Feedback Dialogues in Elementary Mathematics: An Exploratory Study

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Dedication

This dissertation is dedicated to my children: Teddy, Brooks, and Ellie.

Watching you learn and grow inspires me continue to support the work of educators everywhere. I hope you seek out feedback at every point along the way in your life. I encourage you to embrace the challenges and to dialogue with others so that you may never stop learning.

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Chapter 1: Introduction

Feedback has been recognized as a powerful tool used in education; and research has shown the powerful effect feedback can have on learning (e.g. Black & William, 1998; Hattie & Timperley, 2007; Klueger & DeNisi, 1996). In a synthesis of over 800 meta-analyses, John Hattie (2009) examined the impact of over 100 various influences on learning such as parental involvement, diversity of school, peer tutoring, small group learning, and direct instruction. In his analysis, Hattie found feedback to be highly influential and impactful with regard to student achievement. However, conflicting research has also shown feedback can have little or even a negative impact on learning (Bangert-Drowns et al., 1991; Klueger & DeNisi, 1996; Shute, 2008). Although there have been decades of research, including several meta-analysis around the effectiveness of feedback, there is still much that remains unknown about the specific characteristics of effective feedback and how it can be used to improve student learning. Additionally, feedback practice continues to be one of the weakest components of teachers' classroom assessment (Ruiz-Primo & Li, 2004, 2013a, 2013b;). Questions still remain as to why, when teachers provide valid statements that address the performance of student work improvement does not always follow (Sadler, 1989). Deciding what feedback consists of, how it is delivered, and what is done after the feedback is given, are all questions that teachers must consider (Wiliam, 2011).

One of the challenges in understanding what makes feedback more effective has been the varied ways in which feedback has been studied and the aspects of feedback focused on during the study (Ruiz-Primo & Brookhart, 2018). These different aspects can

include factors such as, how the feedback is given (written, orally, automated computer generated), when the feedback is given (immediate or delayed) and probably the most complex or extensive factor is the content of the feedback or the actual feedback messages themselves. While there are have been some recommendations around these factors in regards to feedback effectiveness (Shute, 2008) which I elaborate on in Chapter 2, most of the suggestions for effective feedback tend to span across all subject and content areas, which leaves researchers wondering if feedback practices differ across different disciplines (Ruiz-Primo & Li, 2013).

A second challenge in understanding how the specific characteristics of feedback impact learning is the way in which the feedback process is conceptualized (Nicol, 2010). Specifically, in the field of mathematics, there have been a limited number of studies focused on feedback practices and within the studies conducted most of them focused on evaluative feedback as opposed to formative feedback (Li, Yin, Ruiz-Primo & Morozov, 2011). Formative feedback is designed to provide information to the learner to help them improve and move forward in their understanding, whereas evaluative feedback simply provides a score or indication of accuracy. Without knowledge and an understanding of what feedback looks like in a mathematical context, many educators struggle to use this process effectively in improving student thinking and learning (Ruiz-Primo & Li, 2013).

I suggest that it is necessary to study more specifically what feedback looks like in a mathematics classroom and how that feedback may create opportunities for students to take action which in term could lead to the improvement of conceptual understanding of mathematical concepts. The National Council of Teachers of Mathematics (NCTM)

(2013) takes the position that "[b]y receiving formative feedback, students learn how to assess themselves and how to improve their own learning" (p. 1).

Much of the prior research around feedback effectiveness has conceptualized feedback as a one-way transmission of knowledge. Instead of feedback as a one-way process, some more of the recent research and literature has suggested that feedback move away from a monologue view and towards an interactive, co-constructed dialogic view of feedback (e.g. Askew 2000, Molloy & Boud 2013, Nicol, 2007, 2010,) Dialogic feedback is based on a co-constructivist model of teaching and learning in which dialogue is central to the taking up and construction of new ideas.

In a Dialogic feedback model, students play a participatory role in the feedback process and the focus is on strategies to improve student performance (e.g., Boud, 2000; DeNisi & Kluger, 2000; Hattie & Timperley, 2007). Dialogic feedback is constructed during interaction between the teacher and student in which meaning is constructed and misunderstandings are shared and clarified during this process.

Researchers have examined and described feedback given to students and some have explored how children take up the feedback, but there has been less research that examines the interaction teachers and students have with the feedback. Additionally, much of the prior research around dialogic feedback has occurred in the field of higher education. Ruiz-Primo and Li (2013) suggest further research is needed in order to understand the details of interaction that aim to describe specific characteristics of feedback dialogues. It is necessary to look more deeply at feedback as a process of communication (Higgens et al., 2001). When feedback is conceptualized as information provided to modify, or influence the learner's performance, it ignores the social aspect of

feedback that is bounded by language (Torres, 2016). Instead, analyzing feedback as interaction has the potential for one to better understand how dialogue is carried out and sustained, opening up new understandings of how feedback functions and is interpreted (Ajjawi and Boud, 2015).

Purpose of the Study

The purpose of this study is to look more closely and more deeply at how feedback is co-constructed with students in regards to specific mathematical tasks-with a particular focus on gaining an understanding of dialogic feedback. The dynamic nature of dialogic feedback varies depending on the context and the learner, but this study sought to understand if there are several key moves that one may follow or hold onto as they dialogue with students around their mathematical thinking. The goal is to study feedback dialogues in a natural and authentic setting in order to describe and analyze what is happening inside these dialogues and the particular shifts in learning that may occur during the interaction. As a result of this study, I aimed to construct a description of potentially effective dialogues with students based on the research of what feedback should be.

This study was guided by the following research questions:

1. Given the conceptual framework based on effective feedback practices and theories of learning from constructivist and sociocultural perspectives, how do these feedback dialogues take shape?

1. What might we notice about the characteristics of moments, within feedback dialogues, that prompt a change in mathematical thinking throughout the unit of study?

Overview of the Chapters

Chapter 1 has provided the rationale for the study and presented the research questions. Chapter 2 provides a review of literature on areas related to the study. Chapter 3 outlines the design of the research, what data was collected and how the data was analyzed. Chapter 4 presents the analysis of the data and answers the research questions. Finally, Chapter 5, summarizes the findings and outlines its implications and shares recommendations for further research.

Chapter 2: Literature Review

Feedback is a complex and multidimensional concept, with varying degrees of impact. Prior research has shared much about the different aspects of feedback and how they might contribute to feedback's overall effectiveness of learning. However there is still much unknow about the actual process and the "how-to" of feedback in the classroom. This review of literature focuses on three topics that influenced the design and implementation of this research. First, I highlight what is currently understood about effective feedback practices, and recommendations about feedback from prior research. In this section I include research centered around how feedback is given, what content is contained in the feedback, and when feedback is given. This review provides a foundational understanding of what is already known about feedback practices and the impact those practices have shown to have on student learning.

The second section shifts to a focus on dialogic feedback and the opportunities it may afford students in ownership over their learning. I share what is already known about dialogic feedback and what components others have theorized are important in co-constructing feedback dialogues. Lastly, I conclude by situating my work within a theoretical framework of how knowledge and learning develop by drawing on constructivist and socio-cultural theories of learning and, finally, present the conceptual framework that guided this study.

How Feedback Is Given

When trying to understand the effectiveness of feedback it is important to consider the form of the feedback or how the feedback is given. Feedback can consist of markings, grades, rubrics, or descriptive comments and can be delivered through various

modes such as oral comments, written symbols and language, or auto generated by a computer (Butler, 1987; Murtagh 2014; Sobahni & Tayebipour 2015; Telceker & Akan 2010; Tunstall & Gipps 1996;) In an attempt to better understand the impact and structure of feedback, researchers have conducted studies that aim to draw out and explore the various ways in which feedback is provided to students. Tunstall and Gipps (1996) observed and recorded the variety of feedback given to primary children by teachers in the classroom. Based on their observations, the researchers developed a typology which divided feedback into two categories: descriptive and evaluative. Evaluative feedback was based on judgments and accuracy and might include comments such as "great work" or a number to indicate how many correct or incorrect answers. Descriptive feedback was explanatory and based on the specification and construction of further learning and could include specific comments about why something is good or provide suggestions on what to do next.

Butler (1998), examined evaluative and descriptive feedback more closely in order to determine the impact on student learning. In the study (Butler, 1998) students were given a set of tasks to complete during the first lesson and then were collected and scored by outside evaluators. Each group was given feedback in one of three ways. The first group was given a score, the second received comments, and the third was given both a score and comments. Students were asked again to complete similar tasks as before, and were scored again. The group that received only a score made no gain in improving their initial score from the first lesson. The students who received feedback in the form of comments scored 30% better. The group with the comments and score also made no improvement; similar to the group that received only a score. Similarly, Elewar

and Como (1985) found that students who received descriptive feedback on their homework scored higher on the post-test in mathematics than students who only received a score for the number of correct answers. Much research around feedback (e.g. Black & Wiliam 1998; Klueger & DeNisi 1996) has found descriptive feedback to be more effective and powerful in impacting learning. Leahy, Lyon, Thompson & Wiliam (2005) suggest "To be effective, feedback needs to cause thinking. Grades don't do that. Scores don't do that. And comments like 'Good job' don't do that either (p. 22). Descriptive feedback has the potential to move learning forward, rather than evaluative feedback, which often presents a view of looking back on what was learned.

There has been some question as to whether the mode in which feedback is delivered matters, but only a few studies have actually aimed to examine this. Descriptive feedback can be given orally, through written comments, or computer generated through programmed responses. Some have suggested that computer generated feedback is more effective because it is seen by students as a neutral source of information without any bias, meaning it is less likely for students to dismiss the feedback due to a lack of mistrust for the feedback giver. (Kuleger and Denisi, 1996; Shute, 2008). However, Lipnevich and Smith (2008) found students who received electronic descriptive feedback from someone they believed to be their course instructor outperformed students who received descriptive feedback from what they believed to be a highly sophisticated computer software program. Students also perceived instructor feedback to be more helpful and accurate than computer generated feedback. This suggests the effectiveness of feedback may depend more on the relationship between student and instructor, rather than the form the feedback takes.

Additional studies have attempted to determine whether it is better to have feedback that is written or provided orally. Sobahni and Tayebipour (2015) compared written vs oral feedback given to students during the writing process. The authors found oral feedback to be more effective than written feedback in regards to writing performance with English language learners. A similar conclusion was reached by Telekar and Akan (2010), when investigating how the teacher's written feedback and one-on-one conferences with students impacted student writing. The authors concluded that written feedback was useful for students in correcting grammatical revisions, but the oral conference with the teacher led to a greater effect in helping students improve the content and organization of their writing. It is important to note that both of these studies were subject-specific and worked specifically with the students who were learning English. The ability to apply their conclusions across all learners and content areas is likely limited However, I suggest that oral feedback tends to be more dialogic in nature, creating a conversation around the feedback and therefore potentially being more impactful, which I discuss later in the chapter.

The Content of Feedback

When feedback is carefully constructed and crafted, there is an opportunity to communicate with students about their current state of understanding in the learning process and where they can move next. Unfortunately, feedback can also send unintentional messages that hinder learning, blocking a way for students to move forward. Due to mixed messages, it is necessary to consider how the content, or the 'what' of feedback impacts student actions and outcomes. Hattie and Timperley (2007) constructed four different levels in order to describe the type of information feedback can

provide. These levels include feedback about the self, the task, the process, and the self-regulation strategies. Additionally, research also addresses the relationship of feedback and learning goals, and the specificity of detail.

Level of Feedback.

Klueger and DeNisi as summarized by Wiliam (2015) found that students can have one of four reactions to feedback. Based on the feedback students receive they can 1) change their behavior by either increasing or decreasing effort; 2) change the goal by making it easier or more difficult; 3) abandon the goal because feedback indicates the task was too easy or too difficult; 4) reject the feedback. As Wiliam points out, only two of the options listed above, increasing effort or increasing the difficulty of the goal, result in a positive outcome towards impacting student learning. Ideally, feedback provided to students will either result in a positive change of behavior, or the student challenging themselves to a more difficult goal.

Some teachers believe that marking students work with ticks or merits helps to motivate children or increase self-esteem (Murtagh, 2014); but there is also considerable evidence that this is not the case (Butler 1987; Kohn, 1993; Mueller & Dweck 1998). Dweck (2006) has completed extensive research on intelligence and motivation and has found that mindset can greatly predict and impact student achievement or success. Through her research, Dweck suggests that those who have a "*fixed mindset*" believe that intelligence is fixed and attribute either success or failure to an innate ability that cannot be changed. However, those with a *growth mindset* believe that with continual effort and challenging opportunities, a person can 'grow their intelligence' and increase their knowledge. In a study with fifth graders, Muller and Dweck (1998) found

that students who were praised for their ability showed less tenacity and task persistence when the tasks become increasingly difficult as compared to students who were praised for effort. Students praised for their ability were also more concerned with performance rather than learning outcomes – and attributed their success and failure to a fixed level of intelligence. When students are given feedback about their ability, there is an increased risk that the student will focus on their intelligence rather than their learning.

Although it may seem counterintuitive to many elementary school teachers who hope to build student confidence and motivation, praise has shown to have little impact on learning (Hattie & Timperley, 2007). Feedback that is focused at the self-level (Hattie & Timperley, 2007) such as "you're doing a great job" has shown to have a negative impact on student achievement later on. This is because students focus on receiving the praise as an indication of their intelligence and therefore avoid challenging tasks that increase learning in fear of failure. Instead, recent literature around feedback practices for teachers suggests using Dweck's mindset theory as a framework to provide feedback to students (Boaler, 2015; Wiliam, 2015). Feedback that uses a growth mindset provides information to the student at the task and process level (Hattie & Timperley, 2007). Feedback is crafted around the learning tasks and processes exhibited by the learner, and can provide ways for students to take action. Growth feedback sends the message to students that ability, intelligence and success is not created in one instance, but rather over time through the process of creating and recreating, writing and rewriting, or solving and resolving.

Feedback in Relation to Learning Intentions.

In a theoretical paper, Sadler (1989) defined feedback in relation to its function in

formative assessment. Sadler expanded on Ramaprasad's (1983) definition, which describes feedback as "information about the gap between the actual level and the reference level of a system parameter which is used to alter the gap in some way" (Ramaprasad, 1983 p. 4). Expanding on Ramaprasad's early definition, Sadler identifies three components that must occur simultaneously in order for feedback to close the learning gap: students must be able to identify the goal, understand where their performance is in relation to the goal, and take action that helps narrow the learning gap.

In order for information to be considered feedback, a person must have a goal that he or she is working towards and is taking action to reach that goal (Hattie, 2012; Wiggins, 2012). When students are unclear about where they are going and what it takes to be successful, they are unlikely to act on the feedback provided to them. Success criteria for the learning goal or intention is developed by the teacher and students, such that each person is aware of what it means to understand the learning intention. When students have a strong understanding of what the learning goal is and what successful performance might look like, they are better able to self-assess and identify the gap by comparing their own learning to the expected learning outcome (Sadler, 1989). In interviewing students, Dahn (2015) observed that low achieving primary students were able to identify what they perceived to be the teacher's learning targets in literacy such as "read it" or "use commas" and were able to suggest areas of their own learning gap. When asking students about mathematics, they were unable to come up with any learning targets and were not able to identify their own learning gap and learning aspirations. One way to ensure students have an understanding of the learning goal and the success criteria is to have students look at exemplars of previous work, and identify areas of success and

areas of improvement based on the learning intentions (Wiliam, 2011).

Once students have a strong sense of the expected learning intentions, feedback should then be provided to students based on those learning intentions in a systematic way, rather than addressing all student errors. Furthermore, Nyquist (as cited in Wiliam, 2013) found that feedback with the greatest effect sizes not only provided information about the gap, but included activities or information intended for closing the gap between the student's current learning and the desired learning goal. It is not enough for students to simply understand what their learning gap is, there has to be an opportunity and a way to move forward in closing the gap in order for feedback to have any effect.

Feedback Specificity.

According to Kulhavy and Stock(1989), Feedback can provide learners with two types of information: verification and elaboration. Verification feedback, also referred to as reinforcement (Wiliam, 2015), simply tells the learner whether the answer is correct or not. Elaborated, or corrective (Wiliam, 2015) feedback, presents more information about why the answer is correct, and if not, additional information is provided in order to guide the learner to the correct answer or desired outcome (Shute, 2008). In some instances, whether then answer is correct or not may be helpful in communicating to learners they are on the right track (Wiliam, 2015), but elaborated or corrective feedback has been shown to be more effective in increasing learning (Banger-Drowns, et al., 1991).

Elaborated feedback can provide information in various ways in regards to the learning by: 1) providing an explanation of the correct answer; 2) focusing on the student's errors; 3) providing additional content information; or 4) scaffolding the task through cues and prompts (Shutte, 2008). In an experiment with third graders learning to

use a balance scale, Day and Cordon (1993) found that students who were given a scaffolded response based on the learner's ability did better on the post assessment, and displayed a higher rate of learning transfer as compared to students who received a full explanation of how to solve the problem. Essentially, the students who were given just enough information to help get 'unstuck' or move forward, performed better than students who were given all the information needed to solve the problem. This suggests that more feedback is not always better. In interviews with students (Hargreaves, 2013), children indicated a preference for cues and prompts provided by the teacher, rather than being directly told what the answer was.

Although some have suggested that feedback should contain specific actionable steps that students can take to improve performance, Wiliam (2015) cautions the use of feedback that is too prescriptive. Feedback that is too specific runs the risk of only being useful for that one particular task and therefore is not sustainable for future learning. For example, when giving feedback about adding two fractions together, the intent is not only for the student to be able to solve that one problem correctly, but also so they are able to solve any problem involving the addition of fractions. Knight and Yorke (2003) propose that "it is general feedback that has the greater power to stimulate learning" (p. 32). Additionally, Hargreaves (2013) documents an instance in which a student was shown how to use an online thesaurus. The student perceived this as one of the most useful and effective pieces of feedback given to her. The feedback consisted of a tool in which the student could continue to use with future tasks, and focused more on the process of learning than the product or outcome. Hattie and Timperley (2007) claim that feedback that is aimed at the level of process and self-regulation is more beneficial in the long term

than feedback aimed at the level of the task.

It should also be noted, however, that feedback that is too general runs the risk of not being understood or put into action by the learner. When children were asked about their experience with feedback, some indicated a frustration when the teacher did not provide enough information or the information was not specific enough to help them identify next steps (Hargreaves, 2013). This indicates that there must be a balance between feedback that provides actionable steps, and feedback that is also sustainable beyond the current task.

The Timing of Feedback

Answering the question about when feedback should be delivered in order to have the greatest impact on learning has resulted in mixed conclusions. Some studies have suggested that immediate feedback is more effective (Corbett & Anderson, 2001; Dihoff, Brosvic, & Epstein, 2003), while others have suggested that delayed feedback has more potential in impacting the transfer of learning (Butler et al., 2007; Metcalf Kornell & Finn 2009; Schroth, 1992). Additionally, it is difficult to compare studies due to the inconsistent definition of delayed feedback. Immediate feedback is most often defined as occurring right after the task or assessment. Delayed feedback is defined in relation to immediate feedback and therefore could occur minutes, hours, days or weeks later (Shute, 2008). A better question to consider, then, is how the timing of feedback acts in relation to the individual learner and the learning task or processes.

Fyfe, Rittle-Johnson & Decaro (2012) looked at the effect feedback had during an exploratory phase before formal instruction. They found that children with limited prior knowledge benefited from feedback during the exploratory phase and did better on the

post assessment, as compared to children with limited prior knowledge who did not receive feedback during exploration. However, students with moderate prior knowledge did not benefit from feedback during the exploration phase and actually did worse on the post assessment. Students with moderate prior knowledge were better off not receiving the feedback during exploration.

These findings suggest that it is important to consider the learner's prior knowledge when deciding the timing of feedback. If feedback is given too early, before students have had time to construct their own answer, feedback can have a negative effect on learning (Bangert-Drowns et al., 1991; Kulhavy 1997). Bangert-Drowns and colleagues found when individuals received feedback before they had the chance to construct their own response, it prevented a mindful reflection of the information-thus resulting in a negative effect of the feedback. When students are in the middle of actively constructing information, feedback may obstruct or hinder the learning process (Corno & Snow, 1986). Additionally, Hargreaves (2013) found students became frustrated when their teacher gave feedback too quickly without giving the students a chance to work through and process the task on their own.

Dialogic Feedback

Much of the prior research around feedback effectiveness has viewed feedback as a one-way transmission of knowledge (Nicol, 2010), which I refer to as fixed feedback. Askew (2002) suggests when feedback is given in this way it is perceived as a gift. This assumes that the teacher has the most knowledge, and therefore presents students with this gift of feedback that is presumably supposed to improve the student's work. When feedback is positioned as telling, there is an assumption that the student has no input or

insight about what is best for their learning;(Boud & Molloy, 2013). Additionally, fixed feedback places the learner as a passive recipient instead of an active constructor of information and knowledge (Ajawi & Boud 2015; Nicol, 2007). Building off Askew's (2002) original analogy of feedback as a gift, fixed feedback does not guarantee whether or not the gift of feedback is opened, or even removed from its packaging. Fixed feedback cannot ensure if the feedback is ever used by the student or was in fact valuable for the student's learning.

Instead of feedback as a one-way process, recent research and literature has suggested that feedback move away from a monologue view and towards an interactive, co-constructed dialogic view (e.g. Askew 2000; Molloy & Boud 2013, Nicol, 2007, 2010). Dialogic feedback is based on a co-constructivist model of teaching and learning in which dialogue is central to the taking up and construction of new ideas (Askew 2000). Carnell and Lodge (2002) discuss the assumptions of dialogue below:

> Dialogue is grounded in the assumption that learners are teachers and teachers are learners. Hierarchies are broken down and boundaries less evident. The role of the teacher is to instigate a dialogue between and with their students, based on their common experiences, but often the roles of teacher and learner are shared. (p. 16)

Dialogic feedback therefore invites students into the feedback process and positions them as active agents over their own learning. Feedback is not given, but rather constructed, between the teacher and learner. Feedback is at risk of being merely perceived as a thing rather than a process, however I argue that dialogic feedback has the ability to engage students as part of the process providing opportunities to:

1. Create a shared context for understanding feedback;

2. Provide feedback that is contingent to the learner's needs; and
3. Allow space for students to be active agents over their own learning

Creating a Shared Context

Researchers have explored why feedback is sometimes ignored and discarded by students. Some have suggested that students simply are not interested in the feedback, but others have found that this is not the case. Instead, researchers have discovered that when students are provided with the opportunity to use feedback and choose to ignore it, it is often because students are unable to make sense of the comments provided by the teacher (Hattie & Timperley, 2007; Higgins et al., 2001; Wiliam 2011). Nicol (2010) emphasizes that within dialogue there is often a shared context that is established at the beginning of, or during, conversation enabling the listener to interpret and make sense of what the speaker is conveying. Without a shared context, it is difficult for students to decode and understand the feedback message in order to use it. This may be due to differing perceptions of the assessment criteria held by students and teachers (Hounsell, 1997).

As mentioned earlier in the paper, it is important that feedback is related to the learning intentions or goals. However, Rust, Price, & O'Donovan (2003) suggest that simply sharing the learning goals and success criteria does not allow students to make sense of the often implied or unspoken knowledge around the learning task. Assessment criteria and feedback can often be conveyed in academic language that is not yet available to students, leaving students unable to use the feedback (Lea & Street 2000). As part of a shared context, it is important that teachers and students hold similar

understandings of success criteria, therefore allowing students to make sense of the feedback provided in relation to the learning goals.

Additionally, there must be a chance for students to make sense of the feedback once provided by the teacher. Nicol (2010) suggests "the meaning of feedback comments is not transmitted from the teacher to the student; rather meaning comes into being through interaction and dialogue" (p. 507). Often students are unable to make sense of the feedback provided to them. Feedback as a dialogic two-way process can provide space for the student to respond to the feedback and further question how it can be used in their work. This can also enable time and space for the teacher to elaborate on their feedback and find out if the student understands where to go next with their work. The teacher may ask the student to identify the next steps they are going to take during the revision process to ensure the student is able to put the feedback into action (Boud & Molloy, 2013).

Contingent on the Learner's Needs

Feedback that does not address the needs of the learner is highly unlikely to be effective. For example, if the goal is for students to solve an addition problem using the standard algorithm, and the student does not understand place value, reminding the student to line up their ones and ten places correctly will have little meaning for the student and will not likely result in use or understanding of the feedback. In order for feedback to have a greater chance of being effective, it must be contingent and responsive to the learner's needs (Nicol, 2011). Optimal feedback is not based on a formula, but varies in accordance to the learning gap presented by students. Hargreaves (2013) found that there was a delicate balance between providing enough feedback that allowed

students to take actionable steps, and overburdening students with too much information or detail. Feedback that is highly fruitful and productive for one student may be completely unattainable and unproductive for another.

Contingent feedback also acknowledges "feedback is a social practice in which the management of relationships represents a source of emotions influencing learners' ways of studying" (Yang & Carless, 2013 p. 289). An important aspect of the feedback process that is often overlooked is how students emotionally take up the feedback they have received. Previous feedback experiences, whether receiving praise for work produced or receiving a failing grade from an assessment, will impact how students take up feedback in the present and future. Building a relationship of trust with students is essential to effective feedback (Wiliam 2015). Yeager et al. (2013) found that when students received feedback accompanied with a message about the teacher's high standards and belief that the student could reach them, they were more likely to use the feedback in future revisions. Dialogic feedback can allow teachers to have conversations and enquire deeply about the concerns and frustrations students may have before constructing feedback that will be beneficial. Dialogic feedback also has the potential to build trust between the teacher and student. If the student does not trust the person giving feedback or believes that person has the student's best interest at heart, it is at a high risk of being disregarded. (Wiliam, 2015).

Promoting Learning Agency

When feedback is dialogic, students have the opportunity to discuss and negotiate the terms and expectations of their learning along with the expectations of the teacher. Dialogic feedback is based on the assumption that learners are not simply recipients of

information, but rather co-constructors of knowledge. It is also assumed that the learner has some awareness of their own learning goals and terms of success. Boud and Molloy (2013) suggest that some learning outcomes are identified by the teacher, but others may be negotiated by the student.

When feedback is provided in a one-way transmission, it assumes that the teacher understood what the learner was intending or trying to do through their work. Many have had the experience in which the feedback that was given to them misinterprets or goes against what they, as the author or creator were trying to do or portray through their work. This can be a moment of frustration for the person receiving the feedback as someone other than themself decided the agenda for feedback. There is a gap between what the learner is trying to do, and how the teacher perceives what the learner has actually done. When feedback occurs as dialogue it can open up space for the learner to share their goals and intentions of what they are trying to do through their work. This helps to close the gap between what the learner was trying to do and what the teacher perceived the learner was trying to do. The teacher may clarify *with* the learner about their intentions and with this new information, better craft feedback that moves learning forward in a way that matters to the student.

Taking Action

Much of the research around feedback identifies the need for students to respond to, and use the feedback, in order for it to have a positive effect on learning. Sadler (1989) asserts that without action taken in order to close the gap between the learner's current state and the desired state, feedback simply becomes "dangling data" (p. 121). Consider a professional athlete who receives countless moments of feedback throughout

their career that include coaching, watching video, and strength and skill assessments. Many would find it odd if an athlete received all this feedback, but never had the opportunity to put the feedback into action during the next game or the next practice or the next competition. Wiliam (2011) suggests that if there is no opportunity to use the feedback, that any comments, no matter the quality, are essentially a wasted effort on the teacher's part. In essence, the 'what,' the 'when' and the 'how' of feedback only matter if students are doing something with the feedback. "To be effective, a feedback system must be designed with an implicit theory of action, where the information about the current state is identified only once the way in which it will increase student achievement is already determined" (Ruiz-Primo & Li, 2013). Without knowing how feedback will provide the student with an opportunity for action, there is little to be gained from this process.

Perspectives of Learning and Knowledge Construction

Askew and Lodge (2002) write that "Any evaluation of the usefulness of feedback must rest on an analysis of its purpose, the assumptions about learning on which it is based and a recognition that feedback has different purposes" (p.3). It is here in this dissertation that I aim to elucidate the assumptions of learning and communication which ground the concept of feedback dialogues. The purpose of this section is to develop the theoretical perspectives that guide this study and provide a grounding in which to interpret the feedback dialogues situated in an elementary mathematics setting, and to evaluate the importance of particular features of the development of these dialogues.

I begin with a discussion around theories of how knowledge and learning develop by drawing on constructivist and socio-cultural theories of learning. These theories set

the foundation for why I believe dialogic feedback is necessary in order for feedback to have an impact on student's mathematical thinking.

Constructivism

Stemming from behaviorist theories of learning, traditional mathematics instruction and curriculum has focused on the transmission view of learning in which students are passive recipients of knowledge that is presented by others in power or authority (Post, Clements & Batista, 2009). The constructivist perspective presents an opposing view suggesting children construct their own knowledge through active experience and reflection, in which new knowledge is made meaningful when integrated with pre-existing knowledge (Clements & Batista 2009; Ernst 1989; von Glasserfeld 1991).

Piagetian theory was fundamental in building the foundation of the constructivist perspective (Piaget 1964) and many mathematics educators continue to draw from Piaget's theories of the mind and cognitive development (Noddings, 1990). Piaget theorized that conceptual development occurred as children moved from physical action with the environment around them, to a mental representation of those ideas and experiences. Specifically, Piaget's concepts of assimilation and accommodation were important to the field of education. Piaget (1964) posed that children do not simply absorb new information, but instead filter and make sense of the information in regards to their previously existing knowledge. The concept of assimilation posits that "(u)nderstanding cannot be imposed on children" (Baroody & Ginsburng 1990, p 56), which is often a failure of direct instruction in which the teacher presents new knowledge and students then practice this new knowledge by mimicking the steps taken by the

teacher. Children must be afforded the opportunity to assimilate new information through active engagement and sense-making activities.

Part of knowledge construction also involves reconstructing prior knowledge, which Piaget termed accommodation. Accommodation describes the process children go through when new knowledge conflicts with previously held information, and as a result, children modify or reorganize their existing structures to make sense of this new knowledge. An example of this process can be found in Kieran's (1981) work with students in solving algebraic equations. Kieren found students often had difficulty with solving algebraic equations due to their existing understanding of the equal sign as a symbol that means "to do something." In her work, she was able to sequence activities that allowed students to reorganize their understanding of the equal sign to see the symbol as a representation of relational meaning, rather than a sign that indicates computation is needed.

As the constructivist perspective in mathematics education has continued to evolve and be shaped by research and debate, it has moved from an individualist perspective of knowledge construction towards an interactionist perspective of knowing and learning (Steffe & Kieren, 1994). This suggests that children's conceptual structures do not arise from being in an environment, but rather from children's actions within that environment, which include reflection, abstraction and interaction with others. Knowledge is not communicated, but "constructed out of our in-context experience of each other's speech and action" (Goldin 1990). Therefore, knowledge is dependent upon many factors that include time, space, and prior experience and is unique to each person.

Sociocultural Theory of Learning

In a sociocultural perspective of learning, communication and language are seen as central to the ways in which people learn. Learning and knowledge are seen as social activities and therefore dialogue is an important factor in explaining learning, rather than just the learning potential of a student or the knowledge and skills held by the teacher (Mercer, 2004). A foundational premise of sociocultural theory in the learning of mathematics suggests that "Language and learning are intertwined and that understanding mathematical learning, assessing students' mathematical learning and designing mathematical learning environments require examining the role of language in mathematical activity" (Edwards, Esmonde, & Wagner, 2011, p. 62). A sociocultural perspective seeks to understand the relationship between language and thinking. A central focus of the sociocultural perspective is the belief that knowledge development is not solely located within the individuals, but rather learning is seen as a social process in which knowledge is co-constructed among members of a community.

Lev Vygotsky, an influential theorist contributing to sociocultural theories of learning, focused on the fundamental role culture and social interactions played in the development of knowledge. A central premise of Vygotskian learning is that the individual cannot be separated from his or her environment (Cole, 1985). Vygotsky believed psychological functioning is mediated by cultural tools and placed emphasis on the important role language has in the development of cognition.

Vygotsky (1978) believed language functioned in two ways: first occurring at the social level which he called the intermental level, and second, at the individual level also referred to as the intramental level. Language occurring between people at the

intermental level provides access for children to make sense of speech and then internalize new understandings at the individual cognitive level (Mercer & Howe, 2012). Vygotsky emphasized that it was during these social interactions children have with other social agents, that higher mental functioning can be considered, and a child's developmental level must take account both his "actual" and "potential" levels of development (Hickmann, 1985). Vygotsky defined the distance between these two levels of development, as the "zone of proximal development" (1978). Within the space of the ZPD, is where internalization occurs. Thus, for Vygotsky, the analysis for development consisted not within the child himself, but instead in the interaction of the child and a more capable peer. However, this does not imply that knowledge is simply passed on to the child from an adult, but rather children actively construct knowledge from the social interaction in which they participate in (Walsh, 2016). He suggests that "what the child is able to do in collaboration today, he will be able to do independently tomorrow" (Vygotsky, 1978, p.211). Ideally, this is the premise in which the role of feedback stands on, where the student and teacher jointly construct knowledge together so that the student may internalize the feedback and act upon the feedback in subsequent occasions when the teacher is no longer there.

Constructivism and Sociocultural Theories in Tandem

After examining theories of cognitive thinking from constructivism and sociocultural learning, it is clear that the two perspectives seemingly contradict one another. While Vygotsky and sociocultural learning theories emphasized the social process as a means of internalizing speech in order to construct higher cognitive functioning, Piaget and constructivist theories focused on the individual's interactions

with physical reality as a means of internalization (Wertsch, 1985). It may then seem antithetical to draw from both these perspectives, however, some (e.g. Confrey 1995; Saxe & Bermudez, 1996; Wertsch, 1985) suggest an integrated theory of both perspectives is possible, and necessary, in order to address questions regarding mathematics teaching and learning. Saxe and Bermudez (1996) capture this in the following statement:

> An understanding of the mathematical environments that emerge in children's everyday activities requires the coordination of two analytic perspectives The first is a constructivist treatment of children's mathematics; children's environments cannot be understood apart from children own cognizing activities...The second perspective derives from sociocultural treatments of cognition...Children's construction of mathematical goals and subgoals are interwoven with the socially organized activities in which they are participants. (p. 52)

As Confrey (1995) points out, mathematics consists of both a language which includes mathematical symbolism, and a model that describes relationships between actions and objects. Thus, it would make sense that both Piagetian and Vygotskian perspectives are considered together. For Vygotsky, interaction with a more knowledgeable other is a foundation for the ways in which children internalize new understanding, however, Confrey cautions that without active construction on the part of the student, the interaction can result in the student taking on the 'other' perspective with weak connections to their own viewpoints and understanding. Both Piaget and Vygotsky suggested the importance of interaction. However, Vygotsky did not view the

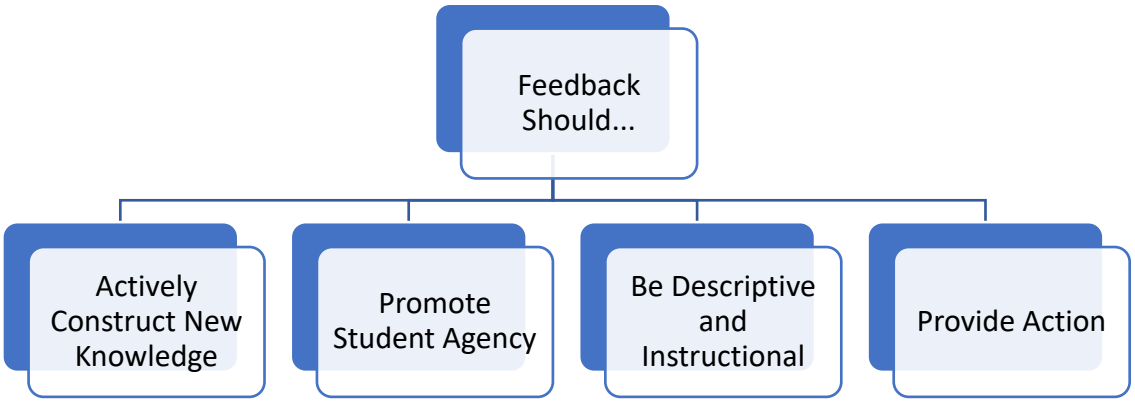
development of the child as independent from the social interaction, but instead proposed that language may include both social and cognitive processes (Hickmann, 1985).

Constructivists recognize that children bring prior actions and experience with them to any interaction. Equally important, sociocultural theorists recognize that our interactions with others in a cultural context determine what knowledge is meaningful and how it is applicable to our experiences. Therefore, when trying to understand how knowledge is constructed, one cannot only look at the individual experience, or only look at the social experience, but rather how both the individual and the social activities are understood in relationship with one another.

Conceptual Framework

Drawing from both the research around effective feedback practices and theories of learning from constructivist and sociocultural perspectives, the following framework *(Figure 1)* guided this study. This conceptual framework provides four major tenants or principles of what I propose dialogic feedback should be in order to be potentially effective at furthering students learning. These four tenants influenced both the way the dialogues were carried out during the study as well as the analysis of the feedback dialogues.

Figure 1: Feedback Framework



The first tenant suggests feedback should *actively construct new knowledge*. The feedback dialogue should be based on of the child's prior learning and allow students to construct knowledge through explaining their thinking and use of models rather than passively absorbing information. Secondly, feedback should help *promote student agency* by positing students as active contributors to the dialogue and allowing students to determine areas of feedback they believe would be most useful. Feedback should also *be descriptive and instructional*. Descriptive feedback emphasizes elaboration on student strategies and thinking rather than solely providing an evaluation of the accuracy of the answer. Additionally, dialogic feedback is instructional, and as Heritage describes it "in a regular classroom and on the everyday basis, there is a blurred distinction between feedback and instruction" (Heritage, 2010*)*. The teacher provides effective questioning and scaffolding in order to co-construct the feedback and new learning. Lastly, feedback should *provide action.* Research suggests without opportunity for students to use the feedback, there is less likelihood the feedback will be effective at impacting student learning.

Conclusion

This chapter began by laying out what is currently understood about effective feedback practices, and recommendations about feedback from prior research. The next section focused on more recent research and theories around dialogic feedback and how dialogic feedback is potentially more powerful in impacting student learning and ownership. Lastly, I concluded by situating my work within a theoretical framework of how knowledge and learning develop by drawing on constructivist and socio-cultural theories of learning and presented the conceptual framework that guided this study. Tin the next chapter I layout the research design and methodology used to answer the research questions.

Chapter 3: Methodology

This is a qualitative study designed to explore and describe how a teacher and student co-construct feedback dialogues in order to improve student learning in mathematics. The goal was to study feedback dialogues in a natural and authentic setting in order to analyze how the dialogues are carried out and the characteristics of the moments when students moved forward in their mathematical understanding.

Research Design

This research was designed as a descriptive case study (Yin, 2014) in order to look deeper at how feedback is co-constructed with students in regards to specific mathematical tasks. I participated in a series of dialogues with students and as a result, I have constructed a description of potentially effective dialogues with students based on the research of what feedback should be and how they are enacted between teacher and student.. The unit of analysis is a small group of fourth grade students from one classroom participating in the classroom formative assessments and feedback dialogues.

This study was guided by the following research questions:

2. Given the conceptual framework based on effective feedback practices and theories of learning from constructivist and sociocultural perspectives, how do these feedback dialogues take shape?

3. What might we notice about the characteristics of moments, within feedback dialogues, that prompt a change in mathematical thinking throughout the unit of study?

Context and Participants

This study took place in a large Midwest suburban district during the 2017- 2018 academic year. Data was collected during 3 weeks of mathematics instruction. Each class was approximately 60 minutes in length.

The school consisted of a diverse population with over 50% of the students enrolled in free or reduced lunch. The study occurred in a fourth grade classroom during the math block of instruction. I was an instructional coach who co-taught with the classroom teacher during math throughout the school year and maintained this role during the study. By spending three days a week in the classroom as a co-teacher through-out the year, the students saw me as another teacher during math time. Although it was made known to the students about the research I was doing, they continued to view my role as teacher rather than researcher. Because the goal of this study was to learn deeply about feedback dialogues that both follow the recommendation of previous feedback literature as well as pedagogy for how children learn mathematics, I participated as the provider of feedback.

Four students participated in the feedback dialogues. I anticipated that the dialogue may differ with students at varying levels of achievement and therefore this study included two students typically performing below benchmark level and two students performing at benchmark level.

Table 1 Description of Participants

Student*	Gender	Typical Performance in Mathematics
Kate	Female	Below Benchmark Level
Chip	Male	Below Benchmark Level
Mary	Female	At Benchmark Level
Cameron	Male	At Benchmark Level

*Pseudonym for Student in Study

The students were selected based on their typical performance in fourth grade mathematics as agreed upon by both the co-teacher and me and based on the availability

of a signed consent form to participate in the research study. A brief description of each participant is included in Table 1.

Overview of the Curriculum

The curriculum used during this study consisted of lessons from *The Rational Number Project*, which was developed from a multi-university cooperative research project funded by the National Science Foundation (Cramer, Behr, Post & Lesh, 2009). The curriculum focuses on a conceptual understanding of fractions and decimals through the use of multiple representations which include; manipulatives, verbal symbols, written symbols, real life situations and pictures. The classroom teacher and I co-taught these lessons during the students' core math block of instruction. An understanding of the lesson overview and objectives is important to the study as they inherently influenced the feedback dialogues I had with students (see Table 2).

Table 2 Overview of Lessons Used During The Study

Lesson #	Lesson Title	Lesson Overview
Lesson 8	*Fraction Circles and Equivalence*	Students explore fraction equivalence by naming fractions equal to 1/2 with fraction circles and by finding other fraction equivalences with fraction circles.
Lesson 9	*Fraction Circles, Paper Folding and Equivalence*	Students continue to explore equivalence with pictures and fraction circles.
Lesson 10	*Paper Folding and Equivalence*	Students explore equivalence ideas with paper folding.
Lesson 11	*Comparing Fractions to One-half*	Students use fraction circles to order 2 fractions by comparing them to one-half.
Lesson 12	*Introducing the Chip Model*	Students are introduced to chips as a fraction model. They learn to represent a given fraction using different sets of chips as a unit.
Lesson 13	*Using the Chip Model*	Students continue practicing showing fractions with chips. They determine several units that can be used to model a fraction and what units can't be used to model fractions.
Lesson 14	*Using the Chip Model*	Students continue to model fractions with chips. They determine possible fractions that can be shown with different sets of chips.
Lesson 15	*Chips and Equivalence*	Students explore fraction equivalence using chips.
Lesson 18	*Fraction Circles and Names for One-half*	Students look at the numerical relationship between the numerators and denominators of fractions equal to 1 2 . They use this number pattern to determine if a given fraction is less than or equal to 1 2 .

Lesson #	Lesson Title	Lesson Overview
Lesson 19	*Estimation and Fraction Addition*	Students are introduced to fraction addition through familiar contexts and estimating reasonable answers (by comparing sum to 1 2 and 1).
Lesson 20	*Fraction Circles and Fraction Addition*	Students use fraction circles to obtain exact answers to fraction addition.

Lessons from Rational Number Project (Cramer, Behr, Post & Lesh, 2009)

Data Collection

The Data used in this study included (a) formative assessments, (b) a final post-assessment, (c) feedback dialogues with each student, and (d) a reflective journal constructed by the researcher. These instruments are described in the following sections.

Formative Assessments and Final Post-Assessment

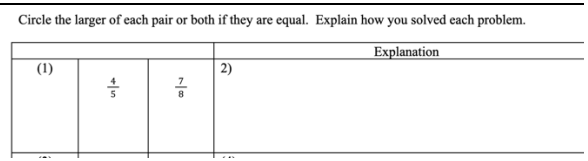
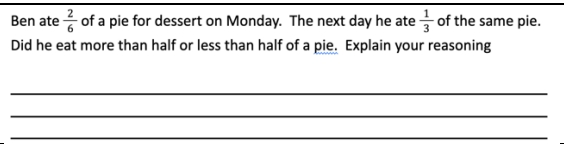
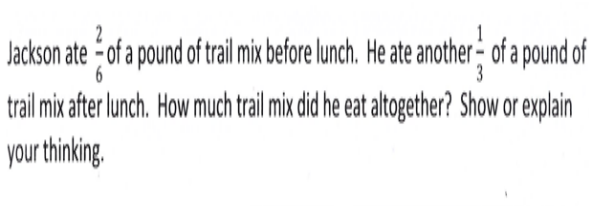
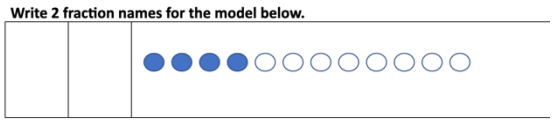
An important component of the feedback process is the design of the task in which the feedback was based off of. It is suggested that formative assessment tasks should be designed to elicit student thinking and focus not only what students know, but also how they know it (Ruiz-Primo & Brookhart, 2018). This emphasis falls in line with leading research around mathematics learning which emphasizes the importance of identifying student thinking beyond just the correctness of an answer (ex; Hiebert et al., 1997). Reeuwijk and Wijers (2003) suggest that "only if an answer shows some kind of reasoning, thinking, or a strategy, feedback can be given that enhances the learning" (p. 191). The importance of having a strong task or assessment that elicits student thinking is critical to providing valuable dialogic feedback.

There were four formative assessments used during this study occurring throughout the unit and a final cumulative post-assessment. These were paper assessments students completed during the classroom math block of instruction and occurred before each of feedback dialogues. The assessments were developed based on questions and tasks used in previous studies within the Rational Number Project and were

shown to be productive in revealing student thinking (Cramer, Behr, Post & Lesh, 2009). The formative assessments were also designed so that students had multiple opportunities to show thinking around similar problems across the assessments. New concepts were included in each formative assessment as students engaged with these throughout the unit, but also solved problems similar to tasks from prior assessments. This was intentional so that I could notice how student's thinking changed over the course of the unit and the feedback dialogues. I wanted to be able to notice *if and how* student's solved similar problems differently after participating in a feedback dialogue. The post-assessment included concepts from all of the assessments with tasks that were again similar to the formative assessments but with different numbers. Table 3 provides a summary of the tasks on each assessment and copies of the entire assessments are provided in the appendix.

Copies of the student assessment were made following each dialogue and scanned electronically for analysis. The students completed the formative assessments in pencil. During the feedback dialogues students had the opportunity to edit their assessment and made these edits in a different color pen. The reason for this was twofold. The first reason was so that students could visually see how their work and answer changed from their initial completion of the assessment to after the feedback dialogue. Secondly, during analysis it allowed me to capture the students' work both before and after the dialogue.

Table 3 Overview of Assessments

	Fraction Concepts	**Expected Understandings**	**Potential Mis-understandings**	**Sample Task**
Assessment 1	Comparing two Fractions Ordering multiple fractions	Uses benchmarks of 0, ½ and 1 Compares size of residual piece Identify fractions that are equivalent to 1/2	Uses whole number thinking by looking at numerator or denominator in isolation	
Assessment 2	Estimating addition of two fractions Comparing two fractions	Use a picture Uses benchmarks of 0, ½ and 1	Adds numerators and denominators-use whole number thinking	
Assessment 3	Estimating addition of two fractions Using Fraction Circles to add two fractions Comparing two fractions	Uses Fraction circles to find equivalent fractions to add using the same size piece (denominator	Adds numerators and denominators-using Fraction Circles	
Assessment 4	Fraction equivalence with Chip Model Estimating adding two fractions	Can identify multiple units Identify # of equal parts an # in each part	Student confuse the number of groups with the number in each group.	
Post-Assessment	All concepts included above	Includes all Expected understandings Assessments 1-4	Includes all potential mis-understandings Assessments 1-4	Includes all tasks above

Feedback Dialogues

The main source of data used to answer the first research question which focused on understanding the nature of the feedback dialogues were the actual feedback dialogues I had with each student. For this study, I define feedback dialogues by drawing from Carless' (2012) definition of dialogic feedback. He writes,

> I define dialogic feedback as: interactive exchanges in which interpretations are shared, meanings negotiated and expectations clarified... Dialogic feedback is facilitated when teachers and students enter into trusting relationships in which there are ample opportunities for interactions about learning and around notions of quality. (p.90)

One might ask how dialogic feedback is different from other types of dialogue such as a math conference (Munson, 2018) or a tutoring session. Although there are certainly commonalties across different types of dialogues with students, feedback dialogues are unique in some specific ways.

First, a feedback dialogue is initiated from work that students have already produced or completed and during the dialogue students identify how their work compares to the desired outcome or expectations. Additionally, there is opportunity to further clarify the expectations for the type of task the student has completed so they may deepen their understanding of the criteria for success. Secondly, a feedback dialogue is centered around how the student might improve their current and future work by building new understanding and then considering how they might use this new understanding moving forward. This means that a feedback dialogue must contain actionable steps the

student can take as they work on similar types of tasks, whereas a tutoring session or math conference might only focus on advancing student thinking without calling attention to these actionable steps a student might take moving forward.

Each student participated in four dialogues over the course of the unit and research study, resulting in a total of 16 dialogues. Following each formative assessment, each student individually engaged in a feedback dialogue with me around the assessment and their performance. The dialogues were audio and video recorded in order to be transcribed following the study. The formative assessment task was used as a starting point for the feedback dialogue, but did not exclusively dictate the dialogue as it was co-constructed by the student and me. The structure and format of the dialogues were not prespecified but were guided by several principles of effective feedback practices as cited by the literature and included in my conceptual framework. The use of video recording allowed me to document and add to the transcription moments where students were using the fraction circles or pointing to a model.

The feedback dialogues were intended to take place on the same day students completed the formative assessment during the 30 minute math intervention time. However, this was not always possible and most often there was not enough time to dialogue with all four students on the same day. As a result, the dialogues occurred anywhere from 1-3 days after the formative assessment. The timeline of the assessments and dialogues can be seen in Table 4.

Table 4 Timeline of Assessments and Dialogues

Assessment 1	April 25	
	April 25-27	**Dialogue 1**
Assessment 2	April 30	
	May 1 & 2	**Dialogue 2**
Assessment 3	May 2	

	May 3-7	**Dialogue 3**
Assessment 4	May 9	
	May 9 & 10	**Dialogue 4**
Post Assessment	May 11	

Researcher Constructed Reflective journal. Bazeley (2007) suggests a reflective journal written by the researcher that documents the emergent ideas and concepts and thinking processes, can help to serve as the beginning of data interpretation. Additionally, a reflective journal can be beneficial when things did not happen as planned and as the researcher's thinking changed, this reflective journal served as a place in which I could be reflexive about the noticings and to some extent pre-analysis of the dialogues. Because I actively participated in the research study as the provider of feedback, I naturally reflected after the dialogues around moments that I thought were interesting, or caught me off guard which ultimately impacted the dialogues moving forward. The reflective journal allowed for these internal thoughts and noticings to be transparent as the research was happening and the decisions I was making as I engaged with students in dialogue.

Data Analysis

The data analysis addressed each research question separately. A constant comparative analysis method was used to explore the data, generate themes regarding the feedback dialogues, and describe the strategies present in feedback dialogues to support further student learning in mathematics.

Analyzing Data For Research Question #1

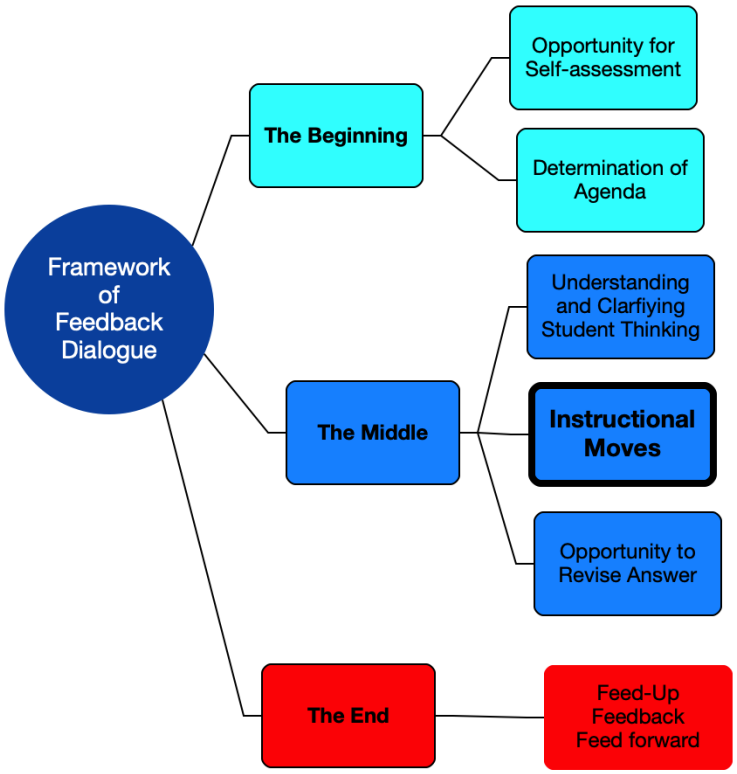
Audio recordings of the feedback dialogues were transcribed and coded by me using the QSR NVivo 12 software. The software provided an organized database in which the transcripts from the dialogues could be imported and analyzed in the same space. The

use of visualizations of frequencies were useful when noticing patterns across codes. The data excerpts were coded using a mixture of inductive and deductive coding. Beginning with open coding (Straus & Glaser, 1998), I generated codes that were specific to the data themselves. Open coding is designed so that one enters the data without any ascribed codes so that as much as possible the meaning comes from the data. An example of this is during open coding, I initially coded student reflection of performance. This was directly taken from my noticings and only later after returning to the literature and second phases of coding did I ascribe "self-assessment".

Charmaz (2006), suggests that as researchers work to make comparisons within data of single events as well as comparison between events, making sequential comparisons can help this part of the analytic process. During this initial phase of coding I noticed that there were patterns among the dialogues that related to the sequence of the dialogue. As I moved across dialogues I found that there were commonalities in the way the dialogues began and then again when the dialogues shifted to a discussion about the tasks. I used this analytic noticing to develop the framework of beginning, middle and end for second cycle coding. During the second cycle of coding, I looked to find patterns that would address three questions: 1)how did the dialogues begin; 2) how did they progress; and 3) how did they end? During this cycle of coding, pattern coding was used in order to "develop a statement that describes a major theme, a pattern of action…from the data" (Saldana 2013, p. 212). I also returned to the literature around feedback practices as well as my conceptual framework in order to connect and further analyze what was happening within moments throughout the dialogue. This was an iterative process moving from the data to the literature and theory back to the data in

order to describe the dialogues and answer the research questions. As a result, Figure 2 depicts both the three major parts of the dialogue which I refer to as *Beginning, Middle* and *End,* as well as the themes that emerged in regard to what was happening in each part of the dialogue. I used this framework both during the analysis as well as during the writing and organization of the data in Chapter 4.

Figure 2 Framework of Feedback Dialogue



Notice in the middle of the dialogue, one of the themes occurring was *Instructional Moves.* In Chapter 4, I discuss zooming in on this part of the dialogue where the instructional moves were happening in order to describe the moments where students shifted in their mathematical thinking around the tasks we were discussing from the assessment. One other thing to note was the presence of two additional themes that did not fit cleanly within the framework of beginning, middle and end of the feedback

dialogues. Rather, these two additional themes were threaded throughout a single dialogue as well as across dialogues and it therefore did not make sense to place them in any one place. I present these themes in Chapter 4 as additional pedagogical noticings.

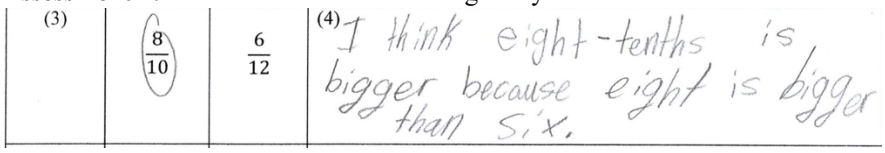
Analyzing Data For Research Question #2

My second research question sought to identify moments in the feedback dialogues where students had a shift in their mathematical thinking. The purpose of this questions was to better understand the characteristics that surrounded these moments in the specific feedback dialogue and also throughout the unit of study. The primary data used to answer this question was the feedback dialogues as well as the student work from the both the formative assessments and post assessment. Each of the 4 assessments as well as the post assessment were analyzed and descriptively coded-question- by question and coded based on the strategy used, the fraction ideas understood, and any misunderstandings that remained.

In order to analyze and notice how student thinking changed over the course of the four dialogues and how it might relate to the dialogues themselves, I summarized and reorganized the data based on the sequence in which it occurred. This enabled me to look across and notice connections between the student performance on the assessment to the preceding dialogue(s). Table 5 provides an example of what this looked like for one student and includes a sample of their work. Under the assessment, I summarized the student's understandings, misconceptions and also the strategies they used to solve the tasks.

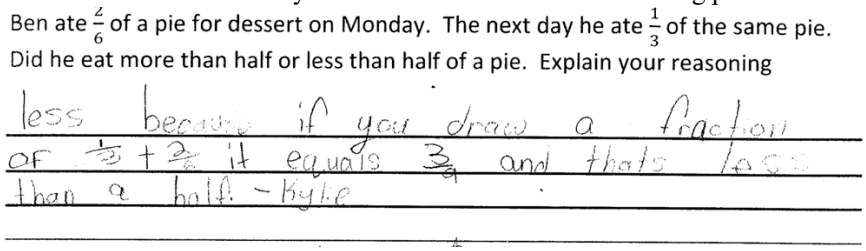
Table 5 Example of Analysis Process for Research Question #2

Assessment 1: Used whole number thinking. Only considers the numerator or denominator in isolation



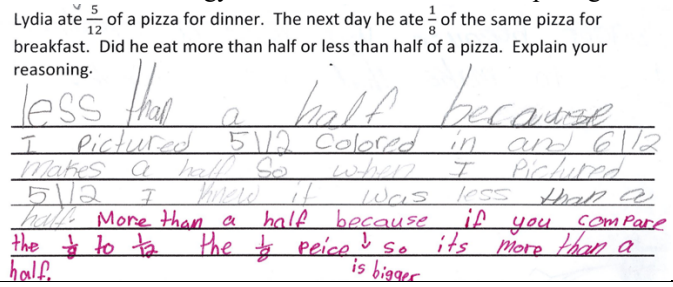
Dialogue 1: Focus on size of pieces and indirectly consider the numerator and denominator relationship by asking if two fractions are equal if they are both missing one piece. Focus on the missing piece.(residual strategy.) Second part of dialogue focused on one half and more or less than a half. By using her fraction circles and asking what she noticed…she came to the conclusion that they both would be equal if they were a half. She constructed that idea herself by looking at the fraction pieces. Then with some additional prompting from teacher researcher, she was able to determine which fraction was more than a half

Assessment 2 Uses whole number thinking when estimating the sum of two fractions. Uses a picture model for fractions one away from a whole but focuses on missing piece.



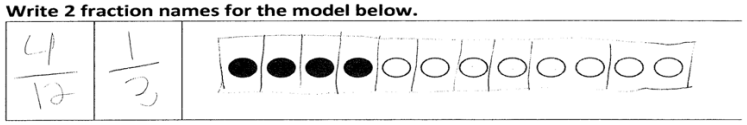
Dialogue 2: Explicitly calls out she felt better on comparing fractions by focusing on the missing piece. First Problem is centered around estimating. Uses fraction circles to notice it is less than 1 and also notices 1/8 is half of one fourth. Did not specifically call out that her ¾ + 1/8 equals 4/12 does not make sense. Second part of dialogue focuses on using ½ as a benchmark to compare fractions as another way to explain reasoning besides just a model.

Assessment 3: uses thinking around a ½ when estimating but forgets to estimate the second number. Uses residual strategy. Doesn't use a ½ when comparing 6/10 and 4 /12



Dialogue 3: knows 5/15 is less than half need 6 /12 to make a half. See that 1/8 is smaller than 1/12 and with some prompting knows that it would be more than a half. Second part…revisit explanation for residual strategy which is correct and student self corrects answer. Third part: Has correct answer but pushing thinking around a ½. Is able to determine more than a half with fraction circles.

Assessment 4: Can use chip model. Didn't have strong reasoning on estimating
Dialogue: Ran out of time for estimating problem. Recognizes 3/6 is half and that you need 2 more tenths to get a whole. Again understand when it is more than a half less than a half with prompting. Suggestion around using a half



Dialogue 4: Estimating using ½ as benchmark

Post Assessment: Generalized using a half strategy to all situations. Thinks you just need to add denominators. Has difficulty estimating fractions..again tries to compare to a half inappropriately.

For each Dialogue, I summarized and coded the strategies the student built understanding around as well as any insight the student shared about their performance. I then compared these codes to focus on whether and how each dialogue related to the strategies students used on each of the assessments. While the purpose of the research was not to measure efficacy, I wanted to see whether or not strategies discussed during the dialogue showed up on the student's assessment as a way to further understand and characterize moments in the dialogue that impacted student thinking.

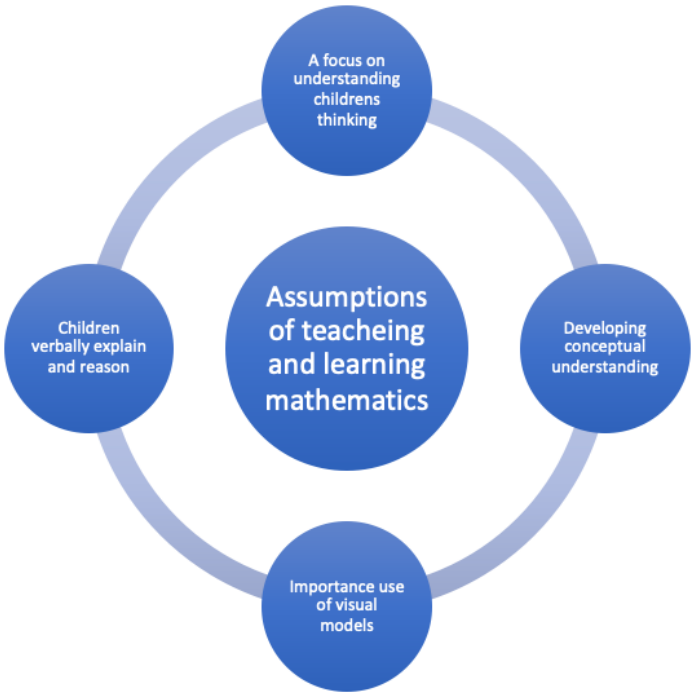
Analytic Memos.

According to Charmaz (2006), "memo-writing is a pivotal step between data collection and writing drafts of papers" (p. 72). Memo writing is particularly useful during the early stage of the analysis process. During the initial coding phases, I used memos to help make comparisons between data and codes. As I moved into secondary rounds of coding and phases of analysis I used memo-writing to articulate conjectures of the comparisons I was noticing in the data.

Positionality of the Researcher

Foote and Bartel (2011) point out the important role the positionality of the researcher has in the research process. The positionality of the researcher may influence both how the research is conducted as well as how results are interpreted. Due to my experience, as both a math specialist and scholar of math education, I entered into this research and specifically the feedback dialogues with a level of pedagogical content knowledge (Shulman, 1986) that is perhaps more developed than an average elementary classroom teacher. Because of this, I brought with me particular assumptions of how children learn mathematics described in the figure below. These beliefs and assumptions about how children learn mathematics will most likely be evident in the feedback dialogues and will ultimately influence the nature of these dialogues.

Figure 3 Assumptions of Teaching and Learning Mathematics



Trustworthiness

Trustworthiness can be described as the confidence a reader has with what the researcher has written and presented for others to interpret (Stahl & King, 2020). Several methods were used to increase the confidence and credibility of this study. The first method includes the process of triangulation for both data and theory. Multiple form of data were collected throughout the study which included audio and video recordings of the feedback dialogues, Student work from the formative assessments, Student work and revisions after engaging in dialogues, post-assessment from the unit, and a Research Constructed reflective Journal. Additionally, multiple theories which drew from constructivist and sociocultural learning theories as well as theories of dialogic feedback were used to analyze and interpret the data.

Another strategy to enhance the credibility of this study was to provide "thick description in the crafting of the narrative. Thick description is more than just providing enough detail for the reading of what a person is doing. Instead, Schwandt (2001) writes thick description is to "actually to begin to interpret it be recording the circumstances, meanings, intentions, strategies, motivations, and so on that characterize a particular episode. It is this interpretive characteristic of description rather than detail per se that makes it thick" (p. 255). During analysis, I was able to go between the data from the dialogue transcripts and the researcher constructed reflective journal to illuminate reflections, intentions and motivations that I had as the study was occuring. Additionally I used analytic memoing to record early ideas and assertions as well as document the steps of analysis from codes, to categories to themes. These processes together helped to contribute to thick description during the write-up of the findings in Chapter 4.

This chapter summarized the research design, participants, data collection and data analysis procedures. The following chapter will present the data and findings in order to answer the research questions for this study.

Chapter 4: Findings

As described in Chapter One, although there have been decades of research, which include several meta-analyses around the effectiveness of feedback, there is still much that remains unknown about the specific characteristics of effective feedback and how it can be used to improve student learning. Furthermore, models of dialogic feedback have been proposed but there has been less research that actually describes what the process of dialogic feedback looks like, particularly in an elementary classroom. The purpose of this study is to look deeper at how feedback is co-constructed with students in in regards to specific mathematical tasks and was guided by the following research questions:

1. Given the conceptual framework, how do these feedback dialogues take shape?

2. What might we notice about the characteristics of moments, within feedback dialogues, that prompt a change in mathematical thinking throughout the unit of study?

The data collected from this study consisted of four formative assessments that were each followed by a feedback dialogue with four students from a fourth grade classroom. I conducted these feedback dialogues with individual students that were unstructured but guided by effective feedback practices as cited in the feedback literature and conceptual framework for this study. During the initial phases of coding and analysis, there were noticeable patterns that occurred sequentially within the dialogue. As I moved forward in additional phases of analysis I used the framework of Beginning, Middle, and End that would address these three questions:

- How did the dialogues begin?

- How did the dialogues progress?
- How did the dialogues end?

In this chapter I present the data and findings around these three parts of the dialogue mentioned above. When I move to discussing the middle of the dialogue, I spend more time here addressing my second research question that looked to identify characteristics of moments when students moved forward in their mathematical thinking. This is where I describe the specific instructional moves that occurred in the dialogue that prompted a change in the thinking or strategies students used to solve the tasks. In the last section of this chapter I present additional pedagogical noticings that did not fit within the beginning, middle and end of the feedback dialogue framework. Rather these noticings or themes I present in this section are threaded throughout individual dialogues as well as across dialogues that occurred throughout the study.

The Beginning of the Dialogue: Self-assessment and Determining Agenda

I define the beginning of the dialogue as the interaction that took place before moving to discuss the first task. There were two major acts that consistently occurred at the beginning of the dialogues. The first act encouraged students to reflect on their performance and thereby self-assess and attribute a cause to their believed outcome. Following this reflection was the second act which positioned students as active agents in determining the initial agenda for the feedback dialogue. In this section I describe what these acts looked like in the dialogue and the opportunities they afforded both the student and my role as the teacher providing feedback.

Student Reflection of Assessment Performance

In 14 of the 16 dialogues, students began by reflecting on how they performed on the assessment. In most instances, this reflection was prompted by me, asking the students about their performance. The language used to prompt this reflection varied, with questions such as " how do you think it went, how do you feel about this, or how do you think you did?" In three of the dialogues, the student first initiated this reflection by inquiring about whether their performance was good or bad; but in all dialogues, students were asked to provide their own reflection without any evaluative feedback from me. I did not go into these feedback dialogues with the intent of starting the conversation this way. However, in the first round of dialogues, two of the students began by inquiring about their performance. In those moments, I realized that students were interested in how well they did. Given my conceptual framework I knew it was important to refrain from starting with an evaluation as prior research suggests that feedback is often ignored once students are given a grade or a score (Butler, 1998). So instead, in those moments I responded to the student with questions asking them to share how they believed they did on the assessment. I realized this was a valuable opportunity to hear from the students and also opened up further conversation about their performance, which I discuss in the following paragraphs. Thus, in the remaining dialogues I began with this type of performance reflection question. Students responded to this question in one of three ways (Table 5).

Most often students responded with an overall level of performance on the assessment as a whole, which they described using the words "good" or "bad."

Similarly, in two of the dialogues, the student responded by giving a level of performance for each question.

Table 6 Student Response to Prompted Reflection of Performance

Type of Response	**Occurrence in Dialogue**	**Example Response**
Overall Level of Performance	C1, C2, C3, C4, M1, M2, CH1 K4	Teacher: All right. How did this one go? C: Good
Level of performance on each task	CH3, CH4	**Teacher:** All right. Let's look at this. How do you feel about this? **Ch:** (points)That one, good. **Teacher:** Yep **Ch:** (points)That one, Good. **Teacher**: Ok **Ch:** (points) This one good and bothered
Progress made since before assessment	CH1, K2, K3, M4	**Teacher:** What do you think? How do you think you did? **Ch**: umm Pretty good. I think it's improving

The last type of response students gave referenced the progress they believed they had made, presumably since the last assessment or prior learning. In the example response above Chip believed that his performance is "improving," and in another dialogue Kate replied with "I think it's better." In both of these examples, the students reflected on how their performance had changed.

This first question, although seemingly mundane and obvious, was an important first move in the dialogue. It was an invitation to students to engage in this feedback dialogue around their performance on the assessment. In a traditional model of feedback, it is generally the teacher or the person with the most knowledge sharing information about how the student performed and becomes the driver of the conversation. Some researchers consider this type of traditional feedback to be a monologue. However, if

teachers are to move away from a monologue view and towards an interactive, co-constructed dialogic view of feedback (e.g. Nicol, 2007, 2010, Askew 2000, Molloy & Boud 2013), then this question is the first move to invite students to co-construct the dialogue and make known to the student that what they think matters and will be part of the conversation.

In all types of responses students did not elaborate beyond their initial evaluation of their performance. Notice in the example responses (Table 6), students answered the question about their performance in as little as one word and at most seven words. In 11 of the dialogues I asked students to expand on their reflection and share *why* they thought their performance was in their words "good, bad or improving". Again, there were consistencies in how students responded to this follow up question discussed below.

How Students Attributed Performance

In five of the dialogues, students attributed their performance to their use of a specific strategy related to the math. In the dialogue below, K attributes her performing "better" to using a strategy in which she looks at the missing piece of each fraction when comparing two fractions that are close to a whole. Similarly, in another dialogue when Ch was asked why he thought he had good reasoning, he attributed it to using a strategy that compared fractions to one-half as a way to determine which one was larger.

In two of the dialogues with the same student, Mary attributed her performance to her mindset about math. Notice in the example dialogue above (Table 6), Mary thinks she did "bad" because she believes she is bad at math. She attributes her performance to a broader belief about her ability in math and not to anything specifically related to the assessment.

Table 7 What Students Attribute Performance To

Student Attribute Performance to:	Dialogue	Sample Excerpt
Mathematical Practice	C3 Ch3, Ch4 K3, K4,	*T: All right. How did this one go* *C: Good,* *T: Good.... What do you think was good?* *C: That um* *T: What felt good about it?* *C: Um… being able to know how to explain more*
Mindset	M1, M2	*T: You tell me, what do you…how do you think it went?* *M: I don't know. I think it was bad.* *T: Why do you think that?* *M: Because I always think that.* *T: Why do you always think that?* *M: I dunno. Because I am bad at math*
Specific Math Strategy	Ch1, Ch3, K2, K3, M4 C4	*K: I did better on these ones than last time* *T: You think you did better on those ones? Why do you think you did better on these ones?* *K: Because I did them down here and I saw which ones were umm larger and i looked at those ones and saw which ones were smaller....and it was this one* *T: Ohh* *K: I did better on that* *T: Oh, OK. So you looked at that (pointing to student's drawing)* *K : Yeah* *T: This missing piece*

In the third type of response, students implicitly attributed their performance to one of the Standards for Mathematical Practice (CCSSM, 2010). For example, below (Table 8) are excerpts, from two different students. In each dialogue, the student said that they felt good or confident about their performance. When asked why, both Cameron and Chip shared they were able to explain or provide good reasoning for their answer.

Table 8 Example

Camron Dialogue 3	Chip Dialogue 3
T: All right. How did this one go *C: Good,* *T: Good.... What do you think was good?* *C: That umm* *T: What felt good about it?* *C: Um… being able to know how to explain more*	*T: Ok --Did you feel -- how did you feel about that answer?* *Ch: Confident* *T: You felt confident. What made you feel confident?* *Ch: I had good reasoning*

One of the standards for mathematical practice is that students can *construct viable arguments and critique the reasoning of others (*CCSSM, 2010*).* Although the students did not knowingly make reference to this Standard for Mathematical Practice, they suggested their performance was good because they were able to explain their thinking and provide reasoning. This practice is valued by the mathematics community and one that the classroom teacher and I encouraged throughout the year. This practice was also indirectly referenced in prior dialogues with each of these students. For example, in the prior dialogue with Cameron, toward the end of the dialogue I focused the feedback around this practice by saying *"You've got the answer. But now you're trying to think about how am I going to convince somebody? How am I going to explain this to somebody?"* (Cameron, Dialogue 2)

Students did not only refer to this practice when they did well. In the fourth dialogue with Chip, he considered how he did on each task and he pointed to one of them and said he felt "good and bothered." When I followed up to find out what he meant by that statement, he provided the answer to the task and then said "*But I can't umm--like -- I can't really explain the reasoning right*" (Chip Dialogue 3). He understood that having

the answer was not enough; he also needed to have good reasoning, which he did not believe he had.

Something important is happening here in these moments when students were connecting their performance to one of the standards of mathematical practice. Recall from the literature review, Sadler (1989) identified three components that must occur simultaneously in order for feedback to close the learning gap. Students must be able to:

- Identify the goal
- Understand where their performance is in relation to the goal
- Take action that helps narrow the learning gap

The first two of those components listed above are present in the excerpts shared earlier (Table 6) with Cameron and Chip. Both students relate their performance to being able to explain or reason. In essence, they have identified a goal related to the assessment and shared their performance in relation to this goal. Feedback is more effective when students understand the learning intentions or goals and know when they have met those goals and when they fall short (Nicol, 2007; Sadler 1989). I discussed the goal of being able to reason and convince others of their answer in at least one dialogue with each of the four students, but it was coded over 10 times throughout the 16 dialogues. However, some researchers suggest that simply sharing the learning goals and success criteria is not enough (e.g. Rust, Price, & O'Donovan 2003). Dialogic feedback however, provides the opportunity to co-construct *an understanding of the goal* so that students have a strong sense of what the learning goal is and what their performance might look like when they have met that goal. It is not my intent to discuss how students built this understanding of the goal in this section as I discuss this more and provide examples of when this

happened in the middle and end of the dialogue later in the chapter. Rather my point here, is that in some of the dialogues students had an awareness of the learning goal which is an important component of the feedback process (Hattie, 2012; Wiggins, 2012) and it is here, in the beginning of the dialogue, where we see students self-assessing their progress toward that learning goal.

Engaging in acts of self-assessment

Looking back at the consistencies that occurred at the beginning of the dialogues, it became apparent through analysis that students were engaging in acts of self-assessment. I asked students to reflect on how they performed on the assessment and to share what they believed attributed to that performance. Nicol (2007) suggests one of the principles of effective feedback is to facilitate the development of self-assessment (reflection) in learning. He noted that structured opportunities are needed in order for students to get better at self-monitoring the gap between where the student is and what the desired goals are.

Although I did not enter into these dialogues with the intent of facilitating self-assessment, the two questions related to performance provided a natural way for this to occur in the dialogue and was a pivotal moment for both student and teacher in the co-construction of the feedback. When students are able to effectively self-assess, Hattie (2007) writes "they can evaluate their levels of understanding, their effort and strategies used on tasks, their attributions and opinions of others about their performance, and their improvement in relation to their goals and expectations" (p. 94). I saw examples of this in the types of responses students gave when asked to reflect on their performance. Students attributed their performance to specific strategies, suggested their understanding

was improving, and in some cases evaluated their performance in relation to the mathematical practices.

When I specifically asked students to attribute or identify a cause for their performance, it encouraged students to identify what was working or what had changed. Weiner (1985) posits that how students explain their success or failure strongly impacts the effort students exert as well as the success they have on future tasks. At a more micro level and as it relates to learning in mathematics in these dialogues, if the learner cannot recognize or make a connection between the strategies they used and their performance outcome, they do not have a systematic way of understanding what strategies they should stop doing, start doing or continue doing.

While some research has shown that students may disengage with self-assessment practices because they believe only the teacher has the ability and authority to assess student work (Brown & Harris, 2013), interestingly that was not the case for any of the students in these dialogues. When asked about their performance no one responded with "I don't know." Each student was able to provide some type of evaluation of their performance whether they indicated if it was good, bad or improving as we saw in the sample responses.

Additionally, as students engaged in these acts of self-assessment, their responses provided feedback to me which in turn better enabled me to craft my feedback comments and questions to the student. It provided valuable information as to what the student believed attributed to their success and whether or not they valued or applied previous feedback. For example, when Mary shared her belief about her ability in math, this

opened up the opportunity to provide valuable feedback that was not task related but equally important. I elaborate more on this theme later in the chapter.

The Agenda for Feedback

The other noticeably consistent act that occurred during the beginning of the dialogue was a determination or the initiation of the agenda. The intent of dialogic feedback is to position students as active agents over their learning, rather than passive recipients of knowledge given by others. Therefore, as part of the analysis, I looked to identify how the agenda for the feedback dialogue was determined. In the first set of dialogues with each student, the agenda for feedback was determined by me, the teacher. Toward the beginning in each of these dialogues, I initiated the agenda by simply saying "let's look at this one" or "there are two that I want to look at." As part of my reflective journal after the first set of dialogues, I returned to my framework and noted the importance of having students identify their own areas for feedback. Although students still participated in the dialogue, I had not given them the opportunity to co-construct the agenda.

As I moved forward in the remaining dialogues, after students engaged in acts of self-assessment, I asked students to select problems they wanted to talk about first. This question alone shifted the power and positioned students as a co-constructor of the dialogue. Students did not hesitate with this shift in power and immediately chose problems they wished to discuss. In some cases, students chose problems they struggled with and in other moments they chose to talk about a problem they felt good about in order to verify their thinking. I elaborate more on these two noticings below.

Students Looked for Verification. When asked to select problems to talk about, sometimes students began talking about problems they felt confident in. Notice in the dialogue below (Chip, Dialogue 3) how Chip selects a problem and shares his thinking around it. After being asked how he felt about his answer, he shared that he was confident.

Ch: Oh, I want to talk about that one.
T: This one?
Ch: Yeah,
T OK, tell me, how did you feel about those ones?
Ch: I did six tenths bigger because I knew that five tenths is a half of ten tenths. And I know that six is bigger than five, so it would be six tenths.
T: Ok --Did you feel -- how did you feel about that answer?
Ch: Confident

Recall in the literature review, feedback tends to provide two types of information--verification or corrective. Verification feedback provides information as to whether or not the answer is correct. William (2015) similarly calls this feedback reinforcement. The intent of verification feedback is to simply let the learner know whether they are on the right track and does not elaborate on the correctness of the answer. Although corrective feedback has shown to have greater impact in learning (e.g., Banger-Drowns, et al 1991), it is important to note that sometimes learners want to verify that things are going well (William, 2015). This is the power in co-constructing the dialogue with students and giving them the space to determine what kind of feedback is valuable to them. In the dialogue with Chip, we can presume that it was important for him to verify and ensure that his answer was correct even though he felt confident about it. In my

conceptual framework, one of the tenants I relied on as I engaged in this study was providing descriptive feedback. Therefore, had I not given Chip the opportunity to choose and have input over which problems were discussed, I would not have addressed this problem, thereby missing an opportunity for feedback that was valuable from the student's perspective.

Students were perceptive and open about tasks they struggled with. When students were asked to reflect on their assessment and to select which tasks they would like to discuss, it was evident that students had a strong awareness of where they struggled or problems they found to be difficult. This was true for all four students regardless of whether they typically performed at benchmark level or below benchmark level in math. This is contradictory to some research that has shown that higher performing students tended to be more accurate in assessing their own work than lower performing students (Brown & Harris, 2013). Equally important to noticing that students were aware of problems they may have fallen short on, was the observation that students were open and willing to disclose this information. Research has shown in some instances that students are afraid to share an honest self-evaluation in fear of the teacher's response and the fact that their misunderstandings will now be known by others (Brown & Harris, 2013). However, self-assessment can lead to a constructive dialogue when students have developed trusting relationships and are willing to share information about areas where they lack knowledge and seek potential feedback and help (Raider-Roth, 2005).

In the examples below, Chip discloses that he was confused on one of the problems and wants to start by discussing that problem. Mary reveals that one problem

took a lot of thinking and she needed her fraction pieces and so she wanted to discuss this problem since she was unsure of her answer.

Table 9 Example Responses of Students Sharing Their Misunderstandings

Chip Dialogue 2	*T: All right. So is there any one you want to talk about first?* *C: I was confused--on this one*
Mary Dialogue 3	*M: I wanted to talk about this one, because this one took a lot of thinking and because i had to use my fraction pieces a lot on it.* *T: OK, so what are you thinking about this one? Are you unsure or...* *M: I'm unsure*

In a traditional model of feedback it is assumed that the teacher holds the most knowledge and therefore decides what feedback to give. However, we can see from these dialogues that when given the chance, even these fourth grade students had an awareness of their own struggles and could identify tasks on the assessment they may not have performed well on.

Additionally, all of these students were willing to identify these areas of misunderstandings in order to receive feedback. This is perhaps a key uncovering that I took for granted as I engaged in these dialogues. In order to co-construct a feedback dialogue students have to be *willing* participants that engage and take ownership in the conversation. Without knowing directly, we can infer that students had established a relationship of trust with me and knew that openly sharing their lack of understanding was not going to impact them negatively. While it was not the intent of the research to understand how this trust was built, it is notable to at least say that trust is a necessary component in order for students to openly and honestly engage in the reflection and self-assessment part of these dialogues. This finding is consistent with recommendations from others in the field (William, 2015; Yang & Carless, 2013)

Positioning Students as Active Contributors Towards the Agenda

The simple but straightforward question of asking students which problems they wanted to talk about encouraged students to lead and take an active role in the dialogue. This question was an enactment of the idea Bloxham and Campbell (2010) discuss, which suggests students are more empowered when they are able to request feedback specific to their interests or apprehensions, rather than receiving feedback based on how the teacher has interpreted their understanding. This, however, did not mean that I as the teacher was not able to also contribute to the agenda. In some cases, students *don't know what they don't know,* which was evident in these dialogues. In several of the dialogues, after an initial discussion of the problems selected by the student, I was able to move to discussing a problem I had selected for feedback. Although in many cases, because of the strong awareness students had in regards to their performance and understanding, many of the problems students selected to discuss were also the same problems I desired to give feedback on. We also saw when given the opportunity, students opted to discuss problems they believed they answered successfully, possibly to verify that their thinking was on the right track.

Due to the active contribution students had in determining the agenda, there was a sense of negotiation or compromise. There was not always enough time to dialogue about all the areas of concern I had based on a student's performance. Some educators or researchers may find it difficult or concerning that students would walk away from the feedback dialogue without addressing all areas of student performance that did not meet the learning goals or criteria for success. However, there is evidence that it is not beneficial to overburden students with too much feedback, as this can result in the

feedback being unmanageable and left unused (Hargreaves, 2013). Because of this negotiation and limited time, in some of the dialogues the student and I did not get to discussing every task in which the student had only partial understanding. As I noted in my reflective journal, "there will be opportunities during class to continue and explore the strategies around comparing fractions and estimating adding fractions…this will be important especially for Chip since we did not get to everything in the dialogue" (Dialogue 2, *Reflective Journal,* 2 May 2018). Because of my position as co-teacher in the classroom, I knew there would be opportunities during whole class instruction to address some of the other areas I did not get to during the individual dialogues. I also noted that because of the way the assessments were designed, students had multiple opportunities to engage in problems focused on using similar strategies.

Summary of the Beginning of the Dialogue

If we refer back to the research questions that explore the nature of these feedback dialogues, there were two major consistencies that took place in the beginning of the dialogues. The first consistency was students participated in acts of self-assessment that were facilitated by my questions. I started with an initial question to students about how they thought they performed and followed up by asking students questions that prompted students to attribute their performance to something. For example, students shared how they thought they performed on the assessment and attributed a mindset, a strategy, or a math practice as a reason for their performance. This part of the dialogue was a benefit to both the student and to my role as the teacher giving feedback. Students were given the first opportunity to reflect and evaluate their performance, compare their understanding to prior learning and assessment, and to consider the effectiveness of particular strategies

used. Self-assessment, an element of self-regulated learning, helps students identify the gap by comparing their own learning to the expected learning outcome (Sadler, 1989) which is a key component of effective feedback (Hattie 2007; Nicol, 2005).

The second consistency that occurred in the beginning of the dialogue was the initial determination of the agenda. After the first set of dialogues, students were given the opportunity to co-construct the agenda and therefore helped to determine what feedback they wanted to receive. This occurred when I asked the student to identify problems he/she wished to discuss. From the analysis, we saw students respond in one of two ways, either selecting problems they were confident in, to possibly verify their thinking--or they selected problems that were difficult or knew their answer was not enough. We saw that students had a strong awareness of their performance and/or any misunderstandings and were willing to discuss this during the dialogue. This aligns to theories that suggests students are not passive recipients of knowledge (e.g. von Glasserfeld, 1995) and provides the foundation for building a co-constructed dialogue When we ask students to select their own areas of feedback we rely on the pedagogical assumption that learners do not solely need the input and observation from others, but that students are active agents in their own learning and can identify at least some of their own areas of improvement (Boud & Malloy, 2013).

In addition to students determining the initial agenda, there was still room for me to bring up areas to focus on and provide feedback. It is not only possible, but likely, that students are unaware of some of their own mistakes. This is one of the beauties of a co-constructed dialogue. It is not solely the teacher, or solely the student who decides what

feedback is given. Instead, both teacher *and* student are able to notice, point to and question, as a way to determine which tasks are discussed and what feedback is given.

Ruiz-Primo, Araceli, and Brookhart (2018) write "the highest level of feedback is that which involves the student as a learner who can reflect on his or her own learning. It is feedback that helps the student make connections about what has been learned at any given point" (p. 69). Although this was very much present in the beginning of the dialogues, through analysis, it is apparent that there were opportunities to develop this reflection further. One thing that was missing was a reflection on how students were putting to use feedback from the previous dialogue. Although some students, when reflecting, shared strategies they believed attributed to their performance, which were strategies discussed from a previous dialogue, there was an opportunity to make this connection more explicit. In the future, there would be benefit in having students look across assessments or look back to the prior one and notice how their performance and thinking had changed. This would again engage students in acts of self-assessment and contribute to developing students' internal feedback messages as a component of self-regulation. I discuss this further in Chapter 5.

The Middle of the Feedback Dialogue

In the beginning of the dialogue, as discussed above, students engaged in acts of self-assessment and determined the initial agenda by selecting tasks to discuss. As I moved to analyze the next part of the dialogue, I noticed at the broadest level, this is where new understanding was occurring for the student. Recall from my conceptual framework that I view instruction as a necessary part of the feedback process (Hattie, 2007) and as Heritage writes, "in a regular classroom and on the everyday basis, there is a

blurred distinction between feedback and instruction" (Heritage, 2010*).* This part of the dialogue is where specific tasks were discussed and therefore naturally diverged and varied based on the particular student and their current understandings. The questions and instructional moves were dependent on the task at hand and how the students answered the problem. These are the moments that I focus on, in order to answer my second research question, which examines changes in students' mathematical thinking. However, if we zoom back out in order to look at the overall shape and flow which is the focus of my first research question, it was evident that the middle part of the dialogue had consistencies and an overall structure even though the content varied.

After the student and I determined which task would be discussed, the middle part of the dialogue generally followed this pattern or structure: 1) the student answer was read aloud creating a shared context and reference point, 2) scaffolding questions and instructional moves were used to build understanding, 3) students were asked to change or add to their previous answer. This sequence often repeated itself if more than one task was discussed. I first elaborate on the benefits and opportunities that arose from starting with the student answer. I then move to examining the scaffolding and instructional moves in detail. In essence, I zoom in on the *middle of the middle* of the dialogue in order to answer my second research question which examines the moves and characteristics occurring when there was a change in mathematical thinking. Finally, I discuss the end of the middle part of the dialogue which gave students the opportunity to revise their answer.

The Beginning of the Middle: Starting with the students' answers. As the dialogues transitioned into the instructional part of the feedback dialogue, it was

noticeable that the middle part of 14 dialogues started with reading the student answer. In the two dialogues that this did not occur, it was because the student had an unfinished answer or shared they were confused and so we jumped right into making sense of the problem. In the 14 dialogues, it varied as to who read the student answer. In some cases, I asked the student to read aloud their own writing and other times I read their answer out loud. This was not planned nor intentional and only after analysis was this illuminated. Although at first glance this seems to be such a minor move in the dialogue, it provided the foundation for the feedback and instructional moves that followed, and allowed both the student and I a common reference point to refer back to when building new understanding. It was the entry point into this middle part of the dialogue and a way to build the feedback off of what the student had already done. After the student answer was read aloud, two things typically followed. One outcome was students caught their own mistake or had questions about what to do next. A second outcome involved the opportunity to clarify student thinking. The benefits and opportunities from each of these outcomes are discussed below.

Identifying mistakes. After the student answer was read aloud, in three of the dialogues the students were able to catch their own misunderstanding. This was a powerful act in itself and again positioned students as active contributors to the feedback. Take for example Dialogue 4 with Mary. After she read her answer out loud, she comments about her answer being "weird." Here in this moment, Mary recognizes for herself that her answer does not make sense, without any input from me. This continues to align with the theme noticed in the beginning of the dialogue which observed that when given the opportunity to reflect and weigh in, students had an awareness of their

own shortcomings and could be active participants in the feedback without having to only rely on the input from the teacher/assessor. In a traditional model of feedback, I would have likely told Mary that her answer did not make sense. Notice here in this excerpt, after Mary read her answer out loud that I did not provide any evaluation or comment beyond a simple "ok" to signify that I was listening. She then goes on to notice and state that what she had written down was "weird" because she gave two contradicting answers.

T: *So read me what you wrote.*

M: *I think it is more than a candy bar because three sixths…Sixths are big and tenths are small, but I think it's less*

T: *Ok .*

M: *That was kind of weird…(laughs)*

T: *(Laughing) What do you think is weird about it?*

M: *I think what is weird is I said it's more in my first, but then I said it's kind of less.*

This example again reiterates the important role of student reflection and self-assessment as a means to co-constructing the dialogue. Mary did not sit there as a passive recipient of feedback information about whether or not her answer made sense. Instead, Mary had the first opportunity to dialogue about her own noticings after reading her answer out loud. After Mary read her answer I refrained from any sort of evaluation or comments about her answer. Although this restraint of the teacher not providing any evaluative comment first does not guarantee that a student will fill in the space and indeed contribute to the dialogue, as Mary did in this example, at the very least there was space for it.

If the intent is for students to co-construct the feedback dialogue, there has to be an opening or in some cases a more explicit invitation to do so. As seen from the

example with Mary, one way to do that is to start with reading what the student wrote and then pause before making any sort of comment or evaluation. This opens up space for a student to reflect and self-assess. When students are able to recognize or self-monitor their own mistakes, it sets up a greater likelihood they will engage with the feedback and use it to further their learning and performance outcomes. If we are aware of our own misunderstanding, we are more open to receiving the feedback we need to move forward (Hattie, 2011).

Clarifying Student Thinking. After the student answer was read aloud, I followed with probing questions in order to clarify student thinking. Examples of these questions taken from the dialogues are included in Table 8 below. Although the phrasing of each question varies, the goal behind each was the same. Each question sought to find out more of what the student was thinking about or intended to say but did not say.

Table 10 Example of Probing Questions

Dialogue	*Probing Question*
C1	*"What do you mean by there's less to go?"*
Ch3	*"Can you tell me more… what do you mean fifteen is bigger than ten?"*
K4	*"OK…can you tell me more about your reasoning because I wasn't quite sure I understood what you were thinking."*
M1	*"Can you tell me about some of the things that you were thinking about, how did you start?"*

As students responded to these types of questions, it provided more insight into what they understood and how they made sense of the problems. In many cases, students understood more than what they had indicated through their answer. An example of this is shown in Table 9 below. At first, we see an image of Chip's answer to the task that

asked students to compare two fractions. His original answer is written in pencil and is very limited in explaining how he knows that the fraction 4/6 is closer to a whole. Notice in the dialogue below, when I asked a question in order to clarify his thinking, he went on to talk about how he knows 4/8 is a half and that 4/6 is bigger than a half. In this example, it was not clear from his original answer that Chip had a strategy for justifying how he knew which fraction was larger. However, after clarifying with him about his answer, it was evident that he did in fact have a strategy. Therefore the feedback was not focused on developing a strategy but instead the feedback focused on adding language about one half in order to explain his strategy.

Table 11 Chip Dialogue 1, Assessment 1

$\frac{4}{8}$	$\frac{4}{6}$	(6) 4/6 is bigger because it is closer to a whole. 4/6 is greater than a half but 4/8 is not.

T: You said four sixths is bigger because it's closer to a whole. What do you mean by that?

Ch: Closer to a whole because…That I know that's half of that…But this is over half. It's over a half, three sixths. But it's four sixths so. And I know four is over three and so four is bigger than three. So that would make it four sixths and that would make it bigger than that..

T: OK, so what I just heard you talking about is this idea about a half.

A benefit of dialogic feedback is the opportunity to craft feedback that is contingent on and responsive to the learner's needs. When engaging in dialogue there is opportunity to clarify with the learner their intentions in order to better understand what the learner was trying to do versus what the teacher perceived the learner trying to do.

The teacher may clarify *with* the learner about their intentions, and with this new information better craft feedback that moves learning forward in a way that matters to the student. In the example with Chip, it was not necessary to model or think through a strategy in order to make sense of 4/6 as the larger fraction. This would have been a misuse of time during the feedback dialogue. He already had an effective strategy so instead, the feedback needed to focus on helping him communicate his strategy.

Reading the student answer played an important role in the dialogue in two ways. First, it established a common ground or shared context in which both the teacher and student were aware of what was said or not said as part of the answer the student gave. The feedback that followed was not based on any assumptions by the teacher or student, but rather the context that was established when reading the answer out loud. Secondly, based on what was read aloud, there was opportunity to clarify what students were thinking and for students to elaborate beyond what they had written. This allowed for the feedback to be more finetuned to what the student already knew and where I thought students needed to go in their understanding next, based on research around the progression of children's mathematical thinking with fractions (e.g. Cramer, Behr, Post & Lesh, 2009)

The Middle of the Middle: Zooming In On Instructional Moves

My second research question sought to better understand the characteristics and moves in the dialogue that prompted a change in mathematical thinking. In my analysis, I looked to identify moments where students had a shift or made progress in their thinking and reasoning around the assessment tasks. Recall from Chapter 2, a sociocultural perspective on learning emphasizes the analysis of the social interaction with a more

knowledgeable other as a means to understanding cognitive development, while constructivists examine the way in which children construct their own knowledge through active experience and reflection. I suggested that an integration of these two perspectives was necessary and therefore in my analysis I examine the moments where students are actively constructing new knowledge in relation to the scaffolding moves provided by a more knowledgeable other which in this study was my role as the teacher.

As I analyzed the characteristics and instructional moves through the phases of coding I noticed that many of the instructional moves corresponded with some of the Standards of Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The Standards for Mathematical Practice (SMP) describe the habits and processes in which students who are mathematically proficient engage in the math content. The SMP's reach across all levels of education, and educators should be concerned with developing these in their students as they learn the math content standards (CCSSM, 2010).

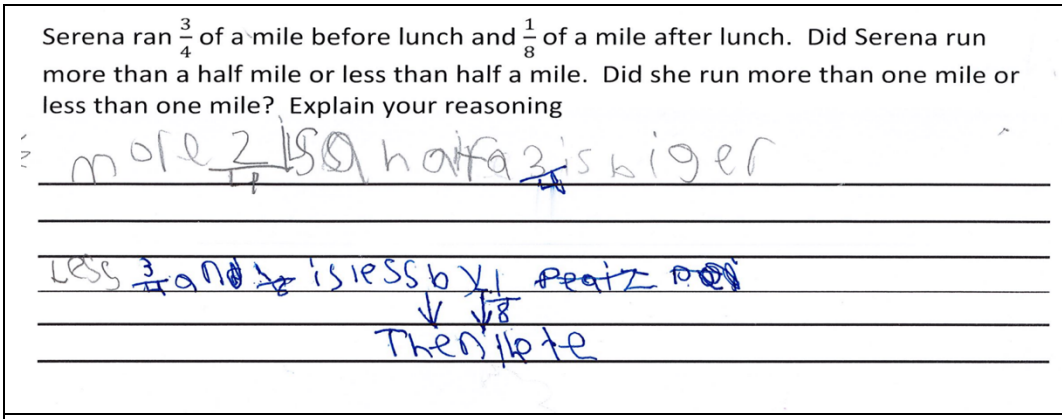
As noted in my analytic memo, I did not design the instructional moves ahead of time or go into these dialogues with a conscious attempt to develop these practices in students. Rather, my background and experiences as a math educator subconsciously drew on these practices as a means to develop and deepen student understanding of fractions. However, now that this has been brought to light, it provides a framework for outlining the instructional moves one might consider as they engage in feedback dialogues with students

Pressing for Justification. One of the SMP's is for students "to construct viable arguments and critique the reasoning of others" (National Governors Association Center

for Best Practices & Council of Chief State School Officers, 2010). In other words, students can justify their results and communicate their reasoning to others. As I analyzed the instructional moves, one of the most common codes was *pressing for justification.* The phrases occurring under this code included variations of "how do (did) you know, why do you think…and what reason can you give." As students responded to these questions, two things were occurring. First, as students expressed their reasoning out loud it helped to make their implicit knowledge explicit, which then supported students in the articulation of their reasoning and justification. Additionally, it gave me an opportunity to use scaffolding questions to help students clarify their own thinking and understanding. An example of this occurs in the dialogue with Cameron as he works to add reasoning to answer the world problem included in Table 12.

Cameron had indicated in his answer that Serena ran less than 1 mile. When I asked him "how do you know", he continued to make sense of his own understanding as he explained his reasoning out loud. You see in line 3 and 6 he modeled with the fraction pieces that he needed one more eighth to make a whole, but then realized another way to explain this would be to start with the ¾ and notice you need another fourth to make a whole. I then built on his thinking and used scaffolding questioning to help him articulate what he was noticing with the fraction circles. He sees that 3/4 + 1/4 equaled a whole and because 1/8 is less than 1/4, then 3/4 + 1/8 is less than a whole.

Table 12 Cameron Dialogue 2



T:	*ok and then how do you know it's less...you wrote less, but now you just got to have your reasoning. So how do you know that it's less than one?*
C:	*because...(opens up the fraction circles and shows 3/4 and 1/8) so... so this is what it would look like.*
T:	*ok*
C:	*And you need one more eighth to get a whole (pause) So… And… Or you could also put you need one more fourth to make a whole.*
T:	*OK. All right, so you need one more fourth to make a hole, but what do you have?*
C:	*Three fourths and one eighth*
T:	*And is one eighth more than a fourth or less than a fourth?*
C:	*Less*
T:	*OK. So how are you going to write that?*
C:	*three fourths and one eighth is less than a whole... by one piece... by one eighth piece*

A similar type of interaction occurred with Kate around this same problem. Just before this excerpt from the dialogue, Kira noticed that 1/8 is half of a fourth. In this exchange, Kira is trying to explain her reasoning, but is having some difficulty. Notice in the transcription she started to explain, but then paused and looked at me for help. I was able to structure sequences of questions (Kyriacou and Issit, 2008) to help her make explicit or verbalize what she was noticing in her model.

(Kate, Dialogue 2)

T:	*So what reason could you tell somebody why it's less than one, and it's not more than one*
K:	*Less than one because if you...you have three fourths… you have one more left and then you put that one on there (inaudible) (Looks at teacher)*

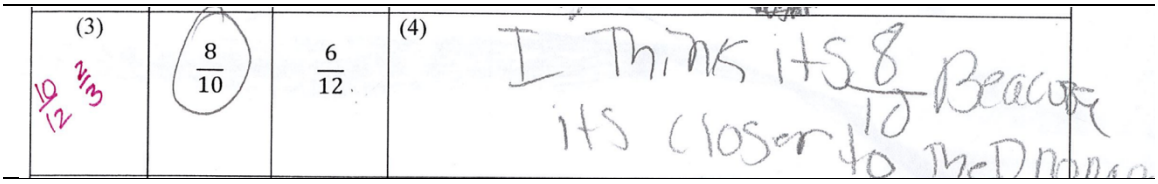
T *So I think I know, I understand. I think I know where you're going. So you said if you have one more fourth, that would make what...*
K: *a whole*
T: *but you don't have one more fourth. You have…*
K: *one half of a fourth*
T: *you have one half of a fourth. So is that going to be enough to make another whole?*
K: *No,*
T: *How could you how could you write your explanation about what we just talked about with the fraction circles*

These findings are consistent with the findings from [Rojas-Drummond & Mercer, 2004]. When examining the interactions teachers had with students, the authors noticed that students with higher outcomes in reading comprehension and mathematical problem solving had interactions with teachers that were characterized by an intent to build on children's initial thinking. Teachers used questioning to help students make explicit their thinking and to guide their understanding (Mercer 2008). As I pressed students to justify their thinking, there was the opportunity to co-construct the dialogue. The students provided an explanation and then through a sequence of questions and responses, students were able to further extend their understanding of fractions.

Providing Counter Examples. Another instructional move related to helping students construct viable arguments and critique the reasoning of others present in the dialogues was the use of counter examples or false propositions. When students make a claim or conjecture, counter examples can be used in order to illuminate conflict with the students current reasoning or justification. An example of this occurred in the first dialogue with Mary. The task asked students to circle the larger fraction and provide an explanation for how they determined their answer. Notice in Table 11 Mary had circled the larger fraction of 8/10 but her explanation presents a potential misunderstanding. She suggested 8/10 was larger because it is closer to the denominator. In the dialogue I asked

Mary to clarify this statement and she went on to say that 8 is closer to 10 than 6 is closer to 12 further cementing her misuse of whole number thinking to compare the fractions. My next instructional move in the dialogue presented two new fractions (represented to the left of the original problem).

Table 13 Mary Dialogue 1--Using Whole Number Thinking



T: Which one do you think is bigger?
M: I think…think…10/12,
T: Why do you think that?
M: I think 10/12 is bigger because um it has a smaller piece to get to the denominator and two thirds is actually one more to the denominator, but it's a bigger piece to get to it.
T: Oh, OK, so your reasoning over here when you said, I think it's 8/10 because it's closer to the denominator, does that reasoning work. For this one?
M: I mean…
T: (…) Which one is closer to the denominator?
M: Mmm two thirds actually.
T: Is closer. Right?
M: Yeah
T: But you told me that you think 10/12 is bigger.
M: Yeah, because, I mean, it's smaller pieces to get to it, but two-thirds is brown (color of fraction piece) so it would take bigger pieces
T: I want you to think about this reasoning. Does this reasoning always work?
M: No

I asked Mary to consider these two fractions and determine which one was bigger. I chose fractions that she would be familiar with based on the work she had done in class and that would also cause conflict with the current reasoning she had for 8/10 and 6/12. You can see in the dialogue when Mary has the moment of realization in Line 8 where she notices the 2 in 2/3 is closer to the denominator, yet she did not choose this as the bigger fraction. What is interesting is that once she realizes this, she does not revert back

to her whole number thinking in which she determines the larger fraction by comparing the difference between the numerator and denominator. In line 10 she continues to defend why she thinks 10/12 is bigger even though it conflicts with her original reasoning given for the fraction pair on the assessment and then concluding that her reasoning will not always work.

The dialogue continued with Mary as she built understanding around using a half as a benchmark to determine which fraction is larger. Before Mary built this new understanding, it was important that she first saw a flaw in her own reasoning before she would consider the need for providing another justification. If I were to have approached this moment with a transmission view of feedback, I would have told Mary that her reasoning would not always work, and possibly given an explanation why. However by providing a counter example to Mary's reasoning and having her use the models to make sense of this new problem, she was able to actively construct why her reasoning would not always work and created a dialogue between teacher and student around this uncovering. Von Glasserfeld (1995) writes "only when students can be led to see as their own a problem in which their approach is manifestly inadequate, will there be any incentive for them to change it (p. 9).

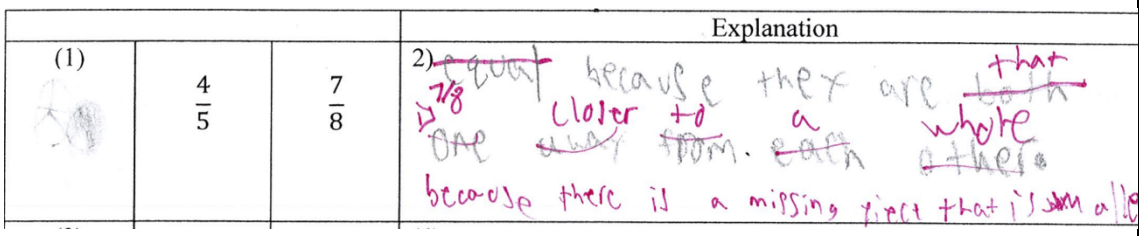
This is an example of Sociocultural and Constructivism theories of development working together as shared in my conceptual framework and as it relates to dialogic feedback. Mary moves through what Piaget (1974) describes as accommodation as she makes sense of the new problem presented to her which conflicts with her previous method of determining the larger fraction by comparing the difference between numerator and denominator. However it was through the interaction with a more

knowledgeable other (Vygotsky, 1978) that Mary became aware of the conflict with her proposed reasoning and through the active feedback dialogue Mary was able to reorganize her thinking around comparing fractions in a mathematically justifiable way.

Use of a Model. Another instructional move, used during the middle part of the dialogues, focused on the importance of models. I directed or suggested students use a model in 12 different moments throughout the 16 dialogues. Most often the model used was the fraction circles, a manipulative students used often in the classroom throughout the unit and were very familiar with. Another model students used was a pictorial drawing of the fraction circles. I directed students to take out the fraction circles when I wanted them to notice their answer was incorrect or if their explanation revealed a deep misunderstanding or misconception of fractions. After students modeled the problem with fraction circles or drawings, on every occasion, I started by asking students what they noticed. I used this as a starting point in order to determine what feedback and instructional moves came next, and to give students the first opportunity to make sense and actively construct knowledge based on what they were seeing. Notice this occurring in the example below with Chip (Table 12).

Chip's original explanation in pencil was that the two fractions were equal since they were both one away from each other. I asked Chip to model these two fractions using the fraction circles. Then I asked him to share what he noticed when looking at the fraction circles. He quickly determined 7/8 was the larger fraction because it was missing a smaller piece to make a whole. By using the model, Chip was able to construct a new understanding of comparing fractions by looking at the missing piece.

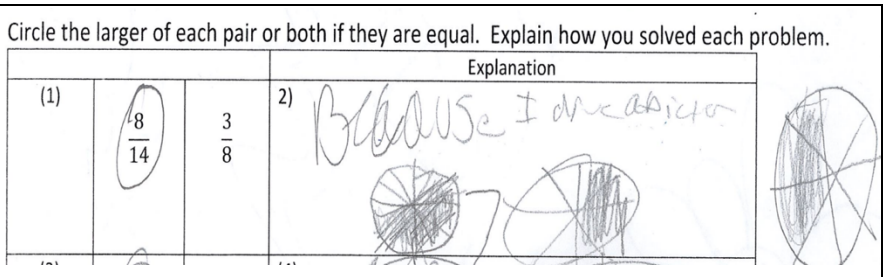
Table 14 Chip Dialogue 1--Noticing with Model



T:	*Ok what do you notice?*
Ch:	*I notice that this one is bigger*
T:	*OK. Why-- Why do you think that?*
Ch:	*Because it's is closer to a whole*
T:	*Oh, how do you know it's closer to a whole?*
Ch:	*Because it's because this one is smaller and that one is bigger*
T:	*So this piece that's missing?*
Ch:	*Yeah*

In Dialogue 2 with Mary, I directed her to the pictures she had drawn on her paper to compare 3/8 and 8/14. She had the correct answer and an accurate model so my next instructional move was to ask Mary to communicate with words and symbols what she noticed when she looked at her drawing. As she answered this question, she made explicit the idea of comparing fractions to a half. She *had* the correct answer and had already created a pictorial model. Here the use of a model was geared to strengthening Mary's ability to justify her reasoning.

Table 15 Mary Dialogue 2--Using a Model To Strengthen Reasoning

Mary's Original Response	

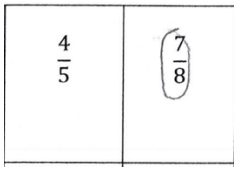
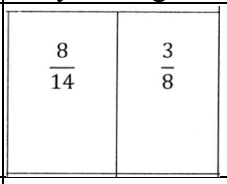
This is different than how I used a model to support Chip. In the example with Chip, I directed him to a use a model to notice his answer was incorrect and to build new understanding of comparing fractions. The use of a model as an instructional move, in these two different ways, was significant in helping each student move forward in their thinking about fractions.

In both of the examples shared, it is important to notice that the student is still actively constructing and making sense of the math. It is not solely the model or solely the dialogue with me that is providing the student with the correct answer or thinking. Instead it is the *interaction* students are having with the model and my questioning that is propelling thinking forward. Prior research has shown the important role concrete models have in the development of fraction concepts and number sense (Cramer & Henry, 2002). These finding from this study suggest the use of models also play an important role in helping construct feedback during the dialogues. As seen in the examples discussed above, the feedback developed out of the noticings students had with the model, coupled with my scaffolding questions, allowed students to synthesize information in a deeper and more meaningful way than the model alone could offer.

Attending to Precision. As students worked to explain and justify their reasoning, there were moments where the feedback and instructional moves focused on the use of more precise language. This is another move that relates to the Standard of Mathematical Practices which emphasize students using precise language to communicate their thinking and mathematical explanations to others (CCSS, 2010). As I examined these moments, I noticed two different ways in which this happened in the

dialogue. The first way I initially coded as "pressing for precision." I asked questions to encourage students to be more precise in their communication and to push students to use vocabulary and math language to be more specific. An example of this questioning is shown with two different students in Table 14.

Table 16 Example of Pressing for Precision in the Dialogues

Cameron Dialogue 1	Mary Dialogue 2
	
T: What do you mean by there's less to go, *C: Like less of a whole to go.* *T: OK, so…* *C: So like there's like less of a fraction piece like a fraction to go* *T: to make what?* *C: A whole*	*M: That one's a half (points to 3/8).* *T Is this one exactly a half?* *M: Not exactly a half, but it's almost a half.* *T: Oh, it's almost a half?* *M: Because three and there's still one more piece to make a half*

In the first example with Cameron, he compares the fraction 4/5 and 7/8 and reasons 7/8 is bigger "because the pieces are smaller and there is less to go." When I asked him what he meant by this he was able to add in language that included *whole* and *fractional piece* in order to make his explanation more clear of what he was trying to communicate. In the second example, Mary looked at her picture she drew to represent 3/8 and I asked her to share what she noticed. She first shares that it is a half, but when I question her if it's exactly a half, she is able to articulate that it is one piece less than a half.

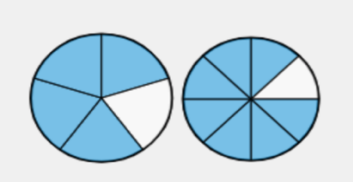
Similarly, in the exchange with Chip, I press him to be more specific when talking about the size of what he calls "one more piece." Notice in the beginning of the exchange I ask him what we call the piece that he is referring to in his explanation. When Chip is

still unsure, I continue to scaffold by referring him back to the fraction in the problem and then further asking him what size is the fractional piece.

The other instructional move that focused on the precision and language students used to communicate their thinking was through the use of rephrasing what students shared or I modeled language for students to use. When comparing fractions that both had a numerator that one piece away from the denominator, such as 4/5 and 7/8, it was common for students to explain their thinking by simply saying 7/8 is closer to a whole. As part of the feedback dialogues and discussed earlier, I pushed students to justify their reasoning and explain how they knew it was closer to a whole. This was the case for three of the students. You can see in the examples of Kate and Chip (Table 15), both of them were trying to communicate the idea they noticed with their model (an example of a model is included for reference in Table 15.)

These two students were comparing the piece that was missing in each of the fractions in order to make a whole. As both Kate and Chip reasoned out loud, they used the words "this one" and "that one" to describe what they noticed.

Table 17 Example of Teacher Rephrasing Student Language



Chip Dialogue 1	*T:*	*Oh, how do you know it's closer to a whole?*
	Ch:	*Because this one is smaller and that one is bigger*
	T:	*So this piece that's missing?*
	Ch:	*Yeah (Writing reasoning down on assessment)*
	T:	*Ok so you wrote seven eighths is closer to a whole. And how do you know it's closer to a whole?*
	Ch:	*Because that's one small --because that -- cause that missing piece is smaller.*
	T:	*OK, why don't you add that ("missing piece") to your explanation and now it's going to be a solid explanation.*

Kate Dialogue 1	*K:*	*Yeah but that one's bigger so it's like those ones are taking up like less space.*
	T:	*So when you say that one, why don't we say the missing piece,*

In the exchange with Chip, I rephrased what he said by saying "this piece that is missing" to name what he described. It was evident that Chip noticed the language I provided and used it later in the dialogue. As he verbally described what he was going to write, he started to say "that one" is smaller. He then paused and restarted by adding in the language "missing piece" in order to be more specific in what he was referring to as being smaller. This served as an important reminder that when teachers offer new, or different, language to students, it has the potential to influence their future thinking and use of that language.

Additional Considerations of Instructional Moves. There are two additional considerations that are related to the instructional moves that I believe are important to share. The first consideration revolves around the overlap of the instructional moves in some of the excerpts. For example, in one exchange I was directing a student to use a model and then followed by asking them to justify their thinking. Even though an excerpt may have been included under a particular section to highlight an instructional move, in many instances multiple instructional moves were happening sequentially and therefore the exchange could have fit under multiple headings. I did not want to cut off the excerpts too quickly as it was often necessary to keep several lines in order to see how student thinking had evolved and changed. Although discussed independently, most often these instructional moves did not occur in isolation and were complimentary to each other.

The second consideration draws out an important aspect of the feedback process in which the instructional moves were dependent on the feedback from students. In a feedback dialogue, feedback is happening in both directions—from teacher to student and from student to teacher. Feedback should not only inform the student but also the teacher and the teacher should focus on using the interaction to learn about student thinking and understanding (Ruiz-Primo, Araceli & Brookhart, 2018). Hattie (2009) goes as far as suggesting that it is feedback from the student to the teacher that is "most powerful" in terms of impacting the relationship between teaching and learning (p. 173). However, I would argue that while feedback to the teacher is incredibly important, the teacher must know how to act on that feedback in a productive way that not only continues to further the student learning, but also helps to maintain the role of student voice and agency in their learning.

The important role of Pedagogical Content Knowledge is highlighted during these moments in which the teacher makes an instructional move based on the feedback from students. Pedagogical Content Knowledge includes knowledge of representing the subject matter as well as the knowledge of learners and their developmental conceptions of the content (Shulman, 1987). Ruiz-Primo, Araceli and Brookhart (2018) explain even further by saying "teachers need deep domain knowledge to interpret the student thinking represented in student work or student discourse. Then, they need a flexible repertoire of instructional moves that they can apply strategically based on their appraisal of the students' next steps in learning." (p. 62).

During the middle of the dialogue, I made decisions and instructional moves that were dependent on what the student already understood or what misconception they

presented. Because of my background as a math educator and experience in researching the strategies children use when working with fractions (Cramer, Monson, Ahrendt, Wyberg, & Fagerlund, 2019), I was better prepared to make some of these *in-the-moment* instructional decisions that could move students from their current understanding to new understanding. This was a balance of trying to engage the student in the thinking as much as possible without prescribing what the student should do to find the answer.

The Final Part of the Middle: An Opportunity to Revise Using Feedback

If you recall, I outlined that there were 3 parts to the middle of the dialogue. The first part involved creating a shared context by reading what students had on their paper. The next part, what I called the middle of the middle of the dialogue, consisted of the instructional moves in order to build new understanding. This is where I spent time zooming in on the moments where a change in mathematical thinking occurred. And finally, it is here that I discuss the third or final part of the middle. Students were given the opportunity to revise their answer based on the feedback constructed around the task. This happened in all 16 dialogues and always directly followed the instructional moves around the specific task being discussed. This is not surprising as it a major tenet of my conceptual framework.

Going into the dialogues I knew there was going to be a place for students to revise their thinking, as research supports the idea that feedback without opportunity for action has minimal to no impact (e.g Sadler 1989, William, 2015). However, what research has yet to reveal as fully is what this process looks like, particularly as part of a co-constructed dialogue. Through my analysis, I wanted to understand when this

happened and how this took shape. I have shared when this occurred in the dialogue, but I discuss the noticings around "how" this occurred below.

Table 18 provides a sample of the prompts used as a signal in the dialogue for students to revise their answer. As you can see from the prompts, I was still asking students to actively participate and construct their answer. I had anticipated that after my prompt or direction the student would revise their answer and then we would move on. It was surprising that this was seldomly the way it was enacted. Instead, there was still active exchange and co-construction between the student and me.

Table 18 Sample Prompts to Student for Revising Answer

	"How would you change this now?
	"How could you write your explanation about what we just talked about with the fraction circles?"
	"So where can we add that *(new thinking)* in there *(student's previous answer*)? Where would it make sense to write that part?"
	"OK, so on here, when you write your explanation, what could you say instead of this *(students initial answer)*?
	"How could you finish that last part that you didn't feel so good about?"

I had predicted that students would make sense of the feedback earlier in the dialogue as we built new understanding during the instructional move, which then led students to be able to adjust their answer. However I recognized this was not true, both as I engaged in the dialogue and during my analysis.

As students were revising their answer, there was still an opportunity to provide feedback and ask follow up questions when students were unsure. In the dialogues below (Table 19) both are examples of a moment when the students started writing and fixing their answer but got stuck. In the dialogue with Kate, I provided a follow up question to

help her think through the rest of what she wanted to express in her reasoning. Similarly, Chip started writing his revised answer and acknowledged that he was getting confused.

Table 19 Example of Students Clarifying Next Steps

Kate Dialogue	CH 3 Dialogue
K: Should I write it down? *K: (Reading answer to self)* *T: So let's see what you have… (reading student answer) "Less than one because you have three fourths and you have half of a fourth."* *K: So it would be…* *T: So would that be enough to make a whole or not?* *K: No (Writes and adds to answer)*	*CH: umm wait what?* *CH: I'm getting too confused that I don't remember what I said.* *T: What can you say about the missing piece for 11/12?* *CH: It's smaller than (inaudible)* *T: Ok so that's what I want you to write.*

A strength of dialogic feedback shines through in these moments. Even as students were revising their answer, there was opportunity for students to make sense of the feedback and clarify their next step. As a result, it would be important to make time for this to occur in a feedback dialogue, knowing that as students put the feedback to use, they may need additional clarity.

The act of changing or adding to the answer was doing two things. First, it sent the message that the learning does not stop after completing the assessment. Students can add to and change as a way of showing they are still learning. Secondly, it gave students the opportunity to do something with the feedback right there in the middle of the dialogue. A noticing that cannot be ignored came from the 3rd dialogue with Mary. As we were reflecting on how she did with the problems, she shared she felt good about both of the tasks on the assessment in which she was comparing two fractions.

T: *OK, I thought that your explanation this time were really strong because we talked about you--*

M: *It was the same day we talked about it*

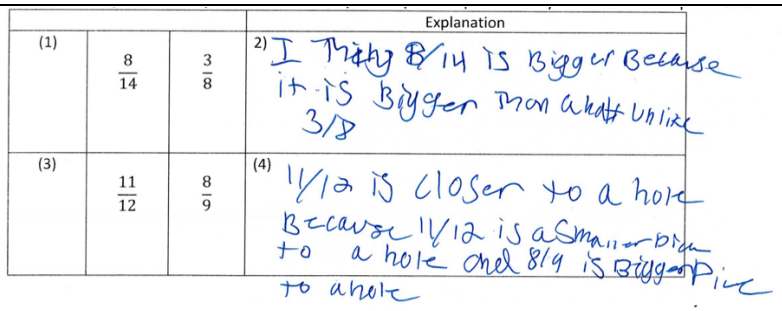
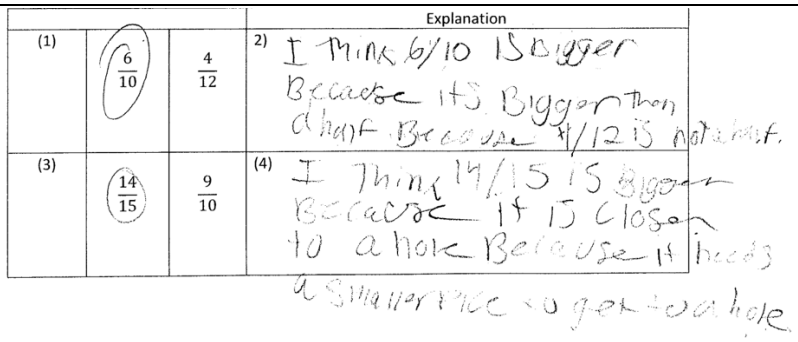
T: *Yes.*

M: *So I could remember.*

The day Mary took the third assessment, we had a feedback dialogue from her second assessment earlier that day. As you see from the excerpt, she notes that because we had discussed feedback on the same day she took the next assessment, she could "remember" what we discussed and could apply it to these new problems. This stands out amongst and across all of the assessments and dialogues for two reasons. First, it was the only time in which I had engaged in a feedback dialogue the same day as the student took the next assessment. There was generally a small gap of 1-3 days between the previous feedback dialogue and the next assessment. Secondly, as I analyzed all of the assessments sequentially for each student, this was the strongest representation in which the student directly made use of the feedback we had discussed from a prior dialogue.

I examined each students "revised responses" and then looked to see how their response compared on the next assessment with a similar problem. In general, students' responses progressed but were still not as strong as their revised response they had fixed on the prior assessment during our dialogue, except for this one instance with Mary. Her edited responses on Assessment 2 developed from Feedback Dialogue 2 were nearly identical to the responses from Assessment 3 with similar problems. The only difference was that she was comparing different fractions.

Table 20 Comparison of Similar Answers on Mary's Assessments

Mary's Revised Answer from Assessment 2 Feedback Dialogue	
Mary's Answer on Assessment 3	

I discussed this noticing in my analytic memo and consider whether or not giving the students the opportunity to revise their answer based on the feedback dialogue was enough. I wrote "perhaps instead of, or in addition to, only revising their previous answer based on the feedback dialogue, students need the opportunity to apply that feedback to a new problem right away, and not wait until the next assessment". Although students interacted and made use of the feedback, maybe it was not enough and they needed an opportunity to try out the feedback on their own and with new context without waiting until the next assessment. This is something I consider as I discuss areas for future research in chapter 5.

The End of the Dialogue

I define the end of the dialogue as the remaining conversation that happens after students had the opportunity to revise their answer and no further tasks were discussed.

This part of the dialogue was the shortest, in terms of length and number of exchanges between myself and the students, and closed the dialogue fairly quickly. I noted in my analytic memo that this part of the dialogue seemed to be the least co-constructed and the most teacher directed. I write "this part reminds me of the concluding remarks of an essay. A last sentence to summarize the key points."

Feed-up, Feedback and Feed-forward

Following the initial cycle of opening coding and memoing, I used Hattie and Timperley's (2007) model of feedback that focuses on three components: *feed-up* (where am I going), *feedback* (how am I progressing), and *feed forward* (where to next).

In order for feedback to be effective, Hattie and Timperley suggest that it must address one of these questions. My intent in using this framework was to better understand how these types of feedback occurred in the dialogue and what surrounded the particular use of the type of feedback. Table 21 provides examples of what each of these components looked during the end of the dialogue with students.

Table 21 Examples of Feed-Forward, Feedback and Feed-up From Dialogues

Type of Feedback Based on Hattie &Timperley (2007)	**Teacher Response During End of Dialogue**	**Dialogue**
Feed-up (Related to the goal)	*T: But sometimes as our fractions we've talked about before, as they get bigger, as they get more complex, sometimes our pictures, it's really hard to to get that. But if we can have our mental pictures…*	K4
	T: OK, so sometimes a picture can work really well, but as our fractions get larger, right, like if I... *M: it's harder to draw.* *T: You already recognize that and it becomes harder to draw. And so another way that mathematicians can use their explanation is through its reasoning, in their words*	M2
	T: But sometimes as our fractions we've talked about before, as they get bigger, as they get more complex, sometimes our pictures, it's really hard to get that. But if we can have our mental pictures…	K4

Feedback (How am I progressing)	*T: You have a really great model*	K2
	T: OK, so umm. You just had to work on your explanation on this one, right? You had the right thinking, but what you told me was not what you wrote down.	Ch1
Feed-forward (Where to next?)	*T: OK, so one thing that I think that's going to be helpful for you that you've started to notice is this idea about a half... and thinking about fractions that might be close to half and fractions that might be more than half and fractions that might be less than half, alright?*	M1
	T: Mmm. So I want to keep encouraging to see if you can use that reasoning to also help you, especially when fractions get harder and harder to draw.	K3

The most common type of feedback addressed the question of "where am I going next" or feed-forward. The feed-forward was based on the strategies we focused on in the dialogue. In the initial round of coding, I differentiated between what I called *suggestive feed-forward* and *directed feed-forward.* Suggestive feed-forward was a suggestion to students to continue working on a strategy that we had dialogued about as they moved forward. I introduced this feed-forward with phrases like "I think it will be helpful…" or "sometimes it might help you to…". I offered the feed-forward as a suggestion so that students still had the space to make their own decisions as they were engaging in mathematical tasks. As a math educator I wanted students to continue to see math as creative and flexible (Boaler 2018) and therefore did not want to box them into having to use one strategy.

Directed feed-forward on the other hand was more direct and I was explicit about what I wanted them to do moving forward. In these instances I used a phrase like "I want you to think about…". I noted in the analysis, I most often used directed feed-forward when it was around strategies we had dialogued about previously. An example of this occurred with Kate. In all four dialogues with Kate, part of the dialogue and feed-forward focused on thinking about ½ as a strategy to compare fractions and then

eventually estimate adding fractions. This also happened with Chip as I noticed in the second dialogue we were focusing on strategies we had discussed during the first dialogue. After the second round of dialogues, I wondered in my reflective journal if this was because I wasn't direct enough in the feedback dialogue. I wrote "maybe I need to be more specific or explicit about what students should do moving forward." (*Reflective Journal*, 2 May 2018)

I felt this tension over the course of the 16 dialogues with students as to how the feed-forward should be provided to students, not knowing which was better or offered an increased chance that students would use the feed-forward. When offered as a suggestion, it still allowed students the agency and ownership over their own learning, something I valued as part of my conceptual framework and the way feedback is co-constructed. However, as I moved through the dialogues and there were repeated areas of feedback that came up with the same student, I worried that I was not explicit enough in asking them to try out some these strategies we discussed during the dialogue and wondered if this is why students were not using them on the next assessment.

Perhaps what I did not have time or the space to notice and process during the dialogues was that even though the dialogues focused on some of the same strategies over multiple dialogues, this was not necessarily a failure of the feed-forward. Rather it was an indication that students needed more time with these mathematical ideas and strategies before they were able to use them independently. In other words students were in the "Zone of Proximal Development," (Vygotsky, 1978) able to use strategies in the presence and guidance of a "more knowledgeable other," but not yet on their own.

Related to this tension of suggestive feed-forward or directive feed-forward was a noticing during the analysis of the post-assessment. As mentioned earlier, in all of the dialogues with Kate, some of the feed-forward focused on using the strategy of ½ as a benchmark when comparing or estimating fractions. On Kate's post assessment, she applied the strategy of using ½ as a benchmark on all tasks even when it was not an effective strategy. In some cases, she used one strategy but then went on to explain how you could also use a half to compare the fractions. It was as if she went out of her way to include thinking around ½ which resulted in her applying some incorrect whole number thinking. This was of course alarming when I analyzed her assessment and I found myself asking the questions "did I over emphasize the use of this strategy, or was I too direct in telling her to use this strategy?" (C. Fagerlund, 6 February 2021).

Out of the four dialogues with Kate, I used directed feed-forward in dialogue 2, and used suggestive feedforward in all the other dialogues. While it would be difficult to determine all the ways in which the phasing of feedback might affect the way students use the feedback and is certainly out of the scope of this study, it does bring up a larger issue which is concerned with the feedback messages that students take away. Regardless of whether I suggested the feed-forward or took a more direct approach is stating what I wanted students to work on, feedback can send messages to students whether those messages were intended or not (Hargreaves 2012). In the example with Kate, I had not intended for her to use the strategy around ½ on every single problem, but it is apparent from her post-assessment this is likely the message she took away.

A similar noticing related to unintended feedback messages also came from Cameron's post-assessment. In all of the dialogues with Cameron, much of the feedback

centered around improving his written explanation or reasoning for the assessment tasks. However, on his post-assessment he used all pictures under the space designated for students to write their explanation. When I went back through the dialogues, I noticed in Dialogue 3 at the very end I mentioned how I could see his thinking from his picture and emphasized mathematicians can explain things in different ways.

> T: *And I could tell from your picture right because you could also explain it through your thinking, your umm picture as well. Because we've said, Mathematicians can explain things in different ways and we're just trying to work on multiple ways to do it. So you've been working on your writing part and you said you felt like that was getting better. And I think it's getting better too. (Cameron Dialogue 3)*

It is possible that he heard what I said about explaining through pictures and processed this as something he could do moving forward and therefore on his post-assessment included only pictures for his reasoning. However, the feedback I intended for Cameron was to continue working on improving his written justifications. I knew it was more difficult for him to write his explanations and this is where I centered my feedback in order to move him forward in his learning. Notice in the excerpt I call this out by saying "you've been working on your writing part." It would seem though, that Cameron made sense of my feedback message in a different way that allowed him to show his thinking through pictures which is something he was more comfortable doing and was easier for him. This is perhaps another example of the unintended messages students receive from the feedback.

Maintaining Active Co-Construction

Earlier in this section, I pointed out that the end of the dialogue was the least co-constructed. It was evident that students played a more passive role during this part of

the dialogue and this perhaps relates to the unintended messages students took away from the dialogue. Higgens et al. (2001) write that "instead of asking if the student will take notice of feedback or whether it relates explicitly enough to assessment criteria, or whether the quantity is sufficient, we should be asking… how the student understands the feedback (how they make sense of it)" (p. 273).

As the feedback giver, it was unclear what feedback students took away or valued from the dialogue and relates back to my concern through the dialogues of whether or not I was explicit enough in communicating the feed-forward. This suggests a valuable opportunity at the end of the dialogue for students to reflect and share what they believe or understand their next steps to be, an implication I discuss further in Chapter 5.

The examples with Kate and Cameron around the possible unintended messages they took away from the dialogue reiterates the importance of the active role students must have in co-structing the dialogue. Confrey (1995) warns us that without active construction on the part of the student, the interaction can result in the student taking on the "other" perspective with weak connections to their own viewpoints and understanding. It is possible that Kate understood the importance of using the fraction ½ as a strategy to compare and estimate other fractions as a result of our feedback dialogues without a strong understanding of when to apply this strategy and for what type of tasks. I shared the active role students played during the beginning of the dialogue as they engaged in acts of self-assessment and during the middle as they built understanding around the math. The findings from the section remind us that there is a constant need for students to actively participate by sharing their understanding throughout the entire dialogue, even at the end when the dialogue is closing.

Additionally these findings emphasize the well-known challenge in communicating feedback that is understandable and used by students moving forward. It reiterates the need for clear and explicit feedback, while also posing a challenge in not being overly direct and giving space for students to take ownership over their learning

Pedagogical Noticings of In the Moment Feedback

Throughout this chapter, I have discussed the features and characteristics that occurred in the beginning, middle and end of the dialogue. This provided a framework that was helpful when thinking about how a teacher might be intentional in the structure of the dialogues and the key moves they hold onto as they engage in feedback dialogues with students. With that said, there are two additional noticings that did not fit cleanly in the framework of beginning, middle and end, but are equally important to consider and further implicate the dynamic nature of co-constructed feedback dialogues. I first discuss the unpredicted moments in the dialogue when feedback deviated from focusing on the student's performance and mathematical thinking. Second, I dive deeper into examining and describing what it meant to co-construct the dialogue with students by leading and following.

Feedback Beyond The Tasks

I identified this first theme as *feedback beyond the tasks* which draws on the noticing that sometimes the feedback did not always focus on the math content. In other words, there were moments of feedback that did not have to do with the actual tasks on the assessment. Instead the feedback focused on other learning behaviors or math identity issues. I describe two examples of this occurring in the dialogues below.

In the first example, K shared that she had to go fast on a particular part of the assessment because the class was finishing up or moving on. When I asked her what she could do if she needed more time, she did not know. My response included two pieces of feedback. On the first part, I suggested that next time she can ask whoever the teacher is for more time. This was an actionable piece of feedback she could use moving forward. In the second part of my response, I referenced previous conversations we had in class that indicated to students that mathematicians are slow deep thinkers. The intent of this feedback was to reiterate to K that it is ok to need more time to solve the problems and to debunk the common myth that to be good at math you have to be fast.

K: *But I had to go fast on this part because we were finishing up*

T: *You had to go fast on this part… because… ohhh, ok, so you felt like you kind of ran out of time. OK, well, what could you do if you feel like you're running out of time? You feel like you need more time. What can you do?*

K: *I don't know*

T: *So you could always just ask whoever the teacher is to say, can I get a little bit extra time because we don't want you…Remember we've talked about mathematicians are really slow, deep thinkers. And so we don't want you to have to worry about rushing and trying to finish it*

Another example of feedback unrelated to the tasks occurred in the dialogue with Mary as she reflected about her performance. Through a sequence of questions posed by me, as well as Mary's responses, it was illuminated that she believed she was bad at math because it is harder for her than reading. Similar to the example with Kate shared earlier, the feedback I gave focused on common myths around what it means to be good at math. I used this opportunity to remind the student of some of the discussions we had previously in class about how our brains grows when things are difficult. I wanted to

reinforce the ideas from Carol Dweck's work on Growth Mindset (2006), which emphasizes that we are not born with a set level of intelligence and that our brain can grow as we learn new skills and make mistakes.

M: I dunno Because I am bad at math

T: Why do you think you're bad at math?

M: Because I know reading is my best subject of school.

T: And what makes you think reading is your best subject?

M: because it's easier for me than math.

T: Oh, so you're thinking because math is harder, that means you're not good at it?

M: (Nods)

T: Ohhh. Have we talked about, though, that sometimes things are hard and that means our brain is growing. And so when things are hard, that means we are learning. And there's lots of people, mathematicians that have to work really hard at it because that's what your brain is doing. It's growing when you're learning

Before engaging in the dialogues with these students, I did not anticipate that there may be these moments where my feedback would deviate from discussing the tasks and the math strategies. However as teachers, if we are actually open to listening and co-constructing the dialogue with kids, we will learn from them what we could not have anticipated. We can go into a feedback dialogue with an intent and end result in mind, but must also provide space for other areas of feedback to surface that may not be concerned with the assessment. These are the moments where the teacher is following the student, giving space and responding to where the student takes the dialogue.

As this theme came to light during my analysis, I was able to step back and notice places where I missed opportunities to provide feedback unrelated to the tasks, but were important areas of learning to follow up on. This occurred in a dialogue with Cameron when I asked him what was difficult about a problem he did not finish answering. He

responded by saying, “it was getting super loud in the classroom so I couldn’t concentrate really.” My response moved into discussing the task, but if I could go back I would have slowed down and provided feedback around strategies Cameron could have used when unable to concentrate due to noise. Had Cameron had some strategies to know what to do or how to address the problem when it was getting too loud in the classroom to concentrate, the outcome of his assessment may have been different. Therefore when engaging in feedback dialogues, the feedback cannot only be concerned with the math

Even though I did not always slow down and spend a lot of time in the dialogue about these broader concerns of math identity and learning behaviors because the dialogue was focused on the influence on mathematical thinking, it is important to note that these opportunities occurred and the space to address it was important and necessary. The intent of feedback is to move the learner forward in their understanding and a dialogue opens the space to move the learner forward in other ways than what was predetermined based on their performance. This, however relies on the pedagogical assumption that the teacher is *really* listening and following up on opportunities. This ties to my second theme discussed below, that focuses on leading and following as a means to co-constructing the dialogue.

Co-construction as a Balance of Leading and Following

Research around dialogic feedback has emphasized the importance of moving away from viewing feedback as *telling*, which tends to position students as passive recipients of knowledge. Instead, recommendations for dialogic feedback focus on the co-construction of the dialogue between teacher and student. As part of this study, I sought to understand what co-construction looks like in a feedback dialogue with

elementary students in mathematics. During the analysis, I wanted to open up the roles both the student and I played during the dialogue, and how these roles contributed to co-constructing the feedback.

I reexamined the dialogues to notice where, when and how students were playing an active role and when student were passive in the conversation. The same was true for the way I examined my role as teacher. As I moved through this part of the analysis, I found that the active/passive dichotomy did not capture the back and forth relationship that occurred in the dialogue. As I examined the moments in which one of us had a more passive role, it did not feel as though it was completely passive or that one person was an inactive participant. Instead I found that in any one moment while one person was leading the other person was actively following creating this reciprocal back and forth relationship throughout the dialogue. In this next section, I discuss the ways in which the roles of leader and follower were shared in the dialogue between the student and myself. I intentionally placed this section towards the end of chapter as it draws on some of the themes and noticings discussed earlier as part of the features in the beginning, middle and end of the dialogue.

In order to better communicate what I mean when I say the reciprocal back and forth relationship between leading and following, I have included two different excerpts from the dialogues. I use the examples in order to highlight how the reciprocal moments of lead and follow were set up and carried out in the dialogue.

Table 22 Example of Authentic Questions from the Dialogues

Chip Dialogue #1	*T:*	*What do you think? How do you think you did?*
	Ch:	*umm Pretty good. I think it's improving*
	T:	*You think it's improving. Why do you think it's improving?*

Mary Dialogue #1	*T: Can you tell me about some of the things that you were thinking about, how did you start?* *M: I started so I was looking at one-twelfth I was like, well, that's one small because it's only you only need 11 more to get to 12. So I thought it's just the smallest. So then I said, 3/4 for that one because because umm it was one more piece of one fourth piece away from the fourth being a whole. So and then I just put that there. So and then I put four ninths because it only needs five more pieces and then seven fourteenths because it's… it's like a half because 7 plus 7 is 14 so that I just put that there..* *T: OK. So it's interesting that you talked about that half again and you talked about seven fourteenths as being a half.* *M: I noticed that one was a half* *T: You noticed that one was a half. I'm wondering four-ninths. Do you think that's greater than a half or less than a half?*

One of the important characteristics of this reciprocal lead-follow dynamic was the way each of these moments were often initiated. In both of the excerpts, I asked each student an open-ended question, and at this point, I am leading the dialogue. In the first example I am asking the student about their own thoughts on their performance and in the second example I ask the student to share what they were thinking about as they were ordering fractions. What is similar in both of these examples is that fact that I asked a question to the student in which I did not have the answer to. Nystrand and Gamoran (1991) call these *authentic questions*—these are questions that teachers do not have a prespecified answer to. An important feature of these questions is that they indicate to students that the teacher is interested in what the students think rather than whether students can recall information.

In the excerpts above I use some variation of the phrase "what do you think." This was not the only way these authentic questions began. Examples of other authentic questions include "what do you notice, can you tell me what you mean by that, or how did you decide that." When I asked these questions, because there was not a prescribed answer I was looking for, it set up the opportunity for the student to lead and influence

the path of the discourse and as result position me as follower in this moment of the dialogue. Of course authentic questions were not the only types of questions I asked. In many cases, depending on how students responded to these authentic questions, I led the dialogue with a follow-up question that was more specific in helping students notice, draw attention to, or reason about the mathematical understanding I was trying to move the student towards. I discussed these instructional moves earlier in the chapter during the middle of the dialogue. Therefore, authentic questions were important in setting up the exchange of roles of leader and follower between the student and teacher but were certainly not the only questions used throughout the dialogue. However, I would argue that authentic questions alone did not and *cannot* guarantee that the teacher follows. One could invariably ignore what the student says and proceed with their own agenda. This point leads us now to the importance of responding to students in the dialogue.

The response to the student becomes a critical point of analysis in whether or not the student was actually leading and I was actually following and relenting some control over the dialogue. If you refer back to the excerpts from Table 22, notice after students responded to an authentic question, my next question included part of a student's response. Collin (1982) calls this *uptake,* a process in discourse where one speaker builds on the contribution of another speaker. In the examples with Chip and Mary, I actually quote what each student said as part of my response, though this does not have to happen for uptake to occur. Uptake has been shown to increase student engagement in classroom discourse, a contributing factor to academic achievement (Nystrand & Gamoran, 1991). The use of uptake has been more often studied in English or reading lessons, although

revoicing, a particular form of uptake is a common move emphasized in leading mathematical discourse (Franke, Kazemi, & Battey 2007; O'Connor & Michaels, 2009).

As I used uptake as a discourse move in the dialogue, it meant that I was actually listening to what the student was saying and following them in the dialogue as I crafted my next question, feedback or instructional move. I would be remiss if I did not point out that in order to do this well, I needed the pedagogical content knowledge about how children think and make sense of fraction concepts in order to follow their thinking and move them forward in their learning progression. This also meant being open to following students in ways that I could not have predicted. Recall the example I discussed earlier where one of the students shared her belief about being bad at math. In this moment, it was important to follow this student as she shared this belief and perception about her ability in mathematics. During that moment, I took the time to remind the student about growth mindset characteristics and what it meant to be good at mathematics. Likewise, I also discussed examples where I missed opportunities to address other areas of feedback that I had not anticipated and moved on in the dialogue, without following up on what the student was saying. In essence, these were moments where I continued to lead in the dialogue and overlooked an opportunity to follow the student.

The use of authentic questions and uptake in the dialogue help to describe the reciprocal back and forth relationship between leading and following. The use of authentic questions were important because it positioned students as leader in the dialogue. These types of questions are essential given the power dynamics between teacher and student, the student is far less likely to ask questions (Nystrand & Gamoran,

1991) or lead in dialogue unless given the invitation to do so. Subsequently, it is up to the teacher on whether or not they follow the student's lead in this moment by drawing on the student's response in the following question, statement, suggestion or directive.

Conclusion

This chapter presented the results of the feedback dialogues with four 4th grade students focusing on student performance and understanding of fraction ideas as assessed through formative assessment tasks. The framework of beginning, middle and end of dialogue was used to describe how these feedback dialogues took shape and to open up the consistencies happening in each of these parts of the dialogue. Additionally, a closer look at the middle of the dialogue highlighted the characteristics and moves that prompted a change in students' mathematical thinking. Finally, additional pedagogical noticings were shared and discussed which were important in understanding the co-construction of dialogue between teacher and student. In the next chapter the results will be extended along with a discussion of the implications and limitations of this study. Areas of future research will also be shared.

Chapter 5: Discussion

Instead of feedback enacted as a one-way process from teacher to student, recent research and literature has suggested that feedback move away from a monologue view and towards an interactive, co-constructed dialogic view of feedback (e.g. Askew 2000; Molloy & Boud 2013; Nicol, 2007, 2010). Although there have been recommendations and conceptual frameworks (Nicol 2006; Yang & Carless, 2013), much of this has centered around Higher Education and does not consider the nuances of working with elementary students. Additionally, much is still unknown about the actual interaction that students and teachers have around the feedback.

The purpose of this study was to look deeper at how feedback, with regard to specific mathematical tasks, is co-constructed with students. In this chapter I first provide a brief summary and discussion of the findings. Next, I present and discuss the implications from this study followed by areas for future research. Finally, I close this chapter by providing concluding remarks.

Summary and Discussion of Findings

As discussed in Chapters 3 and 4, during the analysis phases of this research it became noticeable that there were consistencies across the dialogues that related to how the dialogues began, how they progressed and how they ended. The use of the framework Beginning, Middle and End, was useful in order to answer Research Question #1, which aimed to describe how the feedback dialogues took shape. Additionally, the findings from each part of the dialogue (Beginning, Middle, End) directly relate to the implications of these dialogues and can help teachers and teacher educators to be more

intentional in how they conceptualize and carry out feedback dialogues with elementary students.

Before I discuss and summarize the findings from the three parts of the dialogue, I want to reiterate a point I shared in Chapter 3. An important component of the feedback process is the design of the task used in order to initiate the feedback dialogue. The importance of having a strong task or assessment that elicits student thinking is critical when providing valuable dialogic feedback. This point has become more salient as I reflect on these dialogues I carried out with students. Without tasks that included multiple pathways for solving and enabled students to show their reasoning, the opportunity for feedback and dialogue would have been potentially limited to a mere evaluation of performance. This is perhaps why there has been less research around dialogic feedback in a mathematical setting, as it depends greatly on the types of questions and tasks teachers and curriculum pose to students to solve. Currently, many traditional math curricula emphasize computation over reasoning and conceptual understanding. The depth of each feedback dialogue in this study would not have been possible without first starting with strong formative assessment tasks which were aligned with *The Rational Number Project* (Cramer, Behr, Post & Lesh, 2009) and were shown to be productive in revealing student thinking around fraction concepts and could be solved using multiple strategies. This is something important to consider as teachers are making decisions about what assessment tasks they use in order to provide dialogic feedback and whether or not a task requires dialogic feedback.

The Beginning

In the beginning of the dialogue students were invited to reflect and participate in acts of self-assessment. With an initial prompt to students about their performance, students shared how they believed they performed on the assessment and attributed a mindset, a strategy, or a math practice as a reason for their performance. An effective component of the feedback process is that students can identify the gap between their own learning and the expected learning outcomes (Hattie 2007; Sadler, 1989) The beginning of the dialogue was a key moment for this part of the feedback process to happen as students were given the first opportunity to reflect and evaluate their performance, compare their understanding to prior learning and assessment and to consider the effectiveness of particular strategies used.

Additionally, we saw examples of students identifying a goal related to the assessment and sharing their performance in relation to this goal. Recall the examples with Cameron and Chip. Both of these students suggested their performance was related to whether or not they had good reasoning and explanations for their answers. They understood that performing well meant not just having the right answer but that it was also important to communicate a strong justification or reason for their answer. It was unclear as to how students identified that this was a goal of the assessment as this was not ever explicitly shared with students throughout the study. However, it was noticeable that as the students and I dialogued about specific tasks, I often referenced the importance of having good reasoning and during the middle of the dialogue when applicable, the feedback focused on building an understanding of what it looked like to have good reasoning. When students are able to identify and assess where their performance is in

relation to the learning goal, they are better suited to requesting feedback that is valuable and meaningful to them. Dialogic feedback takes time, which is often a concern from educators at all levels. Secondly, as students request feedback, it expresses to students that they have some agency over their learning and what feedback they receive. This latter points led to the second finding that occurred during the beginning of the dialogue in which students helped determine the agenda.

It was evident from the data that students generally had a strong awareness of areas they needed more support with and when given the opportunity, selected tasks from the assessment they wished to discuss during the feedback dialogue. This allowed students to co-craft the agenda as they chose problems to dialogue about first while still allowing me to add in feedback around additional areas not suggested by the students. Note that this happened after the first round of dialogues only after reflecting in my journal on being intentional about allowing students to select areas for feedback moving forward in the remaining dialogues. In some instances, students chose problems they were confident about to perceivably verify they had the correct answer. Often students selected tasks that they were confused about or did not believe they answered the question well. This was an important moment to recognize as students were trusting and willing to share their shortcomings with me, an evaluator of their work. We know that in order for feedback to be effective, trust from the student perspective is an important determinant of whether or not the student values the feedback or believes the assessor has the student's best interests in mind (Yang & Carless, 2013)

The Middle

As shared in Chapter 4, I broke apart the middle into three sections. The first part of the middle involved creating a shared context by reading what students had on their paper. This was the entry point into this middle part of the dialogue and a way to build the feedback off of what the student had already done. After the students or I read aloud what students had answered on the assessment, two acts generally followed. In some instances, students recognized a mistake on their own. This was a powerful moment for students to lead in the dialogue by then asking questions or recognizing their own misunderstanding. The other act that occurred after the student answer was read aloud was a clarification of the student's thinking. I often asked questions such as "what do you mean by…" or "can you tell me more about your reasoning?" These questions brought forward additional information about the student's thinking and allowed me to generate feedback that was contingent and responsive to their understanding.

The next part, what I called the middle of the middle of the dialogue, consisted of the instructional moves in order to build new understanding. This is where I examined the instructional moves that provoked a change in mathematical thinking. I found that many of the instructional moves were tied to the Standards of Mathematical Practice (CCSSM, 2010). This may come as a relief for some teachers as it suggests that the use of good teaching practices for mathematics can be carried over into feedback dialogues and not another practice they must learn. These findings further suggest that instructional moves during feedback dialogues that help students model, justify and communicate their thinking can work to build new mathematical understanding when working with fractions.

The final part of the middle of the dialogue can be summarized as taking action with the feedback. I provided students with the time to essentially 'do something' with the feedback- as without action, feedback has little impact (Sadler, 1989; William, 2011). During this part of the dialogue students had a chance to edit their responses and apply the feedback. In some of the dialogues, when students were unsure of how to edit their response, they asked for clarification or I continued to scaffold the feedback. This is a benefit of dialogic feedback as it provides opportunities for students to further question how the feedback can be used moving forward (Nicol 2010) leaving less of a chance for the feedback to be discarded.

The End

The end of the dialogue was the most brief in terms of content as well as overall exchanges in dialogue. During analysis, I used Hattie and Timperley's (2007) model of feedback which focuses on three components: *feed-up* (where am I going), *feedback* (how am I progressing), and *feed forward* (where to next). I found that I most often provided feed-forward and differentiated between feed-forward that was more suggestive in nature vs directed feed-forward which explicitly called out what I wanted students to do moving forward. Throughout of the dialogues I felt a tension between deciding what was better, suggestive feedback that still honored the student's ability to make decisions about their learning, or directed feedback that pushed student's to think in a new way. However, as I have reflected on this, I came to the conclusion that rather than pit these types of feed-forward up against each other, it is indeed necessary to have both types of feed-forward. In some instances, we might suggest ideas or strategies that students can use moving forward as there are multiple paths to the desired outcome. In other cases, it

may be necessary that we are more direct and explicit with students about how they are to move forward. I argue that the type of feed-forward we provide is based on many factors including where the student is in the learning process, and the inherent nature of the tasks or outcome that is desired. An example of this can be seen by looking back at two students from this study. Chip, a student who typically performed at grade level was able to effectively use various strategies with comparing and estimating fractions. Our dialogues focused around refining his written explanations and we discussed suggestions to help him explain his thinking more clearly.

This was different than the conversations I had with Kate who typically performed below grade level and was not yet able to accurately compare fractions. She often drew a picture but this strategy became unreliable when the fractions were bigger or had an odd number of total pieces. Here, I wanted Kate to continue working on using the strategy of identifying fractions that are close to a half as a way to compare and was more direct in encouraging her to use this strategy. Perhaps I was more direct also because I understand that students can grow more when they embrace challenges or strategies that are beyond what they are comfortable with (Dweck, 2006).

Both Chip and Kate were at different places in their learning, which is something teachers must take into consideration as they provide feed-forward. However, from this study it was difficult to know which offered a better approach because there were examples from the data in which students misunderstood or did not use the feed-forward on subsequent assessments. This leads to a second noticing related to the messages students take away from the feedback.

As I analyzed how student's thinking and responses changed across the four formative assessments and in particular on the final post-assessment, I noticed places where it seemed that students misunderstood the feedback we previously dialogued. This brought to light a missing piece in the dialogue which I believe is necessary and essential to the student's use of feedback. After providing feed-forward to students, there must be an opportunity for the student to reflect and share what feedback they are taking away and how they might proceed forward. This provides an opportunity for the giver of feedback to better understand the message or perhaps unintended messages students take away from the dialogue and the ability to clarify how students can move forward in their progress towards meeting the goal or learning outcome. I suggest that we must continue to hear from the student throughout the entire dialogue, even when it seems we are nearing the end and provided the next steps.

Co-constructing as Leading and Following

In addition to the findings I shared around the 3 parts of the dialogue, (beginning, middle and end) I shared additional pedagogical noticings that were present throughout each dialogue as well as across dialogues. In order to better understand the roles both the student and I had in the dialogue and how those roles may or may not have led to co-constructing the dialogue, I shared examples in which there was a reciprocal back and forth relationship in the dialogue. I describe how the roles of leading and following were set up through the use of authentic questions and uptake. The use of authentic questions invited students to lead and control the dialogue, and in some ways presented an opportunity to follow the student and provide feedback that I did not predict. Examples of

this occurred when students shared information around their math identity or other challenges related to the learning environment.

In other parts of the dialogue I was leading with more closed questioning as a means to help students notice and uncover particular mathematical connections or conceptual understandings. Authentic questions are powerful as they allow the student to lead in the dialogue. However, sometimes students need the "support of closed questions to guide further articulation" (Boyd, 2015). In these moments I was leading often through a series of scaffolding questions and students were following as they provided and responded to these prompts. Theorists of dialogic feedback focus on the co-construction of dialogue. In order to better describe what it means to co-construct, I propose one way to visualize co-construction is the constant exchange of the dual roles both the student and teacher have in the dialogue as leader and follower. In the next section, I further discuss how educators might invite and explicitly teach students how to co-construct feedback.

Implications

Based on the findings discussed above, this study provides three implications that are important in helping teachers and teacher educators conceptualize and carry out feedback dialogues in Elementary Mathematics.

Implication #1: Inviting and explicitly teaching students how to co-construct feedback dialogues.

The findings from this study suggest that it is important to invite students to lead in the dialogue as well as explicitly teaching students how to co-construct feedback dialogues. Before unpacking this implication, I want to take a moment to reflect on my

own experience in feedback dialogues as a doctoral student first starting out in a Ph.D. program and moving to the final phases of completing this dissertation. I had the fortunate opportunity to participate in multiple feedback dialogues during course work as well as the preliminary examination and dissertation phases of my own doctoral research. When I think back to my very first dialogue, I can remember dialoguing about a paper I submitted and mostly listening to what my advisor was noticing and pointing out, and answering questions when I was prompted. I was participating in the dialogue but constructing very little of it.

If I compare my first dialogue as a doctoral student to my most recent dialogue with my advisor, there are several key differences. The first is that in the most recent dialogue, I began by leading the dialogue, providing areas that I was struggling with in my writing and providing information about what I was thinking about and what I thought understood. I had an active role in determining what would be discussed during the dialogue and specified what I needed support with in order to move forward. Furthermore, as the dialogue continued, and I received feedback I was able to recognize when I did not understand the feedback and request further clarification. Alternatively, when I felt like the feedback didn't match what my intentions were, I was able to further clarify what I was intending to do in my writing in order to get feedback that was more valuable to me and the work I was trying to do.

A question to consider related to both this study and my own experiences is how does one move from being a mostly passive recipient of information to an active co-constructor of feedback? As a doctoral student engaging in feedback dialogues as well as

a researcher studying feedback dialogues I suggest at a minimum, the three 'R's that need to happen in order for students to co-construct dialogue. Students need to be able to:

- Recognize the Gap
- Request Feedback,
- Respond to Feedback

Recognizing The Gap. Scholars have suggested an important component of the feedback process that must occur in order for feedback to close the learning gap is the understanding a student has about where their current performance is in relation to the desired learning goal (Hattie, 2010; Sadler, 1989) I would add to this by suggesting that before one can understand the gap between their understanding and the desired outcome they first have to be able to recognize the gap. If I refer back to my own experience and reflection with feedback dialogues as a doctoral student, one of the things that contributed towards my growth in being able to co-construct feedback from a student perspective was my ability to recognize and understand the gap between where I was and where I needed to be. To provide an even more concrete example, when I engaged in recent feedback dialogues I understood the criteria for presenting research findings and ultimately what the desired outcome was intended to look like.

The data from the study presented similar findings. As discussed in Chapter 4, we saw specific examples of when students were able to recognize the gap between their performance and the desired outcome. Recall the example with Chip as he projected being bothered by his performance on one of the tasks. "*But I can't umm--like -- I can't really explain the reasoning right*." He had felt good about having the correct answer but struggled with having a strong enough reasoning. He understood the gap between what he had currently written down and the criteria for having a strong explanation.

I suggest one of the ways students are able to co-construct the dialogue is to recognize the gap between their current performance or understanding and the desired outcome as indicated by success criteria. This means teachers have to support and in some instances explicitly teach students how to recognize their gap by understanding the success criteria and how to compare their performance to the desired outcome. This process of when students compare their own work with the outlined criteria for success is a key component of self-assessment practices (Ruiz-Primo, Araceli, & Brookhart, 2018). In this study, as part of the feedback dialogues we saw examples in which the middle of the dialogue supported and worked towards building students understanding of what good reasoning and explanation looked like as a mathematician, which was a criteria for each of the assessments

Although a strength of dialogic feedback is the ability to further students' understanding of the learning outcomes or success criteria within the dialogue, in some examples, those dialogues lasted anywhere between 12-17 minutes. I understand that time is of the essence in any classroom and it is not feasible to engage in dialogues with every student lasting much more than a few minutes. This is why I believe then, that moving forward it is necessary to help students build an understanding of the success criteria and learning outcomes before the dialogues. This would serve as a catalyst to helping them recognize the gap between their learning and the intended final product. The use and development of success criteria is supported by recent literature which provides specific ways and examples of how to help students develop a clear understanding of success criteria (e.g. Clarke, 2021). If this process is done prior to the dialogues, it is possible that the efficiency of feedback dialogues will increase while the

time it takes will decrease. Students will enter in the dialogue having both an understanding of the desired outcome and can in essence recognize their "gap."

I further support this assertion by drawing attention back to an example in the data. Recall in many instances at the beginning of the dialogue when I prompted students to reflect on their performance and select tasks to discuss during the dialogue, students often had a strong awareness of their performance and the tasks they struggled with. In one dialogue in particular, Mary jumped immediately into the dialogue leading right away with the "*I wanted to talk about this one, because this one took a lot of thinking and because I had to use my faction pieces a lot on it*" (Mary, Dialogue 3). We quickly moved to discuss this task and the total length of the dialogue was approximately 5 ½ minutes. I believe the efficiency of this dialogue was influenced partly by Mary's ability to recognize her gap and as a result was able to specifically request feedback. This moves me to the next 'R' that we must support students as they learn to co-construct dialogues which is "Request feedback."

Requesting Feedback. As teachers support students in recognizing the gap, this has to be followed by helping students *Request Feedback* that will in fact help them take action to narrow the gap. When engaging in my own feedback dialogues as a doctoral student, after recognizing where my current understanding or progress towards the desired outcome was at that moment, I was able to request feedback to help move my work forward and close the gap. My ability to request feedback was pivotal in positioning me as a co-constructor of the feedback dialogue rather than a passive recipient. Having student's request feedback based on their own interests and understanding prior to receiving feedback has been encouraged by others in the field as a

way to shift the balance of power from teacher to student during the feedback process (Bloxham & Campbell, 2008; Nicol, 2009).

In this study, except for the one example with Mary, students only requested feedback after being asked to select tasks to discuss during the dialogue. This supports the implication of not only inviting students to lead, but also explicitly teaching students how to co-construct feedback and more specifically teaching students how to *Request feedback.* As a reminder, when students are requesting feedback, they are positioning themselves as co-constructors of the dialogue and are taking on a more active role. In a feedback dialogue, I suggest the work of facilitating the dialogue cannot only be on the teacher and thus reinforces the role students must play as a co-constructor. When students are requesting feedback they are ultimately helping to co-craft the agenda for the dialogue as I discussed in Chapter 4. This supports the assumption that learners do not solely need the input and observation from others, but that students are active agents in their own learning and can identify at least some of their own areas of improvement (Boud & Malloy, 2013).

As I mentioned earlier, time is limited, especially when educators have large class sizes, and many students to support. Helping students learn to request feedback provides multiple benefits for both student and educator. The first benefit being that even if educators only have a passing moment to provide some sort of feedback, if students are able to enter into dialogue with already having assessed their current progress and can request the type of feedback that is most helpful, the student and educator can quickly drop into the conversation allowing there to be just enough feedback and support to move students forward. Secondly, a student who is able to request feedback will undoubtedly

receive more feedback than if they only wait for it to be initiated by someone else. When students request feedback, this also provides a benefit to the teacher by giving her insight and information as to what a student perceives to be challenging and thus can inform future instruction.

Respond to Feedback. The last "R" that is important to help students learn to co-construct feedback dialogues is *Respond to feedback.* One of the frustrations and/or challenges when teachers provide feedback to students is that students do not always use it to improve their work. When students do not apply feedback they have received, it is possible they are just choosing to ignore it, however researchers have found that it is more likely that students are unable to make sense of the comments provided by the teacher (Hattie & Temperley, 2007; Higgins et al. 2001; Wiliam 2011). Consider for a moment a time when you have received feedback from an editor, reviewer or another scholar in the field. It is likely that many of us at some point have received feedback that we did not understand what the other person meant or how we were supposed to adjust our work given the feedback we were provided. Furthermore, we may have also received feedback that we disagree with because we do not believe the other person has a strong understanding of the work we are trying to do. These are just some of the ways in which feedback is at risk of being discarded which is why I suggest it is important to teach students how to *Respond to feedback.*

I use the word *Respond* to mean two different actions in the feedback dialogue. First, we need to support students to respond to feedback by clarifying their own understanding of the feedback. In Chapter 2, I suggested that feedback as a dialogic two-way process can provide space for the student to respond to the feedback and further

question how it can be used in their work. However, based on the findings from this study, although there may be space for students to respond and question the feedback, it does not guarantee that students will enter in this space. This is problematic as we noticed in this study sometimes students perceived the feedback messages differently than they were intended. Teachers need to support students in their ability to recognize when the feedback does not make sense or potentially align with what the student intended to portray in their work. Additionally teachers must help students understand that it is okay to question the feedback as a means for advocating for themselves.

The second way I used the word respond is to describe the actual response and action students must take in order to use the feedback to improve their learning. Using the feedback in order to move forward is critical to the impact feedback can have. We understand from the literature that feedback has the potential to be highly effective, however without opportunity to use the feedback, William (2011) describes this as a wasted effort on behalf of the teacher. Helping students learn to respond to feedback in terms of questioning or clarifying the information constructed during the dialogue as well as how to actually respond by taking action in order to apply the feedback cannot be overlooked.

Moving Forward with the 3 "R's". By supporting student's ability to Recognize the Gap, Request Feedback and Respond to Feedback, teachers are able to set the stage for engaging in a feedback that moves away from a transmission view of feedback to an active co-constructed dialogue. How teachers might invite or explicitly teach students how to do the 3 "R's" is further describe in the next implication.

Implication #2 Focusing on "HOW" to operationalize feedback dialogues

A second implication from this study is the importance of focusing on *how* to operationalize feedback dialogues. I would argue much of the literature available presents the "what" around dialogic feedback. Scholars have written and theorized about *what* dialogic feedback is, *what* the components of dialogic feedback are, and *what* opportunities for learning dialogic feedback can afford, as well as other areas that focus on describing what dialogic feedback is and is not. (e.g. Boud & Malloy, 2013; Nicol, 2010, 2006; Yang & Carless, 2013;).

I believe knowing the "*what"* around dialogic feedback is an important first step in helping teachers understand and move towards using dialogic feedback in their classrooms. However, I strongly believe that it is critical that we also focus on describing the "*how"* of dialogic. In my experience working with teachers, many express the strong desire and need for more explanation around *how* to enact suggested teacher practices from the literature. I, myself as an educator, also have had this desire to better understand *how* dialogic feedback occurs in the classroom with elementary students, and *how* to facilitate a feedback dialogue that supports students in co-constructing feedback. This ultimately led to this study. Recall, a goal of this research was to more fully understand *how* feedback dialogues with students in elementary mathematics take shape. Therefore, as part of my findings I described how the dialogues began, how they progressed and how they ended– and discussed what was important about each of these components. Now, I move to describing more specifically *how* teachers can invite, teach and usher students into the dialogues using the three "R's".

Inviting students to *Recognize the Gap*. As part of the first implication, I discussed the importance of helping students learn to recognize the gap between their current performance and the desired outcome. So now we must answer the question of how do teachers support students in recognizing their gap? Although it may seem obvious, one way to do this at the beginning of the dialogue is to directly invite students to reflect on their current performance. When students were asked about their performance in this study, students gave one of three responses: 1) as an indicator of their overall performance; 2) as progress made since the last assessment; or 3) level of performance on each task. As teachers have these dialogues in their own classroom, they may consider turning the types of responses students gave in this study into guiding questions. Teachers can ask students to reflect on their overall performance and more specifically how their performance compares to the success criteria, to think about how their performance has changed since the last assessment and how they performed on each task. A follow up question would be to then have students attribute a reason or use of strategy that led to their performance. These questions could be asked at the beginning of the dialogue as was the case in this study, however because of the limited amount of time teachers have to dialogue with students, I would suggest that these guiding questions could be given to students in advance to reflect and write about before engaging in a feedback dialogue.

As teachers continue to invite and coach students to recognize the gap, the goal would be that eventually students are able to self-assess without prompting. Additionally, I believe the above recommendations provide teachers with specific examples of the structured opportunities Nicol (2007) suggested are needed in order for students to get better at self-monitoring the gap between where they are and what the desired goals are. Finally, when teachers invite and support students to recognize the gap, students are better positioned to advocate for and request feedback that is responsive to their current level of understanding.

Inviting students to *Request Feedback.* A component of co-constructing feedback is the opportunity for students to request feedback that is valuable in helping the student move from their current performance or understanding to the desired outcome or learning goal. This differs from a traditional view of feedback in which it is the feedback giver that determines what feedback is and is not provided to the student. Teachers can move away from traditional feedback by explicitly asking students what areas of the assessment or task the student wants to talk about. Again, one might be thinking this is an obvious step teachers would do, however I argue that without making it explicit for teachers and being very intentional, this opportunity might be missed as it was during the first round of dialogues during this study. Even as a researcher who understood the importance of promoting student agency by allowing students to determine areas of feedback, I fell into a traditional role of feedback where the teacher leads and makes decisions about what areas of feedback to discuss. Although students still participated in the dialogue, they were not invited to lead. It took an intentional and conscious effort as I

moved forward to ask students to participate in co-creating the agenda for feedback until it eventually became an expected part of the dialogue for me *and* for the student.

Inviting students to *Respond to Feedback.* How teachers invite students to respond to feedback is an important set-up for students to take action with the feedback. Recall that in a feedback dialogue that is co-constructed, some of the feedback is constructed as a result of the new thinking and understanding students built as they engage in the instructional moves and scaffolding provided by the teacher. Therefore, at the end of the dialogue it is important for teachers to invite students to share their takeaways and the actions they are going to take moving forward. This not only provides an important opportunity for students to reflect on their new learning, but it also provides an opportunity for the teacher to better understand the messages that students are taking away. Secondly, as the teacher provides suggestive or direct feedforward, asking students what questions they still have, what their next steps are, or if there is something they disagree with, again invites students to state what still remains unclear. Given the power dynamic between teacher and student, without an explicit invitation, students may remain passive, assuming they are supposed to understand and agree with the feedback given to them. I would argue this would also be true of many feedback situations occurring outside of elementary schools in which there is an imbalance of power such as between a doctoral student and professor, an employee and manager, or a healthcare provider and a patient. Teachers need to provide students with an opening to clarify the feedback and then give students the opportunity to use the feedback. Teachers can do this by giving students time to modify and edit their current assessment or performance

task, or provide a new task that is similar for the students to apply or "try out" the feedback.

Implication #3 Teacher Education must continue to support building Teacher Pedagogical Content Knowledge

Teacher Pedagogical Content Knowledge (PCK) is a crucial component of feedback dialogues. I was hesitant to write about this implication as the importance of PCK is not a new finding nor has there been a lack of literature and discussion around it. However, I would argue that in order for teachers or educators to truly engage in Dialogic Feedback, they must have a level of PCK that enables them to understand how children's mathematical thinking develops and what to do to help students continue to develop their mathematical thinking. A central premise or argument for Dialogic Feedback is to be more responsive to the student and to increase the chances and impact feedback can have on learning. Pedagogical Content Knowledge includes knowledge of representing the subject matter as well as the knowledge of learners and their developmental conceptions of the content (Shulman 1987). The recognition on behalf of the pedagogue as to what the long term goal is, where students are currently in their learning, how children's thinking progresses from one concept to another and what instructional strategies or moves can be used to move students forward is essential to providing dialogic feedback.

In my experience working with educators, too often the feedback and instruction provided does not take into consideration what the student conceptually understands, and is instead based on the desired outcome without considering the learning steps in between. For example, in this study, I had to possess a strong understanding of how children develop fraction concepts to provide what I would now call "just right"

feedback. To me, *just right* feedback includes: 1) noticing what the student currently understands; 2) knowing the strategies that will help students further their understanding; and 3) determining the next step or two that can move them closer to being able to use these same pedagogical strategies, independently, in the future. Similarly, the feedback provided to a doctorate student on their writing during the first year of course work is drastically different than the feedback provided as they are writing a dissertation. Although the desired outcome in both instances is the same, to be able to think, analyze, apply theory and write about their own research , providing feedback that supports this outcome requires an understanding of the steps or progression one makes as they move towards becoming a writer of research.

Teacher noticing is an additional framework to consider when examining the important role teacher PCK plays during feedback dialogues. Teacher noticing, an important component of teacher expertise, "is at the crux of developing responsive interactions focused on students' ideas" (van Es & Sherin, 2021, p. 17). Teacher noticing as proposed by van Es and Sherin (2021) includes the teacher's ability to *attend to* noteworthy features of an interaction, *interpret* what they are noticing based on their knowledge of teaching and learning*, and shape* the interaction to gain additional information. Teachers must have the PCK in order to attend to and interpret specific aspects of students mathematical thinking and then shape the feedback dialogue by proposing the next question, or instructional move that will support the student's progress to the desired outcome.

The research on teacher noticing combined with what I have suggested throughout this dissertation with regard to feedback dialogues could potentially lead to

some interesting implications for professional development to support teachers in learning how to implement feedback dialogues.

Limitations

This study has limitations that must be considered when applying the findings described in this paper The first limitation of this study relates to limited data. Although this study captured the dialogue occurring specifically during the feedback dialogues, it did not capture the classroom discourse and learning in between dialogues. This study looked to identify characteristics around moments in the dialogue where students changed their thinking as well as to some extent how their thinking changed across all of the dialogues, however it is likely that the normal classroom discussion during math also impacted students thinking. Therefore this study does not look at the efficacy of the feedback dialogues on student performance of fraction concepts over time.

Secondly, student's perceptions of the feedback dialogues, a valuable component in understanding the experience and impact as seen from the student perspective, were not included in this study. This is a limitation as it missed the opportunity to understand how students receive the feedback and the decisions they make about using the feedback to improve their learning.

Future Research

The findings as well the limitations from this study have provided several opportunities for further research and work related to feedback dialogues in elementary mathematics. The most prominent I would suggest would be looking at feedback dialogues with students from different grade levels in order to describe how the dialogues vary and how they might be co-constructed with both younger and older elementary

students. It is likely the dialogues would vary in terms of how the teacher might scaffold and invite students to lead in the dialogue.

In addition to studying feedback dialogues with different groups of students, it is important to continue to focus on the *how* of feedback dialogues. Feedback dialogues are time consuming and so looking at how teachers can facilitate these given a classroom full of 20-30 students would be critical in terms of teachers engaging in feedback dialogues. Considering how a teacher might facilitate these dialogues in small groups would be important as well as contemplating when a feedback dialogue may or may not be necessary. I would argue that not every moment in the students learning continuum would require a co-constructed feedback dialogue, rather it is possible that in some instances corrective or descriptive feedback developed solely by the teacher is enough.

Lastly, a next step from this research might include increasing the scale of this study. It would be interesting to have a team of trained educators around effective feedback practices conduct these dialogues at a larger scale in order to generalize more broadly how feedback dialogues take shape and to the potential development of curriculum. This could be eventually followed by a study that assesses the effectiveness of the dialogues on children's mathematical understanding.

Closing Remark

When learning is viewed from constructivist and sociocultural point of views, knowledge is not communicated but instead is actively constructed by the individual in relation to his or her environment and interactions with others. Feedback has long been considered an important part of the learning process and therefore I believe must reflect how students construct new knowledge and understandings. Additionally, there is

extensive research in the general field of mathematics education that emphasizes the active participation and construction of new math ideas through discourse. Both of these perspectives help draw out the need for dialogic feedback that is co-constructed between teacher and student.

This study sought to focus specifically on dialogic feedback and how it might be constructed with students in 4th grade around fraction concepts. The findings described how the dialogues began, progressed and ended and showed how the specific use of instructional moves that relate to the Standards of Mathematical Practice can support students in building new understanding. This study also brought to light the importance of explicitly inviting students to lead in the dialogue, and the important role they must play in co-constructing the feedback. Moving forward, researchers must continue to focus on describing the *how* of dialogic feedback in order to move from theory to practice, thus impacting more students in elementary classrooms.

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Appendix A: Formative Assessment 1

Name:_____

Circle the larger of each pair or both if they are equal. Explain how you solved each problem.

			Explanation
(1)	$\frac{4}{5}$	$\frac{7}{8}$	2)
(3)	$\frac{8}{10}$	$\frac{6}{12}$	(4)
(5)	$\frac{4}{8}$	$\frac{4}{6}$	(6)
(7)	$\frac{2}{5}$	$\frac{9}{15}$	(8)

7. **Order the fractions from least to greatest**

$\frac{5}{8}$ $\frac{2}{6}$ $\frac{11}{12}$ $\frac{2}{9}$

_____ _____ _____ _____ _____

$\frac{3}{4}$ $\frac{1}{12}$ $\frac{4}{9}$ $\frac{7}{14}$

Appendix B: Formative Assessment 2

Ben ate $\frac{2}{6}$ of a pie for dessert on Monday. The next day he ate $\frac{1}{3}$ of the same pie. Did he eat more than half or less than half of a pie. Explain your reasoning

Serena ran $\frac{3}{4}$ of a mile before lunch and $\frac{1}{8}$ of a mile after lunch. Did Serena run more than a half mile or less than half a mile. Did she run more than one mile or less than one mile? Explain your reasoning

Circle the larger of each pair or both if they are equal. Explain how you solved each problem.

			Explanation
(1)	$\frac{8}{14}$	$\frac{3}{8}$	2)
(3)	$\frac{11}{12}$	$\frac{8}{9}$	(4)

Appendix C: Formative Assessment 3

Ben ate $\frac{5}{12}$ of a pizza for dinner. The next day he ate $\frac{1}{8}$ of the same pizza for breakfast. Did he eat more than half or less than half of a pizza. Explain your reasoning

Serena ran $\frac{3}{4}$ of a mile before lunch and $\frac{1}{8}$ of a mile after lunch. Did Serena run more than a half mile or less than half a mile. Did she run more than one mile or less than one mile? Explain your reasoning

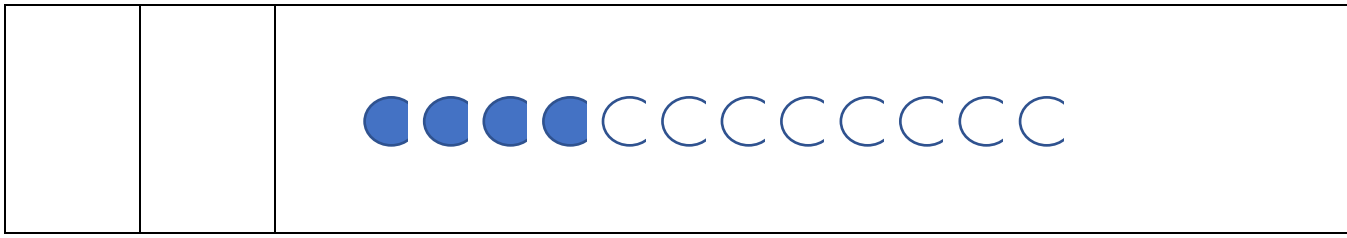
Circle the larger of each pair or both if they are equal. Explain how you solved each problem.

			Explanation
(1)	$\frac{6}{10}$	$\frac{4}{12}$	2)
(3)	$\frac{14}{15}$	$\frac{9}{10}$	(4)

Appendix D: Formative Assessment 4

Show $\frac{1}{4}$ with chips. Use 8 chips in all. Draw a picture of your display

Write 2 fraction names for the model below.



Mackenzie had two chocolate bars. On Friday she ate $\frac{3}{10}$ of the first candy bar. The next day she ate $\frac{3}{6}$ of the second candy bar. Estimate how much she ate all together. Did she eat more than 1 whole candy bar or less than a 1 whole candy bar? Explain your reasoning.

Appendix E: Post-Assessment

Name:_____

Circle the larger fraction of each pair or both if they are equal. Explain how you solved each problem.

			Explanation
(1)	$\frac{6}{8}$	$\frac{5}{12}$	2)
(3)	$\frac{11}{12}$	$\frac{9}{10}$	(4)
(5)	$\frac{4}{9}$	$\frac{4}{6}$	(6)
(7)	$\frac{3}{4}$	$\frac{6}{8}$	(8)

7. **Order the fractions from least to greatest**

$\frac{5}{6}$ $\frac{2}{4}$ $\frac{3}{8}$ $\frac{14}{15}$

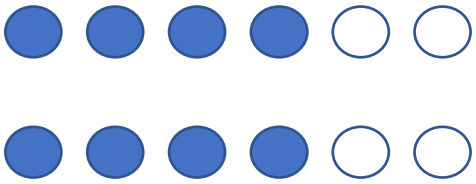
_____ _____ _____ _____

Lilly wanted to make two different kinds of cookies. For the first recipe she needed ate $\frac{4}{6}$ of a cup of sugar. For the second recipe she needed $\frac{5}{8}$ cup of sugar. Estimate how much sugar she used in all. Did she use more than 1 cup or less than 1 cup of sugar. Explain your reasoning.

Jackson ate $\frac{4}{12}$ of a pound of trail mix before lunch. He ate another $\frac{1}{3}$ of a pound of trail mix after lunch. How much trail mix did he eat altogether? Show or explain your thinking.

Show $\frac{2}{3}$ with chips. Use 15 chips in all. Draw a picture of your display

Write 2 fraction names for the model below.

Appendix F: Example Transcript (C2)

00:00 Teacher: All right, should we look at what you got here?

00:07 C: yeah. I wasn't able to finish explaining here and right here

00:08 Teacher: ok

00:09 Teacher: OK, so how do you how do you feel about it?

00:15 C: good

00:16 Teacher: Yeah, how about these ones?

00:21 Teacher: Yeah, and I noticed that you were able to really have a strong explanation for these ones, which I think.

00:31 Teacher: Has been an improvement since your last one. What do you think?

00:37 Teacher: Yeah, all right, so. Let's Talk about these ones, what was… talk to me about these two problems. What was hard about that?

00:46 C: It was getting supper loud in the classroom so I couldn't concentrate really

00:50 Teacher: ok

00:59 C: but I was on this one and then I started to get a headache and stuff

01:04 Teacher: OK so... umm.. how about this one? Let's talk about this one?

01:25 C: I I know 2/6 is a third--equals up to a third. And so i thought. And--. then i knew two thirds was more than a half

01:34 Teacher: ok

01:34 C: and I got my answer

01:59 Teacher: OK, so what what can you write here to explain your reasoning to did he eat more than a half hour or less than a half

02:14 C: um

02:14 Teacher: So what did you just tell me to?

02:25 C: two sixths equals a third.

02:26 Teacher: Ok

02:26 C: And...(pause) 2/3 is more than a half

02:26 Teacher: Where do you get two thirds from?

02:27 C: from the two sixth and the one third.

02:33 Teacher: OK, so you get so, you know, two six is equal to a third. And if you add another third,

02:42 C: (Starts writing)

02:42 Teacher: is that what you saying? OK.

03:34 C: (continues to write)

04:45 Teacher: Ok so two sixth plus one third equals two thirds. And how do you know that again?

04:49 C: cause...2/6 equals up to one whole.

05:00 Teacher: Oh, OK, so that part, I think you can add to your explanation because you told me that part, but then when you went to write it, you left that part out. So why don't you just kind of put it like an arrow down here? And so you can start with I know that...

06:25 Teacher: I know that 2/6 equals one third (reading student response)

06:28 Teacher: OK, let's talk about this one, down here. So you started this one. One, so it's more because two 4ths is the half and three is bigger (reading student's response)

06:43 Teacher: Three what?

06:54 C: um three fourths

06:55 Teacher: ok make sure you add that in there

07:04 Teacher: ok and then how do you know it's less...you wrote less, but now you're just got to have your reasoning. So how do you know that it's less than one?

07:15 C: because...(opens up the fraction circles and shows 3/4 and one eight) so...

08:03 C: So this is what it would look like.

08:08 Teacher: ok

08:08 C: And you need one more eighth to get a whole.

08:15 C: So. And. Or you could also put you need one more fourth to make a whole.

08:27 Teacher: OK. All right, so you need one more fourth to make a hole, but what do you have?

08:34 C: Three fourths and one eighth

08:38 Teacher: And is one eighth More than a fourth or less than a fourth?

08:41 C: Less

08:41 Teacher: OK? So how are you going to write that?

08:44 Teacher: What do you think? So when you're thinking about your writing, this is what you're working on and thinking about.

08:51 Teacher: You've got the answer. But now you're trying to think about how am I going to convince somebody? How am I going to explain this to somebody? I want you to kind of almost be just what you just talked about. But now go back to the beginning with the first part that you're going to tell somebody.

09:08 C: (Starts writing)

09:45 Teacher: So what's the first part you're writing down

09:49 C: three fourths and one eighth is less than a whole...by one piece...by cut one eighth piece

10:02 Teacher: by... There you go.

10:11 C: (starts writing)

10:58 C: I am messing up

11:00 Teacher: OK, so now I want you to reread it, to read it, read aloud to yourself.

11:11 C: (Reads to self quietly)

11:16 Teacher: Will you read it out loud so I can hear?

11:20 C: Three fourths, and. Is less. Three, fourth and one eighth is less by one eighth piece but i was messing up

11:36 Teacher: That's OK. So is less.

11:40 Teacher: If I were to ask you a question, what might I ask you?

11:54 Teacher: Is it less than two.

11:57 C: Two wholes?

11:58 Teacher: yeah

11:58 C: yeah

12:00 Teacher: is it Less than one?

12:02 C: yes

12:02 Teacher: is it less than a half?

12:02 C: No

12:04 Teacher: So what could you add to make it really clear?

12:09 C: It's not less than a half but it's over half.

12:11 Teacher: It's over a half. So you talked about that up here. So what are you trying to say down here that it's less than? By one eighth piece?

12:21 C: a whole

12:21 Teacher: it's less than a whole.

12:29 C: (Starts writing)

12:34 Teacher: You know what you can do if you want to add, like you reread it to yourself and you say, oh, I don't want to add this little piece. You got it. You have an arrow. And then. Add what you want them to say, oh.

12:59 Teacher: We read it again and then figure out what you want to add to it.

13:49 Teacher: OK.

13:51 Teacher: So I want you to keep working on your explanation.

13:54 Teacher: All right, you really started to think about how are you going to write this? OK, and now what I want you to think about is if somebody else were to read this, would they understand what I'm talking about here? And and so reading that out loud to yourself again, helps see if you missed anything. Oh, wait. I might need to add in this piece.

14:18 Teacher: OK. all right you are all set my friend.