Foundations of Mathematics Achievement: Associations Between Executive Function and Specific Numerical Skills in Early Childhood

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Abstract

The importance of early numerical and executive function (EF) skills is well established, with each skill set positively and specifically predicting later mathematics achievement, income, postsecondary education, and more. Less is known about the relations *between* EF and numerical skills, however. Thus, Research Question 1 examined the concurrent and predictive relations between EF and numerical skills in 4.5- to 9-year-olds (*N*=205). I found positive concurrent relations between EF and all six numerical skills, including nonsymbolic magnitude comparison, verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving. There were unidirectional longitudinal relations between EF and four of the six numerical skills, after controlling for covariates and prior performance on the skill of interest. Bidirectional relations were found for EF and nonsymbolic magnitude comparison. I also found the concurrent relation between EF and count on was higher for children with typical, compared to persistently low mathematics achievement. All other concurrent and predictive EF and numerical skill relations were similar for children with typical and persistently low mathematics achievement. Research Question 2 took a different approach by examining concurrent associations between EF, vocabulary, and numerical skills and mathematics achievement in children with Turner syndrome (*N*=44), from Low SES households (*N*=130), or with No Known Risk (*N*=204) for mathematics learning difficulties. The Turner syndrome and Low SES groups represented examples of biological and socioenvironmental mathematics learning difficulty risks, respectively. I found a distinct cognitive profile for children with Turner syndrome, indicating the numerical and non-numerical skills associated with mathematics achievement vary by some mathematics learning difficulties risk factors. Overall, this work can inform mathematics interventions and instruction for children who are typically achieving and at risk for mathematics learning difficulties.

Keywords: executive function, numerical skills, mathematics, early childhood

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Chapter 1: Introduction

A report from 2020 estimated nearly 30% of U.S. adults have difficulty calculating with whole numbers and percentages, estimating quantities, and interpreting simple statistics (National Center for Education Statistics at IES, 2020). The numerical and non-numerical skills underlying the development of these proficiencies emerge early, persist throughout the lifespan, and predict life course and well-being outcomes (e.g., Blair & Raver, 2015; Watts et al., 2014). One set of non-numerical skills implicated in mathematics is executive function (EF), which are neurocognitive skills involved in goal-directed problem solving and behaviors (reviewed by: Carlson et al., 2013; Diamond, 2012). Despite evidence linking both EF and numerical skills to mathematics achievement independently, there is limited work examining the relations between EF and numerical skills specifically. Understanding the early relations between these sets of foundational skills can inform educational instruction and supports available for children with varying levels of mathematical proficiency.

Here I examined two primary research questions. In **Research Question 1**, I examined the concurrent and predictive relations between EF and specific numerical skills in a racially and socioeconomically diverse sample of preschool to third graders. I also examined whether relations between EF and numerical skills vary for children with typical and persistently low mathematics achievement. In **Research Question 2,** I examined the contribution of EF, vocabulary, and numerical skills to mathematics achievement in groups of children experiencing a biological, socioenvironmental, or no known risk factor for mathematics learning difficulties.

Importance of Early Mathematics

Mathematics development begins before school entry and early mathematics is a predictor of later mathematics achievement (Claessens & Engel, 2013; Clements & Sarama, 2004; Watts et al., 2014). Several studies report that mathematical skills prior to and at school entry are moderate to strong predictors of subsequent mathematics achievement (Duncan et al., 2007; Nguyen et al., 2016; Watts et al., 2014), mathematics skill growth (Jordan et al., 2009; Ribner et al., 2023), and enrollment in higher level mathematics classes later in life (Davis-Kean et al., 2022). From a skill-building theoretical framework, the link between early and later mathematics is attributable to early skills facilitating learning of later skills (Aunola et al., 2004; Clements & Sarama, 2004; Jordan et al., 2009; Watts et al., 2018). Mathematical content is hierarchical in structure; therefore, mastering basic concepts is necessary before one can move on to higher-level concepts.

Beyond early childhood, mathematics skills continue to be important predictors of multiple academic and life course outcomes, including enrollment and completion of postsecondary education (Lee, 2012), income (Ritchie & Bates, 2013; Rose & Betts, 2004), and perceptions of health-based risks and benefits (reviewed by: Peters et al., 2007; Reyna et al., 2009). Thus, individuals who struggle with mathematics early in life might face challenges both inside and outside the classroom and these challenges are likely to persist beyond the elementary school years (Geary et al., 2013; National Center for Education Statistics at IES, 2020).

Within the domain of mathematics, early numerical skills are thought to play a particularly important role in later mathematics achievement (Chu et al., 2016; Mazzocco & Thompson, 2005; Nguyen et al., 2016). The focus on early numerical skills comes from prior work showing preschool, kindergarten, and first grade numerical skills are associated with mathematics achievement in in the primary (Jordan et al., 2009; Watts et al., 2018), middle (Claessens & Engel, 2013), and high school years (Watts et al., 2014). The significance of numbers and numerical concepts for later mathematics is unsurprising given that children and adults rely on these foundational skills when doing various tasks, such as counting the sides to identify a shape or measuring a piece of furniture to see if it will fit in a particular location. Importantly, even though numerical skills might play a role doing many mathematical and everyday tasks, mathematics is still thought to be multidimensional (e.g., Clements et al., 2008; Common Core State Standards, 2010; Milburn et al., 2019; National Research Council, 2001). For instance, the goals, activities, and skills to support understanding and learning of patterning are not *only* numerical, but a child might use their numerical skills in patterning.

Individual differences in mathematics achievement are thought to be related to various factors, including domain-specific (e.g., numerical; Geary, 2011; Halberda et al., 2008; Tosto et al., 2017) and domain-general skills (e.g., EF, verbal skills; Geary et al., 2017; Peng et al., 2019; Ribner, 2020; Ribner et al., 2023). In addition to their importance for mathematics, these domain-specific and domain-general skills are thought to be interrelated with one another (Chan & Scalise, 2022; Coolen et al., 2021; Purpura & Ganley, 2014). Thus, in the current study I aimed to provide a more nuanced examination of the relations between EF and numerical skills, given their established importance for mathematical and life course outcomes.

Executive Function Skills and Their Role in Mathematics

EF skills play a role in regulating attention, thoughts, and behaviors to achieve a goal (reviewed by: Carlson et al., 2013; Diamond, 2013). These skills undergo a rapid period of development during the preschool years and continue to develop at a slower rate into adulthood (e.g., Ahmed et al., 2022; Best et al., 2011; Zelazo et al., 2013). EF skills are theorized to vary along a continuum from “hot” (emotional) to “cool” (cognitive). In the current research I focused on cool EF skills, which are typically assessed in contexts that are emotionally neutral. Cool EF skills include, but are not limited to, inhibitory control, working memory, and cognitive flexibility (Miyake et al., 2000; Miyake & Friedman, 2012). Inhibitory control refers to inhibiting an automatic response in favor of a behavior that is more appropriate for achieving a goal. Working memory refers to holding and manipulating information in the mind. Cognitive flexibility refers to shifting between tasks, switching between rules, or changing perspectives/approaches when solving a problem. These three components are theoretically distinct; however, they are interconnected with one another and difficult to separate empirically in the early childhood years (Miyake & Friedman, 2012; Wiebe et al., 2008, 2011). Factor analyses on the structure of cool EF skills ranging from preschool through the elementary years suggest a unitary factor structure is a better fit than multiple distinct components (Brydges et al., 2012; Wiebe et al., 2008, 2011; Willoughby, Blair, et al., 2012).

Doing Mathematical Tasks

EF skills are theorized to play a role in *doing* mathematics by enabling children to focus on the task at hand, remember and manipulate relevant information in the mind, and shift between aspects of a problem (e.g., Blair et al., 2015; Bull et al., 2008; Ribner, 2020). EF skills might also enable children to enact their mathematical knowledge when a doing mathematical task. This means children with higher and lower EF skills might have similar mathematical knowledge but children with higher EF might be better able to deploy this knowledge on mathematics assessments and in the classroom (reviewed by: Zaitchik et al., 2016). The discrepancy between knowing and doing is commonly seen when younger preschoolers do EF tasks that involve rule switching, like the Dimensional Change Card Sort (DCCS), wherein the child is taught a card sorting rule and enacts it successfully, then the rule switches and the child is taught the new rule. Despite passing verbal rule checks for the new rule, when asked to enact the new rule many younger preschoolers err by using the old rule instead (Zelazo et al., 1996). This might occur also while doing mathematics tasks, such that a child doing arithmetic might continue to engage in addition after switching from addition to subtraction, despite understanding subtraction, in part due to their EF skills (reviewed by: Blair et al., 2008).

Additionally, some mathematical activities might invoke greater EF demands than others, and the kinds of mathematical activities that invoke greater EF skills might vary across development and individuals. For example, in a study including children aged 5 to 17 years, solving word problems and learning novel strategies and procedures was more strongly associated with EF than rote calculations (Mononen & Niemivirta, 2023). Developmental differences are inevitable in part because new mathematical concepts, strategies, and skills are addressed throughout and across every school year (Clements & Sarama, 2014; Common Core State Standards, 2010), just as EF skills continue to develop over the school years (Ahmed et al., 2022; ; Willoughby, Wirth, et al., 2012). Once a particular skill is overlearned and automatized (such as fluent mental arithmetic), the skill is no longer as effortful as when the task was novel. This is important because EF skills are thought to be particularly important when engaging with academic content that is more *effortful,* rather than automatic (reviewed by Medrano & Prather, 2023; Peng et al., 2018; Spiegel et al., 2021).

Relatedly, Spiegel et al. (2021) proposed a framework that emphasizes the developmental element of the relation between EF and academic skills. Specifically, they propose that the EF components important for academic activities will vary across development and as EF skills shift from a unitary factor to multiple distinct components. With repeated exposure and practice to a particular academic activity the demand on one's cognitive resources while completing the activity will likely decrease. Cross sectional imaging data support the notion that brain activation shifts from anterior to posterior over time, as automatization occurs, such as for simple arithmetic (Ansari et al., 2005; Rivera et al., 2005), suggesting decreasing involvement of frontally-mediated EF. These changes across development have been interpreted as increased specialization in areas of the brain that support numerical cognition and decreased dependence on areas of the brain associated with working memory and attentional skills. Thus, EF skills might be relied upon less while doing number activities as the numerical skills become more automatic.

Importantly, individual differences also play a role and not all individuals benefit from the same level of "overlearning" of mathematics relative to their peers. Specifically, the level at which a skill is "overlearned" varies with individuals. This could be due to the child needing more practice, more frequently spaced practice, or diversity in the kind of practice (e.g., different types of problems). Thus, it is not possible to determine based solely on amount of practice or instructional exposure whether a particular skill is "overlearned" on an individual level. Mathematical skills that become automatized for some people continue to remain *effortful* throughout the school age years for others (Geary et al., 2011; Mazzocco et al., 2008; Stickney et al., 2012) and therefore might continue to invoke EF skills. These individual differences often begin with early numerical skills, which are shown to be challenging for children with low mathematics achievement (Chu et al., 2019; Mazzocco et al., 2013; Namkung & Fuchs, 2012). These individual differences also extend to other skills, including number combination and calculation. For instance, in a series of studies examining the efficacy of number combination interventions for third graders with mathematics learning difficulties, overall effects for number combination skills were significant but a subset of children that received the intervention did not improve (reviewed in Fuchs et al., 2008; Fuchs et al., 2009). Thus, it could be that the children who did not improve needed more practice per day, more weeks of the intervention, or a different type of instructional program.

Learning Mathematical Content

EF is also theorized to play a role in the learning process (reviewed by: Clements et al., 2016). EF skills are domain-general skills that might make it possible for children to learn academic material, including mathematics content, by being as a prerequisite for learning (e.g., Blair et al., 2015; Fuhs et al., 2014). Therefore, EF skills might place constraints on how much a child is able to learn because EF skills are directly invoked by learning and doing academic tasks. Ribner (2020) supported this hypothesis with the finding that EF skills in the fall of kindergarten moderated mathematics learning from instruction over the school year, such that children with higher EF benefited more from mathematics instruction than their lower EF peers (see also, Hassinger-Das et al., 2014). Thus, it is possible EF demands are placed on individuals during mathematical learning and students without the EF skills to meet these demands might not optimally learn from educational instruction. Prior work also indicates EF might play a compensatory role for children with lower mathematics skills (Ribner et al., 2017, 2023). In other words, children with lower mathematics but higher EF skills might be able to rely on their EF skills when struggling to do a mathematical problem or learn a new mathematical concept and this could help them catch up to their higher achieving peers (e.g., Blair et al., 2016; Ribner et al., 2017). Relatedly, there are also hypotheses that there is an age effect on the relation between EF and mathematics, such that EF and mathematics might be more strongly related in older children (reviewed by: Peng & Kievit, 2020). However, few developmental changes in the magnitude of relations between EF and mathematics have been found across age and grade when examined in single studies from 8 to 25 years of age (Cragg et al., 2017), 5 to 20 years of age (Kahl et al., 2021), or when examined meta-analytically in elementary school children (Spiegel et al., 2021).

Further, on average children who are older have higher mathematics skills, so age might also be a proxy for mathematics skill level. Thus, some work also focuses on mathematical skill level in addition to developmental changes across age and grade. For instance, Dong et al. (2020) cross-sectionally examined the predictive strength of EF skills at different mathematics skill levels using quantile regressions. They found EF skills within the preschool and kindergarten years predicted mathematics skills regardless of mathematics skill level, but that EF was a *stronger* predictor of mathematics for children with low, rather than moderate or high mathematics skills. Therefore, higher EF skills could be particularly beneficial for children with low mathematics skills, whereas children with low EF and mathematics skills might be particularly challenged by doing and learning mathematics. In other words, some work has suggested EF might be protective and supportive of mathematics learning for children with lower mathematics skills.

Several researchers have expanded upon the theoretical EF and mathematics link by considering the possibility of bidirectional relations, in other words, that early mathematics skills might be important for EF development, too (reviewed by Blair et al., 2008; Van der Ven et al., 2012). More recently, Peng and Kievit (2020) articulated this point through the theory of mutualism, which postulates a bidirectional relation between EF and mathematics due to mutually beneficial interactions among the skill sets. Higher EF skills might facilitate greater mathematics performance and learning due to the direct and indirect contributions of EF to mathematics. In addition, mathematics tasks might facilitate EF by providing an opportunity for children to practice EF skills. In other words, mathematics activities might provide an everyday life context where children can utilize EF because effortful mathematics invokes these domain-general skills.

Partial support has been found for these theorized relations. There is consistent support for the concurrent relations between EF and mathematics achievement from 2 to 25 years of age (Alloway et al., 2014; Alloway & Alloway, 2010; Bierman et al., 2008; Blair & Razza, 2007; Bull et al., 2011; Cameron et al., 2012; Welsh et al., 2010), which remains significant in studies controlling for intelligence (Bull et al., 2011; Clark et al., 2010; Prager et al., 2016). However, longitudinal findings in early childhood are mixed (Blair & Razza, 2007; Fuhs et al., 2014; Schmitt et al., 2017). Many previous longitudinal studies of EF and mathematics test and report a unidirectional relation (EF → Mathematics), but fewer studies test for a bidirectional relation (Blair & Razza, 2007; Bull et al., 2008; McClelland et al., 2007). When bidirectional relations are examined the results are mixed, such that mathematics *does not* consistently predict later EF (Cameron et al., 2019; Ellis et al., 2021; Fuhs et al., 2014; Mazzocco & Kover, 2007; McKinnon & Blair, 2019; Miller-Cotto & Byrnes, 2019; Schmitt et al., 2017; Welsh et al., 2010). Some possible factors contributing to these mixed bidirectional findings include age or other characteristics of the sample, measures, mathematics domain and EF component assessed, and variation in analytic strategies. Most studies focus on mathematics achievement broadly, which captures *various* mathematical skills and subdomains (e.g., patterning, counting, telling time) that vary by measure and across development. One way to begin to unpack why EF and mathematics achievement relations might vary by study is to examine how EF relates to specific mathematical skills. Therefore, here I focused on the relations between EF and numerical skills.

Relations Between Executive Function and Numerical Skills

Early numerical skills involve various symbolic and nonsymbolic skills that children begin to engage with prior to formal schooling (Purpura & Lonigan, 2013). In early childhood children practice numerical skills in classroom and home contexts (Daucourt et al., 2021; Engel et al., 2016) and these skills are predictive of later mathematics achievement (for review see: Raghubar & Barnes, 2017). In the current study, I focused on six numerical skills – nonsymbolic magnitude comparison, verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving. In the following sections I provide a brief overview of each of these numerical skills, including developmental trajectories and relations with mathematics achievement and EF. Importantly, there is some variation in terminology and definitions used to describe each of these skills in the field and across development. Thus, the terms and descriptions provided below are specific to this study and are predominately focused on early childhood, rather than a comprehensive description of each numerical skill and its development.

Nonsymbolic magnitude comparison. Nonsymbolic magnitude comparison of large (i.e., ≥ 5) numerosities refers to being able to determine which of two groups of objects (e.g., dots) is larger without counting. Nonsymbolic magnitude comparison tasks, such as the Panamath, intend to measure acuity of the Approximate Number System (ANS; reviewed by: Odic & Starr, 2018). The ANS is proposed to be the mental system that supports nonverbal number representations (reviewed by: Dehaene, 2009; Feigenson et al., 2004). However, there is disagreement in the field about the role of nonsymbolic skills in mathematics and measures used to estimate nonsymbolic magnitude comparison.

Specifically, there is disagreement surrounding the significance of nonsymbolic versus symbolic skills for formal mathematics. It has been proposed that nonsymbolic skills serve as the foundation for symbolic numerical skills and formal mathematics (reviewed by Odic & Starr, 2018; Piazza, 2010). However, this proposal is not widely accepted. Importantly, some researchers find weak relations between ANS acuity and mathematics and instead argue the focus should be more on symbolic number skills and their contribution to mathematics (Caviola et al., 2020; reviewed by: De Smedt et al., 2013). Meta-analytic findings support the notion that there is a stronger relation between symbolic number skills and mathematics (r = .302) than nonsymbolic number skills and mathematics (*r* = .241; Schneider et al., 2017). However, this finding does not negate the significance of nonsymbolic magnitude skills overall, it merely highlights the importance of symbolic number skills for mathematics achievement. Multiple empirical studies report nonsymbolic magnitude comparison tasks are positively associated with concurrent and later mathematics achievement (Chen & Li, 2014; Halberda et al., 2008; Libertus et al., 2013; Mazzocco et al., 2011b; Wang et al., 2016). In addition, there is some evidence of bidirectional relations between nonsymbolic magnitude comparison and mathematics ability over time in 3 to 5 year old children using cross-lagged panel models (Elliott et al., 2019).

There is also debate in the field surrounding the tasks used to measure nonsymbolic magnitude comparison. Specifically, it has been proposed that the relation between nonsymbolic magnitude comparison tasks and mathematics are due to the EF demands of nonsymbolic magnitude comparison tasks (e.g., Wilkey et al., 2022). Specifically, other visual parameters of the stimuli are either congruent or incongruent with the numerosity of the dots (Gilmore et al., 2013). When the visual parameters conflict with numerosity, the participant might rely on their inhibitory control skills to attend to numerosity while ignoring other non-numerical stimuli, which might mean nonsymbolic magnitude comparison tasks capture EF instead of, or in addition to, ANS acuity (Fuhs & McNeil, 2013; Piazza et al., 2018; Wilkey et al., 2017, 2020, 2022). For example, inhibitory control skills might be used to inhibit focusing on the surface area, in favor of dots, when the more numerous set of dots has lesser surface area. However, if non-numerical features, like surface area, are not controlled for when creating stimuli then participants can complete the task accurately using non-numerical information. Thus, removing these control conditions is not ideal.

Empirical work examining whether EF demands in nonsymbolic magnitude comparison tasks are driving the relation between nonsymbolic magnitude comparison and mathematics is mixed. Specifically, the relation between nonsymbolic magnitude comparison tasks and mathematics is explained by EF in some (Fuhs & McNeil, 2013; Gilmore et al., 2013), but not all studies (Decarli et al., 2023; Keller & Libertus, 2015; Starr et al., 2017). Thus, this topic is still widely debated and requires further examination.

Despite these disagreements in the field, a large body of literature has reported developmental and individual differences in performance on nonsymbolic magnitude comparison tasks. Individual differences in nonsymbolic magnitude comparison performance emerge early and remain stable over time, with performance at 6 months of age predicting performance in preschool (Starr et al., 2013). Interestingly, students that meet criteria for a mathematics learning difficulty (mathematics achievement ≤ 10th percentile) receive significantly poorer scores on nonsymbolic magnitude comparison tasks than children with low (11th-25th percentile), typical (25th – 95th percentile), and high (>95th percentile) mathematics achievement, on average (Mazzocco et al., 2011a). Thus, ANS deficits are thought to be *one of many* potential pathways to mathematics learning difficulties (Chu et al., 2013; Mazzocco et al., 2011a; Piazza et al., 2010).

Developmental differences in childhood have also been reported (Chu et al., 2016; Halberda & Feigenson, 2008). Specifically, nonsymbolic magnitude comparison tasks involve discriminating sets of dots that vary in difficulty level based on the ratio bin (e.g., easier: 1:2; harder: 3:4). Prior work indicates there is a protracted developmental trajectory, such that developmental differences are evident in preverbal infants, performance continues to improve throughout the school years, and peaks around approximately 30 years of age (Halberda et al., 2012; Halberda & Feigenson, 2008). Specifically, from infancy into adulthood individuals' ability to discriminate increasingly difficult ratios increases, such that typical college-aged students can discriminate between sets of 25 to 22 dots (Halberda et al., 2012). There are many hypothesized sources of these individual differences, including genetic heritability and formal education (Piazza et al., 2013; Tosto et al., 2014), but there are likely other unexplored factors as well.

As described above, the role of EF skills in nonsymbolic magnitude comparison is debated due to the hypothesized EF demands of the tasks. It might also be that EF skills contribute to performance on the task by supporting a child's ability to remember the "rule" (focus on numerosity) on all trial types and sustain attention on the task while inhibiting any distracting elements in the task or broader environment wherein the task taking place. Concurrent relations between EF and nonsymbolic magnitude comparison vary by study with some reporting significant relations for total accuracy (i.e., including both congruent and incongruent trials; Chu et al., 2016; Keller & Libertus, 2015) and others reporting significant correlations between EF and incongruent trials, but not congruent trials (e.g., Wilkey et al., 2020). Longitudinal relations have also been found. For instance, nonsymbolic magnitude comparison was correlated with EF one year later in preschoolers, but this analysis did not control for earlier EF (Chu et al., 2016).

Verbal counting. Verbal counting refers to reciting number words in the accurate order. Many children begin to practice the counting sequence prior to school entry and have knowledge of the counting sequence as early as 2 or 3 years of age (Gelman & Gallistel, 1978). According to Litkowski et al. (2020), around half of children can count to 10 by age 3 and count to 20 by age 5, which aligns with the benchmarks described by Clements and Sarama (2014). In English, a predictable number naming pattern emerges beyond twenty, whereas learning smaller number words is somewhat less predictable. As a result of this predictable pattern, many English-speaking children can count to 100 by age 6; however, individual differences are present with some children learning this skill earlier or later (reviewed by: Clements & Sarama, 2014). Correctly stating the count list is a precursor to becoming a cardinal principal knower, which refers to understanding that the last word counted in a set refers to the total number of items in the set (reviewed by Carey, 2004, 2009; Gelman & Gallistel, 1978; Spelke, 2017). Further, verbal counting proficiency at the beginning of preschool is associated with mathematics achievement at the end of the preschool year (Geary & vanMarle, 2016).

Reciting the counting sequence in the correct order is thought to be a procedural task that involves some memorization, particularly for numbers 1-20 in English. However, EF skills might help support young children in continuing the counting sequence beyond 20 if it is still effortful for the individual to use information about the pattern and structure of the count sequence to determine which number comes next. Positive concurrent relations between verbal counting and inhibitory control, but not working memory or cognitive flexibility skills have been found in samples of preschoolers (i.e., 3 to 5 years of age; Purpura et al., 2017) and preschool and kindergarteners (i.e., 4 to 6 years of age; Purpura & Ganley, 2014). As children proceed through school and continue to practice the counting sequence, verbal counting is thought to become more rote or automatic. Therefore, for verbal counting to be effortful in older childhood an individual might have to count to a higher number than they would have in earlier childhood, on average. This might need to be a particularly high number for some children especially if they have sufficient understanding of base 10.

Numerical literacy. Numerical literacy refers to reading and writing numerals. Children often become familiar with number words through verbal counting before they begin reading and writing numerals. Next, children can identify or read numbers. By 4 years of age, approximately 25% of children can accurately identify numerals 1 to 9 (Ginsburg & Baroody, 2003). Identification of two and three-digit numbers is more advanced than one-digit number identification (Claessens et al., 2014), and many children begin to identify the digit “10” prior to other two-digit numbers (Litkowski et al., 2020). Further, reading numbers in kindergarten is associated with later mathematics achievement, in second and third grade (Mazzocco & Thompson, 2005). As children age, being able to read and write numbers accurately is necessary for engaging in various types of higher-level numerical problem solving, such as addition, subtraction, and algebra.

EF might play a role in mapping digits to number words before this skill is “mastered.” Specifically, children must hold a verbally stated number word in their mind while attempting to map the number word onto a digit as a means to accurately identify or write the digit (Benoit et al., 2013; Lira et al., 2017). After repeated practice and exposure to reading and writing of specific numerals, completing these tasks might place greater demands on retrieving previously learned information rather than using mathematical knowledge and other domain-general cognitive skills, like EF, to find a solution to a novel problem (reviewed by: Blair et al., 2008). In a previous study, numeral identification was not significantly concurrently related to EF in early childhood when inhibitory control, working memory, or cognitive flexibility were measured separately (Purpura et al., 2017; Purpura & Ganley, 2014), but it was significantly related to an EF composite in preschool (Chan & Scalise, 2022). Longitudinal relations were also examined by Chan and Scalise (2022) and they found earlier EF was associated with later numerical identification in preschool and kindergarten samples; however, once earlier numerical skills were accounted for this relation was no longer significant. It might be that differences in measurement of EF and numerical skills are contributing to these mixed findings.

Count on. Count on refers to naming the next number after a specified number in a count. For example, if one said, “18, and then comes…” the accurate response would be “19.” Count on and verbal counting both fall within the realm of counting, but children begin to verbally count prior to identifying the next number in a sequence or counting from a number other than 1 (reviewed by: Clements & Sarama, 2014). Previous work proposes that identification of a number before and after a specified number (other than 1) begins around age 6, but that there is variability in performance (e.g., if a child can only verbally count to 20, it would be unlikely they would be able to identify what number comes after 45; Clements & Sarama, 2014).

EF skills might support children’s ability to mentally manipulate the counting sequence to be able to identify the next number, which might require deeper understanding of the counting sequence and relations between numbers than engaging in verbal counting from one. Positive concurrent relations between count on and working memory, inhibitory control, and cognitive flexibility have been reported in preschool and kindergarteners (Purpura et al., 2017; Purpura & Ganley, 2014).

Non-rote counting. I conceptualized non-rote counting to involve the following counting-based skills: counting backwards (from 20), dot enumeration, and providing the examiner a set of a given quantity (i.e., "Give me exactly 19"). I included reciting the counting sequence backwards as a non-rote counting skill because this task is thought to be more effortful in early childhood than reciting the counting sequence forward, which might be becoming more automatic during these years due to practice. Developmentally, children can accurately verbally count to 20 prior to counting backwards from 20 (Clements & Sarama, 2014). Therefore, timing of counting backward from 20 varies from one individual to the next, but 5 and 7 years of age are proposed as benchmarks for counting backward from 10 and 20, respectively (Clements & Sarama, 2014). With repeated practice, counting backwards is also thought to become more automatic for most individuals.

Additional items included in the non-rote counting category involved counting dots and identifying the total number of dots in the set as well as being given a set of tokens and providing the examiner the specified amount. Both of these items involved counting sets and aimed to capture cardinality (Clements & Sarama, 2014; Krajewski & Schneider, 2009). Before children can accurately connect number words to specific quantities, they must first be able to verbally count as high as the set total. In addition, one-to-one correspondence, which refers to counting each item in the set only once, is thought to support cardinality understanding (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1990). Thus, cardinality understanding occurs over a protracted period and many kindergarteners are still developing their cardinal understanding (Gelman & Gallistell, 1978; Litkowski et al., 2020).

Children are thought to use their EF skills for various non-rote counting tasks, including to remembering the target number, stopping once they hit the target, and counting each item only once when counting a set of items. When counting backward children might use EF to mentally manipulate the often forward stated counting sequence (i.e., 1, 2, 3, etc.) to backwards (i.e., 20, 19, 18, etc.). Along these lines, prior work has found a positive concurrent relation between set counting and EF (e.g., Chan & Scalise, 2022; Purpura & Ganley, 2014; Purpura et al., 2017). Further, Chan and Scalise (2022) found earlier EF was associated with set counting three to five months later in preschool; however, once earlier set counting skills were accounted for this relation was no longer significant. Interestingly, in a slightly older sample that were kindergarten-aged, there was no longitudinal relation between EF and set counting, which the authors interpreted to be due to children becoming more skilled at set counting and thereby the task invoking lower EF demands with age (Chan & Scalise, 2022).

Numerical problem solving. Numerical problem solving in early childhood may include nonsymbolic mental arithmetic problems, verbal story problems, and formal addition and subtraction. The specific elements within this skill vary developmentally. For example, multiplication and division based word problems might be included within the numerical problem solving umbrella but are not described here because these topics are often not covered in U.S. schools until second or third grade (Common Core State Standards, 2010), which is near the end of the early childhood period.

It is theorized that children become cardinal principal knowers before they engage in higher level formal problem solving (Krajewiski & Schneider, 2009), but many children in preschool can engage in numerical problem solving with small numbers (Clements & Sarama, 2014; Levine et al., 1992; Litkowski et al., 2020). Children often begin engaging with numerical problem solving through nonsymbolic mental arithmetic, which involves adding or subtracting by physically moving the referents in a set. In a study examining the development of calculation skills from 4 to 6 years of age, Levine et al. (1992) found children began to successfully complete nonsymbolic mental arithmetic problems as young as 4 years of age (Levine et al., 1992). By ages 5.5 to 6.5 years of age, on average, the participants reached similar levels of success on verbal story problems that involved addition and subtraction of small numbers (Levine et al., 1992). Next, children engage in formal addition and subtraction of small numbers (e.g., 2 + 2 = ___). Some children are beginning to do formal addition and subtraction by age 5, but many children continue developing this skill beyond the preschool years (Clements & Sarama, 2014; Litkowski et al., 2020). For instance, Litkowski et al. (2020) found that only 44.6% of children correctly responded to the formal addition problem 1+1 by 5 years of age.

Various elements of numerical problem solving might invoke EF skills. Engaging in operations with “hidden” materials is thought to invoke greater working memory demands than when working with manipulatives or images that are visible in the environment. Further, switching from one operation to the another (e.g., addition to subtraction) might invoke demands on inhibitory control and cognitive flexibility skills. Ultimately, numerical problem solving items often involve multiple systems and steps and maintaining information in the mind to accurately solve the problem, which is why

EF skills are thought to be useful when doing these numerical problems. In support of this, Purpura et al. (2017) found evidence for a positive concurrent relation between inhibitory control and verbal story problems in preschoolers. Further they found a marginal, but not statistically significant, relation between verbal working memory and formal addition skills (1 + 1 = ___). In addition, EF predicted mathematics achievement three to five months later, above and beyond age and gender, in a preschool sample wherein mathematics achievement was derived by compositing two addition measures (i.e., story problems, forced choice addition problems). There was also a significant relation between EF and story problem scores when examined separately from the composite (Chan & Scalise, 2022). Meta-analytic findings also converge on this pattern of findings, such that small to moderate positive associations have been reported for word problems and inhibitory control (r = .33), working memory (r = *.43*), and cognitive flexibility (r =.35) in elementary school children (Spiegel et al., 2021).

Current Gaps in the Literature

Several studies have examined the concurrent and predictive relations between EF and mathematics achievement; however, there are fewer studies that have examined the interrelations between EF and numerical skills specifically. Studies that have focused on the EF and numerical skill relation predominantly examine concurrent relations or only test for a unidirectional association (EF → numerical skills) longitudinally, which limits understanding of the early relations between EF and specific numerical skills. Recent work on the longitudinal associations between EF and numerical skills conducted by Coolen et al. (2021) included typically developing 3- and 4- year-old preschool children in the UK. They found EF predicted later symbolic number knowledge and growth in symbolic mathematics. Symbolic number knowledge was correlated with later EF; however, symbolic number knowledge did not predict EF growth. Their use of latent factors provided insight into symbolic mathematical skills but limited their ability to examine how EF related to specific numerical skills within these broader numerical skill categories. Thus, additional longitudinal examinations of the relations between EF and various numerical skills in early childhood are still needed.

Many of the studies examining interrelations between EF and numerical skills do not examine whether these findings differ for children with typical versus persistently low mathematics achievement. This research is important for informing how best to support children with lower mathematics performance. Thus, the current study aims to extend prior work by examining a set of specific numerical skills– nonsymbolic magnitude comparison, verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving – and how they relate to EF skills concurrently and the directionality of these relations longitudinally. Further, I identified children with typical versus persistently low mathematics achievement and examined the relations between EF and numerical skills to determine whether the pattern of relations varies by mathematics achievement level.

Chapter 2: Research Question 1

In the first research aim, I examined the relations between EF and specific numerical skills in a sample of preschool to third graders to address the following research questions:

RQ 1a. Which specific numerical skills are concurrently associated with EF?

Consistent with prior work, I hypothesized there would be positive concurrent relations between EF and count on (e.g., naming the next number in a sequence), non-rote counting (e.g., counting backwards, enumerating dots), numerical problem solving (e.g., addition, subtraction, story problems), and nonsymbolic magnitude comparison, after controlling for relevant covariates (age, maternal education, vocabulary; e.g., Purpura et al., 2017). I hypothesized the relations between EF and numerical literacy (e.g., reading/writing numbers) and verbal counting would not reach significance. This is because these numerical skills are likely to become more automatic, and less effortful, following the practice many children experience during the early childhood years.

RQ 1b. Which specific numerical skills at assessment 1 are associated with EF skills at assessment 2 (and vice versa)? I hypothesized EF at assessment 1 (A1) would predict the following specific numerical skills at assessment 2 (A2): count on, non-rote counting, numerical problem solving, and nonsymbolic magnitude comparison. Consistent with the hypothesis that more effortful mathematics will invoke greater EF skills, I predicted bidirectional relations would emerge for EF and the following numerical skills: numerical problem solving and non-rote counting (see Figure 1). I hypothesized verbal counting and numerical literacy skills would be less effortful, on average, in the age range of this sample; therefore, following an effort-based hypothesis I did not predict significant longitudinal relations between verbal counting or numerical literacy and EF.

RQ 1c. Do the concurrent and predictive relations between EF and numerical skills vary for children with typical mathematics achievement versus persistently low mathematics achievement? This question is exploratory. I hypothesized EF and specific numerical skills would be more strongly associated with one another for children with persistently low mathematics achievement versus children with typical mathematics achievement. This aligns with some work by Dong et al. (2020) indicating EF was more strongly associated with mathematics achievement for children with lower, compared to higher, mathematics competencies. This is because there are likely differences in relative *effortfulness* of these numerical skills based on ability-based groups. Specifically, the numerical skills might be more challenging for children with persistently low mathematics performance, thereby invoking greater EF demands.

Method

Participants

Participants were drawn from a larger prospective, longitudinal study examining the pathways to mathematics learning difficulties in preschool to third graders (M.M. Mazzocco, PI). All participants were required to be proficient English speakers. Participants included in this analytic frame had no known developmental disorders.

Participants

The analytic frame for Research Question 1 included 205 typically developing children primarily recruited from three schools in the Minneapolis Public School (MPS) district. Some children were also recruited from the Institute of Child Development Participant Pool (IPP), which is a departmental database of local children and families that parents can opt into to receive invitations to participate in research studies. A total of 360 children started at least one assessment of data collection. The interruption of in-person data collection by start of the COVID-19 pandemic in 2020, resulted in 152 children not being able to complete a second time point of data collection. Due to the longitudinal nature of the first research question, I included only the 208 children who completed two time points of data collection. Within this subsample of 208, I excluded 3 children due to missing date of birth (*n* = 2) or the participant stopping early in the data collection sessions (*n* = 1).

Participants in the analytic frame (*N* = 205) included 4.67-8.75-year-old children (M_{age}= 6.68 years; *SD* = .95; 43.9% Female) at A1. A little over half the children in this sample were White, non-Hispanic (51.2%), with an additional 18% Multiracial, 6.3% Hispanic, 12.2% Black, 2% American Indian/Alaska Native, 4.9% Asian, 1% not otherwise listed, and 4.39% had missing data. As a proxy for socioeconomic status (SES) parents reported whether their child was eligible for Free/Reduced Price Lunch (FRL) and the number of years of maternal education. Most participants were not eligible for FRL (62.9%), 33.7% received FRL, and 3.4% had missing data. On average, mothers were likely college educated, with an average of 16.06 (*SD* =2.45; range = 2-18) years of education. Over a third of children (37.6%) were regularly exposed to a language other than English, 59% were regularly exposed to English only, and 3.4% had missing data. The same set of children had missing data for FRL and exposure to another language.

Attrition Analyses

Data on key variables of interest (e.g., mathematics, EF) were available for 205 participants at A2. A series of *t*-tests and chi-squared tests comparing children with A1 and A2 (*N* = 205) to children with only A1 data (*n* = 146) demonstrated no significant differences between groups in terms of child FRL eligibility (χ^2 *(*1, *n* =334) = 3.39 *p* = .059) or gender (χ^2 (1, *n* =351) =3.56 *p* = .065). Participants with A2 data were older (*t* =

4.19, p <.001) from households with greater maternal education (t = 3.12, p <.001), and had higher EF composite (t = 4.14 , p <.001), vocabulary composite (t = 4.87, p <.001), TEMA-3 (t = 5.03 , p <.001), and WJ-Applied Problem scores (t = 5.34, p <.001) at A1, than those lost to attrition. After accounting for age and maternal education, there were no longer significant differences in EF composite (p =.195) or vocabulary composite scores (p = .060); however significant differences for TEMA-3 (p =.046) and WJ-Applied Problems (p = .005) remained.

Some of the attrition in this study was due to COVID-19. I determined if the attrition was due to COVID-19 by identifying children who did not have the option to attend A2 because their scheduled session date was after the United States COVID-19 stay at home orders were put into effect. This included children who were scheduled for sessions any time after late March 2020. Therefore, I ran the prior set of analyses to examine differences in children lost due to COVID-19 (n = 104) versus those who were retained. Children lost to COVID-19 were younger in age (t = 4.76, p <.001), had lower maternal education (t = 2.61, p =.01), and lower scores on EF composite (t = 3.74, p <.001), vocabulary composite (t = 4.39, p <.001), TEMA-3 (t = 5.04, p <.001), and WJ-Applied Problems (t = 5.65, p <.001); however, after controlling for age and maternal education, the only significant effect that remained was for WJ-Applied Problems (p = .007). There were no significant differences in terms of child FRL eligibility (χ^2 (1, n =298) = .76, p = .383) or gender (χ^2 (1, n =309) = 2.73, p = .098).

Next, I ran another set of analyses to examine those lost to attrition “naturally” (i.e., due to a reason other than COVID-19). Children lost to attrition for a reason other than COVID-19 (n = 42), were more likely to be FRL eligible (χ^2 (1, n =234) = 7.1, p =

.008), have lower maternal education (*t* = 2.15, *p* = 0.014), and lower scores on the vocabulary composite (*t* = 3.03, *p* = .004), TEMA-3 (*t* = 2.33, *p* = .023), and WJ-Applied Problems (*t* = 2.01, *p* = .043); however, after controlling for age and maternal education, no significant effects remained for the academic or cognitive skills examined (*ps* > .08). There were also no significant differences in terms of child age (*t* = 1.20, *p* = .233), EF composite (*t* = 1.73, *p* = .088), or gender (χ^2 (1, *n* =247) =1.66, *p* = .198).

Measures

Mathematics

Woodcock-Johnson Achievement Test, Third Edition (WJ-III, Woodcock et al., 2001). The WJ-III is a standardized academic achievement test appropriate for ages 2-90 years. The Applied Problems (WJ-III AP) subtest was administered as a measure of mathematical reasoning. Early items involve providing an answer to the examiner verbally, while in later items children are given a paper and pencil to solve problems. Difficulty increases as the subtest progresses, with the subtest being complete when the child answered at least six consecutive questions incorrectly at the end of a given page. Good internal reliability for the WJ-III AP subtest has been established for children in the preschool to third grade age range (α = .92; Mather & Woodcock, 2001). Age-normed (*M* = 100, *SD* = 15) and raw scores (out of 63) were examined.

The Test of Early Mathematical Ability, Third Edition (TEMA-3; Ginsburg & Baroody, 2003). The TEMA-3 is a standardized measure of early mathematics skills designed for children ages 3 to 8 years. This test assesses formal and informal mathematical knowledge and skills. Good internal reliability has been established in the early childhood years (α = .94; Ginsburg & Baroody, 2003). Age-normed scores (*M* =

100, *SD* = 15) are available for children up through 8 years, 11 months. As a result, children 9 years of age or older at the second time point did not have TEMA-3 provided age-normed scores. To determine percentiles for children older than this threshold, raw TEMA-3 scores were regressed on age and predicted scores were used. Age-normed and raw scores were examined.

Numerical Skills. To gain additional information about specific numerical skills, the TEMA-3 items were expanded upon in various ways. In the TEMA-3, the estimated start point is identified by child age and scores are identified through a pass/fail on trials. To expand on this, items were scored beyond a pass/fail standard. Specifically, I scored accuracy on individual trials that comprised each item. For instance, to pass Item 26 using traditional scoring the child only needs to get two of three trials correct. I expanded upon this by scoring by looking at accuracy on each trial. Next, some items were administered to children even if it was below their typical TEMA-3 entry point and/or above their standard ceiling. Lastly, in a few cases additional trials were added to expand the number and range of trials for a particular scale. This expansion was done to increase variation for the specific numerical skills of interest beyond what was in the TEMA-3. These additional trials were combined with items from the TEMA-3 items to create a more comprehensive raw score for a particular specific numerical skill. The following five specific numerical skills were derived primarily from the TEMA-3: numerical literacy, count on, verbal counting, non-rote counting, and numerical problem solving. An independent measure of nonsymbolic magnitude comparison was also administered.

Nonsymbolic Magnitude Comparison. The Psychological Assessment of Numerical Ability (Panamath, Halberda et al., 2008) was administered as a computerized measure of nonsymbolic magnitude comparison of large numerosities (i.e., ≥ 5). This task was administered on a 13-inch MacBook Air laptop using the version available at Panamath.org. During this task, participants view two dot arrays simultaneously on separate sides of the computer screen for a 2128 ms per trial, followed by a 200ms backward mask. For each trial participants indicate which side of the screen had more dots. All participants completed the same two practice trials and 88 test trials, appearing in a fixed order. The numerosity of the two sides varied by trial according to a specific ratio, with each trial being one of four ratios: 1.3, 1.5, 1.8, and 2.8. All trials that conform to each ratio are referred to collectively as a ratio bin. Half of the trials were controlled for surface area and the other half were controlled for average dot size. These controls were included as a means to prevent participants from relying upon non-numerical strategies while completing this task. This version of Panamath has a split-half reliability of $\alpha = 0.72$ in a sample of kindergarten children (Libertus et al., 2013). Based on recommendations for this age group, percent accuracy was the primary score of interest (Inglis & Gilmore, 2014).

Verbal Counting. Verbal counting was derived from the TEMA-3 trial wherein the participant counted as high as they could (starting from 1). If the child stopped counting the examiner said, “What comes next?” The trial ended when the child stopped counting, made an error, or reached 110. The primary score of interest was how high the child counted without error.

Numerical Literacy. Numerical literacy was derived from 21 TEMA-3 trials. Numerical literacy trials involved reading and writing two-, three- and four digit numbers (range: 23 to 4073). Children were only administered trials where they read four-digit numbers if they passed all three trials with three-digit numbers. Similarly, a stop rule was implemented with writing numbers wherein children did not move up to three-digit numbers if they could not accurately demonstrate writing two-digit numbers. Children in kindergarten, first, second, and third grade, or who reached Item 66 in the TEMA-3 prior to the test stop rule were administered trials where they identified the smallest and largest numbers of a particular digit-length (e.g., “What is the smallest 1-digit number?”). Children were also administered all one-digit numbers from 1 to 9, but due to high accuracy rates these were not included in the numerical literacy skill score. Internal consistency for included items was good at both time points (Cronbach’s alphas ≥ .91).

Count On. Count on was derived from eight TEMA-3 trials involving naming the next number after a specified number in a count of two- and three-digit numbers (e.g., “10 and then comes *(pause and wait for response*)?”). A stop rule was implemented wherein if children missed two items in a row, they were deemed to have reached their ceiling. These items were comprised of two or three trials. Internal consistency for these items was good at both time points (Cronbach’s alphas ≥ .82).

Non-Rote Counting. Non-rote counting on was derived from six TEMA-3 trials involving counting backwards from 20, dot enumeration of sets ranging from nine to 16, and set enumeration through providing the examiner a given set quantity (i.e., 19) from a larger set of available tokens. Internal consistency for these items was acceptable at A1 (Cronbach’s alpha =.711) but fell below the .70 threshold at A2 (Cronbach’s alpha = .538).

Numerical problem solving. Numerical problem solving on was derived from 12 trials total (11 from the TEMA-3 and 1 additional trial). There were six trials involving nonverbal addition and subtraction (5 trials from the TEMA-3, the additional trial added was 5 minus 4). In these trials the examiner places tokens under a mat then they add additional items under their mat or remove tokens from under their mat. The child watches and for each trial are prompted to produce the same solution as the examiner (i.e., “Make yours just like mine”). An additional six trials were story problems from the TEMA-3. This included the story problem trials in Item 25, which was administered to all children in kindergarten, first, second, and third grade and preschoolers who reached this item on the TEMA-3 prior to the test stop rule. Internal consistency for these items was good at both time points (Cronbach’s alphas ≥ .74).

Executive Function

Two measures of EF skills that call on working memory, inhibitory control, and cognitive flexibility skills were administered. In prior work the selected EF measures have been moderately, positively correlated with one another (e.g., Carlson, 2021; Distefano et al., 2018).

Minnesota Executive Function Scale (MEFS^{TM}; Carlson & Zelazo, 2014). The MEFS is a standardized measure of EF administered on a tablet. This task is reliable, valid, and normed based on a sample of 51,424 typically developing children ages 2-17.9 years old (Carlson, 2021). The MEFS is an adaptive task in which the recommended starting level is based on age. In total, there are seven levels of increasing difficulty. Children were seated next to an experimenter with the tablet in front of them. Participants were asked to sort virtual cards based on different dimensions (e.g., color or shape) by dragging them into the boxes on the screen. Children continued to move up a level until they failed or completed Level 7. If the child did not pass the starting level, they moved down a level, and continued to move down until they passed or completed Level 1. The MEFS takes approximately 5 minutes. An algorithm within the MEFS app takes trial accuracy and reaction time into account to calculate total scores for each participant (see Appendix for scoring details). Total scores (range: 0 -100) were used as the primary score of interest.

Head Toes Knees Shoulders (HTKS; McClelland et al., 2014). The HTKS task is appropriate for children ages 4 to 8 years. In Part 1 of this task the examiner first presents the child with two commands: “touch your head” and “touch your toes.” Then the children are told they will be playing a “silly” game, wherein they do the opposite of what the examiner says (e.g., if the examiner says “touch your toes” the accurate response would be to touch one’s head). The task demands increase in part 2 as new commands are introduced including, “touch your knees” and “touch your shoulders.” Similar to part 1, the child is then asked to do the opposite of the stated command. In part 3, the earlier commands from both previous parts were scrambled, such that when told to “touch their head” the accurate response is to touch their knees. Each part involved a series of practice trials with corrective feedback and 10 test trials without feedback. For each item children received 0 for an incorrect response, 1 for a self-corrected accurate response, or 2 for an accurate response without a self-correction. HTKS has an internal reliability of $\alpha = 0.94$ for children in their spring of the kindergarten year (McClelland et al., 2014). Upon inspection of the test trials, negative skew was identified. Thus, I opted to include the 17 practice trials and 30 test trials, resulting in a total score out of 94 in subsequent analyses given the greater variance of scores (Fuhs et al., 2014).

Vocabulary

The Boston Naming Task, Second Edition (BNT-2; Kaplan et al., 2001). The BNT-2 was used to estimate expressive vocabulary. This measure is commonly used in standard neuropsychological assessments of language, including assessments of individuals with learning difficulties (Rabin et al., 2016). BNT-2 is a confrontation picture naming measure that involves an experimenter presenting images of line drawings and prompting the participant to name the image. Participants were prompted by the experimenter if they provided an answer that was semantically aligned with the correct answer (e.g., calling a soup pot a slow cooker). The original test has 60 items, but one item was removed for this study due to cultural insensitivity, resulting in a score out of 59. Raw scores were used because the BNT-2 scores are not age-normed.

The Kaufman Brief Intelligence Test, Second Edition (KBIT-2; Kaufman & Kaufman, 2004). The KBIT-2 is a standardized measure of intelligence appropriate for individuals 4 to 90 years of age. The Verbal Knowledge subtest (KBIT-2 VK) was administered to estimate receptive vocabulary. On each trial, participants view a set of 6 photographs/illustrations and the examiner prompts them with questions relating to item identification (e.g., Point to *X*) or general information (e.g., Point to the item that *X*). The KBIT-2 VK subtest has reliability of .74-.87 for children in the preschool to third grade years (Kaufman & Kaufman, 2004). Age-normed (*M* = 100, *SD* = 15) and raw scores were examined, but ultimately raw scores were used as the primary score of interest.

Parent Questionnaire

Parents were asked to complete a brief demographic questionnaire to gather information pertaining to child age, race and ethnicity of the parent(s) and child, parent(s) occupation, parent(s) highest education, child FRL status, languages exposed to beyond

English, etc. This information served to describe the sample and select items served as covariates in analyses. Parents also provided information pertaining to relevant exclusionary criteria.

Procedure

Participants completed an extensive battery of mathematical and non-mathematical tasks during two to three sessions per year, across three waves of data collection. Participants each completed a maximum of three time points. Only relevant data from assessments one and two were considered, due to the interruption in data collection by COVID-19. All the measures described and considered for this study were collected at *both* time points (see Table 1 for summary of constructs and corresponding measures).

There were three cohorts of participants with data for their first time point collected in 2017-2018, 2018-2019, and 2019-2020, respectively. The first wave included only new enrollees; the two subsequent time points including new enrollees and longitudinal assessments of children seen at the prior time point. Children from Research Question 1 were primarily recruited locally from three schools in the MPS district. A small group of preschoolers were also recruited from the University of Minnesota Institute of Child Development Participant Pool (IPP). As such, data collection for each assessment was conducted primarily in schools, with some children doing data collection in the lab (e.g., IPP children). Data collection each year was completed in two to three sessions (depending on timing/child age). The testing battery was administered in the following order: vocabulary, EF, then mathematics tasks.

Results

Analytic Approach

Prior to analysis, data were screened for outliers (< 2% for all variables, except verbal counting which was 3.9%) and extreme values were winsorized. Winsorizing of outliers is a transformation that minimizes the influence of outliers on the overall results while still retaining the outlier values in the sample. Here I winsorized by replacing scores more than 3 *SD* from the mean with values computed at 3 *SD* from the mean. The skew and kurtosis improved on multiple EF and mathematics variables after winsorizing. Missing data were treated as missing at random and imputed through a multi-step process (see subsequent Missing Data section for details).

Descriptive and correlational analyses were conducted to examine the distributions of the data, associations among the variables of interest, and examine potential covariates. Finally, OLS, logistic, and ordinal logistic regression models were conducted as the main analyses. Benjamini–Hochberg corrections were applied to each regression to account for multiple comparisons (Benjamini & Hochberg, 1995).

The primary analyses were conducted in R (R Core Team, 2020). I used the *mice* package (van Buuren & Groothuis-Oudshoorn, 2011) for multiple imputation and the *lme4* (Bates et al., 2015) and *miceadds* (Robitzsch & Grund, 2020) packages for linear regressions. The *MASS* (Venables & Ripley, 2002) package was used for ordinal logistic regressions.

Missing Data

A two-step process was used to handle missing data for the numerical skills of interest. Calculating numerical skills required summing scores from responses to individual trials on the TEMA-3 (e.g., 5 trials make up Item 8 on the TEMA-3).

Therefore, the first step of the missing data process involved examining missing data at the trial level (0-11.38% missing). Trial level numerical missing data was handled by examining other relevant items for the individual, including examining performance on trials that were conceptually similar as well as ceiling level for the individual on the TEMA-3 overall. For example, an individual who did not pass trials that required naming one- or two-digit numbers was not administered trials that involved identifying three-, or four-digit numbers; the three-, and four-digit number trials were considered above the individual's performance ceiling, so these items were scored as incorrect. Alternatively, if a child was consistently accurate in reading three- or four-digit number items, the two-digit number items that were not administered were scored as accurate. This process is in line with other empirical work on the development of numerical skills (e.g., Litkowski et al., 2020) and with standardized psychoeducational assessments (e.g., Ginsburg & Baroody, 2003; Woodcock et al., 2001). In addition, items on the TEMA-3 increase in difficulty as the test progresses and when administered traditionally the tasks ends when five consecutive items in a row are answered incorrectly (Ginsburg & Baroody, 2003). Further, the TEMA-3 has age specific start points and the basal is established when five consecutive items in a row are answered correctly. In cases of uncertainty (e.g., inconsistent responses), trial level data remained missing.

The second step of the missing data process was multiple imputation. Numerical skill sum scores with remaining missing data were imputed at the skill level, alongside key variables of interest and auxiliary variables. The percentage of missingness the entire test was between 0 and 5.4%, in the analytic frame (*N* = 205) for RQ1. An entire test was deemed missing if the whole test was not administered or the child stopped mid-test and a total score could not be calculated. In total 178 out of 341 records (52.2%) were incomplete; however, 136 of these records were incomplete due to the child having no A2 data, which was predominantly due to COVID-19. Importantly, only 42 out of 205 (20.48%) records were incomplete for participants that attended both assessments and therefore fell within the analytic frame for RQ1.

Further, the data were assumed to be missing at random. Specifically, children who were younger, from lower SES households (as measured by lower maternal education and eligibility for FRL), and with lower verbal, EF, and mathematics achievement were more likely to have missing data relative to their counterparts. A set of auxiliary variables, not part of the main analyses, were also included in the imputation model to reduce bias and increase power (Graham, 2009). The auxiliary variables included gender, timing of testing during the school year (i.e., fall or spring), number of months in school, and grade in school. Importantly, values were not imputed at A2 for individuals who did not attend the second wave of data collection sessions. I used multiple imputation to create and analyze 20 imputed datasets. Multiple imputation improves accuracy and statistical power relative to other missing data techniques. Incomplete variables were imputed under fully conditional specification, using the *mice* 3.14.0 package (van Buuren & Groothuis-Oudshoorn, 2011). The parameters of interest were estimated in imputed datasets separately and combined using Rubin's rules. Main analyses were also performed on complete cases and the pattern of findings was similar.

Descriptive Analyses

Descriptive statistics on non-imputed data are displayed in Tables 2 and 3. The means, standard deviations, and skew indicate ceiling effects for some numerical variables, particularly at A2. Table 4 presents the bivariate and partial correlations between each measure at assessments 1 and 2. EF, numerical, and general mathematics measures were positively and significantly correlated with one another. Various studies report a one factor solution for EF is a better fit than multiple distinct components in early childhood (Wiebe et al., 2008, 2011). Additionally, the moderate, positive Spearman correlations between the MEFS and HTKS ($r_{A1} = .40$, $r_{A2} = .40$) and between KBIT-2 VK and BNT-2 ($r_{A1} = .74$, $r_{A2} = .70$), at both time points, indicated it was appropriate to create composites for EF and Vocabulary, respectively. Composite scores were created by averaging the *z*-scores of individual measures at each assessment.

I also examined ANOVAs to identify additional covariates for substantive aims. I determined semester of data collection (i.e., fall or spring; *p*s ≥ .084) and cohort (*p*s ≥ .07) were not significantly related to key variables of interest. Gender differences were not found for most key variables (*p*s ≥ .062). Gender effects were present for numerical literacy at A2, but when other covariates (i.e., age, maternal education, vocabulary composite, and earlier numerical literacy skills) were accounted for gender ($p = .236$) was not uniquely related to numerical literacy. Further, grade effects were also found on some mathematics and EF variables, but not others (*p*s = 0.0001 - .9); however, due to the high correlation between age and grade ($r_s = .94$), I opted to include only age in subsequent analyses to prevent multicollinearity. Therefore, none of these variables were included as covariates in the main analyses.

Additionally, I examined nesting by school program and individual schools. Preschoolers at A1 were recruited from programs serving children that meet one or more of the following criteria: eligible for FRL, English Language Learner, homeless, has an

Individualized Education Plan or Interagency Intervention Plan, is at potential risk for learning difficulties, identified as at risk through the local public school system, or lives in the school's attendance zone, which is a zone serving low-SES households. Given these criteria, it was important to examine whether this subsample of participants differs from those of the same age that likely did not attend programs serving *only* families meeting these criteria. Therefore, I examined the kindergarteners seen at A2 (i.e., those that attended a preschool serving families meeting the above criteria; $n = 20$) to kindergarteners at A1 (i.e., those that likely *did not* attend preschool meeting the criteria listed above; $n = 129$) to quantify group differences and determine what needed to be further statistically accounted for in subsequent analyses. Through a series of *t*-tests I found significant differences in age ($t = 3.1$, $p = .005$) and performance on the count on task at A1 ($t = 2.23$, $p = .032$), indicating the importance of including age as a covariate and accounting for school effects in substantive analyses involving count on at A1.

To assess clustering at the school level, I computed intraclass correlations (*ICC*s) for all dependent variables (e.g., mathematics and EF). I found some evidence for between-school level variability (*ICC*s: .02-.08). Specifically, ICCs for the following variables indicated nesting ($ICC > .05$): A1 non-rote counting, A1 verbal counting, A1 numerical literacy, A1 count on, and A2 numerical problem solving. Taking both preschool effects and *ICC*s into account, I clustered the standard errors at the school level in subsequent analyses for the variables listed above to reduce bias. Multilevel models were underpowered for this sample size and as such unlikely to converge, making standard error clustering a viable alternative.

Concurrent Relations between Executive Function and Numeracy Skills

To examine the concurrent EF and numerical relations (RQ1a), I conducted a series of OLS regressions predicting the following numerical skills: nonsymbolic magnitude comparison, verbal counting, numeral literacy, count on, and non-rote counting, numerical problem solving, from concurrent EF composite scores at A1 while controlling for age, maternal education, and vocabulary composite score. EF composite scores were positively associated with all six numerical skills - nonsymbolic magnitude comparison, verbal counting, numeral literacy, count on, and non-rote counting, numerical problem solving - above and beyond covariates at A1 (Table 5).

Paralleling the initial regression analyses, I entered MEFS and HTKS scores into the separate models, rather than as an EF composite score. Findings for HTKS replicated EF composite scores, such that HTKS was positively associated with each numerical skill after accounting for covariates (ps < 0.001). Conversely, MEFS scores were positively associated with five of the six numerical skills examined (ps $\leq .015$). MEFS was not significantly associated with verbal counting ($p = .08$).

Numerical Problem Solving Item Analysis

I also conducted planned comparisons on select items within the numerical problem solving skill sum score. Specifically, I conducted additional analyses on the two trials within Item of the TEMA-3 and the one additional supplemental item I administered. For these three trials children are prompted to put as many tokens underneath an opaque paperboard mat as the experimenter has under their mat. Some trials in Item 8 involve adding items (addition) and other involve taking items away (subtraction). Here, I was interested in whether EF predicted the mathematical operation children used on trials following a switch from one mathematical operation to another (i.e., addition to subtraction). Therefore, I focused on the three trials where children were needed to engage in switching from addition to subtraction to successfully answer the problem. I hypothesized EF would be positively related to successful operation switching, such that children with lower EF would be more likely to perseverate on the previous operation (e.g., addition) rather than switch to the new operation (e.g., subtraction) and children with higher EF would switch operations accurately more often, on average.

I calculated the proportion of trials children used the correct operation on a switch trial out of the total number of trials where it was possible to determine which mathematical operation was used by the child. The total number of switch trials where it was possible to determine the mathematical operation used by the child was not always three because there were some cases where it was not possible to determine the child's mathematical operation based on their response (e.g., if the child responded to 5-4 = ___ with 5 then there was not enough evidence to determine whether the child engaged in addition or subtraction). This is hereafter referred to as the proportion of accurate operational switches and was the dependent variable of interest. Two children were excluded from this analysis. The first child had missing data on all three trials of interest and because data was imputed at the numerical problem solving skill level, I did not have trial level data to examine. Another child was excluded because they never engaged in subtraction; therefore, lack of understanding of subtraction and switching were confounded. All other children demonstrated some evidence of engaging in subtraction and were retained in the analytic frame. I retained 203 children from the 205 children within the analytic frame.

Proportion of accurate switches were regressed on child age, maternal education, vocabulary composite, and EF composite. Consistent with my hypothesis, EF composite scores were positively associated with proportion of accurate switches, above and beyond covariates (Table 6). I also examined MEFS and HTKS scores in separate models. I found HTKS ($B = 0.002$, $SE = 0.001$, $p = .015$), but not MEFS ($B = 0.001$, $SE = 0.001$, $p = 0.52$), scores were associated with proportion of accurate switches. These findings were no longer significant when I focused only on trials where children erred (ps $\geq .055$).

Predictive Relations between EF and Numeracy Skills

To examine predictive relations between EF and numerical skills (and vice versa; RQ1b), OLS regressions predicting A2 numerical skills were conducted. A1 EF was examined as a predictor while controlling for A1 performance on the numerical skill of interest, age, maternal education, and vocabulary composite. OLS regressions revealed children's EF skills at A1 were significantly associated with later performance on numerical literacy, count on, non-rote counting, and nonsymbolic magnitude comparison, but not problem-solving skills or verbal counting, above and beyond the covariates (Table 7).

Paralleling the initial regression analyses, I entered MEFS and HTKS scores into the separate models, rather than as an EF composite score. Regressions revealed earlier HTKS scores were associated with later performance on each of the six numerical skills examined (ps $\leq .046$), whereas no significant relations were found for MEFS scores and numerical skills of interest (ps $\geq .189$).

Due to ceiling effects on select numerical skills at A2, I conducted subsequent analyses to examine whether the findings for numerical literacy, count on, verbal counting, non-rote counting, and numerical problem solving were robust to various decisions made during the analysis process. First, I repeated these analyses dropping children who had no room for growth (i.e., got all items correct at A1) on the skill of interest. For example, in the analysis predicting A2 numerical literacy I dropped 32 participants who got all 21 items that comprised this skill correct at A1. The pattern of findings changed, such that EF skills at A1 only significantly predicted later performance on non-rote counting (*B* = 0.37, *SE* = .15, *p* = .034*)*, after accounting for covariates.

Next, I examined the score distributions at A2 for verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving, to assign each participant to a skill level subgroup to be used for group comparisons in a series of logistic regressions (Table 8). The data distributions for each of these numerical scores were used to determine the number of levels that were appropriate. All participants were familiar with *verbal counting*, up to at least 11. Most children correctly counted aloud up to 110 (148; 72.9%), which was the ceiling on this task. Therefore, I assigned the 72.9% of children who counted to 110 to the high group (1), and the 27.1% of children who counted between 11 and 109 to the low group (0). Next, of the 203 children who completed numerical problem solving items, 29.3% of children scored at ceiling and were assigned to the high achieving subgroup, 54.2% of children scored between 9 and 11 and were assigned to the medium group, and 16.5% of children correctly solved between 0 and 8 and were assigned to the low subgroup. For non-rote counting, of the 203 children who completed these items, 53.2% of children scored at ceiling and were assigned to the high achieving subgroup, 29.8% of children scored a 5 and were assigned to the medium group, and 17% of children correctly solved between 0 and 4 items and were assigned to the low subgroup. Next, of the 203 children who completed count on items, 56.6% of children scored at ceiling and were assigned to the high achieving subgroup, 29.2% of children scored between 6 and 7 and were assigned to the medium group, and 14.2% of children correctly solved between 0 and 5 and were assigned to the low subgroup. Lastly, of the 203 children who completed numerical literacy items, 46.8% of children scored at 20 or 21 (ceiling) and were assigned to the high achieving subgroup, 34.2% of children scored between 13 and 19 and were assigned to the medium group, and 19% of children correctly solved between 1 and 12 and were assigned to the low subgroup (Table 8).

These numerical skill subgroups were then used as dependent variables in a series of binary and ordinal logistic regressions. First, I conducted a binary logistic regression and regressed A2 Verbal Counting (Low (0) or High (1)) subgroup on age, maternal education, vocabulary composite, A1 verbal counting skills, and A1 EF composite. I found that each one-unit increase in EF Composite score was associated with 2.23 times greater odds of verbally counting to 110 ($p = 0.036$), after accounting for covariates.

Next, I examined a series of ordinal logistic regressions with numerical problem solving, non-rote counting, count on, and numerical literacy subgroups at A2 as dependent variables. First, I regressed non-rote counting subgroup performance (i.e., low, medium, high) on age, maternal education, vocabulary composite, A1 non-rote counting skills, and A1 EF composite. I found that for each one unit increase in the EF composite score the odds of higher non-rote counting subgroup achievement is multiplied 1.99 times holding constant all other variables ($p = 0.025$). Parallel models were run for numerical problem solving, count on, and numerical literacy subgroups and no significant associations with A1 EF composite scores were found.

Next, I conducted OLS regressions predicting A2 EF Composite with A1 numerical skills while controlling for earlier EF skills and the same set of demographic and verbal covariates. This series of OLS regressions revealed numerical problem solving and nonsymbolic magnitude comparison were significantly associated later performance on EF skills, above and beyond child age, maternal education, vocabulary composite, and A1 EF Composite (Table 9).

I found a similar pattern of results when HTKS scores were examined in separate models, rather than as an EF composite score. When MEFS scores were examined separately, OLS regressions revealed numerical problem solving, nonsymbolic magnitude comparison, numerical literacy, and count on, but not non-rote counting or verbal counting, were associated with later MEFS, above and beyond covariates.

Moderation Analyses

To examine whether these associations varied by mathematics performance (RQ1c), the concurrent and predictive models described in the prior sections were built upon to determine if these relations were moderated by mathematics achievement subgroup (i.e., children with typical versus persistently low mathematics achievement). I examined TEMA-3 and WJ percentiles to identify children with *persistent* low mathematics performance at both time points (≤ 25th percentile on either TEMA-3 or WJ at one time point and ≤30th percentile at the other point).[1] The ≤30th percentile threshold was important because many children who were lower performers at one time point might be right on the cusp of meeting the criteria at the other assessment. These children

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[1] Norms for the TEMA-3 are provided from 3 years, 0 months through 8 years, 11 months. To determine percentiles for children older than this threshold, raw TEMA-3 scores were regressed on age and predicted scores were used (Assessment 1: *n* = 1, Assessment 2: *n* = 23).

would be excluded if a strict ≤ 25th percentile cutoff was employed. I identified an additional 5 children when I used the above criteria, compared to requiring ≤ 25th percentile at both time points.

Previous work has examined groups of children performing between the 11th and 25th percentiles and below the 10th percentile separately and reported differences in mathematics and related non-mathematics skills by group (Murphy et al., 2007). However, I did not have enough children below the 10th ($n = 13$) and 11-25th ($n = 28$) percentiles to parse out differences between these groups. Children at the 26th percentile or higher comprised the typical mathematics achievement group. This resulted in 41 children with persistently low mathematics achievement and 161 children with typical mathematics achievement. Three children were dropped because missingness in raw data made it impossible to sort them into either of the two categories.

I conducted OLS regression analyses with age, maternal education, A1 vocabulary composite, A1 EF Composite, mathematics achievement subgroup (0: typical mathematics achievement, 1: persistently low mathematics achievement), and EF Composite × mathematics achievement subgroup. I found the effect of A1 EF Composite on A1 count on was moderated by mathematics achievement subgroup. The interaction effect ($B = -0.68$, $SE = .24$, $p = .007$) indicated that for children with persistently low mathematics achievement, a one-unit difference in EF was generally associated with a 0.68-point lower difference in count on skills than children with typical mathematics achievement, for the same change in EF (Figure 2). In other words, there was a stronger concurrent relation between EF and count on skills for children with typical ($r = .52$, $p < .001$) compared to persistently low mathematics achievement ($r = .39$, $p = .010$). No other significant interactions were found for concurrent EF and numerical relations at A1. Next, I examined longitudinal relations wherein I examined whether earlier EF was associated with later numerical skills. In these regressions I also controlled for earlier performance on the numerical skill of interest. There were no significant moderation effects between EF Composite and mathematics achievement subgroups in these longitudinal analyses. Finally, I examined longitudinal relations in the opposite direction, specifically whether earlier numerical skills were associated with later EF skills after controlling for earlier EF skills and previously described covariates. In these models I examined interactions between each numerical skill and mathematics achievement subgroups, such as A1 numerical problem solving × mathematics achievement. No significant moderation effects were found.

Discussion

The current study leveraged a prospective, longitudinal study of preschool to third graders in the United States to investigate the relations between EF and specific numerical skills. First, I examined the concurrent associations between EF and each of the following numerical skills: nonsymbolic magnitude comparison, verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving. Second, I examined the longitudinal associations approximately one year later between EF and numerical skills and vice versa. Third, I identified children with typical mathematics achievement and persistently low mathematics achievement and examined whether the concurrent and predictive relations between EF and numerical skills vary in for these mathematics achievement subgroups. Collectively, the current study contributes to the existing literature on the relation between EF and mathematics skills in early childhood.

Concurrent Relations

Given robust associations between EF and mathematics achievement in early childhood and the importance for early numerical skills for later mathematical learning, it is important to identify which specific numerical skills are related to EF. Consistent with my hypothesis, EF was positively associated the following numerical skills - nonsymbolic magnitude comparison, count on, non-rote counting, and numerical problem solving. Similarly, MEFS and HTKS were concurrently associated with each of these four numerical skills when examined in separate models. These findings align with work showing a positive association between EF and specific numerical skills, such as number line estimation, addition, story problems, number comparison, and cardinal knowledge (e.g., Chan et al., 2022; Chan & Scalise, 2022; Purpura et al., 2017; Purpura & Ganley, 2014). Many of these prior studies have focused predominantly on the preschool or kindergarten years, therefore my work extended these findings into the early elementary grades.

Contrary to my hypothesis that EF would *not* be significantly related to verbal counting or numerical literacy, I found evidence EF was positively and significantly related to these two numerical skills as well, even after controlling for vocabulary. Prior work has found verbal counting was positively related to inhibitory control, but not cognitive flexibility or working memory, in preschoolers (Purpura et al., 2017) and kindergarteners (Purpura & Ganley, 2014). Therefore, my findings suggest that even though my sample was older and likely had more opportunities to practice reciting the verbal counting sequence, EF was still a significant predictor. I replicated prior work that verbal counting was positively related to inhibitory control (as measured by HTKS) but not cognitive flexibility (as measured by MEFS). Inhibitory control might support children's ability to count in sequence by resisting the urge to skip a number or repeat numbers, whereas cognitive flexibility skills might not be as supportive of verbal counting because shifting or switching between rules or approaches is not needed during this mathematics task.

Further, prior work on the links between reading numerals and EF skills is mixed with some studies finding a significant positive association (Chan & Scalise, 2022) and others reporting a null effect (Purpura et al., 2017; Purpura & Ganley, 2014). I found numerical literacy was concurrently associated with EF skills when operationalized as an EF composite or examining MEFS and HTKS separately. There a few reasons my findings might differ from prior null results. My numerical literacy task included two-three-, and four-digit numbers, whereas prior work included either one-digit or one-and two-digit numbers. In addition, I included both reading and writing numbers, due in part to the older age range of some of the participants in this study. Thus, it is possible that because I included more challenging items in general or items that involved writing some of these items were still effortful for the children in my sample and therefore invoked their EF skills. For instance, when asked to read the digit "13" children might need to use their EF skills to inhibit reading each digit individually as "1" and then "3" and shift their thinking to read the combination of the two digits as "13." This might be particularly challenging for younger children who generally have less experience engaging with two-digit numbers. Thus, EF skills might still be important for accurately reading and writing numbers beyond the preschool and kindergarten years.

Numerical Problem Solving Item Analysis

EF skills were positively associated with proportion of accurate switches during numerical problem solving trials at A1, above and beyond covariates. This finding was significant in separate models using the EF composite and HTKS but was not significant for MEFS. This finding provides support for the hypothesis that EF plays a role in doing mathematical tasks (e.g., Zaitchik et al., 2016). Specifically, I limited children in this analysis to those who showed some evidence of understanding subtraction. Therefore, their lack of switching from addition to subtraction was likely not due to being unfamiliar with subtraction but instead seemed to be at least partially explained by perseverating on addition rather than switching to subtraction when appropriate. Children with lower EF skills might struggle with executing their mathematical knowledge partially due to the EF demands of the task (reviewed by: Blair et al., 2008).

Given that many children are introduced to addition first and the first two trials within the TEMA-3 item of interest involved addition, the children might have needed to use their EF skills to support switching mathematical operations. The direction of EF effects was similar when I isolated my analyses to trials where children made errors, but the EF variables did not reach significance. In fact, the effect of HTKS was significant before applying the Benjamini–Hochberg ($p = .03$), but this finding did not hold after the adjustment significant (ps $\geq .055$). Overall, these findings indicate that EF is associated with accuracy on numerical problem solving and switching between mathematical operations when completing a sequence of numerical problem-solving items.

Predictive Relations

Earlier EF skills were associated with later performance on four of the six numerical skills examined - nonsymbolic magnitude comparison, numerical literacy, count on, and non-rote counting - after controlling for prior numerical skills and other covariates. When HTKS and MEFS scores were examined in separate models, HTKS, but not MEFS was associated all six numerical skills. Further, two of the six earlier numerical skills – nonsymbolic magnitude comparison and numerical problem solving – were associated with later EF above and beyond earlier EF and additional covariates. I found a similar pattern of findings when HTKS was examined separately; however, when MEFS scores were examined separately four of six numerical skills - nonsymbolic magnitude comparison, numerical literacy, count on, and numerical problem solving - were associated with later MEFS scores. Overall, these findings highlight the robustness of the association between EF and these early numerical skills because many of these positive associations remained even after controlling for earlier numerical and EF skills.

I found a bidirectional relation between EF and nonsymbolic magnitude comparison skills such that earlier EF was associated with later nonsymbolic magnitude comparison and earlier nonsymbolic magnitude comparison was associated with later EF. Interpretation of these findings is complex given the disagreement in the field on whether nonsymbolic magnitude comparison tasks are capturing ANS acuity and/or other factors, such as inhibitory control or attention to numerosity. On the measure of nonsymbolic magnitude comparison I used (Panamath), the trials were either controlled based on average dot size or surface area. It has been proposed that trials with incongruency between the visual stimuli and the number of dots require a participant to focus on numerosity while ignoring another non-numerical visual parameter, and that this conflict might invoke inhibitory control skills. However, some, but not all, work supports this hypothesis. Wilkey et al. (2017) used fMRI to examine neural activity during a nonsymbolic magnitude comparison task wherein some conditions were average dot size controlled and others were area controlled and found ratio-dependent neural activity did not differ by congruency condition. However, they did report congruent and incongruent trials recruit different neural mechanisms. In a behavioral study with a sample of first graders, Wilkey et al. (2022) found evidence for similar increases in accuracy rate on congruent and incongruent trials over the course of the year. Further, inhibitory control skills, as measured by performance on Head Toes Knees Shoulders, did not moderate growth on congruent or incongruent trials. Overall, these findings suggest improvement in number discrimination might be due to sharpening numerical representations.

Odic (2018) found no significant differences between performance on congruent and incongruent trials nor did they find a relation between age and the difference between congruent and incongruent trials. Thus, these findings, in combination with previous work (e.g., Gilmore et al., 2013) indicate the difference in performance between congruent and incongruent trials *might* be partially due to inhibitory control, but that measures of nonsymbolic magnitude comparison are likely also measuring elements of ANS acuity. Further, given that this bidirectional relation was found for the EF composite and HTKS, but not MEFS, it might be that that this effect is capturing elements of the relation between EF and ANS acuity as well as *some* inhibitory control demands of the task. Thus, future work is needed to continue to explore the mechanisms underlying performance on tasks like the Panamath.

Next, I found earlier EF was not associated with later verbal counting skills, after accounting for covariates. This could potentially be because the increase in practice with reciting the counting sequence over the course of the year contributed to high performance on this task for most children. Consistent with the effort hypothesis, the effect of EF on growth in verbal counting might decrease over time. By A2, 72.9% of the sample could verbally count to 110, which was the ceiling for this task. When I dropped children from the analysis that had ceiling effects at A1, there was still no significant effect of EF; however, the sample size also decreased. Thus, I also conducted logistic regressions to examine this relation and I found a positive association between EF and being able to count to 110. Thus, these findings indicate I might need to assess this relation among a sample of children who were younger, as they might have less verbal counting experience, to get a better understanding of when and whether there are predictive relations between EF and verbal counting.

Contrary to my hypothesis, earlier EF was predictive of later numerical literacy in the full analytic frame. However, when children who were at ceiling at A1 were dropped the relation between numerical literacy and the EF composite was no longer significant. The latter varies slightly from prior work indicating EF was a stronger predictor of mathematics for children with low, rather than moderate or high mathematics skills (Dong et al., 2020). Based on the findings from Dong et al. (2020) one might hypothesize a stronger relation between EF and numerical literacy after removing the highest performers. Importantly, Dong et al. (2020) focused on mathematics skills broadly and this might not apply to numerical literacy skills. Thus, more work is needed to further examine the relation between EF and children with high, medium, and low numerical literacy skills across a wide achievement range. It might also be that this discrepancy in findings in this study is partially due to the reduction in power due to the decreased sample size after dropping those who had no room for growth at A1 compared to the entire analytic frame. Alternatively, it could be that removing the highest performers on numerical literacy also excluded higher EF performers and the findings changed due to the range restriction.

Further, the significant predictive relation between earlier EF and later numerical literacy in the full sample slightly varies from some similar work by Chan and Scalise (2022). They found a longitudinal relation between earlier EF and later identification of numbers, but the effect was not significant after controlling for earlier number identification skills. However, this numerical measure did not include any elements of writing numbers so this difference might explain the discrepancy in these findings. Taken together, my findings indicate children with greater EF skills at the first time point might have been able to rely on these skills to support their learning of and integration of place values and individual numerals when reading and writing numbers. In other words, EF helped children improve in numerical literacy performance over time.

Next, I found earlier EF was associated with later count on skills, which was consistent with my hypothesis. This might suggest EF skills support children's ability to identify the next number in the counting sequence, beyond stating a memorized verbal counting sequence in order. This level of engagement with numerical sequences might require mentally manipulating the counting sequence (working memory) and resisting the impulse to recite the verbal count sequence beginning with 1 (inhibitory control). However, similar to the findings for numerical literacy, the relation between earlier EF and later count on skills was no longer significant in analyses limited to participants that still had room for growth in A1 (i.e., dropping children who got all eight items that comprise this numerical skill correct at A1).

As hypothesized, earlier EF was associated with later non-rote counting. This pattern of findings was robust, such that EF was still significantly associated with non-rote counting after I dropped children who were at ceiling at A1 and when I assigned children to low, medium, and high groups for non-rote counting and examined the effect of EF through ordinal logistic regressions. This aligns with prior work indicating EF plays an important role in counting sets (e.g., Scalise & Ramani, 2021). For instance, inhibitory control skills might support children's ability to count each item in the set once and only once (e.g., one-to-one correspondence) and to resist the urge to recount an item that has already been counted. Additionally, while counting backwards children might rely on working memory to support mental manipulation of the forward courting sequence and inhibitory control to resist the automatic response to count forward rather than backwards. Prior work by Chan and Scalise (2022) did not find a significant relation between earlier EF and later set counting skills after accounting for earlier counting skills in preschoolers or kindergarteners. Both measures used in the study by Chan and Scalise involved showing children a page with a set of items and asking them to count the set. These tasks were similar to the dot enumeration items I used, wherein children were shown a set of dots and instructed to count them. Importantly, I also included counting items where the child was given a bunch of tokens and asked to count out an exact amount. It is possible that these two forms of set counting have differing relations with EF or that the addition of backward counting drove these effects. I am unable to meaningfully tease apart these effects here due to the low number of items children completed for dot enumeration, providing an exact quantity, and backward counting, but this is a worthy area of future study.

Further, based on prior research and theoretical rationale I hypothesized earlier EF would be associated with later numerical problem solving skills. This is because some items within this skill set were thought to place demands on working memory skills by requiring the child to work with hidden manipulatives and engage with story problems which might be more abstract (e.g., Purpura et al., 2017). Thus, EF skills could possibly limit children's engagement in some kinds of higher-level mathematical numerical problem solving due to the inherent EF demands of the task. Thus, children with higher EF skills at A1 would have had greater time available to engage in higher-level numerical problem solving and thereby EF skills were hypothesized to predict numerical problem solving skills over time. This hypothesis has been supported in some concurrent and longitudinal in prior work (Lee et al., 2018; Lubin et al., 2013; Ramani & Scalise, 2020). However, the predictive relations between EF and numerical problem solving were not found here when EF was operationalized as a composite for the full analytic frame, after dropping children that were at ceiling at A1, or when MEFS scores were examined separately. Importantly, this hypothesis was supported when HTKS was examined separately, indicating it might be inhibitory control that is driving this association, which aligns with work from Purpura et al. (2017). However, it is also likely that working memory is playing a role (e.g., Bull et al., 2008; Peng et al., 2016), but this cannot be tested here given the lack of a working memory specific measure in this study.

Interestingly, I found a significant, positive relation between earlier numerical problem solving skills and later EF; thus, my hypothesis was partially supported. This is consistent with prior theoretical work proposing that doing and learning mathematics is a context where children are practicing and thereby growing their EF skills. Items within this numerical problem solving skill were likely novel to the children and therefore required some degree of in the moment engagement in mathematical problem solving. This differs from being prompted to verbally count, which might be novel for some children but is also likely familiar content to many children. Thus, stating the number sequence in order might place greater demands upon recalling previously learned information, whereas numerical problem solving items might require recalling and manipulating previously learned information and judging if it is relevant to a new mathematical problem. This aligns with prior work showing word problems (a form of numerical problem solving) and learning new strategies and procedures are associated with EF more strongly than rote calculations (Mononen & Niemivirta, 2023). Thus, mathematical content might vary in its demands on EF skills and opportunities for children to practice EF skills while doing mathematics. Overall, the predictive relation between earlier numerical problem solving and later EF skills indicates engagement in numerical problem solving could be a promising avenue for supporting both mathematics and EF skills. Future experimental work is needed to examine whether this is the case.

Finally, the significant predictive relations between earlier HTKS, but not MEFS, and all six numerical skills can be interpreted in multiple ways. It might be that inhibitory control is a broad predictor of later performance on various numerical skills, which aligns with some prior work by Purpura et al. (2017). Alternatively, it might be that working memory demands are greater in HTKS than the MEFS and that the HTKS scores are capturing working memory skills to a greater degree than MEFS scores. Specifically, the MEFS has several prompts built into the task to remind the child of the rules, which *reduces* the working memory demands of the task. However, in HTKS reminders and feedback stop after completing the practice trials. Unfortunately, without a working memory task I cannot parse out these differing interpretations in this study.

Moderation Analyses

I found the effect of A1 EF Composite on A1 count on was moderated by mathematics achievement subgroup. Specifically, the effect of A1 EF Composite on A1 count on was stronger for children with typical mathematics achievement compared to children with persistently low mathematics achievement, holding all other variables constant. Notably, the greatest difference in the relation between EF and count on skills between children with persistently low and typical mathematics achievement was for children with higher EF skills, rather than lower EF skills. These findings diverge from my exploratory hypothesis that EF would be more strongly related to numerical skills for children with persistently low mathematics achievement due to greater relative effortfulness of numerical tasks for lower mathematics performers. There a few possible reasons count on skills might be less strongly related to EF for lower mathematics performers. It is possible that lower mathematics performers are using alternative skills in addition to or instead of EF to support performance on count on items. For instance, it might be that they are recruiting their verbal skills or relying on other numerical skills to a greater degree or differently than their typically achieving peers. Further, higher EF skills might help compensate for lower mathematical knowledge in children with persistently low mathematics achievement to some degree, but not to the same degree as children with typical mathematics achievement. Specifically, it is possible that EF skills are less helpful for children with persistently low mathematics achievement because they do not have the necessary foundational mathematical knowledge to complete the count on task. In other words, EF skills might contribute to how children do mathematics, such as being able to select and shift between strategies or demonstrate knowledge, but EF might not be enough without a certain amount of mathematical knowledge to understand the concept. Importantly, future work is needed to tease out some of these possibilities.

The moderation effects were not significant in any other models examining interactions between EF and mathematics achievement subgroups and numerical skills and mathematics achievement subgroups, indicating the relations between EF and numerical skills - nonsymbolic magnitude comparison, verbal counting, numerical literacy, non-rote counting, and numerical problem solving - did not vary for children with persistently low mathematics achievement and typical mathematics achievement. Thus, future work is needed to determine if these results replicate and if so, to understand the mechanisms contributing to why this moderation effect was specific to EF and count on.

Limitations and Future Directions

Several limitations must be considered when interpreting these findings. First, my moderation analyses were underpowered, which might have limited my ability to identify a statistically significant effect. This is a common challenge when studying children with persistently low mathematics achievement or mathematics learning difficulties due to the lower size of this population overall and lack or representation of these groups in WEIRD psychological datasets. Future work should consider pooling participants from various samples or leveraging large publicly available datasets (e.g., ECLS-2011) to increase power. Second, these data are longitudinal, but causality cannot be inferred. I controlled for important child (e.g., vocabulary and age) and family characteristics (e.g., maternal education), but it is possible that an unmeasured characteristic is driving the relations between EF and numerical skills I found in this study.

Third, there were measurement limitations for both EF and numerical skills. Regarding EF, I lacked a direct assessment of working memory skills. Therefore, the EF composite did not equally represent inhibitory control, working memory, and cognitive flexibility. This is important to note because it is possible that each EF component contributes to numerical development differently (e.g., Purpura et al., 2017) and I was unable to capture the full scope of this without a direct assessment of working memory. Regarding numerical skills, there were ceiling effects on various numerical skills, particularly at A2. Thus, it is possible that some of the numerical items were not challenging enough to be effortful for some children. This could mean that if I had more challenging items for a particular skill then EF demands might have been greater. Some of my follow-up analyses aimed to examine whether this was impacting my pattern of findings (e.g., drop those who got all items correct at A1 from predictive analyses, ordinal regressions); however, future work with a more extensive numerical testing battery is needed.

Fourth, most of the items used to create numerical skill scores were correlated with one another, as indicated by their good Cronbach’s alphas ($\geq .70$). Non-rote counting skills had an acceptable A1 Cronbach’s alpha (.71); however, A2 did not meet an acceptable level of intercorrelations based on Cronbach’s alphas (.54). This lower alpha might be due to only having six trials that made up this numerical skill in combination with many children increasing in their performance on these items from A1 to A2. Future work is needed including more items and items of greater difficulty level to measure non-rote counting skills. Nevertheless, non-rote counting was one of the most robust tasks for showing relations to EF, such that the longitudinal relation remained significant across various analytic decisions.

Fifth, children in this sample were recruited from the same metropolitan area and are not representative of demographics across the state or country. Despite the limited geographical region, this sample was diverse in terms of race, ethnicity, SES, and multi-language exposure. Sixth, I would be remiss if I did not acknowledge the differences in age, maternal education, EF, mathematics, and vocabulary between children in the analytic frame (*n* = 205) compared to those lost to attrition (*n* = 146). Importantly, the only difference that remained after accounting for child age and maternal education was lower performance on mathematics measures (i.e., WJ-Applied Problems and TEMA-3) for children lost to attrition. Most of the children lost to attrition were due to the study stopping data collection for everyone in 2020 due to the COVID-19 pandemic, rather than the children and families opting out. Upon further inspection, analyses revealed it was those lost to attrition due to COVID-19, that had lower A1 mathematics performance (WJ-Applied Problems), whereas no mathematics differences were found in children lost to attrition for other reasons, after accounting for age and maternal education covariates.

The differences in mathematics performance found in the attrition analyses for children lost to attrition due to COVID-19 was somewhat unexpected. One possibility is that the families who signed up for the study in September compared to January when the study team members came into the child's school might have varied from one another. For instance, families that were already familiar with or interested in research might have been interested immediately whereas others might have wanted more time to consider.

However, testing order was not only based on when children's families enrolled them in the study. Testing order was based on children's age, such that older children were tested first, within grade, to maximize the similarity in age across the sample and to decrease biasing enrollment and time scheduled. Thus, this is merely speculative and might be one of several factors contributing to these group differences among participants.

Further, it is possible some of the children lost to attrition due to COVID-19 would have also been lost to attrition regardless. For instance, over 100 children were scheduled come in for A2 in the three months after the COVID-19 lockdown began in the United States. It is possible the group of children lost to attrition due to COVID included some children who would have been lost to attrition for other reasons, such as moving from one school to another, which is a risk factor for lower math achievement (Mehana & Reynolds, 2004). Given that there were some differences between those retained in the sample versus lost to attrition it is important to attempt to replicate these findings in the future with a different sample.

Conclusions

Overall, I found evidence for concurrent associations between EF and all six numerical skills I measured; however, differential predictive associations emerged between EF and numerical skills. My results indicated most EF and numerical skills I measured were unidirectionally related to one another over time (EF → numerical skills). Importantly, for nonsymbolic magnitude comparison skills and EF I found a bidirectional relation. Further, in several predictive analyses HTKS, but not MEFS scores, were related to numerical skills, which aligns with some prior work highlighting the contribution of inhibitory control to concurrent performance on various mathematics and literacy skills

(Purpura, et al., 2017). However, this might also reflect some of the working memory elements of the HTKS task. Further in several other studies utilizing more general mathematics achievement measures, there were associations with the MEFS (Carlson, 2021; Ernst et al., 2022; Prager et al., 2016), and training on the MEFS transferred to improvements in numerical tasks among preschoolers (Prager et al., 2023). Lastly, I found partial support for my effort-based hypotheses on the relation between EF and numerical skills. For instance, in some cases I found a significant relation between earlier EF and later numerical skills (e.g., count on) that I hypothesized would be correlated from one time point to another. Alternatively, I hypothesized some bidirectional relations that did not emerge (e.g., EF and numerical problem solving). This is not to say effort does not play a role in the relation between EF and mathematics, because prior work indicates it likely does (e.g., Mazzocco & Kover, 2007). Instead, there are likely other factors, such as mathematics achievement in other domains and relative effortfulness of the task on an individual level, that are also contributing to the relations among these skills. Thus, future work is needed to understand the other contributing factors.

These findings contribute to our understanding of the link between EF and foundational numerical skills, which can be used to inform instructional efforts to support mathematical learning. If educators and curricula developers know which numerical skills are calling upon stronger EF demands they can make appropriate instructional modifications for children struggling to master that skill, whereas if the numerical skill is unrelated to EF an alternate instructional modification might be more appropriate. In addition, the weak evidence for Number → EF relations over time indicates the importance of not relying only on numerical activities to support EF development.

Rather, intentional practices to integrate EF into the classroom during mathematics activities as well as in other classroom activities might be a more effective approach. In sum, this study provides a nuanced examination of the relations between EF and numerical skills in early childhood and highlights the contribution of EF to the development of various foundational numerical skills.

Chapter 3: Research Question 2

Rationale

In Research Question 1 I examined the concurrent and predictive relations between EF and numerical skills. I found EF was concurrently related to all six numerical skills I measured – nonsymbolic magnitude comparison, non-rote counting, verbal counting, count on, numerical problem solving, and numerical literacy. Longitudinal analyses indicated unidirectional relations for many skills, such that earlier EF was associated with later numerical skills for three of the six skills measured, after controlling for prior numerical skills. Unidirectional relations in the opposite direction were found for numerical problem solving and EF, such that earlier numerical problem-solving skills predicted later EF, after controlling for prior EF skills. Bidirectional relations were found for nonsymbolic magnitude comparison skills. Next, I examined subgroups of typical mathematics achievement and low mathematics achievement. For most numerical skills, the relations between concurrent and predictive EF and numerical skills were similar for children with persistently low and typical mathematics achievement. One notable exception was the interaction effect of mathematics achievement subgroup and EF skills on concurrent count on skills at A1.

Research Question 2 extended these findings in multiple ways. First, I broadened the scope by examining the contribution of numerical and non-numerical skills (e.g., EF and vocabulary) to mathematics achievement. Second, I expanded the sample to include children with Turners syndrome, who show specific impairments in numerical processing. This enabled me to examine whether the relation between EF and numerical skills and mathematics achievement varied for subgroups that differed in presence and "source" of risk for mathematics learning difficulties. To do this, I examined three groups of children including: biological risk (i.e., Turner syndrome), socioenvironmental risk (i.e., Low SES households), and No Known Risk for mathematics learning difficulties. Importantly, I acknowledge it is unlikely that a risk factor with a known biological basis is only biological or that a known socioenvironmental risk factor is only socioenvironmental. Biological and socioenvironmental interactions shape one's experience. For instance, the experience of growing up in a low SES household is an socioenvironmental risk factor, but this experience might also contribute to developing biological systems (Miguel et al., 2023). Thus, I am using the terms biological and socioenvironmental to simply refer to the one known "source" of the risk factor.

The *socioenvironmental risk factor* for mathematics learning difficulties I examined was growing up in a low SES household. It is well-documented that children in low SES households have lower EF, mathematics, and verbal skills, on average, compared to their higher-income peers (Jordan & Levine, 2009). These findings are robust, such that this has been replicated with various conceptualizations of SES, such as maternal education, family income, and eligibility for free or reduced price lunch (FRL; e.g., Davis-Kean, 2005; Reardon & Portilla, 2016; Starkey & Klein, 2008). The reasons for these SES disparities in cognitive and academic skills are multifaceted and vary from one individual to another. This is because SES has various ripple effects on the home (e.g., parental beliefs and behaviors), neighborhood (e.g., community violence, environmental toxin exposure), and school (e.g., school quality and funding) experiences of an individual (reviewed by: Davis-Kean et al., 2019; Evans, 2004). Thus, SES is thought to shape the context in which children develop (reviewed by: McLoyd, 1998).

Alternatively, some children have a *biological* risk for mathematics learning difficulties. Research on the cognitive phenotypes of children with specific biological conditions as well as congenital and acquired neurodevelopmental disorders associated with lower mathematics achievement has provided valuable insight into the variability in cognitive skills, correlates, mechanisms, and pathways to mathematics learning difficulties. Past work in this area has examined Fragile X syndrome, spina bifida, chromosome 22q11.2 deletion syndrome, and Turner syndrome (reviewed by Berch & Mazzocco, 2007). Here I focus on Turner syndrome as a model of children with a biological risk for mathematics learning difficulties, in part due to the documented variability within the population on mathematical and other cognitive skills.

Turner syndrome is a chromosomal condition resulting from complete or partial loss in one of two X chromosomes (1:2500 live Female births; Lippe, 1991; Rieser & Underwood, 1989). Turner syndrome generally does not run in families, rather it seems to be sporadic (Wolff et al., 2010). There are several physical features or symptoms of Turner syndrome including but not limited to: short stature, non-functioning ovaries, a wide neck, low or indistinct hairline, feet and hands that are swollen at birth, skeletal abnormalities, and heart defects (Huang et al., 2021; Wolff et al., 2010). The presence of these physical features is one reason medical professionals might suggest running tests to determine if a diagnosis of Turner syndrome is appropriate. Some individuals are diagnosed at prenatally or at birth, but many individuals are diagnosed in early childhood or near the onset of puberty, due to slower growth rates or lack of pubertal onset, respectively. Turner syndrome is diagnosed through a karyotype analysis which involves analyzing complete sets of chromosomes in an individual (Saenger et al., 2001).

Regarding cognitive skills, there is great variability across individuals with Turner syndrome particularly in the realm of mathematics. Children with Turner syndrome are more likely to meet mathematics learning difficulties criteria than those in the general population, but the prevalence rates reported vary based on stringency of criteria. For instance, Mazzocco et al. (2001) reported 43% of children with Turner syndrome met mathematics learning difficulties criterion, compared to 10% of controls matched on gender, age, grade, and IQ in a sample of 5- and 6-year-old children. This study was based on stringent mathematics learning difficulties criterion (i.e., below the 10th percentile on the Test of Early Mathematics Ability – second edition). Therefore, mathematics skills are often considered a difficulty of children with Turner syndrome on average, but there are also documented individual differences within the Turner syndrome population (e.g, Baker & Reiss, 2016; Brankaer et al., 2017; Murphy et al., 2006).

Children with Turner syndrome do not differ in performance compared to their age-matched peers on all mathematical tasks. For instance, a meta-analysis found a difference in performance between individuals with Turner syndrome and neurotypical peers on calculation items (e.g., addition, subtraction, multiplication, division, geometry), but no significant difference on non-calculation items (e.g., counting, digit comparison, writing numbers; Baker & Reiss, 2016). However, single study findings indicate that it might be more complex arithmetic problems (e.g., problems that require multiple steps) where children with Turner syndrome have difficulty, rather than all calculation items (Murphy et al., 2006; Rovet et al., 1994).

Murphy et al. (2006) examined mathematics skills among children aged 5 to 9 years of age with Turner syndrome and a grade-matched normative comparison group and found no difference in average performance on various mathematics skills, such as verbal counting, simple arithmetic, and reading and writing numbers. Matched peers significantly outperformed children with Turner syndrome on a measure of one-to-one correspondence when counting, but even still many children with Turner syndrome passed this item (89%). In another study, Lukowski et al. (2020) found 4.5 to 9 year old children with Turner syndrome were less accurate that their peers matched on age and verbal skills on a measure of nonsymbolic magnitude comparison. Overall, these findings indicate that one numerical deficit underlying lower mathematics performance in all children with Turner syndrome has not been identified and is thought to be unlikely given the documented variability within the Turner syndrome community.

Multiple studies indicate some numerical difficulties in children with Turner syndrome are present into adulthood, despite an increase in mathematics achievement from elementary to middle school (Bruandet et al., 2004; Mazzocco & Hanich, 2010; Molko et al., 2003). It is possible the skills contributing to mathematics achievement vary for children with and without Turner syndrome which could explain why mathematics achievement scores increase despite numerical processing difficulties. Lukowski et al.

(2020) found some support for this with regard to nonsymbolic magnitude comparison. Specifically, individual studies of typically developing infants (Decarli et al., 2023) and children (Mazzocco et al., 2011) as well as meta-analyses (Schneider et al., 2017) report a positive association between nonsymbolic magnitude comparison and mathematics achievement; however, this relation did not emerge in 4.5 to 9 year old children with Turner syndrome (Lukowski et al., 2020).

Therefore, many questions remain, including to what extent mathematics learning difficulties are specific, correlated, or comorbid with other phenotypic features in children with Turner syndrome. Specifically, EF and verbal skills and their relation to mathematics have been examined. Regarding EF skills, Mauger et al. (2018) conducted a meta-analysis on children (ages 6 to 18 years) with Turner syndrome and found evidence of impairments in all three EF components including inhibitory control (Hedge's g = -0.44), cognitive flexibility (Hedge's g = -0.57), and working memory (Hedge's g size = -0.89) compared to their peers. The variability of impairments by EF component is challenging to interpret given prior work highlighting the unity of these three components in early childhood and distinct components later in childhood and into adulthood (Brydges et al., 2012; Wiebe et al., 2011). There is also variability within and across EF tasks used (e.g., McGlone, 1985; Romans et al., 1997, 1998; Temple et al., 1996; Kirk et al., 2005; Mazzocco & Hanich, 2010). When examining the trajectory of EF skills from first through seventh grade using the Contingency Naming Task differences emerged in typically developing children and children with Turner syndrome (Kirk et al., 2005), indicating the development of EF might vary for those in the Turner syndrome community compared to peers without Turner syndrome.

Further, ocular motor findings from participants ages 7 to 20 years of age with Turner syndrome showed poorer performance on memory-guided saccades (i.e., look where a target previously was), visually guided saccades (i.e., look at an appearing target), and anti-saccades (i.e., suppress directing gaze toward where stimulus will appear by looking in another location) compared to their peers. No significant differences were found between individuals with Turner syndrome and their peers on a predictive saccade (i.e., look back and forth between two lights at the appropriate time) or gap/overlap task (i.e., disengage fixation on a target). These findings indicate slower response times on selective tasks and widespread, but subtle, deficits in various brain regions. This aligns with prior work indicating children with Turner syndrome have specific EF difficulties.

Lukowski et al. (2020) examined whether EF skills contributed to mathematics ability and achievement in 4.5 to 9 year old children with Turner syndrome and peers matched on either verbal or nonsymbolic magnitude comparison skills. Partially supporting their hypothesis, they found a significant relation between EF and mathematics measures in both comparison groups, but not the Turner syndrome group. It might be that both the development of EF and the relation between EF and mathematics varies for children with Turner syndrome compared to children without Turner syndrome. Thus, this study will add to this line of work by examining whether EF emerges as a unique predictor of mathematics achievement in models including vocabulary, EF, and varying numerical skills, and after accounting for age and maternal education as covariates.

Further, the hypothesis that children with Turner syndrome do mathematics differently or use different skills to support doing mathematics also includes using verbal skills. The cognitive profile of children with Turner syndrome includes average or above average verbal skills (Temple, 2002; Temple & Carney, 1996). Specifically, verbal strengths have been found for phonological processing and vocabulary (reviewed by Murphy, 2009; Temple, 2002; Temple & Carney, 1996) and receptive and expressive vocabulary skills in children ages 5 to 12 years of age (Ross et al., 1997).

In an fMRI study of individuals with Turner syndrome and age-matched typically-developing peers (M_{age}: ~14 years old), those in the Turner syndrome group showed greater activation in regions of the brain associated with verbal processing while solving arithmetic problems compared to their peers (Kesler et al., 2006). Accuracy rates were similar on the arithmetic problems indicating that the Turner syndrome group might be doing mathematics differently, possibly through relying more heavily on verbal strategies than their age-matched peers. Both groups also showed activation in regions of the brain associated with arithmetic processing. Therefore, Lukowski et al. (2020) hypothesized that children with Turner syndrome might use alternate strategies that more heavily rely on their verbal strengths to *compensate* on mathematics achievement tests. Thus, this might partially contribute to the differential pattern of predictors of mathematics achievement. This hypothesis was partially supported as indicated by the unique association between verbal skills and mathematics on one of two mathematics measures examined.

Together, the specificity in mathematical and numerical difficulties and potential for elucidating compensatory strategies for mathematics make Turner syndrome a unique opportunity to understand pathways to mathematics learning difficulties. Children with Turner syndrome might be more prone to use verbal compensatory strategies, but it is possible these strategies are also effective for children without Turner syndrome that have a similar profile of verbal strengths (Lukowski et al., 2020). It might also be that other domain-general or specific numerical skills that are unimpaired are contributing to mathematics achievement to a different degree in children with and without Turner syndrome.

Prior work has examined either environmental or biological risk for mathematics learning difficulties, but *no study of the relations between EF and mathematics has included both risk groups*. Thus, I addressed this gap by examining the pattern of concurrent relations between EF, vocabulary, and specific numerical skills and mathematics achievement in groups of children with Turner syndrome (biological risk for mathematics learning difficulties), from Low SES households (socioenvironmental risk for mathematics learning difficulties) or with No Known Risk for mathematics learning difficulties.

Current Study

In the second research question, I examined the numerical and non-numerical (i.e., EF, vocabulary) skill associations with mathematics in preschool to third graders varying in mathematics learning difficulties risk to address the following research question:

> *RQ2. Which cognitive skills (i.e., EF, verbal, numerical) are associated with mathematics in children with Turner syndrome, from low SES households, and with No Known Risk for mathematics learning difficulties?* I hypothesized different profiles of skills would be associated with mathematics for the Turner

syndrome (biological risk), Low SES (socioenvironmental risk), and No Known Risk for mathematics learning difficulties subgroups.

Turner Syndrome: In the Turner syndrome group, I predicted verbal skills (as measured by performance on vocabulary tasks), but not EF, would be related to mathematics. This is partially based on the work of Lukowski et al. (2020), which had some overlap in the sample I am using here (described in participants section) and found no difference in EF skills between children with and without Turner syndrome and no relation between EF and mathematics. Based on prior work indicating children with Turner syndrome have verbal strengths, I hypothesized greater reliance on verbal rather than EF skills while doing mathematics. Thereby, I hypothesized no unique association between EF and mathematics would be found.

I hypothesized some, but not all, numerical skills would be associated with mathematics. Specifically, I predicted numerical skills most conducive to relying upon verbal skills or strategies would be positively associated with mathematics (i.e., numerical literacy, verbal counting), whereas the numerical skills where it is more challenging to rely on verbal skills or strategies would not be related to mathematics (i.e., non-rote counting, count on, numerical problem solving). For instance, there are some numerical tasks with higher working memory demands that are also more challenging to use verbally-based strategies to solve, such as count on. It is possible to count from 1 to solve count on (e.g., “49 and then comes?”) items; however, the items in this set involved two and three-digits, which makes it less efficient, more time-intensive, and possibly leaves more room for error when using this strategy than if the count on items involved smaller digits. Additionally, I hypothesize a "replication" of the uncoupling of nonsymbolic magnitude comparison and mathematics in Turner girls previously reported in in Lukowski et al (2020), which include the same children in this sample.

Low SES and No Known Risk: In the Low SES and No Known Risk subgroups, I hypothesized EF and vocabulary would be associated with general mathematics, representative of the general population of primary school age childhood in the United States. Additionally, numerical skills fall within the mathematics domain and are foundational to mathematics overall, thus numerical and mathematics relations were expected.

Method

Participants

Group 1: Turner Syndrome (*n* = 44). There were only Females included in this Turner syndrome group. Turner syndrome participants were recruited by distributing flyers at Turner syndrome clinics and support groups in Minnesota and across the continental United States. Interested parents completed the inclusionary criteria screener, which included a Turner syndrome karyotype review and general background review. Of this group, some potential participants were excluded due to having a karyotype associated with general intellectual disability or another chromosomal abnormality in addition to Turner syndrome (*n* = 10) or a history of developmental or neuropsychological diagnoses with potential cognitive implications (*n* = 20) presumably unrelated to Turner syndrome. The remaining 50 participants were eligible, and of these 44 children from 22 states were enrolled and completed at least one time point of data collection. This sample is identical with Turner syndrome sample described in Lukowski et al. (2020).

The Turner syndrome participants included 4.67-8.62-year-old children (M_{age} = 6.86 years; *SD* = 1.38; 100% Female). Over half the children in this sample were White, non-Hispanic (72.7%), with an additional 15.9% Multiracial, 9.1% Hispanic, and 2.3% Black. Most participants were not eligible for FRL (81.8%), 13.6% received FRL, and 4.5% indicated FRL was not applicable. On average, mothers were likely college educated, with an average of 16.47 (*SD* = 1.97; range = 12-18). Most participants were only regularly exposed to English (77.3%) and 22.7% were regularly exposed to a language other than English.

Group 2: Low Socioeconomic Status (Low SES, *n* = 130). Participants were recruited primarily from three schools in the Minneapolis Public School (MPS) district. Some preschool children were also recruited from the Institute of Child Development Participant Pool (IPP). Two children were not included in the analytic frame due to ending early in data collection sessions and completing very few tasks (*n* = 1) or having scores and school service reports that were consistent with an intellectual disability (*n* = 1).

Eligibility for the FRL was used as a proxy for low SES. In the United States, students meet criteria for free lunch if household income is at or below 130% of the poverty line and reduced-price lunch if household income is between 130% and 185% of the poverty line (USDA, 2017). I used FRL eligibility rather than parent education for two reasons. First, FRL status is a unique and stronger predictor of academic achievement beyond household income (Domina et al., 2018), which is speculated to be due to FRL capturing broader elements of educational disadvantage than income. Second, parent educational information was limited, such that I had data on years of parent education, but not highest degree earned. This is an important distinction because the differences between 16 and 17 years of school might be due to an individual taking four versus five years to complete a bachelor's degree, completing a bachelor's degree and starting an advanced doctorate degree, or many other variations. Years of parental education is a useful SES proxy for capturing parenting experiences and beliefs because it is likely the educational experience rather than obtainment of the degree itself is what is driving variation in parenting (reviewed by: Davis-Kean et al., 2019); however, it is challenging to use years of education to inform categorizations of parental education as low or high.

The Low SES participants included 4.67-9-year-old children (M_{age} = 6.64 years; *SD* = 1.029; 47.7% Female). A little less than one third of participants were Black (30.8%) with an additional 23.1% Multiracial, 20% White, non-Hispanic, 16.2 % Hispanic, 6.9% Asian, 2.3% American Indian/Alaska Native, and .8% missing. On average, mothers likely had a high school education and a few years of college, with an average of 13.93 (*SD* =3.21; range = 2-18) years of education. More than half of participants (60%) were regularly exposed to a language other than English and 40% were regularly exposed to English only.

Group 3: **No Known Risk** (*n* = 204). Participants that were not eligible for FRL, did not have a Turner syndrome diagnosis, and met the previously described inclusionary criteria were primarily recruited from three MPS schools. A subsample of preschoolers was also recruited from the IPP. Five children were not included in the analytic frame due to ending early in data collection sessions and completing very few tasks (*n* = 2), COVID-19 interrupting the A1 assessment series (*n* = 2), or validity concerns from the experimenter (*n* = 1).

The No Known Risk participants included 4.58-8.75-year-old children (M_{age} = 6.43 years; *SD* = .936; 47.1% Female). Around two thirds of children in this sample were White, non-Hispanic (66.2%), with an additional 16.2% Multiracial, 6.9% Asian, 3.4% Black, 3.9% Hispanic, 0.5% American Indian/Alaska Native, 1% not otherwise listed, and 2% missing. On average, mothers were likely college educated, with an average of 16.71 (*SD* =1.65; range = 6-18). Around a third of children (34.8%) were regularly exposed to a language other than English, 64.7% were regularly exposed to English only, and 0.5% missing data.

Measures

Children in Research Question 2 completed the same battery of measures in an identical order as described in Research Question 1.

Procedure

The procedure for Research Question 2 was similar to Research Question 1, with a few exceptions. First, only relevant assessment data from A1 were considered for Research Question 2 due to the substantial decrease in sample size for the Turner syndrome subgroup from A1 (*N* = 44) to A2 (*N* = 26). Second, the Turner syndrome girls were recruited nationwide, and therefore attended various schools throughout the U.S. As such, data collection for the children with Turner syndrome was conducted in the lab given that some children traveled from out of state, or at locations in the participants' hometown.

Results

Analytic Approach

The preliminary analytic approach for RQ2 was similar to what was described for RQ1. The missing data for children in the Low SES and No Known Risk groups were imputed simultaneously with the data with RQ1 due to the sample overlap (see Missing Data sections for details). The analytic frame between RQ2 and RQ1 varied in two ways: 1) the Turner syndrome group was excluded from RQ1 and 2) the sample size was larger for the groups of children without Turner syndrome in RQ2 because this research question only required data at A1, whereas RQ1 required data at A1 and A2.

I followed a similar preliminary data analysis with the Turner syndrome group as described in RQ1. First, I screened for outliers and determined winsorizing was not warranted. Next, I examined descriptives, correlations, and created composites in the Turner syndrome group using the same process described in RQ1. Due to the distinct numerical and non-numerical cognitive profile of Turner syndrome, it was important to examine relations with mathematics separately from participants who were typically developing. Missing data in the Turner syndrome group was 0-2.27% per variable. In total 2 out of 44 records (4.54 %) were incomplete in the Turner syndrome group. Given the low proportion (<5%) of missingness and results of the Little’s MCAR test indicating data were likely MCAR ($\chi^2(23) = 22.63$, $p = 0.483$), I opted not to impute because bias due to listwise deletion is likely inconsequential (Schafer, 1999). Therefore, non-imputed data were used for all subsequent analyses in the Turner syndrome group.

Analyses for the Low SES and No Known Risk groups were conducted on non-imputed and imputed datasets. The Turner analyses were only computed on non-imputed datasets. Next descriptive and correlational analyses were conducted to examine the distributions of the data and associations among the variables of interest and potential covariates. Finally, OLS regression models were conducted as the main analyses. Benjamini–Hochberg corrections were applied to each regression to account for multiple comparisons (Benjamini & Hochberg, 1995). Participants with Turner syndrome were recruited nationwide, rather than in local schools meaning that nesting in schools was not necessary in these analyses. In the No Known Risk group there was limited evidence of clustering on mathematics variables of interest (*ICC*s: ≤ .01); therefore, clustered standard errors were not needed. I found some evidence for between-school level variability (*ICC*s: .11-.12) on mathematics variables the Low SES group; therefore, I clustered the standard errors in this group at the school level in subsequent analyses to reduce bias.

The primary analyses were conducted in R (R Core Team, 2020). I used the *mice* package (van Buuren & Groothuis-Oudshoorn, 2011) for multiple imputation and the *lme4* (Bates et al., 2015) and *miceadds* (Robitzsch & Grund, 2020) packages for linear regressions.

Missing Data

First, trial level missing data on the items required to derive numerical sum scores were examined. The same process of examining and handling item level missing data described in RQ1 was used here (see prior Missing Data section). Similarly, due to sample overlap, the Low SES and No Known Risk subgroups were imputed during the multiple imputation process described in RQ1. There was no multiple imputation for the

Turner syndrome group due to low rates of missing data. These analyses were also performed on data prior to multiple imputation and the pattern of findings was similar.

Descriptive Analyses

Descriptive statistics on non-imputed Low SES, No Known Risk, and Turner syndrome groups are displayed in Tables 10 and 11. The skew was acceptable for all variables in the Turner and Low SES groups (skew: between -1 and +1). There was slight negative skew for verbal counting (skew = -1.22) and EF composite scores (skew = -1.42) in the No Known Risk group. The moderate, positive Spearman correlations between the MEFS and HTKS (r_{Turner}= .59, $r_{\text{Low SES}}$ = .49, $r_{\text{No Known Risk}}$= .42) and between KBIT-2 VK and BNT-2 (r_{Turner}= .78, $r_{\text{Low SES}}$ = .78, $r_{\text{No Known Risk}}$= .68), in each group, indicated it was appropriate to create composites for EF and Vocabulary, respectively. Composite scores were created by averaging the *z*-scores of individual measures at each assessment. Table 12 presents the partial correlations between EF and numerical skills and mathematics in each group controlling for age. Additionally, because numerical skills of interest were positively correlated with one another, it was necessary to examine the relations between numerical skills and mathematics in separate models to prevent multicollinearity among predictors. Thus, it was not possible to determine which of the six numerical skills were the uniquely predicting mathematics, above and beyond other numerical skills.

I also examined ANOVAs to identify additional covariates that were related to key variables of interest. In cases of significant findings, I examined whether these effects still held after controlling for age and maternal education. I examined timing of testing (i.e., fall versus spring) and determined this was only uniquely related to verbal counting (*p* = .01), such that students who were tested in the fall had higher scores than those tested in the spring in the No Known Risk group. Given that timing of testing was not related to most key variables in each group, I opted not to include it as a covariate in subsequent analyses. Cohort was examined and was not related to key variables of interest. There were no gender differences for many key variables of interest; however significant differences were found for TEMA-3 overall scores. Thus, gender was only included as a covariate in the No Known and Low SES groups because the Turner sample was 100% Female.

Associations between Numerical and Non-Numerical Skills and Mathematics

To examine the associations between numerical and non-numerical skills and mathematics in the three groups (i.e., Turner syndrome, Low SES, No Known Risk) experiencing variation in risk for mathematics learning difficulties, a series of OLS regressions predicting WJ-Applied Problems and TEMA-3 scores were conducted. For each risk group, I regressed WJ Applied Problems scores on the numerical skill of interest (e.g., numerical literacy), vocabulary composite, and EF composite, while controlling for age, gender, and maternal education at A1. Similar regressions were conducted with TEMA-3 scores as the dependent variable to examine which findings were robust to multiple measures of mathematics.

Associations between EF, number, and vocabulary and mathematics were robust in the Low SES and No Known Risk groups. Specifically, the vocabulary composite, EF composite, and various numerical skills, including nonsymbolic magnitude comparison, verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving, were associated with WJ-Applied Problems. The pattern of findings was slightly different when mathematics was operationalized as TEMA-3 overall scores. In several analyses the vocabulary composite was associated with TEMA-3 scores in the No Known Risk group, but not the Low SES group. Overall, EF and numerical skills were often associated with TEMA-3 scores for both Low SES and No Known Risk groups (Tables 13-16).

In the Turner syndrome group, neither EF nor vocabulary composite scores were related to WJ-Applied Problems or TEMA-3 scores. Numerical problem solving, numerical literacy, and count on, were associated with WJ-Applied Problems and TEMA-3 scores. Non-rote counting and verbal counting were associated mathematics as measured by TEMA-3, but not WJ-Applied Problems. Nonsymbolic magnitude comparison was not associated with WJ-Applied Problems or TEMA-3 (Tables 17 and 18, respectively). Notably, a subset of children in the Turner syndrome group qualified for FRL (n = 6). Therefore, I re-ran my analyses without these children given that these individuals were experiencing both living in a low SES household and Turner syndrome as mathematics learning difficulties risk factors. For models with WJ-Applied Problems as the dependent variable, one important difference in the pattern of findings emerged. Specifically, there were three models where significant relations were found between the vocabulary composite and WJ-Applied Problems. No significant EF or vocabulary composite effects were found for TEMA-3 overall scores in this subsample. Non-rote counting (B = 3.40, SE =1.23, p = .03) and verbal counting (B = .17, SE =.04, p < .00) were associated with TEMA-3 overall scores in this subsample.

Parallel analyses for each group were run examining MEFS and HTKS scores separately, rather than as an EF composite. The pattern of findings remained the same as when the EF composite was used.

Robustness Tests

Substantial power differences exist in the RQ2 analyses due to variation in sample size across groups. In particular, the Turner syndrome group has limited power (N = 44), whereas the Low SES (N = 130), and No Known Risk (N = 204), groups have higher power. Thus, I selected a random sample of 44 children from the Low SES and No Known Risk groups, respectively, and ran parallel sets of analyses within these subsamples. The findings did not consistently replicate, exemplifying that power differences between groups might have contributed more to the lack of significant findings in the Turner syndrome group compared to the Low SES and No Known Risk groups. For example, in the No Known Risk subsample with WJ-Applied Problems as a dependent variable, the vocabulary composite findings did not replicate, but many of the EF and numerical skill relations were still significant. Conversely, in the Low SES subsample with WJ-Applied Problems as a dependent variable, some of the EF, Verbal, and numerical skill findings replicated, except for verbal counting, nonsymbolic magnitude comparison, and non-rote counting.

Additionally, I re-analyzed the No Known Risk and Low SES groups with only the Female participants to match the Turner syndrome group on gender. In the Low SES and No Known Risk groups, the overall pattern of findings was similar but less robust. For instance, the EF and vocabulary composites were significantly associated with WJ-Applied Problems in some, but not all models. Several numerical skills were associated with WJ-Applied problems, but not all. Specifically, verbal counting was not associated with WJ-Applied Problems in the Low SES group and neither verbal counting nor nonsymbolic magnitude comparison were associated with WJ-Applied Problems in the No Known Risk group. Similar patterns of associations were found for EF, vocabulary, and numerical skills and TEMA-3 in the Low SES group. In the No Known Risk group, vocabulary skills were no longer associated with TEMA-3 scores, but EF and many numerical skills, except nonsymbolic magnitude comparison were associated with TEMA-3 scores. Given the decrease in sample size these analyses should be interpreted with caution.

Discussion

The current study investigated the domain-general and domain-specific correlates of mathematics in early childhood in groups of children varying by mathematics learning difficulties risk factor. My study is the first to include children from all three of the following groups: No Known Risk, socioenvironmental risk (e.g., Low SES households), and biological risk (e.g., Turner syndrome) for mathematics learning difficulties. I examined the relations between EF, vocabulary, and six numerical skills (e.g., nonsymbolic magnitude comparison, verbal counting, numerical literacy, count on, non-rote counting, and numerical problem solving) and two indices of general mathematics. Overall, the current study contributes to the existing literature on whether and how the cognitive profiles of young children vary by mathematics learning difficulties risk status.

I found positive, significant associations between EF, vocabulary, and all six numerical skills and WJ-III Applied Problems performance in the Low SES and No Known Risk groups. Similarly, all six numerical skills were associated with TEMA-3 performance in both groups. EF composite scores were associated with TEMA-3 performance in all models for the Low SES group, and all models except the one including numerical literacy for the No Known Risk group. Thus, the overall pattern of relations between numerical and EF skills were positive, significant, and similar across both mathematics measures and in the Low SES and No Known Risk groups.

Interestingly, in both the Low SES and No Known Risk groups, the relations between vocabulary and mathematics varied based on the mathematics measure used. Specifically, vocabulary was not significantly related to TEMA-3 performance in the Low SES group, in fact the beta coefficient for vocabulary was negative. The vocabulary composite was positively and significantly related to TEMA-3 performance in three of six models in the No Known Risk group. Thus, these findings might indicate WJ-III Applied Problems has greater verbal demands than the TEMA-3. Face value inspection of items within each of these mathematics measures supports this explanation. Specifically, two coders (including myself) examined the first 40 items of WJ-Applied Problems (range in this sample: 7-40) and found five items (12.5%) could be responded to accurately without a verbal response, whereas 15 of the 72 items (20.83%) in the TEMA-3 (range in this sample: 0-72) could be responded to accurately without a verbal response (Cohen’s kappa = .94). Importantly, both tasks provide instructions and prompts verbally and therefore involve some verbal demands on the participant, but this item analysis as well as my diverging findings indicate the verbal demands might be higher for the WJ-Applied Problems than the TEMA-3.

Next, I conducted parallel sets of analyses for the Turner syndrome group. I found no significant relations between EF or vocabulary and WJ-II Applied Problems or

TEMA-3 performance. The lack of a significant relation between EF and mathematics achievement is consistent with Lukowski et al. (2020), but stands in contrast to the robust positive relation between EF and mathematics in typically developing samples (Peng et al., 2016; Spiegel et al., 2021). There are several possible explanations for these findings. It might be that EF does not uniquely contribute to mathematics achievement in children with Turners syndrome. It could also be that the relation between EF and mathematics emerges later in life for children with Turner syndrome and thus was not detected here due to my focus on early childhood. These findings also could be due to the EF tasks I used, given the variability in performance on EF tasks within the Turner syndrome community (Mauger et al., 2018). Finally, given that Turner syndrome is characterized by individual differences and variability, it is possible that the relation between EF and mathematics achievement is stronger for some children than others. Future work is needed to disentangle these possibilities.

Regarding numerical skills in children with Turner syndrome, there was no significant relation between nonsymbolic magnitude comparison and WJ-III Applied Problems or TEMA-3 scores (as reported in Lukowski et al., 2020 using the same Turner syndrome participants and similar, but not identical models). Three of the six numerical skills examined - numerical literacy, count on, and numerical problem solving - were associated with WJ-Applied Problems and TEMA-3 scores, whereas the remaining two numerical skills - non-rote counting and verbal counting – were only associated with TEMA-3 scores. However, the later finding might be an artifact of the numerical skills being derived from specific items from the TEMA-3, thus the relations between non-rote counting and verbal counting and general mathematics needs to be replicated using differing measures.

These numerical relations with WJ-III Applied Problems in the Turner syndrome group partially supported my hypothesis that number skills most conducive to applying verbal strategies or involving use of verbal skills would be positively associated with mathematics achievement (e.g., numerical literacy). However, I did not predict a significant relation between count on or numerical problem solving and mathematics. It could be that other numerical or non-numerical skills support children's ability to do count on or numerical problem solving items and as these numerical skills grow they might contribute to doing mathematics more broadly. For instance, identifying the next number after 29 is not as taxing on working memory if the individual is proficient in understanding the verbal count sequence. Regarding numerical problem solving, the findings from Kesler et al. (2006) are indicative of children with Turner syndrome relying on verbal strategies to solve visually presented formal arithmetic problems. However, these children were older (*M* = ~14 years of age) and the problems were visually presented so it was not necessary to also remember the numbers while engaging in verbal numerical problem solving strategies. The items included in this study were often presented verbally in a story context or were nonverbal and involved hidden items. Thus, I hypothesized it might be harder to extract only the relevant elements of the problem, hold the elements of the problem in mind, and engage in a verbal strategy. It is possible some children with Turner syndrome did engage in this process. Alternatively, children with Turner syndrome might have compensatory strategies that are effective for solving these kinds of problems that might not be verbally-based. In either case, these findings suggest numerical literacy, count on, and numerical problem solving skills positively concurrently contribute to mathematics in children with Turner syndrome and it is worth examining whether these numerical skills also predict growth in mathematics. It is also important to determine whether these numerical and mathematics relations vary developmentally and if more complex items are included (e.g., adding multiplication into numerical problem solving composite) in the numerical scores.

When I examined the subset of children with Turner syndrome that did not qualify for FRL (*n* = 38), the pattern of findings slightly varied. Most notably, the vocabulary composite was associated with WJ-III Applied Problems in three of six models. This finding lends partial support to my hypothesis that a relation between vocabulary and mathematics achievement would emerge in children with Turner syndrome. This hypothesis was based on the idea that children with Turner syndrome might leverage their average or above average verbal skills (reviewed by: Hong et al., 2009) to support mathematical numerical problem solving to compensate for lower EF and specific numerical difficulties (Baker & Reiss, 2016). However, it might also be that this effect is due to the change in sample size given that there was no difference in the relations between vocabulary and mathematics for children in the Low SES versus the No Known Risk group.

Limitations and Future Directions

This study had several limitations. First, my Turner syndrome (*n* = 44) sample size was comparable to prior work with this population (e.g., Bruandet et al., 2004; Mazzocco, 2001), but my analyses were still underpowered. Thus, findings should be interpreted with caution. Second, these findings are correlational and cross-sectional and as such causality cannot be inferred. Future experimental work is needed to better understand the relations between the numerical and non-numerical skills contributing to mathematics performance. Third, I lacked a direct assessment of working memory skills. Therefore, my EF composite did not adequately represent the various EF components. Fourth, typically developing children were recruited from the same metropolitan area and were not representative of demographics across the state or country. Despite the limited geographical region where I recruited, the sample was diverse in terms of race, ethnicity, SES, and multi-language exposure. Fifth, examination of demographics in Research Question 2 revealed the differences between the Low SES, No Known Risk, and Turner syndrome groups were not only eligibility for FRL or a diagnosis of Turner syndrome. The racial and ethnic breakdown within each group varied greatly, with children in the Low SES group being more racially diverse than the No Known Risk or Turner Syndrome group.

Conclusions

Taken together, the findings across all three groups indicate the cognitive skills related to mathematics achievement vary for children in No Known Risk and Low SES groups versus children with Turner syndrome. Specifically, my results showed numerical, EF, and vocabulary skills were associated with mathematics achievement in Low SES and No Known Risk groups, but that only select numerical skills - numerical literacy, count on, and numerical problem solving–were associated with both indices of mathematics in children with Turner syndrome. This study filled a critical gap in the literature as prior studies have focused primarily on either socioenvironmental or biological risks for mathematics learning difficulties, but no prior work has included all three groups. Future work is needed to examine whether, which, and for whom EF, vocabulary, and numerical skills predict growth in mathematics achievement over time.

These findings contribute to the existing body of literature highlighting a distinct cognitive profile for children with Turner syndrome, which is characterized by average or above average verbal skills (Temple, 2002; Temple & Carney, 1996), lower performance on specific EF tasks (Mauger et al., 2018), slower speed of doing mathematics tasks (Baker & Reiss, 2016), and growth in mathematics achievement over time (Bruandet et al., 2004; Mazzocco & Hanich, 2010; Molko et al., 2003) despite difficulties on specific numerical tasks (e.g., nonsymbolic magnitude comparison; Lukowski et al. 2020). Specifically, these findings shed some light on a few of the specific numerical skills that might contribute to mathematics achievement in Turner syndrome, which has implications for future research and practice. Research that helps to uncover which skills are associated with mathematics achievement in children with Turner syndrome can be used to inform effective screening and instructional supports in the classroom. In sum, this study provides a nuanced examination of the relations between domain-general and domain-specific skills and mathematics achievement in children with varying risk factors for mathematics learning difficulties. This work also highlights the distinct cognitive profile of children with Turner syndrome compared to children with from Low SES homes and with No Known Risk for mathematics learning difficulties.

Chapter 4: General Discussion

Taken together, these two research questions further elucidate the relation between EF and numerical skills, and how these skill sets relate to mathematics achievement, in early childhood. These studies build on prior literature showing moderate relations between EF and mathematical skills in the early childhood years and beyond (e.g., Best et al., 2011; Cragg et al., 2017; Jacob & Parkinson, 2015), by demonstrating that EF is concurrently related to all six numerical skills I measured, but that predictive relations *vary* by numerical skill in a typically developing sample (RQ1). Further, I replicated prior work on the positive, significant relations between EF, vocabulary, and numerical skills and mathematics in children with No Known Risk for mathematics learning difficulties or from Low SES households. I also extended work showing children with Turner syndrome have a different cognitive profile from children with No Known Risk or from Low SES households (RQ2). Specifically, select numerical skills - numerical literacy, count on, and numerical problem solving skills - but not EF, vocabulary, nonsymbolic magnitude comparison, non-rote counting, or verbal counting, were associated with mathematics in children with Turner syndrome. Thus, there was an uncoupling of various numerical skills and mathematics achievement within the Turner syndrome group specifically.

Overall, this investigation added to our understanding of the intersection of domain-general and domain-specific skills and how these skills contribute to mathematics achievement in typically developing children and children with Turner syndrome. Both studies provided a nuanced look at numerical skills rather than only including a broader measure of general mathematics achievement. These findings fit within the broader framework of how children do and learn mathematics. Specifically, the National Research Council (2001) identified five interwoven strands of mathematical proficiency including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, all of which can apply to numerical or non-numerical skills. Further, it is thought that EF skills might play a role in several of these strands, such as coming up with several approaches to solve a problem when the solution is not immediately known (strategic competence). Thereby, mathematical proficiency is thought to involve domain-specific mathematical knowledge and skills and EF. Thus, the findings from these studies on which numerical skills are related to EF and for whom EF is associated with mathematics achievement have implications for research and practice.

Specifically, it is beneficial for educators to know which numerical activities have EF demands and how EF contributes to doing numerical tasks and learning about numbers. This information can be particularly valuable when designing interventions, classroom materials, or tailoring instruction to account for the role of EF because not all numerical tasks have the same EF demands. Additionally, to support EF and number development it is important to know which EF and number skills are mutually supportive of one another over time and which are not. I found primarily unidirectional predictive relations (EF → Number), indicating limited evidence of mutual enhancement of EF and numerical skills over time. Thus, even though a child might use EF skills to do a numerical task, it is also important to provide opportunities to practice EF in other activities within the classroom to adequately support EF skills. Similarly, numerical tasks are needed to support numerical development. These findings also have implications for researchers. Given that numerical problem solving and nonsymbolic magnitude comparison predicted later EF skills it is worth experimentally examining if engaging in these numerical activities can benefit both EF and numerical development.

The varying cognitive profile of skills related to mathematics achievement in the children with Turner syndrome compared to the children in the Low SES and No Known

Risk groups, suggests it is necessary to consider that pathways and skills related to mathematics achievement vary between children. This is important to consider for those screening and supporting children who might be at risk for mathematics learning difficulties. Finally, future work is needed to identify potentially avenues or pathways to support mathematics achievement in children with Turner syndrome. It might be that children with Turner syndrome do and learn mathematics slightly differently, and it is worth continuing to examine which skills children with Turner syndrome could leverage to engage in effective alternative strategies for mathematics achievement.

Figure 1

Visual Depiction of Longitudinal Hypotheses for Research Question 1

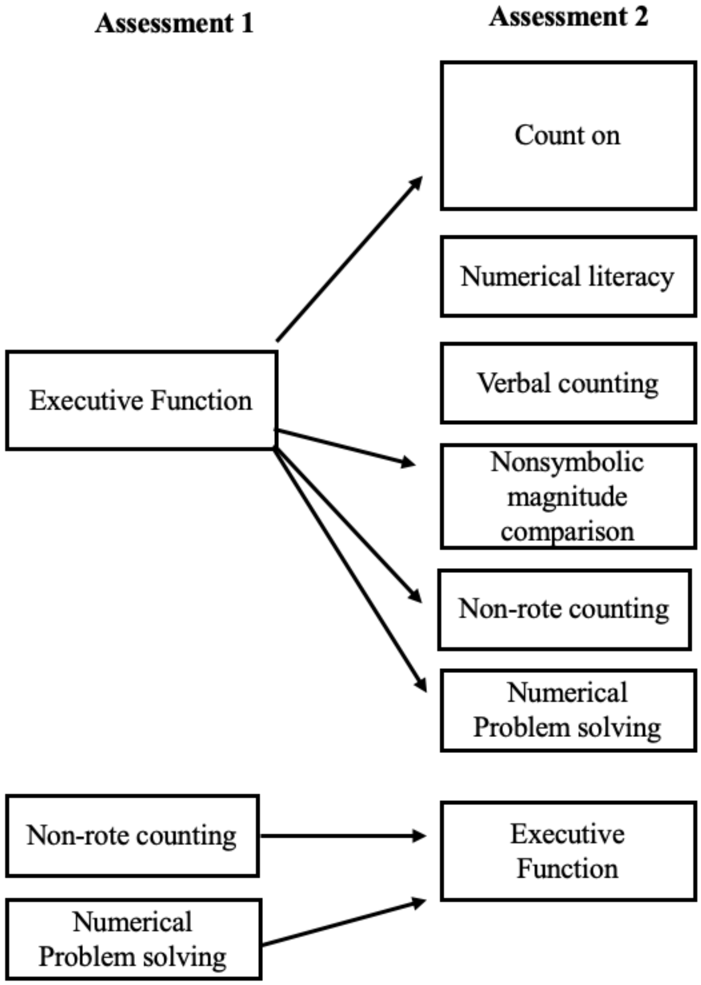
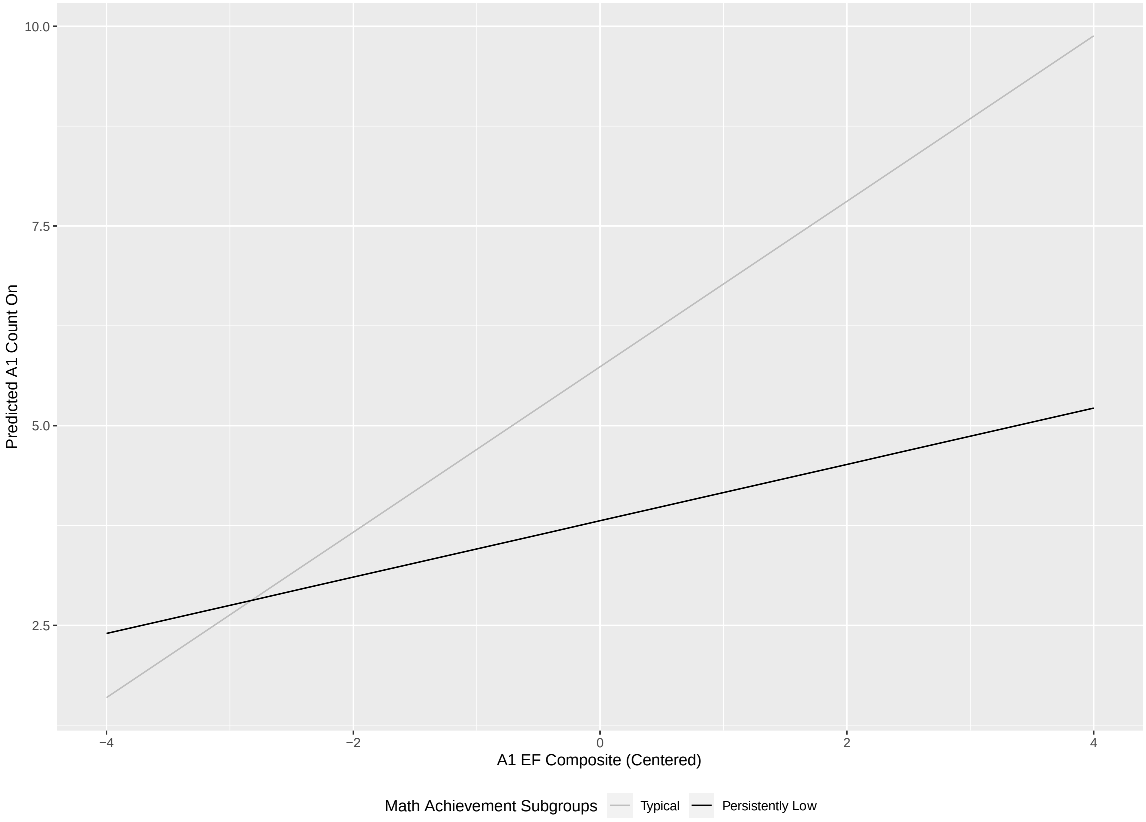


Figure 2

Mathematics Achievement Subgroup Moderates the Effect of Assessment 1 Executive Function on Count On



Note. A1 = Assessment 1. EF = Executive Function.

Table 1

Summary of the Domain-Specific and Domain-General Constructs of Interest and their Corresponding Measures

Construct	Corresponding Measures
Domain-specific (mathematics)	
Mathematics Achievement	Woodcock Johnson-III Applied Problems
Mathematics ability	TEMA-3
Specific Numerical Skills	
Numerical literacy	21 TEMA-3 trials
Count on	8 TEMA-3 trials
Verbal counting	1 TEMA-3 trial (count as high as you can)
Non-rote counting	6 TEMA-3 trials
Numerical problem solving	11 TEMA-3 trials and 1 supplemental trial
Nonsymbolic magnitude comparison	Panamath
Domain-general	
Executive Function	Minnesota Executive Function Scale
	Head Toes Knees Shoulders
Vocabulary	
Receptive vocabulary	Kaufman Brief Intelligence Test-2 Verbal Knowledge
Expressive vocabulary	Boston Naming Task

Note. TEMA-3 = Test of Early Mathematics Ability.

Table 2

Descriptive Statistics for Child Executive Function, Mathematics, and Vocabulary

	N	*M*	*SD*	*Range*	*Skew*	*Kurtosis*
Assessment 1						
A1 Vocabulary composite	205	0	.94	-2.6 3- 2.12	-.52	.02
A1 KBIT Verbal	205	21.64	5.85	5 - 39	-0.19	0.36
A1 BNT	205	32.86	9.50	5 - 51	-0.64	-0.15
A1 EF composite	205	.14	.76	-2.83-1.25	-1.3	1.8
A1 MEFS	204	71.59	10.78	35 - 89	-0.95	1.47
A1 HTKS	204	72.16	16.67	17 - 94	-1.58	2.77
A1 Nonsymbolic magnitude comparison	194	87.97	8.59	62 - 100	-1.19	1.24
A1 Numerical literacy	204	13.27	6.46	0 - 21	-0.56	-0.81
A1 Count on	205	5.61	2.78	0 - 8	-0.87	-0.75
A1 Verbal counting	204	90.16	33.65	4 - 110	-1.32	0.04
A1 Non-rote counting	204	4.76	1.51	0 - 6	-1.31	1.20
A1 Numerical problem solving	202	8.44	2.97	0 - 12	-0.86	-0.02
A1 WJ-Applied Problems	202	23.72	5.55	7 - 36	-0.45	0.36
A1 TEMA-3	196	41.65	14.14	1 - 72	-0.05	-0.30
Assessment 2						
A2 Vocabulary composite	205	0	.93	-2.78-1.92	-.513	.18
A2 KBIT Verbal	205	25.42	5.89	9 - 40	-0.03	0.07
A2 BNT	205	36.79	8.58	11 - 52	-0.72	0.26
A2 EF composite	205	-.01	.85	-2.78 - 1.92	-.88	.82
A2 MEFS	203	76.85	9.04	48 - 91	-0.74	0.12
A2 HTKS	204	80.84	9.50	50 - 94	-0.91	0.50
A2 Nonsymbolic magnitude comparison	201	91.45	6.21	72 - 100	-1.30	1.26
A2 Numerical literacy	201	17.26	4.41	4 - 21	-1.13	0.19
A2 Count on	203	6.96	1.59	2 - 8	-1.70	2.22
A2 Verbal counting	203	102.15	18.88	41 - 110	-2.37	4.22
A2 Non-rote counting	203	5.28	1.00	2 - 6	-1.61	2.39

A2 Numerical problem solving	203	10.09	2.00	4 - 12	-1.27	1.16
A2 WJ-Applied Problems	200	27.90	5.94	10 - 46	0.08	0.35
A2 TEMA-3	200	52.63	13.39	19 - 72	-0.21	-1.10

Note. Non-imputed data were used for these analyses. KBIT = Kaufman Brief Intelligence Test. BNT = Boston Naming Test. EF = Executive function. MEFS = Minnesota Executive Function Scale. HTKS = Head Toes Knees Shoulders. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

Table 3

Descriptive Statistics for Age-Normed Child Vocabulary, Executive Function, and Mathematics Tests by Grade

All grades	*N*	*M*	*SD*	*Minimum*	*Maximum*
Assessment 1					
KBIT SS	205	105.95	13.74	65	135
WJ SS	205	107.45	13.83	64	152
TEMA-3 SS	196	103.52	15.50	64	145
MEFS SS	204	100.20	8.91	61	118
Assessment 2					
KBIT SS	205	107.22	13.06	65	135
WJ SS	200	105.74	15.78	65	143
TEMA-3 SS	200	102.66	16.61	67	146
MEFS SS	203	99.89	7.98	77	116
Preschool	*N*	*M*	*SD*	*Minimum*	*Maximum*
Assessment 1					
KBIT SS	20	101.25	16.54	70	125
WJ SS	20	105.20	13.29	78	127
TEMA-3 SS	19	101.05	15.54	65	132
MEFS SS	20	99.95	11.50	61	113
Assessment 2					
KBIT SS	20	103.5	18.57	65	125
WJ SS	19	107.11	17.73	65	134
TEMA-3 SS	19	102.16	12.86	71	127
MEFS SS	18	102.11	9.83	79	114
Kindergarten	*N*	*M*	*SD*	*Minimum*	*Maximum*
Assessment 1					
KBIT SS	65	108.85	12.77	70	130
WJ SS	65	112.95	11.63	80	152
TEMA-3 SS	65	108.26	14.57	76	144
MEFS SS	65	105.23	6.87	86	118
Assessment 2					
KBIT SS	65	109.92	11.61	70	135
WJ SS	63	110.76	14.54	77	140
TEMA-3 SS	64	107.47	17.40	76	146
MEFS SS	65	102.49	7.59	85	116

First Grade	*N*	*M*	*SD*	*Minimum*	*Maximum*
Assessment 1					
KBIT SS	62	103.15	13.59	65	125
WJ SS	62	105.81	15.71	64	149
TEMA-3 SS	58	102.03	15.60	64	145
MEFS SS	61	98.23	8.26	72	114
Assessment 2					
KBIT SS	62	104.27	11.97	65	125
WJ SS	60	103.67	16.64	75	143
TEMA-3 SS	59	102.25	17.61	74	135
MEFS SS	62	99.06	7.07	82	111

Second Grade	*N*	*M*	*SD*	*Minimum*	*Maximum*
Assessment 1					
KBIT SS	58	107.33	13.32	65	135
WJ SS	58	103.81	12.53	66	124
TEMA-3 SS	54	100.26	15.53	68	128
MEFS SS	58	96.72	8.28	65	111
Assessment 2					
KBIT SS	58	108.62	12.90	75	135
WJ SS	58	101.98	14.34	67	125
TEMA-3 SS	58	97.93	14.54	67	123
MEFS SS	58	97.17	7.83	77	112

Note. Non-imputed standard scores were used for these analyses. KBIT = Kaufman Brief Intelligence Test. SS = Standard Score. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability. MEFS = Minnesota Executive Function Scale. KBIT scores were prorated because only one of two verbal subtests were administered.

Table 4

Spearman Correlation Matrix of Study Variables for Research Question 1 (N = 205)

	Variable	1	2	3	4	5	6	7	8	9	10
1	Age in Months	-	-	-	-	-	-	-	-	-	-
2	Maternal education	-0.09	-	0.36**	0.46**	0.25**	0.31**	0.38**	0.35**	0.16	0.29**
3	A1 KBIT	0.51**	0.29**	-	0.69**	0.43**	0.36**	0.33**	0.47**	0.16*	0.40**
4	A1 BNT	0.29**	0.43**	0.74**	-	0.48**	0.39**	0.44**	0.57**	0.23**	0.45**
5	A1 HTKS	0.40**	0.22**	0.55**	0.53**	-	0.39**	0.51**	0.52**	0.43**	0.51**
6	A1 MEFS	0.16*	0.30**	0.39**	0.42**	0.40**	-	0.45**	0.53**	0.28**	0.38**
7	A1 TEMA-3	0.67**	0.22**	0.56**	0.51**	0.65**	0.44**	-	0.72**	0.50**	0.66**
8	A1 WJ	0.66**	0.21**	0.65**	0.59**	0.63**	0.49**	0.87**	-	0.36**	0.65**
9	A1 Verbal counting	0.39**	0.19**	0.36**	0.33**	0.52**	0.36**	0.65**	0.57**	-	0.46**
10	A1 problem solving	0.56**	0.22**	0.60**	0.53**	0.62**	0.39**	0.79**	0.79**	0.54**	-
11	A1 Numerical Literacy	0.65**	0.21**	0.54**	0.53**	0.60**	0.44**	0.92**	0.83**	0.62**	0.76**
12	A1 Count on	0.60**	0.14*	0.46**	0.40**	0.54**	0.37**	0.85**	0.72**	0.62**	0.71**
13	A1 Non-rote	0.47**	0.23**	0.37**	0.38**	0.49**	0.31**	0.67**	0.60**	0.55**	0.61**
14	A1 Nonsymb mag comp	0.31**	0.17*	0.33**	0.37**	0.44**	0.38**	0.55**	0.57**	0.38**	0.59**
15	A1 EF Composite	0.34**	0.31**	0.54**	0.56**	0.79**	0.86**	0.64**	0.66**	0.52**	0.58**
16	A1 Vocab Composite	0.43**	0.38**	0.93**	0.93**	0.58**	0.43**	0.58**	0.67**	0.37**	0.61**
17	A2 KBIT	0.46**	0.32**	0.78**	0.72**	0.53**	0.43**	0.59**	0.65**	0.40**	0.56**
18	A2 BNT	0.22**	0.43**	0.70**	0.93**	0.47**	0.44**	0.48**	0.56**	0.31**	0.47**
19	A2 HTKS	0.23**	0.28**	0.43**	0.37**	0.56**	0.27**	0.45**	0.48**	0.30**	0.47**
20	A2 MEFS	0.21**	0.26**	0.39**	0.42**	0.50**	0.52**	0.41**	0.46**	0.37**	0.39**
21	A2 Nonsymb mag comp	0.23**	0.12	0.23**	0.20**	0.44**	0.26**	0.45**	0.46**	0.31**	0.51**
22	A2 Verbal Counting	0.19**	0.24**	0.29**	0.41**	0.45**	0.32**	0.52**	0.47**	0.46**	0.46**
23	A2 problem solving	0.44**	0.22**	0.56**	0.49**	0.56**	0.39**	0.70**	0.66**	0.51**	0.70**
24	A2 Numerical Literacy	0.50**	0.22**	0.55**	0.55**	0.62**	0.40**	0.79**	0.74**	0.56**	0.75**
25	A2 Count on	0.43**	0.26**	0.47**	0.50**	0.58**	0.39**	0.76**	0.69**	0.63**	0.68**
26	A2 Non-rote	0.23**	0.15*	0.24**	0.32**	0.40**	0.22**	0.43**	0.40**	0.33**	0.44**
27	A2 WJ	0.52**	0.26**	0.59**	0.59**	0.64**	0.45**	0.86**	0.84**	0.52**	0.77**
28	A2 TEMA-3	0.57**	0.25**	0.57**	0.54**	0.64**	0.43**	0.89**	0.83**	0.60**	0.80**
29	A2 EF Composite	0.24**	0.31**	0.47**	0.45**	0.61**	0.45**	0.50**	0.54**	0.40**	0.50**
30	A2 Vocab Composite	0.38**	0.40**	0.81**	0.88**	0.54**	0.47**	0.59**	0.66**	0.39**	0.57**

	Variable	11	12	13	14	15	16	17	18	19	20
1	Age in Months	-	-	-	-	-	-	-	-	-	-
2	Maternal education	0.33**	0.21**	0.28**	0.19*	0.34**	0.44**	0.37**	0.43**	0.29**	0.26**
3	A1 KBIT	0.32**	0.19*	0.21**	0.23**	0.46**	0.90**	0.70**	0.69**	0.35**	0.32**
4	A1 BNT	0.46**	0.27**	0.31**	0.31**	0.51**	0.93**	0.66**	0.93**	0.33**	0.37**
5	A1 HTKS	0.46**	0.42**	0.42**	0.39**	0.77**	0.50**	0.42**	0.46**	0.51**	0.49**
6	A1 MEFS	0.44**	0.33**	0.29**	0.36**	0.87**	0.40**	0.42**	0.43**	0.24**	0.51**
7	A1 TEMA-3	0.83**	0.72**	0.52**	0.46**	0.57**	0.42**	0.41**	0.45**	0.40**	0.40**
8	A1 WJ	0.67**	0.54**	0.43**	0.50**	0.61**	0.57**	0.51**	0.58**	0.45**	0.46**
9	A1 Verbal counting	0.53**	0.56**	0.54**	0.33**	0.40**	0.21**	0.19*	0.25**	0.19*	0.33**
10	A1 problem solving	0.66**	0.65**	0.53**	0.54**	0.50**	0.47**	0.39**	0.44**	0.41**	0.38**
11	A1 Numerical Literacy	-	0.73**	0.52**	0.48**	0.54**	0.43**	0.38**	0.49**	0.34**	0.37**
12	A1 Count on	0.79**	-	0.52**	0.43**	0.44**	0.26**	0.25**	0.31**	0.28**	0.31**
13	A1 Non-rote	0.63**	0.61**	-	0.35**	0.42**	0.30**	0.23**	0.31**	0.27**	0.19*
14	A1 Nonsymb mag comp	0.53**	0.50**	0.44**	-	0.42**	0.29**	0.27**	0.31**	0.39**	0.40**
15	A1 EF Composite	0.62**	0.53**	0.47**	0.48**	-	0.53**	0.49**	0.52**	0.40**	0.59**
16	A1 Vocab composite	0.58**	0.46**	0.40**	0.38**	0.59**	-	0.74**	0.88**	0.36**	0.38**
17	A2 KBIT	0.60**	0.50**	0.40**	0.37**	0.57**	0.81**	-	0.68**	0.31**	0.37**
18	A2 BNT	0.51**	0.40**	0.34**	0.34**	0.54**	0.87**	0.70**	-	0.32**	0.41**
19	A2 HTKS	0.43**	0.36**	0.33**	0.43**	0.44**	0.43**	0.39**	0.34**	-	0.37**
20	A2 MEFS	0.41**	0.33**	0.24**	0.42**	0.63**	0.43**	0.40**	0.42**	0.40**	-
21	A2 Nonsymb mag comp	0.43**	0.42**	0.39**	0.62**	0.40**	0.23**	0.26**	0.17*	0.46**	0.35**
22	A2 Verbal Counting	0.49**	0.49**	0.43**	0.38**	0.43**	0.37**	0.36**	0.41**	0.35**	0.39**
23	A2 problem solving	0.64**	0.62**	0.49**	0.55**	0.54**	0.56**	0.52**	0.46**	0.50**	0.43**
24	A2 Numerical Literacy	0.79**	0.73**	0.55**	0.53**	0.60**	0.60**	0.57**	0.51**	0.44**	0.40**
25	A2 Count on	0.73**	0.70**	0.52**	0.50**	0.57**	0.53**	0.46**	0.49**	0.44**	**0.38
26	A2 Non-rote	0.39**	0.33**	0.41**	0.38**	0.34**	0.30**	0.29**	0.26**	0.31**	0.24**
27	A2 WJ	0.82**	0.73**	0.59**	0.53**	0.64**	0.64**	0.64**	0.58**	0.49**	0.43**
28	A2 TEMA-3	0.85**	0.78**	0.57**	0.58**	0.63**	0.61**	0.61**	0.51**	0.49**	0.46**
29	A2 EF Composite	0.50**	0.40**	0.31**	0.50**	0.62**	0.50**	0.45**	0.44**	0.81**	0.84**
30	A2 Vocab Composite	0.61**	0.49**	0.41**	0.39**	0.60**	0.91**	0.93**	0.91**	0.40**	0.44**

	Variable	21	22	23	24	25	26	27	28	29	30
1	Age in Months	-	-	-	-	-	-	-	-	-	-
2	Maternal education	0.14	0.13	0.24**	0.28**	0.26**	0.14	0.34**	0.35**	0.33**	0.43**
3	A1 KBIT	0.16*	0.76**	0.43**	0.34**	0.29**	0.13	0.45**	0.37**	0.40**	0.76**
4	A1 BNT	0.17*	0.86**	0.42**	0.49**	0.42**	0.24**	0.55**	0.46**	0.41**	0.86**
5	A1 HTKS	0.43**	0.48**	0.48**	0.52**	0.47**	0.38**	0.50**	0.51**	0.58**	0.48**
6	A1 MEFS	0.26**	0.46**	0.36**	0.38**	0.37**	0.21**	0.41**	0.43**	0.44**	0.46**
7	A1 TEMA-3	0.43**	0.47**	0.58**	0.69**	0.67**	0.36**	0.77**	0.80**	0.47**	0.47**
8	A1 WJ	0.46**	0.59**	0.53**	0.60**	0.54**	0.33**	0.73**	0.70**	0.54**	0.59**
9	A1 Verbal counting	0.29**	0.24**	0.47**	0.53**	0.60**	0.35**	0.30**	0.39**	0.30**	0.24**
10	A1 problem solving	0.52**	0.46**	0.64**	0.70**	0.64**	0.42**	0.64**	0.70**	0.46**	0.46**
11	A1 Numerical Literacy	0.40**	0.48**	0.55**	0.77**	0.70**	0.34**	0.66**	0.75**	0.43**	0.48**
12	A1 Count on	0.39**	0.31**	0.56**	0.71**	0.68**	0.28**	0.55**	0.65**	0.35**	0.31**
13	A1 Non-rote	0.37**	0.29**	0.39**	0.50**	0.46**	0.38**	0.41**	0.41**	0.24**	0.29**
14	A1 Nonsymb mag comp	0.62**	0.31**	0.51**	0.49**	0.46**	0.38**	0.42**	0.48**	0.47**	0.31**
15	A1 EF Composite	0.39**	0.38**	0.47**	0.52**	0.49**	0.33**	0.54**	0.55**	0.58**	0.54**
16	A1 Vocab composite	0.18*	0.89**	0.46**	0.46**	0.40**	0.21**	0.55**	0.44**	0.44**	0.89**
17	A2 KBIT	0.20**	0.91**	0.36**	0.38**	0.28**	0.19*	0.52**	0.48**	0.39**	0.91**
18	A2 BNT	0.17*	0.92**	0.43**	0.49**	0.43**	0.21**	0.56**	0.42**	0.42**	0.92**
19	A2 HTKS	0.46**	0.33**	0.44**	0.34**	0.35**	0.29**	0.41**	0.41**	0.80**	0.33**
20	A2 MEFS	0.33**	0.42**	0.42**	0.35**	0.31**	0.21**	0.36**	0.44**	0.83**	0.42**
21	A2 Nonsymb mag comp	-	0.20**	0.42**	0.41**	0.34**	0.36**	0.39**	0.33**	0.47**	0.20**
22	A2 Verbal Counting	0.33**	-	0.34**	0.45**	0.47**	0.39**	0.30**	0.64**	0.33**	0.33**
23	A2 problem solving	0.42**	0.44**	-	0.56**	0.55**	0.37**	0.57**	0.72**	0.52**	0.43**
24	A2 Numerical Literacy	0.45**	0.51**	0.63**	-	0.80**	0.38**	0.59**	0.68**	0.41**	0.48**
25	A2 Count on	0.37**	0.54**	0.61**	0.78**	-	0.40**	0.57**	0.40**	0.39**	0.39**
26	A2 Non-rote	0.35**	0.41**	0.40**	0.39**	0.40**	-	0.35**	0.81**	0.29**	0.21**
27	A2 WJ	0.45**	0.51**	0.72**	0.75**	0.72**	0.44**	-	0.44**	0.45**	0.59**
28	A2 TEMA-3	0.49**	0.55**	0.75**	0.80**	0.76**	0.48**	0.88**	-	0.50**	0.50**
29	A2 EF Composite	0.48**	0.43**	0.56**	0.49**	0.48**	0.32**	0.53**	0.55**	-	0.43**
30	A2 Vocab Composite	0.23**	0.41**	0.53**	0.59**	0.52**	0.29**	0.66**	0.61**	0.48**	-

Note. Bivariate (lower) and partial (upper) correlations controlling for age in months are displayed. Imputed data were used for these analyses. Nonsymb mag comp = Nonsymbolic magnitude comparison. KBIT = Kaufman Brief Intelligence Test. BNT = Boston

Naming Test. EF = Executive function. MEFS = Minnesota Executive Function Scale. HTKS = Head Toes Knees Shoulders. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

*$p < .05$, **$p < .01$.

Table 5

Results of Regression Analyses Predicting Numerical Skills at Assessment 1 (N = 205)

	A1 Non-rote Counting	**A1 Verbal Counting**	**A1 Numerical Literacy**	**A1 Count On**	**A1 Nonsymbolic Magnitude Comparison**	**A1 Numerical problem solving**
Age in months	0.05***	0.84***	0.28***	0.12***	0.12	0.09***
	(0.01)	(0.20)	(0.02)	(0.02)	(0.06)	(0.02)
Maternal Education	0.07	1.69	0.17*	0.04	-0.18	0.00
	(0.04)	(0.79)	(0.07)	(0.07)	(0.30)	(0.08)
A1 Vocabulary composite	0.05	-1.14	0.71	-0.02	1.28	0.68**
	(0.12)	(3.71)	(0.39)	(0.24)	(0.90)	(0.24)
A1 EF Composite	0.70***	17.25***	3.10***	1.24***	5.13***	1.40***
	(0.06)	(4.17)	(0.33)	(0.26)	(1.07)	(0.26)
Constant	-0.25	-6.22	-12.27***	-4.68***	79.66***	0.48
	(1.11)	(29.96)	(2.37)	(1.94)	(7.66)	(1.94)
R^2	0.41	0.34	0.62	0.47	0.33	0.55

Note. EF = Executive Function. Unstandardized beta coefficients are presented with standard errors in parentheses. Clustered standard errors are presented for A1 non-rote counting, A1 verbal counting, A1 numerical literacy, and A1 count on. Benjamini–Hochberg adjustments were made to *p*-values.

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 6

Results of Regression Analyses Predicting Proportion of Accurate Operational Switches at Assessment 1 (N = 203)

	B	*SE*	*p*	R^2
Age in months	0.84	0.00	0.037	0.11
Maternal Education	0.00	0.00	0.965	
Vocabulary composite	0.00	0.01	0.995	
EF Composite	0.03*	0.02	0.037	
Constant	0.84**	0.12	<.001	

EF = Executive Function. Benjamini–Hochberg adjustments were made to *p*-values.

**p < .05, **p < .01.*

Table 7

Results of Regression Analyses Predicting Numerical Skills at Assessment 2 (N = 205)

	A2 Numerical problem solving	**A2 Verbal Counting**	**A2 Num-erical Literacy**	**A2 Count On**	**A2 Nonsymbolic Magnitude Comparison**	**A2 Non-rote Counting**
Age in months	-0.00	-0.09	-0.03	-0.01	0.02	-0.00
	(0.01)	(0.14)	(0.02)	(0.01)	(0.04)	(0.01)
Maternal Education	-0.07	0.19	-0.06	0.01	-0.10	-0.03
	(0.04)	(0.67)	(0.08)	(0.04)	(0.17)	(0.03)
A1 Vocabulary composite	0.43***	2.98	0.65*	0.20	-0.60	0.09
	(0.07)	(2.10)	(0.27)	(0.13)	(0.57)	(0.10)
A1 EF Composite	0.35	5.08	0.74*	0.40*	2.16**	0.32*
	(0.21)	(2.31)	(0.32)	(0.14)	(0.67)	(0.11)
A1 Numerical problem solving	0.40***					
	(0.07)					
A1 Verbal Counting		0.13*				
		(0.05)				
A1 Numerical Literacy			0.51***			
			(0.04)			
A1 Count on				0.35***		
				(0.04)		
A1 Nonsymbolic Magnitude Comparison					0.36***	
					(0.05)	
A1 Non-rote Counting						0.22***
						(0.05)
Constant	7.82***	93.04***	13.87***	5.42***	60.33***	4.72***
	(1.16)	(17.47)	(2.22)	(1.06)	(6.03)	(0.85)
R^2	0.63	0.22	0.73	0.58	0.43	0.27

Note. A1 = Assessment 1. A2 = Assessment 2. EF = Executive Function. Unstandardized beta coefficients are presented with standard errors in parentheses. Clustered standard errors are presented for A2 numerical problem solving. Benjamini–Hochberg adjustments were made to *p*-values.

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 8

Descriptive Statistics from Assessment 2 Numerical Skill Subgroups

				Observed Range for Subgroups^a			
Numerical Skills	*N*	*M*	*SD*	Full Group	Low	Med	High
Verbal counting	203	101.76	20.33	11 – 110	11 – 109	--	110
Numerical problem solving	203	10.07	2.1	0 – 12	0 – 8	9 – 11	12
Non-rote counting	203	5.26	1.1	0 – 6	0 – 4	5	6
Count on	203	6.93	1.69	0 - 8	0 – 5	6 – 7	8
Numerical Literacy	201	17.24	4.46	1 – 21	1 – 12	13 – 19	20 – 21

Note. Non-imputed data are presented in this table.

Table 9

Results of Regression Analyses Predicting Executive Function Composite at Assessment 2 (N = 205)

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	-0.00	-0.01	-0.00	-0.00	0.00	-0.01
	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Maternal Education	0.00	0.00	0.01	0.01	0.01	0.01
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
A1 Vocabulary Composite	0.14	0.13	0.14	0.12	0.14	0.09
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
A1 EF Composite	0.61***	0.58***	0.59***	0.53***	0.66***	0.54***
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
A1 Verbal Counting	0.00					
	(0.00)					
A1 Numerical Literacy		0.02				
		(0.01)				
A1 Count on			0.04			
			(0.02)			
A1 Nonsymbolic Magnitude Comparison				0.02**		
				(0.01)		
A1 Non-rote Counting					-0.03	
					(0.04)	
A1 Numerical problem solving						0.07**
						(0.02)
Constant	-0.24	-0.00	-0.07	-1.85*	-0.25	-0.28
	(0.59)	(0.60)	(0.59)	(0.74)	(0.59)	(0.57)
R^2	0.48	0.49	0.49	0.51	0.48	0.51

Note. A1 = Assessment 1. A2 = Assessment 2. EF = Executive Function. Unstandardized beta coefficients are presented with standard errors in parentheses. Benjamini–Hochberg adjustments were made to *p*-values.

$^{***}p < 0.001$; $^{**} p < 0.01$; $^{*} p < 0.05$

Table 10

Descriptives for Mathematics, Verbal, and EF Skills For Each Group at Assessment 1

	No Known Risk *N* = 204	**Low SES** *N* = 130	**Turner syndrome** *N* = 44
Vocabulary composite			
M (SD)	0.28 (0.81)	- 0.41 (0.97)	0 (.94)
Range	-2.15 - 2.51	-2.81 - 2.09	-2.31 - 1.66
EF Composite			
M (SD)	0.17 (0.79)	- 0.28 (0.95)	0.01 (.89)
Range	-2.81 - 1.27	-2.81 - 1.22	-1.96 - 1.24
Nonsymbolic magnitude comparison			
M (SD)	87.21 (9.18)	83.58 (11.25)	78.62 (13.7)
Range	62 - 100	62 - 98.86	47.73 - 97.73
Numerical Literacy			
M (SD)	12.31 (6.96)	10.45 (6.77)	10.59 (8)
Range	0 - 21	0 - 21	0 - 21
Count on			
M (SD)	5.19 (2.87)	4.6 (3.15)	4.45 (3.09)
Range	0 - 8	0 - 8	0 - 8
Verbal Counting			
M (SD)	88.89 (33.66)	79.62 (39.82)	69.68 (42.64)
Range	10 - 110	4 - 110	4 - 110
Non-rote counting			
M (SD)	4.58 (1.53)	4.15 (1.78)	4.07 (1.8)
Range	0 - 6	0 - 6	0 - 6
Numerical problem solving			
M (SD)	8.35 (2.99)	6.88 (3.5)	6.23 (3.91)
Range	1 - 12	0 - 12	0 - 12
TEMA-3			
M (SD)	40.51 (14.51)	35.46 (15.1)	34.7 (17.79)
Range	9 - 72	0 - 69	2 - 72
WJ-Applied Problems			
M (SD)	23.57 (5.63)	20.83 (6.12)	21.82 (6.66)
Range	9 - 39	7 - 34	10 - 40

Note. Non-imputed data were used for these analyses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

Table 11

Descriptive Statistics for Age-Normed Child Vocabulary, Executive Function, and Mathematics Tests for Each Group at Assessment 1

	N	*M*	*SD*	*Minimum*	*Maximum*
No Known Risk					
KBIT SS	204	109.24	11.78	65	140
WJ SS	203	110.93	11.66	64	152
TEMA-3 SS	199	106.22	14.20	64	145
MEFS SS	200	102.25	7.73	61	118
Low SES					
KBIT SS	130	97.96	14.73	55	130
WJ SS	128	98.85	12.80	59	128
TEMA-3 SS	124	94.70	13.73	63	139
MEFS SS	130	96.25	9.59	61	114
Turner Syndrome					
KBIT SS	44	105.45	10.83	75	125
WJ SS	44	99.02	10.83	74	122
TEMA-3 SS	44	90.77	12.76	65	120
MEFS SS	43	97.72	7.80	82	113

Note. Non-imputed standard scores were used for these analyses. KBIT = Kaufman Brief Intelligence Test. SS = Standard Score. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability. MEFS = Minnesota Executive Function Scale. KBIT scores were prorated because only one of two verbal subtests were administered.

Table 12

Partial Correlations between Mathematics Achievement and Verbal, Numerical, and Executive Function Skills by Group at Assessment 1

	No Known Risk		**Low SES**		**Turner syndrome**	
	WJ-Applied Problems	TEMA-3	WJ-Applied Problems	TEMA-3	WJ-Applied Problems	TEMA-3
Vocabulary composite	0.52**	0.37**	0.51**	0.30**	0.42**	0.28
EF Composite	0.55**	0.49**	0.62**	0.38**	0.32*	0.21
Nonsymbolic magnitude comparison	0.42**	0.35**	0.54**	0.36**	0.34*	0.25
Numerical Literacy	0.61**	0.8**	0.51**	0.74**	0.52**	0.73**
Count on	0.53**	0.77**	0.47**	0.73**	0.55**	0.74**
Verbal Counting	0.36**	0.46**	0.41**	0.66**	0.44**	.60**
Non-rote counting	0.39**	0.48**	0.4**	0.65**	0.30	.46**
Numerical problem solving	0.54**	0.58**	0.59**	0.57**	0.66**	.61**

Note. Partial correlations presented are controlling for age in months. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

**p* < .05, ***p* < .01.

Table 13

Results of Regression Analyses Predicting Mathematics as Measured by WJ-Applied Problems in No Known Risk Group (N = 204)

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	0.22***	0.21***	0.14***	0.16***	0.18***	0.20***
	(0.03)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)
Female	-0.78*	-0.76	-0.20	-0.59	-0.53	-0.75
	(0.36)	(0.37)	(0.35)	(0.35)	(0.35)	(0.36)
Maternal Education	0.15	0.14	0.09	0.08	0.15	0.09
	(0.12)	(0.12)	(0.11)	(0.11)	(0.10)	(0.11)
Vocabulary composite	1.52***	1.73***	1.51***	1.66***	1.34***	1.62***
	(0.34)	(0.34)	(0.31)	(0.32)	(0.33)	(0.33)
EF Composite	2.02***	2.01***	1.54***	1.86***	1.84***	2.04***
	(0.33)	(0.33)	(0.31)	(0.31)	(0.32)	(0.32)
Nonsymbolic Magnitude Comparison	0.09** (0.03)					
Numerical Literacy			0.29***			
			(0.04)			
Numerical problem solving					0.48***	
					(0.09)	
Non-rote counting						0.57***
						(0.15)
Verbal Counting		0.02*				
		(0.01)				
Count on				0.49***		
				(0.09)		
Constant	-3.81	2.91	7.37**	6.52*	2.69	3.65
	(3.41)	(2.76)	(2.64)	(2.71)	(2.62)	(2.74)
R^2	0.80	0.80	0.83	0.82	0.82	0.80

Note. Unstandardized beta coefficients are presented with standard errors in parentheses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

$^{***}p < 0.001$; $^{**} p < 0.01$; $^{*} p < 0.05$

Table 14

Results of Regression Analyses Predicting Mathematics as Measured by TEMA-3 in No Known Risk Group (N = 204)

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	0.69***	0.60***	0.25***	0.36***	0.53***	0.57***
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
Female	-3.31**	-3.30**	-0.52	-2.37**	-2.44*	-3.24**
	(1.11)	(1.07)	(0.83)	(0.85)	(1.02)	(1.05)
Maternal Education	0.67	0.61	0.39	0.26	0.67*	0.38
	(0.35)	(0.34)	(0.26)	(0.27)	(0.32)	(0.34)
Vocabulary composite	2.11	2.56**	1.49	2.18**	1.14	2.05*
	(1.00)	(0.97)	(0.73)	(0.78)	(0.94)	(0.96)
EF Composite	4.46***	3.47***	1.11	2.49**	3.30***	3.71***
	(0.98)	(0.95)	(0.73)	(0.75)	(0.89)	(0.91)
Nonsymbolic Magnitude Comparison	0.19* (0.08)					
Numerical Literacy			1.42***			
			(0.10)			
Numerical problem solving					1.78***	
					(0.25)	
Non-rote counting						2.64***
						(0.45)
Verbal Counting		0.10***				
		(0.02)				
Count on				2.64***		
				(0.22)		
Constant	-40.12***	-24.94**	-2.79	-5.26	-25.87***	-21.49*
	(10.42)	(8.05)	(6.27)	(6.66)	(7.63)	(7.94)
R^2	0.73	0.75	0.86	0.84	0.78	0.76

Note. Unstandardized beta coefficients are presented with standard errors in parentheses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

$^{***}p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 15

Results of Regression Analyses Predicting Mathematics as Measured by WJ-Applied Problems in Low SES Group (N = 130)

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	0.17***	0.17***	0.09***	0.11***	0.10***	0.16***
	(0.02)	(0.01)	(0.02)	(0.02)	(0.03)	(0.01)
Female	-0.71	-1.00	-0.47	-0.53	-0.72	-0.94
	(0.48)	(0.70)	(0.59)	(0.48)	(0.53)	(0.64)
Maternal Education	0.12	0.05	0.02	0.04	0.07	0.10
	(0.06)	(0.06)	(0.08)	(0.09)	(0.06)	(0.07)
Vocabulary composite	1.01*	1.13*	1.12*	1.16*	1.05*	1.23**
	(0.39)	(0.41)	(0.46)	(0.43)	(0.36)	(0.38)
EF Composite	1.96***	2.25***	1.97***	2.10***	1.61***	2.13***
	(0.19)	(0.21)	(0.27)	(0.17)	(0.20)	(0.16)
Nonsymbolic Magnitude Comparison	0.13***					
	(0.01)					
Numerical Literacy			0.32***			
			(0.06)			
Numerical problem solving					0.68***	
					(0.08)	
Non-rote counting						0.61***
						(0.13)
Verbal Counting		0.03*				
		(0.01)				
Count on				0.61***		
				(0.07)		
Constant	-3.34	6.40*	11.25***	10.20***	8.55***	5.67*
	(2.27)	(2.49)	(2.07)	(2.27)	(1.82)	(2.34)
R^2	0.79	0.78	0.79	0.80	0.80	0.78

Note. Unstandardized beta coefficients are presented with standard errors in parentheses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

$^{***}p < 0.001$; $^{**} p < 0.01$; $^{*} p < 0.05$

Table 16

Results of Regression Analyses Predicting Mathematics as Measured by TEMA-3 in Low SES Group (N = 130)

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	0.71***	0.57***	0.24***	0.36***	0.46***	0.55***
	(0.04)	(0.07)	(0.05)	(0.03)	(0.03)	(0.04)
Female	-3.47***	-4.18***	-1.55	-2.02	-3.23***	-3.85***
	(0.52)	(0.67)	(1.03)	(1.18)	(0.74)	(0.57)
Maternal Education	0.55*	0.15	0.11	0.22	0.41*	0.44*
	(0.23)	(0.17)	(0.19)	(0.17)	(0.18)	(0.20)
Vocabulary composite	-0.63	-0.53	-0.54	-0.32	-0.67	0.08
	(1.42)	(0.44)	(0.92)	(0.39)	(1.04)	(0.61)
EF Composite	3.74***	3.12***	2.08***	2.85***	2.01*	2.44***
	(0.48)	(0.24)	(0.40)	(0.76)	(0.81)	(0.55)
Nonsymbolic Magnitude Comparison	0.24* (0.10)					
Numerical Literacy			1.53***			
			(0.10)			
Numerical problem solving					2.06***	
					(0.39)	
Non-rote counting						3.58***
						(0.30)
Verbal Counting		0.16***				
		(0.02)				
Count on				2.72***		
				(0.34)		
Constant	-45.68**	-22.07**	0.14	-6.97	-19.43*	-26.31***
	(15.23)	(6.79)	(5.74)	(4.62)	(7.18)	(5.70)
R^2	0.75	0.83	0.86	0.86	0.80	0.83

Note. Unstandardized beta coefficients are presented with standard errors in parentheses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

$^{***}p < 0.001$; $^{**} p < 0.01$; $^{*} p < 0.05$

Table 17

Results of Regression Analyses Predicting Mathematics as Measured by WJ-Applied Problems in Turner Syndrome Group

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	0.18^{**}	0.07	0.09	0.17^{*}	0.15^{*}	0.10
	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)
Maternal Education	0.13	0.14	0.07	0.27	0.12	0.11
	(0.26)	(0.24)	(0.22)	(0.27)	(0.25)	(0.23)
Vocabulary composite	2.00	1.56	1.41	1.98	1.97	1.52
	(0.85)	(0.81)	(0.75)	(0.91)	(0.82)	(0.79)
EF Composite	1.15	1.19	0.49	0.87	0.94	1.14
	(0.81)	(0.72)	(0.70)	(0.85)	(0.78)	(0.71)
Nonsymbolic Magnitude Comparison	0.06 (0.05)					
Numerical Literacy		0.35^{*}				
		(0.13)				
Numerical problem solving			0.86^{***}			
			(0.22)			
Non-rote counting				0.55		
				(0.48)		
Verbal Counting					0.04	
					(0.02)	
Count on						0.82^{*}
						(0.26)
Constant	0.23	9.52	7.76	1.39	4.80	7.67
	(6.72)	(6.41)	(5.65)	(6.71)	(6.26)	(6.05)
R^2	0.81	0.84	0.86	0.81	0.82	0.84
N	43	43	43	42	43	43

Note. Unstandardized beta coefficients are presented with standard errors in parentheses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 18

Results of Regression Analyses Predicting Mathematics as Measured by TEMA-3 in Turner Syndrome Group

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
Age in months	0.68^{***}	0.22	0.45^{**}	0.57^{***}	0.53^{***}	0.36^{**}
	(0.13)	(0.13)	(0.13)	(0.13)	(0.12)	(0.11)
Maternal Education	-0.05	-0.16	-0.24	0.59	-0.25	-0.25
	(0.67)	(0.49)	(0.57)	(0.64)	(0.56)	(0.48)
Vocabulary composite	3.43	1.39	1.84	2.55	3.07	1.36
	(2.20)	(1.67)	(1.92)	(2.12)	(1.84)	(1.63)
EF Composite	1.48	1.00	-0.39	-0.43	-0.15	0.84
	(2.10)	(1.48)	(1.81)	(1.97)	(1.75)	(1.45)
Nonsymbolic Magnitude Comparison	0.12					
	(0.12)					
Numerical Literacy		1.48^{***}				
		(0.26)				
Numerical problem solving			2.21^{***}			
			(0.56)			
Non-rote counting				3.08^{*}		
				(1.12)		
Verbal Counting					0.16^{***}	
					(0.04)	
Count on						3.24^{***}
						(0.54)
Constant	-30.06	3.46	-12.00	-34.26^{*}	-15.89	-5.33
	(17.39)	(13.24)	(14.47)	(15.67)	(14.06)	(12.39)
R^2	0.82	0.90	0.87	0.85	0.88	0.91
N	43	43	43	42	43	43

Note. Unstandardized beta coefficients are presented with standard errors in parentheses. EF = Executive function. WJ = Woodcock Johnson. TEMA-3 = Test of Early Mathematics Ability.

$^{***}p < 0.001$; $^{**} p < 0.01$; $^{*} p < 0.05$

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Appendix

Details about Minnesota Executive Function Scale Scoring

In January 2021 the MEFS released an update to the algorithm and norms used to calculate total scores and standard scores (Carlson, 2021). The algorithm update still uses accuracy and reaction time to calculate total scores, but the algorithm used slightly changed. The standard scores are still the age-adjusted total scores. As more individuals take the MEFS within a given age range, the age-adjustments can increase in precision. Therefore, updates to the norms are released when appropriate. In April 2019 there were 32,800 typically developing individuals in the U.S. aged 2-17.9, plus 1,370 adults in the MEFS pool for norming; in 2021 there were 51,424 individuals aged 2-17.9, plus 1,815 adults in the MEFS pool for norming (Carlson, 2021).

There has been one publication using this dataset thus far using the 2019 norms (Lukowski et al., 2020), and due to the timing the older MEFS algorithm and norms were used. To determine if it was possible to utilize the newer MEFS score, I compared the older versus the newer total and standard scores from the MEFS. Correlations among total and standard scores were significant and high (≥.929) at A1 and A2. There were significant differences in means, such that 2021 the total scores for were significantly higher than the 2019 total scores at A1 and A2. The standard scores from 2021 were significantly lower than the 2019 standard scores. The 2021 scoring algorithm was used for the current study given the larger representative sample that informed it.