Schrödinger's cat from a modern viewpoint see B. S. De Witt, "Quantum Mechanics and Reality," *Physics Today* (September 1970): 30–35.

19. See note 15.
20. I am indebted to Professor A. Taub for this point.
21. H. Müller zum Hagen, P. Yodzis, H.-J. Seifert, "On the Occurrence of Naked Singularities in General Relativity," *Communications in Mathematical Physics* 34 (1973): 135–148; 37 (1974): 29–40.
22. By the "boundary of space-time", I mean the *b*-boundary (B. G. Schmidt, "A new Definition of Singular Points in General Relativity," *General Relativity and Gravitation* 1 (1971): 269–280). Data which can be expressed as scalars on the frame bundle sometimes have limiting values on this boundary (C. J. S. Clarke, "The Classification of Singularities," *General Relativity and Gravitation* 6 (1975): 35–40), and it might be hoped that this would generalise to genuinely singular situations.
23. See, for example, J. C. Graves, *The Conceptual Foundations of Contemporary Relativity Theory* (Cambridge, Mass.: M. I. T. Press, 1971).

JOHN EARMAN

Till the End of Time

1. Introduction

What could it mean to say that time has a beginning or an end? Is it possible that time has a beginning or an end? In this paper I shall not be concerned with these questions in their full generality, for I shall be concerned only with physically interesting possibilities. I cannot specify at the outset what is to count as a physically interesting possibility in the present context—substantial discussion will be needed to uncover the factors relevant to such a specification. In the sense in which I am using it, the notion of a physically interesting possibility is broader than that of a physical possibility; any actual physical possibility is a physically interesting possibility, but not conversely, although every physically interesting possibility must be intimately related to actual physical possibilities. It would seem good strategy to discuss physical possibilities first, before proceeding to the murkier concept of physically interesting possibilities. This would indeed be sound strategy, except for the fact that we do not know what counts as a physical possibility in the present context. Thus it is necessary to plunge right into murkier waters.

The particular approach that I shall explore is certainly not the only one, nor do I claim it is the best. However, it does have a virtue, albeit a negative one: it reveals that we are not now in a position to give meaningful answers to the questions posed above, and that in order to arrive at such a position it is necessary to settle a number of other questions first, some of which belong to mathematics, some to physics, and some to metaphysics.[1] Since the recognition of ignorance is often the first step toward wisdom, it is to be hoped that the way will be paved for more positive results.

2. Aristotle and Leibniz on the Beginning and End of Time

Initially, Aristotle's theory of time seems to allow for the possibility of a beginning or an end for time. According to Aristotle, time is the measure