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ON THE SIZE AND CONNECTIVITY OF THE k-CORE OF A RANDOM GRAPH

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Abstract. Let $G(n,p)$ be a graph with n labelled vertices in which each edge is present independently with the probability $p = p(n)$ and let $C(k\,; n,p)$ be the maximal subgraph of $G(n,p)$ with the minimal degree at least $k = k(n)$. In this paper we estimate the size of $C(k\,; n,p)$ and consider the probability that $C(k\,; n,p)$ is k-connected when $n \to \infty$.

1. Introduction. For a graph G let $C(k\,; G)$ be the maximal subgraph of G with the minimal degree at least k. It is not hard to see that $C(k\,; G)$ is well defined and can be obtained from G as a result of the process of removing from a graph vertices of degree less than k. Following [2] we call $C(k\,; G)$ the k-*core* of G. If no subgraph H of G has the property $\delta(H) \geq k$ we say that the k-core of G is empty.

Now let $G(n,p)$ be a random graph with n labelled vertices in which each from $\binom{n}{2}$ possible edges is present independently with probability p. In this paper we shall study the size and the k-connectivity of the k-core of $G(n,p)$, denoted by $C(k; n,p) = C(k; G(n,p))$ (for other properties of $C(k\,; n,p)$ see [2, 7]). We shall assume that both k and p may depend on n and we shall consider only the asymptotical properties when $n \to \infty$. Instead of p we shall use also average degree of $G(n,p)$ defined as $c = c(n) = (n-1)p(n)$ as another parameter characterizing density of $G(n,p)$.

2. The size of the k-core. Let $v(k\,; n,p)$ be a random variable equal 0 when the k-core of $G(n,p)$ is empty or the number of vertices of $C(k\,; n,p)$ otherwise. In this part of paper we shall proof three theorems stating that either $v(k\,; n,p)$ is equal 0 or it is a.s. very large (here and below a.s. means "with the probability tending to 1 as $n \to \infty$").

THEOREM 1. *If $k(n) \geq 3$ then a.s. either $v(k\,; n,p) = 0$ or $v(k\,; n,p) \geq 0.0002n$.*

THEOREM 2. *For every $\varepsilon > 0$ there is a constant d, such that for $c = c(n) > d$ and $k = k(n) \leq c - c^{0.5+\varepsilon}$ we have $v(k\,; n,p) \geq n - n\exp(-c^{\varepsilon})$ a.s.* .

THEOREM 3. *For every $\varepsilon > 0$ there is a constant d such that for every k and p we have a.s. either $v(k\,; n,p) = 0$ or $v(k\,; n,p) > n - nc^{-0.5-\varepsilon}$.*

In the proofs of above theorems the following result about density of $G(n,p)$ will be useful.

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LEMMA. *Let $a = a(n)$, $c = c(n)$ be functions of n such that $1.1 \leq a \leq 0.5c$ and either $c(n) = o(n)$ or $a(n) \to \infty$. Then, for $p(n) = \dfrac{c(n)}{n}$, $G(n,p)$ a.s. contains no subgraphs with s vertices, $s \leq 0.35\left(\frac{2a}{c}\right)\exp\left(-\frac{2}{a-1}\right)n$, and more than as edges.*

Furthermore, if $c \leq 3$ then a.s. for each subgraph of $G(n,p)$ with s vertices and at least $1.5s$ edges we have $s > 0.006n$.

Since Lemma can be easily shown using the first moment method, standard for random graph theory, we omit the proof here.

Proof of Theorem 1. Let us consider firstly the case when $c(n) \leq 3k(n)$. The number of edges in $C(k\,;n,p)$ is at least $\frac{k}{2}$ times greater than the number of its vertices, thus, from Lemma, the k-core, if nonempty, has a.s. at least $0.35 \cdot 3^{-3} \cdot e^{-4}n > 0.0002n$ vertices. Moreover, for $c(n) = 3k(n)$ $G(n,p)$ contains a.s. at least $0.9\binom{n}{2}p > 1.1nk$ edges. Since one can show easily that the k-core of any graph with n vertices and at least $n(k-1)$ edges is nonempty for $c(n) = 3k(n)$ we have $v(k\,;n,p) > 0.0002n$ a.s.. However, the property that graph contains a subgraph on at least $0.0002n$ vertices with the minimal degree at least k is increasing, so the lower bound for $v(k\,;n,p)$ remains valid also for $c(n) > 3k(n)$ (see Theorem II.1.1 from [1]). □

Let us notice here an interesting consequence of Theorem 1. In 1960 Erdős and Rényi proved [4] that $G(n, \frac{1}{n})$ contains a.s. a cycle of odd length, i.e. the chromatic number of $G(n, \frac{1}{n})$ is at least 3 but they did not find the exact value of this number. This problem remained open until 1986 when Łuczak and Wierman showed in [8] that actually a.s. $\chi(G(n, \frac{1}{n})) = 3$. From Theorem 1 we can obtain easily a bit stronger result

COROLLARY 1. *If $1 \leq c \leq 1.0001$ then a.s. $\chi(G(n,p)) = 3$.*

Proof. For $1 \leq c \leq 1.0001$, $G(n,p)$ consists of isolated trees, unicyclic components and an unique "dense" component of the size less than $0.00015n$ (see [4] or Ch.V of [1]). Thus, due to Theorem 1, for such a graph a.s. $v(3\,;n,p) = 0$. Since it is well known (see for example [5]) that each graph with the chromatic number at least 4 has nonempty 3-core, so the assertion follows. □

Proof of Theorem 2. In the proof we use the idea presented in [2, 3]. Let us define an increasing sequence $\{U_i\}_{i=0}^{s}$ of subsets of vertices of $G(n,p)$ in the following way:

(i) U_0 is the set of vertices of degree at most k;

(ii) for $U_0, U_1, \ldots, U_\ell$, $U^1_{\ell+1}$ is the set of those vertices outside U_ℓ which have at least two neighbours in it. If $U^1_{\ell+1}$ as $U_\ell \cup \{v\}$ where v is the first vertex from $U^1_{\ell+1}$.

If $|U_s| < n$ then a subgraph induced in $G(n,p)$ by the set W of all vertices outside U_s has the minimal degree at least k, so it is contained in $C(k\,;n,p)$. Thus, it is enough to show that $|U_s| < n\exp(-c^{\epsilon})$.

Let us start with the estimation of the size of U_0. If $c(n) \geq n^{0.5}$ then $U_0 = \emptyset$ a.s. (see Theorem III.1.3′ in [1]). So let $c(n) < n^{0.5}$ and X be the number of vertices of degree at least k, $k \leq c - c^{0.5+\varepsilon}$. Then

$$\begin{aligned} \mathrm{E}X &= n \sum_{i=0}^{k+1} \binom{n-1}{i} p^i (1-p)^{n-i-1} \leq \\ &\leq 2n \left(\frac{ec}{k}\right)^k \cdot e^{-c} \leq 2n \exp(-c^{0.5+\varepsilon}) \left(1 + \frac{c^{0.5+\varepsilon}}{c - c^{0.5+\varepsilon}}\right)^{c - c^{0.5+\varepsilon}} \leq \\ &\leq 0.01 n \exp(-c^{\varepsilon}) . \end{aligned}$$

Similarly, we can obtain that $\mathrm{Var}X = o((EX)^2)$ so, from Chebyshev inequality, we have a.s. $|V_0| < 0.1 n \exp(-c^{\varepsilon})$.

From the definition of $\{U_i\}_{i=0}^{s}$ the subgraph induced in $G(n,p)$ by U_i, where $i = 1, 2, \ldots, s$, has at least $2(|U_i| - |U_0|)$ edges. Thus if for some i, $|U_i| = \lfloor n \exp(-c^{\epsilon}) \rfloor$ then the subgraph induced by this set would have

$$2(|U_i| - |U_0|) \geq 2(|U_i| - 0.2|U_i|) > 1.5|U_i|$$

which is impossible due to Lemma. Thus a.s. $|U_s| < n\exp(-c^{\epsilon})$ and $v(k; n, p) < n - n\exp(-c^{\epsilon})$. □

Proof of Theorem 3. For $k < c - c^{0.5(1+\varepsilon)}$ the assertion follows from Theorem 2. So, let $k > c - c^{0.5(1+\varepsilon)}$ and let S be the set of all vertices outside the k-core. If $v(k; n, p) \neq 0$ then, from Theorem 1, $|S| < 0.9998n$ a.s. . Let us suppose that $|S| > nc^{0.5+\varepsilon}$. We shall show that in this case a.s. a half of vertices of the k-core has more than $0.5c^{0.5+\varepsilon}$ neighbours in S.

Indeed, for fixed S and $v \notin S$, the number of vertices in S adjacent to v is binomially distributed with parameters $|S|$ and p. For Bernoulli random variable X with parameters m and p the following inequalities hold for every $\varepsilon > 0$ [6]:

$$(*) \qquad \begin{aligned} \mathrm{Prob}(X > (1+\varepsilon)mp) &< \exp\left(-\frac{\varepsilon^2 mp}{3}\right) \\ \mathrm{Prob}(X < (1-\varepsilon)mp) &< \exp\left(-\frac{\varepsilon^2 mp}{2}\right) . \end{aligned}$$

Thus, the probability of the event that

$$(**) \qquad |N(v) \cap S| \leq 0.5c^{0.5+\varepsilon} \leq 0.5p|S|$$

is less than $\exp(-0.1c^{0.5+\varepsilon})$. Hence, the probability that there exists such S that for at least $0.5(n - |S|)$ vertices outside it $(**)$ holds is at most

$$\sum_{s=nc^{-0.5+\epsilon}}^{0.9998n} \binom{n}{s} \binom{n-s}{0.5(n-s)} \exp(-0.1c^{0.5+\varepsilon} \cdot 0.5(n-s)) \leq n 2^n 2^n \exp(-c^{0.5} n) = o(1) .$$

and a.s. at least half of all vertices of the k-core have more than $0.5c^{0.5+\varepsilon}$ neighbours outside it.

Now note that calculations similar to these from the proof of Theorem 2 shows that a.s. less than $n\exp(-c^{0.5\varepsilon}) > 0$ vertices of $G(n,p)$ has more than $c + c^{0.5(1+\varepsilon)}$ neighbours. Thus at least one third of vertices of $C(k\,;n,p)$ are adjacent to less than

$$c + c^{0.5(1+\varepsilon)} - 0.5c^{0.5+\varepsilon} < c - 2c^{0.5(1+\varepsilon)}$$

vertices in it. But each vertex of $C(k\,;n,p)$ must have degree at least $c - c^{0.5(1+\varepsilon)}$. This contradiction completes the proof of Theorem 3. □

From Theorem 3 and the fact that the number of vertices of degree at least $c + c^{0.5+\varepsilon}$ is smaller than $n\exp(-c^{\varepsilon})$ we obtain immediately the following result.

COROLLARY 2. *For every $\varepsilon > 0$ there is a constant d such that for $c = c(n) > d$ and $k = k(n) > c + c^{0.5+\varepsilon}$ a.s. $C(k\,;n,p)$ is empty.*

3. The connectivity of the k-core. In this section we shall study the connectivity of $C(k\,;n,p)$. Since the k-core is the maximal subgraph with the minimal degree k so $C(k\,;n,p)$ is the largest possible subgraph of $G(n,p)$ which may be k-connected. Our main result solves the problem of the connectivity of the k-core.

THEOREM 4. *There is a constant d such that for $c(n) > d$ and $k(n) \geq 3$ the graph $C(k\,;n,p)$ is a.s. either empty or k-connected. Moreover, if $c(n) \to \infty$ then also $C(2\,;n,p)$ is a.s. 2-connected.*

Bollobás, in [2], proved Theorem 4 in a very special case when $c(n)$ is a constant larger than 67 and $k \in \langle 8, \frac{c}{2}\rangle$. In fact, he introduced the notion of the k-core seeking for large, k-connected subgraphs of random graphs.

Proof of Theorem 2. To proof the result we must show that the set of vertices of $G(n,p)$ can not be divided onto four sets S_1, S_2, T_1, U in such a way that S_1, S_2, T are vertices of $C(k\,;n,p)$ and $T, |T| \leq k-1$, is a cutset of the k-core, i.e. there are no edges between S_1 and S_2 in $G(n,p)$. We shall set $s_1 = |S_1|$, $s_2 = |S_2|$, $t = |T|$, $u = |U|$ and assume that $s_2 \geq s_1 \geq 2$.

We split the proof into 6 cases.

Case 1. $d < c(n) < \log^4 n$.

It is well known (see Theorem IV.1.1 in [1]) that for such $c(n)$, $G(n,p)$ a.s. contains no subgraphs on less than 500 vertices with more edges than vertices. Hence a.s. no two vertices of S_1 have three common neighbours so for every k we have $s_1 \geq 2k-2-t \geq k-1$. Moreover, for $3 \leq k \leq 50$ set $S_1 \cup T$ has more edges than vertices, thus we have $s_1 \geq 10t$ in this case.

Let us consider the subgraph induced by $S_1 \cup T$. It has $s_1 + t$ vertices and at least $0.5ks_1$ edges where

$$0.5ks_1 \geq \begin{cases} 1.25(s_1+t) & \text{for} \quad k > 50 \\ 6(s_1+t) & \text{for} \quad 3 \leq k \leq 50 \,. \end{cases}$$

Thus, as a consequence of Lemma for $a = 1.25$ and $a = 6$ we obtain that $s_1 + t > 4ne^{-c}$ for $c \leq k \leq 50$ and $s_1 + t \geq 4nc^{-1.5}$ for $k > 50$. Hence, since $s_1 \geq k - 1 \geq t$ we have $s_1 \geq 2nc^{-6}$ for small k and $s_1 \geq 2nc^{-1.5}$ for k large.

However one can show easily using the first moment method that a.s. if S is a subset of $G(n,p)$ and $nc^{-6} \leq |S| \leq nc^{-1.4}$ then $|N(S)| > 0.5c|S|$. Hence, for $3 \leq k \leq 50$ Theorem 2 implies that

$$|N(S_1)| > nc^{-5} > n\exp(-c^{0.1}) + k > u + t.$$

Similarly, for $k > 50$ using Theorem 3 and Corollary 2 we arrive at

$$|N(S_1)| > nc^{-0.5} > nc^{-0.5-\epsilon} + c + c^{0.5+\epsilon} > u + t.$$

Thus we have $N(S_1) \cap S_2 \neq \emptyset$. This completes the proof of Case 1.

Case 2. $\log^4 n \leq c(n) \leq 0.75n$.

We shall find first the lower bound for the sum $s_1 + t$. First observe that for $c(n) < 0.75n$ there are a.s. no pairs of vertices in $G(n,p)$ with more than $0.8c$ common neighbours. Indeed, for $c(n) < n^{0.6}$ it is well known and for $c(n) < n^{0.6}$ the probability of existence of such a pair is from $(*)$ smaller than

$$n^2 \exp(-0.0001np^2) \leq n^2 \exp(-n^{0.1}) = o(1)\,.$$

Now notice that for $c(n) \geq \log^4 n$ we have a.s. $\delta(G(n,p)) > 0.99c$ (see Theorem III.1.3′ in [1]). Hence, since $s_1 \geq 2$ we have

$$s_1 + t > 0.99c + 0.99c - 0.8c > 1.15c\,.$$

Since it is enough to consider k for which $|k - c| < 0.01c$ we obtain that $s_1 > 0.1t$ and the subgraph induced by $S_1 \cup T$ has at least

$$0.5ks_1 \geq 0.04k(s_1 + t) \geq 0.03c(s_1 + t)$$

edges. Then, from Lemma, we have $s_1 + t > 0.002n$ and so $s_1 > 0.001n$. However, the probability that in $G(n,p)$ there are sets S_1, S_2, $|S_2| \geq |S_1| \geq 0.001n$ with no edges between them is at most

$$2^n 2^n (1-p)^{(0.001n)^2} \leq 4^n \exp(-10^{-6}c^2 n) = o(1).$$

This completes the proof of this case.

Case 3. $c(n) \geq 0.75$ but $\overline{c(n)} = n - c(n) \geq \log^4 n$.

Since we have (see Theorem II.1.3′ from [1])

$$n - 1.01\bar{c} \leq \delta(G(n,p)) \leq \Delta(G(n,p)) \leq n - 0.99\bar{c}$$

we may consider only those k for which $|n - \bar{c} - k| \leq 0.01\bar{c}$. Moreover, since $s_2 \geq s_1$ using the first moment method we obtain immediately that $s_1 < \log^4 n$. Thus, the fact that $t \leq k-1 \leq n - 0.99\bar{c}$ implies that for every $v_1, w \in S_1$ we have

$$\begin{aligned}|N(v) \cap N(w)| &> n - 1.01\bar{c} - 1 + n - 1.01\bar{c} - 1 - 2\log^2 n - (n - 0.99\bar{c}) > \\ &> n - 1.04\bar{c}.\end{aligned}$$

But in this case in the complement of $G(n,p)$, which is nothing else but $G(n, 1-p)$, vertices v, w would have at least

$$0.99\bar{c} + 0.99\bar{c} - 1.04\bar{c} = 0.94\bar{c}$$

common neighbours and the probability of such an event tends to 0 as was shown in the proof of the previous case.

Case 4. $\dfrac{1}{\log\log n} \leq \overline{c(n)} \leq \log^4 n$.

Since in the complement of $G(n,p)$, i.e. in $G(n, 1-p)$, each vertex of S_1 is adjacent to all vertices from S_2 and, for $c(n) \leq \log^4 n$, $G(n, 1-p)$ contains no subgraphs on 5 vertices with more edges than vertices must have $s_1 = s_2 = 2$. Moreover, the k-core of a graph has at least $k+1$ vertices, so $k + 3 \leq t \leq k+1$. Then the minimal degree of a subgraph induced in $G(n, 1-p)$ by $S_1 \cup S_2 \cup T$ is at most 3 so $G(n, 1-p)$ should contain at least

$$t + 4 - u\Delta(G(n, 1-p)) \geq n - 1.01\log^4 n - (1.01\log^4 n)^2 < n - n^{0.1}$$

vertices of degree less than 4.

However, it can be easily observed that a.s. $G(n, 1-p)$ contains more than $n^{0.2}$ vertices of degree at least 4. Indeed, let us count the vertices with label less than $\frac{n}{2}$ which has more than three neighbours of label at least $\frac{n}{2}$. The number of such vertices is binomially distributed with parameters $\frac{n}{2}$ and $\tilde{p}$ where

$$\tilde{p} = \sum_{i=4}^{\frac{n}{2}} \binom{\frac{n}{2}}{i} (1-p)^i p^{\frac{n}{2}-i} \geq n^{-0.1}$$

so $G(n, 1-p)$ contains more than $n^{0.5}$ of vertices of degree at least 4.

Case 5. $\overline{c(n)} < \dfrac{1}{\log\log n}$.

Since $s_2 \geq s_1 \geq 2$ so either $C(k\,; n, p)$ is k-connected or $G(n, 1-p)$ contains a cycle of length 4 and the probability of the latter event tends to 0 as $n \to \infty$.

Case 6. $k = 2$, $c(n) \to \infty$.

We shall show that for $c(n) \to \infty$, $C(2\,; n, p)$ a.s. is 2-connected. Since for $c(n) \geq 2\log n$ $G(n,p)$ is a.s. 2-connected (see [1] Theorem VII.2.6) and then, obviously, $C(2\,; n, p) = G(n,p)$, thus it is enough to prove this fact for $c(n) < 2\log n$.

It can be easily observed that every graph consists of its the 2-core, isolated trees and some trees rooted at the 2-core, i.e. such trees that each from their vertices except one "root" are adjacent to no vertices outside them. Now suppose that $C(2\,; n, p)$ is not 2-connected and let $\overline{S}$ be the sum of S_1, T and all vertices of trees rooted in $S_1 \cup T$. Since a subgraph induced by $\overline{S}$ is connected and contains at least one cycle so the number of its edges is at least as large as the number of its vertices. Moreover, every vertex of $\overline{S}$, except, maybe, the one from T, has no neighbours outside $\overline{S}$. Since from Theorem 2

$$|\overline{S}| < n - \frac{v(2\,; n, p) - 1}{2} < 0.1n$$

it is enough to show that $G(n,p)$ a.s. contains no sets S, $|S| < 0.1n$ with all these properties. The expectation of the number of such sets can be estimated from above by

$$\sum_{k=3}^{0.1n} \binom{n}{k} \binom{\binom{k}{2}}{k} p^k k (1-p)^{(k-1)(n-k)} \leq$$

$$\leq \sum_{k=3}^{0.1n} \left(\frac{en}{k}\right)^k k \left(\frac{ekc}{2n}\right)^k e^{-0.2ck} \leq$$

$$\leq \sum_{k=3}^{0.1n} k(e^2 c e^{-0.2c})^k = o(1)\,.$$

This completes the proof of case 6 and thus Theorem 4. Let us observe that it does not remains valid when $k = 2$ but $c(n)$ tends to a constant as $n \to \infty$ since then the probability that $G(n,p)$ contains an isolated triangle tends to $1 - \exp(-\frac{c^3 e^{-3c}}{6})$. □

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