Vector Mesons and an Interpretation of Seiberg Duality

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Motivations

- We will interpret the dynamics of Supersymmetric QCD (SQCD) in terms of ideas familiar from the hadronic world. (The reason for which YM theory was invented is realized.)
- Mysterious properties of the supersymmetric theory, such as the emergent magnetic gauge symmetry, have analogs in QCD.
- Several phenomenological concepts from nuclear physics, such as “hidden local symmetry” and “vector meson dominance,” are rigorously realized in SQCD.
- On the way we will obtain new results about vector mesons in QCD and propose general properties of supersymmetric theories with weakly coupled duals.

Pions

Several theoretical tools are indispensable in our understanding of the hadronic world. The chiral limit is, perhaps, the most important one.

If $m_u = m_d = m_s = 0$ the underlying theory has $SU(3)_L \times SU(3)_R$ global symmetry, which is spontaneously broken as

$$SU(3)_L \times SU(3)_R \to SU(3)_{diag}$$

In the real world the masses of these quarks are small compared to the strong coupling scale, so we should find 8 light pseudo-scalars in the adjoint of $SU(3)_{diag}$. These are identified with the familiar pions, kaons, and eta.

Pions

For simplicity we discuss $SU(2)$ groups.

$$U = e^{i\pi^a T^a}$$

The $SU(2)_L \times SU(2)_R$ symmetry is realized by $U' = g_L U g_R^\dagger$. There is a unique invariant Lagrangian at the two derivative level

$$L = \frac{1}{4} f_\pi^2 Tr \left(\partial_\mu U \partial^\mu U^\dagger \right)$$

The diagonal symmetry with $g_R = g_L$ acts linearly on the pions but the axial transformations do not. Expanding in the number of pions we get

$$L = \frac{1}{2} f_\pi^2 \left((\partial \vec{\pi})^2 - \frac{1}{2} \vec{\pi}^2 (\partial \vec{\pi})^2 + \cdots \right) .$$

Pions

Equivalently, we can factorize the matrix $U(x)$ at each point in terms of two special unitary matrices ξ_L and ξ_R as follows

$$U(x) = \xi_L(x)\xi_R^\dagger(x) \ .$$

This factorization is redundant. The gauge invariance is $\xi_L \to \xi_L h(x)$, $\xi_R \to \xi_R h(x)$. The global $SU(2)_L \times SU(2)_R$ transformations are $\xi_L \to g_L\xi_L$, $\xi_R \to g_R\xi_R$.

In terms of these redundant degrees of freedom

$$L = -\frac{f_\pi^2}{4} Tr\left[\left(\xi_L^\dagger \partial_\mu \xi_L - \xi_R^\dagger \partial_\mu \xi_R\right)^2\right]$$

It is easy to check that this Lagrangian is gauge invariant and it is also invariant under global symmetry transformations.

Pions

To calculate we can fix a gauge. For example, we can choose $\xi_L = \xi_R^\dagger$. $SU(2)_L \times SU(2)_R$ transformations take us out of this gauge, but it can reinstated by a gauge transformation.

The $[SU(2)]$ hidden gauge symmetry endows the model with a quiver-like structure $SU(2)_L \times [SU(2)] \times SU(2)_R$. The vacua are parametrized by constant matrices ξ_L, ξ_R, modulo gauge transformations. We can always choose $\xi_L = 1$ and ξ_R is a general special unitary matrix. $\xi_L = 1$ breaks the gauge symmetry but a diagonal flavor symmetry coming from a mixture of the global transformations in $SU(2)_L$ and (global) gauge transformations in $[SU(2)]$ remains. Then, ξ_R breaks the flavor symmetry to $SU(2)_{diag}$.

Vector Mesons

Next we find the vector mesons, consisting of the ρ mesons (with masses around 770 MeV) and their $SU(3)_{diag}$ partners. The analysis of the chiral limit does not place interesting constraints on their dynamics. However, there are phenomenological hints of an underlying structure.

- The ρ mesons couple equally strongly to many different hadrons (such as the pions). (And perhaps their self coupling is also similar.)
- Many processes are saturated by vector meson exchanges. This is usually referred to as "vector meson dominance."
- There are curious empirical relations. The most striking one is

$$m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2 \ .$$

Vector Mesons

One can attempt to account for these properties by imagining that the rho mesons are the gauge fields of a hidden local $[SU(3)]$ gauge symmetry.

- Coupling universality is explained by the universality of gauge interactions.
- The relation $m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$ can be interpreted in terms of the usual formula $m_V^2 \sim g^2 v^2$, suggesting that the hidden $[SU(3)]$ symmetry is higgsed at the scale f_π.
- With a little more work, vector dominance can be reproduced too.

Vector Mesons

The second version of the theory of pions had a redundancy but no gauge fields associated to this redundancy. Consider adding such a triplet of real vector fields ρ_μ^a transforming as

$$\rho_\mu \equiv \rho_\mu^a T^a \to h^\dagger \rho_\mu^a T^a h + i h^\dagger \partial_\mu h \ .$$

We can construct two covariant objects

$$\rho_\mu^L = \rho_\mu - i\xi_L^\dagger \partial_\mu \xi_L \ , \qquad \rho_\mu^R = \rho_\mu - i\xi_R^\dagger \partial_\mu \xi_R$$

At the two derivative level the most general Lagrangian symmetric under $L \leftrightarrow R$ can be written as

$$L = -\frac{1}{g^2}(F_{\mu\nu}^a)^2 + \frac{f_\pi^2}{4} Tr\left[\left(\rho_\mu^L - \rho_\mu^R\right)^2\right] + a\frac{f_\pi^2}{4} Tr\left[\left(\rho_\mu^L + \rho_\mu^R\right)^2\right]$$

So far, a, g are undetermined positive real numbers.

Vector Mesons

Denoting $\xi_L = e^{i\pi_L^a T^a}$, $\xi_R = e^{i\pi_R^a T^a}$ and expanding to quadratic order we get

$$L = -\frac{1}{g^2}(F_{\mu\nu}^a)^2 + \frac{f_\pi^2}{2}(\partial_\mu(\pi_L^a - \pi_R^a))^2 + \frac{af_\pi^2}{2}\left(\partial_\mu(\pi_L^a + \pi_R^a) + 2\rho_\mu^a\right)^2 + \cdots$$

In order to find the physical spectrum we pick unitary gauge $\pi_L = -\pi_R \equiv \pi$. We find a triplet of massless pions and a massive gauge field with mass

$$m_\rho^2 = ag^2 f_\pi^2$$

Vector Mesons: Regime of Validity

An effective action for massive particles is a subtle thing. One needs a small parameter that justifies truncating the effective action. To make sense of the effective theory we will assume that the gauge coupling g is parametrically small.

Eventually, we would like the massive spin one particles in the theory above to be identified with the rho mesons of QCD. In nature, the gauge coupling of the rho mesons is by no means small. In spite of this, the theory reproduces some of the properties of QCD remarkably well.

Vector Mesons

Note that $a = 1$ is special in

$$L = -\frac{1}{g^2}(F^a_{\mu\nu})^2 + \frac{f_\pi^2}{4} Tr\left[\left(\rho^L_\mu - \rho^R_\mu\right)^2\right] + a\frac{f_\pi^2}{4} Tr\left[\left(\rho^L_\mu + \rho^R_\mu\right)^2\right]$$

In this case ξ_L interacts with ξ_R only through gauge fields. As a consequence, when $g = 0$, the global symmetry is enhanced due to the global gauge transformations to $SU(2)_L \times SU(2)^2 \times SU(2)_R$.

This symmetry argument has led Georgi to propose the importance of $a = 1$. Unfortunately, $a = 1$ does not describe well the phenomenology of QCD.

Vector Mesons

Let us evaluate the global $SU(2)_{diag}$ currents $(J^a_{diag})_\mu$. The complete gauge invariant expression for the conserved current is

$$(J^a_{diag})_\mu = \frac{f_\pi^2}{4} Tr\left[\left(\rho^L_\mu - \rho^R_\mu\right)\left(\xi^\dagger_L T^a \xi_L - \xi^\dagger_R T^a \xi_R\right)\right]$$

$$+\frac{af_\pi^2}{4} Tr\left[\left(\rho^L_\mu + \rho^R_\mu\right)\left(\xi^\dagger_L T^a \xi_L + \xi^\dagger_R T^a \xi_R\right)\right]$$

In unitary gauge the expression above becomes

$$(J^a_{diag})_\mu = 2af_\pi^2\rho^a_\mu + 2f_\pi^2(a-2)\epsilon^{abc}\pi^b\partial_\mu\pi^c + \text{three particles} + \cdots$$

Setting $a=2$, the coefficient of the second term vanishes. The fact that $a=2$ is special is inconspicuous in the original Lagrangian. $a=2$ is the value which best describes the phenomenology of QCD.

Vector Mesons

From this we also see a general relation between the physical coupling of the rho meson to pions $g_{\rho\pi\pi} \equiv \frac{1}{2}ga$, and the amplitude for photon - rho meson mixing $g_{\rho\gamma} \equiv gaf_\pi^2$,

$$g_{\rho\gamma} = 2g_{\rho\pi\pi}f_\pi^2$$

The unknown parameters a, g cancel from this relation. One can test this relation in QCD. The agreement is about 10%, which is remarkable.

For $a = 2$ we get $m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$, which holds in nature to 5%. We also get, $g_{\rho\pi\pi} = g$ which means that rho mesons couple to pions and themselves equally strongly.

Vector Mesons

Consider the electromagnetic form factor of the charged pion.

$$\langle \pi(p) \left| J_\mu^{QED}(0) \right| \pi(p') \rangle = (p+p')_\mu F(q^2) \ .$$

Recall

$$(J_{diag}^a)_\mu = 2af_\pi^2 \rho_\mu^a + 2f_\pi^2(a-2)\epsilon^{abc}\pi^b \partial_\mu \pi^c + + \cdots \ .$$

Thus $F(q^2)$ has two contributions: a direct contact term and a conversion into a rho meson.

$$F(q^2) = (1 - \frac{1}{2}a) + \frac{\frac{1}{2}am_\rho^2}{m_\rho^2 - q^2} \ .$$

For $a = 2$ the effect of scattering a photon on a pion target is fully accounted for by a ρ exchange. This is “vector dominance.”

Vector Mesons

Why does nature pick $a = 2$?

Vector Mesons

One can provide an argument akin to the Weinberg sum rules. Asymptotic freedom implies

$$\lim_{q^2 \to -\infty} F(q^2) \sim \frac{1}{q^2} \; .$$

Therefore integrating $F(q^2)/q^2$ over a large contour we get zero.

Following Weinberg, we ignore all the contributions besides the lightest resonances. Then, for γ that encircles the origin and the rho meson pole we get

$$\int_\gamma d(q^2) \frac{F(q^2)}{q^2} = 0 \; .$$

Since $F(0) = 1$ and the residue at $q^2 = m_\rho^2$ has been calculated, we find

$$a = 2$$

Vector Mesons

Vector dominance is often regarded as an input. Our point of view is that it is not a random fact about the hadronic world, rather, under some circumstances it could have been predicted by sum rules, in the same way that Weinberg predicted the axial vector mesons.

(A similar sum rule also allows to calculate $g_{\rho\pi\pi}$ and we get 5.1 while the correct answer is around 6.)

Vector Mesons: Summary

- Vector mesons are included in the chiral Lagrangian by introducing a redundancy and adding gauge fields for this redundancy.
- First, the gauge symmetry is broken but the full $SU(2)_L \times SU(2)_R$ flavor symmetry survives as a linear combination of flavor generators and gauge generators. Subsequently, the flavor symmetry is further broken to $SU(2)_{diag}$.
- The ρ mesons can be created from the vacuum by the action of unbroken flavor symmetry generators.
- Despite the fact that this description makes physical sense only for small values of the gauge coupling (with fixed f_π), it works pretty well.

Question

Are there theories in which the ideas about ”hidden local symmetry,“ and “vector meson dominance” can be made precise?

SQCD

Consider $SU(N_c)$ gauge theory with N_f flavors $Q^i, \tilde{Q}_i$, $i = 1...N_f$. The the theory is in an IR-free non-abelian phase if $N_c + 1 < N_f < \frac{3}{2}N_c$. The symmetry group is $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$.

At energies much below the strong coupling scale, this theory flows to the Seiberg dual $SU(N_f - N_c)$ IR-free gauge theory with N_f magnetic quarks $q_i, \tilde{q}^i$ and a gauge-singlet matrix M^i_j in the bi-fundamental representation of the flavor group.

These degrees of freedom are very different from the original variables, but the vacua agree upon introducing the superpotential

$$W = \tilde{q}^j M^i_j q_i \ .$$

SQCD

For small VEVs, these “magnetic” fields have a canonical Kähler potential, albeit with an unknown normalization. Neither the $SU(N_f - N_c)$ magnetic gauge fields nor the magnetic quarks appear as well defined local operators in the UV. This must be so because they are charged under a hidden local symmetry group.

We will provide evidence for the claim that the gauge fields should be thought of as rho mesons and the magnetic quarks are analogous to ξ_L, ξ_R.

SQCD

At the origin of the moduli space of SQCD the magnetic gauge fields are massless, so to test our proposal we need to move away (slightly) from the origin. Let us consider the following direction in moduli space

$$q \equiv \left(\chi_{(N_f-N_c)\times(N_f-N_c)}, \varphi_{N_c\times(N_f-N_c)}\right) \ , \qquad \chi = 1 \ .$$

Along this flat direction, the symmetry is broken $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \to$ $SU(N_f - N_c)_L \times SU(N_c)_L \times SU(N_f)_R \times U(1)'_B \times U(1)'_R$. The massless particles consist of $2N_f N_c - 2N_c^2 + 1$ Goldstone bosons and some massless mesons. The magnetic gauge fields are massive with mass $\sim gv$.

SQCD

We already see some *superficial* similarities to QCD: The flavor symmetry $SU(N_f - N_c)_L$ survives at the IR because in the magnetic description it mixes with global gauge transformations. And the way the magnetic gauge fields appear can be thought of gauging a redundancy of the coset $SU(N_f)_L/(SU(N_f - N_c)_L \times SU(N_c)_L)$.

These similarities can be made precise by studying the global symmetry currents of the theory. Consider the $SU(N_f - N_c)_L$ global symmetry current superfields:

$$J^a_{SU(N_f-N_c)_L} = \chi^c_j (e^V)^d_c (\chi^\dagger)^i_d (T^a)^j_i - \text{meson bilinears}$$

The meaning of this expression becomes transparent once we fix a unitary gauge for the magnetic group.

SQCD

Unitary gauge is achieved by

$$\forall a. \ \langle \chi_i^c \rangle (T^a)_c^d (\delta\chi^\dagger)_d^i = 0$$

In our case $\langle \chi_i^c \rangle = v\delta_i^c$ and unitary gauge therefore means that $\chi_i^c = v\delta_i^c + \pi\delta_i^c$.

Evaluating the current in this gauge we find

$$J^a_{SU(N_f - N_c)_L} = v^2 (e^V)_c^d (T^a)_d^c + v(\pi + \pi^\dagger)(e^V)_c^d (T^a)_d^c - \text{mesons} + \cdots$$

We can now compute the form factor of the Goldstone bosons φ. Only the piece linear in V from the first term contributes at tree level. This gives

$$F_\varphi(q^2) = \frac{m_V^2}{m_V^2 - q^2}$$

This is complete vector dominance.

SQCD

We see that not only is the identification between the Seiberg dual gauge fields and the rho mesons manifest, SQCD also satisfies vector dominance (and relations among $m_\rho, g_{\rho\pi\pi}, f_\pi, g_{\rho\gamma}$.). So SQCD seems to sit at a point analogous to $a = 2$ in QCD.

We thus see that SQCD slightly deformed from the origin has a rich structure that in some respects resembles QCD, especially in the way vector mesons appear and the way they dominate physical processes. The conclusion that the Seiberg dual gauge fields are the analogs of the ρ mesons, and the magnetic quarks' role is similar to those of ξ_L, ξ_R, is unavoidable.

SUSY Breaking in SQCD

We must subject the identification we are proposing to further tests.

Consider again the free magnetic phase with

$$W_{electric} = mQ^i\tilde{Q}_i \ .$$

The dynamics of this theory for small field VEVs has unfolded only in recent years, starting with ISS. The symmetry group is $SU(N_f) \times U(1)_B$. Near the origin we use the Seiberg dual variables

$$W_{magnetic} = \tilde{q}^j M^i_j q_i - \mu^2 M^i_i \ ,$$

where $\mu^2 = -m\Lambda$.

The symmetry group of the theory contains $SU(N_f)$, but we see only $(N_f - N_c)^2 - 1$ gauge bosons in the IR. This apparent contradiction is resolved by carefully studying the dynamics.

SUSY Breaking in SQCD

The F-term equations for the meson field $q^j q_i - \mu^2 \delta_i^j = 0$ cannot all be satisfied because the ranks do not match. There are no SUSY vacua and the vacuum energy density is (at least) of order μ^4 To trust these configurations we focus on the regime $m \ll \Lambda$.

One finds that the classical energy density is minimized by setting $\chi_i^c = \mu \delta_i^c$, $\tilde{\chi}_c^i = \mu \delta_c^i$, while all the other fields are set to zero. The symmetry is *broken* as follows

$$SU(N_f) \times U(1)_B \hookrightarrow SU(N_f - N_c) \times SU(N_c) \times U(1)'_B \ .$$

We now see how the apparent contradiction mentioned above is going to be resolved. The vacuum close to the origin spontaneously breaks the $SU(N_f)$ global symmetry.

From here the story proceeds in parallel to QCD.

SUSY Breaking in SQCD

The currents of the unbroken $SU(N_f - N_c)$ symmetry are candidates for creating the magnetic gauge fields from the vacuum. We can write an explicit expression for the $SU(N_f - N_c)$ currents

$$J^a_{SU(N_f-N_c)} = \chi^c_j (e^V)^d_c (\chi^\dagger)^i_d (T^a)^j_i - (\tilde{\chi}^\dagger)^c_j (e^{-V})^d_c \tilde{\chi}^i_d (T^a)^j_i + mesons \; .$$

The natural choice of unitary gauge in this case is

$$\forall a. \;\; \langle \chi^c_i \rangle (T^a)^d_c (\delta\chi^\dagger)^i_d - \langle \tilde{\chi}^i_d \rangle (T^a)^d_c (\delta\tilde{\chi}^\dagger)^c_i = 0 \; .$$

Plugging the VEVs of χ, $\tilde{\chi}$, we find $\delta\chi - \delta\tilde{\chi} \sim 1$. In this gauge the current multiplet becomes

$$J^a_{SU(N_f-N_c)} = \mu^2 V^a + \text{two particles} + \cdots \; .$$

SUSY Breaking in SQCD

Hence, we see again that the unbroken $SU(N_f - N_c)$ currents create the magnetic gauge fields from the vacuum. Since at energy scales above μ a quadratic term in the NG bosons $\sim \varphi T^a \varphi^\dagger$ is absent, the form factor for the Goldstone bosons satisfies vector dominance.

This completes our analysis of the SUSY-breaking case, corroborating our proposal.

Open Questions

- It would be nice to check whether the phenomenon we found here is general or not. To address this, one would need to study other examples, such as the orthogonal and symplectic cases. Examples such as adjoint SQCD are also interesting.
- One can identify the emergent gauge bosons with rho vector mesons only if the number of emergent gauge bosons is not larger than the number of flavor symmetry generators. Thus, we obtain an inequality between two important quantities. It can be checked in adjoint SQCD, and indeed, it is miraculously satisfied (a nontrivial interplay between the beta function, the proposed dual, and global symmetries).
- Theories with light axial vector mesons? ρ'?
- Many other questions...