

# Technical Report

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A Taxonomy for Task Allocation Problems with Temporal and Ordering  
Constraints

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## Abstract

Previous work on assigning tasks to robots has proposed extensive categorizations of allocation of tasks with and without constraints. The main contribution of this paper is a more specific categorization of problems that have both temporal and ordering constraints. We propose a novel taxonomy that builds on the existing taxonomy for multi-robot task allocation and organizes the current literature according to the temporal nature of the tasks. We summarize widely used models and methods from the task allocation literature and related areas, such as vehicle routing and scheduling problems, showing similarities and differences.

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## 1. Introduction

2     What is multi-robot task allocation? Think of a shipping company that sells  
3     an item every hour; a robot at the warehouse could receive that order, fetch the  
4     item, pack it, and prepare it for pick-up by a postal service. What happens when  
5     the company sells 20 items every hour? What about 20 items every minute?  
6     What about 20 items a second? Amazon, a popular shopping website, sold 36.8  
7     million items on an especially popular shopping day in 2013. With 426 items  
8     ordered per second that day, a single robot would be hard-pressed to keep up  
9     with the orders. If the warehouse used a large team of robots, each robot would  
10    have to plan an efficient route through the warehouse to fetch items for shipping  
11    without colliding with other robots, without taking items that another robot is  
12    handling, all while planning its route around fetching items that are out-of-stock  
13    but will be restored soon.

14    Allocation of tasks with constraints on when, where, and in what order they  
15    need to be done by groups of robots is an important class of problems with many  
16    real-life applications. Applications include warehouse automation, pickup and  
17    delivery, surveillance at regular intervals, space exploration, search and rescue,  
18    and much more.

19    The nature of the temporal constraints in this class of problems is very broad;  
20    for example, in search and rescue domains the tasks are discovered over time and  
21    have to be done as quickly as possible. In dynamic environments, robots might

22 end up arriving late to some tasks and even miss some. On the other hand,  
23 success in surveillance tasks requires not to arrive to tasks late. Additionally,  
24 tasks may need to be executed in a specific order, such as in urban disaster  
25 scenarios in which police must clear blockades from roads before firetrucks can  
26 find and put out fires. Other tasks may need to be done synchronously, as in  
27 surveillance where robots have to track multiple people at the same time. The  
28 different nuances of temporal and ordering constraints lead to different models  
29 and solutions.

30 Previous taxonomies, such as Gerkey and Mataric [2004]’s key taxonomy  
31 and more recently the iTax taxonomy [Korsah et al., 2013], devote only lim-  
32 ited attention to tasks that have temporal and ordering constraints. Our work  
33 attempts to fill this gap. We address the following research questions:

- 34 • What are the predominant types of temporal constraints in multi-robot  
35 task allocation?
- 36 • What are the most commonly used optimization objectives? Are they  
37 predominantly temporal-based, distance-based, or multi-objective?
- 38 • What models and methods from related areas can be applied to this class  
39 of problems?
- 40 • Which questions or variants have been answered well, and which remain  
41 largely open in this class of problems?

42 Our main contribution is a novel taxonomy that extends Gerkey and Mataric  
43 [2004], Korsah et al. [2013] taxonomies according to the nature of the temporal  
44 and ordering constraints considered. Gerkey’s taxonomy is based on three main  
45 characteristics of robots, tasks, and time. It considers the following axes:

- 46 • *Single-task robots (ST) vs. multi-task robots (MT)*: ST robots can do at  
47 most one task at a time, while MT robots can work on multiple tasks  
48 simultaneously.
- 49 • *Single-robot tasks (SR) vs. multi-robot tasks (MR)*: SR tasks require ex-  
50 actly one robot in order to be completed, while multiple robots are needed  
51 to complete an MR task.
- 52 • *Instantaneous (IA) vs. time-extended (TA) assignments*: In IA, tasks are  
53 allocated as they arrive, while in TA, tasks are scheduled over a planning  
54 horizon (defined in Section 2.1).

55 Our taxonomy focuses on time-extended task allocation problems with tem-  
56 poral and ordering constraints. Consequently, we drop the IA and TA distinc-  
57 tion and replace it with our classification axes. However, when appropriate, we  
58 highlight if a problem or solution is of IA nature.

59 *1.1. Organization*

60 We begin by defining the class of multi-robot task allocation problems with  
61 temporal and ordering constraints (MRTA/TOC) in Section 2. In Section 3  
62 we relate this class of problems to problems in other areas, setting the ground  
63 for our exploration of the models and methods in those areas. In Section 4 we  
64 present commonly used temporal and ordering models. In Section 5 we review  
65 the most common optimization objectives considered in the literature. Our  
66 taxonomy is introduced in Section 6. Task execution and the dynamics therein  
67 are discussed in Section 7. Solutions are introduced in Section 8. We discuss  
68 open issues, future directions, and final thoughts in Section 9.

69 Next, we formally define the task allocation problem with temporal and  
70 ordering constraints (MRTA/TOC), and summarize the terminology we use.

71 **2. MRTA/TOC: Multi-robot Task Allocation with Temporal and Or-**  
72 **dering Constraints**

73 *2.1. Terminology and Abbreviations*

74 We define the terminology we use informally as follows:

- 75 • A *robot* is an autonomous agent responsible for performing some actions.  
76 Alternative names for robots are physical agents, unmanned vehicles, and  
77 rovers. Robots in MRTA/TOC are often modeled as holonomic and point  
78 robots, since the focus is not in low level control of robot motion.
- 79 • A *team* is a set of robots that work together. A team is called a *coalition*  
80 when it is formed to do some tasks and disbanded after that [Parker and  
81 Tang, 2006].
- 82 • A *task* is an action to be performed, also referred to as a work unit,  
83 activity, waypoint, or customer request. In some scheduling literature  
84 tasks are divided into jobs [Davis and Burns, 2011], while in other cases  
85 jobs consist of tasks [Balas et al., 2008].
- 86 • A *time window* is a time interval composed of the earliest time a task can  
87 start, and the latest time it can end. If an earliest time is not given, the  
88 latest time is referred to as a deadline constraint.
- 89 • A *schedule* is a timetable in which each task has a specific time to start,  
90 end, or both. In some cases each robot has its own individual schedule,  
91 while in others all robots share a single schedule.
- 92 • The *scheduling horizon* is the time period for which schedules are created.  
93 Alternatively, it is the end time, after which robots are not allowed to  
94 start or end tasks.
- 95 • The *planning horizon* is the time period over which plans are created.

- 96 • The *makespan* is the time difference between the end of the last task and  
97 the start of the first task.
- 98 • A *route* is a sequence of locations to visit. Routes and schedules are often  
99 used interchangeably, but schedules always concern time, while routes  
100 concern physical locations.
- 101 • A *task release* refers to a task becoming available for execution. Task  
102 release can be deterministic if the release time is known upfront, dynamic  
103 if the release time is stochastic, or sporadic if it is governed by unknown  
104 probabilities; task release is also called periodic when the same task is  
105 released at regular intervals.

106 We use the following acronyms:

- 107 • MRTA/TOC for Multi-Robot Task Allocation with Temporal and Order-  
108 ing Constraints.
- 109 • MIP for Mixed Integer Programming and MILP if the objective function  
110 and constraints are linear.
- 111 • TOPTW for Team Orienteering Problem (TOP) with Time Windows.
- 112 • VRPTW for Vehicle Routing Problem (VRP) with Time Windows.
- 113 • JSP for job-shop scheduling problems.

## 114 2.2. Problem Formulation

115 All the problems herein considered allocate tasks with either temporal or  
116 ordering constraints, or both. In MRTA/TOC we assume there is a finite set of  
117 robots and a set of tasks. A robot may have a location, velocity, route, and/or  
118 schedule. A task is defined by a subset of the following parameters: location,  
119 expected duration, cost, demand, reward, earliest start, and latest finish time.

120 Ordering constraints express a dependency between tasks, and are usually  
121 encoded as directed acyclic graphs. Each node in the graph represents a task,  
122 and each edge indicates precedence or simultaneity in the order of execution of  
123 the tasks.

124 The objective is to optimize some function of the cost (or reward) for doing  
125 the tasks for all the robots. Cost can be a time measure (e.g. makespan), or a  
126 spatial measure (e.g distance traveled). Commonly used optimization functions  
127 are more thoroughly described later in Section 5.

## 128 3. Connections with Other Problems

129 Multi-robot task allocation (MRTA) started in earnest in the 90's, when re-  
130 searchers started pulling together teams of robots to accomplish multiple tasks.  
131 MRTA draws from a variety of areas in mathematics and operations research

132 as well as computer science and robotics, including assignment problems, dis-  
133 tributed computing, distributed AI, and scheduling.

134 The search for robust approaches to MRTA and related problems focused  
135 on how the robots perform in complex environments, leading researchers to add  
136 features like uncertainty with probabilistic and stochastic models, time windows  
137 for tasks, and spatial constraints. Solutions take different approaches, such  
138 as auctions, market-based planning, Markov Decision Processes, decentralized  
139 scheduling algorithms, and distributed constraint optimization.

140 In this paper we cover a subset of MRTA problems, which we call MRTA/TOC,  
141 to highlight the importance of temporal and ordering constraints among tasks  
142 and to shed light on how the inclusion of temporal and ordering constraints  
143 increases the complexity of task allocation.

144 Similar types of problems include the vehicle routing problem [Dantzig and  
145 Ramser, 1959], the job shop scheduling problem, and the team orienteering  
146 problem. Overall, multi-robot task allocation diverges from each of these prob-  
147 lems on key points, including assumptions on the number of robots, robot and  
148 task homogeneity, environment dynamics caused by failures or interference from  
149 other robots, and communication restrictions.

150 We are now prepared to discuss the relationship between MRTA/TOC prob-  
151 lems and vehicle routing problems with time windows (VRPTW), team orien-  
152 teering problem with time windows (TOPTW), and job-shop scheduling prob-  
153 lems (JSP).

154 **MRTA/TOC vs. VRPTW:** like the MRTA/TOC problem, the vehicle rout-  
155 ing problem with time windows (VRPTW) [Kolen et al., 1987, Solomon and  
156 Desrosiers, 1988, Desrochers et al., 1988, Toth and Vigo, 2002] studies problems  
157 which require solving allocation, routing, and scheduling subproblems simul-  
158 taneously. Vehicles and robots are often treated as points in space, ignoring  
159 kinematic constraints, but kinematic [e.g. Cheng et al., 2008, for unmanned  
160 aerial vehicles] and sometimes dynamic [Pecora and Cirillo, 2012, for ground  
161 vehicles] constraints can be considered.

162 The solutions to several variants of VRPTW – such as multi-depot [Kang  
163 et al., 2005, Polacek et al., 2004], dynamic and stochastic [Taş et al., 2013,  
164 Pavone et al., 2011, Laporte et al., 1992], and precedence and synchronization  
165 constrained [Korsah et al., 2012, Bredström and Rönnqvist, 2008] – have been  
166 extended to MRTA/TOC settings. An example of VRP similarities is the on-  
167 line pickup and delivery problem with transfers, where a team of vehicles has to  
168 pick up a set of items at a location and deliver them to another location [Coltin  
169 and Veloso, 2014a]. This problem is a generalization of the pickup and delivery  
170 problem which is well studied in operations research. However, the proposed  
171 solution is a typical MRTA approach. The authors combine a centralized tempo-  
172 ral planner, which creates initial schedules, with auctions, which are used to  
173 repair the plans when delays or failures occur. In the same vein, Korsah et al.  
174 [2012] studied a MRTA problem that can be framed as a vehicle routing problem  
175 with temporal, precedence and synchronization constraints. The authors offer  
176 a MILP-based model and an optimal Branch-and-Price solution.

177 Despite their similarities, these problems differ in some ways. First, VRPTW

178 assumes an infinite number of vehicles is always available, with a few exceptions  
179 [e.g. Lau et al., 2003]). This assumption is not practical in robotic systems  
180 where the number of robots is usually fixed and can even decrease due to fail-  
181 ures. VRPTW problems usually assume that all vehicles start from the same  
182 depot and return to the depot after work. In MRTA/TOC problems, robots  
183 may start at different locations and do not need to return to their initial lo-  
184 cations. VRPTW problems mostly assume homogeneous vehicles with respect  
185 to their capabilities and capacities [for exceptions, see Bettinelli et al., 2011,  
186 Dondo and Cerdá, 2007]. In MRTA/TOC, however, robots are not necessar-  
187 ily homogeneous and their capacities and types can differ [Ponda et al., 2010,  
188 Schneider et al., 2005, Xu et al., 2005]. Lastly, MRTA/TOC problems, unlike  
189 VRPTW problems, usually use communication, often with constraints. In [Mer-  
190 cker et al., 2010] the communication graph is unknown (hence the algorithm does  
191 not always converge), while in [Ponda et al., 2012a] the communication graph  
192 is maintained by using specialized robots or robots not working on a task to  
193 act as communication relays. While in the previous two works convergence is  
194 guaranteed only for complete communication graphs, Jackson et al. [2013] and  
195 Smith and Bullo [2007] proposed distributed algorithms that converge with only  
196 local communication.

197 **MRTA/TOC vs. TOPTW:** in TOPTW, an origin and destination pair is  
198 given, and the goal is to search for control points to visit between the origin and  
199 destination such that the profit (or score function) is maximized while respecting  
200 all constraints. Each control point is associated with a profit (or score), and  
201 each edge connecting control points is weighted by the cost of moving between  
202 the control points [Labadie et al., 2012]. Control points are equivalent to tasks  
203 for robots in MRTA/TOC.

204 When TOPTW considers the origin and destination pairs to be the same  
205 point, then we have sub-tours similar to those for VRPTW problems, which  
206 can be described as vehicle routing problems with profit [Archetti et al., 2014].  
207 One application of TOPTW problems, dial-a-ride, has gained some popularity  
208 in MRTA/TOC [e.g. Coltin and Veloso, 2014a, Rubinstein et al., 2012, Bourous  
209 et al., 2011], In dial-a-ride, the problems are over-constrained [Carrabs F., 2007,  
210 Cordeau and Laporte, 2007], which means that not all the tasks can be per-  
211 formed, and thus the goal is to find the subset of tasks that maximizes the total  
212 profit [Rubinstein et al., 2012].

213 **MRTA/TOC vs. JSP:** job-shop scheduling problems are concerned with al-  
214 locating groups of activities, called jobs, to a set of machines with the goal of  
215 minimizing the cost of completing the jobs, alone or in combination with other  
216 objectives [Allahverdi et al., 2008, Graham et al., 1979]. The problem can be  
217 decomposed into sequencing the activities and assigning start and end times  
218 to them (scheduling), which are solved simultaneously. Certain MRTA/TOC  
219 problems can be modeled as job-shop scheduling with setup times, deadlines  
220 and precedence constraints [Cesta et al., 2000, Balas et al., 2008, Oddi et al.,  
221 2011]; these problems include [Nunes and Gini, 2015, Gombolay et al., 2013,  
222 Dahl et al., 2009], although [Nunes and Gini, 2015] does not consider precedence  
223 constraints. In order to model MRTA/TOC problems as job-shop scheduling

224 problems, tasks are treated as jobs and robots as machines. We can map simple  
225 tasks to a job with only one activity, and complex tasks with subtasks to a job  
226 with multiple activities.

227 The mathematical models for job-shop scheduling do not apply directly to  
228 MRTA/TOC problems, because job-shop scheduling does not account for travel  
229 time. When setup times are used in job-shop scheduling, the setup time typically  
230 depends on the machine and not on the time needed for the job to reach the  
231 machine. The equivalent of travel time would be to use setup times that depend  
232 on the specific job [Korsah et al., 2013].

233 Modeling MRTA/TOC problems as job-shop scheduling problems is most  
234 useful when models and methods developed for scheduling [Cesta and Oddi,  
235 1996, Cesta et al., 1999, Lee et al., 2009, Shah et al., 2009] are combined with  
236 MRTA solution techniques. In [Gombolay et al., 2013] a centralized approach  
237 is proposed in which a central temporal network is used and integrated with  
238 a MILP-based planner yielding near optimal schedules. In the decentralized  
239 approach of Barbulescu et al. [2010] each robot forms its own simple temporal  
240 network [Dechter et al., 1991], encoding both temporal constraints and preced-  
241 ence constraints in the network. To enforce precedence constraints a robot has  
242 to know which other robots depend on its schedule, so a high communication  
243 overhead is required to keep all robots up-to-date. The distributed approach in  
244 [Nunes and Gini, 2015] cuts down on communication costs by having each robot  
245 keep its own independent local temporal network and uses sequential single-item  
246 auction for allocation.

247 Having outlined the differences and similarities between MRTA/TOC and re-  
248 lated problems, we now turn our attention to temporal and ordering constraints  
249 on MRTA problems.

## 250 4. Temporal Models and Task Ordering

251 Time models are outlined in Subsection 4.1 and Subsection 4.2, the nature  
252 of ordering constraints is presented in Subsection 4.3, while in Subsection 4.4  
253 we discuss the nature of temporal constraints.

### 254 4.1. Relationships between time intervals

255 In general terms, time can be modeled using points or intervals [Allen, 1983].  
256 An example time point is 10 am, while an interval is a continuous set of val-  
257 ues bounded below and above by some time point, for example [10 am-12 pm].  
258 When representing temporal constraints we may use either representation; how-  
259 ever, the interval representation is much more common and is referred to as a  
260 time window. Time windows of tasks in general are allowed to overlap [e.g. Bar-  
261 bulescu et al., 2010, Gombolay et al., 2013, Heilporn et al., 2010, Koes et al.,  
262 2005, Nunes and Gini, 2015, Ponda et al., 2010, Luo, 2014].

263 The seminal paper by Allen [1983] proposed a set of relationships that hold  
264 between any two time intervals, as depicted in Fig. 1.

265 While the relationships originally were described between qualitative time  
266 intervals, they are also useful to describe the ordering between quantitative time

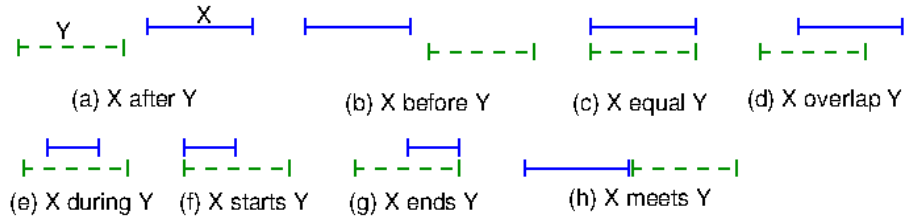


Figure 1: All possible relationships between pairs of time intervals [Allen, 1983]

267 intervals. The relationships can be used to model partial or complete ordering  
 268 constraints between tasks, – for example, task  $X$  should be done before, after,  
 269 or at the same time as task  $Y$ . The  $X$  before  $Y$  operator can be used to describe  
 270 precedence constraints between tasks, while the  $X$  equal  $Y$  operator describes  
 271 a simultaneity constraint between the intervals or time points of two tasks.

#### 272 4.2. Simple Temporal Networks (STN)

273 Equally influential is Dechter’s approach [Dechter et al., 1991], which pro-  
 274 posed to represent temporal constraints with a graph, called a simple temporal  
 275 network (STN). An example is in Fig. 2.

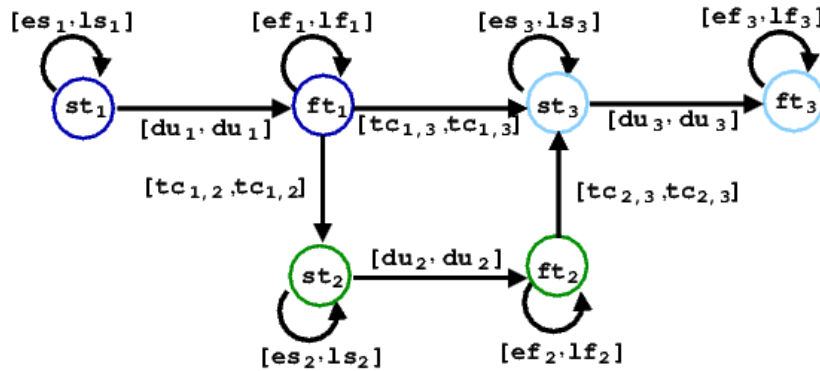


Figure 2: A simple temporal network with three tasks 1, 2 and 3. The self-loops on tasks indicate the absolute start and end times for the task. Task 1 is done first, and then there is a choice of doing 2 and 3 or just 3.  $st$  and  $ft$  are the actual start and finish times for each task,  $es$  and  $ls$  are the earliest and latest times tasks can start, similarly  $ef$  and  $lf$  are the earliest and latest times tasks can finish, and  $du$  represent tasks’ durations.  $tc_{k,k'}$  is a time cost defined as the sum of the travel and wait times, which constrain when the next task  $k'$  can be started.

276 Nodes represent time point variables or time events, and weighted edges  
 277 represent inequality constraints between time points. To reduce computational  
 278 complexity, this model requires exactly one constraint between every pair of  
 279 time point variables. This allows a solution to the scheduling problem to be

280 computed in polynomial time using the Floyd-Warshall algorithm. In an STN  
281 the relationship between time windows can be represented by establishing con-  
282 straints between start and finish times of tasks. While there are more complex  
283 models, for instance [Stergiou and Koubarakis, 2000, Block et al., 2006], in  
284 general these are NP-hard and can be approximated by solving several simple  
285 temporal problems [Boerkoel and Durfee, 2013].

286 STNs are commonly used in MRTA problems [Nunes and Gini, 2015, Gom-  
287 bolay et al., 2013, Barbulescu et al., 2010] because constraint consistency can be  
288 efficiently verified in polynomial time [Planken et al., 2008, Xu and Choueiry,  
289 2003, Dechter et al., 1991]. An important feature of STNs is that new time  
290 points and constraints can be dynamically added in polynomial time [Coles  
291 et al., 2009, Cesta and Oddi, 1996], which is beneficial in dynamic domains  
292 where new tasks can appear and disappear.

293 STNs have been successfully extended to multi-agent settings [Boerkoel and  
294 Durfee, 2012, Boerkoel and Planken, 2012, Hunsberger, 2002] and to scenarios  
295 with uncertainties. Vidal [1999] uses set bounded uncertainty to model dura-  
296 tion uncertainty of temporal events in an STN, and introduces the STN with  
297 uncertainty (STNU). Tsamardinou [2002] and Fang et al. [2014] extend STNUs  
298 by modeling uncertainty as probabilities. The former attempts to minimize the  
299 risk of temporal inconsistencies occurring, and the latter attempts to bound the  
300 probability of not meeting a schedule, respectively.

### 301 *4.3. Task ordering*

302 Precedence and simultaneity constraints are common in MRTA/TOC prob-  
303 lems [Korsah et al., 2010, Gombolay et al., 2013, Barbulescu et al., 2010]. Or-  
304 dering constraints force MRTA/TOC solutions to follow the partial or complete  
305 ordering of the tasks. Any solution violating the ordering of tasks is consid-  
306 ered illegal. If time windows are pairwise disjoint, except possibly for the end-  
307 points [Melvin et al., 2007], and the robots move in 2D then the strict order of  
308 the tasks simplifies the solution and allows for special cases where a polynomial  
309 solution exists.

310 The introduction of temporal and ordering constraints increases the com-  
311 plexity of task allocation, because solutions might contain assignments of tasks  
312 that depend on each other to different robots, creating execution dependencies  
313 among robots. This is undesirable because exogenous events affecting one robot  
314 will also affect all the robots that depend on the affected robot. The complexity  
315 of ordering constraints is further discussed in Jones et al. [2011], where intra-  
316 path precedence constraints among robots have been shown to add complexity  
317 to time-extended coordination solutions, when more than one task is assigned  
318 to each agent in domains that have intra-path constraints, such a disaster re-  
319 sponse. Luo et al. [2011] present an alternative model that divides tasks into  
320 disjoint sets with precedence constraints between the sets, each task takes the  
321 same time, and each robot can do only one task from each set.

322 A rare case is when executing a task precludes the execution of another.  
323 This type of problem is typically addressed at the planning stage, enforcing  
324 precedence constraints between the tasks [e.g. Olawsky and Gini, 1990].

#### 325 4.4. Hard vs. Soft Constraints

326 Temporal constraints can be characterized as hard or soft constraints. Hard  
327 temporal constraints require that no temporal constraint is violated [Borning  
328 et al., 1992]. MRTA/TOC and related areas frequently require rigid time win-  
329 dows for tasks such as surveillance, routing for blood supply, and order fulfill-  
330 ment by warehouse robots, so many works focus on hard temporal constraints.

331 Soft temporal constraints allow some temporal constraints to be violated or  
332 some tasks to be skipped entirely, as long as the robot incurs a penalty for doing  
333 so [Bistarelli et al., 2007, Domshlak et al., 2006, Gerevini and Long, 2005].

334 Common types of soft temporal constraints include:

- 335 1. agents can start tasks early and/or finish tasks late with some penalty  
336 (called *soft* constraints in real time system terminology);
- 337 2. comply with the deadlines with some probability [Zheng and Woodside,  
338 2003];
- 339 3. skip a number of consecutive tasks entirely or skip some percentage of  
340 tasks entirely [Bernat et al., 2001] without penalty (called *weakly hard*  
341 constraints in the real time systems terminology);
- 342 4. finish a task late without reward, or skip without penalty (called *firm*  
343 tasks [Bernat et al., 2001]);
- 344 5. use a mix of positive and negative preferences as constraints [Bistarelli  
345 et al., 2007, Domshlak et al., 2003], usually found in constraint and logic  
346 programming research.

347 The penalty incurred may differ depending on which constraint was violated;  
348 for example, finishing tasks late may be penalized more severely than doing  
349 tasks early. All non-hard constraints are called soft. Soft here is equivalent  
350 to modeling constraints as preferences, which encompasses all the non-hard  
351 constraints. These preferences do not take temporal constraints into account,  
352 but provide a way of comparing plans when several agreeable solutions are  
353 available.

354 Having discussed temporal models and constraints, and the nature of order-  
355 ing constraints, we switch focus to optimization objectives. Determining these  
356 objectives is another important aspect to consider when building models for  
357 MRTA/TOC problems.

## 358 5. Optimization Objectives

359 Applications of MRTA/TOC problems require the robots to achieve a given  
360 optimization objective. In the rest of this paper we will refer to  $f(\cdot)$  as a generic  
361 function representing one of these objectives. There can be a single or multiple  
362 objectives [Jozefowicz et al., 2008]. Depending on the deterministic or stochastic  
363 nature of the problem, objectives will either be over actual or expected values.  
364 Optimization objectives might require a quantity to be minimized, usually a cost  
365 [Nunes and Gini, 2015, Gombolay et al., 2013, Chopra and Egerstedt, 2012] or  
366 regret [Heap and Pagnucco, 2014, Wu and Jennings, 2014], or to be maximized,

367 usually a score [Mercker et al., 2010, Ponda et al., 2010] or a reward [Korsah  
368 et al., 2012, Melvin et al., 2007, Koes et al., 2005]. Single optimization objectives  
369 may be of spatial nature (e.g. minimize total distance traveled) or of temporal  
370 nature (e.g. minimize makespan).

371 Common optimization objectives for MRTA/TOC problems include:

- 372 • MiniSUM, i.e. minimize the sum of the robot path costs over all the robots  
373 [Lagoudakis et al., 2005]. Minimizing the distance traveled is common [e.g.  
374 Coltin and Veloso, 2014b, Chopra and Egerstedt, 2012, MacKenzie, 2003])  
375 but some instead minimize a time measure over robot paths [e.g. Heap and  
376 Pagnucco, 2014, Barbulescu et al., 2010]).
- 377 • MiniMAX, i.e. minimize the maximum path cost of a robot over all the  
378 robots [Lagoudakis et al., 2005]. Instead of minimizing the maximum  
379 path cost, a similar objective function is to minimize the makespan, i.e.  
380 the time difference between the start of the first and the end of the last  
381 task [Graham et al., 1979]. In [Nunes and Gini, 2015] the makespan is  
382 minimized in a decentralized manner while in [Gombolay et al., 2013] the  
383 makespan, along with other objectives, is minimized using a near-optimal  
384 centralized MILP-based planner.
- 385 • MiniAVE: i.e. minimize over all the tasks the average cost of the path  
386 for a robot from its initial location to the task location, assuming each  
387 task is visited by a single robot [Lagoudakis et al., 2005]. This is known  
388 as the Traveling Repairman Problem [Fakcharoenphol et al., 2007], where  
389 the objective is to minimize the wait time of the customers (or tasks) for  
390 a repairman (or robot).
- 391 • Minimize lateness or tardiness, which is the difference between the earliest  
392 start time of a task and the actual arrival time of the robot [Ponda et al.,  
393 2010, Rubinstein et al., 2012, Beck and Refalo, 2003]. A similar objective  
394 is to minimize the idle time of the robots [Hasgül et al., 2009].
- 395 • Maximize the number of tasks completed [Lau et al., 2003, Colorni and  
396 Righini, 2001] or minimize the number of tasks missed [Hasgül et al., 2009].
- 397 • Minimize the number of robots used. This is very common in vehicle  
398 routing problems, where there is an unlimited number of vehicles available  
399 [Luo and Schonfeld, 2007, Bräysy and Gendreau, 2005a, Desrochers et al.,  
400 1988].
- 401 • Maximize profit, measured as the difference between the reward of tasks  
402 and their respective costs [Korsah et al., 2012, Melvin et al., 2007], or  
403 as the team utility [Amador et al., 2014, Ponda et al., 2010, Koes et al.,  
404 2005].

405 While not extensively covered here, multi-objective problems are common  
406 [Jozefowiez et al., 2008], especially when objectives are combined through linear  
407 aggregation. For example, makespan and distance are minimized in [Ponda

408 et al., 2010, Nunes and Gini, 2015], while [Gombolay et al., 2013] also minimizes  
 409 workspace overlap.

410 The problems in the subcategories of our taxonomy can, in most cases,  
 411 be formalized using MIP programs. In other cases, where tasks' locations or  
 412 durations, or travel times are probabilistic, stochastic models (e.g. Markov  
 413 Decision Processes) are more commonly used. We give specific examples of  
 414 different objectives within the subcategories, using  $f(\cdot)$  as a generic objective  
 415 function. The constraints include coverage constraints that dictate the number  
 416 of robots required to complete a task as well as the number of tasks a robot is  
 417 allowed to complete at a time; ordering constraints, for example precedence and  
 418 simultaneity constraints; and side constraints, such as resource constraints.

419 We summarize the notation we use in Table 1.

Agents	
$A$	set of agents
$a$	agent in set $A$
$q_a$	capacity of agent $a$
Tasks	
$K$	set of tasks
$k$	task in set $K$
$es_k$	earliest start time of task $k$
$ls_k$	latest start time of task $k$
$ef_k$	earliest finish time of task $k$
$lf_k$	latest finish time of task $k$
$st_k$	actual start time of task $k$
$ft_k$	actual finish time of task $k$
$du_k$	duration of task $k$
Optimization	
$f(\cdot)$	generic optimization function
$x_k^a$	indicator assignment of task $k$ to robot $a$
$o_{kk'}^a$	indicator that agent $a$ performs task $k'$ directly after $k$
$tt_{kk'}$	travel time between tasks $k$ and $k'$
$w_k^a$	workload for task $k$ when performed by robot $a$
$v_k^a$	indicator that robot $a$ performs task $k$ first
$z_k^a$	indicator that robot $a$ performs task $k$ last
$Y_a$	set of possible routes for agent $a$
$\hat{x}_y^a$	indicator assignment of route $y$ to agent $a$
$b_{yk}^a$	indicator that task $k$ is in route $y$ of agent $a$
$c_y^a$	cost to agent $a$ of route $y$
$u_k^a$	reward agent $a$ collects for performing task $k$

Table 1: Notation used in the paper

420 **6. Taxonomy**

421 We are now ready to introduce our extensions to the taxonomy of Gerkey and  
 422 Mataric [2004] and focus on time-extended assignments, in which robots build  
 423 schedules for the tasks. We categorize the literature according to the temporal  
 424 constraint types, while keeping the ST-MT and SR-MR axes to describe the  
 425 robot capabilities and the task needs, respectively. We add the following new  
 426 axis in our taxonomy:

- 427 • *Hard temporal constraints (HC) vs. Soft temporal constraints (SC)*. Hard  
 428 temporal constraints require that no temporal constraint for any task is  
 429 violated. Soft temporal constraints allow some temporal constraints to be  
 430 violated or some tasks to be rejected entirely with a penalty.

431 We now illustrate our taxonomy in terms of single- vs. multi-task robots  
 432 (SR - MR), single- vs. multi-robot tasks (ST - MT), and hard vs. soft con-  
 433 straints (HC - SC). Most categories are further broken down as deterministic  
 434 vs. stochastic problems. We begin with the least complex problem settings, in  
 435 which single-task robots are allocated to single-robot tasks.

436 *6.1. ST-SR-HC: Single-Task robots, Single-Robot tasks, Hard Constraints*

437 *6.1.1. Deterministic allocations*

438 Deterministic ST-SR-HC problems typically assume that there are more  
 439 tasks than robots, and that all tasks are known in advance; they require time-  
 440 extended assignments. These problems are comprised of three intertwined sub-  
 441 problems: (1) an assignment subproblem, to find the assignment of tasks to  
 442 robots that optimizes the given objective function  $f(\cdot)$ ; (2) a task sequencing  
 443 subproblem, to find feasible orderings of tasks that result in optimal assign-  
 444 ments, and (3) a scheduling subproblem, to assign times to tasks in a way that  
 445 optimizes  $f(\cdot)$ .

446 Tasks have to be scheduled so that no temporal or assignment constraints  
 447 are violated. Temporal constraints are violated when robots do tasks at times  
 448 that are not consistent with the temporal constraints on the tasks. Assignment  
 449 violations occur when two or more robots are assigned to the same task, or two  
 450 or more tasks are scheduled to be done at the same time by the same robot.

451 In the mixed integer linear programming formulation in Fig. 3,  $x_k^a$  is an  
 452 indicator variable that takes the value 1 if robot  $a$  is assigned task  $k$  and 0  
 453 otherwise,  $o_{kk'}^a$  is an indicator variable that takes the value 1 if robot  $a$  performs  
 454 task  $k$  followed directly by task  $k'$ , and 0 otherwise.  $q_a$  is the capacity of robot  
 455  $a$ ,  $st_k$  and  $ft_k$  are respectively the actual start and finish times for task  $k$ ,  $tt_{kk'}$   
 456 is the travel time between tasks  $k$  and  $k'$ ,  $w_k^a$  is the amount of work robot  $a$  has  
 457 to perform when assigned task  $k$ ,  $v_k^a$  is a binary variable that is 1 if task  $k$  is the  
 458 first task in robot  $a$ 's schedule and 0 otherwise, and  $z_k^a$  is a binary variable that  
 459 is 1 if task  $k$  is the last task in robot  $a$ 's schedule and 0 otherwise. We assume  
 460 all times are strictly positive.

461 Since robots start at their own initial location, we create an empty task for  
 462 each robot  $a$  at its initial location. The empty task starts at time 0, and has

$$\begin{aligned}
& \text{minimize or maximize } f(\cdot) \\
& \text{subject to} \\
& \text{(a) } \sum_{a \in A} x_k^a = 1 && \forall k \in K_a^+ \\
& \text{(b) } \sum_{k \in K_a^+} v_k^a = 1 && \forall a \in A \\
& \text{(c) } \sum_{k \in K_a^+} z_k^a = 1 && \forall a \in A \\
& \text{(d) } \sum_{k \in K_a^+} w_k^a x_k^a \leq q_a && \forall a \in A \\
& \text{(e) } \sum_{k \in K_a^+} o_{kk'}^a + v_{k'}^a = x_{k'}^a && \forall a \in A, k' \in K_a^+ \\
& \text{(f) } \sum_{k' \in K_a^+} o_{kk'}^a + z_k^a = x_k^a && \forall a \in A, k \in K_a^+ \\
& \text{(g) } es_k \leq st_k \leq ls_k && \forall k \in K_a^+ \\
& \text{(h) } ef_k \leq ft_k \leq lf_k && \forall k \in K_a^+ \\
& \text{(i) } ft_k - st_k \geq du_k && \forall k \in K_a^+ \\
& \text{(j) } ft_k + tt_{kk'} - M * (1 - o_{kk'}^a) \leq st_{k'} && \forall a \in A, k \in K_a^+, k' \in K_a^+ \\
& \text{(k) } x_k^a \in \{0, 1\} && \forall a \in A, k \in K_a^+ \\
& \text{(l) } o_{kk'}^a \in \{0, 1\} && \forall a \in A, k \in K_a^+, k' \in K_a^+ \\
& \text{(m) } v_k^a \in \{0, 1\} && \forall a \in A, k \in K_a^+ \\
& \text{(n) } z_k^a \in \{0, 1\} && \forall a \in A, k \in K_a^+
\end{aligned}$$

Figure 3: Mixed integer linear programming formulation of assignment of tasks with time windows

463 a duration of  $\epsilon$ , ending at time  $\epsilon$ . We indicate the set of all the tasks plus the  
464 empty task at the start location of robot  $a$  as  $K_a^+ = K \cup \{\text{start location of } a\}$ .  
465  $\epsilon$  should be smaller than the early start time of any task.

466 The objective function  $f(\cdot)$  in the optimization formulation can be a cost  
467 function to be minimized [e.g. Gombolay et al., 2013]), or a value function to  
468 be maximized [e.g. Koes et al., 2005]). It can also be single or multi-objective.  
469 For example, in [Alighanbari et al., 2003]  $f(\cdot)$  is a multi-objective function that  
470 minimizes the maximum and average task completion times, as well as total idle  
471 times.

472 For instance, to minimize the makespan the optimization function would be

$$\text{minimize}_{x_k^a, o_{kk'}^a, z_k^a, st_k, ft_k} \max_{a \in A} \max_{k \in K_a^+} ft_k \quad (1)$$

473 For ST-SR-HC problems the coverage constraints in Fig. 3 enforce that (a)  
474 each task gets at most one robot, each robot has a first (b) and last (c) task and  
475 that (d) each robot does as many tasks as its capacity allows. Capacity here  
476 means maximum workload a robot is allowed to perform.

477 The sequencing constraints require that (e) every task  $k$  assigned to robot  
478  $a$  except the first has a predecessor, and that (f) every task except the last  
479 has a successor. Temporal constraints (g)–(j) are constraints on the service  
480 times of tasks. Constraint (j) ensures that the interval between two consecutive  
481 tasks is large enough for the robot to travel to it. The constraint includes a  
482 sufficiently large constant  $M$  to make the formulation a mixed-integer linear  
483 program. Constraints (k)–(n) bound the values for the indicator variables.

484 Ordering constraints in ST-SR-HC problems, such as synchronization con-  
485 straints, do not differ significantly from other types of temporal constraints,  
486 thus we will only show one example of such constraints here. Eq. 2 and Eq. 3  
487 are an example of the formulation of these constraints. Let  $k, k' \in P$  where  $P$   
488 is a set of task pairs with ordering constraints, and  $P^{sync} \subseteq P$  is the subset  
489 of tasks that have to be performed simultaneously, i.e. start at the same time.  
490 Eq. 2 states that regardless of which robot(s) is assigned to tasks  $k$  and  $k'$ , task  
491  $k'$  should start  $\epsilon$  time units after the finishing time of task  $k$ . If  $\epsilon > 0$   $k, k' \in P$   
492 (Eq. 2), and if  $\epsilon = 0$  then  $k, k' \in P^{sync}$  (Eq. 3).

$$\sum_{a \in A} st_{k'} x_{k'}^a - \sum_{a \in A} ft_k x_k^a > \epsilon + M(1 - o_{kk'}^a) \quad \forall a \in A, k, k' \in P, \epsilon > 0 \quad (2)$$

$$\sum_{a \in A} st_{k'} x_{k'}^a - \sum_{a \in A} st_k x_k^a = 0 \quad k, k' \in P^{sync} \quad (3)$$

493 Advances in MILP formulations for VRPTW [Barnhart et al., 1998, Feillet,  
494 2010] and more recently for MRTA problems [Korsah et al., 2012] have pro-  
495 posed set covering and set partitioning-based formulations. These formulations  
496 assign routes, instead of tasks, to robots. The allocation problem is decomposed  
497 into what is known as the master problem, and a pricing subproblem. One of  
498 the gains of such formulations is that the master problem can be restricted to  
499 evaluating subsets of tasks at a time, instead of the entire set of tasks. Pric-  
500 ing subproblems solve temporally constrained shortest path problems rooted at  
501 robot locations, in which routes can be computed via heuristic methods, such  
502 as D\* lite as in [Korsah et al., 2012]. Such formulations benefit from the insight  
503 that for very large problems many routes are not part of any optimal solution.  
504 Thus, selectively incrementing candidate routes decreases computational and  
505 memory costs. Feillet [2010] provides a technically rigorous tutorial of such  
506 formulations and their advantages for VRPTW problems.

507 As defined in Table 1, let  $Y_a$  be a set of routes for robot  $a$  computed using  
508 the shortest path algorithm with resource constraints;  $\hat{x}_y^a$  is an indicator variable  
509 that assumes a value of 1 if robot  $a$  is assigned route  $y \in Y_a$  and 0 otherwise;  
510  $C_y^a$  is the expected cost robot  $a$  incurs for performing route  $y$ ; finally,  $b_{yk}^a$  is  
511 an indicator variable that is 1 if task  $k$  is performed in route  $y \in Y_a$  belonging  
512 to robot  $a$  and 0 otherwise. An example of a set partitioning formulation of  
513 ST-SR-HC problems is shown in Fig. 4.

514 One of the main limitations of deterministic ST-SR-HC problems is their  
515 applicability. These problems do not take uncertainty and partial knowledge  
516 into account, and these are important properties of many robotics problems.  
517 Next, we discuss ST-SR-HC problems that handle uncertainty in planning by  
518 modeling uncertainty as stochastic processes.

### 519 6.1.2. Stochastic allocations

520 In stochastic ST-SR-HC problems, it is assumed that a model of uncertainty  
521 is available. Stochastic ST-SR-HC, like other stochastic problems in our taxon-  
522 omy, are usually modeled as pure or mixed stochastic integer programs, or as

$$\begin{aligned}
& \text{minimize } \sum_{a \in A} \sum_{y \in Y_a} C_y^a \hat{x}_y^a \\
& \text{subject to} \\
& \text{(a) } \sum_{a \in A} \sum_{y \in Y_a} \hat{x}_y^a \leq 1 \quad \forall a \in A \quad \text{Every robot gets only 1 route.} \\
& \text{(b) } \sum_{a \in A} \sum_{y \in Y_a} \hat{x}_y^a b_{yk}^a = 1 \quad \forall k \in K \quad \text{Each task is on 1 route.} \\
& \text{(c) } \hat{x}_y^a \in \{0, 1\} \quad \forall a \in A, y \in Y_a \quad \text{Indicator: route to robot.} \\
& \text{(d) } b_{yk}^a \in \{0, 1\} \quad \forall a \in A, y \in Y_a, k \in K \quad \text{Indicator: task to route.}
\end{aligned}$$

Figure 4: Set partitioning formulation example for MRTA/TOC problems.

523 Markov Decision Processes (MDPs) [Gendreau et al., 1996]. When modeled as  
524 stochastic integer programs [Ponda et al., 2012b] they assume the form in Eq. 4  
525 with the constraints shown in Fig. 3 or stochastic constraints [Shen et al., 2009].  
526 In Eq. 4 the objective function is the expected reward,  $\theta \in \Theta$  is the uncertainty  
527 model that is available to the robots, and  $u_k^a$  is the reward that agent  $a$  gets for  
528 doing task  $k$ .

$$\text{maximize } \mathbb{E}_\theta \left( \sum_{a \in A} \sum_{k \in K} u_k^a x_k^a \right) \quad (4)$$

529 Examples of uncertainty models include probability distributions for task ar-  
530 rival, robot travel time, task availability, and more [Miao et al., 1991]. Stochastic  
531 formulations other than MDPs are not, to the best of our knowledge, widely  
532 used in the MRTA literature, so we turn to the dynamic and stochastic VRPTW  
533 literature for examples of such models. For instance, [Bopardikar et al., 2014]  
534 studied a dynamic VRP problem in which demands (or tasks) with deterministic  
535 time constraints arrive randomly. A Poisson process generates the time when a  
536 task appears, while a uniform distribution is used for the demand location. The  
537 main goal of the work is to maximize the fraction of demand met.

538 Similarly, in [Pavone et al., 2009] demand is stochastic; however, they also  
539 consider time window constraints. They study stochastic and dynamic VRPTW  
540 problems, with the objective of minimizing the number of utilized vehicles and  
541 maximizing the demand satisfied. Both [Bopardikar et al., 2014] and [Pavone  
542 et al., 2009] analyze a different number of requirements, such as bounds on the  
543 number of vehicles used and maximum number of tasks that can be missed.  
544 In both, temporal constraints cannot be violated. However, in order to prove  
545 properties about their solutions, some strong assumptions are made, such as all  
546 time windows have the same length [Pavone et al., 2009].

547 An alternative way of modeling uncertainty uses MDPs. In [Dean et al.,  
548 1993, Beynier and Mouaddib, 2007] MDP states are locations in a map with  
549 obstacles, tasks and robots. In [Beynier and Mouaddib, 2007] a state is a triplet  
550 representing the previously visited state, the amount of resources left, and the  
551 time window. The goal is to search for policies that maximize a value function for  
552 the augmented states. Dolgov et al. [2007] poses the problem as a combinatorial  
553 resource scheduling problem with uncertainty, which can be easily extended to

554 include locations, forming an MRTA problem.

555     Uncertainty models for ST-SR-HC problems are, to the best of our knowl-  
556 edge, rarely explored in the vast MRTA literature, although stochastic planning  
557 could lead to practical gains in terms of finding sound and robust allocation  
558 policies for robots. Instead, it is far more common to find papers that address  
559 stochasticity by model-free methods, such as reinforcement learning or that deal  
560 with uncertainty by simply replanning during task execution.

## 561 *6.2. ST-SR-SC: Single-Task robots, Single-Robot tasks, Soft Constraints*

### 562 *6.2.1. Deterministic allocations*

563     Deterministic ST-SR-SC problems and deterministic ST-SR-HC problems  
564 are very similar, except that they differ in the hardness of the time constraints.  
565 Classic problems include vehicle routing and scheduling problems, task sequence,  
566 and soft constraint scheduling subproblems. We will also look at problems and  
567 solutions in the real time systems and artificial intelligence literature to assist  
568 in modeling temporal and ordering constraints in MRTA.

569     In soft time window constraints for vehicle routing, the goal is to find the  
570 best agent-task assignments that minimize the cost function  $f(\cdot)$  of servicing  
571 some number of clients. The total cost of assigning a set of agents or vehicles  
572 and departure times is equal to the fixed cost of operating the agents, plus the  
573 cost of operating the agents on the specific routes, plus the penalty cost for  
574 arriving early or late to the clients on the routes [Taş et al., 2013, Hsu et al.,  
575 2007, Ando and Taniguchi, 2006, Taillard et al., 1997]. Penalty costs for arriving  
576 early may be different than for arriving late, and these may vary by domain.  
577 Frequent assumptions are that agents may operate more than one route a day,  
578 each client must be serviced exactly once and by one agent (see Fig. 3), clients  
579 have time windows in which to be serviced for a specific amount of time, and  
580 agents have capacity constraints.

581     Though some soft constraint types are not widely used in MRTA, such as  
582 the weakly hard constraints in real time systems, they transfer quite sensibly  
583 into MRTA problems. In a weakly hard system with periodic task release, the  
584 distribution of met and missed deadlines in a time period is precisely bounded  
585 [Bernat et al., 2001]. Other approaches to skipping some deadlines for periodic  
586 tasks include degradation policies in overloaded systems [Beccari et al., 1999]  
587 or exploiting skips to improve response time for aperiodic tasks [Caccamo and  
588 Buttazzo, 1997]. MRTA problems most frequently have sets of tasks that must  
589 be assigned and completed once; the tasks may be similar or near each other  
590 spatially or temporally, but in general, there are no repeatedly released tasks.  
591 Real world robot-task assignment problems, however, might demand periodic  
592 tasks. A Mars rover, for example, has a regularly scheduled self-maintenance  
593 period, as well as periodic deadlines to finish uploading data or downloading  
594 instructions. These deadlines are usually hard deadlines, so the robot can shut  
595 down overnight and clear memory caches; other regularly scheduled robotic  
596 activities are not so sensitive to the time of execution.

597     A periodic task’s worst case met and missed deadlines can be considered by  
598 the number of random or consecutive missed or met deadlines [Bernat et al.,

599 2001]. There are four constraints to consider: making any  $n$  in  $m$  deadlines,  
600 making  $n$  in a row in  $m$  deadlines, missing any  $n$  in  $m$  deadlines, and missing  
601  $n$  in a row in  $m$  deadlines. In this way, any regularly scheduled sequence of  
602 tasks can allow some missed deadlines without penalty, while still allowing the  
603 agent responsible for those tasks to schedule and make most of its deadlines.  
604 Agricultural drones, for example, may have regularly scheduled sampling, such  
605 as fertilization, weed picking, or soil testing responsibilities that allow to skip a  
606 few deadlines.

### 607 6.2.2. Stochastic formulations

608 Stochastic versions of ST-SR-SC problems have an uncertainty model avail-  
609 able, as in Section 6.1; now, however, we use soft windows and allow agents to  
610 gain value even when performing tasks outside their original time window. Our  
611 objective is still to minimize the cost function given earlier in the determinis-  
612 tic formulation, with the inclusion of some probability model; often these are  
613 probabilities of travel delay between tasks and therefore travel times. The cost  
614 function now includes the cost of using vehicles and the cost of arriving to a  
615 task outside the proper time window.

616 Work from [Taş et al., 2013, Ando and Taniguchi, 2006] distinguishes be-  
617 tween fixed costs of operating a vehicle and service costs, which vary depending  
618 on the route and are impacted by the distribution of travel times. Work in [Taş  
619 et al., 2013] models travel time delays with several distribution types, which  
620 change the variable service cost of operating a vehicle. Besides changing the  
621 cost of using a specific vehicle, stochastic formulations impact the arrival times  
622 and the cost of doing a task early or late. If we use soft time windows, we can  
623 vary the cost of arriving early (e.g. early arrival is a small penalty) or of arriving  
624 late (e.g., late arrival is a large penalty). The travel time probability directly  
625 impacts whether the agent arrives early or late, which is why we frequently see  
626 stochastic formulations in soft time windows but no other kinds of preferred  
627 constraints.

## 628 6.3. ST-MR-HC: Single-Task robots, Multi-Robot tasks, Hard Constraints

### 629 6.3.1. Deterministic allocations

630 In ST-MR-HC allocation problems, agents are scheduled to work simultane-  
631 ously on tasks as coalitions. Coalition-based task allocation occurs when tasks  
632 cannot be executed by a single agent, or when task execution is more efficient  
633 when done by more agents [Vig and Adams, 2006, Shehory and Kraus, 1998]. In  
634 disaster rescue, for instance, fire fighters working in coalitions may extinguish  
635 the same number of fires earlier than if these rescuers had to work individually  
636 on each fire [Parker et al., 2015]. Moreover, in scenarios where the number of  
637 agents is limited, coalition-based allocations may enable a higher task comple-  
638 tion rate [Ramchurn et al., 2010b].

639 Coalition formation, in general, requires dealing with two subproblems: coali-  
640 tion value computation and coalition structure generation [Sandholm et al.,  
641 1999]. The former is concerned with computing the expected utilities (or costs)

642 of forming all possible coalitions, whereas the latter is concerned with partitioning  
643 the set of agents into exhaustive and disjoint groups that maximize the total  
644 utility. In MRTA, the coalition value is typically a combination of the utility  
645 gained and the coordination cost necessary to perform a task. Coalition size  
646 may be restricted by the physical constraints which limit the number of agents  
647 that can simultaneously work on the same task.

648 Let  $2^A$  be the set of agent coalitions that may be formed with the agents in  
649  $A$  (i.e., all subsets of  $A$ ) and  $x_k^c$  be an indicator variable that takes the value  
650 of 1 if coalition  $c \in 2^A$  is assigned to task  $k$  and 0 otherwise. For simplicity's  
651 sake, we assume that all agents start their tours from an initial node 0 and  
652 finish at node  $m + 1$ . Let  $o_{kk'}^a = 1$  when agent  $a$  visits task  $k'$  directly after  
653  $k$ .  $o_{0k}^a = 1$  denotes the fact that  $a$  visits  $k$  at the very beginning of the route,  
654 and 0 otherwise. Similarly,  $o_{k(m+1)}^a = 1$  when agent  $a$  visits task  $k$  at the end  
655 of its route, and 0 otherwise. ST-MR-HC allocation problems can generally be  
656 formalized by the MILP in Fig. 5.

$$\begin{aligned}
& \text{minimize or maximize } f(\cdot) \\
& \text{subject to} \\
& \text{(a) } \sum_{c \in 2^A} x_k^c \leq 1 && \forall k \in K \\
& \text{(b) } \sum_{a \in c} x_k^a = |c| x_k^c && \forall c \in 2^A, k \in K \\
& \text{(c) } \sum_{k \in K} o_{0k}^a = 1 && \forall a \in A \\
& \text{(d) } \sum_{k \in K} o_{k(m+1)}^a = 1 && \forall a \in A \\
& \text{(e) } \sum_{k \in K, k \neq k'} o_{kk'}^a - \sum_{k'' \in K, k' \neq k''} o_{k''k'}^a = 0 && \forall a \in A, k' \in K \\
& \text{(f) } st_k + du_k + tt_{kk'} - M * (1 - o_{kk'}^a) \leq st_{k'} && \forall a \in A, k, k' \in K \\
& \text{(g) } es_k \leq st_k \leq ls_k && \forall k \in K \\
& \text{(h) } ef_k \leq ft_k \leq lf_k && \forall k \in K \\
& \text{(i) } ft_k - st_k \geq du_k && \forall k \in K \\
& \text{(j) } x_k^a \in \{0, 1\} && \forall a \in A, k \in K \\
& \text{(k) } x_k^c \in \{0, 1\} && \forall c \in 2^A, k \in K \\
& \text{(l) } o_{kk'}^a \in \{0, 1\} && \forall a \in A, k \in K, k' \in K
\end{aligned}$$

Figure 5: Standard mixed integer formulation of the task allocation problem with single-task robots, multiple-robot tasks, and hard temporal constraints

657 Constraint (a) guarantees allocations of no more than one coalition per task.  
658 As the problem maybe over-constrained, not all tasks may be allocated. Con-  
659 straint (b) guarantees if a coalition is assigned to a task then all the agents in  
660 the coalition are assigned to that task too. Constraint (c) guarantees that all  
661 the agents start from the initial location, and constraint (d) ensures that they  
662 finish their routes at the final location. Constraint (e) guarantees the connec-  
663 tivity of the routes, so that a robot reaches all its assigned tasks in sequence.  
664 Constraints (f)–(i) ensure that the visit time-line is feasible and the time win-  
665 dows are respected (as in MILP for ST-SR-HC in Fig. 3). An extra waiting  
666 time may be imposed after the task's earliest start time to form the coalition.  
667 When coalition work affects the task execution efficiency, the task duration ( $du_i$ )

668 should be computed accordingly T[Ramchurn et al., 2010b]. Constraints (j)–(l)  
 669 bound the values for the indicator variables.

670 An alternative model for ST-MR-HC problems is the set partitioning model.  
 671 When cast as a set partitioning problem, a set of coalitions  $S = \{c_1, \dots, c_{|S|}\}$   
 672 corresponds to a set partition of  $A$  if and only if  $\bigcup_{c_i \in S} c_i = A$  and the elements  
 673 of  $S$  are pairwise disjoint (i.e.,  $\forall c_i, c_j \in S$  s.t.  $i \neq j : c_i \cap c_j = \emptyset$ ). The  
 674 solution to the set partition problem is a partition of  $A$  that maximizes the  
 675 utility  $u : S \rightarrow \mathbb{R}^+$ . The NP-hard nature of problems in this class require  
 676 approximate solutions for practical coalition-based MRTA problems.

677 Certain side constraints are very important in this class of MRTA/TOC  
 678 problems. Examples of such constraints include capability and resource con-  
 679 straints. Agents may have limited resources, especially in the case of small  
 680 robots. For instance, in disaster rescue scenarios, fire trucks may need a certain  
 681 amount of fuel to travel to a fire and a certain amount of water to extinguish it.  
 682 Tasks might require coalitions of agents with certain capabilities. For instance,  
 683 a fire might require a coalition of fire fighters, while police and ambulances can  
 684 collaborate to dig out and carry survivors to refuge centers [Kitano and Satoshi,  
 685 2001].

686 MRTA researchers have proposed coalition-based frameworks for heteroge-  
 687 neous robotics. Examples include the ASyMTRe [Parker and Tang, 2006] ar-  
 688 chitecture. ASyMTRe is a reasoning system for heterogeneous robots to form  
 689 coalitions to do tasks that require tight robot coordination. The architecture  
 690 uses a collection of schemas for perception and motor control, which are con-  
 691 nected at run time, enabling the robots to share information as needed to com-  
 692 plete the tasks. The architecture has been extended [Zhang and Parker, 2013a]  
 693 to ensure that only feasible coalitions are formed. Efficient scheduling heuristics  
 694 for coalitions are proposed in [Zhang and Parker, 2013b].

695 To the best of our knowledge, no literature addresses stochastic ST-MR  
 696 problems with either hard or soft constraints. We will now examine tasks with  
 697 soft constraints.

#### 698 6.4. ST-MR-SC: Single-Task robots, Multi-Robot tasks, Soft Constraints

699 ST-MR-SC assumes that multiple agents can work simultaneously on the  
 700 same task and are allowed to violate some temporal constraints, as long as they  
 701 incur a penalty for the violation. ST-MR-SC can be likened to ST-MR-HC  
 702 problems. The difference is that in the ST-MR-SC’s case the objective function  
 703 takes a temporal violation penalty into account (Eq. 5).

$$704 \quad \underset{S \in 2^A}{\operatorname{argmax}} \sum_{c \in S} \sum_{k \in K} x_k^c u(c, k) \pi(k, st_k) \quad (5)$$

705 In Eq. 5  $S$  is a coalition structure,  $u(c, k)$  is coalition  $c$ ’s utility for performing  
 706 task  $k$  and  $\pi(k, st_k) \in [0, 1]$  is the utility decay coefficient function for task  $k$ .  
 707 This coefficient is set as  $\pi(k, st_k) = 1$  when task  $k$  is started and/or finished  
 within the time window (i.e.,  $es_k \leq t \leq ls_k$  and  $st_k + du_k \leq lf_k$ ). Early and/or

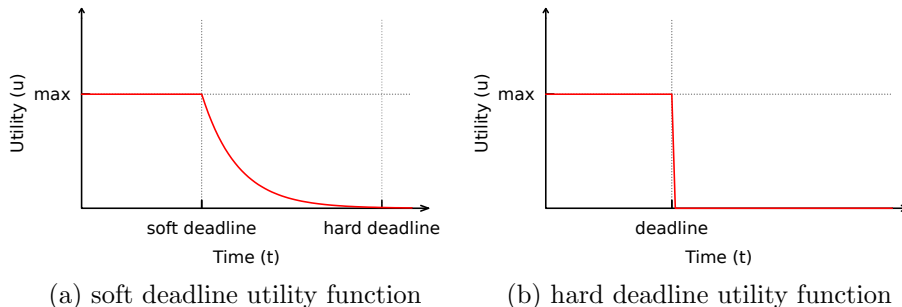


Figure 6: Utility of soft deadlines vs. hard deadlines. The maximum utility is earned before the deadline. An exponentially decaying utility can be gained if the task finishes between the soft and hard deadlines (case (a)). No utility is gained after the (hard) deadline (case (b)).

708 late task executions are penalized by setting  $\pi(\cdot) \in [0, 1]$ . In particular, we set  
 709  $\pi(k, st_k) = 0, \forall k \in K$  when  $st_k + du_k > ls_k$  [Koes et al., 2005, Amador et al.,  
 710 2014]. Fig. 6 illustrates the difference between a soft deadline utility function  
 711 (left) and a hard deadline utility function (right).

712 There is very little research on ST-MR-SC problems. Most of this research  
 713 is motivated by application areas such as urban search and rescue [Koes et al.,  
 714 2005, Scerri et al., 2005], or law enforcement where police officers are assigned to  
 715 crime events in a city [Amador et al., 2014]. In [Scerri et al., 2005] an expected  
 716 utility model is used to allocate interdependent tasks. Late task executions  
 717 are penalized by subtracting the delay cost from the total utility. The work  
 718 subdivides large tasks into smaller subtasks that are linked with simultaneous  
 719 execution interdependency, and coalitions of agents execute the smaller sub-  
 720 tasks. The coalition formation problem is simplified by fixing the coalition size  
 721 and reducing the number of allowed coalitions.

722 In [Koes et al., 2005] the task utility decays over time from the beginning  
 723 of the mission and becomes zero by the mission deadline. Likewise, in [Amador  
 724 et al., 2014] the utility of tasks delayed beyond the soft deadline decays expo-  
 725 nentially over time. The coalition value depends on the number of agents and  
 726 is a function of the agents' fitness in performing a task.

## 727 6.5. MT-SR-HC & MT-SR-SC: Multi-task robots, Single-robot tasks, Hard and 728 Soft Constraints

### 729 6.5.1. Deterministic allocations

730 The MT-SR problem with hard temporal constraints is no more common  
 731 now than it was in [Gerkey and Mataric, 2004], but we can provide some addi-  
 732 tional context for multi-task robots regardless of hard or soft time constraints.  
 733 Gerkey's work likens the MT-SR problem to the ST-MR problem, using the  
 734 same mathematical formulation for both problems but switching the role of  
 735 tasks and agents in the formula. Multi-task robots do exist in real life, however,  
 736 such as the mission-driven Mars rover Curiosity, so the multi-task robot prob-

737 lem merits deeper discussion before we consider it in terms of hard or soft time  
738 windows.

739 A multi-tasking robot can perform located tasks, such as grasping and ma-  
740 nipulating objects, and non-located tasks, such as taking pictures of nearby  
741 objects. Furthermore, located tasks can be near to or far away from the robot;  
742 for example, an object two feet in front of the robot is close, but an object ten  
743 feet away is probably considered far from the robot. Nearby objects should be  
744 relatively easy to grasp, but farther-away objects will require larger or longer  
745 actuators and thus more complex kinematic calculations to properly manipulate  
746 them. Consider unmanned aerial vehicles; reconnaissance drones may track ob-  
747 jects and take pictures (a relatively easy task) or may need to track objects on  
748 the ground and drop packages (a more difficult task that includes more intense  
749 object manipulation).

750 Another complexity for the multi-tasking robot arises in the solution meth-  
751 ods used for task assignment, which will be addressed more thoroughly in Sec-  
752 tion 8. Are the tasks assigned in a centralized or de-centralized fashion? A  
753 centralized system can produce optimal assignments, but if the robot decides  
754 on its own which tasks to perform at once, it must schedule its own resources  
755 according to the time frame and load on its system.

756 Lastly, a multi-tasking robot can either preempt tasks or not; preemptable  
757 tasks require priority knowledge and may require task rescheduling, whereas a  
758 simpler system of non-preemptive tasks may miss important tasks that arrive  
759 during execution. In preemptive cases where the robot was physically manip-  
760 ulating the environment, additional overhead time is required to restore the  
761 robot’s pose and to continue grasping or other movement [Groth and Henrich,  
762 2014]. Additionally, the robot must deal with failures; not only must the robot  
763 prioritize tasks, but it must decide (or have a plan for) what to do when the  
764 preempting task fails. Does the robot retry the failed task, move directly back  
765 to the preempted task, or drop into some kind of re-calibration or maintenance  
766 mode? Consider the Mars rover – if it runs into a rock or becomes stuck while  
767 navigating to a site where it has to perform chemical analysis, it should stop  
768 and get unstuck (or consult Earth-based humans for assistance), then return  
769 to navigation towards its earlier goal. If instead a piece of the rover’s chemical  
770 analysis fails due to hardware problems, it should probably stop all analyses  
771 until it can relay its problems and receive solutions from Earth.

772 Very limited literature exists on multi-tasking robots; much of the work  
773 focuses instead on robots that have many tasks to do very close together tem-  
774 porally, for example a UAV that searches for objects, targets an object, and  
775 releases a bomb. Groth and Henrich [2014] discusses a multi-tasking robot with  
776 nearby tasks (taking pictures of people and finding objects, or taking pictures  
777 of walls and greeting humans) with preemption; the robot stops taking pictures  
778 of walls if a human is in the way, for example.

779 In general, the MT-SR problem, regardless of the type of time window, can  
780 be approached heuristically as a bin packing problem, where each robot is a bin  
781 and each task is assigned to a robot that has available capacity and resources to  
782 perform that task. Other approaches to scheduling tasks are inspired by operat-

783 ing systems, such as shortest job first and priority scheduling; however, because  
784 context switching is more time-intensive for robotics than it is for processors,  
785 these methods may be highly suboptimal.

786 Lastly, as far as we know, no literature has explored the addition of stochastic  
787 formulations with temporal constraints on the multi-task robot problem.  
788 Probabilistic events, such as a delivery robot’s travel times due to traffic, would  
789 affect the agent’s ability to perform single tasks, just as stochastic formulations  
790 do in the ST-SR problem.

#### 791 *6.6. MT-MR-HC & MT-MR-SC: Multi-Task robots, Multi-Robot tasks, Hard* 792 *and Soft Constraints*

793 Multi-task robots and multi-robot task problems remain sparsely explored  
794 [Korsah et al., 2013, Gerkey and Mataric, 2004], even when additional temporal  
795 constraints are not considered. This class of problems can be modeled as an  
796 overlapping coalition formation problem [Chalkiadakis et al., 2010] combined  
797 with a routing and scheduling problem. Standard coalition formation methods  
798 produce either a super-coalition (with all the robots) or a set of non-overlapping  
799 subsets of robots.

800 In cooperative games with overlapping coalitions, agents can do more than  
801 one task at a time. This may lead robots to commit to the task assigned to more  
802 than one coalition. Overlapping coalitions have been used to model collaborative  
803 smartphone sensing in [Di et al., 2013]. In that work, smartphone users form  
804 overlapping networks, and an incentive function rewards users’ contributions  
805 to different tasks. Unfortunately, finding the optimal overlapping coalition is  
806 NP-complete.

807 MT-MR-HC problems are comprised of the following subproblems: (1) as-  
808 signing coalitions to tasks, (2) assigning different coalitions to the same robot  
809 as long as no resource constraints are violated, and (3) assigning values to the  
810 start and finishing times of tasks. Each of these subproblems is NP-hard.

811 Like MT-MR-HC, MT-MR-SC also lacks coverage in the MRTA literature.  
812 The models used for MT-MR-HC can be extended to this class of problems,  
813 the only difference being that temporal constraints are allowed to be violated.  
814 This requires that one of the optimization objectives minimizes penalties from  
815 violating the constraints.

#### 816 *6.7. Summarizing the Taxonomy*

817 The coverage of our taxonomy is illustrated in Table 2. ST-SR-HC prob-  
818 lems, both deterministic and stochastic, are by far the most commonly inves-  
819 tigated. This is not surprising, given that problems in many application areas  
820 can be modeled as single-robot and single-task problems with hard temporal  
821 constraints. However, exogenous events do occur in robotic applications, hence  
822 the lower prevalence of formal models for problems with soft constraints is sur-  
823 prising.

Category	Deterministic	% Refs	Stochastic	% Refs
ST-SR-HC	[Nunes and Gini, 2015] [Coltin and Veloso, 2014a] [Gombolay et al., 2013] [Barbulescu et al., 2010] [Ponda et al., 2010] [Jones et al., 2009] [Alighanbari et al., 2003]	48	[Campbell et al., 2013] [Ponda et al., 2012b] [Pavone et al., 2011] * [Laporte et al., 1992] *	21
ST-SR-SC	[Taş et al., 2013] * [Hsu et al., 2007] * [Ando and Taniguchi, 2006] * [Taillard et al., 1997] *	11	[Taş et al., 2013]	2
ST-MR-HC	[Zhang and Parker, 2013b] [Zhang et al., 2014] [Zhang and Parker, 2013a] [Ramchurn et al., 2010b] [Parker and Tang, 2006]	13	[Parker et al., 2015]	0
ST-MR-SC	[Amador et al., 2014] [Scerri et al., 2005] [Koes et al., 2005]	3		0
MT-SR-HC	[Groth and Henrich, 2014]	1		0
MT-SR-SC		0		0
MT-MR-HC	[Di et al., 2013] *	1		0
MT-MR-SC		0		0

Table 2: Select papers from each category, with the percentage of papers that fall under each category (% Refs). The percentages are rough estimates from our references. Papers with a \* symbol are not MRTA papers, but are included for completeness.

## 824 7. Dynamic Task Release and Execution

825 Execution of tasks in MRTA/TOC problems vary according to the dynamics  
826 considered. Dynamics may be due to faulty robots, changes in estimated cost  
827 of tasks due to uncertainties, changes in task definitions, online arrival of tasks,  
828 addition of robots to the team, and other changes made by external agents  
829 [Sariel-Talay et al., 2009]. While the execution aspect is outside of the task  
830 allocation scope, the planning-execution-replanning of tasks forms a planning  
831 loop that is usually addressed at once in dynamic domains. Here we consider  
832 dynamics caused by task arrival and during task execution separately.

833 Some dynamics are caused by the arrival of tasks over time without further  
834 knowledge of future tasks. Usually when a new task arrives at any given time  
835 there is already an existing solution for previously not yet performed but sched-  
836 uled tasks. Thus, replanning occurs at task arrivals, while robots are executing  
837 previously assigned tasks [Cordeau and Laporte, 2007]. In [Nunes and Gini,  
838 2015] both deterministic and dynamic task arrivals are considered, assuming  
839 the robots have perfect knowledge of the map where tasks appear. In contrast,

840 problems usually defined as online pickup and delivery problems or dial-a-ride  
841 include not only online arrival of tasks but other uncertain events, such as ve-  
842 hicle breakdowns and delays [Cordeau and Laporte, 2007]. Recent examples of  
843 online pickup and delivery consider transfers, in addition to the arrival of tasks  
844 with hard temporal constraints [Coltin and Veloso, 2014b,a, Bouros et al., 2011].

845 The dynamics that occur during plan execution [Usug and Sariel-Talay, 2011,  
846 Barbulescu et al., 2010, Ponda et al., 2010, Shah et al., 2009, Block et al., 2006]  
847 are very important for the practical use of robots, because execution can fail  
848 due to a host of reasons and replanning is essential to maintain some level of  
849 efficiency. In [Barbulescu et al., 2010] dynamics during execution are created  
850 by unexpected events and changes in costs and constraints; in [Ponda et al.,  
851 2010] dynamics are caused by breaks in communication links, which may cause  
852 conflicting assignments, as more than one robot could be assigned the same  
853 task. In [Usug and Sariel-Talay, 2011] temporary failures are considered, such  
854 as obstacles, which can be overcome by replanning.

## 855 8. Typical Solution Approaches

856 So far, we have proposed a taxonomy for MRTA/TOC problems; now we  
857 discuss the most popular solutions and how to map these to our taxonomy.  
858 Here we simply divide the methods into centralized vs. decentralized. Cen-  
859 tralized methods are further separated into exact and approximate methods,  
860 while decentralized methods are grouped into distributed constraint-based and  
861 market-based according to nature of the proposed solutions.

### 862 8.1. Centralized Solutions

863 Centralized methods rely on a central controller that allocates tasks to  
864 robots. The autonomy of the robots in pure centralized methods is limited  
865 or non-existent, as they solely execute the dispatched orders and do not make  
866 decisions on what tasks to do.

867 MRTA/TOC is intractable for a non-trivial number of robots and tasks.  
868 Optimal centralized solutions are intractable because they need to evaluate a  
869 large number of candidate solutions in order to guarantee optimality. Thus, the  
870 focus of MRTA/TOC solutions is largely on approximation and heuristic solu-  
871 tion methods. We discuss some of the common centralized exact and heuristic  
872 methods next.

#### 873 8.1.1. Exact Solutions

874 Exact solutions are optimal, but their computation time is impractical for  
875 realistic robotics applications. The most naive way to search for such solutions  
876 is to exhaustively search for all possible allocations that do not violate the tem-  
877 poral constraints. This is, however, intractable, because an exhaustive search  
878 leads to worst-case  $O(|K|!)^{|A|}$  complexity for  $|K|$  tasks and  $|A|$  robots. We have  
879 to search through all the possible sequences of tasks and all possible allocations  
880 of tasks to robots, and in addition to all feasible assignments of times to tasks.

881 A more sophisticated method for MRTA/TOC problems is Branch-and-  
882 Bound (B&B) [Clausen, 1999]. B&B searches the state space of candidate  
883 solutions represented as a tree and uses upper and lower bounds of the op-  
884 timal solution to prune the branches of the search tree that have costs higher  
885 than the computed lower bounds. Optimal solutions can be found using the  
886 B&B algorithm and its variants: Branch-and-Cut [Ropke et al., 2007, Bard  
887 et al., 2002], Branch-and-Price [Korsah et al., 2012, Feillet, 2010, Dohn et al.,  
888 2009, Barnhart et al., 1998], and Branch-Price-and-Cut [Barnhart et al., 2000].  
889 Branch-Price-and-Cut is becoming more popular in VRPTW [Bettinelli et al.,  
890 2011, Archetti et al., 2011, Desaulniers, 2010, Ropke and Cordeau, 2009], but,  
891 as far as we know, this technique has not been used in MRTA/TOC problems.

892 For many of these exact methods problems are solved using use tools such as  
893 CPLEX [ILOG, 2006], Gurobi [Gurobi Optimization, 2014], ABACUS [Jünger  
894 and Thienel, 2000], lp\_solve [Berkelaar et al., 2004] or other tools to build and  
895 solve the underlying MILP formulations. MILP-based formulations and solu-  
896 tions have been predominantly used in ST-SR-HC problems [Korsah et al., 2012,  
897 Alighanbari et al., 2003], but these models have also been used in other parts  
898 of the taxonomy (e.g. ST-MR-HC [Ramchurn et al., 2010b, Koes et al., 2005]).

### 899 *8.1.2. Approximate and Heuristic Solutions*

900 To reduce computation time, MILP-based heuristics are used to find approx-  
901 imate partial allocations while searching the state-space tree. Such approaches  
902 have been used to address problems in our taxonomy (e.g. ST-SR-HC [Gom-  
903 bolay et al., 2013, Korsah et al., 2012]). As far as we know, these methods  
904 do not provide any theoretical guarantees, but in some cases (e.g. [Gombolay  
905 et al., 2013]) they experimentally achieve results that are only 10% away from  
906 the optimal value (makespan).

907 Another way to gain computational efficiency is to use metaheuristic ap-  
908 proaches. Metaheuristics are algorithmic templates that approximately solve  
909 hard combinatorial optimization problems. Unlike other combinatorial opti-  
910 mization algorithms, metaheuristics may allow lower quality solutions in the  
911 search process to escape local optima, and often embed off-the-shelf heuristics  
912 to solve the problem [Bräysy and Gendreau, 2005b].

913 Metaheuristic approaches to VRPTW, TOPTW and related routing and  
914 scheduling problems have been shown to outperform many other methods (e.g.  
915 construction heuristics and local search) for standard benchmarks [Bräysy and  
916 Gendreau, 2005b, Hu and Lim, 2014]. Recent trends in the metaheuristic lit-  
917 erature seek to reduce the computation time and improve the solution quality  
918 by using parallelization and hybridization of different heuristics and exact tech-  
919 niques. However, metaheuristic parameters remain hard to tune [Birattari, 2009,  
920 Bräysy and Gendreau, 2005b].

### 921 *8.2. Decentralized Solutions*

922 Decentralized approaches vary widely; a detailed categorization is outside the  
923 scope of this paper and we refer the reader to [Dias et al., 2006, Pentico, 2007,

924 Koenig et al., 2010] for more thorough taxonomies on MRTA methods. Here we  
925 focus on distributed constraint optimization and market- and negotiation-based  
926 algorithms since these have received a great deal of attention in the MRTA  
927 community.

### 928 *8.2.1. Distributed Constraint (DCOP)-Based Methods*

929 MRTA/TOC problems can be modeled as a Distributed Constraint Opti-  
930 mization Problem (DCOP) [Maheswaran et al., 2004] and solved using DCOP  
931 methods. Solving DCOP exactly is NP-hard and impractical even for uncon-  
932 strained MRTA problems [Junges and Bazzan, 2008]. Thus, approximate meth-  
933 ods such as Max-Sum [Farinelli et al., 2008] have been used for task allocation  
934 in sensor networks and in RoboCup Rescue [Ramchurn et al., 2010a].

935 Ramchurn et al. [2010a] proposed the Fast Max-Sum algorithm, which was  
936 shown to be robust in situations where the number of tasks is dynamic; the  
937 approach reduced the computation time, number and size of messages sent com-  
938 pared to Max-Sum. The work in [Macarthur et al., 2011] improved upon the  
939 DCOP solution proposed in [Ramchurn et al., 2010a] by using online domain  
940 pruning and branch-and-bound. Their solution uses less computation overhead  
941 in a dynamic environment.

942 Another method is the LA-DCOP [Farinelli et al., 2006] algorithm, which  
943 is an approximation that uses token passing [Xu et al., 2005] as follows: when  
944 an agent perceives a task, it creates a token to represent it. It can decide to  
945 do the task or pass the token to a randomly chosen agent. This tends to guide  
946 the search quickly towards a greedy solution, which is reasonable for ST-SR-HC  
947 problems.

948 In [Ferreira et al., 2008] LA-DCOP and Swarm-GAP are compared in RoboCup  
949 settings. In Swarm-GAP an agent chooses a task according to a probability that  
950 depends on the stimulus generated by the task and the agent’s threshold. Re-  
951 sults show that both DCOP approaches behave similarly, and both perform  
952 better than a greedy task allocation. Their approach works for ST-MR-SC  
953 problems, where agents are allowed to arrive late to tasks. Recently, to facili-  
954 tate comparing the performance of DCOP algorithms, RMA SBench, a system  
955 that provides a library of state-of-the-art solvers for DCOP and for comparing  
956 them, has been created in [Kleiner et al., 2013].

### 957 *8.2.2. Market and Negotiation-Based Methods*

958 Among the decentralized algorithms, sequential auction- and negotiation-  
959 based algorithms [e.g. Nunes and Gini, 2015, Sariel-Talay et al., 2009, Ponda  
960 et al., 2010] are more prevalent than other methods. Sequential auction algo-  
961 rithms produce solutions that are two away from optimal in the worst-case in  
962 both single-item [Lagoudakis et al., 2004] and multi-item auctions [Choi et al.,  
963 2009]. This, together with the ease of implementation and extension to dynamic  
964 scenarios and robust execution [Nanjanath and Gini, 2010] makes sequential auc-  
965 tions an attractive solution. However, the greedy nature of sequential auctions  
966 and the complex structure of most MRTA/TOC problems cause the addition  
967 of temporal constraints to auction algorithms to produce suboptimal solutions

968 [Nunes et al., 2012]. Temporal modeling and balancing between temporal- and  
969 distance-based objectives can help auctions perform better [Nunes and Gini,  
970 2015, Ponda et al., 2010]. In [Amador et al., 2014] Fisher markets are used in a  
971 decentralized online solution to dynamic ST-MR-SC problems where agents are  
972 considered buyers and tasks are the goods.

973 Auctions distribute the computation to individual agents but require com-  
974 munication to share bids and results. To reduce the need for communication,  
975 several approaches use consensus algorithms [Zavlanos et al., 2008, Choi et al.,  
976 2009, Ponda et al., 2010], where each agent determines independently which  
977 tasks it should do. An equilibrium is reached by iteratively sharing informa-  
978 tion with neighbors and re-allocating tasks if needed. [Godoy and Gini, 2012]  
979 extended the Consensus Based Bundle Algorithm (CBBA) [Choi et al., 2009]  
980 to optimize the number of completed tasks for tasks with temporal constraints  
981 in ST-SR-HC problems. A different method, called emergent task allocation  
982 [Atay and Bayazit, 2003], distributes the computation of task allocation for a  
983 surveillance task to individual robots, by sharing intentions and directives with  
984 1-hop away neighbors. The method has been shown to converge to the optimal  
985 solution as the number of iterations of information sharing increases.

986 Despite the development of many decentralized methods for MRTA/TOC  
987 problems, very limited work offers theoretical analysis of the quality of these so-  
988 lutions. There is a need for theoretical performance bounds for both centralized  
989 and decentralized heuristics for the MRTA/TOC problem.

990 There are other decentralized approaches to task allocation that are not  
991 market- or DCOP-based. For instance, [Chapman et al., 2010] formulated ST-  
992 MR MRTA as a stochastic game and used overlapping potential games to ap-  
993 proximate an optimal solution. Their approach is robust to restricted agent  
994 communication and observation range.

995 Swarm-based approaches have been proposed for various tasks, such as for-  
996 aging, where robots need to find food and bring it to the nest [Lerman et al.,  
997 2006, Brutschy et al., 2014] or where swarms of robots are allocated different  
998 monitoring tasks without any communication among the robots [Berman et al.,  
999 2009]. Swarm methods often work well but do not have theoretical guarantees.

## 1000 9. Summary, Open Issues, Direction for Future Research

1001 Problems that consider temporal and ordering constraints relate to many  
1002 well studied problems, such as vehicle routing, job-shop scheduling, and multi-  
1003 robot task allocation. A large portion of the literature in MRTA/TOC focuses  
1004 on ST-SR-HC problems, some address the soft constraint version of this class  
1005 of problem; however, the literature is sparser for other classes of problems that  
1006 consider multi-task robots and multi-robot tasks.

### 1007 9.1. Summary

1008 We surveyed the multi-robot task allocation literature related to problems  
1009 where tasks have constraints on where, when and possibly the order in which

1010 they have to be performed. We built on a previous taxonomy and added a  
1011 classification axis that separates the literature according to the hard versus soft  
1012 nature of the constraints. Where appropriate, we gave a generic mathemat-  
1013 ical formulation of problems both in deterministic and stochastic cases, and  
1014 offered an account of some common execution dynamics. We briefly discussed  
1015 the methods applied to the problems in our taxonomy, and split solutions into  
1016 centralized and decentralized approaches. In addition, our work drew paral-  
1017 lels between multi-robot task allocation with temporal and ordering constraints  
1018 with other areas of research, and throughout the paper we discussed models and  
1019 solutions coming from these areas. Lastly, our work discussed areas that are still  
1020 sparsely covered, and provided directions for future research in the area.

### 1021 *9.2. Open Issues and Future Research*

1022 There are several open issues that need to be addressed, which we did  
1023 not exhaustively address here. Progress in the following topics would greatly  
1024 advance research in MRTA/TOC: (1) models and algorithms for stochastic  
1025 MRTA/TOC problems, (2) study of theoretical guarantees of approximate so-  
1026 lutions, (3) richer and more complex temporal models with provably good and  
1027 efficient algorithms, (4) models and algorithms for multi-task robot task alloca-  
1028 tion problems, (5) studies on the effects of time scales and time sensitivity in  
1029 MRTA/TOC problems and (6) the development of a research platform to make  
1030 software and data available to researchers.

1031 Research in stochastic MRTA/TOC problems is still very sparse. The de-  
1032 velopment of MRTA methods that take advantage of simulation and stochastic  
1033 models to better plan under uncertainty is an endeavor worth pursuing because  
1034 robots often operate in uncertain environments. Important research questions  
1035 can be asked here; for example, in an uncertain environment is it more benefi-  
1036 cial to build a complex model that incorporates uncertainty, or is it enough to  
1037 build less well-informed plans and replan as often as needed to quickly react to  
1038 unexpected events?

1039 There is also a need for work on theoretical guarantees for many heuristic  
1040 schedulers developed for MRTA/TOC problems. The NP-complete nature of the  
1041 problem and the need for relatively fast planners has generated many heuristics.  
1042 However, such heuristics typically lack performance guarantees, which can be  
1043 crucial for safety critical systems, to ensure that robots work effectively even in  
1044 the worst possible scenarios.

1045 More work needs to be done to address more complex temporal constraint  
1046 types, such as disjunctive temporal models. The literature could also benefit  
1047 from work that combines soft and hard time windows, and precedence with  
1048 simultaneity constraints. A mix of these constraints might produce more ex-  
1049 pressive models for a larger set of real-world problems.

1050 The challenge of allocating tasks to multi-task robots, which are robots that  
1051 can perform more than one task at a time, remains open. As we can imagine this  
1052 is difficult for many existing robots because they lack the necessary actuators.  
1053 The lack of literature might also be due to the lack of practical applications  
1054 of multi-task robots. We are not aware of any practical problems that strictly

1055 requires robots to perform multiple tasks concurrently; one example of such  
1056 problems could be in military domains where a drone robot could be required  
1057 to strike a target while at the same tracking other targets in nearby areas.

1058 Another interesting, yet not so explored topic, regards time scales and sensi-  
1059 tivity. Robots that move and navigate in an environment can be on a short time  
1060 scale, as in exploring a building in hours, or a large time scale of exploring a  
1061 planet for years. Even for a single robot, tasks can have varied time sensitivities;  
1062 some tasks may have short hard time constraints, whereas others may have long  
1063 time horizons with soft time windows. For instance, the Mars rover Curiosity  
1064 has periodic tasks that occur every day for years (data upload), constantly run-  
1065 ning tasks (temperature regulation), and sporadic tasks (chemical analysis of  
1066 collected material and drilling). Each of these tasks have different time sensi-  
1067 tivities; for example, data upload needs to occur when the receiving orbiters are  
1068 within view of the rover. Temperature regulation requires constant vigilance,  
1069 and drilling can be postponed, but needs to occur when the rover is within reach  
1070 of the material. Chemical analysis in the rover’s internal chambers can occur  
1071 regardless of location. This single robot has tasks with hard time windows, soft  
1072 time windows, varied scheduling horizons, and varied sensitivities. Considering  
1073 tasks in terms of time scales and task sensitivities in the same robotic system  
1074 thus holds value for any researcher interested in real world problems.

1075 Lastly, we are concerned with the public availability of research data and  
1076 methods. We advocate for a computational infrastructure for MRTA problems  
1077 (in general, or in particular problems with temporal and ordering constraints).  
1078 A tool identical to the Computational Infrastructure for Operations Research  
1079 COIN-OR [2015]) could greatly benefit MRTA researchers. COIN-OR is an  
1080 open source software project in which many operations research algorithms are  
1081 implemented and maintained by scholars in the area. That in combination with  
1082 datasets would help researchers verify their results on publicly available data  
1083 and methods, allowing for richer comparisons among methods.

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