THE SPECTRUM OF RESTRICTED RESOLVABLE DESIGNS WITH $r = 2$

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Abstract. We construct restricted resolvable designs for the remaining eighteen orders, thus completing the spectrum for these designs.

1. Introduction. A **restricted resolvable design** $R_rRP(p,k)$ is a resolvable edge-decomposition of the complete graph K_p into K_rs and K_{r+1}s in which there are k parallel classes. These designs arise quite naturally in the consideration of certain covering problems (see [8]). An $R_1RP(p,k)$ is just a proper k-edge colouring of K_p and so exists whenever $k \geq p$, or $k = p-1$ and p is even. Necessary and sufficient conditions for the existence of $R_2RP(p,k)$, covering all but eighteen values of p, have been determined in a recent series of papers [6,7,8]:

THEOREM 1.0. *Let* $p > 1, p \notin \{31,35,37,41,43,47,49,53,55,59,61,65,67,71,79,83,85,89\}$. *There exists an* $R_2RP(p,k)$ *if and only if* $\lfloor \frac{p}{2} \rfloor \leq k \leq p-1$ *and* $p(k-p+1) \equiv 0$ *modulo 3, with the following exceptions:*

(i) $p \equiv 1$ *modulo* 6 *and* $k = \frac{p-1}{2}$,
(ii) p *is odd and* $k = p-1$,
(iii) $p \equiv 3$ *modulo* 6, $p \neq 3$ *and* $k = p-2$,
(iv) $p \equiv 3$ *modulo* 6, $p \neq 9$ *and* $k = p-3$, *and*
(v) $(p,k) = (6,3)$ *or* $(12,6)$.

In this paper we show that the restriction $p \notin \{31,35,\ldots,89\}$ can be dropped from the hypothesis of theorem 1.0, thus completing the spectrum for restricted resolvable designs with $r = 2$. In section 2 we deal with the orders $p = 31,35,37,41,43,47$ and 59; in section 3 the orders $p = 79,83,85$ and 89; and in section 4 the orders $p = 49,53,55,61,65,67$ and 71. It will be assumed that the reader is familiar with the terminology, notation and constructions from [5,6,7]. For brevity, R_2RP will be condensed to RRP. We will make direct use of the following.

CONSTRUCTION 1.1. *([6, Construction 2.2]) Suppose that there is a resolvable* 2,3*-GDD of type* S *with replication number* k *and that for each* $s_j \in S$ *there is an* $RRP(s_j,k')$. *Then there is an* $RRP(\Sigma s_j, k+k')$.

CONSTRUCTION 1.2. *([6, Construction 2.4]) Let* (X,G,B) *be a* 2,3*-frame in which hole* G_i *has degree* d_i, $i = 1,\ldots,j$. *Suppose that for each* $i = 1,\ldots,j-1$ *there is an*

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$RRP(|G_i| + w,\ d_i + d) - RRP(w, d)$ and that there is an $RRP(|G_j| + w, d_j + d)$. Then there is an $RRP(w + \Sigma|G_i|, d + \Sigma d_i)$.

LEMMA 1.3. *([4, section 3C]) There exists an $RRP(p, p-4)$ for every odd integer $p \geq 9$.*

2. p=31, 35, 37, 41, 43, 47, 59.

RRP(31, 18) Point set $(\mathbb{Z}_3 \times \{1,2,3,4,5,6,7\}) \cup \{a_i : 1 \leq i \leq 10\}$.

Twelve parallel classes:

$$\begin{array}{ccccccccc}
a_1 2_4 2_5 & & a_1 1_3 1_6 & & a_1 2_2 1_7 & & a_1 0_1 & & \\
a_2 0_5 2_6 & & a_2 2_4 2_7 & & a_2 0_3 0_1 & & a_2 2_2 & & \\
a_3 1_6 1_7 & & a_3 1_5 2_1 & & a_3 1_4 1_2 & & a_3 0_3 & & \\
a_4 0_7 0_1 & & a_4 0_6 0_2 & & a_4 1_5 1_3 & & a_4 1_4 & & \\
a_5 2_1 2_2 & & a_5 0_7 0_3 & & a_5 0_5 & & a_5 2_4 2_6 & & \\
a_6 0_2 0_3 & ; & a_6 0_1 1_4 & ; & a_6 1_6 & ; & a_6 0_5 0_7 & ; & \bmod 3 \\
a_7 1_3 1_4 & & a_7 1_7 & & a_7 2_6 2_1 & & a_7 1_2 1_5 & & \\
a_8 0_4 1_5 & & a_8 1_1 & & a_8 2_3 0_6 & & a_8 0_2 1_7 & & \\
a_9 1_2 & & a_9 2_5 2_6 & & a_9 0_4 2_7 & & a_9 1_1 2_3 & & \\
a_{10} 2_3 & & a_{10} 2_2 0_4 & & a_{10} 1_1 2_5 & & a_{10} 1_6 2_7 & & \\
1_1 0_6 2_7 & & 1_2 2_3 0_5 & & 0_2 2_4 0_7 & & 2_1 1_3 0_6 & & \\
 & & & & & & 0_4 2_5 & &
\end{array}$$

Six classes on $\mathbb{Z}_3 \times \{1,2,3,4,5,6,7\}$ are

$$\begin{array}{cccccc}
0_1 1_1 2_2 & 2_3 1_7 & & 0_1 0_5 & 0_3 2_5 1_7 & \\
2_1 1_4 & 0_5 2_5 & & 1_1 0_7 & 1_3 2_3 & \\
0_2 1_5 2_6 & 0_6 1_6 & ; & 2_1 2_4 & 0_4 1_6 & \bmod 3. \\
1_2 0_3 & 0_7 2_7 & & 0_2 2_2 & 1_4 0_6 & \\
1_3 0_4 2_4 & & & 1_2 2_6 & 1_5 2_7 &
\end{array}$$

Now construct an $RRP(10, 6)$ on $\{a_1, a_2, \ldots, a_{10}\}$.

RRP(31,21) Point set $(\mathbb{Z}_3 \times \{1,2,3,4,5,6,7\}) \cup \{a_i : 1 \leq i \leq 10\}$.

Fifteen parallel classes:

$$\begin{array}{cccccccc}
a_1 0_3 0_4 & & a_1 0_6 0_7 & & a_1 0_1 & 1_2 2_6 & \\
a_2 0_2 0_5 & & a_2 2_3 2_7 & & a_2 1_6 & 1_3 2_4 0_7 & \\
a_3 0_1 0_6 & & a_3 0_2 0_4 & & a_3 0_3 & 0_4 1_4 2_5 & \\
a_4 1_4 1_5 & & a_4 1_7 1_1 & & a_4 0_2 & 0_4 1_4 2_5 & \\
a_5 1_3 1_6 & & a_5 0_1 0_5 & & a_5 1_7 & 1_5 0_6 & \\
a_6 1_2 1_7 & ; & a_6 1_3 1_5 & ; & a_6 2_1 & & ; \\
a_7 2_5 2_6 & & a_7 2_1 2_2 & & a_7 2_7 & & \\
a_8 2_4 2_7 & & a_8 1_2 1_6 & & a_8 1_1 & & \\
a_9 2_1 2_3 & & a_9 2_4 2_6 & & a_9 0_5 & & \\
a_{10} 1_1 & & a_{10} 1_4 & & a_{10} 2_2 2_3 & & \\
2_2 0_7 & & 0_3 2_5 & & & &
\end{array}$$

$$\begin{array}{ll} a_1 0_2 & a_8 1_3 \\ a_2 1_4 & a_9 0_7 \\ a_3 2_5 & a_{10} 1_5 1_7 \\ a_4 0_3 & 0_1 1_1 2_7 \\ a_5 2_2 & 2_1 1_2 0_5 \\ a_6 0_6 & 2_3 1_6 \\ a_7 2_4 & 0_4 2_6 \end{array} \quad ; \quad \begin{array}{ll} a_1 2_5 & a_8 1_5 \\ a_2 0_1 & a_9 0_2 \\ a_3 2_7 & a_{10} 1_6 \\ a_4 2_6 & 2_1 1_3 \\ a_6 0_4 & 1_2 0_7 \\ a_7 2_3 & 2_2 0_5 \\ a_5 1_4 & 0_3 1_7 \\ & 1_1 2_4 0_6 \end{array} \qquad \bmod 3$$

Six classes on $\mathbb{Z}_3 \times \{1,2,3,4,5,6,7\}$ are

$$\begin{array}{c} 0_1 0_4 \\ 0_2 1_3 2_6 \\ 0_5 1_7 \end{array} \; ; \; \begin{array}{c} 0_1 1_2 \\ 1_3 0_4 \\ 0_6 1_7 \\ 0_5 1_5 2_5 \end{array} \; ; \; \begin{array}{c} 0_1 1_6 \\ 0_2 1_4 \\ 0_3 1_5 \\ 0_7 1_7 2_7 \end{array} \; ;$$

$$\begin{array}{c} 0_1 1_3 \\ 0_4 2_7 \\ 0_5 1_6 \\ 0_2 1_2 2_2 \end{array} \; ; \; \begin{array}{c} 0_2 2_4 \\ 0_1 2_5 \\ 0_6 2_7 \\ 0_3 1_3 2_3 \end{array} \; ; \; \begin{array}{c} 0_5 2_7 \\ 0_1 2_4 \\ 0_2 2_3 \\ 0_6 1_6 2_6 \end{array} \qquad \bmod 3.$$

Now construct an $RRP(10,6)$ on $\{a_1, a_2, \ldots, a_{10}\}$.

RRP(31,24). Point set $(\mathbb{Z}_3 \times \{1,2,3,4,5,6\}) \cup \{a_i : 1 \leq i \leq 13\}$.
Fifteen parallel classes:

$$\begin{array}{cc} a_1 0_1 0_2 & a_8 1_1 \\ a_2 0_3 0_4 & a_9 1_5 \\ a_3 0_5 0_6 & a_{10} 2_4 \\ a_4 1_4 & a_{11} 2_5 \\ a_5 1_6 & a_{12} 2_2 \\ a_6 1_2 & a_{13} 2_1 \\ a_7 2_3 & 1_3 2_6 \end{array} \; ; \; \begin{array}{cc} a_1 1_3 & a_8 1_6 \\ a_2 1_5 & a_9 2_4 \\ a_3 1_1 & a_{10} 2_5 \\ a_4 0_2 0_3 & a_{11} 2_6 \\ a_5 0_4 0_5 & a_{12} 2_1 \\ a_6 0_6 0_1 & a_{13} 2_2 \\ a_7 1_2 & 2_3 1_4 \end{array} \; ; \; \begin{array}{cc} a_1 1_4 & a_8 0_2 0_5 \\ a_2 2_2 & a_9 0_3 0_6 \\ a_3 1_2 & a_{10} 2_6 \\ a_4 2_5 & a_{11} 2_1 \\ a_5 1_1 & a_{12} 1_6 \\ a_6 1_3 & a_{13} 2_3 \\ a_7 0_1 0_4 & 2_4 1_5 \end{array}$$

$$\begin{array}{cc} a_1 1_5 & a_8 1_4 \\ a_2 1_1 & a_9 2_1 \\ a_3 1_3 & a_{10} 0_1 0_3 \\ a_4 1_6 & a_{11} 2_3 \\ a_5 1_2 & a_{12} 2_4 \\ a_6 2_5 & a_{13} 0_4 0_6 \\ a_7 0_5 & 0_2 2_2 2_6 \end{array} \; ; \; \begin{array}{cc} a_1 1_6 & a_8 2_3 \\ a_2 2_6 & a_9 2_2 \\ a_3 1_4 & a_{10} 1_2 \\ a_4 1_1 & a_{11} 0_2 0_4 \\ a_5 1_3 & a_{12} 0_3 0_5 \\ a_6 2_4 & a_{13} 1_5 \\ a_7 0_6 & 0_1 2_1 2_5 \end{array} \qquad \bmod 3$$

Nine classes on $\mathbb{Z}_3 \times \{1,2,3,4,5,6\}$ are

$$\begin{array}{ccccccccccc}
0_1 1_2 & & 0_1 1_4 & & 0_1 1_3 & & 0_1 2_2 & & 0_2 2_3 & \\
0_3 1_3 2_3 & & 0_2 1_3 & & 0_2 1_5 & & 0_3 1_4 & & 0_4 1_5 & \\
0_4 1_4 2_4 & ; & 0_5 1_6 & ; & 0_4 1_6 & ; & 0_5 2_6 & ; & 0_6 1_1 & ; \\
0_5 1_5 2_5 & & & & & & & & & \\
0_6 1_6 2_6 & & & & & & & & &
\end{array}$$

$$\begin{array}{cccccccc}
0_1 2_3 & & 0_3 1_5 & & 0_1 2_4 & & 0_2 2_4 & \\
0_2 2_5 & ; & 0_2 1_4 & ; & 0_2 1_6 & ; & 0_3 2_6 & \text{mod } 3 \\
0_4 2_6 & & 0_6 2_1 & & 0_3 2_5 & & 0_1 1_5 &
\end{array}$$

Now construct an $RRP(13,9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(31,27). This design exists by lemma 1.3; however, we shall require an $RRP(31,27)-RRP(13,9)$ later in the paper and so we construct this design here.

Point set: $\mathbb{Z}_{18} \cup \{a_i : 1 \leq i \leq 13\}$.

Eighteen parallel classes:

$$\begin{array}{ccccc}
0,2,6 & a_4 7 & a_9 13 & & \\
4,12 & a_5 8 & a_{10} 14 & & \\
a_1 1 & a_6 9 & a_{11} 15 & & \text{mod } 18 \\
a_2 3 & a_7 10 & a_{12} 16 & & \\
a_3 5 & a_8 11 & a_{13} 17 & &
\end{array}$$

Nine classes on $\mathbb{Z}_{18}$: two one-factors can be formed by developing each of the pairs $0,1$ $0,3$ $0,5$ and $0,7$ mod 18; a ninth one-factor is obtained by developing the pair $0,9$ mod 18.

RRP(35,19). Take a resolvable $TD(3,12)$ and remove a point. Apply Construction 1.1, filling in $RRP(12,7)$s and an $RRP(11,7)$.

RRP(35,22). Point set $(\mathbb{Z}_6 \times \{1,2,3,4\}) \cup \{a_i : 1 \leq i \leq 11\}$.

Fifteen parallel classes:

$$\begin{array}{cccccccc}
a_1 0_2 1_3 & a_7 0_4 1_1 & & a_1 0_4 & a_7 1_2 & 1_1 1_3 & & \\
a_2 2_2 5_3 & a_8 2_4 5_1 & & a_2 1_4 & a_8 3_2 & 3_1 5_3 & & \\
a_3 4_2 3_3 & a_9 4_4 3_1 & & a_3 5_4 & a_9 4_2 & 0_2 2_3 & & \\
a_4 0_3 1_4 & a_{10} 0_1 1_2 & ; & a_4 2_1 & a_{10} 3_3 & 5_2 3_4 & & \text{mod } 6 \\
a_5 2_3 5_4 & a_{11} 2_1 5_2 & & a_5 4_1 & a_{11} 4_3 & 0_3 2_4 & & \\
a_6 4_3 3_4 & 4_1 3_2 & & a_6 5_1 & 0_1 2_2 4_4 & & &
\end{array}$$

$$\begin{array}{cccc}
a_1 i_1 (3+i)_1 & a_4 i_2 (3+i)_2 & a_7 i_3 (3+i)_3 & a_{10} i_4 (3+i)_4 \\
a_2 (1+i)_1 (2+i)_1 & a_5 (1+i)_2 (2+i)_2 & a_8 (1+i)_3 (2+i)_3 & a_{11} (i+1)_4 (i+2)_4 \\
a_3 (4+i)_1 (5+i)_1 & a_6 (4+i)_2 (5+i)_2 & a_9 (4+i)_3 (5+i)_3 & (i+4)_4 (i+5)_4
\end{array}$$

for $i = 0, 2, 4$.

Seven classes on $\mathbb{Z}_6 \times \{1,2,3,4\}$:

$$\begin{array}{cc} 0_1 2_1 0_2 & 2_2 1_4 \\ 1_2 3_2 1_3 & 3_3 5_1 \\ 2_3 4_3 2_4 & 4_4 4_2 \\ 3_4 5_4 3_1 & 4_1 5_3 \\ 1_1 0_3 & 5_2 0_4 \end{array} \quad ; \quad \begin{array}{c} 0_1 3_3 \\ 0_2 3_4 \end{array} \qquad \bmod 6$$

Now construct an $RRP(11,7)$ on $\{a_1, a_2, \ldots, a_{11}\}$.

RRP(35,25). Point set $(\mathbb{Z}_6 \times \{1,2,3,4\}) \cup \{a_i : 1 \le i \le 11\}$.

Eighteen parallel classes:

$$\begin{array}{cc} a_1 0_2 1_3 & a_7 0_4 1_1 \\ a_2 2_2 5_3 & a_8 2_4 5_1 \\ a_3 4_2 3_3 & a_9 4_4 3_1 \\ a_4 0_3 1_4 & a_{10} 0_1 1_2 \\ a_5 2_3 5_4 & a_{11} 2_1 5_2 \\ a_6 4_3 3_4 & 4_1 3_2 \end{array} \quad ; \quad \begin{array}{cc} a_1 1_4 & a_9 4_2 \\ a_2 2_4 & a_{10} 3_3 \\ a_3 4_4 & a_{11} 4_3 \\ a_4 1_1 & 5_1 5_3 \\ a_5 3_1 & 5_2 5_4 \\ a_6 4_1 & 0_1 0_2 2_3 \\ a_7 2_2 & 1_2 1_3 3_4 \\ a_8 3_2 & 0_3 0_4 2_1 \end{array} \quad ; \quad \begin{array}{cc} a_1 2_1 & a_9 4_3 \\ a_2 3_1 & a_{10} 1_4 \\ a_3 4_1 & a_{11} 4_4 \\ a_4 0_2 & 1_1 2_3 \\ a_5 4_2 & 5_1 3_2 \\ a_6 5_2 & 1_2 2_4 \\ a_7 0_3 & 3_3 5_3 \\ a_8 1_3 & 3_4 5_4 \\ & 0_1 2_2 0_4 \end{array} \qquad \bmod 6$$

Seven classes on $\mathbb{Z}_6 \times \{1,2,3,4\}$:

$$\begin{array}{cccc} 0_1 3_1 & 0_2 3_2 & 0_3 3_3 & 0_4 3_4 \\ 1_1 2_1 & 1_2 2_2 & 1_3 2_3 & 1_4 2_4 \\ 4_1 5_1 & 4_2 5_2 & 4_3 5_3 & 4_4 5_4 \end{array} \; ; \; \begin{array}{c} 0_1 2_4 \\ 2_2 0_3 \end{array} \; ; \; \begin{array}{c} 0_1 3_3 \\ 0_2 3_4 \end{array} \; ; \; \begin{array}{c} 0_1 5_3 \\ 0_2 5_4 \end{array} \; ; \; \begin{array}{c} 0_1 2_1 4_1 \\ 0_2 2_2 4_2 \\ 0_3 4_4 \end{array} \qquad \bmod 6$$

Now construct an $RRP(11,7)$ on $\{a_1, a_2, \ldots, a_{11}\}$.

RRP(35,28). Point set $\mathbb{Z}_{21} \cup \{a_i : 1 \le i \le 14\}$.

Twenty-one parallel classes:

$$\begin{array}{cccc} 0,3,7 & a_3 4 & a_7 12 & a_{11} 16 \\ 8,10 & a_4 5 & a_8 13 & a_{12} 17 \\ 11,20 & a_5 6 & a_9 14 & a_{13} 18 \\ a_1 1 & a_6 9 & a_{10} 15 & a_{14} 19 \\ a_2 & & & \end{array} \qquad \bmod 21$$

Seven classes on $\mathbb{Z}_{21}$ are obtained by developing the blocks $0,1,6$ $2,12$ and $3,11$ $\bmod 21$ (note that these blocks cover each residue class $\bmod 7$ exactly once). Now construct an $\mathrm{RRP}(14,7)$ on $\{a_1, a_2, \ldots, a_{14}\}$.

RRP(35,31). Lemma 1.3.

RRP(37,21). Point set $(\mathbb{Z}_3 \times \{1,2,\ldots,9\}) \cup \{a_i : 1 \le i \le 10\}$ Fifteen parallel classes:

$$
\begin{array}{cc}
a_1 2_5 1_6 & a_7 2_2 2_3 \\
a_2 0_6 0_7 & a_8 1_3 1_4 \\
a_3 2_7 2_8 & a_9 0_4 0_5 \\
a_4 1_8 0_9 & a_{10} 2_4 1_5 \\
a_5 2_9 2_1 & 0_1 0_3 2_6 \\
a_6 1_1 1_2 & 0_2 1_9 \\
 & 1_7 0_8
\end{array}
\; ; \quad
\begin{array}{cc}
a_1 0_4 0_7 & a_7 2_1 2_4 \\
a_2 0_5 0_8 & a_8 1_2 1_5 \\
a_3 2_6 2_9 & a_9 1_3 1_6 \\
a_4 1_7 1_1 & a_{10} 0_6 2_7 \\
a_5 2_8 2_2 & 0_2 1_4 \\
a_6 0_9 0_3 & 0_1 1_9 \\
 & 2_3 2_5 1_8
\end{array}
\; ; \quad
\begin{array}{cc}
a_1 2_3 2_8 & a_7 0_5 0_9 \\
a_2 2_4 2_9 & a_8 1_6 1_1 \\
a_3 1_2 0_4 & a_9 2_1 0_8 \\
a_4 0_2 0_6 & a_{10} 2_2 1_3 \\
a_5 0_3 0_7 & 0_1 1_5 1_7 \\
a_6 1_4 1_8 & 2_5 2_6 \\
 & 2_7 1_9
\end{array}
\; ;
$$

$$
\begin{array}{cc}
a_1 1_2 0_9 & a_9 2_2 0_7 \\
a_2 0_2 & a_{10} 0_1 \\
a_3 2_1 2_5 & 1_1 0_8 \\
a_4 0_3 1_5 & 0_4 1_6 \\
a_5 0_5 & 1_3 2_3 \\
a_6 2_6 & 1_4 2_4 \\
a_7 0_6 1_8 & 1_7 2_7 \\
a_8 2_8 & 1_9 2_9
\end{array}
\; ; \quad
\begin{array}{cc}
a_1 2_1 & a_9 1_9 \\
a_2 1_1 0_3 & a_{10} 0_8 0_9 \\
a_3 2_3 & 0_1 1_2 \\
a_4 2_4 & 1_3 0_4 \\
a_5 1_4 0_6 & 0_2 2_2 \\
a_6 1_5 0_7 & 0_5 2_5 \\
a_7 2_7 & 1_6 2_6 \\
a_8 1_7 2_9 & 1_8 2_8
\end{array}
\qquad \bmod 3
$$

Six classes on $\mathbb{Z}_3 \times \{1,2,\ldots,9\}$ are

$$
\begin{array}{cc}
0_1 2_2 2_7 & 0_4 1_7 2_8 \\
2_1 0_6 1_9 & 0_5 1_8 2_9 \\
1_3 2_4 0_9 & 1_1 2_3 \\
0_2 1_5 2_6 & 0_3 2_5 \\
1_2 1_4 0_7 & 1_6 0_8
\end{array}
\; ; \quad
\begin{array}{c}
0_1 1_1 2_1 \\
1_2 2_3 0_8 \\
0_4 0_6 2_9 \\
1_5 2_7
\end{array}
\; ; \quad
\begin{array}{c}
0_1 1_4 2_5 \\
1_2 2_6 2_8 \\
0_3 1_7 1_9
\end{array}
\; ; \quad
\begin{array}{c}
0_1 2_4 0_8 \\
0_2 2_5 0_9 \\
0_3 1_6 2_7
\end{array}
\qquad \bmod 3
$$

Now construct an $RRP(10,6)$ on $\{a_1, a_2, \ldots, a_{10}\}$.

RRP(37,24). Point set $(\mathbb{Z}_3 \times \{1,2,\ldots,8\}) \cup \{a_i : 1 \le i \le 13\}$.
Fifteen parallel classes:

$$
\begin{array}{cc}
a_1 0_7 1_8 & a_8 0_6 1_7 \\
a_2 0_8 1_1 & a_9 2_1 \\
a_3 0_1 1_2 & a_{10} 1_6 \\
a_4 0_2 1_3 & a_{11} 2_7 \\
a_5 0_3 1_4 & a_{12} 2_2 2_3 \\
a_6 2_4 0_5 & a_{13} 2_8 \\
a_7 1_5 2_6 & 0_4 2_5
\end{array}
\; ; \quad
\begin{array}{cc}
a_1 2_1 & a_8 0_1 1_3 \\
a_2 2_3 2_5 & a_9 0_2 1_4 \\
a_3 2_7 & a_{10} 0_3 1_5 \\
a_4 1_8 & a_{11} 0_4 1_6 \\
a_5 0_6 2_8 & a_{12} 0_5 1_7 \\
a_6 0_7 1_1 & a_{13} 2_4 \\
a_7 0_8 1_2 & 2_2 2_6
\end{array}
\; ; \quad
\begin{array}{cc}
a_1 0_3 1_6 & a_8 2_4 \\
a_2 1_4 2_7 & a_9 2_5 \\
a_3 0_5 1_8 & a_{10} 2_2 \\
a_4 0_6 1_1 & a_{11} 2_3 \\
a_5 0_7 1_2 & a_{12} 2_8 \\
a_6 0_8 1_3 & a_{13} 0_2 2_6 \\
a_7 1_7 & 2_1 0_4 \\
 & 0_1 1_5
\end{array}
\; ;
$$

$$\begin{array}{cc} a_1 0_5 & a_8 0_2 2_5 \\ a_2 2_6 & a_9 1_3 0_6 \\ a_3 2_3 & a_{10} 1_4 0_7 \\ a_4 2_4 & a_{11} 1_5 0_8 \\ a_5 2_1 & a_{12} 1_6 0_1 \\ a_6 2_2 & a_{13} 0_3 1_7 \\ a_7 1_1 0_4 & 1_2 2_7 \\ & 1_8 2_8 \end{array} \; ; \; \begin{array}{cc} a_1 1_2 0_4 & a_8 2_8 \\ a_2 0_2 & a_9 2_7 1_8 \\ a_3 1_4 0_6 & a_{10} 2_1 0_8 \\ a_4 1_5 0_7 & a_{11} 0_1 2_2 \\ a_5 2_5 & a_{12} 2_4 \\ a_6 2_6 & a_{13} 1_1 0_5 \\ a_7 2_3 & \end{array} \qquad \bmod 3$$

Nine classes on $\mathbb{Z}_3 \times \{1, 2, \ldots, 8\}$ are

$$\begin{array}{cc} 0_1 2_3 & 1_4 2_8 \\ 1_1 2_1 & 2_4 1_8 \\ 0_2 2_2 & 1_5 2_5 \\ 1_2 0_3 & 0_6 2_6 \\ 1_3 0_5 & 1_6 0_7 \\ 0_4 0_8 & 1_7 2_7 \end{array} \; ; \; \begin{array}{cc} 0_1 0_7 & 0_3 1_8 \\ 1_1 2_7 & 1_3 0_4 \\ 2_1 2_6 & 2_3 1_7 \\ 0_2 0_5 & 1_4 2_4 \\ 1_2 2_5 & 1_5 0_6 \\ 2_2 0_8 & 1_6 2_8 \end{array} \; ; \; \begin{array}{c} 0_1 0_5 \\ 0_2 0_7 0_8 \\ 0_3 0_4 0_6 \end{array} \; ; \; \begin{array}{c} 0_2 1_6 \\ 0_1 0_3 0_8 \\ 0_4 0_5 0_7 \end{array} \; ; \; \begin{array}{c} 0_3 0_7 \\ 0_1 0_2 0_4 \\ 0_5 0_6 0_8 \end{array} \qquad \bmod 3$$

Now construct an $RRP(13, 9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(37,27). Point set $(\mathbb{Z}_6 \times \{1, 2, 3, 4\}) \cup \{a_i : 1 \le i \le 13\}$.

Eighteen parallel classes:

$$\begin{array}{cc} a_1 0_1 1_2 & a_8 2_4 \\ a_2 0_2 1_3 & a_9 4_1 \\ a_3 0_3 1_4 & a_{10} 4_2 \\ a_4 0_4 1_1 & a_{11} 5_4 \\ a_5 2_1 3_3 & a_{12} 5_3 \\ a_6 2_2 3_2 & a_{13} 5_1 3_2 \\ a_7 4_3 & 3_1 5_2 \\ & 2_3 4_4 \end{array} \; ; \; \begin{array}{cc} a_1 3_3 & a_8 1_2 0_3 \\ a_2 4_4 & a_9 1_3 0_4 \\ a_3 2_1 & a_{10} 1_4 0_1 \\ a_4 2_2 & a_{11} 3_1 2_3 \\ a_5 4_2 & a_{12} 3_2 2_4 \\ a_6 4_1 & 5_1 5_3 5_4 \\ a_7 1_1 0_2 & a_{13} 4_3 \\ & 5_2 3_4 \end{array} \; ; \; \begin{array}{cc} a_1 1_4 & a_{10} 5_3 \\ a_2 2_1 & a_{11} 5_2 \\ a_3 1_2 & a_{12} 5_1 \\ a_4 1_3 & a_{13} 5_4 \\ a_5 2_4 & 0_1 3_2 \\ a_6 3_3 & 1_1 4_3 \\ a_7 4_4 & 4_1 4_2 \\ a_8 3_1 & 2_3 0_4 \\ a_9 2_2 & 0_2 0_3 3_4 \end{array} \qquad \bmod 6$$

Nine classes on $\mathbb{Z}_6 \times \{1, 2, 3, 4\}$ are

$$\begin{array}{cc} 0_1 2_4 & 1_2 3_4 \\ 1_1 4_4 & 2_2 5_3 \\ 2_1 0_3 & 3_2 1_3 \\ 3_1 1_4 & 4_2 5_2 \\ 4_1 5_1 & 3_3 4_3 \\ 0_2 2_3 & 5_4 0_4 \end{array} \; ; \; \begin{array}{c} 0_1 3_1 \\ 0_2 3_2 \\ 0_3 3_3 \\ 0_4 3_4 \end{array} \; ; \; \begin{array}{c} 0_1 2_3 \\ 0_2 0_4 \end{array} \; ; \; \begin{array}{c} 0_1 2_1 4_1 \\ 0_2 2_2 4_2 \\ 0_3 2_3 4_3 \\ 0_4 2_4 4_4 \end{array} \qquad \bmod 6$$

Now construct an $RRP(13, 9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(37,30). Point set $\mathbb{Z}_{21} \cup \{a_i : 1 \le i \le 16\}$. Twenty-one parallel classes: take the blocks $0, 1, 17$ and $7, 14$ together with the sixteen pairs formed by matching off the a_is to the remaining residues $\bmod 21$ in any order. Develop this class $\bmod 21$.

Nine classes on $\mathbb{Z}_{21}$:

$$\begin{array}{cc}
2+i, 4+i, 12+i & 13+i, 16+i \\
6+i, 18+i & 1+i, 7+i \\
i, 3+i & 8+i, 17+i \\
9+i, 15+i & 14+i, 11+i \\
10+i, 19+i & 5+i, 20+i
\end{array} \quad ; \quad 9+i, 11+i, 19+i \quad ; \quad 16+i, 18+i, 5+i$$

for $i = 0, 3, 6, 9, 12, 15, 18$.

Now construct an $RRP(16, 9)$ on $\{a_1, a_2, \ldots, a_{16}\}$.

RRP(37,33). Lemma 1.3.

RRP(41,22). Take a resolvable $TD(3, 15)$ and remove four points from a group. Apply construction 1.1, filling in $RRP(15, 7)$s and an $RRP(11, 7)$.

RRP(41,25). Point set $(\mathbb{Z}_9 \times \{1, 2, 3\}) \cup \{a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2, \infty_1, \infty_2, \infty_3, \infty_4, \infty_5\}$.

Eighteen parallel classes: develop each of the following two classes $\bmod 9$ (subscripts on a, b and c are to be evaluated $\bmod 3$)

$$\begin{array}{cc}
a_0 3_1 4_1 & \infty_1 2_3 \\
a_1 3_2 4_2 & \infty_2 5_3 \\
a_2 1_1 0_2 & \infty_3 0_1 \\
b_0 2_2 7_2 & \infty_4 7_1 \\
b_1 7_3 8_3 & \infty_5 2_1 0_3 \\
b_2 5_2 4_3 & 5_1 6_3 \\
c_0 1_2 & 6_1 8_1 \\
c_1 6_2 & 1_3 3_3 \\
c_2 8_2 &
\end{array} \quad ; \quad \begin{array}{cc}
a_0 3_3 & \infty_1 4_1 1_2 \\
a_1 8_3 & \infty_2 8_1 6_2 \\
a_2 7_3 & \infty_3 7_2 4_3 \\
b_0 1_1 & \infty_4 8_2 6_3 \\
b_1 3_1 & \infty_5 5_2 \\
b_2 2_1 & 7_1 1_3 \\
c_0 0_1 5_1 & 0_2 3_2 \\
c_1 0_3 5_3 & 2_2 4_2 \\
c_2 6_1 2_3 &
\end{array}$$

Seven classes on $\mathbb{Z}_9 \times \{1, 2, 3\}$ are obtained by developing each of $0_1 0_2 0_3$, $0_1 1_2 2_3$, $0_1 2_2 4_3$, $0_1 3_2 6_3$, $0_1 4_2 8_3$, $0_1 3_1 6_1$ $0_2 5_3$, and $0_3 3_3 6_3$ $0_1 5_2$ $\bmod 9$.

Now construct an $RRP(14, 7)$ on $\{a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2, \infty_1, \infty_2, \infty_3, \infty_4, \infty_5\}$.

RRP(41, 28). Point set $(\mathbb{Z}_6 \times \{1, 2, 3, 4\}) \cup \{a_i : 1 \le i \le 17\}$.

Eighteen parallel classes:

$$
\begin{array}{ll}
a_1 2_3 & a_{10} 4_2 \\
a_2 3_3 & a_{11} 5_2 \\
a_3 1_4 & a_{12} 4_3 \\
a_4 3_4 & a_{13} 0_1 1_3 \\
a_5 5_4 & a_{14} 2_1 0_3 \\
a_6 1_1 & a_{15} 3_1 5_3 \\
a_7 4_1 & a_{16} 1_2 2_4 \\
a_8 5_1 & a_{17} 2_2 4_4 \\
a_9 0_2 & 3_2 0_4
\end{array}
\ ;\quad
\begin{array}{ll}
a_1 5_4 & a_{10} 5_3 \\
a_2 2_1 4_2 & a_{11} 4_1 2_4 \\
a_3 0_1 1_2 & a_{12} 1_1 0_4 \\
a_4 3_1 & a_{13} 2_2 \\
a_5 5_1 & a_{14} 3_2 \\
a_6 0_2 1_3 & a_{15} 3_4 \\
a_7 5_2 & a_{16} 3_3 \\
a_8 2_3 4_4 & a_{17} 4_3 \\
a_9 0_3 1_4 &
\end{array}
\ ;\quad
\begin{array}{ll}
a_1 1_1 0_2 & a_{10} 1_4 0_1 \\
a_2 4_4 & a_{11} 3_3 \\
a_3 2_3 & a_{12} 5_2 \\
a_4 1_2 0_3 & a_{13} 2_4 \\
a_5 2_2 4_3 & a_{14} 3_4 \\
a_6 5_4 & a_{15} 3_2 \\
a_7 1_3 0_4 & a_{16} 5_1 \\
a_8 4_2 & a_{17} 3_1 \\
a_9 4_1 & 2_1 5_3
\end{array}
\qquad \bmod 6
$$

Ten classes on $\mathbb{Z}_6 \times \{1,2,3,4\}$:

$$
\begin{array}{ll}
0_1 3_2 4_2 & 1_3 1_4 \\
0_2 3_3 4_3 & 2_1 2_4 \\
0_3 3_4 4_4 & 2_2 2_3 \\
0_4 3_1 4_1 & 5_1 5_3 \\
1_1 1_2 & 5_2 5_4
\end{array}
\ ;\quad
\begin{array}{l}
1_1 0_3 \\
1_2 0_4
\end{array}
\qquad \bmod 6
$$

$$
\begin{array}{ll}
0_1 3_1 & 0_3 3_3 \\
2_1 4_1 & 2_3 4_3 \\
5_1 1_1 & 5_3 1_3 \\
0_2 2_2 4_2 & 0_4 2_4 4_4 \\
1_2 3_2 5_2 & 1_4 3_4 5_4
\end{array}
\ ;\quad
\begin{array}{ll}
1_1 4_1 & 1_3 4_3 \\
3_1 5_1 & 3_3 5_3 \\
0_1 2_1 & 0_3 2_3 \\
0_2 3_2 & 0_4 3_4 \\
1_2 4_2 & 1_4 4_4 \\
2_2 5_2 & 2_4 5_4
\end{array}
\ ;\quad
\begin{array}{ll}
2_1 5_1 & 2_2 0_4 \\
4_1 0_1 & 3_2 1_4 \\
1_1 3_1 & 4_2 2_4 \\
2_3 5_3 & 5_2 3_4 \\
4_3 0_3 & 0_2 4_4 \\
1_3 3_3 & 1_2 5_4
\end{array}
$$

Now construct an $RRP(17,10)$ on $\{a_1, a_2, \dots, a_{17}\}$.

RRP(41,31). Point set $(\mathbb{Z}_6 \times \{1,2,3,4\}) \cup \{a_i : 1 \le i \le 17\}$.

Proceed as in the previous design, but replace the first six classes on $\mathbb{Z}_6 \times \{1,2,3,4\}$ by the nine classes

$$
\begin{array}{ll}
0_1 1_1 & 4_2 4_3 \\
2_1 0_2 & 5_2 5_4 \\
3_1 3_2 & 0_3 0_4 \\
4_1 4_4 & 1_3 2_3 \\
5_1 5_3 & 3_3 1_4 \\
1_2 2_2 & 2_4 3_4
\end{array}
\ ;\quad
\begin{array}{l}
0_1 3_2 \\
0_3 3_4
\end{array}
\ ;\quad
\begin{array}{l}
3_1 0_4 \\
0_2 3_3
\end{array}
\ ;\quad
\begin{array}{l}
0_1 2_4 \\
2_2 0_3
\end{array}
\qquad \bmod 6.
$$

Then construct an $RRP(17,13)$ on $\{a_1, a_2, \dots, a_{17}\}$.

RRP(41,34). Point set $\mathbb{Z}_{24} \cup \{a_i : 1 \le i \le 17\}$. Form twenty-four parallel classes by taking the blocks $0,3,9 \quad 1,6$ and $2,12$ together with the seventeen pairs obtained by matching off the a_i's with the remaining residue classes in $\mathbb{Z}_{24}$, and developing this class mod 24.

Ten classes on $\mathbb{Z}_{24}$ are obtained by developing the blocks $0,1 \quad 2,4 \quad 3,11$; $0,4,11$ and $0,12 \pmod{24}$ (the first set of pairs covers each residue class $\bmod 6$ once, while the triple covers each residue class $\bmod 3$ once). Construct an $RRP(17,10)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(41,37). Lemma 1.3.

RRP(43,24). Take a resolvable $TD(3,15)$ and remove two points from a group. Apply construction 1.1, filling in $RRP(15,9)$s and an $RRP(13,9)$.

RRP(43,27). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\}) \cup \{a_i : 1 \le i \le 13\}$. Eighteen parallel classes:

$$
\begin{array}{ll}
a_1 0_3 1_4 & a_9 2_1 4_2 \\
a_2 0_4 1_5 & a_{10} 2_2 4_3 \\
a_3 0_5 1_1 & a_{11} 5_3 3_4 \\
a_4 0_1 1_2 & a_{12} 5_4 3_5 \\
a_5 0_2 1_3 & a_{13} 5_5 3_1 \\
a_6 2_3 4_4 & 5_1 3_2 \\
a_7 2_4 4_5 & 5_2 3_4 \\
a_8 2_5 4_1 &
\end{array} ;
\begin{array}{ll}
a_1 1_2 0_5 & a_9 2_5 4_3 \\
a_2 1_3 0_1 & a_{10} 2_1 4_4 \\
a_3 1_4 0_2 & a_{11} 5_2 3_5 \\
a_4 1_5 0_3 & a_{12} 5_3 3_1 \\
a_5 1_1 0_4 & a_{13} 5_4 3_2 \\
a_6 2_2 4_5 & 5_5 3_3 \\
a_7 2_3 4_1 & 5_1 3_4 \\
a_8 2_4 4_2 &
\end{array} ;
\begin{array}{ll}
a_1 0_1 & a_{11} 4_1 \\
a_2 1_2 & a_{12} 4_2 \\
a_3 0_3 & a_{13} 3_3 \\
a_4 0_4 & 2_4 5_5 \\
a_5 0_5 & 1_1 0_2 1_5 \\
a_6 3_1 & 2_1 3_4 3_5 \\
a_7 3_2 & 5_1 5_2 4_3 \\
a_8 1_3 & 2_2 2_3 1_4 \\
a_9 4_4 & 5_3 5_4 4_5 \\
a_{10} 2_5 &
\end{array}
\qquad \bmod 6
$$

Nine classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ are

$$
\begin{array}{l}
i_1(3+i)_2 i_3 \\
i_4(3+i)_4 \\
i_5(3+i)_5
\end{array} ;
\begin{array}{ll}
0_1 1_1 & i_2 i_4 \\
2_1 3_1 & i_3(3+i)_5 \\
4_1 5_1 &
\end{array} ;
\begin{array}{ll}
0_2 1_2 & i_3 i_5 \\
2_2 3_2 & i_4(3+i)_1 \\
4_2 5_2 &
\end{array} ;
$$

$$
\begin{array}{ll}
0_3 1_3 & i_4 i_1 \\
2_3 3_3 & i_5(3+i)_2 \\
4_3 5_3 &
\end{array} ;
\begin{array}{ll}
0_4 1_4 & i_5 i_2 \\
2_4 3_4 & i_1(3+i)_3 \\
4_4 5_4 &
\end{array} ;
\begin{array}{lll}
0_5 1_5 & 0_1 5_1 & 0_3 5_3 \\
2_5 3_5 & 1_1 2_1 & 1_3 2_3 \\
4_5 5_5 & 3_1 4_1 & 3_3 4_3 \\
 & & i_2(i+3)_4
\end{array} ;
$$

$$
\begin{array}{l}
i_1(i+2)_1(i+4)_1 \\
i_2(i+2)_2(i+4)_2 \\
i_3(i+2)_3(i+4)_3 \\
i_4(i+2)_4(i+4)_4 \\
i_5(i+2)_5(i+4)_5
\end{array} ;
\begin{array}{ll}
0_4 5_4 & i_1(i+3)_1 \\
1_4 2_4 & i_2(i+3)_2 \\
3_4 4_4 & i_3(i+3)_3 \\
0_5 5_5 & \\
1_5 2_5 & \\
3_5 4_5 &
\end{array} ;
\begin{array}{l}
0_2 5_2 \\
1_2 2_2 \\
3_2 4_2 \\
i_1(i+3)_5 \\
i_3(i+3)_4
\end{array}
$$

where in each case $i \in \mathbb{Z}_6$.

Now construct on $RRP(13,9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(43,30). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\}) \cup \{a_i : 1 \le i \le 13\}$. Twenty-one parallel classes:

$$
\begin{array}{ll}
a_1 0_3 1_4 & a_{10} 0_1 1_2 \\
a_2 2_3 4_4 & a_{11} 2_1 4_2 \\
a_3 5_3 3_4 & a_{12} 0_2 1_3 \\
a_4 0_4 1_5 & a_{13} 2_2 4_3 \\
a_5 2_4 4_5 & 5_1 3_2 \\
a_6 5_4 3_5 & 5_2 3_3 \\
a_7 0_5 1_1 & \\
a_8 2_5 4_1 & \\
a_9 5_5 3_1 &
\end{array}
\; ; \;
\begin{array}{ll}
a_1 0_2 & a_{10} 0_5 \\
a_2 3_2 & a_{11} 1_5 \\
a_3 5_2 & a_{12} 0_1 \\
a_4 1_3 & a_{13} 2_1 \\
a_5 4_3 & 1_1 1_2 0_3 \\
a_6 5_3 & 2_2 2_3 1_4 \\
a_7 0_4 & 3_3 3_4 2_5 \\
a_8 2_4 & 4_4 4_5 3_1 \\
a_9 5_4 & 5_5 5_1 4_2 \\
 & 4_1 1_5
\end{array}
\; ; \;
\begin{array}{ll}
a_1 2_5 & a_{10} 1_3 \\
a_2 5_5 & a_{11} 5_3 \\
a_3 3_5 & a_{12} 1_4 \\
a_4 1_1 & a_{13} 5_4 \\
a_5 3_1 & 2_1 4_3 \\
a_6 5_1 & 2_2 4_4 \\
a_7 0_2 & 2_3 4_5 \\
a_8 4_2 & 2_4 4_1 \\
a_9 5_2 & 1_2 1_5 \\
 & 0_3 3_4 \\
 & 3_3 0_5 \\
 & 0_1 3_2 0_4
\end{array}
\qquad \text{mod } 6
$$

and

$$
\begin{array}{ccc}
a_i i_1 (i+3)_1 & a_6 (i+4)_2 (i+5)_2 & a_{11} (i+4)_4 (i+5)_4 \\
a_2 (i+1)_1 (i+2)_1 & a_7 i_3 (i+3)_3 & a_{12} i_5 (i+3)_5 \\
a_3 (i+4)_1 (i+5)_1 & a_8 (i+1)_3 (i+2)_3 & a_{13} (i+4)_5 (i+5)_5 \\
a_4 i_2 (i+3)_2 & a_9 (i+4)_3 (i+5)_3 & (i+1)_4 (i+2)_4 \\
a_5 (i+1)_2 (i+2)_2 & a_{10} i_4 (i+3)_4 & (i+1)_5 (i+2)_5
\end{array}
$$

for $i = 0, 2, 4$.

Nine classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ are

$$
\begin{array}{lll}
0_1 1_3 & 2_1 4_4 & 3_1 5_1 \\
0_2 1_4 & 2_2 4_5 & 3_2 5_2 \\
0_3 1_5 & 2_3 4_1 & 3_3 5_3 \\
0_5 1_2 & 2_5 4_3 & 3_5 5_5 \\
0_4 1_1 & 2_4 4_2 & 3_4 5_4
\end{array}
\; ; \;
\begin{array}{ll}
0_1 3_4 & 0_4 3_1 \\
1_1 1_3 & 4_1 4_3 \\
2_1 5_3 & 2_3 5_1 \\
2_2 2_4 & 5_2 5_4 \\
4_2 2_5 & 1_2 5_5 \\
1_4 4_5 & 4_4 1_5 \\
0_2 3_3 3_5 & 3_2 0_3 0_5
\end{array}
\qquad \text{mod } 6.
$$

Now construct on $RRP(13,9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(43,33). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\}) \cup \{a_i : 1 \le i \le 13\}$.

Twenty-four parallel classes:

$$
\begin{array}{lll}
a_1 0_1 1_2 & a_6 2_1 3_3 & a_{11} 5_1 4_2 \\
a_2 0_2 1_3 & a_7 2_2 3_4 & a_{12} 5_3 4_4 \\
a_3 0_3 1_4 & a_8 2_3 3_5 & a_{13} 5_5 4_1 \\
a_4 0_4 1_5 & a_9 2_4 3_1 & 5_2 4_3 \\
a_5 0_5 1_1 & a_{10} 2_5 3_2 & 5_4 4_5
\end{array}
\; ; \;
\begin{array}{lll}
a_1 1_3 & a_8 3_4 & 3_1 0_4 \\
a_2 1_4 & a_9 1_5 & 0_3 3_5 \\
a_3 0_5 & a_{10} 4_1 & 4_4 4_5 \\
a_4 2_1 & a_{11} 5_3 & 0_1 2_2 2_4 \\
a_5 1_2 & a_{12} 2_5 & 0_2 2_3 2_5 \\
a_6 3_2 & a_{13} 5_2 & 3_3 5_4 5_1 \\
a_7 4_3 & 1_1 4_2 &
\end{array}
$$

$$\begin{array}{ccc}
a_1 1_4 & a_8 1_1 & 3_2 5_4 \\
a_2 0_5 & a_9 1_2 & 3_3 5_5 \\
a_3 0_1 & a_{10} 2_3 & 3_4 5_1 \\
a_4 0_2 & a_{11} 4_4 & 3_5 5_2 \\
a_5 0_3 & a_{12} 2_1 & 4_1 1_3 \\
a_6 2_4 & a_{13} 4_3 & 4_2 1_5 \\
a_7 4_5 & 3_1 5_3 & 2_2 0_4 2_5
\end{array} \; ; \quad
\begin{array}{ccc}
a_1 1_5 & a_8 2_2 & 4_2 5_5 \\
a_2 2_1 & a_9 4_3 & 1_3 1_1 \\
a_3 1_2 & a_{10} 1_4 & 4_4 5_2 \\
a_4 3_3 & a_{11} 3_5 & 4_5 5_3 \\
a_5 0_4 & a_{12} 3_2 & 0_1 0_2 \\
a_6 2_5 & a_{13} 2_4 & 0_3 3_4 \\
a_7 5_1 & 4_1 5_4 & 3_1 2_3 0_5
\end{array} \qquad \text{mod } 6$$

Nine classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ are

$$\begin{array}{ccc}
0_1 2_5 & 2_4 0_5 & 3_1 4_1 \\
1_1 1_5 & 1_2 3_2 & 4_2 5_2 \\
2_1 0_2 & 1_3 3_3 & 4_3 5_3 \\
2_2 0_3 & 1_4 3_4 & 4_4 5_4 \\
2_3 0_4 & 5_1 3_5 & 4_5 5_5
\end{array} \qquad \text{mod } 6$$

and

$$\begin{array}{c}
0_1 3_1 \\
2_1 4_1 \\
5_1 1_1 \\
i_2 (i+3)_3 (i+3)_4 \\
i_5 (i+2)_5 (i+4)_5
\end{array} \; ; \quad
\begin{array}{c}
1_1 4_1 \\
3_1 5_1 \\
0_1 2_1 \\
i_2 i_3 \\
i_4 (i+3)_5
\end{array} \; ; \quad
\begin{array}{c}
2_1 5_1 \\
4_1 0_1 \\
1_1 3_1 \\
i_2 (i+3)_2 \\
i_3 (i+3)_3 \\
i_4 (i+3)_4 \\
i_5 (i+3)_5
\end{array} \qquad i \in \mathbb{Z}_6.$$

Now construct on $RRP(13,9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(43,36). Point set $\mathbb{Z}_{24} \cup \{a_i : 1 \le i \le 19\}$. Twenty-four parallel classes are formed by taking the blocks $0,1,6$ and $2,10$ together with the nineteen pairs obtained by matching the a_i's with the remaining residues in $\mathbb{Z}_{24}$, and developing this class mod 24. Twelve classes on $\mathbb{Z}_{24}$ are obtained by developing the blocks $0,9 \;\; 2,4 \;\; 3,6 \;\; 5,15$; $0,4,11$ and $0,12$ mod 24 (the first set of pairs covers each residue class mod 8 once and the triple covers each residue class mod 3 once). Then construct an $RRP(19,12)$ on $\{a_1, a_2, \ldots, a_{19}\}$.

RRP(43,39). Lemma 1.3.

RRP(47,25). Remove a point from a resolvable $TD(3,16)$ and apply Construction 1.1, filling in $RRP(16,9)$s and an $RRP(15,9)$.

RRP(47,28). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\}) \cup \{a_i : 1 \le i \le 15\} \cup \{b_0, b_1\}$.

Eighteen parallel classes (subscripts on b are to be evaluated mod 2):

$$\begin{array}{cccc}
a_1 0_3 1_4 & a_5 0_2 1_3 & a_9 5_3 2_4 & a_{13} 4_5 \\
a_2 0_4 1_5 & a_6 1_2 3_4 & a_{10} 5_4 5_5 & a_{14} 3_1 \\
a_3 0_5 1_1 & a_7 5_1 2_2 & a_{11} 2_3 & a_{15} 5_2 \\
a_4 2_1 3_2 & a_8 0_1 4_4 & a_{12} 4_1 4_2 & b_0 3_3 4_3 \\
 & & & b_1 2_5 3_5
\end{array} \; ;$$

$$\begin{array}{cccc}
a_1 0_2 1_5 & a_5 0_1 1_4 & a_9 2_2 2_5 & a_{13} 3_2 3_3 \\
a_2 0_3 1_1 & a_6 2_1 4_5 & a_{10} 4_1 4_3 & a_{14} 5_3 3_4 \\
a_3 0_4 1_2 & a_7 2_4 & a_{11} 5_1 5_5 & a_{15} 5_4 3_5 \\
a_4 0_5 1_3 & a_8 4_2 2_3 & a_{12} 4_4 & b_0 3_1 \\
 & & & b_1 5_2
\end{array} \quad ;$$

$$\begin{array}{cccc}
a_1 0_1 & a_5 0_5 & a_9 2_1 & a_{13} 4_1 4_4 \\
a_2 0_2 & a_6 4_3 & a_{10} 4_2 & a_{14} 2_2 4_5 \\
a_3 2_3 & a_7 0_3 2_5 & a_{11} 1_2 1_4 & a_{15} 3_1 1_3 \\
a_4 5_4 & a_8 3_5 & a_{12} 5_3 5_5 & b_0 3_2 0_4 \\
 & 3_3 1_5 & 1_1 5_2 3_4 & b_1 5_1 2_4
\end{array} \qquad \bmod 6$$

Ten classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ are

$$\begin{array}{ccc}
0_1 2_2 1_3 & 0_2 3_3 & 4_1 5_1 \\
0_3 2_4 1_5 & 2_3 5_5 & 3_2 4_2 \\
0_4 2_5 1_1 & 1_4 4_5 & 5_2 3_5 \\
0_5 2_1 1_2 & 3_1 5_3 & 4_3 4_4 \\
 & & 3_4 5_4
\end{array} \; ; \quad
\begin{array}{c}
0_1 3_5 \\
0_2 2_3 1_4
\end{array} \; ; \quad
\begin{array}{c}
0_1 3_1 \\
0_2 3_3 \\
0_3 3_3 \\
0_4 3_4 \\
0_5 3_5
\end{array} \qquad \bmod 6$$

together with

$$\begin{array}{c}
i_1(3+i)_3 \\
i_2(3+i)_5 \\
0_4 1_4 \\
2_4 3_4 \\
4_4 5_4
\end{array} \; ; \quad
\begin{array}{cc}
i_1(2+i)_1(4+i)_1 & 1_4 2_4 \\
i_2(2+i)_2(4+i)_2 & 3_4 4_4 \\
i_3(2+i)_3(4+i)_3 & 5_4 0_4 \\
i_5(2+i)_5(4+i)_5 &
\end{array}$$

where $i \in \mathbb{Z}_6$.

Now construct an $RRP(17,10)$ on $\{a_1, a_2, \ldots, a_{15}\} \cup \{b_0, b_1\}$.

RRP(47,31). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\}) \cup \{a_i : 1 \le i \le 15\} \cup \{b_0, b_1\}$.

Proceed as in the previous design ($RRP(47,28)$), but we change the first seven classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ to the following ten classes:

$$\begin{array}{ccc}
0_1 1_3 & 5_1 2_5 & 0_3 1_5 \\
1_1 0_2 & 1_2 3_3 & 5_3 5_4 \\
2_1 4_3 & 2_2 3_2 & 0_4 4_4 \\
3_1 4_5 & 4_2 3_5 & 1_4 0_5 \\
4_1 3_4 & 5_2 2_3 & 2_4 5_5
\end{array} \; ; \quad
\begin{array}{ccc}
0_1 2_2 & 2_1 4_2 & 4_1 0_2 \\
1_2 0_3 & 3_2 2_3 & 5_2 4_3 \\
1_3 3_4 & 3_3 5_4 & 5_3 1_4 \\
0_4 2_5 & 2_4 4_5 & 4_4 0_5 \\
1_5 3_1 & 3_5 5_1 & 5_5 1_1
\end{array} \qquad \bmod 6$$

$$\begin{array}{cc}
0_1 1_1 & i_2(1+i)_4 \\
2_1 3_1 & i_3(3+i)_5 \\
4_1 5_1 &
\end{array} \; ; \quad
\begin{array}{cc}
1_1 2_1 & i_2(i+4)_5 \\
3_1 4_1 & i_3(i+5)_4 \\
5_1 0_1 &
\end{array} \qquad i \in \mathbb{Z}_6.$$

Then construct on $RRP(17,13)$ on $\{a_1, a_2, \ldots, a_{15}\} \cup \{b_0, b_1\}$.

RRP(47,34). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\} \cup \{a_i : 1 \le i \le 17\}$. Twenty-four parallel classes:

$$
\begin{array}{ccc}
a_1 0_1 1_2 & a_8 2_4 & a_{15} 3_2 \\
a_2 0_2 1_3 & a_9 3_5 & a_{16} 4_2 \\
a_3 0_3 1_4 & a_{10} 3_1 & a_{17} 4_5 \\
a_4 0_4 1_5 & a_{11} 5_3 & 5_1 4_3 \\
a_5 0_5 1_1 & a_{12} 5_4 & 5_2 4_4 \\
a_6 2_2 & a_{13} 5_5 & 3_3 2_5 \\
a_7 2_3 & a_{14} 4_1 & 3_4 2_1
\end{array}
\;;\quad
\begin{array}{ccc}
a_1 3_3 & a_8 0_3 1_5 & a_{15} 5_3 \\
a_2 1_4 & a_9 0_4 1_1 & a_{16} 4_4 \\
a_3 3_5 & a_{10} 0_5 1_2 & a_{17} 4_1 \\
a_4 3_1 & a_{11} 3_4 & 4_2 4_3 \\
a_5 0_2 & a_{12} 4_5 & 5_2 2_3 \\
a_6 0_1 1_3 & a_{13} 5_1 & 5_4 2_5 \\
a_7 2_2 2_4 & a_{14} 3_2 & 5_5 2_1
\end{array}
\;;
$$

$$
\begin{array}{ccc}
a_1 1_4 & a_8 3_1 & a_{15} 0_5 2_1 \\
a_2 1_5 & a_9 3_2 & a_{16} 4_5 \\
a_3 1_1 & a_{10} 3_3 & a_{17} 4_3 \\
a_4 1_2 & a_{11} 0_1 2_2 & 4_1 4_4 \\
a_5 1_3 & a_{12} 0_2 2_3 & 5_1 5_3 \\
a_6 5_4 & a_{13} 0_3 2_4 & 4_2 5_5 \\
a_7 3_5 & a_{14} 0_4 2_5 & 5_2 3_4
\end{array}
\;;\quad
\begin{array}{ccc}
a_1 1_5 & a_8 3_2 & a_{15} 4_4 \\
a_2 3_1 & a_9 3_3 & a_{16} 0_1 2_3 \\
a_3 1_2 & a_{10} 3_4 & a_{17} 0_2 2_4 \\
a_4 1_3 & a_{11} 4_5 & 0_3 2_5 \\
a_5 1_4 & a_{12} 5_1 & 0_4 2_1 \\
a_6 3_5 & a_{13} 5_2 & 0_5 2_2 \\
a_7 4_1 & a_{14} 4_3 & 1_1 4_2 \\
 & & 5_3 5_4 5_5
\end{array}
\qquad \text{mod } 6
$$

Ten classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ are

$$
\begin{array}{ccc}
5_1 3_2 2_3 & 5_5 3_1 2_2 & 0_3 4_5 \\
5_2 3_3 2_4 & 1_1 1_2 1_5 & 1_3 4_4 \\
5_3 3_4 2_5 & 0_1 4_3 & 0_4 4_1 \\
5_4 3_5 2_1 & 0_2 1_4 & 0_5 4_2
\end{array}
\;;\quad
\begin{array}{c}
0_1 3_1 \\
0_2 3_2 \\
0_3 3_3 \\
0_4 3_4 \\
0_5 3_5
\end{array}
\;;\quad
\begin{array}{c}
0_1 2_1 4_1 \\
0_2 2_2 4_2 \\
0_3 2_3 4_3 \\
0_4 2_4 4_4 \\
0_5 2_5 4_5
\end{array}
\qquad \text{mod } 6
$$

together with

$$
\begin{array}{c}
0_j 1_j \\
2_j 3_3 \\
4_j 5_j
\end{array}
\;;\quad
\begin{array}{c}
1_j 2_j \\
3_j 4_j \\
5_j 0_j
\end{array}
\qquad j = 1,2,3,4,5.
$$

Now construct an $RRP(17,10)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(47,37). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5\}) \cup \{a_i : 1 \le i \le 17\}$. Proceed as in the previous design ($RRP(47,34)$), but we change the first seven classes on $\mathbb{Z}_6 \times \{1,2,3,4,5\}$ to the following ten classes:

$$
\begin{array}{ccc}
0_1 3_3 & 5_1 3_2 & 1_3 0_4 \\
1_1 0_2 & 1_2 0_3 & 4_3 2_5 \\
2_1 4_4 & 2_2 4_5 & 5_3 3_4 \\
3_1 5_5 & 4_2 1_5 & 1_4 0_5 \\
4_1 2_3 & 5_2 2_4 & 5_4 3_5
\end{array}
\;;\quad
\begin{array}{ccc}
0_1 3_1 & 0_2 3_2 & 0_3 3_3 \\
1_1 4_4 & 1_2 5_3 & 1_3 4_5 \\
2_1 3_5 & 2_2 3_4 & 4_3 1_5 \\
4_1 1_4 & 4_2 2_3 & 2_4 5_4 \\
5_1 0_5 & 5_2 0_4 & 2_5 5_5
\end{array}
\;;\quad
\begin{array}{c}
0_1 0_2 0_5 \\
0_3 3_4
\end{array}
\qquad \text{mod } 6
$$

Then construct on $RRP(17,13)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(47,40). Point set $\mathbb{Z}_{30} \cup \{a_i, 1 \le i \le 17\}$.

Form a parallel class by taking the blocks $2,14$ $3,13$ $5,11$ $16,29$ $18,27$ $21,22,24$ together with the seventeen pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{30}$. Develop this class mod 30. Ten classes on $\mathbb{Z}_{30}$ are obtained by developing the blocks $0,4$ $1,8$ $3,11$; $0,5,16$; and $0,15$ mod 30 (the first set of pairs covers each residue class mod 6 once, and the triple covers each residue class mod 3 once). Then construct an $RRP(17,10)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(47,43). Lemma 1.3.

RRP(59,31), RRP(59,34). Remove four points from a group in a resolvable $TD(3,21)$. Fill in $RRP(21,10)$s and an $RRP(17,10)$, or $RRP(21,13)s$ and an $RRP(17,13)$.

RRP(59,37). Point set $(\mathbb{Z}_{12} \times \{1,2,3\}) \cup \{a_i : 1 \le i \le 23\}$. Twenty-four parallel classes:

$$\begin{array}{cccccc}
a_1 0_1 2_2 & a_{13} 9_1 2_3 & & a_1 2_3 & a_{13} 7_2 & \\
a_2 1_1 4_2 & a_{14} 4_3 & & a_2 4_3 & a_{14} 7_1 2_2 & \\
a_3 2_1 6_2 & a_{15} 6_3 & & a_3 6_3 & a_{15} 8_1 4_2 & \\
a_4 3_1 8_2 & a_{16} 9_3 & & a_4 8_3 & a_{16} 9_1 6_2 & \\
a_5 4_1 10_2 & a_{17} 11_3 & & a_5 10_3 & a_{17} 10_1 8_2 & \\
a_6 0_2 3_3 & a_{18} 5_1 & & a_6 0_1 & a_{18} 9_2 3_3 & \\
a_7 1_2 5_3 & a_{19} 10_1 & ; & a_7 1_1 & a_{19} 10_2 5_3 & \text{mod } 12 \\
a_8 3_2 8_3 & a_{20} 11_1 & & a_8 6_1 & a_{20} 11_2 7_3 & \\
a_9 11_2 1_3 & a_{21} 5_2 & & a_9 11_1 & a_{21} 3_1 9_3 & \\
a_{10} 6_1 7_3 & a_{22} 7_2 & & a_{10} 0_2 & a_{22} 4_1 11_3 & \\
a_{11} 7_1 10_3 & a_{23} 9_2 & & a_{11} 3_2 & a_{23} 5_1 1_3 & \\
a_{12} 8_1 0_3 & & & a_{12} 5_2 & 2_1 1_2 0_3 &
\end{array}$$

Thirteen classes on $\mathbb{Z}_{12} \times \{1,2,3\}$ are obtained as follows. Construct an $RRP(12,9)$ on each of $\mathbb{Z}_{12} \times \{j\}$, $j = 1,2$ and an $RRP(12,7)$ on $\mathbb{Z}_{12} \times \{3\}$; together with the 48 pairs $0_2 9_3$, $0_2 10_3$, $0_1 9_3$ and $0_1 11_3$ mod 12, form eleven classes. Two more classes are given by developing each of $0_1 0_2 0_3$ and $0_1 1_2 2_3$ mod 12.

Now construct an $RRP(23,13)$ on $\{a_1, a_2, \ldots, a_{23}\}$.

RRP(59,40). Point set $(\mathbb{Z}_{12} \times \{1,2,3\}) \cup \{a_i : 1 \le i \le 23\}$. Twenty-four parallel classes: as in previous design ($RRP(59,37)$).

Sixteen classes on $\mathbb{Z}_{12} \times \{1,2,3\}$ are obtained by constructing an $RRP(12,10)$ on each of $\mathbb{Z}_{12} \times \{j\}$, $j = 1,2$, and an $RRP(12,8)$ on $\mathbb{Z}_{12} \times \{3\}$, and using these together with the 120 pairs $0_2 0_3$, $0_2 1_3$, $0_2 9_3$, $0_2 10_3$, $0_1 0_3$, $0_1 2_3$, $0_1 9_3$, $0_1 11_3$, $0_1 0_2$ and $0_1 1_2$ mod 12. Then construct an $RRP(23,16)$ on $\{a_1, a_2, \ldots, a_{23}\}$.

RRP(59,43). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}) \cup \{a_i : 1 \le i \le 17\}$. Thirty-three parallel classes:

$$
\begin{array}{lll}
a_1 1_3 & a_{10} 3_5 & 0_1 2_2 \\
a_2 1_4 & a_{11} 3_6 & 0_2 2_3 \\
a_3 1_5 & a_{12} 3_7 & 0_3 2_4 \\
a_4 1_6 & a_{13} 3_1 & 5_1 4_4 \\
a_5 1_7 & a_{14} 3_2 & 5_2 4_5 \\
a_6 1_1 & a_{15} 0_4 2_5 & 5_3 4_6 \\
a_7 1_2 & a_{16} 0_5 2_6 & 5_4 4_7 \\
a_8 3_3 & a_{17} 0_6 2_7 & 5_5 4_1 \\
a_9 3_4 & 0_7 2_1 & 5_6 4_2 \\
 & & 5_7 4_3
\end{array}
\;;\qquad
\begin{array}{lll}
a_1 0_2 2_4 & a_{10} 5_6 & 3_2 5_2 \\
a_2 0_3 2_5 & a_{11} 2_7 & 3_5 5_5 \\
a_3 0_4 2_6 & a_{12} 0_1 & 0_6 0_7 \\
a_4 2_3 & a_{13} 2_2 & 1_1 4_3 \\
a_5 3_3 & a_{14} 5_3 & 1_2 4_4 \\
a_6 3_4 & a_{15} 5_7 & 1_3 4_5 \\
a_7 3_6 & a_{16} 3_7 & 1_4 4_6 \\
a_8 5_4 & a_{17} 0_5 & 1_5 4_7 \\
a_9 2_1 & 3_1 5_1 & 1_6 4_1 \\
 & & 1_7 4_2
\end{array}
\;;
$$

$$
\begin{array}{ll}
a_1 3_6 & a_{15} 5_2 \\
a_2 3_7 & a_{16} 5_4 \\
a_3 3_1 & a_{17} 3_4 \\
a_4 0_5 2_7 & 0_1 0_3 \\
a_5 0_6 2_1 & 0_2 0_4 \\
a_6 0_7 2_2 & 2_5 2_6 \\
a_7 3_5 & 3_3 5_3 \\
a_8 5_6 & 1_1 4_4 \\
a_9 5_7 & 1_2 4_5 \\
a_{10} 5_1 & 1_3 4_6 \\
a_{11} 3_2 & 1_4 4_7 \\
a_{12} 2_3 & 1_5 4_1 \\
a_{13} 2_4 & 1_6 4_2 \\
a_{14} 5_5 & 1_7 4_3
\end{array}
\;;\qquad
\begin{array}{ll}
a_1 1_7 & a_{15} 1_3 \\
a_2 1_1 & a_{16} 2_1 \\
a_3 1_2 & a_{17} 4_1 \\
a_4 2_2 & 0_5 0_7 \\
a_5 1_4 & 0_4 2_4 \\
a_6 2_5 & 0_6 2_6 \\
a_7 0_1 2_3 & 2_7 4_7 \\
a_8 0_2 1_5 & 5_1 3_2 \\
a_9 0_3 1_6 & 5_2 3_3 \\
a_{10} 4_2 & 5_3 3_4 \\
a_{11} 4_3 & 5_4 3_5 \\
a_{12} 4_4 & 5_5 3_6 \\
a_{13} 4_5 & 5_6 3_7 \\
a_{14} 4_6 & 5_7 3_1
\end{array}
\;;\qquad
\begin{array}{ll}
a_1 2_5 & a_{14} 0_1 1_4 \\
a_2 2_6 & a_{15} 1_6 \\
a_3 2_7 & a_{16} 0_3 \\
a_4 2_1 & a_{17} 0_2 \\
a_5 2_2 & 4_1 4_2 4_5 \\
a_6 2_3 & 4_3 4_4 4_6 \\
a_7 2_4 & 5_1 3_3 \\
a_8 4_7 & 5_2 3_4 \\
a_9 1_5 & 5_3 3_5 \\
a_{10} 0_4 1_7 & 5_4 3_6 \\
a_{11} 0_5 1_1 & 5_5 3_7 \\
a_{12} 0_6 1_2 & 5_6 3_1 \\
a_{13} 0_7 1_3 & 5_7 3_2
\end{array}
\qquad \text{mod } 6
$$

together with

$$
\begin{array}{cccc}
a_1(1+i)_1(2+i)_1 & a_6(1+i)_6(2+i)_6 & a_{11}(4+i)_4(5+i)_4 & a_{16}i_2(3+i)_2 \\
a_2(1+i)_2(2+i)_2 & a_7(1+i)_7(2+i)_7 & a_{12}(4+i)_5(5+i)_5 & a_{17}i_3(3+i)_3 \\
a_3(1+i)_3(2+i)_3 & a_8(4+i)_1(5+i)_1 & a_{13}(4+i)_6(5+i)_6 & i_4(3+i)_4 \\
a_4(1+i)_4(2+i)_4 & a_9(4+i)_2(5+i)_2 & a_{14}(4+i)_7(5+i)_7 & i_5(3+i)_5 \\
a_5(1+i)_5(2+i)_5 & a_{10}(4+i)_3(5+i)_3 & a_{15}i_1(3+i)_1 & i_6(3+i)_6 \\
 & & & i_7(3+i)_7
\end{array}
$$

for $i = 0, 2, 4$.

Ten classes on $\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}$ are

$$
\begin{array}{llll}
0_1 1_2 2_4 & 0_5 1_6 2_1 & 5_1 4_2 3_4 & 5_5 4_6 3_1 \\
0_2 1_3 2_5 & 0_6 1_7 2_2 & 5_2 4_3 3_5 & 5_6 4_7 3_2 \\
0_3 1_4 2_6 & 0_7 1_1 2_3 & 5_3 4_4 3_6 & 5_7 4_1 3_3 \\
0_4 1_5 2_7 & & 5_4 4_5 3_7 &
\end{array}
\qquad \text{mod } 6
$$

and

$$\begin{array}{cccc}
i_1(3+i)_2 & i_6(3+i)_7 & (1+i)_4(4+i)_5 & (2+i)_2(2+i)_6 \\
i_2(3+i)_3 & i_7(3+i)_1 & (1+i)_5(4+i)_6 & (2+i)_3(2+i)_5 \\
i_3(3+i)_4 & (1+i)_1(4+i)_2 & (1+i)_6(4+i)_7 & (5+i)_1(5+i)_4(5+i)_7 \\
i_4(3+i)_5 & (1+i)_2(4+i)_3 & (1+i)_7(4+i)_1 & (5+i)_2(5+i)_6 \\
i_5(3+i)_6 & (1+i)_3(4+i)_4 & (2+i)_1(2+i)_4(2+i)_7 & (5+i)_3(5+i)_5
\end{array}$$

for $i = 0, 2, 4$, and finally

$$0_1 0_6 \qquad 0_4 0_5 \qquad 0_2 0_3 0_7 \qquad\qquad \text{mod } 6.$$

Now construct an $RRP(17,10)$on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(59,46). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}) \cup \{a_i : 1 \le i \le 17\}$.

Proceed as in the previous ($RRP(59,43)$) case, but replace the first nine classes on $\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}$ by the following twelve classes:

$$\begin{array}{ccccccccc}
0_1 1_2 2_4 & 0_6 1_7 2_2 & 5_4 3_7 & & 1_1 0_2 & 2_1 5_2 & 4_1 3_3 & & \\
0_2 1_3 2_5 & 0_7 1_1 2_3 & 5_5 3_1 & & 1_2 0_3 & 2_2 5_3 & 4_2 3_4 & & \\
0_3 1_4 2_6 & 5_1 3_4 & 5_6 3_2 & & 1_3 0_4 & 2_3 5_4 & 4_3 3_5 & & \\
0_4 1_5 2_7 & 5_2 3_5 & 5_7 3_3 & ; & 1_4 0_5 & 2_4 5_5 & 4_4 3_6 & & \text{mod } 6 \\
0_5 1_6 2_1 & 5_3 3_6 & 4_1 4_4 4_7 & & 1_5 0_6 & 2_5 5_6 & 4_5 3_7 & & \\
 & 4_2 4_6 & 4_3 4_5 & & 1_6 0_7 & 2_6 5_7 & 4_6 3_1 & & \\
 & & & & 1_7 0_1 & 2_7 5_1 & 4_7 3_2 & &
\end{array}$$

Then construct an $RRP(17,13)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(59,49). Point set $\mathbb{Z}_{36} \cup \{a_i : 1 \le i \le 23\}$.

Form a parallel class by taking as blocks $2,16$ $3,5$ $4,14$ $18,35$ $21,32$ $25,26,28$ together with the twenty-three pairs obtained by matching the a_i's with the remaining residues in $\mathbb{Z}_{36}$. Develop this class mod 36. Thirteen classes on $\mathbb{Z}_{36}$ are obtained by developing the blocks $0,4,9$ $1,7,14$ $3,11$ $5,20$ $6,22$; and $0,18$ mod 36 (the first set of blocks covers each residue class mod 12 once). Construct an $RRP(23,13)$ on $\{a_1, a_2, \ldots, a_{23}\}$.

RRP(59,52). Point set $\mathbb{Z}_{36} \cup \{a_i : 1 \le i \le 23\}$.

Proceed as in the previous ($RRP(59,49)$) case, replacing the pair $4,14$ by $23,30$ in the parallel class, and the set $0,4,9$ $1,7,14$ $3,11$ $5,20$ $6,22$ on $\mathbb{Z}_{36}$ by the sets $0,9$ $1,7$ $4,14$ $11,15$ $5,20$ $6,22$; and $0,5,13$. Construct an $RRP(23,16)$ on $\{a_1, a_2, \ldots, a_{23}\}$.

RRP(59,55). Lemma 1.3.

3. p = 79,83,85,89. We construct these designs recursively, in the same fashion as the bulk of the designs in [6] and [7] were constructed, i.e. we construct frames by applying weight 6 to group-divisible designs with block size four. Then we 'fill in the holes' using construction 1.2. We begin by recalling Lemma 3.3 of [5]:

LEMMA 3.1. *Let d_1, d_2, d_3, d_4 be integers, $3 \le d_i \le 6$. There exists a* 2, 3*-frame of type* 6^4 *with holes H_1, H_2, H_3, H_4 such that hole H_i has degree d_i, $1 \le i \le 4$.*

We will employ $4 - GDD$s of types 3^4 and 2^7, according to the following table:

TABLE I

p	4-GDD of type	Remarks
79	3^4	Resulting frames have type 18^4; add 7 ideal points $(79 = 12 \cdot 6 + 7)$.
83	3^4	Resulting frames have type 18^4; add 11 ideal points $(83 = 12 \cdot 6 + 11)$.
85 $(45 \le k \le 63)$	2^7	Resulting frames have type 12^7; add 1 ideal point $(85 = 14 \cdot 6 + 1)$.
85 $(63 \le k \le 81)$	3^4	Resulting frames have type 18^4; add 13 ideal points $(85 = 12 \cdot 6 + 13)$.
89	2^7	Resulting frames have type 12^7; add 5 ideal points $(89 = 14 \cdot 6 + 5)$.

Construction 1.2 is then applied, according to table II:

TABLE II

$p; k$	Input designs	Source
$79; 42 \le k \le 51$	$RRP(25,15) - RRP(7,6)$	Remove two points from a group in a resolvable $TD(3,9)$. Fill in $RRP(9,6)$s.
	$RRP(25,18) - RRP(7,6)$	Appendix
	$RRP(25,15)$	Theorem 1.0
$79; 51 \le k \le 75$	$RRP(25,15) - RRP(7,3)$	Appendix
	$RRP(25,18) - RRP(7,3)$	Appendix
	$RRP(25,21) - RRP(7,3)$	Appendix
	$RRP(25,15), RRP(25,18), RRP(25,21)$	Theorem 1.0
$83; 52 \le k \le 79$	$RRP(29,19) - RRP(11,7)$	[7,Section 3]
	$RRP(29,22) - RRP(11,7)$	[7,Section 3]
	$RRP(29,25) - RRP(11,7)$	Appendix
	$RRP(29,16), RRP(29,19), RRP(29,22), RRP(29,25)$	Theorem 1.0
$85; 45 \le k \le 63$	$RRP(13,7) - RRP(1,1)$	[7,Lemma 3.6]
	$RRP(13,10) - RRP(1,1)$	[4,section 3B]
	$RRP(13,9)$	Theorem 1.0
$85; 63 \le k \le 81$	$RRP(31,24) - RRP(13,9)$	
	$RRP(31,27) - RRP(13,9)$	
	$RRP(31,18), RRP(31,21)$	
	$RRP(31,24), RRP(31,27)$	Section 2
$89; 46 \le k \le 64$	$RRP(17,9) - RRP(5,3)$	There is an NKTS(18) $- NKTS(6)$, by Brouwer [1]. Remove a point from the 'missing' subdesign.
	$RRP(17,12) - RRP(5,3)$	Appendix
	$RRP(17,10)$	Theorem 1.0
$89; 64 \le k \le 85$	$RRP(17,10) - RRP(5,1)$	Appendix
	$RRP(17,13) - RRP(5,1)$	Appendix
	$RRP(17,10), RRP(17,13)$	Theorem 1.0

The only designs that are not constructed by table II are $RRP(83,k)$ where $k = 43, 46, 49$. These are obtained by deleting a point from a resolvable $TD(3,28)$ and applying construction 1.1, filling in, respectively, $RRP(28,15)$s and an $RRP(27,15)$; $RRP(28,18)$s and an $RRP(27,18)$; or $RRP(28,21)$s and an $RRP(27,21)$.

4. p = 49,53,55,61,65,67,71. Most of the designs corresponding to these orders will be constructed recursively, using frames as in section 3; however, in this case we will build our frames by applying weight **four** to group-divisible designs with block size four. We will therefore need the following analogue of Lemma 3.1.:

LEMMA 4.1. *Let d_1, d_2, d_3, d_4 be integers, with each $d_i = 2, 3$ or 4. There is a $2,3$-frame of type 4^4 with holes H_1, H_2, H_3, H_4 such that hole H_i has degree d_i, $i = 1,2,3,4$.*

Proof. We begin with an observation. In each of our frames, a hole of degree two will correspond to two Δ-factors. Now, up to isomorphism, there is only one way to arrange two Δ-factors on 12 points, viz:

ABC	AEI
DEF	DHL
GHI	GKC
JKL	JBF

Note that we can break each triple into its constituent pairs and then arrange these pairs into four 1-factors:

BF	EI	BC	EF
HL	KC	HI	KL
AC	DF	JF	AI
DE	GH	GK	JB
GI	JL	DL	GC
JK	AB	AE	DH

That is, a hole of degree two can always be converted to a hole of degree four. It is therefore, necessary only to construct frames with $d_i = 2$ or 3.

$\mathbf{d_i = 2; i = 1,2,3,4}$

We use the following incomplete orthogonal array (i.e. a $TD(4,6) - TD(4,2)$), which appears in [2] (we have rearranged the columns):

5 5 5 5	6 6 6 6	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
1 2 3 4	1 2 3 4	5 5 5 5	6 6 6 6	1 4 2 3	2 3 1 4	3 2 4 1	4 1 3 2
1 4 2 3	3 2 4 1	1 3 4 2	2 4 3 1	5 5 5 5	6 6 6 6	3 1 2 4	4 2 1 3
2 4 3 1	3 1 2 4	3 4 1 2	4 3 2 1	1 2 3 4	2 1 4 3	5 5 5 5	6 6 6 6

By identifying symbol i in row j with the point i_j ($i = 1,2,3,4,5,6$; $j = 1,2,3,4$) and deleting the points 5_j, 6_j for $j = 1,2,3,4$ we get the following frame:

Holes: $1_j 2_j 3_j 4_j \qquad j = 1,2,3,4$

Holey parallel classes:

$$\begin{array}{cccc}
1_2 1_3 2_4 & 1_1 1_3 3_4 & 1_1 1_2 1_4 & 1_1 3_2 3_3 \\
2_2 4_3 4_4 & 2_1 3_3 4_4 & 2_1 4_2 2_4 & 2_1 2_2 1_3 \\
3_3 2_3 3_4 & 3_1 4_3 1_4 & 3_1 2_2 3_4 & 3_1 4_2 2_3 \\
4_2 3_3 1_4 & 4_1 2_3 2_4 & 4_1 3_2 4_4 & 4_1 1_2 4_3
\end{array}$$

$$\begin{array}{cccc}
1_2 3_3 4_4 & 1_1 2_3 4_4 & 1_1 2_2 2_4 & 1_1 4_2 4_3 \\
2_2 2_3 1_4 & 2_1 4_3 3_4 & 2_1 3_2 1_4 & 2_1 1_2 2_3 \\
3_2 4_3 2_4 & 3_1 3_3 2_4 & 3_1 1_2 4_4 & 3_1 3_2 1_3 \\
4_2 1_3 4_4 & 4_1 1_3 1_4 & 4_1 4_2 3_4 & 4_1 2_2 3_3
\end{array}$$

$\mathbf{d_i = 2; i = 1, 2, 3.}$

$$\text{Points:} \quad (\mathbb{Z}_3 \times \{1,2,3,4\}) \cup \{\infty_1, \infty_2, \infty_3, \infty_4\}.$$

$$\text{Holes:} \quad \{\{j\} \times \{1,2,3,4\} : j = 0,1,2\} \cup \{\{\infty_1, \infty_2, \infty_3, \infty_4\}\}.$$

Holey parallel classes:

$$\begin{array}{c} \infty_1 0_1 1_2 \\ \infty_2 0_2 1_3 \\ \infty_3 0_3 1_4 \\ \infty_4 0_4 1_1 \end{array} \; ; \quad \begin{array}{c} \infty_1 1_3 0_4 \\ \infty_2 1_4 0_1 \\ \infty_3 1_1 0_2 \\ \infty_4 1_2 0_3 \end{array} \qquad \text{mod } 3$$

$$\begin{array}{c} 0_1 1_1 2_1 \\ 0_2 1_2 2_2 \\ 0_3 1_3 2_3 \\ 0_4 1_4 2_4 \end{array} \; ; \quad \begin{array}{cc} 0_1 1_3 & 0_2 1_4 \\ 1_1 2_3 & 1_2 2_4 \\ 2_1 0_3 & 2_2 0_4 \end{array} \; ; \quad \begin{array}{cc} 0_1 2_3 & 0_2 2_4 \\ 1_1 0_3 & 1_2 0_4 \\ 2_1 1_3 & 2_2 1_4 \end{array}$$

$\mathbf{d_i = 2; i = 1, 2.}$

$$\text{Points:} \quad \mathbb{Z}_4 \times \{1,2,3,4\}.$$

$$\text{Holes:} \quad \{\{j\} \times \{1,2,3,4\} : j = 0,1,2,3\}.$$

Holey parallel classes:

$$\begin{array}{c} 1_1 2_2 3_2 \\ 1_2 2_3 3_3 \\ 1_3 2_4 3_4 \\ 1_4 2_1 3_1 \end{array} \qquad \text{mod } 4 \; ; \quad \begin{array}{c} 1_1 2_4 3_3 \\ 1_2 2_1 3_4 \\ 1_3 2_2 3_1 \\ 1_4 2_3 3_2 \end{array} \; ; \quad \begin{array}{c} 0_1 1_3 3_3 \\ 0_2 1_4 3_4 \\ 0_3 1_1 3_1 \\ 0_4 1_2 3_2 \end{array} \; ;$$

$$\begin{array}{cc} 0_1 2_1 & 0_3 2_2 \\ 3_3 0_2 & 3_1 0_4 \\ 2_2 3_4 & 2_4 3_2 \end{array} \; ; \quad \begin{array}{cc} 2_1 3_3 & 2_3 3_1 \\ 0_2 2_2 & 0_4 2_4 \\ 3_4 0_3 & 3_2 0_1 \end{array} \; ; \quad \begin{array}{cc} 0_1 1_4 & 0_3 1_2 \\ 2_2 0_4 & 2_4 0_2 \\ 1_3 2_1 & 1_1 2_3 \end{array} \; ; \quad \begin{array}{cc} 1_4 2_2 & 1_2 2_4 \\ 0_4 1_3 & 0_2 1_1 \\ 2_1 0_3 & 2_3 0_1 \end{array}$$

$\mathbf{d_i = 2; i = 1.}$

$$\text{Points:}\quad \mathbf{Z}_4 \times \{1,2,3,4\}.$$

$$\text{Holes:}\quad \{\{j\} \times \{1,2,3,4\} : j = 0,1,2,3\}.$$

Holey parallel classes:

$$\begin{array}{l} 1_1 3_2 2_3 \\ 1_2 3_3 2_4 \\ 1_3 3_4 2_1 \\ 1_4 3_1 2_2 \end{array} \quad \text{mod } 4 \quad ; \quad \begin{array}{l} 1_1 2_2 3_3 \\ 1_2 2_3 3_4 \\ 1_3 2_4 3_1 \\ 1_4 2_1 3_2 \end{array} \quad ; \quad \begin{array}{ll} 0_1 1_2 & 0_3 1_4 \\ 3_2 0_2 & 3_4 0_4 \\ 1_3 3_3 & 1_1 3_1 \end{array} \quad ; \quad \begin{array}{ll} 1_2 3_2 & 1_4 3_4 \\ 0_2 1_3 & 0_4 1_1 \\ 3_3 0_3 & 3_1 0_1 \end{array} \quad ;$$

$$\begin{array}{ll} 0_1 3_4 & 0_3 3_2 \\ 2_4 0_4 & 2_2 0_2 \\ 3_3 2_3 & 3_1 2_1 \end{array} \quad ; \quad \begin{array}{ll} 3_4 2_4 & 3_2 2_2 \\ 0_4 3_3 & 0_2 3_1 \\ 2_3 0_3 & 2_1 0_1 \end{array} \quad ; \quad \begin{array}{ll} 0_1 2_3 & 0_2 2_4 \\ 1_3 0_3 & 1_4 0_4 \\ 2_1 1_1 & 2_2 1_2 \end{array} \quad ; \quad \begin{array}{ll} 2_3 1_3 & 2_4 1_4 \\ 0_3 2_1 & 0_4 2_2 \\ 1_1 0_1 & 1_2 0_2 \end{array}$$

$\mathbf{d_i = 3; i = 1,2,3,4.}$

$$\text{Points:}(\mathbf{Z}_3 \times \{1,2,3,4\}) \cup \{\infty_1, \infty_2, \infty_3, \infty_4\}.$$

$$\text{Holes:}\{\{j\} \times \{1,2,3,4\} : j = 0,1,2\} \cup \{\{\infty_1, \infty_2, \infty_3, \infty_4\}\}$$

Holey parallel classes:

$$\begin{array}{l} \infty_1 1_1 0_3 \\ \infty_2 1_2 0_4 \\ \infty_3 1_3 0_1 \\ \infty_4 1_4 0_2 \end{array} \; ; \; \begin{array}{l} \infty_1 1_2 \\ \infty_2 0_1 \\ \infty_3 1_4 \\ \infty_4 0_3 \\ 1_1 0_2 \\ 1_3 0_4 \end{array} \; ; \; \begin{array}{l} \infty_1 1_4 \\ \infty_2 0_3 \\ \infty_3 1_2 \\ \infty_4 0_1 \\ 1_1 0_4 \\ 1_3 0_2 \end{array} \; ; \; \begin{array}{l} 0_1 1_2 \\ 0_3 1_4 \end{array} \; ; \; \begin{array}{l} 0_1 1_4 \\ 0_3 1_2 \end{array} \; ; \; \begin{array}{l} 0_1 1_1 2_1 \\ 0_2 1_2 2_2 \\ 0_3 1_3 2_3 \\ 0_4 1_4 2_4 \end{array} \quad \text{mod } 3$$

This completes the proof of lemma 4.1. □

We construct our frames by applying weight 4 to 4-GDDs of types 3^4, 3^5 and 4^4, according to the following table.

TABLE III

p	4-GDD of type	Remarks
49	3^4	Resulting frames have type 12^4; add 1 ideal point $(49 = 12 \cdot 4 + 1)$.
53	3^4	Resulting frames have type 12^4; add 5 ideal points $(53 = 12 \cdot 4 + 5)$.
55	3^4	Resulting frames have type 12^4; add 7 ideal points $(55 = 12 \cdot 4 + 7)$.
61	3^5	Resulting frames have type 12^5; add 1 ideal point $(61 = 15 \cdot 4 + 1)$.
65	4^4	Resulting frames have type 16^4; add 1 ideal point $(65 = 16 \cdot 4 + 1)$.
67	4^4	Resulting frames have type 16^4; add 3 ideal points $(67 = 16 \cdot 4 + 3)$.
71	4^4	Resulting frames have type 16^4; add 7 ideal points $(71 = 16 \cdot 4 + 7)$.

Now apply construction 1.2, according to table IV.

TABLE IV

$p; k$	Input designs	Source
$49; 27 \leq k \leq 36$	$RRP(13,7) - RRP(1,1)$ $RRP(13,10) - RRP(1,1)$ $RRP(13,9)$	[7, Lemma 3.6] [4,section 3B] Theorem 1.0
$53; 28 \leq k \leq 37$	$RRP(17,9) - RRP(5,3)$ $RRP(17,12) - RRP(5,3)$ $RRP(17,10)$	See Table II Appendix Theorem 1.0
$53; 37 \leq k \leq 49$	$RRP(17,10) - RRP(5,1)$ $RRP(17,13) - RRP(5,1)$ $RRP(17,10), RRP(17,13)$	Appendix Appendix Theorem 1.0
$55; k = 39, 42$	$RRP(19,13) - RRP(7,4)$ $RRP(19,12), RRP(19,15)$	[7,Lemma 3.6] Theorem 1.0
$61; 33 \leq k \leq 45$	$RRP(13,7) - RRP(1,1)$ $RRP(13,10) - RRP(1,1)$ $RRP(13,9)$	[7,Lemma 3.6] [4, section 3B] Theorem 1.0
$65; 40 \leq k \leq 52$	$RRP(17,10), RRP(17,13)$	Theorem 1.0
$67; k = 42$	$RRP(19,12) - RRP(3,2)$ $RRP(19,12)$	Appendix Theorem 1.0
$67; 45 \leq k \leq 57$	$RRP(19,12), RRP(19,15)$	Theorem 1.0
$71; 37 \leq k \leq 61$	$RRP(23,12) - RRP(7,4)$	Remove a point from a resolvable $TD(3,8)$ and fill in $RRP(8,4)$s.
	$RRP(23,18) - RRP(7,4)$	[7, Lemma 4.3]
	$RRP(23,13), RRP(23,16),$ $RRP(23,19)$	Theorem 1.0

The following designs have yet to be constructed, not being covered by Table IV or

Lemma 1.3:

$$
\begin{aligned}
&p = 49; k = 39, 42\\
&p = 55; k = 30, 33, 36, 45, 48\\
&p = 61; k = 48, 51, 54\\
&p = 65; k = 34, 37, 55, 58\\
&p = 67; k = 36, 39, 60\\
&p = 71; k = 64
\end{aligned}
$$

RRP(49,39). Point set $\mathbb{Z}_{27} \cup \{a_i : 1 \leq i \leq 22\}$.

Form a parallel class by taking the blocks $0,5,14$ $\;1,13$ together with the twenty-two pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{27}$. Develop this class mod 27. Twelve classes on $\mathbb{Z}_{27}$ are obtained by developing the blocks $0,1,3$ $\;2,8$ $\;4,14$ $\;6,25$; and $0,4,11$ mod 27 (the first set of blocks covers each residue mod 9 once, and the triple covers each residue mod 3 once). Then construct an $RRP(22,12)$ on $\{a_1, a_2, \ldots, a_{22}\}$.

RRP(49,42). Point set $\mathbb{Z}_{30} \cup \{a_i : 1 \leq i \leq 19\}$.

Form a parallel class by taking the blocks $2,14$ $\;3,13$ $\;16,29$ $\;18,27$ $\;21,22,24$ together with the nineteen pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{30}$. Develop this class mod 30. Twelve classes on $\mathbb{Z}_{30}$ are obtained by developing the blocks $0,4,11$ $\;2,8$; $0,5$ $\;1,9$ $\;2,16$; and $0,15$ mod 30 (the first set of blocks covers each residue mod 5 once, and the second set covers each residue mod 6 once). Then construct an $RRP(19,12)$ on $\{a_1, a_2, \ldots, a_{19}\}$.

RRP(55,30). Remove five points from a group in a resolvable $TD(3,20)$ and apply Construction 1.1., filling in $RRP(20,10)$s and an $RRP(15,10)$.

RRP(55,33). Point set $(\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}) \cup \{a_i : 1 \leq i \leq 13\}$.

Twenty-four parallel classes:

$a_1 0_7 2_2$	$a_8 5_7 3_2$	$1_1 1_6$		$a_1 0_3 2_6$	$a_8 5_3 3_6$	$1_1 4_4$	
$a_2 0_1 2_3$	$a_9 5_1 3_3$	$1_2 1_4$		$a_2 0_4 2_7$	$a_9 5_4 3_7$	$1_2 4_5$	
$a_3 0_2 2_4$	$a_{10} 5_2 3_4$	$1_3 1_5$		$a_3 0_5 2_1$	$a_{10} 5_5 3_1$	$1_3 4_6$	
$a_4 0_3 2_5$	$a_{11} 5_3 3_5$	$4_1 4_3$	;	$a_4 0_6 2_2$	$a_{11} 5_6 3_2$	$1_4 4_7$	;
$a_5 0_4 2_6$	$a_{12} 5_4 3_6$	$4_2 4_7$		$a_5 0_7 2_3$	$a_{12} 5_7 3_3$	$1_5 4_1$	
$a_6 0_5 2_7$	$a_{13} 5_5 3_7$	$4_4 4_6$		$a_6 0_1 2_4$	$a_{13} 5_1 3_4$	$1_6 4_2$	
$a_7 0_6 2_1$	$5_6 3_1$	$4_5 1_7$		$a_7 0_2 2_5$	$5_2 3_5$	$1_7 4_3$	

$$
\begin{array}{lll}
a_1 0_4 1_5 & a_8 3_4 2_5 & 4_1 5_1 \\
a_2 0_5 1_6 & a_9 3_5 2_6 & 4_2 5_2 \\
a_3 0_6 1_7 & a_{10} 3_6 2_7 & 4_3 5_3 \\
a_4 0_7 1_1 & a_{11} 3_7 2_1 & 4_4 5_4 \\
a_5 0_1 1_2 & a_{12} 3_1 2_2 & 4_5 5_5 \\
a_6 0_2 1_3 & a_{13} 3_2 2_3 & 4_6 5_6 \\
a_7 0_3 1_4 & 3_3 2_4 & 4_7 5_7
\end{array}
\quad ; \quad
\begin{array}{lll}
a_1 3_1 & a_9 5_2 & 0_7 2_7 \\
a_2 4_2 & a_{10} 5_3 & 0_1 3_2 0_4 \\
a_3 4_3 & a_{11} 4_4 & 0_2 3_3 0_5 \\
a_4 1_4 & a_{12} 4_5 & 0_3 3_4 0_6 \\
a_5 3_5 & a_{13} 5_6 & 5_4 5_5 5_7 \\
a_6 3_6 & 2_1 2_2 & 1_5 4_6 1_1 \\
a_7 3_7 & 2_3 2_4 & 1_6 4_7 1_2 \\
a_8 5_1 & 2_5 2_6 & 1_7 4_1 1_3
\end{array}
\qquad \text{mod } 6
$$

Nine classes on $\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}$:

$$
\begin{array}{ll}
0_1 2_2 1_4 & 5_1 3_2 4_4 \\
0_2 2_3 1_5 & 5_2 3_3 4_5 \\
0_3 2_4 1_6 & 5_3 3_4 4_6 \\
0_4 2_5 1_7 & 5_4 3_5 4_7 \\
0_5 2_6 1_1 & 5_5 3_6 4_1 \\
0_6 2_7 1_2 & 5_6 3_7 4_2 \\
0_7 2_1 1_3 & 5_7 3_1 4_3
\end{array}
\; ; \;
\begin{array}{c}
0_1 2_1 4_1 \\
0_2 2_2 4_2 \\
0_3 2_3 4_3 \\
0_4 2_4 4_4 \\
0_5 2_5 4_5 \\
0_6 2_6 4_6 \\
0_7 3_7
\end{array}
\; ; \;
\begin{array}{c}
0_1 3_1 \\
0_2 3_2 \\
0_3 3_3 \\
0_4 3_4 \\
0_5 3_5 \\
0_6 0_7
\end{array}
\; ; \;
\begin{array}{c}
0_1 0_7 \\
0_2 0_3 \\
0_4 3_5 \\
0_6 3_6
\end{array}
\qquad \text{mod } 6
$$

Now construct an $RRP(13,9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(55,36). Point set $\mathbb{Z}_6 \times \{1,2,3,4,5,6,7\}) \cup \{a_i : 1 \le i \le 13\}$.

Twenty-seven parallel classes:

$$
\begin{array}{lll}
a_1 0_7 2_2 & a_8 5_7 3_2 & 1_1 1_6 \\
a_2 0_1 2_3 & a_9 5_1 3_3 & 1_2 1_4 \\
a_3 0_2 2_4 & a_{10} 5_2 3_4 & 1_3 1_5 \\
a_4 0_3 2_5 & a_{11} 5_3 3_5 & 4_1 4_3 \\
a_5 0_4 2_6 & a_{12} 5_4 3_6 & 4_2 4_7 \\
a_6 0_5 2_7 & a_{13} 5_5 3_7 & 4_4 4_6 \\
a_7 0_6 2_1 & 5_6 3_1 & 4_5 1_7
\end{array}
\quad ; \quad
\begin{array}{lll}
a_1 0_3 2_6 & a_8 5_3 3_6 & 1_1 4_4 \\
a_2 0_4 2_7 & a_9 5_4 3_7 & 1_2 4_5 \\
a_3 0_5 2_1 & a_{10} 5_5 3_1 & 1_3 4_6 \\
a_4 0_6 2_2 & a_{11} 5_6 3_2 & 1_4 4_7 \\
a_5 0_7 2_3 & a_{12} 5_7 3_3 & 1_5 4_1 \\
a_6 0_1 2_4 & a_{13} 5_1 3_4 & 1_6 4_2 \\
a_7 0_2 2_5 & 5_2 3_5 & 1_7 4_3
\end{array}
\quad ;
$$

$$
\begin{array}{lll}
a_1 3_1 & a_9 5_2 & 2_3 2_4 \\
a_2 4_2 & a_{10} 5_3 & 2_5 2_6 \\
a_3 4_3 & a_{11} 4_4 & 1_4 4_5 \\
a_4 5_4 & a_{12} 5_5 & 4_1 5_7 \\
a_5 3_5 & a_{13} 5_6 & 0_1 3_2 0_4 \\
a_6 3_6 & 0_7 2_7 & 0_2 3_3 0_5 \\
a_7 3_7 & 1_3 1_7 & 0_3 3_4 0_6 \\
a_8 5_1 & 2_1 2_2 & 1_5 4_6 1_1 \\
 & & 1_6 4_7 1_2
\end{array}
\quad ; \quad
\begin{array}{lll}
a_1 4_4 & a_{10} 5_6 & 1_6 3_6 \\
a_2 1_5 & a_{11} 2_7 & 2_2 3_1 \\
a_3 0_6 & a_{12} 4_1 & 2_4 3_3 \\
a_4 1_7 & a_{13} 4_2 & 2_5 4_5 \\
a_5 2_1 & 0_1 3_7 & 2_6 3_5 \\
a_6 0_2 & 0_3 2_3 & 4_6 4_7 \\
a_7 1_3 & 1_1 4_3 & 5_1 5_7 \\
a_8 5_4 & 1_2 3_2 & 5_2 5_3 \\
a_9 5_5 & 1_4 3_4 & 0_4 0_5 0_7
\end{array}
\qquad \text{mod } 6
$$

$$\begin{array}{lllll}
a_1(1+i)_5(2+i)_5 & a_6(1+i)_3(2+i)_3 & a_{11}(4+i)_1(5+i)_1 & i_2(3+i)_2 & \\
a_2(1+i)_6(2+i)_6 & a_7(1+i)_4(2+i)_4 & a_{12}(4+i)_2(5+i)_2 & i_3(3+i)_3 & \\
a_3(1+i)_7(2+i)_7 & a_8(4+i)_5(5+i)_5 & a_{13}(4+i)_3(5+i)_3 & i_4(3+i)_4 & i=0,2,4. \\
a_4(1+i)_1(2+i)_1 & a_9(4+i)_6(5+i)_6 & (4+i)_4(5+i)_4 & i_5(3+i)_5 & \\
a_5(1+i)_2(2+i)_2 & a_{10}(4+i)_7(5+i)_7 & i_1(3+i)_1 & i_6(3+i)_6 & \\
 & & & i_7(3+i)_7 &
\end{array}$$

Nine classes on $\mathbf{Z}_6 \times \{1,2,3,4,5,6,7\}$:

$$\begin{array}{llcccc}
0_1 2_2 1_4 & 5_1 3_2 4_4 & & & & \\
0_2 2_3 1_5 & 5_2 3_3 4_5 & & 0_1 2_1 4_1 & & \\
0_3 2_4 1_6 & 5_3 3_4 4_6 & & 1_2 0_3 & & \\
0_4 2_5 1_7 & 5_4 3_5 4_7 & ; & 1_4 0_5 & & \bmod 6 \\
0_5 2_6 1_1 & 5_5 3_6 4_1 & & 1_6 0_7 & & \\
0_6 2_7 1_2 & 5_6 3_7 4_2 & & & & \\
0_7 2_1 1_3 & 5_7 3_1 4_3 & & & &
\end{array}$$

$$\begin{array}{lcll}
0_j 1_{j+1} & & 1_j 2_{j+1} & \\
2_j 3_{j+1} & ; & 3_j 4_{j+1} & j = 1,2,\ldots,7 (\text{addition} \quad \bmod 7) \\
4_j 5_{j+1} & & 5_j 0_{j+1} &
\end{array}$$

Now construct an $RRP(13,9)$ on $\{a_1, a_2, \ldots, a_{13}\}$.

RRP(55,45). Point set $\mathbf{Z}_{34} \cup \{a_i : 1 \le i \le 21\}$

Form a parallel class by taking as blocks $0,16$ $2,17$ $1,3,4$ $4,9,15$ $6,14,26$ together with the twenty-one pairs obtained by matching off the a_i's with the remaining residues in $\mathbf{Z}_{34}$. Develop this class mod 34. Eleven one-factors on $\mathbf{Z}_{34}$ are obtained by developing the pairs $0,5$ $0,7$ $0,9$ $0,11$ $0,13$ and $0,17$ mod 34 (the first five pairs each generate two one-factors). Then construct an $RRP(21,11)$ on $\{a_1, a_2, \ldots, a_{21}\}$.

RRP(55,48). Point set $\mathbf{Z}_{36} \cup \{a_i : 1 \le i \le 19\}$.

Form a parallel class by taking as blocks $1,17$ $3,15$ $4,14$ $6,12$ $19,34$ $20,33$ $22,31$ $25,26,28$ together with the nineteen pairs obtained by matching off the a_i's with the remaining residues in $\mathbf{Z}_{36}$. Develop this class mod 36. Twelve classes on $\mathbf{Z}_{36}$ are obtained by developing the blocks $0,4,11$; $0,5$ $1,9$ $2,16$; $0,17$ and $0,18$ mod 36 (the first set of pairs covers each residue mod 6 once, and the triple covers each residue mod 3 once). Then construct an $RRP(19,12)$ on $\{a_1, a_2, \ldots, a_{19}\}$.

RRP(61,48). Point set $\mathbf{Z}_6 \times \{1,2,3,4,5,6\} \cup \{a_i : 1 \le i \le 25\}$.

Thirty parallel classes:

$$
\begin{array}{llll}
a_1 2_1 0_2 & a_8 0_5 & a_{15} 5_5 & a_{22} 4_6 \\
a_2 1_2 1_3 & a_9 2_6 & a_{16} 3_3 & a_{23} 1_1 \\
a_3 2_3 0_4 & a_{10} 0_1 & a_{17} 3_4 & a_{24} 4_2 \\
a_4 1_4 1_5 & a_{11} 5_6 & a_{18} 3_5 & a_{25} 4_3 \\
a_5 2_5 0_6 & a_{12} 5_1 & a_{19} 1_6 & 4_1 3_2 \\
a_6 2_2 & a_{13} 5_2 & a_{20} 3_1 & 0_3 5_4 \\
a_7 2_4 & a_{14} 5_3 & a_{21} 4_4 & 4_5 3_6
\end{array}
\; ; \quad
\begin{array}{llll}
a_1 3_4 & a_8 1_4 1_6 & a_{15} 4_6 & a_{22} 1_1 \\
a_2 3_5 & a_9 2_1 2_5 & a_{16} 0_6 & a_{23} 0_2 \\
a_3 3_6 & a_{10} 2_2 2_6 & a_{17} 5_1 & a_{24} 2_3 \\
a_4 3_1 & a_{11} 1_2 & a_{18} 5_2 & a_{25} 2_4 \\
a_5 3_2 & a_{12} 4_3 & a_{19} 5_3 & 4_2 3_3 \\
a_6 0_1 0_3 & a_{13} 4_4 & a_{20} 5_4 & 0_4 5_5 \\
a_7 1_3 1_5 & a_{14} 4_5 & a_{21} 0_5 & 5_6 4_1
\end{array}
\; ;
$$

$$
\begin{array}{llll}
a_1 2_3 & a_8 3_2 & a_{15} 1_2 1_4 & a_{22} 4_2 \\
a_2 2_4 & a_9 1_3 & a_{16} 5_5 & a_{23} 4_3 \\
a_3 2_5 & a_{10} 3_4 & a_{17} 5_6 & a_{24} 4_4 \\
a_4 2_6 & a_{11} 0_1 0_4 & a_{18} 5_1 & a_{25} 4_5 \\
a_5 4_1 & a_{12} 0_2 0_5 & a_{19} 5_2 & 2_1 5_4 \\
a_6 3_5 & a_{13} 0_3 0_6 & a_{20} 5_3 & 3_3 1_5 \\
a_7 3_1 & a_{14} 1_1 1_6 & a_{21} 3_6 & 4_6 2_2
\end{array}
\; ; \quad
\begin{array}{llll}
a_1 1_5 & a_8 2_1 & a_{15} 5_3 & a_{22} 4_4 \\
a_2 1_6 & a_9 2_2 & a_{16} 0_1 3_2 & a_{23} 4_5 \\
a_3 1_1 & a_{10} 2_3 & a_{17} 0_2 3_3 & a_{24} 4_6 \\
a_4 1_2 & a_{11} 5_5 & a_{18} 0_3 3_4 & a_{25} 4_1 \\
a_5 4_3 & a_{12} 5_6 & a_{19} 0_4 3_5 & 3_1 1_3 \\
a_6 1_4 & a_{13} 5_1 & a_{20} 0_5 3_6 & 5_2 2_5 \\
a_7 2_6 & a_{14} 5_4 & a_{21} 4_2 & 2_4 0_6
\end{array}
\; ;
$$

$$
\begin{array}{llll}
a_1 1_6 & a_8 2_3 & a_{15} 4_1 & a_{22} 0_3 3_5 \\
a_2 1_1 & a_9 2_4 & a_{16} 5_4 & a_{23} 0_4 3_6 \\
a_3 1_2 & a_{10} 2_5 & a_{17} 5_5 & a_{24} 0_5 3_1 \\
a_4 1_3 & a_{11} 4_3 & a_{18} 5_6 & a_{25} 0_6 3_2 \\
a_5 1_4 & a_{12} 4_4 & a_{19} 2_1 & 5_3 2_6 \\
a_6 4_6 & a_{13} 4_5 & a_{20} 0_2 & 1_5 5_1 \\
a_7 2_2 & a_{14} 4_2 & a_{21} 0_1 3_3 & 5_2 3_4
\end{array}
\qquad \text{mod } 6
$$

Eighteen classes on $\mathbb{Z}_6 \times \{1,2,3,4,5,6\}$:

$$
\begin{array}{lll}
0_1 1_2 2_4 & 0_6 1_1 2_3 & 3_5 4_2 \\
0_2 1_3 2_5 & 3_1 4_4 & 3_6 4_3 \\
0_3 1_4 2_6 & 3_2 4_5 & 5_1 5_2 \\
0_4 1_5 2_1 & 3_3 4_6 & 5_3 5_4 \\
0_5 1_6 2_2 & 3_4 4_1 & 5_5 5_6
\end{array}
\; ; \quad
\begin{array}{lll}
0_1 2_2 & 1_1 3_3 & 5_1 4_3 \\
0_2 2_3 & 1_2 3_4 & 5_2 4_4 \\
0_3 2_4 & 1_3 3_5 & 5_3 4_5 \\
0_4 2_5 & 1_4 3_6 & 5_4 4_6 \\
0_5 2_6 & 1_5 3_1 & 5_5 4_1 \\
0_6 2_1 & 1_6 3_2 & 5_6 4_2
\end{array}
\; ;
$$

$$
\begin{array}{lll}
0_1 3_1 & 0_3 3_3 & 0_5 3_5 \\
1_1 2_1 & 2_3 4_3 & 2_5 4_5 \\
4_1 5_1 & 5_3 1_3 & 5_5 1_5 \\
0_2 3_2 & 0_4 3_4 & 0_6 3_6 \\
1_2 2_2 & 1_4 2_4 & 1_6 2_6 \\
4_2 5_2 & 4_4 5_4 & 4_6 5_6
\end{array}
\; ; \quad
\begin{array}{l}
0_1 2_6 \\
2_2 0_3 \\
2_4 0_5
\end{array}
\qquad \text{mod } 6
$$

$$\begin{array}{cc} 0_j2_j4_j & 4_35_3 \\ 1_j3_j5_j & 0_51_5 \\ 0_31_3 & 2_53_5 \\ 2_33_3 & 4_55_5 \end{array} \qquad j=1,2,4,6; \qquad \begin{array}{c} i_1(3+i)_6 \\ (3+i)_1 i_6 \\ i_2(3+i)_4 \\ (3+i)_2 i_4 \\ (1+i)_3(2+i)_3 \\ (1+i)_5(2+i)_5 \end{array} \qquad i=0,2,4$$

Now construct on $RRP(25,18)$ on $\{a_1, a_2, \ldots, a_{25}\}$.

RRP(61,51). Point set $\mathbb{Z}_{40} \cup \{a_i : 1 \le i \le 21\}$

Form a parallel class by taking the blocks $0,1,3$ $\;2,6,11$ $\;4,10,17$ $\;5,13$ $\;8,18$ $\;9,20$ $\;7,19$ $\;12,28$ together with the twenty-one pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{40}$. Develop this class mod 40. Eleven one-factors on $\mathbb{Z}_{40}$ are obtained by developing the pairs $0,14$ $\;0,15$ $\;0,17$ $\;0,18$ $\;0,19$ and $0,20$ mod 40 (each of the first five pairs generates two one-factors). Then construct an $RRP(21,11)$ on $\{a_1, a_2, \ldots, a_{21}\}$.

RRP(61,54). Point set $\mathbb{Z}_{42} \cup \{a_i : 1 \le i \le 19\}$.

Form a parallel class by taking the blocks $1,21$ $\;2,20$ $\;3,19$ $\;5,17$ $\;6,16$ $\;8,14$ $\;23,40$ $\;24,39$ $\;25,38$ $\;27,36$ $\;30,31,33$ together with the nineteen pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{42}$. Develop this class mod 42. Twelve classes on $\mathbb{Z}_{42}$ are obtained by developing the blocks $0,4,11$; $0,5$ $\;1,9$ $\;2,16$; $0,19$ and $0,21$ mod 42 (the first set of pairs covers each residue mod 6 once, and the triple covers each residue mod 3 once). Then construct an $RRP(19,12)$ on $\{a_1, a_2, \ldots, a_{19}\}$.

RRP(65,34), RRP(65,37). Remove a point from a resolvable $TD(3,22)$ and apply construction 1.1, filling in either $RRP(22,12)$s and an $RRP(21,12)$, or $RRP(22,15)$s and an $RRP(21,15)$.

RRP(65,55). Point set $\mathbb{Z}_{44} \cup \{a_i : 1 \le i \le 21\}$.

Form a parallel class by taking the blocks $0,1,3$ $\;2,6,11$ $\;4,10,17$ $\;5,13$ $\;8,18$ $\;9,20$ $\;7,19$ $\;12,28$ $\;14,32$ $\;15,35$ together with the twenty-one pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{44}$. Develop this class mod 44. Eleven one-factors on $\mathbb{Z}_{44}$ are obtained by developing the pairs $0,14$ $\;0,15$ $\;0,17$ $\;0,19$ $\;0,21$ and $0,22$ mod 44 (each of the first five pairs generates two one-factors). Then construct on $RRP(21,11)$ on $\{a_1, a_2, \ldots, a_{21}\}$.

RRP(65,58). Point set $\mathbb{Z}_{48} \cup \{a_i : 1 \le i \le 17\}$.

Form a parallel class by taking the blocks $1,23$ $\;2,22$ $\;3,21$ $\;4,20$ $\;6,18$ $\;7,17$ $\;9,15$ $\;24,47$ $\;25,46$ $\;26,45$ $\;27,44$ $\;28,43$ $\;29,42$ $\;31,40$ $\;34,35,37$ together with the seventeen pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{48}$. Develop this class mod 48. Ten classes on $\mathbb{Z}_{48}$ are obtained by developing the blocks $0,4$ $\;1,8$ $\;3,11$;

$0, 5, 16$; and $0, 24$ mod 48 (the first set of pairs covers each residue mod 6 once and the triple covers each residue mod 3 once). Construct an $RRP(17, 10)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

RRP(67,36), RRP(67,39). Remove two points from a group in a resolvable $TD(3, 23)$ and apply Construction 1.1, filling in either $RRP(23, 13)$s and an $RRP(21, 13)$, or $RRP(23, 16)$s and an $RRP(21, 16)$.

RRP(67,60). Point set $\mathbb{Z}_{48} \cup \{a_i : 1 \leq i \leq 19\}$.

Form a parallel class by taking the blocks $1, 23$ $2, 22$ $3, 21$ $4, 20$ $5, 19$ $6, 18$ $9, 15$ $24, 47$ $25, 46$ $27, 44$ $28, 43$ $29, 42$ $32, 39$ $34, 35, 37$ together with the nineteen pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{48}$. Develop this class mod 48. Twelve classes on $\mathbb{Z}_{48}$ are obtained by developing the blocks $0, 4$ $1, 6$ $2, 11$ $5, 15$; $0, 8, 19$; and $0, 24$ mod 48 (the first set of pairs covers each residue mod 8 once, and the triple covers each residue mod 3 once). Construct an $RRP(19, 12)$ on $\{a_1, a_2, \ldots, a_{19}\}$.

RRP(71,64). Point set $\mathbb{Z}_{54} \cup \{a_i : 1 \leq i \leq 17\}$.

Form a parallel class by taking the blocks $1, 27$ $2, 26$ $3, 25$ $4, 24$ $5, 23$ $6, 22$ $7, 21$ $8, 20$ $10, 18$ $28, 53$ $29, 52$ $30, 51$ $31, 50$ $32, 49$ $33, 48$ $34, 47$ $36, 45$ $39, 40, 42$ together with the seventeen pairs obtained by matching off the a_i's with the remaining residues in $\mathbb{Z}_{54}$. Develop this class mod 54. Ten classes on $\mathbb{Z}_{54}$ are obtained by developing the blocks $2, 7$ $5, 12$ $6, 17$ $0, 4, 10$; and $0, 27$ mod 54 (the first set of blocks covers each residue mod 9 once). Now construct an $RRP(17, 10)$ on $\{a_1, a_2, \ldots, a_{17}\}$.

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