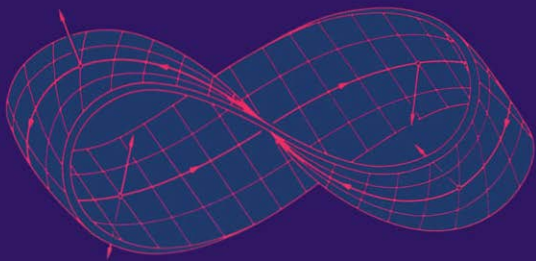


History and Philosophy of Modern Mathematics

William Aspray and Philip Kitcher, editors



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Volume XI

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HERBERT FEIGL, FOUNDING EDITOR

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*History and Philosophy of
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EDITED BY

WILLIAM ASPRAY AND PHILIP KITCHER

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Preface

The purpose of this volume is to bring together a number of different perspectives on modern mathematics, with the aim of understanding how the work of historians, philosophers, and mathematicians can be integrated in an interdisciplinary study of mathematics. Several of the essays adopt such an interdisciplinary approach, whereas others favor an autonomous pursuit of the history of mathematics or of the philosophy of mathematics. In our judgment, this juxtaposition of old and new styles accurately represents the state of the field at this time.

The volume is the outgrowth of a conference held at the University of Minnesota, Minneapolis, 17-19 May, 1985. The conference and the publication of proceedings were supported by generous grants from the Alfred P. Sloan Foundation and the College of Liberal Arts and Institute of Technology of the University of Minnesota. We also wish to thank a number of the scholars who served as commentators at the conference: Thomas Drucker of the University of Wisconsin-Madison; Philip Ehrlich of Brown University; Emily Grosholz of Pennsylvania State University; Joelle Proust of CNRS/Nice; and Helena Pycior of the University of Wisconsin-Milwaukee.

In 1974, Garrett Birkhoff of Harvard University organized and the American Academy of Arts and Sciences sponsored a Workshop on the Evolution of Modern Mathematics. (The proceedings are published in the November 1975 issue of *Historia Mathematica*.) The purpose was to bring together a distinguished group of historians and mathematicians to review and discuss the history of mathematics since 1800, a subject that had received relatively little attention in comparison with earlier periods of mathematics history. The meeting was an unmitigated success, both in providing a forum for discussion between these two bodies of professionals, and in producing a valuable set of proceedings.

In our opinion, in the decade since the 1974 workshop there has been an upsurge of interest in an interdisciplinary approach to historical and philosophical problems connected with mathematics. There have also been recent developments internal to each discipline: historians of mathematics have recognized the need for a more sophisticated historiography and for greater attention to the social context in which the work of mathematicians is set; philosophers of mathematics have moved away from the grand foundational programs of the early twentieth century and the preoccupation with the demise of those programs.

For these reasons, we believed it was important once again to take appraisal of our fields. Looking to the 1974 workshop as our model, we brought together a distinguished group of historians, mathematicians, and for the first time philosophers actively working in this area. Indeed, as a measure of the continuity of this area, seven of the authors represented in this volume were participants in the 1974 workshop. Our authors were requested to keep in mind not only the historical, philosophical, and mathematical significance of their topics, but also to devote special attention to their methodological approach. These approaches came under considerable discussion during the conference. We can only hope that our proceedings will be as influential as those of the 1974 workshop.

W. A. and P. K.

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