VISCOSITY SOLUTIONS WITH SINGULAR INITIAL DATA FOR A MODEL OF ELECTROPHORETIC SEPARATION

By

Joel D. Avrin

IMA Preprint Series # 385

January 1988

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

514 Vincent Hall

206 Church Street S.E.

Minneapolis, Minnesota 55455

Viscosity Solutions with Singular Initial Data for a Model of Electrophoretic Separation*

by

Joel D. Avrin

University of North Carolina at Charlotte
Charlotte, North Carolina 28223

*This research was supported in part by the Institute for Mathematics and its Applications with funds provided by the National Science Foundation.

Viscosity Solutions with Singular Initial Data for a Model of Electrophoretic Separation

Joel D. Avrin

Abstract

Unique global strong solutions of a Cauchy problem arising in electrophoretic separation are constructed with arbitrary initial data in L^1, thus generalizing an earlier result. For small diffusion coefficients, the solutions can be viewed as approximate, or viscosity solutions for the corresponding zero-diffusion Riemann problem.

1. Introduction

Electrophoresis describes various processes by which proteins and other biological materials are separated in solution by an imposed electric field ([5], [6], [12]). The modeling equations for electrophoretic separation developed in [12] are of advection-diffusion type and relate the electric field $E = E(x,t)$ to the chemical species concentrations $u_i = u_i(x,t)$, $i = 1,\dots,m$, as follows:

$$(u_i)_t = d_i(u_i)_{xx} + [z_i \Omega_i E u_i]_x, \qquad (1.1a)$$

$$E_x = (-{}^e/\epsilon) \sum_{k=1}^{m} z_k u_k \qquad (1.1b)$$

Here t is nonnegative and x lies in a suitable domain of $\mathbb{R}$ which depends on the particular separation technique used. Meanwhile e, ϵ and each d_i, Ω_i, and z_i are constants: e is the molar charge and ϵ is the permitivity of the solvent, while d_i and Ω_i are the diffusivity and mobility of the ith species. Each z_i is +1 or -1 depending on whether the ith species is a postive or negative ion. For further background on the physical significance of equations (1.1), please see [6] or [12].

Here, as in [4] and [6], we focus on a particular separation technique known as isotachophoresis, or ITP, in which the reaction column is long and connected at both ends to large electrolyte reservoirs which negate the influence of reactions occuring at the electrodes. This makes the concentrations constant at the column ends and effectively renders the system infinitely long ([6], [12]). Thus x varies over the entire real line in (1.1) and the concentrations satisfy the fixed Dirichlet boundary conditions

$$u_i(-\infty) = \alpha_i \quad , \quad u_i(+\infty) = \beta_i . \qquad (1.1c)$$

As in [4] and [6] we assume the following conditions which also are appropriate for ITP:

$$(u_i)_x\ (\pm\ \infty,\ t) = 0\ , \tag{1.2}$$

$$\sum_{i=1}^{m} z_i\alpha_i = \sum_{i=1}^{m} z_i\beta_i = 0. \tag{1.3}$$

Condition (1.3), in particular, is a natural condition to impose in light of the separation mechanism and it plays an important role in [4] and the present work. Condition (1.2) is appropriate due to the assympotically constant behavior of the concentrations; its connection with various types of initial data will be discussed in the next section.

The remaining boundary conditions apply to the electric field E. It is often the case that the electric current I is constant through the medium; as in [4] and [6] we assume that here. The development in section 2 of [4] shows that the constant current condition allows us to set

$$E(x,t) = (-e/\epsilon) \int_{-\infty}^{x} \sum_{k=1}^{m} z_k u_k(y,t)dy + E_- \tag{1.4}$$

where E_- is a constant that can be determined explicitly by (1.1c), the value of I, and the choice of initial conditions

$$u_i(x,0) = u_i^0\ . \tag{1.5}$$

Similarly it can be shown that $E(+\infty,t)$ is in fact equal to a specifiable constant E_+ (independent of time). We refer the reader to [4, section 2] for details of the derivation of (1.4). Here we only need to know that these details allow us to use (1.4) to define E. Thus we can eliminate (1.1b), plug (1.4) into (1.1a), and thus rewrite (1.1) as a system of m integro-differential equations in the unknows u_i.

In [4] equations (1.1) were handled by first defining functions $w_i(x)$ as follows:

$$w_i(x) = \begin{cases} \alpha_i & , \quad x \le -1 \\ \ell_i(x) & , \quad -1 < x < 1 \\ \beta_i & , \quad x \ge 1 \end{cases} , \tag{1.6}$$

where ℓ_i is such that $\alpha_i \le \ell_i \le \beta_i$ and ℓ_i makes w_i a C^∞ function of x. Note that $(w_i)_x \in C_0^\infty \equiv C_0^\infty(\mathbb{R})$ and that by (1.3) $\Sigma_{k=1}^m w_k \in C_0^\infty$ as well. Setting $u_i = v_i + w_i$ and plugging into (1.1) we obtain the following equations in v_i:

$$(v_i)_t = d_i(v_i)_{xx} + c_i[E(v+w)(v_i + w_i)]_x + d_i(w_i)_{xx} \tag{1.7}$$

where $c_i = z_i\Omega_i$ and $E(v+w)$ is defined by (1.4) with u_k replaced by $v_k + w_k$. Global strong solutions of (1.7) were found in [4] with initial data $v_i^0 \equiv v_i(0) \in W^{2,1}(\mathbb{R})$. In the next sections we will extend this result to allow for singular initial data, establishing global strong solutions for arbitrary $v_i^0 \in L^1(\mathbb{R})$.

This allows for $v_i^0 = f(x) + g(x)$ where $f(x) \in W^{n,1} \equiv W^{n,1}(\mathbb{R})$ with $n \geq 2$ and $g(x) \in L_0^1(\mathbb{R})$, where $L_0^1(\mathbb{R})$ is the set of all functions in L^1 with compact support. Included in the set of all such $f(x) + g(x)$ are locally piecewise constant functions, a class of initial data that arises in practice when (1.1) is considered with each $d_i = 0$ ([8]). Thus one application of our theory is the production (for small d_i) of approximate, or viscosity, solutions to the hyperbolic problem for a wide class of initial data.

Note that for $v_i^0 = f(x) + g(x)$ as specified above, the boundary conditions (1.1c) and (1.2) are automatically satisfied by $u_i(x,o) = v_i^0 + w_i$. We will show in the next section that for each $t > 0$ $u_i(x,t) \in W^{n,1}$ for all $n \geq 2$, thus u_i will satisfy the conditions (1.1c), (1.2) for all $t \geq 0$.

One can regard (1.7) as a pure Cauchy problem, however, independent of (1.1) and the associated boundary conditions. There have been a number of results in recent years on nonlinear parabolic problems with initial data in L^p, see e.g. [3], [9], [10], [13], [14] (in addition to these applications, the Benjamin-Bona-Mahony equation was discussed with L^p initial data in [2]). To the best of our knowledge equation (1.7) is the first nonlinear parabolic equation for which, as shown below, unique global strong solutions v_i exist with initial data in L^1 such that $v_i \in C([0, +\infty); L^1) \cap C^1((0, +\infty); W^{n,1})$ for any $n \geq 1$. We remark that the boundary conditions (1.1c) and (1.2) are satisfied for arbitrary initial data v_i^0 in L^1 when t is positive.

2. Local Existence.

If, for a vector valued function $f = (f_1, \dots, f_m)$ with each $f_i \in L^1$ we define

$$F(f) = (-e/\epsilon) \int_{-\infty}^{x} \sum_{k=1}^{m} z_k f_k(y)dy, \tag{2.1}$$

then we note that

$$E(v+w) = F(v) + E(w) \tag{2.2}$$

where $E(w)$ is obtained by replacing u_k by w_k in the right-hand side of (1.4). Let $W_i(t)$ denote the semigroup generated by $d_i(\cdot)_{xx}$, then equations (1.7) have the corresponding integral equations

$$v_i(t) = W_i(t)\, v_i^0 + G_i(w,t) +$$

$$c_i \int_0^t W_i(t-s) \Big[E(v(s) + w)\, v_i(s) + F(v(s))\, w_i\Big]_x ds \tag{2.3}$$

where F, E are as in (2.1), (2.2) and

$$G_i(w,t) = \int_0^t W_i(t-s)\Big[c_i E(w)\, w_i + d_i (w_i)_x\Big]_x ds. \tag{2.4}$$

Our goal in this section is to solve (2.3) by a contraction-mapping method for $0 \le t \le T$ with $T>0$ suitably chosen.

By the Sobolev embedding theorems ([1], [7]) for each integer $n \geq 0$ there is a constant C_n such that for all $f \in W^{n+1,1}$

$$\|f\|_{n,\infty} \leq C_n \|f\|_{n+1,1} \tag{2.5}$$

where $\|\cdot\|_{n,\infty}$ denotes the norm on $C_B^n(\mathbb{R})$. Direct differentiation of the explicit kernel for $W_i(t)$ shows that there is a constant K_i such that for all $f \in W^{n,1}$ and all $t \in (0,1]$

$$\left\|W_i(t)f\right\|_{n+1,1} \leq K_i \; t^{-1/2} \|f\|_{n,1} \; . \tag{2.6}$$

As in [13] or [14], by considering (2.6) first on dense subsets of smooth functions, we have for all $f \in W^{n,1}$ that

$$\lim_{t \downarrow 0} t^{1/2} \left\|W_i(t)f\right\|_{n+1,1} = 0. \tag{2.7}$$

We note, in fact, that many of the techniques that follow are based on arguments that appeared in [13] and [14], also later in [3].

Since $(w_i)_x$ and $\Sigma_{k=1}^m z_k w_k$ are both in C_0^∞ it follows that $[c_i E(w) w_i]_x$ is in C_0^∞, hence $G_i(w,t)$ is in $W^{j,1}$ for all $j \geq 0$. For fixed $v_i^0 \in W^{n,1}$ it follows by this last remark and (2.6) and (2.7) that there exist positive numbers α, β, T such that $\beta \downarrow 0$ as $T \downarrow 0$ and

$$\left\|W_i(t)v_i + G_i(w,t)\right\|_{n,1} \leq \alpha \tag{2.8a}$$

$$t^{1/2}\left\| W_i(t)v_i^0 + G_i(w,t)\right\|_{n+1,1} \le \beta \tag{2.8b}$$

for all $t \in (0,T]$.

For α, β, T as above let M be the space of all curves $v(t) = (v_1(t),\ldots,v_m(t))$ such that for each i

1) $v_i : [0,T] \longrightarrow W^{n,1}$ is continuous and $\left\|v_i(t)\right\|_{n,1} \le 2\alpha,\ 0 \le t \le T$;

2) $v_i : (0,T] \longrightarrow W^{n+1,1}$ is continuous and $t^{1/2}\left\|v_i(t)\right\|_{n+1,1} \le 2\beta$, $0<t\le T$.

M is a nonempty complete metric space with metric p where, for $v,u\in M$

$$p(u,v) = \sup_{1\le i\le m}\ \sup_{0<t\le T} \left\{ \left\|v_i(t) - u_i(t)\right\|_{n,1},\ t^{1/2}\left\|v_i(t) - u_i(t)\right\|_{n+1,1} \right\}.$$

Let $(Sv)(t) = ((S_1v_1)(t),\ldots,(S_mv_m)(t))$ where for each i $(S_iv_i)(t)$ is the right-hand side of (2.3). In the proof of the following result we obtain a fixed point of S, hence a local solution of (2.3), by showing that S is a contraction on M.

<u>*Theorem 2.1*</u>. *For each integer $n\ge 0$ and each $v_i^0 \in W^{n,1}$ there exists a $T>0$ such that (2.3) has a unique solution $v_i \in C([0,T];\ W^{n,1}) \cap C((0,T];\ W^{n+1})$.*

<u>Proof</u>. Let $\gamma = |-e/\epsilon|$, then from (1.4) and (2.1) note that for $v \in M$

$$\left\|E(v(t) + w)\right\|_\infty \le \gamma\left[\sum_{k=1}^m \left\|v_k(t)\right\|_1 + \left\|\Sigma_{k=1}^m z_kw_k\right\|_1\right] + E_- \le \gamma(2\alpha m) + L_1 \tag{2.9}$$

where L_1 depends only on γ, E_-, and the w_i; meanwhile

$$\left\|F(v(t))\right\|_\infty \le \gamma \sum_{k=1}^{m} \left\|v_k(t)\right\|_1 \le \gamma(2\alpha m), \tag{2.10}$$

and, if $F_x(v(t)) = [F(v(t))]_x$, and $D_j f$ denotes $(d^j f)/(dx^j)$, $0 \le j \le n$

$$\left\|D_j\ f_x(v(t))\right\|_1 \le \gamma \sum_{k=1}^{m} \left\|D_j v_k(t)\right\|_{j,1} \le \gamma(2\alpha m), \tag{2.11}$$

similarly

$$\left\|D_j\ E_x(v(t) + w)\right\|_1 \le \gamma(2\alpha m) + L_2 \tag{2.12}$$

where L_2 depends only on γ, E_-, and the w_i.

If $1 \le j \le n$ we see from 2.5 that

$$\left\|D_j F(v(t))\right\|_\infty = \left\|D_{j-1}\ F_x(v(t))\right\|_\infty$$

$$\le \gamma \sum_{k=1}^{m} \left\|D_{j-1}\ v_k(t)\right\|_\infty \le \gamma \sum_{k=1}^{m} C_n \left\|v_k(t)\right\|_{n,1} \le \gamma C_n(2\alpha m) \tag{2.13}$$

and similarly

$$\left\|D_j\ E(v(t) + w)\right\|_\infty \le \gamma C_n(2\alpha m) + L_3 \tag{2.14}$$

where L_3 depends only on γ, E_-, and the w_i. Meanwhile it is clear that for each i and for $0 \le j \le n$

$$\left\|D_j v_i(t)\right\|_\infty \le C_{n+1}\left\|v_i(t)\right\|_{n+1,1} \le C_{n+1}\,(2\beta t^{-1/2}) \tag{2.15}$$

and that

$$\left\|D_j (v_i)_x(t)\right\|_1 \le \left\|v_i(t)\right\|_{n+1,1} \le 2\beta t^{-1/2}. \tag{2.16}$$

Using (2.9)-(2.16) we thus have for all $v \in M$ and all $i=1,\dots,m$ that

$$\left\|\left[E(v(t)+w)v_i(t) + F(v(t))w_i\right]_x\right\|_{n,1}$$

$$\le \left\|E_x(v(t)+w)v_i(t)\right\|_{n,1} + \left\|E(v(t)+w)(v_i)_x(t)\right\|_{n,1}$$

$$+ \left\|F_x(v(t))w_i\right\|_{n,1} + \left\|F(v(t))(w_i)_x\right\|_{n,1}$$

$$\le \sum_{k=0}^{n}\sum_{j=0}^{k}\begin{bmatrix}k\\j\end{bmatrix}\left\|D_j\,E_x(v(t)+w\right\|_1\left\|D_{k-j}\,v_i(t)\right\|_\infty$$

$$+ \left\|E(v(t)+w)\right\|_\infty\left\|(v_i)_x(t)\right\|_1$$

$$+ \sum_{k=1}^{n}\sum_{j=1}^{k}\begin{bmatrix}k\\j\end{bmatrix}\left\|D_j\,E(v(t)+w)\right\|_\infty\left\|D_{k-j}\,(v_i)_x(t)\right\|_1$$

$$+ \sum_{k=0}^{n}\sum_{j=0}^{k}\begin{bmatrix}k\\j\end{bmatrix}\left\|D_j\,F_x(v(t))\right\|_1\left\|D_{k-j}\,w_i\right\|_\infty$$

$$+ \left\|F(v(t))\right\|_{\infty} \left\|(w_i)_x\right\|_1$$

$$+ \sum_{k=1}^{n} \sum_{j=1}^{k} \left\|D_j \, F(v(t))\right\|_{\infty} \left\|D_{k-j} \, (w_i)_x\right\|_1$$

$$\leq N_1(\gamma(2\alpha m) + L_2) \; C_{n+1} \; (2\beta t^{-\frac{1}{2}})$$

$$+ (\gamma(2\alpha m) + L_2) \; (2\beta t^{-\frac{1}{2}}) + N_2(\gamma \; C_n(2\alpha m) + L_3) \; (2\beta t^{-\frac{1}{2}})$$

$$+ N_3\gamma(2\alpha m) + N_4\gamma(2\alpha m) + N_5\gamma C_n(2\alpha m) \tag{2.17}$$

where N_1 and N_2 depend only on n while N_3, N_4 and N_5 depend only on n and the w_i. Hence there exist constants A, B and C depending only on γ, E_-, n, m, and the w_i such that

$$\left\|\left[E(u(t))+w)v_i(t) + F(v(t))w_i\right]_x\right\|_{n,1}$$

$$\leq (\alpha A+B)\beta^{-\frac{1}{2}} + \alpha C. \tag{2.18}$$

Hence from (2.18) and (2.8) we see that for all $v \in M$ and $t \in [0,T]$

$$\left\|(Sv)(t)\right\|_{n,1} \leq \alpha + |c_i| \int_0^t \left[(\alpha A+B)\beta s^{-\frac{1}{2}} + \alpha C\right]ds$$

$$\leq \alpha + |c_i| \left[(\alpha A+B)2\beta T^{\frac{1}{2}} + \alpha CT\right] \tag{2.19}$$

where we have used the fact that each D_j commutes with $W_i(t)$ for all $j \geq 0$ and hence $W_i(t)$ is a contraction on $W^{n,1}$.

Now if $c, d \in (0,1)$ a simple scaling argument (see [13]) shows that

$$\int_0^t (t-s)^{-c} s^{-d} \, ds = t^{1-c-d} \int_0^1 (1-s)^{-c} s^{-d} \, ds. \tag{2.20}$$

Combining (2.20) with (2.8), (2.18), (2.19) and (2.6) we see that

$$\|(Sv)(t)\|_{n+1,1} \leq \beta + |c_i| \int_0^t K_i (t-s)^{-1/2} \left[(\alpha A+B)\beta s^{-1/2} + \alpha C \right] ds$$

$$\leq \beta + |c_i| \, K_i \left[\alpha C T^{1/2} + \beta(\alpha A+B) \int_0^1 (1-s)^{-1/2} s^{-1/2} ds \right]. \tag{2.21}$$

Recalling that we can arrange that $\beta \downarrow 0$ as $T \downarrow 0$, it is now clear from (2.19) and (2.21) that we can select T and β small enough to that S maps M into M. A similar argument shows that S is a contraction on M, thus completing the proof of Theorem 2.1. We note in particular that the theorem includes the case $n=0$, thus allowing the initial data v_i^0 to be arbitrary functions in L^1.

3. Global Existence and Regularity.

If we set $n=0$ in Theorem 2.1, so that $v_i^0 \in L^1$, then note that, for $0 < t \leq T$ with T as in the theorem, the local solution $u_i(t)$ is in $W^{1,1}$. Fixing $t_0 \in (0,T)$ and setting $v_i^0 = v_i(t_0)$, Theorem 2.1 can now be applied with $n=1$ to obtain a solution $\overline{v}_i(t)$ of (2.3) on some interval $[0, T_0]$ such that $\overline{v_i} \in C((0, T_0]; W^{2,1})$. If we now replace T by $\min\{T, T_0\}$ we see by the uniqueness assertion of Theorem 2.1 that $v_i(t) = \overline{v_i}(t-t_0)$ for $t_0 < t \leq T$. As t_0 is an arbitrary element of $(0,T)$ we can conclude that $v_i \in C((0,T]; W^{2,1})$. But in [4] global strong solutions of (1.7) were found for arbitrary initial data v_i^0 in $W^{2,1}$. Using arguments similar to those above and applying Theorem 2.1 of [4] we thus have established the following global existence result.

Theorem 3.1. *For arbitrary* $v_i^0 \in L^1$ *equations (1.7) have unique global strong solutions* $v_i \in C([0, +\infty); L^1) \cap C^1((0, +\infty); W^{2,1})$.

It is now clear that we can continue the bootstrap process described above for $n \geq 1$, and regularity in t follows from regularity in x by standard arguments (see e.g. [11, p. 42]). We thus can improve Theorem 3.1 as follows:

Theorem 3.2. *For arbitrary* $v_i^0 \in L^1$ *equations (1.7) have unique global strong solutions* $v_i \in C([0, +\infty); L^1) \cap C^j((0, +\infty); W^{\infty,1})$ *for all* $j \geq 1$, *where* $W^{\infty,1} = \bigcap_{n=1}^{\infty} W^{\infty,1}$.

4. Remarks.

We note that in [10], viscocity solutions with singular initial data were produced for conservation lows arising in gas dynamics. For small enough initial data in $L^2 \cap L^\infty$, the solutions could be extended globally. In the present work, we are able to avoid both boundedness and size restrictions on v_i^0 because the special structure of our equations eventually leads to a priori exponential bounds on $v_i(t)$, as demonstrated in [4].

In [8], solutions of (1.1) were found with each $d_i = 0$ with the addtional assumptions of electroneutrality ($\epsilon = 0$) and monotonicity of the initial data. One could relax these assumptions by constructing solutions in the limit as $d_i \downarrow 0$ of the solutions guaranteed by Theorem 3.1. We hope to investigate this limit in a future paper.

References

[1] Adams, R. A., Sobolev Spaces, Academic Press, New York, 1975.

[2] Avrin, J., The generalized Benjamin-Bona-Mahony equation in $\mathbb{R}^n$ with singular initial data, Nonlinear Analysis 11 (1987), 139-147.

[3] Avrin, J., The generalized Burger's equation and the Navier-Stokes equation in $\mathbb{R}^n$ with singular initial data, Proc. A.M.S. 101 (1987), 29-40.

[4] Avrin, J., Global existence for a model of electrophoretic separation, SIAM J. Math. Analysis, to appear.

[5] Deyl, Z., ed., Electrophoresis: A Survey of Techniques and Applications, Elsevier, Amsterdam, 1979.

[6] Fife, P. C., Palusinski, O. A., and Su, V., Electrophoretic traveling waves, to appear.

[7] Friedman, A., Partial Differential Equations, Holt, Rinehart and Winston, New York, 1969.

[8] Geng, X., in preparation.

[9] Giga, Y., Weak and strong solutions of the Navier-Stokes initial-value problem, Publ. RIMS, Kyoto Univ. 19 (1983), 887-910.

[10] Hoff, D., and Smoller, J., Solutions in the large for certain nonlinear parabolic systems, Ann. Inst. Henri Poincaré 2 (1985), 213-235.

[11] Reed, M., Abstract Non-Linear Wave Equations, Springer-Verlag, Berlin, Heidelberg, and New York, 1975.

[12] Saville, D. A., and Palusinski, O. A., Theory of electrophoretic separation, Part 1 and Part 2, AICHE Journal 32 (1986), no. 2.

[13] Weissler, F. B., Local existence and non-existence for semilinear parabolic equations in L^p, Ind. Univ. Math. Jnl. 29 (1980), 79-102.

[14] Weissler, F. B., The Navier-Stokes initial value problem in L^p, Arch. Rat. Mech. Analysis 74 (1980), 219-230.

Recent IMA Preprints

#	Author/s	Title
298	**M. Chipot, F. B. Weissler**	Some Blow-Up Results for a Nonlinear Parabolic Equation with a Gradient Term
299	**L. Kaitai**	Perturbation Solutions of Simple and Double Bifurcation Problems for Navier-Stokes Equations
300	**C. Zhangxin, L. Kaitai**	The Convergence on the Multigrid Algorithm for Navier-Stokes Equations
301	**A. Gerardi, G. Nappo**	Martingale Approach for Modeling DNA Synthesis
302	**D. N. Arnold, L. Ridgway, M. Vogelius**	Regular Inversion of the Divergence Operator with Dirichlet Boundary Conditions on a Polygon Divergence Operator with Dirichlet Boundary Conditions on a Polygon
303	**R. G. Duran**	Error Analysis in $L^p, 1 \leq p \leq \infty$, for Mixed Definite Element Methods for Linear and Quasi-Linear Elliptic Problems
304	**R. Nochetto, C. Verdi**	An Efficient Linear Scheme to Approximate Parabolic Free Free Boundary Problems: Error Estimates and Implementation
305	**K. A. Pericak-Spector, S. J. Spector**	Nonuniqueness for a Hyperbolic System: Cavitation in Nonlinear Elastodynamics
306	**E. G. Kalnins, W. Miller, Jr.**	q-Series and Orthogonal Polynomials Associated with Barnes' First Lemma
307	**D. N. Arnold, R. S. Falk**	A Uniformly Accurate Finite Element Method for Mindlin-Reissner Plate
308	**Chi-Wang Shu**	TVD Properties of a Class of Modified Eno Schemes for Scalar Conservation Laws
309	**E. Dikow, U. Hornung**	A Random Boundary Value Problem Modeling Spatial Variability in Porous Media Flow
310	**J. K. Hale**	Compact Attractors and Singular Perturbations
311	**A. Bourgeat, B. Cockburn**	The TVD-Projection Method for Solving Implicit Numeric Schemes for Scalar Conservation Laws: A Numerical Study of a Simple Case
312	**B. Muller, A. Rizzi**	Navier-Stokes Computation of Transonic Vortices over a Round Leading Edge Delta Wing
313	**J. Thomas Beale**	On the Accuracy of Vortex Methods at Large Times
314	**P. Le Talle, A. Lotfi**	Decomposition Methods for Adherence Problems in Finite Elasticity
315	**J. Douglas, Jr., J. E. Santos**	Approximation of Waves in Composite Media
316	**T. Arbogast**	The Double Porosity Model for Single Phase Flow in Naturally Fractured Reservoirs
317	**T. Arbogast, J. Douglas, Jr., J. E. Santos**	Two-Phase Immiscible Flow in Naturally Fractured Reservoirs
318	**J. Douglas, Jr., Y. Yirang**	Numerical Simulation of Immiscible Flow in Porous Media Based on Combining the Method of Characteristics with Finite Element Procedures
319	**R. Duran, R. H. Nochetto, J. Wang**	Sharp Maximum Norm Error Estimates for Finite Element Approximations of the Stokes Problem in 2-D
320	**A. Greven**	A Phase Transition for a System of Branching Random Walks in a Random Environment
321	**J. M. Harrison, R. J. Williams**	Brownian Models of Open Queueing Networks with Homogeneous Customer Populations
322	**Ana Bela Cruzeiro**	Solutions ET mesures invariantes pour des equations d'evolution Stochastiques du type Navier-Stokes
323	**Salah-Eldin A. Mohammed**	The Lyapunov Spectrum and Stable Manifolds for Stochastic Linear Delay Equations
324	**Bao Gia Nguyen**	Typical Cluster Size for 2-DIM Percolation Processes (Revised)
325	**R. Hardt, D. Kinderlehrer, F.-H. Lin**	Stable Defects of Minimizers of Constrained Variational Principles
326	**M. Chipot, D. Kinderlehrer**	Equilibrium Configurations of Crystals
327	**Kiyosi Itô**	Malliavin's C^{∞} functionals of a centered Gaussian system
328	**T. Funaki**	Derivation of the hydrodynamical equation for one-dimensional Ginzburg-Landau model
329	**Y. Masaya**	Schauder Expansion by some Quadratic Base Function
330	**F. Brezzi, J. Douglas, Jr.**	Stabilized Mixed Methods for the Stokes Problem
331	**J. Mallet-Paret, G. R. Sell**	Inertial Manifolds for Reaction Diffusion Equations in Higher Space Dimensions
332	**San-Yih Lin, M. Luskin**	Relaxation Methods for Liquid Crystal Problems
333	**H. F. Weinberger**	Some Remarks on Invariant Sets for Systems
334	**E. Miersemann, H. D. Mittelmann**	On the Continuation for Variational Inequalities Depending on an Eigenvalue Parameter
335	**J. Hulshof, N. Wolanski**	Monotone Flows in N-Dimensional Partially Saturated Porous Media: Lipschitz Continuity of the Interface
336	**B. J. Lucier**	Regularity Through Approximation for Scalar Conservation Laws
337	**B. Sturmfels**	Totally Positive Matrices and Cyclic Polytopes
338	**R. G. Duran, R. H. Nochetto**	Pointwise Accuracy of a Stable Petrov-Galerkin Approximation to Stokes Problem
339	**L. Gastaldi**	Sharp Maximum Norm Error Estimates for General Mixed Finite Element Approximations to to Second Order Elliptic Equations
340	**L. Hurwicz, H. F. Weinberger**	A Necessary Condition for Decentralizability and an Application to Intemporal Allocation
341	**G. Chavent, B. Cockburn**	The Local Projection P^0P^1-Discontinuous-Galerkin-Finite Element Method for Scalar Conservation Laws
342	**I. Capuzzo-Dolcetta, P.-L. Lions**	Hamilton-Jacobi Equations and State-Constraints Problems

#	Author/s	Title
343	**B. Sturmfels, N. White**, Gröbner Bases and Invariant Theory	
344	**J. L. Vazquez**, C^{∞}-Regularity of Solutions and Interfaces of the Porous Medium Equation	
345	**C. Beattie, W. M. Greenlee**, Improved Convergence Rates for Intermediate Problems	
346	**H. D. Mittelmann**, Continuation Methods for Parameter-Dependent Boundary Value Problems	
347	**M. Chipot, G. Michaille**, Uniqueness Results and Monotonicity Properties for Strongly Nonlinear Elliptic Variational Inequalities	
348	**Avner Friedman, Bei Hu** The Stefan Problem for a Hyperbolic Heat Equation	
349	**Michel Chipot, Mitchell Luskin** Existence of Solutions to the Elastohydrodynamical Equations for Magnetic Recording Systems	
350	**R.H. Nochetto, C. Verdi**, The Combined Use of a Nonlinear Chernoff Formula with a Regularization Procedure for Two-Phase Stefan Problems	
351	**Gonzalo R. Mendieta** Two Hyperfinite Constructions of the Brownian Bridge	
352	**Victor Klee, Peter Kleinschmidt** Geometry of the Gass-Saaty Parametric Cost LP Algorithm	
353	**Joseph O'Rourke** Finding A Shortest Ladder Path: A Special Case	
354	**J. Gretenkort, P. Kleinschmidt, Bernd Sturmfels**, On the Existence of Certain Smooth Toric Varieties	
355	**You-lan Zhu** On Stability & Convergence of Difference Schemes for Quasilinear Hyperbolic Initial-Boundary-Value Problems	
356	**Hamid Bellout, Avner Friedman** Blow-Up Estimates for Nonlinear Hyperbolic Heat Equation	
357	**P. Gritzman, M. Lassak** Helly-Test for the Minimal Width of Convex Bodies	
358	**K.R. Meyer, G.R. Sell** Melnikov Transforms, Bernoulli Bundles, and Almost Periodic Perturbations	
359	**J.-P. Puel, A. Raoult** Buckling for an Elastoplastic Plate with An Increment Constitutive Relation	
360	**F.G. Garvan** A Beta Integral Associated with the Root System G_2	
361	**L. Chihara, D. Stanton** Zeros of Generalized Krawtchouk Polynomials	
362	**Hisashi Okamoto** O(2)-Equivariant Bifurcation Equations with Two Modes Interaction	
363	**Joseph O'Rourke, Catherine Schevon** On the Development of Closed Convex Curves on 3-Polytopes	
364	**Weinan E** Analysis of Spectral Methods for Burgers' Equation	
365	**Weinan E** Analysis of Fourier Methods for Navier-Stokes Equation	
366	**Paul Lemke** A Counterexample to a Conjecture of Abbott	
367	**Peter Gritzmann** A Characterization of all Loglinear Inequalities for Three Quermassintegrals of Convex Bodies	
368	**David Kinderlehrer** Phase transitions in crystals: towards the analysis of microstructure	
369	**David Kraines, Vivian Kraines** Pavlov and the Prisoner's Dilemma	
370	**F.G. Garvan** A Proof of the MacDonald-Morris Root System Conjecture for F_4	
371	**Neil L. White, Tim McMillan** Cayley Factorization	
372	**Bernd Sturmfels** Applications of Final Polynomials and Final Syzygies	
373	**Avner Friedman, Michael Vogelius** Identification of Small Inhomogeneities of Extreme Conductivity by Boundary Measurements: A Continuous Dependence Result	
374	**Jan Kratochvíl, Mirko Křivánek** On the Computational Complexity of Codes in Graphs	
375	**Thomas I. Seidman** The Transient Semiconductor Problem with Generation Terms, II	
376	**Michael A. Trick** Recognizing Single-Peaked Preferences On A Tree	
377	**Michael A. Trick** Induced Subtrees of a Tree and the Set Packing Problem	
378	**Charles J. Colbourn, Ebadollah S. Mahmoodian** The Spectrum of Support Sizes for Threefold Triple Systems	
379	**Bradley J. Lucier** Performance Evaluation for Multiprocessors Programmed Using Monitors	
380	**Tomasz Luczak** On the Size and Connectivity of the k-Core of a Random Graph	
381	**B. Nicolaenko, B. Scheurer and R. Temam** Some Global Dynamical Properties of a Class of Pattern Formation Equations	
382	**Mirko Křivńek** A Note on the Computational Complexity of Bracketing and Related Problems	
383	**Eduard Harabetian** Rarefactions and Large Time Behavior for Parabolic Equations and Monotone Schemes	
384	**Victor Klee and Peter Kleinschmidt** Polytopal Complexes and Their Relatives	
385	**Joel D. Avrin** Viscosity Solutions with Singular Initial Data for a Model of Electrophoretic Separation	
386	**David W. Matula and Rakesh V. Vohra** Calculating the Connectivity of a Directed Graph	
387	**Hubert de Fraysseix, János Pach and Richard Pollack** Small Sets Supporting Fáry Embeddings of Planar Graphs	
388	**Bernardo Cockburn and Chi-Wang Shu** The Runge–Kutta Local Projection P^1- Discontinuous–Galerkin Finite Element Method for Scalar Conservation Laws	
389	**Henry Crapo, Timothy F. Havel, Bernd Sturmfels, Walter Whiteley and Neil L. White** Symbolic Computations in Geometry	