Transition Radius Method

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Abstract

Solving the R_{22} Ricci expression iteratively yields a way to define both a transition radius and a new expression for determining fundamental boson energies. This transition radius can be defined as a radius value where the three spatial dimension f(r) metric component is equal to the same higher dimensional metric component, and the benefit of this transition radius is that it can be used to determine the energy of fundamental boson types and fundamental particle types. This method fits a total of fifteen boson types including the W, Z, and X bosons. Also, letting a higher dimensional radius go to zero for an infinite number of spatial dimensions predicts an energy for a new boson type. The transition radius method seems to be a preferable way to resolve a part of the hierarchy problem because it fits fundamental particle energies, it yields a simple expression for boson energy, and it is derived from general relativity.

1 Introduction

Previously, Schwarzschild solutions have been modified for spatial dimensions higher than three by raising the power of r in (1 − 2M/r) as described in [6]. For example, r would be changed to r^2 for four spatial dimensions, to r^3 for five spatial dimensions. As an alternative, at large distance from a massive object, the higher dimensional Schwarzschild radius could change in a way that is similar to the way a flat space radius changes with higher dimensions.

To explore how general relativity connects to flat space in higher dimensions we examine flat space radius expressions to get an idea of how the r term should be changed for higher spatial dimensions. For Euclidean space, where a three dimensional Cartesian coordinate system applies, the radius squared can be found from the expression $r^2 = (x1-x1^*)^2 + (x2-x2^*)^2 + (x3-x3^*)^2$ for the two end points $(x1, x2, x3)$ and $(x1^*, x2^*, x3^*)$. Extending flat space to five spatial dimensions this radius squared becomes $r^2 = (x1–x1^*)^2 + (x2–x2^*)^2 + (x3–x3^*)^2 + (x4–x4^*)^2 + (x5–x5^*)^2$ by extending the distance formula by two additional terms. We see that this five dimensional radius squared in flat space is corrected for extra dimensions by the addition of two more nonlinear terms. This suggests that the radius term in (1 – 2M/r) should be corrected for higher dimensions by adding nonlinear terms to r squared, and not by raising the power of r. However, such a correction can not come from an expression which describes flat space because the Schwarzschild solution describes curved space.

General relativity offers guidance about nonlinear terms to be added to the radius term in (1 – 2M/r) to correct it for higher spatial dimensions. Going back to the derivation of the Schwarzschild solution, as shown in Robert Wald's book *General Relativity* on page 123, we see that the Ricci expressions R_{22} and R_{33} were not used to derive the Schwarzschild solution [11]. In this work, we explore these two Ricci expressions which were set aside to determine if they can be used to find the nonlinear terms that need to be added to the radius term in (1 – 2M/r) to correct it for higher spatial dimensions. Since $R_{22} = R_{33}$ we only need to look at R_{22} more closely. Integration of R_{22} gives a dimensionally corrected radius expression which matches our expectations because its form is a sum of nonlinear terms.

These nonlinear terms, which correct the radius used in (1 – 2M/r) when there are more than three spatial dimensions, are of the form $A(\ln(r))^n$ where A is a factor that changes for each integer value of n, r is the radius of a thin-shell bubble as in Israel [3], n is the number of spatial dimensions minus three, and $\ln(r)$ is the natural logarithm of the radius. These dimensionally corrected radii are derived from $f_i(r)$ used in an expression which has the form $(SLOPE)_i[f_i(r)]^{-1} + 1$ where best values of SLOPE are used to smoothly connect this new expression to (1 – 2M/r) over a radius range of highest energy. Functions $f_i(r)$ are changed to dimensionally corrected radii by $R_i(r) = -3f_i(r)$ so that $R_i(r)$ has a positive value where i is the number of spatial dimensions. It is important to note that $f_i(r)$ is the dimensionally corrected radius expression $R_i(r)$ divided by – 3 , and $f(r)$ is the factor multiplying dt^2 in the metric; so these two functions are completely different.

To find the value of SLOPE for any spatial dimension requires making a starting guess for the value of (1 – 2M/r). We need to make the best possible starting guess for (1 – 2M/r) which will anchor the lowest possible value of M consistent for Schwarzschild black holes. A suitable value is the approximate maximum neutron-star mass of 2.2 solar masses [10, p.205]. When 2M is in units of Planck lengths it is $(1 - 4.076 \times 10^{38}/r)$ at 2.20 solar masses when M is "nongeometrized". As will be discussed in section 12, the results will apply from 2.2 solar masses up past 3 million solar masses and to the mass of the universe.

Determining the value of SLOPE for each spatial dimension involves averaging a slope estimating the following expression SX (short for slope-estimating expression) for two to five SX values. This SX is given by SX = (1 – 2M/r) ($f_i(r)$), and in all but one case these SX values were only determined for values of radius which changed by an order of magnitude from one radius value to the next. By averaging different numbers of SX values, the range of radius values over which (1 – 2M/r) and (SLOPE) $[\, f_i(r)]^{-1} + 1$ are coupled is varied depending on the number of spatial dimensions involved.

Once a SLOPE value is determined for a given number of spatial dimensions, the transition radius values are determined by finding radius values for 2.20 solar masses which satisfy $(1–4.076 \times 10^{38}/r) = \text{(SLOPE)}\, [f_i(r)]^{-1} + 1$ for each number of spatial dimensions. For five spatial dimensions and for seven through twenty-four spatial dimensions I find two transition radii for each; however, four and six spatial dimensions both have an infinite number of transition radii. The significance of these differences will be discussed later.

I have found that the energy of a boson type is proportional to the reciprocal of its transition radius cubed where a boson type energy is the energy of a specific particle/antiparticle annihilation at rest mass. These boson type energies match masses for fundamental particle pairs or energies for fundamental bosons. A total of fifteen different boson types are fit by this transition radius method. For all of these bosons their TRM fit energies can match their expected values exactly. In addition to these bosons, a boson for an infinite number of spatial dimensions is defined which may represent a boson unification energy.

This transition radius method also gives a transition radius value which relates to the size of the universe at time equals zero. One of the two transition radius values for seven spatial dimension is 1130 Planck lengths which is very close to an estimated bounce radius of 1000 Planck lengths, based on an expected energy density at time equals zero, which was derived by Carmen Molina-París and Matt Visser.[5] Also, the transition radius method implies that our universe was initiated by a 'presingularity' inside a black hole in a parent universe or our universe resulted from a bounce. I use the word 'presingularity' to describe an object having a radius between one Planck length and 30,000 Planck lengths where this object at some time was capable of gravitational collapse.

Finally, I suggest ways that the transition radius method might be used to advance other theories. Throughout this paper I will be using five abbreviations. Ds will be short for spatial dimensions. For example, 3 - Ds is shorthand for three spatial dimensions. Next, Ns is short for the number of spatial dimensions. The shorthand symbol hdf stands for higher dimensional $f(r)$ which means an $f(r)$ for more than 3 - Ds. Another abbreviation is SX which is short for a SLOPE estimating expression. Finally, the overall transition radius method will be abbreviated TRM.

2 Higher demensional metric component

The transition radius method (TRM) begins with solving the Ricci expression R_{22} for f in a way that gives a sum of nonlinear terms, this R_{22} expression is defined in *General Relativity* by Robert Wald on page 123 of his book.[11] This R_{22} is the Ricci expression which was not used to derive the Schwarzschild solution as shown on pages 123 - 124 of the same book.

2.1 *Another look at the Ricci expressions*

Solving expression R_{22} for f is achieved by first simplifying the expression by substituting both $h = \frac{1}{f}$ and $h^{'} = -f^{-2}$ into R_{22}. The R_{22} expression follows:

$$R_{22} = -(\frac{1}{2})(rfh)^{-1}f' + (\frac{1}{2})(rh^2)^{-1}h' + r^{-2}(1 - h^{-1}) \tag{2.1}$$

Expression 2.1 is identical to expression 6.1.37 from reference 5. Substituting $\frac{1}{f}$ for h in 2.1 gives:

$$0 = -(\frac{1}{2})(r^{-1})f^{'} + (\frac{1}{2})(r^{-1})f^2h^{'} + (r^{-1})\frac{(1-f)}{r} \tag{2.2}$$

Substituting $(-f - 2)$ for $h^{'}$ in expression 2.2 and multiplying both sides by 2r simplifies it to:

$$0 = -f^{'} + (-f^{-2})f^2 + 2\frac{(1-f)}{r} \tag{2.3}$$

This expression reduces to:

$$0 = -f^{'}-1 + 2\frac{(1-f)}{r} \tag{2.4}$$

Expression 2.4 does not match expression 6.1.40 on page 123 of Wald's book because it is derived from expression R_{22} only. Let's rearrange expression 2.4 then integrate both sides:

$$f' = 2\frac{(1-f)}{r}-1 \tag{2.5}$$

$$\int df = \int [\ (2\ \frac{(1-f)}{r})-1\ ]\ dr \tag{2.6}$$

$$f + C1 = 2\int [\ \frac{(1-f)}{r}\ ]\ dr - \int\ dr \tag{2.7}$$

Two additional constants of integration will result from integrations on the right hand side of 2.7, but to simplify the expressions I will assume that all constants of integration are zero. Integrating expression 2.7 gives:

$$f = 2\int (\frac{1}{r})\ dr\ -2\int (\frac{f}{r})\ dr - r \tag{2.8}$$

$$f = -r + 2\ \ln(r) - 2\ \int (\frac{f}{r})\ dr \tag{2.9}$$

I have found that expression 2.9 can generate a series of functions by guessing the function f and substituting it into the right hand side of expression 2.9. The series of functions that f can generate will be called f_i expressions.The first guess of f is:

$$f = 2\ln(r) + (\frac{-r}{3})-2\ (\ln(r))^2 \tag{2.10}$$

This first guess is substituted into the right hand side of expression 2.9 giving an expression for f_i:

$$f_i = -r + 2\ln(r) - 2\int [\ (2\ln(r) - \frac{r}{3} + \frac{(-2)(\ln(r))^2)}{r}\ ]\ dr \tag{2.11}$$

$$f_i = -r + 2\ln(r) - 2\ [\ 2\ (\frac{1}{2})(\ln(r))^2 - \frac{r}{3} + (-\frac{2}{3})(\ln(r))^3\ ] \tag{2.12}$$

$$f_i = (\frac{-r}{3}) + (2\ln(r)) + (-2)(\ln(r))^2 + (\frac{4}{3})(\ln(r))^3 \tag{2.13}$$

Successively higher terms of f_i can be generated by using the fourth term of expression 2.13 as a guess for f to be substituted into the third term of expression 2.9. Let's separate out this third term of expression 2.9 so that it stands on its own as a term generating function:

$$\text{Term Generator} = -2\int (\frac{f}{r})\, dr \tag{2.14}$$

$$\text{Term 5} = -2\int [\ (\frac{4}{3})(\frac{\ln(r))^3}{r}]\ dr = -2[(\frac{1}{3})(\ln(r))^4\ ] = (\frac{-2}{3})(\ln(r))^4 \tag{2.15}$$

The process is repeated by substituting Term 5 into expression 2.14 for f to generate Term 6. In general, Term (J + 1) is generated by substituting Term J into expression 2.14 for f and solving. This process generates successively higher terms for f_i shown below:

$$\text{Term 6} = -2\int [\ (\frac{-2}{3})\frac{(\ln(r))^4}{r}\ ]\ dr \text{ so...} \rightarrow \text{Term 6} = (\frac{4}{15})(\ln(r))^5 \tag{2.16}$$

$$\text{Term 7} = (\frac{-4}{45})(\ln(r))^6 \tag{2.17}$$

$$\text{Term 8} = (\frac{8}{315})(\ln(r))^7 \tag{2.18}$$

$$\text{Term 9} = (\frac{-16}{2520})(\ln(r))^8 \tag{2.19}$$

This process can go on to generate an infinite number of terms, but let's derive a general expression for f_i by inspecting the way successive terms above change relative to each other. A repetitive trend starts in expression 2.13 with the second term 2 ln r. By comparing the second term to the higher terms listed above I found that the following expression generates successive terms:

$$\text{Term } (n+1) = [\ \frac{2\,(-2)^{n-1}}{n!}\ ](\ln(r))^n \tag{2.20}$$

A general expression for f_i can be formulated using expression 2.20 as shown below:

$$f_i(r) = (\frac{-r}{3}) + \sum_{n=1}^{k}[\ (2(-2)^n - \frac{1}{n!})(\ln(r))^n\ ] \tag{2.21}$$

In expression 2.21 the left hand side f_i (r) is a function where (r) means $f_i$is a function of the radius r. The subscript i is not yet defined in relation to n. When n = 0 expression 2.21 reduces to −1 so having n = 0 does not generate the next term after $(\frac{-r}{3})$ in expression 2.13. However, when n = 1 for expression 2.21 the term 2 ln r is generated. I noticed that the $(\frac{-r}{3})$ term corresponds to a vector for 3 - Ds (three spatial dimensions) so adding 2 ln r should correspond to 4 - Ds. If we let the subscript i designate Ns (the number of spatial dimensions), then when i = 4 the value of n is n = 1 so n = i – 3 for expression 2.21 above. Therefore, if the summation in expression 2.21 starts with n = 1 this first term of the summation generates f_4 (r). It follows that summing from n = 1 to n = 2 will generate f_5 (r) or in general summing 2.21 from n = 1 to n = k gives f_{3+k} (r) so that i = 3 + k when the summation goes from n = 1 to n = k. This fully defines the relationship between the subscript i and the summation parameter n.

Expression 2.21 combined with the paragraph following it defines a series of higher dimensional expressions that have some relationship to the (3 + 1) dimensional Schwarzschild solution. Notice that the first term on the right hand side of expression 2.21 is equal to r after both sides of expression 2.21 are multiplied by -3. If we let R_i(r) = -3 f_i (r) we see that R_i (r) appears to define a series of higher dimensional radius values. This suggests that the higher dimensional radius R_i (r) should replace the radius in (1-2M/r) to give the modified form (1-KM/R_i) (r)) where K is some constant. If this comparison to (1-2M/r) of the (3 + 1) dimensional metric is correct, then the best way to relate f_i (r) to (1-2M/r) would be to compare (1-2M/r) to $(\frac{1+(SLOPE)}{f_i(r)})$ which contains a plus sign because f_i (r) is less than zero. In my original notes I wrote this expression as (SLOPE) $[f_i(r)]^{-1} + 1$ to put it in the form mx + b so I will use this second form of the expression below.

This term (1-2M/r) comes from the (3 + 1) dimensional Schwarzschild solution which is:

$$s^2 = -(\frac{1-2M}{r})dt^2 + (\frac{1-2M}{r})^{-1}dr^2 + r^2 d\Omega^2 \qquad (2.22)$$

Here s^2 is the spacetime metric, M is the mass of the static spherically symmetric object, t is time, r is the radius, and $d\Omega^2 = (d\theta^2 + sin^2\theta d\phi^2)$ as presented by Wald. This metric is referred to as an exterior metric because it describes spacetime exterior to the massive object. Later, I will show that TRM can be extended to an interior metric.

After attempts at graphically matching hdf = ((SLOPE) $[f_i(r)]^{-1} + 1$) to (1 – 4.076×10^{38}/r), it became apparent that a SLOPE estimating expression abbreviated SX helps to speed up the process of comparing these two expressions. This SX parameter is:

$$SX = (\frac{-2M}{r})(f_i(r)) \tag{2.23}$$

The variables M and r are the same as defined previously. SX will be averaged over a range of radius values for each Ns to determine the most likely value for SLOPE.

Please examine the two tables below numbered 1 and 2 to see how comparison of hdf to $(1 - 4.076 \times 10^{38}/\text{r})$ provides insights into processes occurring over a wide range of radius values. The abbreviation hdf means higher dimensional f (r); I am jumping ahead with my definitions by setting hdf = (SLOPE) $[f_i(r)]^{-1}+1$, but using an abbreviation reduces the amount of typing so I might as well use an abbreviation that is going to apply later. The tables which follow will compare hdf to $(1 - 4.076 \times 10^{38}/\text{r})$ for two numbers of spatial dimensions 9 - Ds and 10 - Ds where M takes on a value corresponding to 2.20 solar masses.

A striking phenomenon seen in these tables is the way hdf changes from a large negative value to a large positive value for radius values below 4 Planck lengths. These changes are labeled inflation triggers, but their effects may not trigger inflation until after the metric flips to its mirror image solution which happens because the spacetime metric is s^2.

Radius (r)	$[f_9(r)^{-}1]$	$(2.08\text{x}10^{39}[f_9(r)^{-}1]+1$	$1\text{-}4.076\text{x}10^{38}/\text{r}$
10^{30}	− 3.00 (−30)	– 6.240 (9)	– 4.076 (8)
10^{10}	− 2.99 (− 10)	– 6.219 (29)	– 6.219 (29)
10^{9}	− 2.946 (− 9)	– 6.128 (30)	– 4.076 (29)
10^{8}	− 2.754 (−8)	– 5.728 (31)	– 4.076 (30)
10^{7}	− 2.154 (−7)	– 4.480 (32)	– 4.076 (31)
10^{6}	− 1.193 (− 6)	– 2.481 (33)	– 4.076 (32)
10^{5}	− 5.094 (− 6)	– 1.060 (34)	– 4.076 (33)
10^{4}\	− 2.282 (− 5)	– 4.747 (34) \(transition radius is	– 4.076 (34)
10^{3}/	− 1.437 (− 4)	– 2.989 (35) 5776 Planck lengths)	– 4.076 (35)
10^{2}\	− 1.881 (− 3)	– 3.912 (36) \(transition radius is)	– 4.076 (36)
10/	− 0.1285	– 2.673 (38) / 90 planck lengths	– 4.076 (37)
5	− 0.8277	– 1.722 (39)	– 8.152 (37)
4	-1.724	-3.586(39)	– 1.019 (38)
3	− 6.694	– 1.392 (40) \Inflation	– 1.359 (38)
2	+ 12.24	+ 2.546 (40) / Trigger	– 2.038 (38)
1.6	+ 13.17	+ 2.739 (40)	– 2.548 (38)
1.4	+ 43.26	+ 8.998 (40) \ Inflation	– 2.911 (38)
1.2	− 10.59	– 2.203 (40) / Trigger	– 3.397 (38)

Table 1: Radius(r) versus (SLOPE) $[f_9(r)]^{-1}+1$ and $(1 - 4.076 \times 10^{38}/\text{r})$ at 2.20 solar masses for 9 spatial* dimensions where (r) is in Planck lengths and $f_9(r)$ is: $f_9(r) = (\frac{-r}{3})+2\ln(r)+(-2\ln(r^2))+(\frac{4}{3}\ln(r^3))+(\frac{-2}{3}\ln(r^4))+(\frac{4}{15}\ln(r^5))+(\frac{-4}{45}\ln(r^6))$
[at 2.20 solar masses the nongeometrized $(1 - (\frac{2M}{r})) = (1 - 4.076 \times 10^{38}/\text{r})$] **

* SLOPE is an average of the 3.5 largest SX values (3 largest SX averaged then averaged with the average of the 4 largest SX) so Nsx = 3.5.

** Columns 3 and 4 are in nongeometrized units per table F.1 on page 471 of General Relativity by Robert Wald [11].

‡ Note that an abbreviated form of scientific notation is used above where − 3.00 (−30) = − 3.00 x 10^{-30}.

* SLOPE is an average of the 3 largest SX values so Nsx = 3.

Radius (r)	$[f_{10}(r)^{-}1]$	(-3.786x10^{39})$[f_{10}(r)^{-1}]$+1	1-4.076x10^{38}/r
10^{30}	− 3.00 (−30)	+ 1.136 (10)	– 4.076 (8)
10^{10}	− 3.069 (− 10)	+ 1.162 (30)	– 4.076 (28)
10^{9}	− 3.358 (− 9)	+ 1.271 (31)	– 4.067 (29)
10^{8}	− 5.546 (− 8)	+ 2.100 (32)	– 4.076 (30)
10^{7}	+ 3.944 (− 7)	– 1.492 (33)	– 4.076 (31)
10^{6}	6.244 (− 7)	– 2.362 (33)	– 4.076 (32)
10^{5}\	2.064 (− 6)	– 7.814 (33) \ [transition radius is 1.32(4)]	– 4.076 (33)
/10^{4}/	1.011 (− 5)	– 3.828 (34) /	– 4.076 (34)
\\$10^{3}$	8. 262 (-5)	– 3.127 (35) \[transition radius is /260 Planck lengths]	– 4.076 (35)
10^{2}	1.704 (− 3)	– 6.451 (36)	– 4.076 (36)
10	+ 1.071	– 4.055 (39) \ Inflation	– 4.076 (37)
5	− 2.009	+ 7.606 (39) / Trigger	– 8.152 (37)
4	− 3.030	+ 1.147 (40)	– 1.019 (38)
3	− 9.967	+ 3.774 (40) \Inflation	– 1.359 (38)
2	+ 11.96	– 4.528 (40) / Trigger	– 2.038 (38)
1.6	+ 13.15	– 4.978 (40)	– 2.548 (38)
1.4	+ 43.23	– 1.637 (41) \ Inflation	– 2.911 (38)
1.2	− 10.59	+ 4.009 (40) / Trigger	– 3.397 (38)

Table 2: Radius r versus (SLOPE) $[f_{10}(r)]^{-1}+1$ and $(1-4.076\times10^{38}/\mathrm{r})$ at 2.20 solar masses for 10 spatial*dimensions where r is in Planck lengths and f10 (r) is: $f_{10}(r) = (\frac{-r}{3}) + 2\ln(r) + (-2)(\ln(r))^2 + (\frac{4}{3})(\ln r)^3 + (\frac{-2}{3})(\ln(r))^4 + (\frac{4}{15})(\ln(r)^5 + (\frac{-4}{45})(\ln(r))^6 + (\frac{8}{315})(\ln(r))^7$ [at 2.20 solar masses the nongeometrized (1 – 2M/r) = $(1-4.076\times10^{38}/\mathrm{r})$]

2.2 Coupling hdf to 1-2M/r

The point of making an effort to couple hdf to (1 – 4.076 x 10^{38}/r) is to get the higher dimensional versions of the metric component f (r) to merge with the value of the 3 - Ds metric component over a range of radius values. For Tables 1 and 2 above the first question that might come to mind is how the SLOPE values were determined for each of these tables. A SLOPE estimating parameter SX is defined to determine SLOPE for hdf.

To determine what form this SX should have the hdf and (1-2M/r) expressions are closely examined to see where they are linked to energy level. For the geometrized (1 – 2M/r) the second term – 2M/r is proportional to energy divided by radius because mass is equal to E/c^2. Also, (SLOPE) $[f_i(r)]^{-1}$ is the part of hdf which corresponds directly to $-2M/r$ so it is also proportional to energy divided by radius or E/r. The ratio of these two E/r expressions gives:

$$\frac{E_1/r_1}{E_2/r_2} = \frac{-2M/r}{(SLOPE)[f_i(r)]^{-1}} \tag{2.24}$$

From previous examination of $f_i(r)$ it was shown that $R_i(r) = -3 f_i(r)$ can be considered a higher dimensional radius. This means that the expression $[f_i(r)]^{-1}$ is proportional to the reciprocal of a higher dimensional radius so that SLOPE should be proportional to energy. To estimate the value for SLOPE the left side of expression 2.24 above can be set equal to one and after rearrangement this gives:

$$SLOPE = \frac{-2M/r}{[f_i(r)]^{-1}} \tag{2.25}$$

This shows that when the energy/radius ratio is equal to one the expression 2.25 gives the SLOPE, but this expression generates a large number of SLOPE values because it gives a different SLOPE for each value of radius. To estimate the best value of SLOPE the SLOPE parameter in expression 2.25 can be replaced by a parameter called SX which will be averaged over a range of radius values to estimate SLOPE. The expression for SX is:

$$SX = \frac{-2M/r}{[f_i(r)^{-1}]} = -(2M/r)[f_i(r)] \tag{2.26}$$

Please notice that this SX expression can be quickly calculated using the values in columns two and four in each table above by using the nongeometrized form of the expression:

$$SX = \frac{[(1\text{–}4.076 \times 10^{38}/r) - 1]}{[f_i(r)]^{-1}} \tag{2.27}$$

Expression 2.27 is calculated for each value of radius in the previous tables and the largest SX values are averaged together to estimate SLOPE. Assuming that the largest SX values are the best estimators for the value of SLOPE is the same as assuming that 3 - Ds and higher spatial dimensions will prefer to couple at the highest shared energy levels where the left side of expression 2.24 is one. However, it was found that the single highest SX value does not give reasonable boson energy predictions for 5 - Ds. This may be due to the need to couple the 3 - Ds metric component to hdf over a range of radius so that the coupling is not limited to too narrow a range of radius. The best number of largest successive SX values to average together (for SX determined for each radius value in the tables above) ranges from 1.1 to five for the spatial dimensions from 5 - Ds to 25 - Ds. The SX values included in this averaging in all but one case are calculated for successive radius values where each radius is ten times larger than the previous radius. The one exception to this rule occurred for 5 - Ds because some of its largest SX values occur below 10 Planck lengths.

I will abbreviate the best number of SX values to average together to estimate SLOPE as Nsx. The table for 4 - Ds is not presented above, but it shows that for 4 - Ds the value of SX remains constant over radius values ranging from 10^6 to 10^{10} Planck lengths. This means that 4 or 5 values of SX can be averaged together over this radius range without changing SLOPE. It was found that for 4 - Ds there are actually an infinite number of transition radius values for which hdf equals $(1 - 4.076 \times 10^{38}/r)$, but it is not necessary to average more than four largest SX values corresponding to successive radius values in column one to determine the best value for SLOPE for 4 - Ds. A table of Ns (the number of spatial dimensions) versus Nsx will be presented later to show how Nsx varies with Ns; this Nsx data is fit assuming Nsx varies smoothly versus odd Ns and versus even Ns separately which was a trend that reinforced itself as more data was fit.

This answers the question concerning how SLOPE values are determined for tables 1 and 2. The noted areas in column three of these tables will be explained in the next section.

3 Transition and inflation trigger radii

Transition radii and inflation trigger radii are two of the three main characteristics common to most of the 5 - Ds through 25 - Ds data tables showing the coupling hdf to $(1 - 4.076 \times 10^{38}/r)$. The third characteristic is whether the SLOPE value is positive or negative. After defining what I mean by a transition radius and an inflation trigger radius, I will present all three characteristics for all spatial dimensions from 5 - Ds to 25 - Ds in tables 3 and 4 below for 2.20 solar masses; one table is for spatial dimensions with positive SLOPE values and the other table is for spatial dimensions with negative SLOPE values.

3.1 Transition Radius Defined

A transition radius is any specific radius value where hdf = (1 – 2M/r) for a given value of nongeometrized M. In most cases there are two transition radii associated with each Ds. However, for 4 - Ds and 6 - Ds there are an infinite number of transition radii because hdf = $(1 - 4.076 \times 10^{38}/r)$ over a wide range of radius values. This phenomenon seems to occur naturally for 4 and 6 spatial dimensions. For 4 - Ds and 6 - Ds a specific transition radius value can not be defined because infinitely many are possible.

I have set Nsx to four for 25 – Ds. The reason I stopped at 25 spatial dimesions is because string theory predicts that it is the highest number of dimensions possible for bosons is (25 + 1) as shown by Yoichiro Nambu. [4, p.93] This limitation to 26 spacetime dimensions means that the maximum number of spatial dimensions should be limited to 25.

I will be referring to transition radii as either r_{LT} or r_{ST} where the former is the symbol for larger transition radius and the latter is for the smaller transition radius. Both of these are equivalent to the transition radius r_T ; I have altered these subscripts merely to keep track of two transition radii for each spatial dimension.

3.2 Inflation Transition Radius Defined

An inflation trigger radius is a radius value where the value of hdf goes from a large negative value to a large positive value with a relatively small decrease in the radius value. Inflation triggers usually occur for radius values less than 4 Planck lengths. However, exceptions to this rule are seen for 4 - Ds and 6 - Ds, as well as, for the even numbered spatial dimensions which are greater than 8 - Ds.

I have not determined if the smaller inflation triggers cause the value of the spacetime metric to change sign or if the bubble shrinks to a radius of one Planck length before the spacetime metric changes sign as the bubble grows. Other theories suggest than the singularity will go below a radius of one Planck length, but TRM seems to be suggesting that the radius should go no lower than one Planck length.

Resolving at what bubble radius collapse should reverse to expansion is beyond the scope of this paper so I will merely comment on the number of inflation trigger radius values predicted by TRM and speculate on the number of stages of inflation this implies

Ns	rLT	rST	SLOPE	Larger Inflation Trigger Radius Range	Smaller Inflation Trigger Radius Range
4	-	-	1.359 (38)	10 – 100	-
5	68.5	5.06	2.979 (38)	-	-
6	-	-	1.359 (38)	103 — 104	-
7	1075	13.3	5.87 (38)	2 – 3	1.2 – 1.4
9	5780	90.1	1.943 (39)	2 – 3	1.2 – 1.4
11	5.50(4)	500	6.999 (39)	2 – 3	1.2 – 1.4
13	5.26 (5)	2.73 (3)	2.6762 (40)	2 – 3	1.2 – 1.4
15	2.12 (6)	3.15 (4)	1.0924 (38)	2 – 3	1.2 – 1.4
17	6.20 (6)	4.97 (5)	4.352 (41)	2 – 3	1.2 – 1.4
19	1.78 (8)	1.132 (6)	1.444 (42)	2 – 3	1.2 – 1.4
21	1.42 (9)	7.65 (6)	5.508 (42)	2 – 3	1.2 – 1.4
23	5.00 (9)	1.078 (8)	2.112 (43)	2 – 3	1.2 – 1.4
25	5.00 (9)	—	8.27 (43)	2 – 37	1.2 – 1.4

Table 3: Ns for Negative SLOPE versus Transition Radii, SLOPE, and Inflation Trigger Radius Ranges*

* The large and small transition radii, as well as, the inflation trigger radii are in units of Planck lengths.

Ns	rLT	rST	SLOPE	Largest Inflation Trigger Radius Range	Medium Inflation Trigger Radius Range	Smaller Inflation Trigger Radius Range
8	2260	33.5	– 8.49 (38)	–	-	1.2 – 1.4
10	1.31 (4)	259	– 3.673 (39)	5 – 10	2 – 3	1.2 – 1.4
12	8.49 (4)	1980	– 1.486 (40)	10 – 100	2 – 3	1.2 – 1.4
14	4.24 (5)	1.93 (4)	– 5.923 (40)	10 – 100	2 – 3	1.2 – 1.4
16	3.40 (6)	1.30 (5)	– 2.396 (41)	10 – 100	2 – 3	1.2 – 1.4
18	3.30 (7)	7.42 (5)	– 8.70 (41)	10 – 100	2 – 3	1.2 – 1.4
20	4.85 (8)	3.03 (6)	– 2.94 (42)	100 – 103	2 – 3	1.2 – 1.4
22	3.30 (9)	2.29 (7)	– 1.10 (43)	100 – 103	2 – 3	1.2 – 1.4
24	7.00 (9)	5.00 (8)	– 3.932 (43)	100 – 103	2 – 3	1.2 – 1.4

Table 4: Ns for Negative SLOPE versus Transition Radii, SLOPE, and Inflation Trigger Radius Ranges*

* The large and small transition radii, as well as, the inflation trigger radii are in units of Planck lengths. For both tables these results are for a static spherically symmetric object of 2.20 solar masses.

3.3 Initial Observations

Tables 3 and 4 contain all of the key characteristics of the tables for the coupling of hdf to $(1 - 4.076 \times 10^{38}/r)$. Showing all of these coupling tables would have been too tedious so I've summarized their features in the two tables above. It is advantageous to split these tables so all positive SLOPE values are in one table and negative SLOPE values are in the other. This split presentation emphasizes that all the spatial dimensions having positive SLOPE values have no more than two inflation trigger radius ranges, and almost all spatial dimensions with negative SLOPE have three inflation trigger ranges. This strongly suggests that there is a significant different between spatial dimensions having positive SLOPE values versus those with negative SLOPE values. In section 2.2 it made sense to think of SLOPE as proportional to energy. If this idea is carried over to tables 3 and 4 it implies that spatial dimensions having a positive SLOPE have positive energy and spatial dimensions having a negative SLOPE have negative energy. This assumption will affect the energy values predicted for bosons later.

Skimming through the two columns of transition radius values in the above two tables, I came across one transition radius which stood out from the rest. In table 3 the larger transition radius for 7 - Ds is 1130 Planck lengths. A bubble having a radius of 1130 Planck lengths may interact in such a way with this transition radius that it is momentarily restricted to this size. As Carmen Molina-París and Matt Visser pointed out in their paper, the energy density at a bounce suggests that the bubble should rebound from a radius of about 1000 Planck lengths [5]. This finding suggests the larger transition radius for 7 - Ds has some influence over the size of the bubble which inflates to become our universe. If this speculation is correct, it may be that the initial formation of large-scale structure is influenced by the larger transition radius for 7 - Ds; this influence may be due to this transition radius causing a momentary slowing of the collapse so the bubble pauses near a radius of 1130 Planck lengths.

Inflation trigger radii are the last bit of characteristic data on which to elaborate. I will not determine how small a collapsing object gets when it obeys the restrictions imposed by this TRM model because such an analysis is beyond the scope of this paper. However, I will speculate that it may be preferable to have the bubble collapse to the 1.2 – 1.4 Planck length radius range so that the smallest radius inflation triggers associated with the negative energy spatial dimensions (in table 4) get a chance to participate in the initiation of inflation. This may be preferred because a very high negative energy density could be generated inside the bubble if the most energetic spatial dimension reached during the collapse is a negative energy spatial dimension, and an energy density dominated by a negative energy density is needed to get the bubble to begin expanding as Alan Guth pointed out in *The Inflationary Universe*. [2, p.266] Analysis of this with other models would be needed to be more confident about this speculation. However, if the bubble were to begin expanding from a radius range of 1.2 – 1.4 Planck lengths it would follow that there should be at least two dominant stages of inflation. One stage of inflation would be dominated by the inflation trigger event at a 1.2 – 1.4 Planck length radius range, and the second stage of inflation would be dominated by the set of inflation triggers at a 2 – 3 Planck length radius range.

A third inflation event which occurs over a much wider range of radius values (associated with the largest inflation trigger radius range in table 4) may also contribute to initiating inflation. Depending on how small the bubble gets before inflation is initiated, there may be as many as three dominant inflation stages, but one or two stages seems most likely. The inflation trigger for 4 - Ds occurs in the same radius range as that for the third inflation event above so its minor effect should merge with the third inflation event. TRM can predict the precise transition radius value where these inflation triggers initiate by looking for a sign change in hdf.

Only the inflation trigger associated with the 6 - Ds stands alone at a radius range which does not match any of the others. This 6 - Ds inflation trigger may momentarily slow the collapse of the bubble long enough for the 1130 Planck length transition radius associated with the 7 - Ds to influence large-scale structure.

4 Nsx distribution for SX

Previously, in section 2.2, it was shown that the best SLOPE value should be determined by averaging several of the largest SX values. The number of SX values that need to be averaged to get the best estimate for SLOPE is called Nsx. A value of Nsx = 4 worked best for 4 - Ds, 5 - Ds, and 6 - Ds. However, the most reasonable way for TRM to get close to predicting a specific boson energy makes it necessary for Nsx to ramp down from Nsx = 4 to Nsx = 1.13 at 16 - Ds. Furthermore, it is advantageous for Nsx values to climb up toward Nsx = 4 again after 16 - Ds to fit a specific boson energy at 22 - Ds.

These restrictions on Nsx are *ad hoc*, but there may be a theoretical explanation for the Nsx distribution below. The following Nsx distribution in table 5 for the spatial dimensions from 4 - Ds up to 25 - Ds is assumed for all initial SLOPE values determined by TRM.

Even Ns	Nsx for Even Ns	Odd Ns	Nsx for Odd Ns
4	4	5	4
6	4	7	4
8	3.5	9	3.875
10	3.25	11	3.75
12	3	13	3.668
14	1.969	15	2.758
16	1.13	17	2.27
18	1.781	19	4.024
20	3.49	21	4.032
22	4	23	4
24	4	25	4
26	4	27	4

Table 5: Even Ns and Odd Ns versus Nsx – when $2M = 4.076 \times 10^{38}$

* See Appendix A for more details

Seeing Nsx = 3.75 in table 5 may be a bit puzzling so I will describe what such a value means when averaging SX values for 7 - Ds. When we average values to determine a mean value we always think of averaging a number of values which is a whole number. When we are asked to average 3.75 values this does not make sense unless we regard it as a shorthand for a more complex averaging technique. In this case, to average 3.75 SX values means to combine averages for whole numbers of SX values. First, find the Nsx = 3 and Nsx = 4 averaged SX values which are $SLOPE_3 = (SX1 + SX2 + SX3)/3$ and $SLOPE_4 = (SX1 + SX2 + SX3 + SX4)/4$, respectively. Next, to find the SLOPE3.5 , which is a 3.5 SX average, we just average the two previous slopes so that $SLOPE_3.5 = (SLOPE_3 + SLOPE_4)/2$. Now averaging 3.75 SX values can be defined as averaging $SLOPE_{3.5}$ with $SLOPE_4$ or $SLOPE_{3.75} = (SLOPE_3.5 + SLOPE_4)/2$ which completes the definition for an average of 3.75 SX values. Those who have a problem with this fractional averaging technique may prefer to think of Nsx = 3.75 as a shorthand way of indicating the complex averaging technique defined above.

The values of Nsx in table 5 are likely to be very close to their true values. If some other theory is developed to explain the smooth shape of this Nsx distribution, fitting the Nsx values listed in table 5 would be an indication that such a theory is on the right track.

5 Boson energy from transition radius

To resolve the part of the hierarchy problem involving fundamental boson energies a model must give accurate boson energy E_B predictions over a wide range of boson energies. Immediately after finding the transition radius values listed in tables 3 and 4, I wondered if the volumes associated with these transition radius values are connected to the energy of some types of particles. I started to think about these transition radius volumes which are (4/3) π (r_T) where r_T is the transition radius defined above which can be either r_{LT} or r_{ST} as defined previously. While thinking of these transition radius volumes and Planck energy I found a simple equation for boson energy.

As the radius of a boson increases from 1.0 Planck lengths to some transition radius value the energy of larger bosons should decrease proportional to the volume ratio given by the expression VR = (4/3) π $(1.0)^3$ /[(4/3) π $(r_T)^3$] or simplifying VR = 1 / (r_T). The maximum energy possible in the smallest region of space is the Planck energy which is Planck energy = 1.22 x 10 GeV. The energy that a boson volume can hold should decline from Planck energy where a boson with a one Planck length radius is at Planck energy, and this boson energy should decline in a way such that boson energy is proportional to VR. I assume this rule holds from one transition radius boson type to the next. Therefore, the energy that a gauge boson volume can hold decreases as the radius increases where the boson's energy can be found by multiplying the Planck energy by VR, giving:

$$E_T = \pm(1.22x10^{19}GeV)VR \rightarrow E_B = E_T = \pm\frac{1.22x10^{19}\text{GeV}}{(r_T)}^3$$

$$E_B = \frac{E_P}{((r_t + M)^3 - A)} \tag{5.28}$$

for $A \neq 0, A \neq 1, r_t \geq 1$ for $r_t > 20$ and $A = -1$ only

or an alternate form $E_B = \dfrac{E_P}{(r_T)^3}$

where E_B is energy of a boson at this transition radius, E_T is the energy corresponding to transition radius r_T (where r_T is in Planck lengths), and E_P is the Planck energy.

When a particle/anti-particle pair annihilates it may produce more than one boson; the E_B value above is the total energy of the rest mass particle/anti-particle pair. Expressions 5.28 which defines transition energy will be used to predict the energy of bosons which have radius values equal to some transition radius. I have assumed that these boson energy values (E_B) can be found from $E_B = E_T$ so expression 5.28 gives a boson energy when a transition radius is entered for r_T. I will assume that all of the particle/anti-particle annihilation energy goes into a single boson of energy E_B. This links a set of fundamental boson energies to transition radii derived from the transition radius method, and TRM is derived from general relativity so it follows that a set of specific annihilation boson energies is linked to general relativity.

Previous methods for finding particle energy have been based on wavelength. One of these methods, called "Special-Scale Relativity" by L. Nottale uses an expression based on wavelength and the Planck scale plus several other factors to predict particle energy. [7] Such an expression is a very good way to make a quick estimate of the energy of a particle or boson, but such methods were not derived directly from general relativity.

Alternatively, TRM uses expression 5.28 to predict annihilation boson energy from transition radius values derived from general relativity. This makes TRM a more basic model because it predicts annihilation boson energies without using particle mass or wavelength.

Expression 5.28 will be used to predict boson energies for the transition radius values for both versions of TRM. Tables 6 and 7 below show the predicted boson energies for TRM. These predicted boson energies are labeled E_{LT} and E_{ST} which correspond to the transition radii r_{LT} and r_{ST}, respectively.

Ns	rLT	ELT ‡ (Boson Energies)	rST	EST (Boson Energies)
4	–	–	–	–
5	69	+ (-) 3.71 (13)	5.05	+ (-) 9.47 (16)
6	–	–	–	–
7	1128 *	+ (-) 8.50 (9)	13.0	+ (-) 5.55 (15)
8	2265	– (+) 1.05 (9)	33.4	– (+) 3.27 (14)
9	7456	+ (-) 2.94 (7)	76.8	+ (-) 2.69 (13)
10	1.494 (4)	– (+) 3.66 (6)	237.6	– (+) 9.10 (11)
11	5.53 (4)	+ (-) 7.21 (4)	499.5	+ (-) 9.79 (10)
12	8.54 (4)	– (+) 1.96 (4)	1969	– (+) 1.60 (9)
13	3.699 (5)	+ (-) 241	3667	+ (-) 2.47 (8)
14	4.237 (5)	– (+) 160.4	1.927 (4)	– (+) 1.705 (6)
15	1.696 (6)	+ (-) 2.50	3.76 (4)	+ (-) 2.295 (5)
16	1.21 (6)	– (+) 6.89	3.264 (5)	– (+) 351
17	7.77 (6)	+ (-) 0.026	4.059 (5)	+ (-) 182.4
18	1.48 (7)	– (+) 3.76 (-3)	1.502 (6)	– (+) 3.60
19	1.781 (8)	+ (-) 2.39 (-6)	1.132 (6)	+ (-) 8.41
20	3.63 (8)	– (+) 2.55 (-7)	3.861 (6)	– (+) 0.212
21	1.49 (9)	+ (-) 4.26 (-9)	7.66 (6)	+ (-) 0.0271
22	3.39 (9)	– (+) 3.13 (-10)	2.287 (7)	– (+) 0.00102
23	1.113 (10)	+ (-) 8.85 (-12)	5.26 (7)	+ (-) 8.38 (-5)
24	4.72 (10)	– (+) 1.16 (-13)	9.75 (7)	– (+) 1.32 (-5)
25	7.462 (10)	+ (-) 2.9363 (-14)	4.137 (8)	+ (-) 1.723 (-7)

Table 6: Ns versus r_{LT}, ELT, r_{ST}, and EST for 2.20 solar masses where r_{LT} and r_{ST} are in Planck lengths and ELT and EST are in GeV energy units (for TRM using table 5 when 2M = 4.076×10^{38}).

‡ The possibility of negative energies are due to $E^2 = p^2c^2 + m^2c^4$ and to sign changes in SLOPE values. * The radius at Ns = 7 per TRM when Nsx = 4 for 2.2 solar masses is 1128 Planck lengths.

Table 7: Ns versus Boson Type, TRM Predicted Boson Energy, and Measured or Estimated Boson Energy where Boson Energies are in GeV

Ns	Boson Type	TRM Predicted Boson Energy (GeV)	Measured or Estimated Boson Energy (GeV)
5	X like boson	+ (-) 3.71 (13) , 9.47 (16)	10^{14} to 10^{15} *
7	X like boson	+ (-) 5.55 (15)	10^{14} to 10^{15} *
13	W boson	241 (triple)	80.2 *
14	W boson	– (+) 160.4 (double)	80.2 *
15	Charm/anti-charm	+ (-) 2.50	2.50
17	Z boson	+ (-) 182.4 (double)	91.2 *
18	Up/anti-up quarks	– (+) 3.76 (-3)	2 (-3) to 10 (-3)
21	e-neutrino/anti-νe annihilation boso	+ (-) 4.26 (-9)	< 6.0 (−9)
22	e− e+ annihilation negative energy photon	– (+) 0.00102	+ 0.00102

* From a paper titled *Elementary Particles and Cosmology* by I L Rozental', Russian Academy of Sciences, Uspekhi Fizicheskikh Nauk, 1997.[9]

The boson energies in table 7 show that TRM gives a reasonable approximation for at least seven specific boson types; very little SLOPE adjustment is needed to fit boson energies.

Getting reasonably close to predicting boson energies for so many different boson types for bosons over a wide range of energies is a good sign that TRM is able to fit fundamental particle energies so that the pattern of particle energies is explained by the smooth change of the single parameter Nsx over even numbered spatial dimensions and by a smooth change of Nsx over odd numbered spatial dimensions.

Table 8 below shows that most New SLOPE values are 6.25 x 10^{32} times larger than the Previous SLOPE values, and this ratio comes from the reciprocal of 1.6 x 10^{-33} centimeters per Planck length which was expected.

To make it easier to duplicate this work I have included an Appendix B where I have listed a BASIC language computer program used to calculate the f_i (r) values. Computerizing the entire TRM process will speed E_B estimation.

Ns	New * SLOPE	New SLOPE Divided by Previous SLOPE **	Ext *** r_{LT}	Ext *** E_{ST} (Gev)	Ext *** r_{ST}	Ext *** E_{ST} (Gev)
7	5.87 (38)	6.10 (32)	1128 ‡	8.50 (9)	13.0	5.55 (15)
13	2.6762 (40)	NA	3.699 (5)	241	3776	2.47 (8)
15	1.0924 (41)	6.25 (32)	1.696 (6)	2.50	3.76 (4)	2.295 (5)
16	– 2.396 (41)	6.25 (32)	1.21 (6)	6.89	3.264 (5)	351
17	4.352 (41)	NA	7.77 (6)	0.0260	4.059 (5)	182.4
18	– 8.70 (41)	6.25 (32)	1.48 (7)	3.76 (-3)	1.502 (6)	3.60
19	1.444 (42)	6.25 (32)	1.781 (8)	2.39 (-6)	1.132 (6)	8.41
20	– 2.94 (42)	6.25 (32)	3.63 (8)	2.55 (-7)	3.861 (6)	0.212

Table 8: . Ns versus New SLOPE, New SLOPE/Previous SLOPE, r_{LT}, E_{LT}, r_{ST}, and E_{ST}

* New SLOPE means the result obtained when the value of 2M used is 4.076×10^{38} Planck lengths.

** Previous SLOPE means the result obtained when the value of 2M used was 652,100 centimeters.

‡ This transition radius is for Nsx = 4 for Ns = 7 at 2.2 solar masses when 2M $= 4.076 \times 10^{38}$.

NA = not applicable due to change from single particle energy to multiple particles.

Ns	Boson Type	Extended TRM Predicted Boson Energy (GeV)	Measured or Estimated Boson Energy (GeV)
5 and 7	X like boson	3.71 (13) , 9.47 (16) , 5.55 (15)	10^{14} to 10^{15} *
13 and 14	W boson	+ (-) **241** (triple) and **160.4**	80.2 *
15	Charm/anti-charm boson	+ (-) 2.50	2.50 **
16	top/anti-top quark boson	– (+) 351	+ 349 **
17	Z boson	+ (-) **182.4** (double)	182.4 *
17	tau-neutrino/anti-t-neutr.	0.026	< 0.036 **
18	Up/anti-up quark negative energy boson	– (+) 3.76 (−3)	2 (−3) to 10 (−3) **
18	tau/anti-tau negative energy boson	– (+) 3.60	+ 3.60 **
19	μ-neutrino/anti-μ- neutrino boson	+ (-) 3.76 (– 3)	< 3.8 (−3) **
19	bottom/anti-bottom quark annihilation boson	+ (-) 8.41	8.40 **
20	muon/anti-muon negative energy boson or strange/anti-strange negative energy boson	– (+) 0.212	+ 0.212 **
21	e-neutrino/anti-νe annihilation boson	+ (-) 4.26 (– 9)	< 6.0 (−9) **
22	e− e+ annihilation negative energy photon	– (+) 0.00102	+ 0.00102

Table 9: Ns versus Boson Type, TRM Predicted Boson Energy, and Measured or Estimated Boson Energy where Boson Energies are in GeV

* From a paper titled *Elementary Particles and Cosmology* by I L Rozental', Russian Academy of Sciences, Uspekhi Fizicheskikh Nauk, 1997. [9] ** Modified from a paper titled Report of The Working Group on The Future of Accelerator-Based Particle Physics in Europe, ECFA/01/213, 13 September 2001.[1]

Even Ns	New SLOPE for Even Ns	Odd Ns	New SLOPE for Odd Ns
4	1.359 (38)	5	2.979 (38)
6	1.359 (38)	7	5.87 (38)
8	– 8.490 (38)	9	1.943 (39)
10	– 3.673 (39)	11	6.999 (39)
12	– 1.486 (40)	13	2.6762 (40)
14	– 5.923 (40)	15	1.0924 (41)
16	– 2.369 (41)	17	4.352 (41)
18	– 8.70 (41)	19	1.444 (42)
20	– 2.94 (42)	21	5.508 (42)
22	– 1.10 (43)	23	2.112 (43)
24	– 3.932 (43)	25	8.270 (43)

Table 10: Even Ns vs. New SLOPE and Odd Ns vs. New SLOPE ($2M = 4.076 \times 10^{38}$ at 2.2 solar masses)

Tables 10 and 11 show the best values of SLOPE, r_{LT} , and r_{ST} for TRM when it is solved assuming a static spherically symmetric object of 2.20 solar masses.

Table 9 shows the best TRM predicted boson energies in comparison to measured or estimated boson energies. The data in this table is for TRM when solved for an object of 2.20 solar masses. It is puzzling that R_{22} which is associated with an exterior metric should give a model which fits boson energies so well; I will explain why in section 9.

TRM can fit exact boson energies for at least thirteen boson types (as shown in table 9). These include the three bosons X, W, and Z boson. Also included are the quark/anti-quark annihilation positive energy bosons for quark types charm, bottom, and top, and the quark/anti-quark annihilation boson energies are nearly identical for the up and down quarks so both are predicted from one value. Two additional bosons which both have negative energy results for TRM are the muon/anti-muon annihilation boson and the strange/anti-strange annihilation boson. Finally, the electron/positron annihilation boson, and the tau/anti-tau annihilation boson can be fit with no SLOPE value adjustment. The boson energy predicted by TRM for the μ-neutrino/anti-μ-neutrino annihilation boson is close to its upper limit.

Thus, TRM can fit the boson energies of all six quark/anti-quark annihilation bosons plus boson energies associated with all six lepton/anti-lepton bosons, plus the W, Z , and X.

There may be other bosons which can be fit by TRM. For example, the electron/positron annihilation energy which fits at Ns = 22 when Nsx = 4 can also be fit at Ns = 17 for Nsx = 3.978. Many of the TRM predicted boson energies above remain unmatched to known particles; these new particles may be found in the future. Additional boson matches may be possible, but I will move on to other concepts.

6 A Boson of infinite dimensions

There is another boson not listed in the above tables which can be predicted by expanding expression 2.21 out to an infinity number of spatial dimensions. Expression 2.21 contains a summation expression which can be summed over an infinite number of terms. Since each term added to this summation describes the next spatial dimension, expanding the summation to an infinite number of terms is the same as examining an infinite number of spatial dimensions.

To find this boson energy for an infinite number of dimensions we start with expression 2.21 from section 2.1 which is:

$$f_i(r) = (-\frac{r}{3}) + \sum_{n=1}^{k}[(\frac{2(-2)^{n-1}}{n!})(\ln(r))^n]$$

Focusing on the summation portion only we have:

$$\text{Summation} = \sum_{n=1}^{k}[\frac{(2(-2)^{n-1}}{n!})(\ln(r))^n] \tag{6.29}$$

Letting $x = \ln(r)$ and simplifying gives:

$$\text{Summation} = -(\frac{1}{2})(2)\sum[(-1)^n(2)^n(\frac{x^n}{n!})] \tag{6.30}$$

or

$$\text{Summation} = -\sum_{n=1}^{k}[(-1)^n(2)^n\frac{x^n}{n!}]$$

when k goes to infinity this summation goes to:

$$\text{Summation} = -(e^{-2x-1}) = (1 - e^{-2x}) \quad (\text{for} k = \infty) \tag{6.31}$$

If x is replaced by ln(r) this becomes:

$$\text{Summation } (k = \infty) \ = (1 - r^{-2}) \tag{6.32}$$

Substituting this into expression 2.21 gives:

$$f_i(r) = (\frac{-r}{3}) + (1 – r^{-2}) \text{ for } i = \infty \tag{6.33}$$

As I pointed out on in the second paragraph after expression 2.21 in section 2.1 when f_i (r) is multiplied by a factor of −3 the result is an expression for the higher dimensional radius which is R_i (r) = −3 f_i (r) or:

$$R_i(r) = r - 3\sum_{n=1}^{k}[(\frac{2(-2)^{n-1}}{n!})(\ln(r))^n] \tag{6.34}$$

An expression for the metric component f (r), which will be derived in a later section, has the form:

$$f(r) = (1 – \frac{KM}{R_i(r)}) \tag{6.35}$$

where K is a constant dependent on i. Expression 6.35 becomes singular when R_i (r) = 0 which occurs when f_i (r) = 0. This shows that finding a value for r where expression 6.33 goes to zero will represent a minimum higher dimensional radius for TRM. Solving f_i (r) = 0 gives:

$$r = 3(1 - r^{-2}) \tag{6.36}$$

Solving 6.36 for r gives r = 2.532089 Planck lengths. When this value of r is substituted into expression 5.28 to solve for energy we get:

$$E_T = \pm 1.22x\frac{10^{19}GeV}{(2.532089)^3} \rightarrow E_B = E_T = \pm 7.515x10^{17}\text{GeV} \tag{6.37}$$

This 7.515 x 10^{17} GeV energy represents the energy [8] for a higher dimensional radius R_i (r) equal to zero for an infinite number of spatial dimensions. Whether this energy corresponds to a unification has yet to be determined, but this value is at an energy which makes it look like a reasonable unification candidate.

7 Determining f (r) expressions

Expression 6.35 has only been assumed based on R_i (r) being like a higher dimensional radius. To derive an expression for f(r) from an hdf expression which incorporates the variable M will require very close inspection of the way variables in these expressions change as M is varied. I will show how I am able to incorporate M into an hdf expression.

Before the form of the hdf expression was known all there was to work with was $[f_i(r)]^{-1}$ and the expression (1 – 2M/r). When I made my first attempts to couple these two expressions a SLOPE value was multiplied by $[f_i(r)]^{-1}$ to achieve a fit to the (1 – 2M/r) expression. To incorporate a geometrized M into this hdf expression a parameter M/(nongeometrized M) can be compared to hdf values as M is varied while the radius is fixed at 1000 centimeters.

Using this comparative method, a table of data was generated, making the initial 2.0 solar mass assumption. The values in table 16 below shows how the comparative method is used with radius fixed at 1000 centimeters.

From table 16 the hdf expression can be multiplied by M/296,400 without changing its magnitude where the hdf expression initially was (SLOPE) $[f_i(r)]^{-1}$ before it was known that the value one should be added to it. The third column of this table can be thought of as equal to the first guess at hdf for the range of radius where the hdf couples to (1 – 2M/r) in a way that the two expressions are nearly equal. From the Schwarzschild solution it is known that the first metric component is f(r) = (1 – 2M/r) so we can also say that f(r) = hdf expression where the two expressions are nearly equal near transition radii.

This results in the following expression for the hdf:

$$f(r) = (\frac{M}{296400})(SLOPE)[f_i(r)]^{-1} \tag{7.38}$$

From table 11 it is obvious that there is a difference of one between columns two and three.

Table 11: M versus (1 – 2M/r), — 2M/r, and (M/296,400) where r = 1000 centimeters for 2.0 solar masses.

M	(1 – 2M/1000)	— M/500 (M/296,400)	
10,000	— 19	— 20	0.0337
25,000	— 49	— 50	0.0843
50,000	— 99	— 100	0.169
100,000	— 199	— 399	— 592
200,000	— 200	— 400	— 593
296,400	0.337	0.675	1.00

The effect of this difference on expression 7.38 is that a value of one must be added to the right hand side of expression 7.38 to get a final f (r) which matches (1 – 2M/r) as closely as possible.

After adding one to the right hand side of 7.38 the result after rearranging and changing units is:

$$f(r) = 1 + [\frac{(SLOPE^*)\frac{M}{1.853x10^{38}}}{f_i(r)}] \tag{7.39}$$

SLOPE* has an asterisk because it is corrected for a change of length scales. The value 1.853 x 10^{38} corresponds to an object of two solar masses so its value will increase in proportion to the mass of the object in solar masses divided by two. Expression 7.39 is an intermediate form of f (r) which incorporates M into an hdf expression, but an unseen factor in expression 7.39 multiplies 1.853 x 10^{38} and this unseen factor is the ratio of the mass of the object in solar masses divided by two.

A preferred form for expression 7.39 substitutes R_i (r) into the expression, and this can be done because we know from a previous definition that f_i (r) = R_i (r)/(—3) . When this substitution is made the final form for the expression for f (r) is:

$$f(r) = 1 - [\frac{(SLOPE^*)\frac{M}{6.177x10^{37}}}{R_i(r)}] \tag{7.40}$$

A set of higher dimensional metric components can be generated for a higher dimensional Schwarzschild solution for a 2.20 solar mass object by using the SLOPE values from table 10 from section 5. For these SLOPE values the value 6.177 x 10^{37} in expression 7.40 must be increased by 2.2/2.0 which gives 6.784 x 10^{37} so the expression for f (r) for table 10 data is:

$$f(r) = 1 - [\frac{(SLOPE*)\frac{M}{6.784x10^{37}}}{R_i(r)}] \quad (7.41)$$

Again SLOPE is changed to SLOPE* to remind us of a change of units for M. Using expression 7.41, the SLOPE data from table 10 is used to generate the set of seven higher dimensional Schwarzschild f (r) metric components for all of the spatial dimensions satisfying 4≤ Ns≤10 to show the trend.

The form of the f (r) expressions in table 17 are nearly identical to (1 – 2M/r), except that r is replaced by Ri (r) (see expression 6.34). The constants multiplying M increase in magnitude as Ns increases, and the constants multiplying M for 8 - Ds and 10 - Ds are both negative. I should also point out that for 2.20 solar masses the constants multiplying M for 4 - Ds and 6 - Ds are equal to two so the f (r) expressions for 4 - Ds and for 6 - Ds match the form of (1 – 2M/r).

Such a matching of the form of (1 – 2M/r) suggests that the extra dimensions associated with 4 - Ds and 6 - Ds are both of infinite extent and are hidden from observation in such a way that Newton's law of gravitation is not changed by having more than three spatial dimensions. There are several ways to hide extra dimensions of infinite extent. The way to hide these dimensions is to imagine that extra dimensions of infinite extent can align with our (3 + 1) dimensions at every point in spacetime in such a way that the overlapping extra dimensions merge into a set of dimensions that look like three spatial dimensions at every point in spacetime. Such a dimensional alignment would require extremely high energies to separate since they have remained undetected. Also, the reason that these aligned dimensions do not change Newton's law of gravitation may be because what we observe every day is a set of three spatial dimensions which is a merged overlay at every point in spacetime.

This explanation for how extra dimensions of infinite extent might go undetected would extend to the other extra dimenions in TRM such as 5 - Ds and spatial dimensions 7 through 25.

Another possibility is that all of these extra dimensions are virtual dimensions which have no influence on Newton's law in our (3 + 1) dimensions.

As was pointed out previously, whenever a negative constant multiplies M it suggests that the associated spatial dimensions prefer negative energy or a negative energy density. Therefore, both 8 - Ds and 10 - Ds should prefer negative energy while spatial dimensions 4 - Ds, 5 - Ds, 6 - Ds, 7 - Ds, and 9 - Ds prefer positive energy. How these negative versus positive energy preferences manifest themselves within the bubble is beyond the scope of this paper.

Each f (r) expression listed in table 17 can be substituted into the Schwarzschild solution to create a new higher dimensional version of the Schwarzschild solution for each value of Ns (where h (r) = 1/f (r)), and the result of making a set of substitutions of this type is to create a set of higher dimensional Schwarzschild solutions.

Ns	**f (r) = 1 − [(SLOPE*) M/6.784 x 10^{37}]/Ri (r)**
4	f (r) = 1 − 2.00 M/R_4(r)
5	f (r) = 1 − 4.39 M/R_5(r)
6	f (r) = 1 − 2.00 M/R_6(r)
7	f (r) = 1 − 8.65 M/R_7 (r)
8	f (r) = 1 + 12.51 M/R_8 (r)
9	f (r) = 1 − 28.64 M/R_9 (r)
10	f (r) = 1 + 54.14 M/R_{10} (r)

Table 12: Ns versus f (r) using expression 8.48 for table 11 data (SLOPE* is adjusted for 2M = 4.076×10^{38} at 2.2 solar masses)

8 Why does an exterior metric work?

It seems quite strange that basing TRM on an R_{22} equation which appears when deriving an exterior solution like the Schwarzschild should result in a model which fits so many boson energies so well, especially since we are located in a universe which should be described by an interior solution. I will examine the equations used to derive an interior solution, which can be found in *General Relativity* by Robert Wald, to see if there is a way to explain why an R_{22} expression yields a TRM model which fits boson energies so closely.

For an interior solution the expressions 6.2.3 and 6.2.4 from *General Relativity* apply [11]. If we add these two expressions the result is:

$$8\pi(\rho + P) = (rh^2)^{-1}h' + (rfh)^{-1}f' \qquad (8.42)$$

where h and f are the metric components of a static spherically symmetric spacetime, r is the interior radius of a spacetime bubble, ρ is density, and P is pressure.

This equation can be simplified and rearranged as:

$$8\pi(rh)(\rho + P) = (\frac{h'}{h}) + (\frac{f'}{f}) \tag{8.43}$$

Let us see what happens to expression 8.43 as the radius r becomes very large. It can be argued that for regions of space exterior to stars and planets when r is sufficiently large the term (ρ + P) in this expression 8.43 will approach a value of zero.

From the right hand side of expression 8.43 the term f '/f is approximately equal to $2M/r^2$ since f is approximately equal to one and f ' is approximately equal to $2M/r^2$. The left hand side of 8.43 must be smaller than $2M/r^2$ for expression 8.43to be valid so let's make the left hand side smaller than $M/(25\ r^2)$ which gives:

$$8\pi(rh)(\rho + P) < M/(25r^2) \tag{8.44}$$

Since the left side of expression 8.43, which is 8π (r h) (ρ + P), goes to zero as r gets large, where expression 8.44 is satisfied, it follows that expression 8.43 becomes:

$$(h'/h) + (f'/f) = 0 \tag{8.45}$$

Expression 8.45 is identical to the intermediate expression 6.1.38 (see page 123 of Wald's General Relativity [11]) that gives the Schwarzschild exterior solution. From this result it follows that h = 1/f and h' = $-\ f^{-2}$ as shown in the same reference.

If we now subtract expression 6.2.4 from expression 6.2.3 (as shown on page 125 in [11]) this gives:

$$8\pi(\rho - P) = (rh^2)^{-1}h' + 2r^{-2}(1–h^{-1}) - (rfh)^{-1}f' \tag{8.46}$$

Rearranging 8.45 gives:

$$8\pi(\rho - P) = -(rfh) - 1f' + (rh2) - 1h' + +2r - 2(1 - h - 1) \tag{8.47}$$

When 8π (r h)(ρ + P) < M/(25 r2) we showed above that h = 1/f and h' = − f^{-2} . Substituting h and h' into expression 8.46 gives:

$$8\pi(\rho - P) = \frac{-f'}{r} + (\frac{f^2}{r})(-f^{-2}) + 2r^{-2}(1–f) \tag{8.48}$$

$$8\pi r(\rho - P) = (-f' - 1) + \frac{2(1-f)}{r} \tag{8.49}$$

For a universe as large as ours P = 0 is very likely, but ρ = 0 needs more inspection. I will briefly assume that both P and ρ go to zero for a universe with a radius as large as the radius of our universe. When ρ and P are both zero the left hand side of 8.49 goes to zero giving:

$$(-f' - 1) + \frac{2(1-f)}{r} = 0 \tag{8.50}$$

Expression 8.50 is identical to expression 2.4 of section 2.1, and this is the expression on which TRM is based.

However, there is a problem with the assumptions made above. If we rearrange expression 8.44 it becomes:

$$r^3 < \frac{M}{(200\pi h)}(\rho + P)^{-1} \tag{8.51}$$

We can substitute the values h = 1, M = 326,000 and P = 0 in the above equation to give:

$$r^3 < \frac{518.8}{\rho} \tag{8.52}$$

To see why there is a problem with expression 8.52 we have to substitute an appropriate value for the density in the denominator of the right hand side.

The average density of our universe if Ω = 1 is five atoms per cubic meter, and this density may be much lower than a five atoms per cubic meter value as Martin Rees pointed out in Just Six Numbers [8, p.73]. If the value of ρ is assumed to correspond to five atoms per cubic meter, the resulting density is about 8.35 x 10^{-30} g/cm3 in units called nongeometrized by Robert Wald. To geometrize this density of 8.35 x 10^{-30} g/cm3 the value must be multiplied by G/c^2 which is 7.41 x 10^{-29} cm/g per table F.1 page 471 of Robert Wald's General Relativity. [11] This gives a geometrized density of 6.19 x 10^{-58} cm^{-2} for our universe if Ω = 1. Substituting this geometrized density value into expression 8.52 for ρ gives:

$$r^3 < 8.38x10^{59}cm^3 \tag{8.53}$$

This is the same as requiring the radius to be less than 9.43 x 10^{19} cm or less than about 100 light-years.

This requirement that r is less than 100 light-years means that the density must be much greater than was assumed. Radiation density can not be ignored for universe radius values less than 100,000 light-years, but even when our universe had a radius of 100,000 light-years its density would be too high for the low density assumption made to reduce expression 8.49 to expression 8.50. This 100 light-year radius requirement of expression 8.53 completely contradicts the low density assumption made for expression 8.50, and we must conclude that the exterior form of TRM does not apply in the most simple way to the interior of the bubble for an early universe. Only when a universe is many billions of years old, where M is extremely large and the density is very low, may the interior form of TRM per expression 8.49 reduce to 8.50. This leaves two possible ways for TRM to influence the energies of annihilation bosons for our early universe:

1. Expressions 8.53 and 8.59 must be solved directly, making the expressions for higher dimensional radius more complicated.

2. Before inflation there was one, a few, or many black holes in our universe. Later, at the time of quark formation, there were many black holes interior to the bubble, where these black holes have nearly identical masses, so quark formation was influenced by being exterior to these black holes where the exterior form of TRM applies.

If the second possibility is correct this means that TRM based on the exterior Schwarzschild solution can be applied directly for our early universe without having to go back to expressions 8.53 and 8.59 .

Therefore, the exterior form of TRM only applies for the interior of a bubble if a pre-inflation bubble was filled with one, a few, or many black holes, and the post-inflation universe was filled with one, a few, or many primordial black holes. For our universe to be as homogeneous as it is one would expect that many black holes existed before and after inflation.

9 Where is the graviton?

The short answer is that TRM does not directly predict a boson energy which is an obvious candidate for the graviton. A boson energy associated with the 25 -Ds is about 9.8×10^{-11} GeV which is far too high for the graviton.

To directly predict a boson energy which is orders of magnitude lower than a photon's energy requires a transition radius which is greater than 10^{15} Planck lengths. However, I have limited TRM to a maximum of 25 spatial dimensions so it will agree with string theory, but a transition radius value above 10^{15} Planck lengths would require more than 25 - Ds. Predicting a specific graviton energy requires specifying a number of spatial dimensions far greater than 25, but there are so many values of Ns > 25 that it is impossible to guess which Ns is the correct one to pick for a graviton. The two-step cancellation mechanism described on page 20 is a way around this problem of fitting the graviton within 25 spatial dimensions.

However, there is another approach to this problem which may make it possible to find the effect of a graviton within the limits of a 25 spatial dimension spacetime. Notice that the larger transition radii for 24 - Ds is 7.00x10^9 Planck lengths and the single transition radius for 25 - Ds is 5.00 x 10^9 Planck lengths. The energy of a boson corresponding to the larger transition radius for 24 - Ds is -3.6x10^{-11} GeV versus 9.8x10^{-11} GeV for the 25 - Ds boson. If the value of Nsx for 24 - Ds were adjusted slightly the absolute values of these two energy values could be much closer together. In that case, the graviton may be the effect of a sum of 24 - Ds and 25 - Ds boson energies where the former has a negative energy boson and the latter has a positive energy boson. If the absolute value of the energies for these two bosons were very close their sum would almost cancel, leaving a very small net energy. These boson energies are extremely sensitive to the value of Nsx used for TRM so a second model would be needed to determine the best Nsx values for 24 - Ds to find a graviton effect. This shows that there is a potential explanation for the graviton effect within a 25 spatial dimension TRM, but it requires additional theory to determine the correct net energy value.

10 Wavelength versus Transition Radius

A relationship between quantum theory wavelength and TRM transition radius which applies at annihilation energy can be derived by equating the expression E = h c/λ with a variation of expression 5.28ff.

In general, expression 5.28ff can be modified to account for the fact that boson energy is usually carried by two bosons of equal energy when annihilation occurs. This modification gives the following expression for boson energy:

$$E_{BK} = \frac{E_P}{(Kr_T^3} \tag{10.54}$$

where K = 1 for an unsplit boson or K = 2 for two bosons of equal energy like electron/positron annihilation photons.

Equating the energy term in E = h c/λ with EBK in expression 10.54 gives an expression which can be rearranged to give an expression for the wavelength λ: E_P $/(Kr_T^3)$ = hc/λ which rearranges to give

$$\lambda = (\frac{Khc}{E_P})(r_T)^3 \tag{10.55}$$

where h is Planck's constant, c is the speed of light, and EP is Planck energy. To get the wavelength λ in expression 10.55 in units of meters the following values are substituted:h = 6.625 x 10–34 J-sec, c = 2.998 x 108 m/sec, and E_P = 1.954 x 10^9 Joules. After making these substitutions, expression 10.55 simplifies to:

$$\lambda = (1.0165x10^{-34})K(r_T)^3 \tag{10.56}$$

where λ is in meters and r_T is in Planck lengths. This expression was checked for the electron which has an energy of 510,000 eV, which is equivalent to 8.17 x 10^{-34} Joules, by using the expression λ = h c /(8.17 x 10^{-34} J) which gives a wavelength of 2.431 x 10^{-12} meters. The same value of λ resulted for expression 10.56 when K = 2 and when r_T = 2.287 x 10^7 Planck lengths. Thus, expression 10.56 gives the connection between wavelength and transition radius provided the correct integer value of K is used.

11 Discussion and conclusions

This transition radius method looks at the definition of higher dimensions in a new way, and this leads to several new findings. First, integrating the Ricci expression R_{22} generates expression 2.21 which defines $f_i(r)$, and the reciprocal of this $f_i(r)$ function can be coupled to (1 – 2M/r) in a way which yields transition radii that can be used to predict many boson energies. When boson energies are predicted from transition radius values, TRM uses the expression $E_B = E_T = \pm$ 1.22 x 10 GeV/(r_T) where E_B is boson energy, E_T is transition energy, and r_T is the transition radius. TRM fit boson energies match the energies of at least fourteen known boson types plus the X bosons. A second finding which results from TRM is a set of inflation trigger radius ranges. These inflation triggers are clustered around three distinctly different radius ranges, but additional work is needed to determine how many of these inflation triggers initiate different stages of inflation. A third finding is that the larger transition radius (r_{LT}) for 7 - Ds predicts a bubble radius of 1130 Planck lengths which is compared with a previous bubble radius estimate giving a most likely bubble radius from 1065 to 1130 Planck lengths which may be the radius when large-scale structure formation was initially influenced. Finally, TRM fits the energies and masses of known fundamental particles, and TRM predicts the existence of particles at energies above 300 GeV. These are the main findings made possible by the development of TRM.

In addition to the above findings, the energy of a boson with an infinite number of dimensions is found in section 6 by deriving expressions 6.29 and 6.33, then solving expression 6.36 to find its transition radius. From this transition radius the boson energy for a boson with an infinite number of dimensions is $E_B = \pm 7.515 \times 10^{17}$ GeV. This boson energy may represent one of the unification energies for bosons. As I suggested previously, an appropriate name for such a boson might be an 'infinitron'.

The next highest boson energy fit by TRM is that for the X-like boson associated with 5 spatial dimensions which has a predicted E_B for its smaller transition radius of 9.47×10^{-16} GeV.

To appreciate the wide range of boson energies that TRM fits one should note that TRM gives a reasonable estimate for the energy of the e-neutrino/anti-e-neutrino annihilation boson. From table 9 the TRM predicted energy for this boson is 4.26×10^{-9} GeV so the energy of a single e-neutrino is predicted by TRM to be 2.13×10^{-9} GeV. Thus, TRM predicts boson energies spanning 26 orders of magnitude when the 'infinitron' is included or it predicts boson energies spanning 25 orders of magnitude when the X-like boson for 5 - Ds is considered the highest boson energy.

In addition to boson energies, inflation trigger radius ranges are predicted by TRM. These events occur in radius ranges shown in tables 3 and 4. This model suggests that a 'presingularity' may never be allowed to collapse to a radius smaller than 1.0 Planck length because these inflation triggers all occur at radius values above a 1.0 Planck length radius. Additional work is needed to determine how these inflation triggers affect inflation, especially any influence these triggers may have on the number of stages of inflation. TRM provides evidence which supports the idea that inflation may have occurred in one, two, or three stages.

Key concepts that I developed which made TRM possible are:

1. Deriving higher dimensional $f_i(r)$ expressions from two of the four Ricci ex-

pressions which had previously been set aside when the Schwarzschild solution was derived.

2. Defining transition radii as the radii where higher dimensional $f(r)$ expressions are equal to the (3 + 1) dimensional $f(r)$ expression (1 – 2M/r).

3. Defining an averaging parameter SX and a distribution for the number of largest SX values (Nsx) to be averaged to determine the first approximations for SLOPE values.

4. Finding an equation which predicts boson energy from a transition radius. This equation is $E_B = \pm$ 1.22 x 10^{19} GeV/$(r_T)^3$ where r_T is the transition radius. This also means that (4 π/3 $(r_T)^3$) $E_B = \pm$ 5.11 x $10^1$9 GeV. Both equations apply for (rest mass) particle/anti- particle annihilation bosons.

These key concepts make it possible for TRM to fit a large number of annihilation boson energies and fundamental particle masses.

A higher dimensional radius R_i (r) defined by expression 6.34 is used in the higher dimensional f (r) expressions. The f (r) expressions for 4 – Ds through 10 - Ds are listed in table 14, and they show that even numbered spatial dimensions starting with 8 - Ds have negative SLOPE values which imply that negative energies occur for these dimensions.

A complete list of these SLOPE values for 4 ≤ Ns ≤ 25 is found in table 10. These SLOPE values affect the degree to which the (1 – 2M/r) metric component for 3 - Ds is coupled to a specific higher dimensional f (r) expression. If this SLOPE parameter were renamed in a way which reflects its function, it might be called an 'interdimensional' intersection factor or IIF. This naming emphasizes the intersection of the higher dimensional metric components to the (3 + 1) dimensional metric component (1 – 2M/r) of the Schwarzschild solution at transition radii.

From the higher dimensional f (r) expressions listed in table 14 a set of higher dimensional Schwarzschild solutions can be derived by substitution.

For 7 - Ds a unique transition radius is obtained which nearly matches a prediction of the size of a bubble at a bounce made by Carmen Molina – París and Matt Visser.[5] I have estimated a bubble radius range of 1065 to 1130 Planck length radius from TRM. At this bubble radius the large-scale structure of our universe may have been first influenced.

Assuming that TRM must apply indirectly through black holes for the interior of the bubble, as pointed out in section 9, also changes the way we have to think about the 1065 to 1130 Planck length bubble radius. TRM requires that this bubble radius be influenced by being external to at least one black hole, but the only place that could happen is when the bubble is in the parent universe. It follows that the parent universe which generated our universe should have had a set of black holes. Therefore, for a 1065 to 1130 Planck length bubble radius to be controlled by TRM the parent universe must interact with the child universe and the parent universe must contain many black holes in its interior.

For the previous TRM version, a possible resonance is seen for the top/anti-top annihilation boson. This annihilation energy can be fit as sum of two bosons of the same energy for the smaller transition radius at 18 – Ds when Nsx = 4.0. If these three boson types having Nsx resonances are excluded, the Nsx values for the remaining boson types have a range which tightens up to $1.78 \leq \text{Nsx} \leq 4$. It seems likely that most boson types will be described by a resonance of two different Nsx values where one value is for a single boson type and the other is for the sum of two boson type energies. This means that many boson types will be fit by a resonance between two different Nsx distributions. Based on the findings for the three boson types above, the Nsx values will be very close to a 4.0 value for the Nsx distribution where two boson type energies are summed. This trend suggests that there are two intersecting Nsx distributions (see Appendix A).

For a given boson type which resonants between two Nsx values it seems that the difference between its Nsx values is related to the massiveness of that boson type. For example, for the previous version of TRM the Nsx differences are 0.023, 1.73, and 2.90 for the e^+e^- boson, Z boson, and top quark/anti-top quark annihilation boson, respectively.

If the total number of positive energy boson types for TRM is the sum of the number of single transition radius positive energy boson types plus the number of possible paired positive energy boson types, then the maximum total number of positive energy gauge boson types (Nb) should be given by Nb = 39 + [(39)(38)/2] + 1 = 781 when negative energy bosons do not interact with positive energy bosons. If negative energy boson types fully combine with positive energy boson types, this Nb value could be as large as Nb = [(780)(780) – 780] + 780 + 1; however, this Nb value seems way too large. In either case, the number of observable gauge boson types (Nobs) should be much less than Nb because one boson type of each pairing may have an energy which is orders of magnitude less than the energy of the other boson type so the paired energy would be indistinguishable from the energy of one of the unpaired boson types. The first case above gives Nobs < 781, assuming that a positive energy boson only combines with a negative energy boson having the same absolute value of energy (see Appendix C for more details).

The transition radius method shows considerable promise based on its ability to fit as many as fifteen particle types over a wide range of energies. Most transition radii occur in pairs for each Ns, and fitting one of these transition radii for a given pair to a known particle energy necessarily anchors a second predicted particle energy.

It is possible that many of the unmatched negative energy bosons could be part of the dark energy which is being investigated today; although, it is likely that these negative energy bosons have much lower energies today.

For TRM the extra dimenions may hide in our three spatial dimensions as a perfectly aligned overlay at every point in spacetime, and these extra dimensions may hide by limiting access to quantum entities provided an equilibrium has been established (to satisfy conservation of energy). For TRM the extra dimensions do not have to curl up at small dimensions to be undetectable. Thus, TRM suggests that our three spatial dimensions may be each a composite of eight infinite dimensions, and one or more infinite dimensions may be free to move between them at a rate comparable to or equal to light speed. A better interpretation of TRM may be to assume that each of our three spatial dimensions is a composite of an infinite number of extra dimensions of infinite extent. Another way that extra dimensions of infinite extent might avoid influencing Newton's Law is if extra dimensions of infinite extent are all virtual.

Either expressions 8.43 and 8.49 must be solved to determine what happens interior to the bubble or we can use the exterior form of TRM (expressions 2.21 through 5.28) if we assume that many black holes of nearly identical masses were present inside a bubble during quark formation.

For the first case, the effect of equation 8.49 may cause boson type energies to be different than they are today, and this implies a contradiction of the uniformity of the atomic mass of hydrogen throughout the observable universe. This apparent contradiction can be resolved if annihilation boson energies matched the energies determined by TRM using expressions 8.43 and 8.49. TRM suggests that particle masses or energies all shifted from early universe values to current values once our universe exceeded a radius of 100,000 light-years.

In the second case, where many black holes are present inside the bubble at the time of quark formation, expressions 2.21 through 5.28 can be used directly to determine annihilation boson energies and inflation triggers. Throughout the period of quark formation these early universe black holes do not have to maintain constant mass for quark masses to remain constant throughout this period due to findings shown below.

The structure of a quark may also restrict a quark to a more probable energy so that the boson type energy restrictions of TRM provide boson type energies close to what is needed for formation of each quark flavor, then quark structure dictates the precise average energy of quark formation from these bosons. Determining how much control TRM has over the precise average energy of a quark will likely require other theories, but it can be said that TRM restricts the allowed energy of each quark flavor (and other particle types) to a narrow range of energy by controlling the energy of the annihilation boson types from which they form.

Finding that TRM allows a wide range of black hole masses to give nearly identical sets of observable boson energies also means that early universe black holes do not need to have nearly identical masses to produce identical patterns of particle masses. Nearly identical black hole masses are not necessary, but they could be nearly identical due to other processes.

The severe spacetime distortions produced by pre-inflation black holes when the radius of their collapsing bodies is equal to an inflation trigger radius are likely intense enough to produce an inflation triggering effect interior to their event horizons. The 'presingularities' within these pre-inflation black holes are likely so tightly packed together before inflation that they are within each other's event horizon when inflation initiates.

TRM suggests that the black hole metric influences rest masses and energies of fundamental particles might be located inside the event horizon of a black hole because all transition radii are much smaller than the radius of the event horizon. There are three ways that TRM may influence particle masses (energies) exterior to the event horizon. If virtual particles inside the event horizon can tunnel out to influence the particle energies of Hawking radiation, then TRM shows how the pattern of rest masses and energies of fundamental particles can be determined by Schwarzschild black holes. A second way for TRM to influence the pattern of particle masses in our universe is for the black holes influencing this pattern to have event horizons which are smaller than the transition radius of most particle pairs;

an event horizon radius less than about 3 x 10^5 Planck lengths would be smaller than the transition radius of the boson associated with a top quark/anti top quark pair. The third way that the pattern of rest masses and energies of fundamental particles throughout our universe can be determined by TRM is if our universe influences the fundamental particles the same way that Schwarzschild black holes influence fundamental particles in their interiors, see section 9.

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As part of the posthumous editing process we did a simplified literature search and found eight other works (references therein and later citing works) relating to predicting the masses of fundamental particles and may be of interest to readers of this manuscript.

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Appendix A

Even Ns	Nsx for Even Ns	Odd Ns	Nsx for Odd Ns
4	4	5	4
6	4	7	4
8	3.5 , 4 **	9	3.875 , 4 **
10	3.25 , 4	11	3.75 , 4
12	3 , 4 ‡	13	3.668 , 4 ‡
14	1.969, 4	15	2.758 , 4
16	1. 13 , 4	17	2.27 , 4
18	1.781 , 4	19	4.024 , 4
20	3.49 , 4	21	4.032 , 4
22	4	23	4
24	4	25	3.999 † , 4
26	4	27	4

Table 5". Even Ns and Odd Ns versus Nsx for TRM ($2M = 4.076 \times 10^{38}$)

$**$ The Nsx distributions where Nsx = 4 for all values of Ns satisfying 8 ≤ Ns ≤ 16 are very speculative

(see tables 14 and 15) while the second Nsx distributions, over this same range of Ns , which are shown in tables 12 and 13 are far less speculative.

‡ Alternative values for Nsx at Ns = 12 and at Ns = 13 can give smoother plots of Nsx versus Even Ns and of Nsx versus Odd Ns. When Nsx = 2.6 for Ns = 12 a smoother plot results, giving E_B energies of 3.56×10^4 GeV and 9.9×10^8 GeV. When Nsx = 3.4 for Ns = 13 a smoother plot results, giving E_B energies of 430 GeV and 1.6×10^8 GeV. Only the 430 GeV energy is likely to be detected at the LHC, and its detection would support the smoothest plots of Nsx versus Even Ns and Nsx versus

Odd Ns. An E_B energy of 430 GeV may be 3 (80.3) + 2 (91.2) = 423 GeV or 2 (80.3) + 3 (91.2) = 434 GeV (Is this a Higgs boson?). Symmetric decay of E_B = 434 GeV gives particle energies at 217 GeV and/or 145 GeV. This note was added on September 10, 2006 and refined on September 21, 2006.

† Positive energy and negative energy particles associated with Ns = 25 may yield net cancellation energy equal to the energy of a graviton when particle energies for Nsx near 3.999 and for Nsx = 4.0 participate in a two-step cancellation as described in section 7 on page 20.

Appendix B

The BASIC language program used to calculate the f_i (r) values follows:

10 REM A BASIC program to find f i (r) for f4 (r) through f25 (r) for single radius inputs in units of Planck 20 REM lengths. The function ln (r) which is a logarithm in base e is written as log (r) in BASIC.

30 REM The radius value 2 x 10^9 must be input in the form 2e9. This was written 11/26/01 by M. D. Holte

40 REM Note that the asterisk ∗ is a multiplication symbol in BASIC and ^ means raised to the power of

50 PRINT "WHAT IS THE BUBBLE RADIUS (PLANCK LENGTHS) WITHOUT COMMAS "; : INPUT R

60 TERM1 = 2 ∗ (LOG(R))

70 TERM2 = −2 ∗ (LOG(R))^2

80 TERM3 = (4/3) ∗ (LOG(R))^3

90 TERM4 = − (2/3) ∗ (LOG(R))^4

100 TERM5 = (4/15) ∗ (LOG(R))^5

110 TERM6 = − (4/45) ∗ (LOG(R))^6

120 TERM7 = (8/315) ∗ (LOG(R))^7

130 TERM8 = − (2/315) ∗ (LOG(R))^8

```
140 TERM9 = (4/2835) * (LOG(R) )^9

150 TERM10 = − (4/14175) * (LOG(R) )^10

160 TERM11 = (8/155925) * (LOG(R) )^11

170 TERM12 = − (4/467775) * (LOG(R) )^12

180 TERM13 = (8/6081075) * (LOG(R) )^13

190 TERM14 = − (16384/8.718E+10) * (LOG(R) )^14

200 TERM15 = (32768/1.3077E+12) * (LOG(R) )^15

210 TERM16 = − (65536/2.0923E+13) * (LOG(R) )^16

220 TERM17 = (131072/3.557E+14) * (LOG(R) )^17

230 TERM18 = − (262144/6.402E+15) * (LOG(R) )^18

240 TERM19 = (524288/1.216E+17) * (LOG(R) )^19

250 TERM20 = − (1048576/2.433E+18) * (LOG(R) )^20

260 TERM21 = (2097152/5.109E+19) * (LOG(R) )^21

270 TERM22 = − (4194304/1.124E+21) * (LOG(R) )^22

280 REM This concludes the term generation.

290 REM Exploring more than 25 spatial dimensions requires additional terms.

300 F4 = (− R/3 ) + TERM1

310 F5 = F4 + TERM2

320 F6 = F5 + TERM3

330 F7 = F6 + TERM4

340 F8 = F7 + TERM5

350 F9 = F8 + TERM6

360 F10 = F9 + TERM7
```

370 F11 = F10 + TERM8

380 F12 = F11 + TERM9

390 F13 = F12 + TERM10

400 F14 = F13 + TERM11

410 F15 = F14 + TERM 12

420 F16 = F15 + TERM 13

430 F17 = F16 + TERM14

440 F18 = F17 + TERM15

450 F19 = F18 + TERM16

460 F20 = F19 + TERM17

470 F21 = F20 + TERM18

480 F22 = F21 + TERM19

490 F23 = F22 + TERM20

500 F24 = F23 + TERM21

510 F25 = F24 + TERM22

520 REM Output to the computer screen follows. For printout rewrite 530 – 650 with PRINT → LPRINT.

530 PRINT “THE SUMMED f i (r) TERMS GIVE THE f i (r) VALUES BELOW”

540 PRINT “RADIUS = “; R

550 PRINT “f4 (r) = “; F4; : PRINT “ f5 (r) = “; F5 : PRINT “ “

560 PRINT “f6 (r) = “; F6; : PRINT “ f7 (r) = “; F7 : PRINT “ “

570 PRINT “f8 (r) = “; F8; : PRINT “ f9 (r) = “; F9 : PRINT “ “

580 PRINT “f10 (r) = “; F10; : PRINT “ f11 (r) = “; F11 : PRINT “ “

```
590 PRINT “f12 (r) = “; F12; : PRINT “ f13 (r) = “; F13 : PRINT “ “

600 PRINT “f14 (r) = “; F14; : PRINT “ f15 (r) = “; F15 : PRINT “ “

610 PRINT “f16 (r) = “; F16; : PRINT “ f17 (r) = “; F17 : PRINT “ “

620 PRINT “f18 (r) = “; F18; : PRINT “ f19 (r) = “; F19 : PRINT “ “

630 PRINT “f20 (r) = “; F20; : PRINT “ f21 (r) = “; F21 : PRINT “ “

640 PRINT “f22 (r) = “; F22; : PRINT “ f23 (r) = “; F23 : PRINT “ “

650 PRINT “f24 (r) = “; F24; : PRINT “ f25 (r) = “; F25

660 END
```