

Development of Individual Tree Based Stand  
Growth Simulators: Progress and Applications

by

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## ABSTRACT

Research since the early 1960's on the development of individual tree based stand growth simulators is reviewed with respect to evolving methodologies for treating overstory growth, competition, mortality, reproduction, stochastic errors, calibration, and validation. The present and potential accuracy and precision of the models is discussed along with applications to date. Strategies for using the models for succession analysis and decision-making regarding choices among management treatments are also considered. It is apparent that these models can be expensive to develop, but represent one of the most powerful approaches available for predicting forest change. They are also a convenient basis for forest managers and ecological scientists to work together effectively. Pitfalls in the process of model development and usage are also discussed.

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## 1. Evolution of forest growth modelling methodologies

There is much past work in the area of forest growth and yield modelling that is germane to the study of forest succession. The earliest record of a study of the relationship of stand development to age is in the form of yield information developed in China in the 17th century (Vuokila 1965). Attempts to develop yield tables for forest management began in earnest in the late 19th century in Europe. The earlier efforts led to what we now call normal yield table methodologies. Such methodology was used heavily until the 1940's through 1950's. Such tables were limited by and based on the availability of temporary or single point in time observations of stand development with respect to age and a site quality variable, usually site index. The development of permanent plot data bases with the emphasis on continuous forest inventory during the late 1940's, 50's, and 60's led to the availability of repeated observations suitable for the application of differential and difference equation models to characterize forest growth as opposed to its integral, yield. Since that period a wide range of methodologies (reviewed by Ek and Monserud 1975) has led to the many variable density yield tables available today. These have been geared primarily to silvicultural alternatives in forest management. They can, however, in their various forms, play a significant role in the study of stand dynamics over longer time periods. A thorough review of these developments is beyond the scope of this report, but it is appropriate here to review the trends in such methodologies. Individual tree based models are one tool for producing variable density yield tables.

The last century of forest growth studies has led to the incorporation of stand description within models in progressively greater detail. Early models indicated stand development and yield for various ages, sites, and a specific density. As capabilities grew, models have evolved to consider the components of change in a forest in terms of survivor tree growth, mortality and ingrowth to the smallest measure size class. Models have further evolved to consider such components by size classes and species groups. Such developments have led recently to models considering individual trees, their status and attributes, and regeneration in general.

With these trends in model detail, researchers also progressed through tabular, graphical and more recently, formalized regression approaches to model specification and fitting. A major step was the use of least squares regression methodology. This produced results that were repeatable (given the same data set) and allowed for the characterization of model reliability. We have further seen a shift from empirical model forms to those constrained by biologically based concepts of growth patterns. It is not uncommon for model components today to be fitted by constrained linear or nonlinear least squares technique.

The evolution in detail and fitting procedures is in part a reaction to a trend in the objectives for growth and yield research. The first objectives were geared specifically to predicting future yields for a narrow range of management situations. Progress in the last century has led to considering the response of the forest to alternative silvicultural treatments and ecological circumstances.

One should not view current methodology as representing a culmination of modelling techniques. Rather, major changes have come about because of a change in the data base; from the availability of temporary and then permanent remeasured plot data. The latter require fewer assumptions about growth patterns and sampling adequacy and have allowed more sophisticated model forms. For many forest types, however, we will soon have completed an entire rotation or longer period of observation that will allow us to study entire real (actual) growth series for specific stands. Such series will be of immense value in validating the model forms. With advances in the theory of model fitting, such data should also allow for more accurate characterization of the forest response.

As a preface to subsequent review of individual tree models, a contemporary form of methodology, it is appropriate to indicate the nature of the objectives underlying the work to be described. The early development of individual tree based models emphasized obtaining an understanding of how the system works and the relationship among its components. More recently models have been oriented towards specific uses in forest inventory projection, the analysis of responses to alternative management treatments and forest succession over long time horizons. This difference in objectives has led to model forms which at times may be criticized for being overly simplistic or excessively detailed. Frequently such criticism is facilitated by imposing a set of objectives different from those intended by the model builders directly involved. In spite of apparent failures to produce a complete and operable model for all types of decisions, many

researchers have contributed to the development of individual tree models. Because of the size of such models, contributions have often been limited to refinement in a single component. The net result of these combined efforts, however, is much refinement in the total modelling concept.

## 2. Individual tree based models

Individual tree based models attempt to describe stand growth as the aggregate of tree origin, growth and mortality processes for individual stems on study plots. A schematic flow diagram of the projection process for such models is illustrated in Figure 1. The sections below describe the nature of processes considered in the diagram.

### 2.1 Construction methodologies

The first individual tree based stand growth model was developed by Newnham (1964) in 1963. Newnham's model for Douglas fir involved the characterization of tree growth on the basis of competing tree sizes and their location on a square lattice arrangement as in a regular plantation. Bella (1970) generalized this spatial arrangement by the consideration of any tree location on a study plot. Subsequent models, for example, those developed by Goulding (1972) and Stage (1973), did not consider individual tree locations as factors in competition. Instead, such models relied on a constant measure of competition or freedom from same developed from the data on a plot. A common measure of such competition is simply basal area per unit area. Munro (1974) categorized these variations in model form as

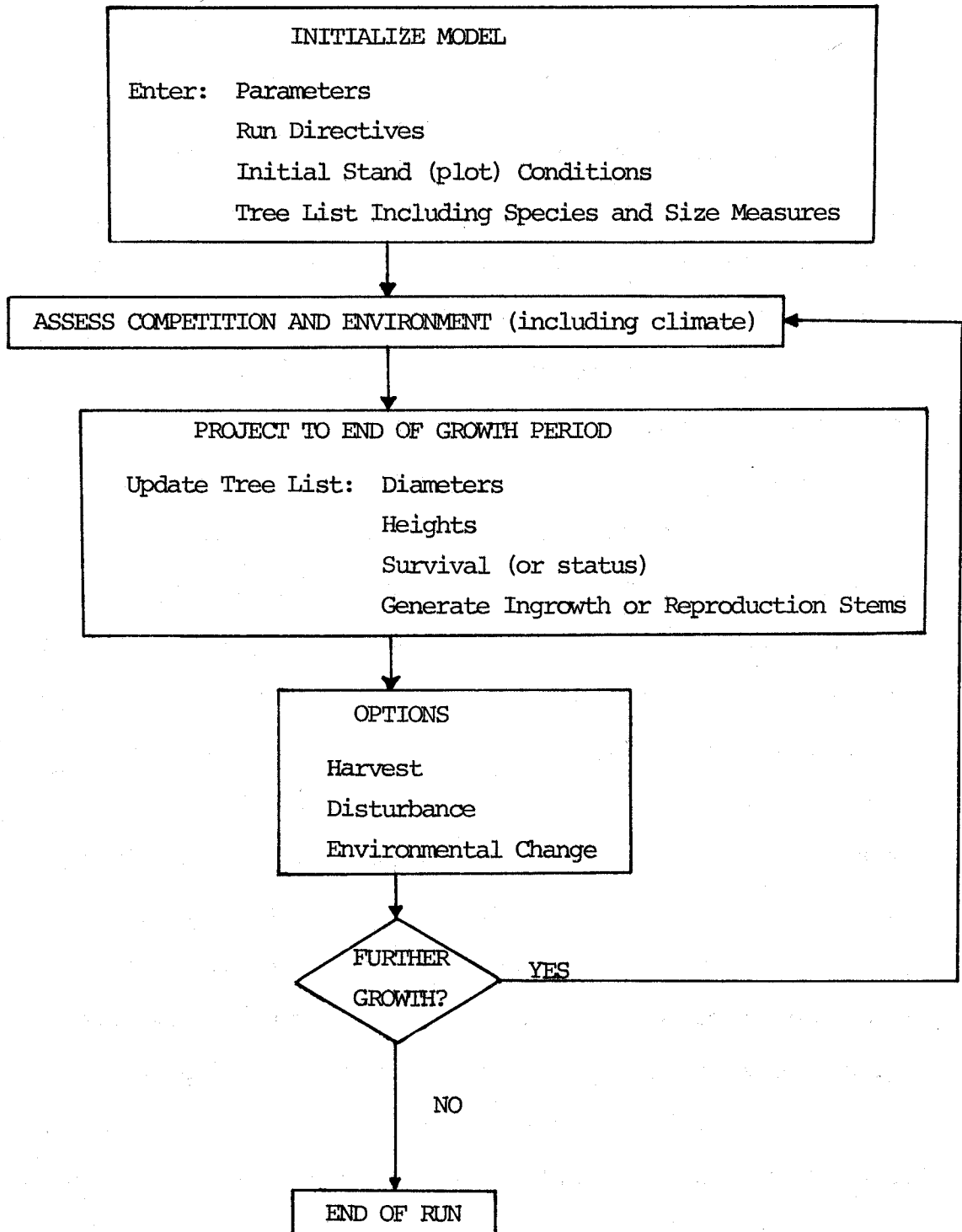


Figure 1. Schematic flow of individual tree based stand growth model projection process.

*distance dependent* and *distance independent*. The former refers to models that consider or depend upon individual tree spatial locations for competition assessment and the latter do not. A bibliography of individual tree model literature by Dudek and Ek (1980) indicates over forty distinct models have been developed with a nearly even split of these efforts among the two model forms.

Investigators have also attempted to develop their model forms in a variety of ways ranging from those that are quite constrained by biological principles (in mathematical form) versus those utilizing rather empirically developed model forms. In general, one can contrast these variations as follows: Biologically constrained model forms tend to be effective in interpolation and extrapolation; however, the constraints in view of the peculiarities of many data sets, can lead to poor fits in a statistical sense. Empirically based model forms frequently fit much better, but are usually more limited in the interpolative and extrapolative power they provide. Some biologically based constraints on model form are definitely desirable. Clearly, one hope in the construction of such models is that the sum of their parts in operational form will lead to greater insight and predictive ability than the component models by themselves can provide. In brief, the model should be capable of telling us more than we already know. Examples of biologically oriented model forms are those by Botkin, Janak, and Wallis (1970) for northeastern hardwood forests and Mitchell's (1975) model for white spruce. Models with a strong empirical orientation are described by Martin (1978) for red pine.

## 2.2 Submodel forms

### 2.2.1 Overstory growth

The majority of attempts to model the overstory development of forest stands on an individual tree basis have postulated a potential growth which is subsequently affected by a multiplier or modifier indicative of the tree's status and competitive situation. The primary advantage of postulating a potential curve or function is that the pattern of tree growth for dominant and open grown trees is fairly easily identified from site curves or existing work on open grown (free of competition) tree development (Krajicek, Brinkman, and Gingrich 1961 and Ek 1974). The fact that such patterns are known from very early to advanced ages particularly for height growth, diminishes extrapolation errors. Applications are aided by restricting multipliers (MULT) to a range of 0 to approximately 1.0. The general model form is:

$$\Delta T = PC \times MULT$$

where

$\Delta T$  = Change in some tree dimension

PC = Potential change (unrestricted growth)

MULT = f( competition measure and tree and stand descriptors)

With experience in developing models, it is evident that the potential function or curve only needs to be in the "ball park" for the estimation of multiplier function coefficients by means of least squares effectively adjusts for inadequacies in the potential curve. Models developed from growth data covering only short periods of time are heavily dependent on

the potential curve for their accuracy in extrapolation, however. Where much data exists over major portions of real growth series, the specification of an explicit potential function becomes less important. Potential functions have been based upon diameter growth patterns (Hahn and Leary 1979), height and/or diameter/age growth patterns (Meldahl 1979), and site insolation and photosynthetic responses considered typical for a species and region (Botkin, Janak, and Wallis 1970). Multipliers for overstory tree height and diameter growth are described in Figure 2. The multipliers or modifiers usually are exponential in nature for the overstocking or heavy competition portion of the relationship. An example is a diameter multiplier of the form:

$$DMULT = b_1 - \exp(b_2/C)$$

where C is some measure of competition and the  $b_i$  are constants.

### 2.2.2. Mortality

The treatment of mortality in individual tree growth models has evolved in two general directions. These are described by Monserud (1976). Briefly, attempts have been made to develop functions that classify tree status, i.e., live or dead. Dead trees are removed from a model and future growth period projections. This amounts to a problem in dichotomous classification. It is difficult, however, to develop a realistic threshold for separating live from dead categories. Some investigators have used a minimal level of growth to suggest a tree's removal and classification as mortality. Such classification has not proved entirely satisfactory, however, since there are numerous causes of mortality in forests--part due

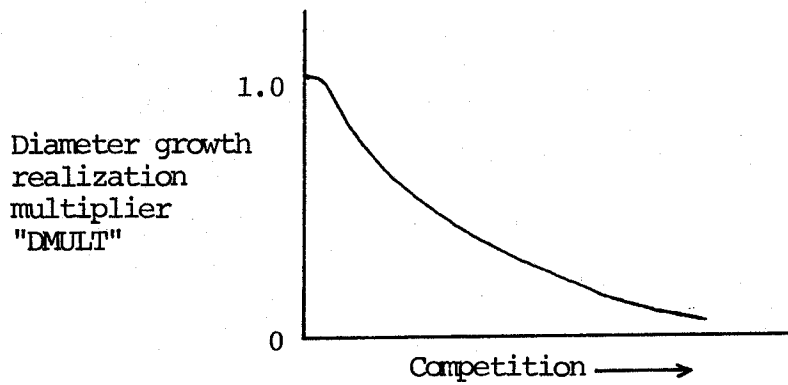
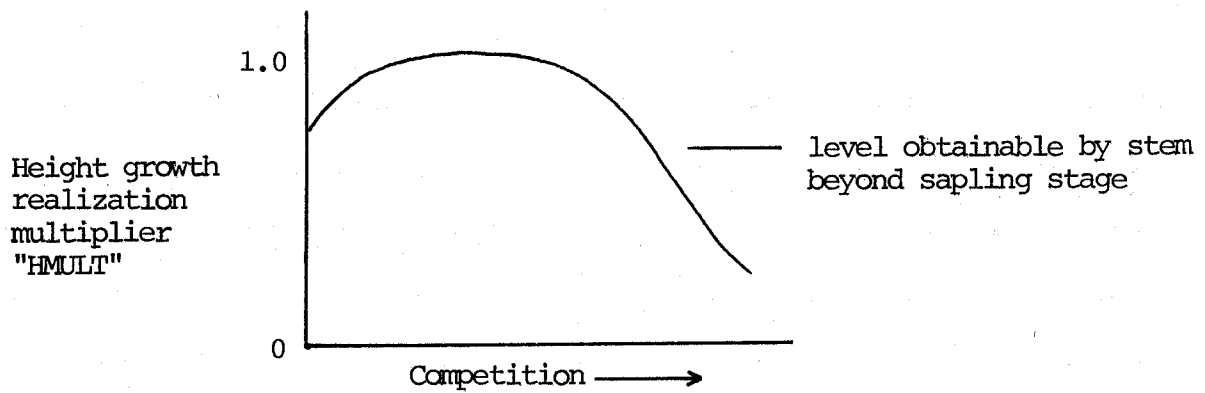


Figure 2. Generalized growth multipliers or modifiers for tree height and diameter growth relationships.

to competition, the remainder might well be considered random in nature. Thus, trees die within all age, size, and species classes, not just the smallest, slowest growing. Mortality is also a rare event and a collection of data to provide a basis for classification is difficult, that is, many more observations are required to develop a mortality classifier than might be necessary to develop an overstory growth potential modifier. Recent developments led by the work of Hamilton (1974) and Hamilton and Edwards (1976) suggests the use of data representing a random sample of trees and the development of probability of mortality expressions. Monserud (1975) indicates probability of mortality and classifier functions can be similar, if not identical, in algebraic form. An example from Hamilton is:

$$P = [1 + \exp (b_1 + b_2 \text{ CR} + b_3 \text{ PCT})]^{-1}$$

where

P = probability of mortality

CR = crown ratio

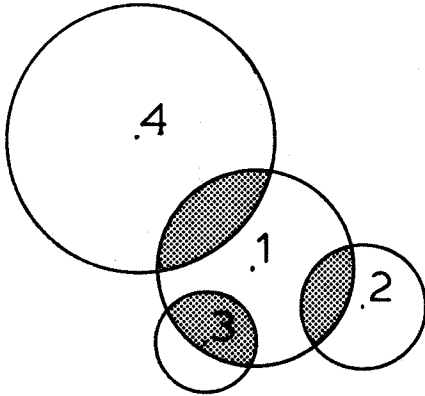
PCT = position in tree basal area distribution

However, their use is quite different. The probability of mortality function has evolved to be used in two ways. In the first, a probability of mortality, frequently based upon the logistic growth model and common predictor variables such as tree diameter, predicted growth and competition, is compared with a uniform (0, 1) random variate. If the probability of mortality is greater than the random variate, the tree is assumed to have died. Otherwise it is retained for projections to the start of the subsequent growth period. The second form of usage considers each stem in a

list of trees on a plot to represent a certain number of trees per unit area via an expansion factor. A deterministic form of mortality can then be generated by using the probability of mortality to reduce the expansion factor or trees per unit area represented by each tree in the list. Ek (1980) has indicated that for short periods, up to 10 years in duration, differences between the stochastic form of mortality where an entire tree either lives or dies and the deterministic treatment produce essentially equivalent results. Long-term studies, however, would be helpful in this area.

### 2.2.3. Competition

Numerous competition models have evolved for individual trees. Some evolved long before the development of individual tree models. Gerrard (1969) describes the development of tree competition measures beginning in the 1950's. These include Staebler's (1951) index of competition based upon linear overlap of competition zones between trees, Spurr's (1962) point density measure, and Gerrard's competition quotient or measure of competitive zone area overlap among nearby trees. Subsequent competition expressions have been described variously as competition indices, stress indices, etc. and alternatively growth potential indices to indicate freedom from competition or potential for growth. Daniels (1976) compared the performance of several competition measures and Alendag (1978) provided an additional and more detailed analysis of alternative measures. An example of a competition measure is given in Figure 3.



$$CIO = \sum_{j=1}^n [(O_{ij}/A_i) (D_j/D_i)^b]$$

where:

CIO = Competitive Influence Zone Overlap for Subject Tree i

n = number of competitors

$O_{ij}$  = Area of zone overlap between subject tree i and competitor j

$A_i$  = Influence zone area of subject tree i

$D_j$  = Diameter of competitor j

$D_i$  = Diameter of subject tree i

b = a constant

Figure 3. An example of a simple tree competition measure: Bella's (1971) Competitive Influence Zone Overlap (CIO). The measure is calculated here for subject tree 1 and its influence zone overlap with competitor trees 2, 3, and 4. The shaded areas indicate influence zone overlap.

Competition is a complex item that involves a consideration of above ground and below ground competition for moisture and nutrients, and indicative as well of temperature and light relationships. The search for the ultimate competition measure seems futile in light of the variability of trees and species, but many approximations have been found that account for a large share of the variation in tree diameter and height growth. Among the most effective to date have been those considering the overlap of zones of influence between nearby trees weighted by relative measures of size. No one measure, however, appears to be most appropriate in all situations. In several studies, for example Martin (1978), it was evident that the detail and competition description for distance dependent models was not entirely warranted. Instead, the use of basal area per hectare or some other aggregate measure of competition was found to work as well or better with much increased efficiency.

#### 2.2.4. Plot edge bias

One of the advantages of a distance independent individual tree growth model is that it involves some  $N$  computations to develop growth for the list of trees on a plot. Conversely, distance dependent models consider competition of each tree with every other tree on the study plot and involve at a minimum some  $N^2/2$  computations (Martin, Ek, and Monserud 1977). In addition to the greater complexity and computer time required for such models, it is usually necessary to estimate off-plot competition, especially for trees near the edges of the plot. Otherwise, bias will result in projections. Several approaches have been suggested for handling potential plot edge bias (see Monserud and Ek 1974) on rectangular plots:

- (1) estimate the competition for trees on the edge of the plot as some function of on plot competition and/or their characteristics as determined from relations developed for trees near the plot center
- (2) reflection, that is, the assumption of a symmetric distribution of trees or competitors on both sides of a line passing through subject trees near the plot edge and parallel to the plot boundary
- (3) translation, that is, the shifting of the study plot image to eight adjacent surrounding locations

All of these approaches have their limitations, however, especially with respect to the physical distance to which they can account for competition (usually one-half the plot width). Translation and reflection also introduce spatial pattern periodicity in the assessment of competition. In view of these problems, Martin, Ek, and Monserud (1977) developed a technique called the linear expansion method to provide efficient unbiased estimates of off plot competition. The technique treats both circular and rectangular study plot.

#### 2.2.5. Understory and regeneration

Attempts to describe natural regeneration and understory-overstory dynamics of forest trees has taken three general forms. An early attempt was that incorporated in the FOREST model (Ek and Monserud 1974). That formulation involved characterization of the understory spatial pattern by sub-plots on a main study plot, the development of a seed crop, its

dispersal to sub-plots, germination, early growth and survival leading to additions to the overstory size class. This sub-model is described by Monserud and Ek (1977). It is analagous to stand table projection methodology (see Husch, Miller and Beers 1972) except that movement ratios to larger size classes are developed by height classes rather than for diameter classes, as with traditional methodology. Competition for each tree in a size class was developed as a function of the overstory and its spatial pattern. The principle difficulty in developing such models is the paucity of data relating understory and overstory stand conditions. Leak et al. (1976) presented another approach where the understory--in fact the entire range of stem diameter classes--is described by tree frequency and equations for movement ratios are used to move the stems to larger size classes as the stand develops. Another approach that provides operational efficiency is the simple substitution of an anticipated species size class distribution at a reasonable age, say 10 to 25 years, after clearcut harvest. This distribution is relatively easy to develop from forest survey statistics. Unfortunately, it does not allow for the overstory to have a direct influence upon the understory development except through model specified ties between overstory and regeneration stand conditions. The MFPS model (Ek, Rose, and Checky 1980) provides an example of this procedure. Clearcut stands are replaced by a young stand of that same species unless otherwise specified at age 25. The NCFES's STEMS model (USDA Forest Service 1979) goes a step further in allowing gradual ingrowth in partially cut stands. Unfortunately, the paucity of data on regeneration modelling is likely to continue to

hinder the rate of development of models of that type. Related kinds of regeneration models have been described for aspen and northern hardwoods by Ek and Brodie (1975) and Ek (1974b), respectively.

### 2.3 Validation

It is relatively easy to specify procedure to be followed for the validation and testing of the components of individual tree models. In the case of an overstory model, for instance, it is important to assess the following statistics in the fitting process: a) coefficient of determination ( $r^2$ ), b) standard error, and c) bias. It is further essential that residuals be examined in some detail. Patterns of residuals plotted over the predictor variables and their various transformations are especially helpful. It is also important to check the interpolative power of the model towards the extremes of the data set used for fitting and to observe whether patterns from extrapolation beyond the basic data are consistent with available knowledge. There is no better test, however, than a trial of prediction capabilities on independent data sets. Frequently an elaborate statistical analysis is not possible, however, because of the deterministic or peculiar stochastic relationships inherent in the predicted results.

More important than the testing of model components, however, is the test on an independent data set of how well the whole model works as a unit. Particularly helpful variables to examine are the number of trees and basal area per unit area, the diameter and possibly height class distributions. With stochastic models it may be possible to develop tests

based on normal theory. As an example, Ek and Monserud (1979) found the FOREST model output to be normally distributed and were thus able to test whether actual and predicted values came from similar distributions. There do not appear to be many helpful tests for the comparisons of size class distributions; however, because of the interdependence of the size classes considered on a plot. Tests such as Chi-square or the Kolmogorov-Smirnov test can serve as indices of fit, however. It is also helpful for the number of trees and basal values to develop regressions of the form

$$y_i = b_0 + b_1 x_i$$

where the  $y_i$  are the actual values from an independent data set and the  $x_i$  are the predicted values. The appropriate test is then whether intercept and slope terms deviate significantly from 0 and 1, respectively. A more penetrating test is to estimate the residuals of the actual versus predicted values as a function of time, initial stand conditions, and important parameters that might change during the run. Helpful initial stand condition descriptors are density, average diameter, and site quality. Resulting fits are indicative of the trends in residuals and can help identify variables contributing to errors.

Tests should also recognize that calibration data sets drawn from a broad region may lead to model fits and projections that fail to fit subsets of the region well. This is the opposite of frequent inference situations, where a localized fit is tested to see if it fits the rest of the population well. The former situation implies that several independent data sets (clusters of plots) may be required to assess model performance.

#### 2.4 Accuracy and precision: Present and Potential

Few papers have described the accuracy of a model in detail. Some examples are Ek and Monserud (1979), Daniels et al. (1979), Martin (1978), Mitchell (1975), Monserud (1975), and USDA Forest Service (1979). From these and other papers describing performance, it is evident that the models developed by least squares fitting procedures and permanent plot growth data can provide reasonable 15 to 30 year projections over a broad range of stand conditions. Typical standard errors of models for growth would appear to lie in the neighborhood of  $\pm 15$  to 45 percent (see for example, Ek and Monserud 1979). It is also evident from various trials that gains of distance dependent over distance independent models are questionable within the usual range of plot data. The work of Ek and Monserud (1979) on northern hardwoods suggests, however, that the individual tree based distance dependent models can have much sensitivity for treatments not included in the data base used for model fitting. Specifically, they were able to project clearcut stand treatments effectively without having clearcut plot data in the fitting set.

The interpretation of model accuracy requires care. In a sense it is a case of comparing dissimilar items. The predictions, because of their data base, represent the predicted average response for an initial set of conditions. The actual plots to which they are frequently compared represent individual observations for such stand conditions.

The sampling error of mean plot growth estimates in the context of double sampling, the usual sampling context for a growth model application, is approximately:

$$S_{\bar{y}} = \left\{ S_Y^2 (1 - \rho_m^2) \left[ \frac{1}{n_2} + \frac{\rho_m^2 S_Y^2}{n_1} \right] \right\}^{1/2}$$

where

$S_Y^2$  = variance of plot growth

$\rho_m$  = correlation coefficient describing the relationship between actual and predicted values for the growth model

$n_1$  = large sample of plot (say in a forest inventory) on which predictor variables are observed

$n_2$  = subsample of  $n_1$  on which growth model was fitted (calibrated)

and

$$S_Y^2 (1 - \rho_m^2) = S_{Y \cdot X}^2$$

is the squared error of the model predicting growth as a function of the vector of stand conditions  $x$ . Note that finite population correction terms are ignored. The sampling error of a prediction for an individual plot, however, is

$$S_{Y_i} = \left\{ S_Y^2 (1 - \rho_m^2) \left[ 1 + \frac{1}{n_2} + \frac{\rho_m^2 S_Y^2}{n_1} \right] \right\}^{1/2}$$

This discussion highlights a frequently overlooked point in the application of such models. The two phase sampling context suggests that there are two sources of error in model application. One is the model fit itself exemplified by its standard error. The other arises from the fact that initial conditions for all plots in the forest are not available, but rather are estimated by a first phase sample or inventory effort. This suggests that models developed with predictor variables which themselves are poorly estimated in an inventory sense, are unlikely to lead to precise predictions of the future forest on an overall basis.

Most of the attempts at prediction with such models in forestry have been limited to 20 to 30 years and/or rotations ages under 200 years. The latter would appear to heavily tax methodology based upon permanent growth plots observed for only short periods of time. The authors are aware of much longer extrapolations for model testing. But these would not seem advisable at this time for management applications. Very constrained models such as the Botkin, Janak, and Wallis (1970) simulator JABOWA have been used for longer term projections. It is important, however, to note that this type of simulator can be indicative of future species-size distributions in general, but it may not be precisely predictive in the sense of describing the development of individual stands over time.

### 3. Applications

For decision or managerial problems, Adams (1975) has pointed out that economic analyses in forestry have been heavily dependent on available forest growth models.

Two central problems in forest management are the determination of the optimal treatment for an individual stand and the optimal choice of treatments or allocation of treatments to stands over a large forest property. The first is referred to as the stand level question. The second is referred to as the forest level problem (Williams 1976). Traditional attempts to handle the stand level problem have led to management guides for specific cover types. Such management guides attempt to suggest best or optimal: (1) stand density, (2) size class distribution, (3) species composition, (4) marking guides, (5) regeneration treatments,

(6) genetic stock, (7) fertilization, (8) harvest regeneration treatments, etc. These are discussed in the context of forest inventory and modelling by Ek and Burkhart (1978). Individual tree models, through either simulation or optimization, have the potential for providing insight on such guides.

With simulation, models may be used to develop careful experiments designed to characterize the response surface for a range of treatments and initial stand conditions. The simulation route in this case is usually a rapid version of traditional field trials. As well, it can allow for experiments which are practically impossible or logistically difficult to conduct in the field. The senior author has recently completed such a comparison for even versus uneven aged management of northern hardwood stands. In addition to interest in the response surface itself, users should also examine the variability about that surface and how flat it is, particularly for regions germane to management. This allows users to understand the consequences of minor deviations from prescribed treatment conditions and the variation in response that might be encountered with applications.

Optimization is a more difficult but potentially rewarding route. Its use at the stand level is exemplified by the work of Adams and Ek (1974) and Adams and Ek (1975). Their work involved the development of optimal stocking and size class structures for northern hardwoods based upon a simple diameter class distribution model, but results are suggestive of advantages over simulation. Specifically, optimization can lead to

treatment suggestions which are unlikely to be found by trial and error simulation. More recently Martin (1978) and Williams (1976) suggest that even the largest individual tree models are amenable to optimization by means of dynamic programming methodology. Adams and Ek (1975), for instance, suggested that optimization problems might eventually consider the choice of species composition and the development of individual tree marking and spacing guides.

In the context of modelling succession, deviations of actual stands from predictions and optimization results for particular objective functions may suggest the degree to which nature's objective function varies in form from that imposed by managerial analyses.

#### 4. Strategies and Logistics for Simulator Construction for Mixed Species Stands

Forest growth models of traditional form and individual tree growth models geared to silvicultural analyses are not as yet refined to the extent of being very useful for long-term simulation of succession. The models are hindered by their data base and weather related growth variation for the short periods of observation available in most cases. Exceptions might lie in the development of very constrained models that would provide better extrapolation, but rather poor fits to very specific stand conditions. The disadvantage with constrained models is that they are likely to tell the user only what the model form implies. In stochastic forms, they might function best to describe an envelope for possible responses.

Conversely, in the area of prediction, modelers have been much more successful in describing undisturbed stand conditions and gradual transitions to new conditions, than in describing response to drastic treatments such as thinning. A thinning response component, for example, estimated by Monserud's (1975) version of FOREST suggested both positive and negative short-term effects depending upon the species. The authors have not found models that provide good fits for both thinned and unthinned stand conditions. It may well be that the study of succession over time is in large part moved by catastrophic events that are difficult to incorporate in deterministic models, i.e., with "smooth" patterns or processes at the plot level. A prerequisite to further definitive work on individual tree based models seems to be the collection of data sets with careful measurement of individual stand conditions and response over an extremely wide range of situations--site qualities, ages, densities, habitats, etc. It would also seem appropriate to assess how to incorporate the random events that can occur over several centuries and profoundly affect results. For such analyses, stochastic models may well be more helpful than deterministic versions, the latter being most definitive for short-term silvicultural analyses. Growth models might also be constructed to project backward in time as well as forward.

Examples of modelling attempts to analyze succession are those with the JABOWA model by Botkin et al. (1970), FORET by Shugart and West (1977), FORAR by Mielke et al. (1978) or Solomon et al. (1980). The latter three models are adaptations of JABOWA, i.e., distance independent and ecologically

based. The growth of trees is developed in accordance with species-specific optimal growth expressions modified by corresponding to soil, moisture, competition, available light, and regional climatic factors.

JABOWA was used for 1400 year simulations of northeastern forest mixed stand dynamics. FORET was used to simulate development of Appalachian deciduous stands for 1000 years. FORAR was used for the simulating development of upland forests in southern Arkansas for 1000 years. The most extended simulation was by Solomon et al. (1980). That work involved simulation of prehistoric forest-stand development in central Tennessee for 16,000 years and comparison of results to pollen data.

It appears from these efforts that such growth models can be helpful in suggesting or confirming patterns of forest development over long time horizons with respect to climatic changes and species migration. That they can consistently provide precise and accurate predictions of change over long time periods, however, is unlikely. There are simply too many possible events in nature that are difficult to predict and/or are not included in the models. They are perhaps most useful in helping to explain observed patterns and suggest possible scenarios for the future.

A significant problem in developing individual tree based models to handle forest succession is that many events in the successional sense are rare in time. We further have little data on them and much difficulty in designing sampling efforts to collect it. In summary, it would thus appear that growth models over the next decade are going to be most effective for helping to explain the successional patterns we have observed.

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