Essays on Optimal Taxation of Carbon Emissions

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Any remaining errors are my own.

Dedication

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Abstract

This dissertation is composed of two essays and studies the optimal taxation of carbon emissions.

In the first essay, I set up an economy where an externality arises from the consumption of an exhaustible resource (oil) and a technology exists to mitigate the externality. I focus on the implications for policy design of assuming social preferences differ from private preferences regarding future generations. In particular, I consider a welfare function that places direct Pareto weights on unborn generations, as opposed to future generations receiving weight only through the altruism of their ancestors. This specification delivers a social discount rate which is lower than that of private individuals. I first show that standard policies, such as price or quantity controls on the *net* emissions of carbon, are insufficient to achieve the social optimum: When social and private discounting differ, more sophisticated policies are necessary. The main results of the chapter characterize these sophisticated policies. I show that an optimal tax scheme requires subsidizing the mitigation technology and taxing carbon emissions, but each at different rates: the optimal subsidy for removing a ton of carbon from the atmosphere will in general not equal the optimal tax for creating a ton of carbon. I also show that an optimal cap and trade system must include a cap on carbon offset allowances.

In the second essay, I study the optimal taxation of carbon emissions in an intergenerational model with imperfect altruism. This means that the current generations discount tradeoffs in the near future more than those which happen in the distant future. As a result, a problem of time inconsistency arises. I study if standard carbon policies are sufficient to control emissions in this economy. I first show that, when society can successfully resolve the inconsistency problem by committing itself to following a climate plan, standard carbon taxes coupled with a subsidy on oil reserves are enough to induce future generations to follow it. However, an initial period of sophisticated policies are required to induce the current generation to abide by it as well. When no commitment technology is available, I solve for the Markov perfect equilibrium of the dynamic game between generations and show that sophisticated policies are always required to implement the constrained social optimum.

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Chapter 1

Introduction

How to design climate policies to control carbon emissions that generate climate change is one of the biggest challenges that policy makers face today. The presence of carbon in the atmosphere is a twofold problem. On the one hand, carbon emissions are a standard externality: the burning of fossil fuel increases the level of carbon in the atmosphere which leads to climate change and reduces our welfare, but we are not taking this into account when we decide how much to burn. On the other hand, each emission that we release today persists in the atmosphere for tens, or hundreds, or even thousands of years into the future. This means that how much oil we burn today will affect also the welfare of people living many years, or centuries, from now. Therefore, the problem has also an intergenerational dimension.

This dissertation focus on the implications for the optimal design of carbon policies of addressing climate change not only as an externality but also as an intergenerational problem. To study intergenerational issues, I set up an economy composed of an infinite sequence of generations. Each generation consists of a continuum of altruistically linked individuals who live for only one period. In Chapter 2, I consider a welfare function that places direct weights on current as well as future unborn generations. By varying these weights, I can recover any point in the Pareto frontier between the current and future generations. Moreover, the efficient allocations associated with strictly positive direct weights on unborn generations imply a social discount rate that, at any point in time, is lower than that of the individuals living in the society. The goal of the chapter is to study the implications for the design of optimal carbon policies of considering social preferences that differ from private preferences. In Chapter 3, I set up a model with imperfectly altruistic individuals, in the spirit of [1]. That is, the current generation values its consumption relatively more than that of generations in the distant future. In this setup, I take the social planner to be a representative government, as opposed to a social architect as in Chapter 2. The planner reflects the preferences of the present generation and social and private preferences coincide. However, this case is complicated since the objective of the social planning problem becomes dynamically inconsistent. The goal of the chapter is to derive the implications for the design of optimal carbon policies of considering a society with time inconsistent preferences.

I describe two types of climate policies in this dissertation: "Standard policies" and "Sophisticated policies". Standard policies are those in which the carbon tax or subsidy is equal to the value of the externality, which I refer to as the social cost of carbon. Standard policies regulate the *net* emissions of carbon (emissions net of sequestration) and it is immaterial how the optimal level of carbon is reached, if by reducing emissions or by offsetting them through carbon offsets projects. On the contrary, sophisticated policies are those in which the carbon tax or subsidy *is not* equal to the value of the externality. Moreover, the subsidy for removing a ton of carbon from the atmosphere is in general not equal to the tax for creating it. Sophisticated policies may as well require additional policy instruments besides a carbon tax, or a standard cap and trade system.

The main results of this dissertation concern the extent to which standard policies are sufficient to implement the optimal path of carbon when the carbon externality entails intergenerational effects. In Chapter 2, I show that standard policies are sufficient in the special case in which only the present generation receives weight on the social welfare, so that social and private discount rates coincide. On the contrary, standard policies are insufficient and sophisticated policies are required whenever unborn generations are also directly valued in social welfare and social and private discount rates differ. In Chapter 3, I show that, when society can successfully resolve the inconsistency problem by means of a commitment technology (such as a Climate bill), standard policies coupled with a subsidy on oil reserves are enough to induce future generation to follow the climate plan. However, an initial period of sophisticated policies is required to induce the present generation to abide by it as well. When no commitment technology is available, sophisticated policies are required both today and for years to come.

Chapter 2

Carbon Taxes, Carbon Trading and Social Discounting

2.1 Introduction

This chapter studies optimal policies to control carbon emissions in an environment where social preferences differ from private preferences regarding future generations. A welfare function that places direct Pareto weights to unborn generations delivers a social discount rate that is lower than that of private individuals. I argue that standard policies to control carbon emissions, such as taxes or caps on the net emissions of carbon, are then insufficient to implement the social optimum. More sophisticated carbon policies have to be designed.

I develop the argument using a simple model of climate change in which an externality (carbon in the atmosphere) arises from the consumption of an exhaustible resource (fossil fuel). The externality can be mitigated by using an available technology (sequestration). In this model, households decide how fast to eat up the resource and how much effort to devote to mitigation activities. Both decisions affect the aggregate stock of carbon but, since households do not take this effect into account, the allocation in the decentralized environment is suboptimal and there is a need for policy intervention.

To address intergenerational issues, I set up an economy composed of an infinite sequence of generations. Each generation consists of a continuum of altruistically linked individuals who live for only one period. I consider a welfare function that places direct weights on current as well as future unborn generations. By varying these weights, I can recover any point in the Pareto frontier between the current and future generations. The main results of the essay focus on how the design of optimal carbon policies is affected by the choice of alternative welfare criteria.

One special such criteria is when only the current generation receives positive weight in the social welfare. In this case, the social discount rate coincides with the private one and the planning problem corresponds to that of a representative infinitely lived individual. I show that in this special case some well known features of optimal taxation of carbon emissions hold. In particular, carbon taxes and sequestration subsidies are equal to the value of the externality, which I refer to as the "social cost of carbon". It is equal to the discounted sum of future marginal damages from carbon emissions. Alternatively, in a cap and trade system in which the government sets a cap on the net emissions of carbon, the social cost of carbon arises as the equilibrium price of carbon permits.

I refer to these policies as "standard policies" since they regulate the net emissions of carbon (emissions net of sequestration). That is, a regulator would typically not care about how the optimal level of carbon is reached, if by reducing emissions or by offsetting them through sequestration. In order to reach the social optimum, it is enough to control the net amount of emissions. Carbon taxes provide incentives for eating up fuel resources at the optimal rate and tax credits on sequestration induce households to devote the right amount of effort to remove the emissions generated by their consumption. As a result, total emissions follow the optimal path. Alternatively, in a cap and trade system, firms that are willing to extract fossil fuels must either buy permits to pollute or pay households for sequestration. The equilibrium price of carbon permits induces the right amount of sequestration and rate of fossil fuel extraction in the decentralized environment.

However, the social optimum associated with valuing future generations only through the altruism of the current generation is only one among many other efficient allocations. These alternative efficient allocations are each associated with positive direct weights on unborn generations in the welfare function and each corresponds to a point on the Pareto frontier. Moreover, these welfare criteria imply a social discount rate that, at any point in time, is lower than that of the individuals in the society. The central contribution of this essay is to show that these efficient allocations can not be implemented with standard policies. First, the direct link between the social cost of carbon in the planning problem and the policy instruments in the decentralized environment no longer holds. And second, it is not enough to control the net emissions of carbon. A social planner that cares about future generations wants to treat differently the emissions of carbon from the emissions offsets.

The first main result of the essay is that, when social and private discounting differ, an optimal tax scheme requires subsidizing sequestration and taxing carbon emissions, but each at different rates: the optimal subsidy for removing a ton of carbon from the atmosphere will in general not equal the optimal tax for creating a ton of carbon. This is true even though both have the same effect on the overall externality. I provide an analytical derivation of the carbon tax formula for this case and show that the ratio between the social and the private discount factors show up as an extra term in the tax rate. The carbon tax is not equal to the shadow cost of carbon. This result is important because it implies that it is not possible to solve for the path of carbon emissions based on a social planner's problem and then associate taxes to the shadow cost of carbon in that problem. When social and private discounting differ, particular attention need to be posed on the decentralized environment and the design of optimal policies.

The second main result of the essay is that an optimal cap and trade system must include a cap on carbon offset allowances. If the government sets net emissions caps to those which correspond to the carbon emitted in the optimal allocation, the economy will exhibit both too much depletion of fossil fuels and too much sequestration. Hence, a cap on carbon offsets allowances has to be coupled with the cap on emissions in order to implement the optimal allocation.

Even though the approach of the dissertation is normative, it is interesting to compare this policy prescription to some of the carbon policies that are currently in place. Setting a cap on sequestration is a policy that, on the face of it, may seem counterintuitive. However, this policy prescription resembles some of the features of actual policies. In particular, the European Union Emissions Trading Scheme (EU ETS) allows firms the use of compliance carbon credits up to a limit, which varies across member countries. California's greenhouse gas (GHG) cap-and-trade program allows the use of offset credits to meet up to 8 percent of the firms' triennial compliance obligation. The Regional Greenhouse Gas Initiative (RGGI) let regulated firms to use offsets to meet up to 3.3 percent of their compliance obligations. Therefore, another way to interpret the results in this essay is that it provides a rationale for why these caps on carbon offsets are optimal.

The basic intuition behind the results in this chapter is the simple rule in public finance by which optimal policies require as many instruments as margins need to be corrected. Standard policies provide only one instrument. If climate policies are meant to deal not only with the environmental externality itself but also with intergenerational equity, an extra lever is missing.

One important contribution of this essay is to provide a unified framework that nests Stern's ("normative") and Nordhaus's ("positive") approach to discounting. When only the current generation receives positive weight in the social criterion, social and private discounting coincide and the problem corresponds to the one solved in positive studies, as in [2]. When future unborn generations receive positive direct weight in the social welfare function, the social discount rate is lower than the private one as advocated by [3]. This essay contributes to the ongoing debate between these two main climate proposals by: (i) providing a framework that makes a clear connection between Pareto weights and discount factors and showing that each proposal corresponds to a different point along the same Pareto frontier; (ii) providing next a mapping from the choice of Pareto weights into optimal carbon policies. That is, I characterize the carbon policies that are required to implement the path of carbon they propose. This is the main contribution of this essay. I show that while Nordhaus's path of carbon can be implemented through standard carbon taxes or a carbon trading system, Stern's one requires sophisticated policies as the ones I propose in this chapter.

The essay is related to the vast literature on discounting in climate change, specially [3], [4] and [5] and also [6] and [7] who provide good summaries of the controversy on discounting . The essay is also related to the literature on optimal taxation of fossil fuels with a climate externality as [8], [9] and [10]. This essay differs from those in that they do not study the interaction between social discounting and optimal taxation. The approach to social discounting adopted in this essay corresponds to the one in [11], [12] and [13], although they work in a different environment.

The remainder of the chapter is organized as follows. Section 2.2 sets up the basic model. Section 2.3 solves the social planning problem. Section 2.4 proposes two alternative market economies. In the first one, the policy instrument is a tax scheme while in the second, it is a cap and trade system. Section 2.5 characterizes optimal policies and presents the main results of the paper. Section 2.6 contains a numerical example illustrating the results. Section 2.7 provides some conclusions from the analysis. Finally, all the proofs are in the Appendix A.

2.2 The Basic Model

Consider the following economy. At any point in time, $t \in \{0, ..., \infty\}$, the economy is populated by a unit mass continuum of identical individuals, who live for one period and constitute generation t. There is a single consumption good k_t. The good (fossil fuel) is exhaustible and thus, at any point in time, must satisfy

$$k_{t+1} \in [0, k_t] \tag{2.1}$$

The economy starts with an initial endowment equal to k_0. Resource feasibility requires

$$c_t + k_{t+1} = k_t \tag{2.2}$$

for every period t, where c_t represents fossil fuel consumption.

Further, the amount of carbon in the atmosphere, S_t increases with consumption and decreases if individuals exert an effort level z_t (sequestration). In particular, the law of motion for carbon in the atmosphere is

$$S_{t+1} = (1-\gamma)\, S_t + k_t - k_{t+1} - z_t \tag{2.3}$$

where $\gamma \in [0, 1)$ is the rate of natural reabsorption of carbon and S_0 is given. The presence of carbon in the atmosphere generates a negative externality which is assumed to take the form of a per period disutility cost. An individual's utility in period t is given by $U(c_t, z_t, S_{t+1}) = u(c_t) - v(z_t) - x(S_{t+1})$. The function u is assumed to be increasing, concave and twice differentiable with $\lim_{c\to 0} u'(c) = \infty$. The disutility cost function v is increasing, convex, twice differentiable and satisfies $\lim_{z\to 0} v'(z) = 0$. The function x is assumed to be increasing, convex, twice differentiable and satisfies $\lim_{S\to 0} x'(S) = 0$. Over time, individuals care about their utility and that of their children. Thus, the utility of an individual born in period t is given by

$$v_t = U(c_t, z_t, S_t) + \beta v_{t+1} \tag{2.4}$$

where $\beta \in (0, 1)$ is the altruistic weight over the child's utility, v_{t+1}. This demographic specification is consistent with one in which households consists of a single infinitely lived individual who care about the value

$$\sum_{t=0}^{\infty} \beta^t U(c_t, z_t, S_{t+1}) \tag{2.5}$$

Social Welfare. When a representative infinitely lived individual inhabits the economy, it is natural to consider a social welfare function that coincides with (2.5). However, when the economy is composed of an infinite sequence of altruistically linked generations, a more general social welfare function involves weighting the utility of each generation separately. Following [13], I consider a utilitarian criterion that weighs current as well as future unborn generations according to the following function

$$\sum_{s=0}^{\infty} \alpha_s \left[\sum_{t=s}^{\infty} \beta^{t-s} U(c_t, z_t, S_{t+1}) \right] \tag{2.6}$$

where $\{\alpha_s\}_{s=0}^{\infty}$ is an arbitrary weighting scheme across generations. We can further simplify the welfare function to get

$$\sum_{t=0}^{\infty} \hat{\beta}_t U(c_t, z_t, S_{t+1}) \tag{2.7}$$

where $\hat{\beta}_t \equiv \sum_{\tau=0}^{t} \alpha_\tau \beta^{t-\tau}$ represents the social discount function. Note that when future generations enter in the calculation of social welfare, even though each individual discounts the future by β, society as a whole does it at a different rate given by

$$\frac{\hat{\beta}_{t+1}}{\hat{\beta}_t} = \beta + \frac{\alpha_{t+1}}{\hat{\beta}_t} \geq \beta \tag{2.8}$$

It becomes clear that the social discount factor is higher than the private one if weights are strictly positive for all generations. For the rest of the paper, I will restrict welfare weights to the ones in Assumption 1 below. This assumption ensures that the social discount function takes the standard geometric form.

Assumption 1 (AI) *The welfare weights $\{\alpha_t\}_{t=0}^{\infty}$ in the social welfare function 2.6 satisfy one of the following conditions:*

(i) $\alpha_0 = 1$ *and* $\alpha_t = 0 \, \forall t > 0$

(ii) $\alpha_0 = \frac{1}{\hat{\beta} - \beta}$ *and* $\alpha_{t+1} = \hat{\beta}^t$ *for some constant* $\hat{\beta} > \beta$ *and* $\forall t > 0$

Condition (i) describes a social welfare criterion that places direct weight on the current generation while future unborn generations are only valued indirectly through the altruism of the current one. This is a special case in which the social and private discount rates coincide. Condition (ii) restricts Pareto weights on unborn generations to be geometric. This ensures that the social discount function is geometric too, see [11]. Furthermore, welfare weights that satisfy condition (ii) induce a social discount rate that is lower than that of the private individuals in the society.

The next section characterizes the social optimal allocation as the solution to a planning problem.

2.3 Social Planning Problem

The *socially optimal allocation* is the path for consumption, fossil fuel, sequestration and carbon level, $\{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^{\infty}$, that maximizes the social welfare function (2.7) subject to the carbon cycle (3.6), the resource constraint (3.5) together with (3.1) and initial conditions $\{k_0, S_0\}$.

It is useful to define $\hat{\beta}^t q_t^*$ and $\hat{\beta}^t p_t^*$ as the social (shadow) cost of carbon and the social value of fossil fuel, respectively. Formally, the first one corresponds to the Lagrange multiplier on the carbon cycle constraint (3.6) and the second to the multiplier on the resource constraint (3.5).

The social cost of carbon is at the center of all economics models of climate change so it is important to understand the basics behind this concept. At the optimal allocation, the first order condition with respect to S_{t+1}^* gives

$$q_t^* = \hat{\beta}\,(1-\gamma)\,q_{t+1}^* + x'(S_{t+1}^*) \tag{2.9}$$

The social cost of carbon is derived from solving this equation recursively and it is equal to

$$q_t^* = \sum_{j=1}^{\infty}[\hat{\beta}\,(1-\gamma)]^{j-1}x'(S_{t+j}^*) \tag{2.10}$$

Essentially, it measures the marginal damage of increasing carbon emission in an extra unit. The fact that it is given by the discounted sum of damages originated from that extra unit over time highlights the dynamic nature of the externality. Moreover, it also highlights the reason why discounting is a controversial issue in economics of climate change. The discount factor directly affects the social value of the externality and, hence, the desirability of any policy aimed at controlling it. Since it will be useful to derive the results, I will denote μ_t^* the social cost of carbon expressed in terms of utils. That is

$$\mu_t^* \equiv \frac{q_t^*}{u'(c_t^*)} \tag{2.11}$$

The other optimality conditions are standard. The marginal cost of removing carbon emissions through sequestration must be equalized to its social value which is the accumulated benefits from having one less unit of carbon in the atmosphere forever after. That is,

$$v'(z_t^*) = q_t^* \tag{2.12}$$

The social value of fossil fuels is driven by a version of the Hotelling rule reflecting the exhaustible nature of fossil fuels, [14].

$$\hat{\beta}\left(p_{t+1}^{*}-q_{t+1}^{*}\right)=p_{t}^{*}-q_{t}^{*} \tag{2.13}$$

The Hotelling rule prescribes that the price of an exhaustible resource must increase at the rate of discount. This ensures that the benefit from extracting the resource is equalized at all dates at which a positive amount of the good is extracted. In this economy, optimality requires that the benefits from extracting must be net from the associated climate damages. Equation (2.13) combined with the first order condition with respect to consumption and sequestration delivers the following Euler equation

$$\hat{\beta}[u'(c_{t+1}^{*})-v'(z_{t+1}^{*})]=u'(c_{t}^{*})-v'(z_{t}^{*}) \tag{2.14}$$

The social cost of carbon (2.10), the first order condition with respect to sequestration (2.12) and the Euler equation (2.14) together with the feasibility constraint (3.5) and the carbon cycle (3.6) fully characterize the socially optimal allocation.

The two key decisions society face are how quickly oil reserves are used up and how much effort each generation should devote to mitigate the associated externality. Since individuals are small to be able to affect the aggregate level of carbon, they will typically get both decisions wrong: they will consume too much and they will fail to make the efficient level of effort on sequestration. A role for policy intervention arises.

2.4 Market Economy

We say that the social planner's allocation is *implementable* if we can find policies and equilibrium prices such that the allocation and prices are a competitive equilibrium given these policies. There are usually two instruments to control carbon emissions: carbon taxes and a cap and trade system. The first is a price-based policy and the second is a quantity control. From a theoretical point of view, both instruments are equivalent in the sense that they are equally capable of implementing the desired allocation. Motivated by this well known result, this section proposes two alternative market economies: one competitive economy with carbon taxes and a second economy with a cap and trade system. In the next section, I use these two decentralized environments to derive the main results of the paper.

2.4.1 Carbon Taxes

Consider the following market economy. A continuum of mass one firms, or a representative firm, operates a linear technology $f(y_t) = y_t$ to produce the consumption good. Firms own the economy's stock of fossil fuel and use it as an input for production. Extraction of fossil fuels creates carbon emissions which can be offset if firms buy sequestration services from households at a market price w_t. Firms face two forms of taxation: a carbon tax, τ_t^k, on emissions and a tax credit, τ_t^z, on sequestration. Hence, per period profits of the firm are given by

$$\pi_t = (k_t - k_{t+1}) - w_t z_t^d - \tau_t^k (k_t - k_{t+1}) + \tau_t^z z_t^d$$

Taxes are defined in units of the consumption good. The problem of the firm is to choose a sequence $\{k_t, z_t^d\}_{t=0}^{\infty}$ in order to maximize discounted profits given by

$$\Pi_0(k_0) = \sum_{t=0}^{\infty} (\prod_{s=0}^{t} \frac{1}{R_s}) \pi_t \tag{2.15}$$

with the initial stock of fossil fuel k_0 given. All profits are rebated to households as dividends.

There is a continuum of mass one households who derive utility from consumption of the single good in the economy and incur in a disutility cost when they provide sequestration services z_t. Households save in a risk free asset, $b_{t+1} \geqslant 0$ which bears a gross one-period rate of return R_t. The asset can be thought of as bequests from individuals in generation t to the ones in the next generation. Households consume, provide sequestration services and save subject to the following set of budget constraints for $t = 0, 1, 2, ...$

$$c_t + b_{t+1} = w_t z_t + R_t b_t + T_t + \pi_t \tag{2.16}$$

where T_t represents a lump sum rebate from the government, π_t are the profits received from the firm and initially $b_0 = 0$. The problem of the households is to choose a sequence $\{c_t, z_t, b_t\}_{t=0}^{\infty}$ in order to maximize (2.5) subject to (3.16), taking prices and taxes as given.

A government collects carbon taxes and pays subsidies. Any surplus (or deficit) is rebated in a lump-sum transfer to households. The sequence of government's budget constraints for $t = 0, 1, ...$ is given by

$$\tau_t^k (k_t - k_{t+1}) - \tau_t^z z_t^d = T_t \tag{2.17}$$

Finally, market clearing for every period t requires that

$$c_t = k_t - k_{t+1} \tag{2.18}$$

$$z_t^d = z_t \tag{2.19}$$

$$b_t = 0 \tag{2.20}$$

A *competitive equilibrium with taxes* $\left\{\tau_t^k, \tau_t^z, T_t\right\}_{t=0}^{\infty}$ is a sequence of prices $\{w_t, R_t\}_{t=0}^{\infty}$ and allocations $\left\{c_t, z_t, z_t^d, k_t, b_t\right\}_{t=0}^{\infty}$ such that: (i) given taxes and prices, the allocation solves the consumer's problem maximizing (2.5) subject to (3.16), and the firm's problem maximizing (2.15), (ii) given the allocation, transfers are such that the government budget constraint (2.17) is satisfied and (iii) prices clear the markets.

I characterize next a competitive equilibrium with taxes. Firms optimizing behavior delivers the following condition on prices

$$\frac{(1 - \tau_{t+1}^k)}{(1 - \tau_t^k)} = R_{t+1} \tag{2.21}$$

which is the market version of the Hotelling rule. It ensures that firms are indifferent between extracting the resource at any date as long as they make the same profits in present value terms. In addition, firms are indifferent between buying sequestration services from households or not if the price they pay for these equals the tax credit they then receive from the government, $\tau_t^z = w_t$.

The household's problem is characterized by two relevant margins: the intratemporal decision between consumption and sequestration and the intertemporal choice regarding how much oil to bequeath to the next generation. Combining the first order conditions with respect to sequestration and consumption delivers the following equilibrium condition

$$\frac{v'(z_t)}{u'(c_t)} = w_t \tag{2.22}$$

This condition says that consumers will decide on consumption and sequestration in order to equalize marginal utilities to the ratio of prices which, in turn, is equal to the sequestration tax credit. It is clear that, absent any tax policy, neither firms nor households have incentives to do sequestration. Once the government sets a price for this missing market, households are willing to offer these services since it is a source of income that allows them to consume. And firms are indifferent between paying for these services or not as long as they are compensated through a tax return.

Combining the first order condition with respect to consumption for two subsequent periods delivers the following Euler equation for this economy

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1-\tau_t^k}{1-\tau_{t+1}^k} \tag{2.23}$$

where I have used the Hotelling rule from the firm to substitute away the interest rate. The Euler equation rules the rate at which fossil fuels are eaten up over time. Note that in an economy without carbon taxes, consumers would be willing to equalize the marginal utility of consumption in present value terms. That is, the marginal utility of consumption would grow at the discount rate. This reflects the exhaustibility nature of the resource and it is a version of the Hotelling rule from the perspective of the consumer.

The intratemporal condition (2.22) and the Euler equation (2.23), together with the condition for prices in the firm's problem, the budget constraint of the government (2.17) and the market clearing conditions fully characterize a competitive equilibrium with taxes.

The next subsection proposes a cap and trade system as an alternative competitive environment.

2.4.2 Cap and Trade System

Consider the following alternative market economy. The demographics, preferences and technologies are identical to the previous setup. Only the policy instruments differ. Instead of carbon taxes, the government introduces a cap and trade system which sets a cap on *net* emissions of carbon $\{\theta_t\}_{t=0}^{\infty}$ together with a cap on sequestration $\{\phi_t\}_{t=0}^{\infty}$. The reason why the introduction of two caps is necessary, as opposed to only one cap on the net emissions of carbon, is at the heart of the results and will become clear in the section 2.5. The government endows households with both carbon permits and sequestration licenses. Sequestration licenses are authorizations to sell sequestration services which firms can count as offsets of their emissions. If a firm wants to use sequestration as a reduction in its emissions, then it must buy sequestration from the authorized households. Households and firms are then allowed to trade carbon permits and sequestration rights at market prices $\hat{\tau}_t$ and $\hat{\tau}_t^z$, respectively.

In order to produce one unit of output, a firm must purchase one carbon permit. Alternatively, the firm can buy sequestration from authorized households and offset the emission. The possibility of offsetting emissions is available as long as sequestration licenses have not been exhausted. Hence, firms face the following constraints for every period t

$$\theta_t^d \geqslant k_t - k_{t+1} - z_t^d \tag{2.24}$$

$$\phi_t^d \geqslant z_t^d \tag{2.25}$$

where θ_t^d and ϕ_t^d corresponds to firm's demand of carbon permits and sequestration rights, respectively. Per period profits of the firm are given by

$$\pi_t = (k_t - k_{t+1}) - w_t z_t^d - \hat{\tau}_t \theta_t^d - \hat{\tau_t^z} \phi_t^d \tag{2.26}$$

The problem of the firm is to choose a sequence $\left\{k_t, z_t^d, \theta_t^d, \phi_t^d\right\}_{t=0}^{\infty}$ in order to maximize discounted profits $\sum_{t=0}^{\infty}(\prod_{s=0}^{t} \frac{1}{R_s})\pi_t$ subject to the constraints (2.24) and (2.25). All profits are rebated to households as dividends.

Households provide sequestration services at a competitive price w_t. Carbon permits and licenses can not be stored. Households consume, provide sequestration services and save subject to the following set of budget constraints for $t = 0, 1, 2, ...$

$$c_t + b_{t+1} = w_t z_t + R_t b_t + \hat{\tau}_t \theta_t + \hat{\tau}_t^z \phi_t + \pi_t \tag{2.27}$$

where prices and taxes are defined in units of the consumption good and π_t are the profits received from the firm. The problem of the households is to choose a sequence $\{c_t, z_t, b_t\}_{t=0}^{\infty}$ in order to maximize (2.5) subject to (2.27).

Finally, market clearing for this economy requires that the following conditions must be satisfied for every period t

$$c_t = k_t - k_{t+1} \tag{2.28}$$

$$b_t = 0 \tag{2.29}$$

$$z_t = z_t^d \tag{2.30}$$

$$\theta_t = \theta_t^d \tag{2.31}$$

$$\phi_t = \phi_t^d \tag{2.32}$$

The last two equations correspond to the markets for carbon emission and sequestration permits.

A *competitive equilibrium with a cap and trade system* $\{\theta_t, \phi_t\}_{t=0}^{\infty}$ is a sequence of prices $\{w_t, R_t, \hat{\tau}_t, \hat{\tau}_t^z\}_{t=0}^{\infty}$ and allocations $\left\{c_t, z_t, k_t, b_t, z_t^d, \theta_t^d, \phi_t^d\right\}_{t=0}^{\infty}$ such that: (i) given prices and the caps, the allocation solves the consumer's problem maximizing (2.5) subject to (2.27), and the firm's problem maximizing (2.26) subject to (2.24)-(2.25), (ii) given the allocation, prices clear the markets.

I characterize next a competitive equilibrium with a cap and trade system. It is easy to see that the equilibrium conditions display prices $\hat{\tau}_t$ and $\hat{\tau}_t^z$ where taxes were before. Firms optimizing behavior delivers a version of the Hotelling rule for this economy:

$$\frac{1 - \hat{\tau}_{t+1}}{1 - \hat{\tau}_t} = R_{t+1} \tag{2.33}$$

which states that firms are indifferent between extracting at any date if the rent they make out of the exhaustible resource (net of payments for carbon permits) grow at the rate of interest. Further, the equilibrium price on sequestration services is given by $w_t + \hat{\tau}_t^z = \hat{\tau}_t$. With the carbon trading scheme, firms pay for sequestration because it is a way to 'save' on carbon permits: a firm that wants to produce a unit of output can avoid paying for a carbon permit by paying for sequestration of the emission associated with its production. The household's intratemporal equilibrium condition is given by

$$\frac{v'(z_t)}{u'(c_t)} = w_t \tag{2.34}$$

Again, consumers choose between consumption and sequestration in order to equalize the marginal utilities to the ratio of prices. Finally, the rate of depletion of fossil fuels is ruled by the following Euler condition

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1-\hat{\tau}_t}{1-\hat{\tau}_{t+1}} \tag{2.35}$$

which is essentially the same that in the previous setup. The price of carbon permits plays here the role that the tax on carbon emissions had in the tax economy. The intratemporal condition (2.34) and the Euler equation (2.35), together with the conditions for prices in the firms's problem and the market clearing conditions (2.28)-(2.32) for every period t fully characterize a competitive equilibrium with a cap and trade system.

2.5 Optimal Policy

This section presents the main results of the paper. I build towards the results in the following way: First, part 2.5.1 studies the business as usual, or laissez faire, economy as a benchmark case. Next, part 2.5.2 shows that in the special case in which only the first generation receives positive weight in the social welfare, so that private and social discounting coincide, standard policies such as carbon taxes and caps on the net emissions of carbon are optimal. Finally, the subsection 2.5.3 shows that standard policies are insufficient when the social and private discount rates differ and it characterizes optimal carbon policies for this case.

2.5.1 No Policy Intervention

Consider the laissez-faire (or business-as-usual) equilibrium in the economies described in the previous section. That is, the decentralized equilibrium outcome without any policy intervention.

The laissez-faire economy collapses to a simple version of a cake-eating problem. Importantly, $z_t = 0, \forall t$. No mitigation of carbon emissions takes place in the economy since households derive disutility from it and get no compensation. The market for sequestration is missing because it does not exist a price on carbon emissions. The next proposition follows.

Proposition 2 *The competitive equilibrium allocation with no policy intervention is not optimal.*

The only relevant decision households face is how much fossil fuel to hand on to the next generation. The intertemporal condition for an interior solution is given by

$$\beta u'(c_{t+1}) = u'(c_t) \tag{2.36}$$

which is a version of the Hotelling rule from the perspective of the consumers: The marginal utility of consumption of an exhaustible good grows over time at the discount rate $1/\beta$. By comparing (2.36) with (2.14), it follows that the rate of depletion of fossil fuels is not optimal in the laissez faire economy. There are two reasons for this. First, individuals do not take into account that, associated to the utility gains and costs derived from moving consumption across generations, there are gains and costs from the environmental damages this consumption generates. Second, when the social welfare criterion induces a low social discount factor, then discounting becomes an extra reason why the rate of depletion in the laissez faire economy is different from the optimal one. This is because the next generation's utility gain from inheriting an extra unit of oil weights β on the current's generation utility but it is worth $\hat{\beta}$ from a social point of view.

The next two sections characterize optimal carbon policies which are designed to make the equilibrium conditions in the market economy coincide with the optimality conditions in the social planning problem.

2.5.2 Standard Policies

Consider a social welfare criterion which values the welfare of future generations only through the altruism of the current ones. The welfare weights satisfy condition AI.i and the social discount factor coincides with that of private individuals. Note that the social planning problem corresponds then to that of a representative infinitely lived individual, which is the problem solved in many positive studies as in [2].

Characterizing optimal policies consists on finding a set of instruments that makes the social optimal allocation arise as the equilibrium outcome of a decentralized market. In this model, optimal policies operate through two channels: the rate of depletion of fossil fuels (which increases carbon) and the amount of sequestration (which reduces it). The following proposition states one of the results of the paper: when social and private discounting coincide, it is sufficient to control the net emissions of carbon. The proof is relegated to the Appendix.

Proposition 3 ***(Standard Policies)*** *Suppose that the social welfare weights are given by AI.i so that* $\beta = \hat{\beta}$. *If* $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ *solves the social planning problem, then it solves the competitive equilibrium with taxes* $\{\tau_t^k, \tau_t^z, T_t\}_{t=0}^{\infty}$ *defined by*

$$\tau_t^k = \tau_t^z = \mu_t^*$$

for every period t and all proceeds from taxation rebated/financed lump-sum through T_t. *Alternatively, it solves the competitive equilibrium with a cap and trade system with a cap on net emissions* $\{\theta_t\}_{t=0}^{\infty}$ *defined by*

$$\theta_t = k_t^* - k_{t+1}^* - z_t^*$$

for every period t and no cap on sequestration $\{\phi_t\}_{t=0}^{\infty}$.

The proposition highlights some basic principles of pricing carbon emissions. Optimal carbon taxes and subsidies in a decentralized economy are set equal to the social cost of carbon in the planning problem, μ_t^*. This result is often used in the literature as a shortcut to characterize optimal taxes. In particular, it is enough to solve for the optimal path of carbon emissions in a social planner's problem and then argue that the carbon tax equals the shadow cost of carbon in that problem. There is no special need to work out the details of a market economy. The next subsection shows that this shortcut is no longer available when the social discount rate differs from the private one.

By condition (3.30), the social cost of carbon is equal to

$$\mu_t^* = \frac{\sum_{j=1}^{\infty} \left[\beta \left(1-\gamma\right)\right]^{j-1} x'(S_{t+j}^*)}{u'\left(c_t^*\right)} \tag{2.37}$$

At any point in time, taxes and subsidies are equal to the discounted sum of future marginal damages from carbon emissions, expressed in units of consumption. This tax formula highlights the dynamic structure of the externality: carbon emissions are cumulative and add to a stock that only depreciates at a low rate. Current generations pay for their emissions an amount equal to the discounted value of marginal costs imposed on the future ones. In the same way, they are compensated for removing carbon an amount equal to the discounted value of benefits created on the future generations. It follows that taxes are sensitive to the discount factor and, for that reason, this last one has been the subject of much controversy.

In an economy with a cap and trade system, if the government sets carbon emissions cap to those which correspond to the carbon emitted in the optimal allocation, the social value of carbon μ_t^* arises as the equilibrium price of carbon permits. To see this, note that equation (2.34) evaluated at the star allocation together with the equilibrium condition for wages imply that the equilibrium price of carbon permits satisfies

$$\hat{\tau}_t = \frac{v'(z_t^*)}{u'(c_t^*)} = \mu_t^* \tag{2.38}$$

In order to burn fossil fuels, firms must either buy permits to pollute or pay households for sequestration. Firms internalize this cost and, as a consequence, the path of sequestration and fossil fuel extraction is the optimal one.

The optimal allocation can therefore be implemented by either carbon taxes or a cap and trade system. This is a well-known equivalence result. It implies in this economy that it is enough to control the net emissions of carbon (emissions net of sequestration). The next subsection shows that when future generations are directly valued in the societal welfare criterion, controlling the net emissions of carbon is not sufficient to achieve optimality.

2.5.3 Sophisticated Policies

This section presents the main results of the paper. The results share the same basic intuition: when controlling a climate externality becomes also a problem of intergenerational equity, two policy goals mix in one. Standard policies provide only one lever which proves to be insufficient to achieve these two policy targets. The policies that I propose in this section provide the extra lever that is missing.

I first show that standard policies are not optimal when social and private discount rates differ. In particular, it is not sufficient to control the net emissions of carbon. A social planner that directly cares about future generations wants to treat differently the emissions of carbon (which increase carbon in the atmosphere) from the carbon offsets (which decrease it).

Proposition 4 (Insufficiency of Standard Policies) *Suppose that the social welfare weights are given by AI.ii so that $\beta < \hat{\beta}$. Assume further that the policy instruments are restricted to have the form $\tau_t^k = \tau_t^z = \mu_t^* \; \forall t$ in the tax economy or to have no cap on sequestration in the cap and trade system. Then the competitive equilibrium allocation is not optimal.*

There are two ways we can deliver the optimal level of carbon to the next generation. One requires us to sequester more; the other to burn less oil. Standard policies treat these two channels as equivalent (i.e. they have the same tax rate and are equally valid as a compliance instrument in the carbon trading scheme). This is because both are equivalent in terms of their effect on the level of carbon. However, by keeping oil underground, future generations inherit not only less carbon but also more oil. Moreover, when the social discount rate is lower than the private one, the social value of leaving more oil for tomorrow is also higher than the private one. This means that, even absent a climate externality, firms in the market economy will be burning too much oil compared to the social optimum.

In a standard cap and trade system, firms burn too much oil compared to the social optimum and, in order to comply with the cap on emissions, they rely on sequestration. Therefore, the market outcome with a standard cap and trade system would exhibit both too much sequestration and too much depletion of oil. Note that future generations would still inherit the optimal level of carbon since this is controlled by the cap on net emissions. This means that the flaw in a standard cap and trade is not the level of carbon it implements, but rather that future generations will be inheriting too little oil compared to the social optimal.

On the other hand, standard carbon taxes are insufficient to induce the optimal path of fossil fuel extraction and sequestration, with too much of the first and too little of the second. Unlike with the standard cap and trade system, the market outcome with standard carbon taxes does exhibit a sub optimally high level of carbon. This is an important result. To highlight its importance, I state it this way: suppose that we agree that the optimal level of carbon is the one proposed by [3] and that we introduce standard carbon taxes to implement it. The proposition says that we will fail to achieve it and the level of carbon will remain sub optimally high despite the policy intervention. Implementing a path of carbon as the one proposed by [3] requires designing sophisticated climate policies as the ones I propose next.

The following proposition presents one of the main results of this paper: when social and private discounting differ, an optimal tax scheme requires subsidizing the mitigation technology and taxing carbon emissions, but each at different rates: the optimal subsidy for removing a ton of carbon from the atmosphere will in general not equal the optimal tax for creating a ton of carbon. The proof is relegated to the Appendix.

Proposition 5 ***(Optimal Taxes)*** *Suppose that the social welfare weights are given by AI.ii so that $\beta < \hat{\beta}$. If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the social planning problem, then it solves the competitive equilibrium with taxes $\{\tau_t^k, \tau_t^z, T_t\}_{t=0}^{\infty}$ defined as follows*

$$1 - \tau_t^k = \left(\frac{\hat{\beta}}{\beta}\right)^t (1 - \mu_t^*) \;\; ; \;\; \tau_t^z = \mu_t^* \tag{2.39}$$

for every period t and all proceeds from taxation rebated/financed lump-sum through T_t.

The optimal tax on emissions is not equal to the value of the externality the emissions generate. Climate policies favor regulating the level of carbon by keeping oil underground and do so by introducing an extra lever. The carbon tax on emissions is now a function not only of the climate externality but also of the ratio of the social and private discount factors. It is worth emphasizing that the social value of carbon is no longer a sufficient indicator of the price that should be charged for carbon emissions and, for this reason, there is no short cut to the design of a carbon tax. In particular, it is not enough to solve for the path of carbon emissions that arise in a social planner's problem and argue that the carbon tax equals the shadow cost of carbon in that problem. When social and private discount rates differ, particular attention need to be posed on the decentralized environment and on the design of the optimal carbon tax rate.

There is a subtle but important point that come out of this proposition. The optimal policy distinguishes between the private and the social discount factors and depends on both. Introducing a high social discount factor in climate change models is not a relabeling of the social planning problem which is just solved with a higher discount factor. The social planning problem where future generations receive direct weight is essentially different from the one in which they do not. Hence, it requires different policies. In order to be able to characterize them, it is necessary to distinctly specify the social and the private discount rates (or more precisely, the weighting scheme attached to future generations).

One drawback of the tax scheme in Proposition 5 is that the tax rate on emissions diverge. However, note that it is only the ratio of taxes which enters into the intertemporal condition (2.23) and this ratio does not diverge. Nevertheless, the next result presents an alternative decentralization which involves the combination of standard policies with a subsidy on the reserves of fossil fuels, none of which diverge.

Corollary 6 ***(A Subsidy on Oil Reserves)*** *The optimal allocation can also be decentralized with standard policies and a subsidy τ_t^s on oil reserves defined by*

$$\tau_t^k = \tau_t^z = \mu_t^* \ ; \quad \tau_t^s = (\frac{\hat{\beta}}{\beta} - 1)(1 - \mu_t^*)$$

for every period t and all proceeds from taxation rebated/financed lump-sum through T_t.

If the tax on emissions is set as a standard carbon tax (that is, a tax equal to the value of the externality), then the optimal tax scheme requires subsidizing firms so that they keep more fossil fuels underground. Per period profits of the firm are then given by

$$\pi_t = (k_t - k_{t+1}) - w_t z_t^d - \tau_t^k (k_t - k_{t+1}) + \tau_t^z z_t^d + \tau_t^s k_t$$

where the last term corresponds to the subsidy on the stock of oil. This subsidy rate, τ_t^s, is composed of two terms: a time-varying term that reflects the value of the externality and a constant one that recovers the divergence in discounting. The intuition for this result is similar to the one before. When social and private discounting differ, there is an extra return on achieving a given level of carbon through less extraction, as opposed to more sequestration. A subsidy on the stock of oil underground is a direct way to make firms internalize this extra return. Note that, absent a climate externality, a constant subsidy on fossil fuels reserves would still be optimal.

The result resembles that in [15], although they work in a different environment. They find that, when the welfare of future generations is directly valued in social welfare, it is optimal to subsidize bequeaths.

The following proposition presents the second main result of the paper: if sequestration can be used to meet compliance obligations within a cap and trade system, then it is optimal to set a cap on it.

Proposition 7 ***(Optimal Cap and Trade System)*** *Suppose that the social welfare weights are given by AI.ii so that $\beta < \hat{\beta}$. If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the social planning problem, then it solves the competitive equilibrium with a cap and trade system defined by*

$$\theta_t = k_t^* - k_{t+1}^* - z_t^*$$

$$\phi_t = z_t^*$$

for every period t.

The intuition for this result comes from the fact that, absent a cap on sequestration, firms will be burning oil too fast compared to the social optimum. Furthermore, since this implies too much emissions, firms will rely on sequestration to meet their cap on emissions. By setting a cap on the amount of carbon offsets that firms can use, the government is indirectly setting a cap on fossil fuel extraction. The cap on sequestration is then the tool that is required to induce firms to deliver not only the optimal level of carbon to the next generation but also the optimal stock of fossil fuels.

Even though the approach of this paper is normative, it is interesting to compare this policy prescriptions to some of the carbon policies that are currently in place. Setting a cap on sequestration is a policy that, on the face of it, seems a little perplexing. Removing carbon from the atmosphere is a social good and it seems counterintuitive that an optimal policy would set a cap on it. However, this policy prescription resembles some of the features of actual policies. In particular, the European Union Emissions Trading Scheme (EU ETS) allows firms the use of compliance carbon credits up to a limit, which varies across member countries. California's greenhouse gas (GHG) cap-and-trade program allows the use of offset credits to meet up to 8 percent of the firms' triennial compliance obligation. The Regional Greenhouse Gas Initiative (RGGI) let regulated firms to use offsets to meet up to 3.3 percent of their compliance obligations. In all cases, carbon offsets must be authorized and meet some regulatory criteria. Another way to interpret the results in this paper is that it provides a rationale for why these types of caps on carbon offsets are optimal.

Finally, the next result presents an alternative cap and trade system.

Corollary 8 ***(A floor on Sequestration)*** *The optimal allocation can also be decentralized with a cap and trade system which sets a floor on sequestration and does not* *allow to use offsets credits as a compliance instrument. In every period t, the optimal cap and floor are defined as*

$$\theta_t = k_t^* - k_{t+1}^*$$

$$\zeta_t = z_t^*$$

With this alternative cap and trade system, firms face the following constraints for every period t

$$\theta_t^d \geqslant k_t - k_{t+1} \tag{2.40}$$

$$z_t^d \geqslant \zeta_t^* \tag{2.41}$$

where θ_t^d corresponds to the firm's demand of carbon permits on *gross* emissions ("gross" since offsets are not allowed as a compliance tool) and ζ_t^* is the floor on sequestration set by the government. In this economy, firms undertake sequestration of carbon emissions only if they obtain some policy-related benefit. Either they receive a tax credit for it or they can use it as tool to meet their compliance obligations in a cap and trade system. If none of these benefits are in place, then firms have no incentives to undertake sequestration. For this reason, a floor on sequestration is required in order to achieve the optimal amount of carbon offsets projects in the market economy.

2.6 An Example Economy

This section presents an example economy. The objective of this numerical exercise is to illustrate the main features of the optimal policies as well as of the path for sequestration, consumption and the stocks of carbon and fossil fuels that solve the social planning problem. It was computed assuming the following functional forms and parameters: $\hat{\beta} = 0.98,\ \beta = 0.96,\ U(c, z, S) = log(c) - \phi_z \frac{z^2}{2} - \phi_s \frac{S^2}{2},\ \phi_z = 500,\ \phi_s = 1, \gamma = 0.001,\ k \in [0, 1],\ S \in [0, 1], k_0 = 1,\ S_0 = .3$.

The picture below shows the time path for consumption and sequestration and for the stocks of carbon and fossil fuels. The red line corresponds to the laissez faire solution and the blue and green lines to the social optimal allocation for different assumptions on discounting. The picture shows that fossil fuels are depleted in all three cases. Depletion occurs at a faster rate in the laissez faire economy. Since fossil fuels are an exhaustible resource, a faster depletion implies that without policy intervention the consumption of later generations is too low. In addition, the laissez faire economy exhibits no sequestration and a too high level of carbon. The picture also shows that a lower discount rate (high discount factor) requires a slower rate of depletion of fossil fuels (the green line in the upper right panel is flatter than the blue one). When future generations are directly valued in social welfare then, they inherit higher reserves of fossil fuels and enjoy a higher consumption. This feature of the optimal allocation is critical and explain the main results of the paper: When society values future generations welfare directly, there is an extra value in delivering a low level of carbon through less consumption rather than through less sequestration (even though both options are technologically equally efficient for doing so). This is reflected in the optimal allocation in a flatter path of consumption and a lower sequestration, compared to the solution when future generations are only value through the altruism of the current one. The path of sequestration in this last case (the blue line in the bottom right panel) has a flavor of the "ramp-up" solution found by [2]. They prescribe a *climate-policy ramp* with mitigation efforts back-loaded on latter generations, which is what the optimal allocation with equal social and private discounting displays in my model.

Figure 2.1: Social Planner's allocation vs Laissez Faire

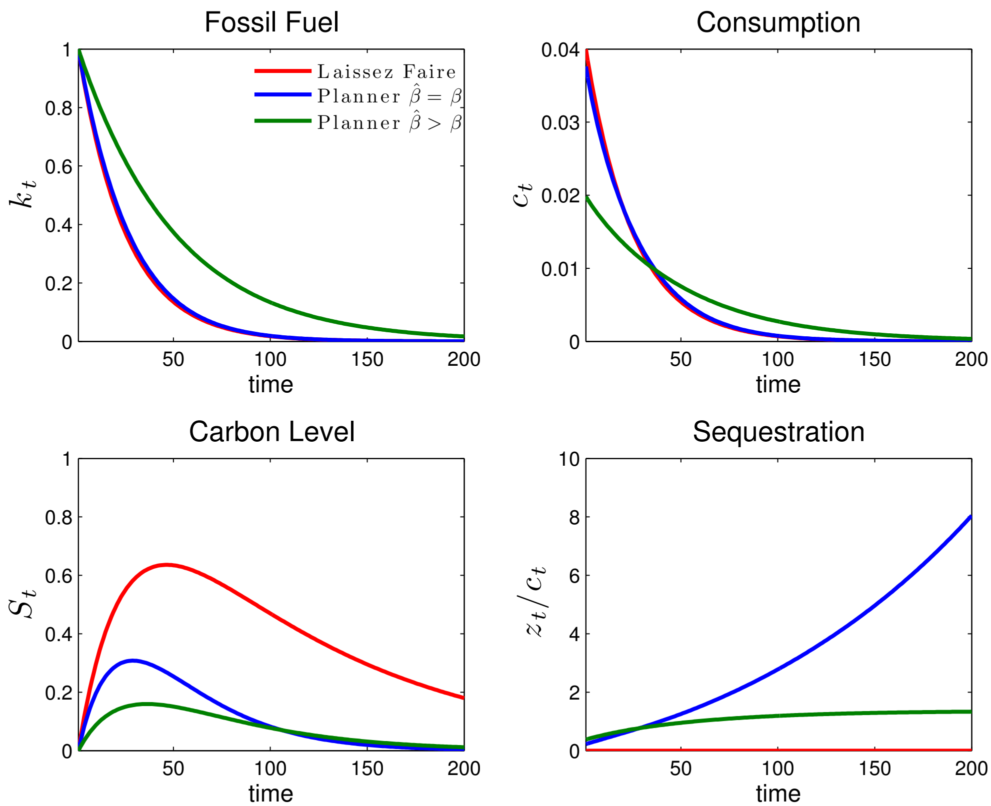
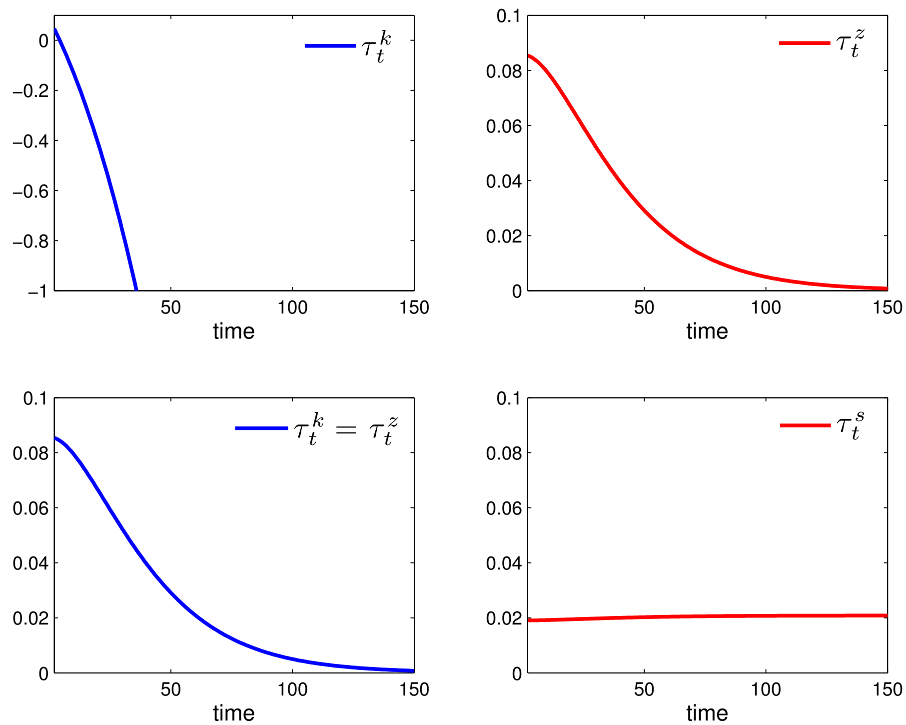


Figure 2.2 illustrates one of the main results of the paper. The upper panel shows the optimal tax on emissions and the tax credit on sequestration that correspond to Proposition 5. The lower panel shows the optimal tax on emissions and the tax credit on sequestration and on fossil fuel reserves that correspond to Corollary 6.

In the upper right panel, the tax credit on sequestration is a carbon tax credit: It is equal to the shadow cost of carbon. For this example, I backed out taxes by plugging the optimal path of $\{c_t^*, z_t^*\}_{t=0}^{\infty}$ into the households intratemporal condition in the market economy with taxes. However, since the tax is equal to the shadow cost of carbon in the social planning problem, an alternative procedure is to compute it as the ratio of minus the derivative of the value function with respect to carbon to the derivative with

Figure 2.2: Optimal Taxes and Subsidies



respect to fossil fuel in the social planner's problem. That is

$$\mu_t^* = -\frac{V_s(k,S)}{V_k(k,S)}$$

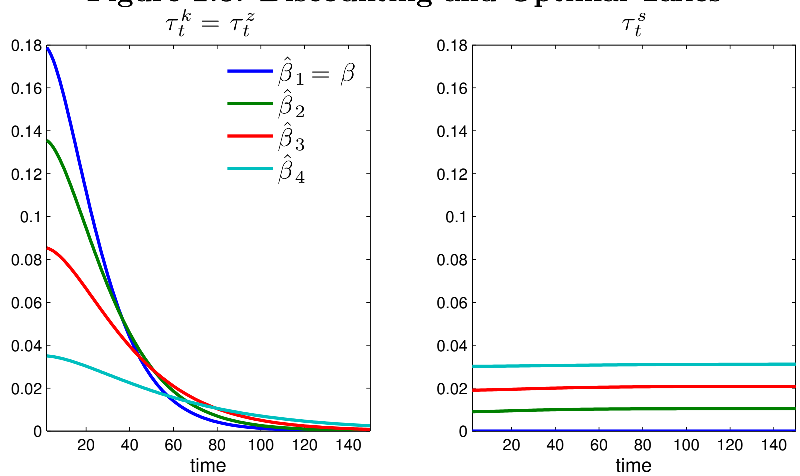
Both procedures deliver the same tax credit for sequestration. This is a shortcut often used in the literature which allows to characterize carbon taxes without specifying the decentralized environment.

However, one of the points of this paper is to show that this shortcut is no longer available when the planning problem is applying a social discount factor different from that of the individuals in the society. The tax on emissions in this case is *not* equal to the shadow cost of carbon. It is not simply a carbon tax. The optimal design of policies requires then to model the market economy explicitly. In the simple market economy

I propose in this paper, the tax on emissions is equal to the blue line in the left upper panel, which is different from the carbon tax credit on sequestration. The lower panels show an alternative tax scheme in which the tax rate on emissions does not diverge (Corollary 6). In that decentralization, the tax on emissions and on sequestration is set equal to the shadow cost of carbon (lower right) and coupled with a tax credit on the stock of fossil fuels, as shown by the red line (lower left).

Finally, Figure 2.3 shows these taxes for different values of the social discount factor. When future generations are valued only through the altruism of the current one, then social and private discounting coincide and taxes correspond to the blue line in the picture. Note that the tax credit on the reserves of oil is zero. This means that standard carbon taxes are sufficient to implement the social optimum in this case. The blue line corresponds to a discount rate of approximately 4% ($\hat{\beta}_1 = 0.96$). Alternatively, when future generations receive direct weight in the social welfare, the social discount factor is higher than the private one and taxes reflect this difference. As the welfare weights on future generations vary from zero to some positive numbers, the optimal taxes vary from the ones in blue to the ones in light blue. The light blue line corresponds to a discount rate of approximately 1% ($\hat{\beta}_4 = 0.99$). The picture shows that a low discount rate actually implies lower taxes on emissions and tax credits on sequestration. The reason is that the optimal allocation with a low discount rate displays a lower level of carbon and taxes are equal to the shadow cost of carbon, which is then lower for a lower carbon. However, in order to achieve this low level of carbon, an optimal tax scheme has to include also a tax credit on the reserves of oil, as the ones displayed in the right panel. This subsidy is higher the lower it is the level of carbon that is desirable to implement. Or equivalently, the lower the discount rate that is applied to compute the social optimum.

Figure 2.3: Discounting and Optimal Taxes



2.7 Conclusion

If the problem of climate change involves not only the environmental damage itself but also, as advocated by [3], a concern about intergenerational equity, then we need to start designing carbon policies capable of dealing with both sides of the same problem. This paper develops a model of climate change and shows that when future generations are directly valued in the social welfare, so that the social discount rate is lower than the private one, standard price or quantity controls are insufficient to implement the social optimum. I characterize the optimal carbon policies in that case. A common feature among them is that they favor reducing emissions directly, rather than relying on carbon offsets projects to control the amount of carbon. In particular, I find that: (i) the optimal carbon tax on emissions is not equal to social cost of carbon and it differs from the optimal subsidy on sequestration; (ii) a subsidy on the reserves of oil may be optimal; (iii) an optimal carbon trading system should limit the award of carbon offsets allowances. I have made these points in a simple model that allowed me to provide theoretical results and analytical derivations for the carbon tax formulas. However, the intuition is broader and the main substantive conclusion of the paper is likely to remain in more complex models.

Chapter 3

Time Consistent Climate Policies

3.1 Introduction

This chapter studies optimal taxation of carbon emissions in a model with *imperfect intergenerational altruism*. By this I mean that the current generations discount trade-offs in the near future more than those which happen in the distant future. The social discount rate varies over time and a problem of time inconsistency arises. I show that, when society can commit future generations to following a climate plan, standard carbon policies coupled with a subsidy on oil reserves are sufficient to induce future generations to follow it. However, an initial period of sophisticated policies are required to get the present generation to abide by it as well. When no commitment technologies are available, sophisticated policies are required both for today and in subsequent periods.

I derive the results using the model of climate change laid out in Chapter 2, in which an externality (carbon in the atmosphere) arises from the consumption of an exhaustible resource (fossil fuel). The externality can be mitigated by using an available technology (sequestration). In this model, households decide how fast to eat up the resource and how much effort to devote to mitigation activities. Both decisions affect the aggregate stock of carbon but, since households do not take this effect into account, the allocation in the decentralized environment is suboptimal and there is a need for policy intervention.

The economy is composed of imperfectly altruistic individuals, in the spirit of [1]: Each generation values its own consumption relative to the next generation's consumption more than the relative consumption of two subsequent generations later in the future. That is, each generation applies a high discount rate for the short run but a low discount rate for the long run. All generations are alike in the sense that they display the same imperfect altruism towards the future ones and the structure of preferences replicates over time. Imperfect altruism introduces a source of disagreement among generations and renders the social problem dynamically inconsistent.

A social planner (government or regulator), who represents the current generation, share its preferences. Therefore, it is subject to the same time inconsistency problem. When designing the optimal carbon tax, the current planner has to take into account that the future planners will design the carbon tax that is best in its eyes. The policy making process becomes a dynamic game where each planner chooses the best carbon policies that he can design, taking as given the policies that future planners will implement. The outcome of this game is typically not optimal in the Pareto sense and therefore, the notion of implementation of the efficient allocation becomes problematic in itself in this environment. I proceed as follows: I first assume that society can successfully resolve the inconsistency problem by committing itself to future policies and I solve for the "social optimal allocation under commitment". I then assume instead that the political arrangement does not contemplate any commitment device. For this case, I abandon the notion of efficiency and solve for the Markov Perfect (time consistent) social allocation. I refer to it as the "constrained optimal allocation" since it is the best outcome society can seek constrained by the fact that current generations can not commit future ones to continuing with a specific climate policy.

The first main result of this essay is that, when society can successfully resolve the inconsistency problem by committing itself to following a climate plan, standard carbon taxes coupled with a subsidy on oil reserves are enough to induce future generations to follow it. However, an initial period of sophisticated policies are required to induce the current generation to abide by the climate plan as well.

The intuition behind this result is that the social allocation with commitment aligns future generations preferences over the stream of consumption, sequestration and carbon level with the preferences of the current one. This requires discounting future trade offs less than they would have wished, absent any commitment. The subsidy on the stock of oil is then required to induce future generations to be more patient towards subsequent ones than what the current generation is towards them. Once the time inconsistency problem is resolved through the introduction of the subsidy, standard policies are required to reach the optimal amount of emissions. Carbon taxes provide incentives for eating up fuel resources at the optimal rate and a subsidy on sequestration induce individuals to devote the right amount of effort to pursue carbon offsets projects.

The second main result of the essay is that, when the political system is such that no commitment technology is available, sophisticated policies are required in the current and in future periods. Moreover, the subsidy for removing a ton of carbon from the atmosphere will in general not equal the tax for creating a ton of carbon.

The intuition behind this result is that, even though creating a unit less of carbon or sequestering a unit more have both the same effect on the amount of the externality, they do not have the same effect on society's welfare. First, a unit less of carbon that the current generation passes on to the next one, has an effect on the overall stream of utilities which is given by the future generations' policy reaction to inheriting less carbon in the atmosphere. This is given by their marginal propensity to consume and sequester out of carbon. In the same way, when the current generation passes on an extra unit of oil underground to the next one, this has an effect on society's welfare which is given by the future generations' policy reaction to having more oil available for extraction (the marginal propensity to consume and sequester out of oil). There is, however, an extra effect associated with leaving oil underground. This extra effect is given by the future generations' marginal propensity to consume and sequester out of carbon since keeping oil underground implies both, bequeathing more oil *and* less carbon to future generations. The overall effect on society's welfare of controlling the level of carbon through less extraction of oil or though more sequestration is not symmetrical and the tax policy reflects it.

One important contribution of this essay is to show that the direct link between the social cost of carbon in the planning problem and the policy instruments in the decentralized environment does not extend to an environment with time inconsistent preferences. When generations are imperfectly altruistic towards each other, the design of carbon policies is more elaborate and particular attention need to be posed on the market environment where policies are to be introduced.

The essay is related to the vast literature on hyperbolic discounting in climate change, specially [16], [17] and [18] and also [19], who provides a review of hyperbolic discounting with an application to environmental issues. The intergenerational framework is borrowed from [1]. In dealing with the time inconsistency problems that arise from using a time-varying discount factor, the paper is related to [20], [11] and also [21] and [22]. The essay is also related to the literature on optimal taxation of fossil fuels with a climate externality as [8] and [10]. It differs from those in that they do not study the interaction between the intergenerational and the externality dimension of carbon emissions.

The remainder of the chapter is organized as follows. Section 3.2 sets up the basic model. Section 3.3 solves the social planning problem. Section 3.4 proposes a market economies with carbon taxes. Section 3.5 characterizes carbon policies with and without commitment and presents the main results of the paper. Section 3.7 provides some conclusions from the analysis. Finally, all the proofs are in the Appendix A.

3.2 The Basic Model

Consider the following economy. At any point in time, $t \in \{0, ..., \infty\}$, the economy is populated by a unit mass continuum of identical individuals, who live for one period and constitute generation t. There is a single consumption good k_t. The good (fossil fuel) is exhaustible and thus, at any point in time, must satisfy

$$k_{t+1} \in [0, k_t] \tag{3.1}$$

The economy starts with an initial endowment equal to k_0. Resource feasibility requires

$$c_t + k_{t+1} = k_t \tag{3.2}$$

for every period t, where c_t represents fossil fuel consumption. Further, the amount of carbon in the atmosphere, S_t increases with consumption and decreases if individuals exert an effort level z_t (sequestration). In particular, the amount of carbon in the atmosphere follows the following process

$$S_{t+1} = (1 - \gamma) S_t + k_t - k_{t+1} - z_t \tag{3.3}$$

where $\gamma \in [0, 1)$ is the rate of natural reabsorption of carbon and S_0 is given. The presence of carbon in the atmosphere generates a negative externality which is assumed to take the form of a per period disutility cost. An individual's utility in period t is given by $U(c_t, z_t, S_{t+1}) = u(c_t) - v(z_t) - x(S_{t+1})$. The function u is assumed to be increasing, concave and twice differentiable with $\lim_{c \to 0} u'(c) = \infty$. The disutility cost function v is increasing, convex, twice differentiable and satisfies $\lim_{z \to 0} v'(z) = 0$. The function x is assumed to be increasing, convex, twice differentiable and satisfies $\lim_{S \to 0} x'(S) = 0$.

Over time, individual's utility is given by

$$U(c_t, z_t, S_{t+1}) + \delta \sum_{j=1}^{\infty} \beta^j U(c_{t+j}, z_{t+j}, S_{t+1+j}) \tag{3.4}$$

That is, individuals are "imperfectly altruistic", in the spirit of [1]. This specification of preferences is commonly known as hyperbolic discounting and it collapses to the standard constant geometric discounting when $\delta = 1$. It implies that the current generation values its consumption relatively more than that of generations in the distant future. That is, the discount rate in the short run, $(\delta\beta)^{-1}$ with $\delta < 1$, is higher than the one in the long run, $(\beta)^{-1}$.

Social Welfare. In this chapter, I will entertain the idea of a social planner which is a representative government. That is, the social planner shares the preferences of the present generation that he represents. In turn, this means that the current planner values the welfare of two subsequent generations in the future different from how future ones will value them. For example, one can imagine a planner who cares about the current generation and their children, but *relatively* less about generations coming after them. This also means that, in general, the current planner can not count on future ones to follow a specific climate policy since the climate plan will not be optimal when evaluated from the future generation's point of view.

The next section characterizes the solution to the social planning problem as the outcome of a dynamic game among social planners.

3.3 Social Planning Problem

Solving the social planning problem with imperfect altruism is complicated since it renders the objective of the social planning problem dynamically inconsistent: the current planner values the welfare of two subsequent generations in the future different from how future ones will value them. As a consequence, the current planner can not count on future ones to follow a specific climate policy since it will not be optimal when evaluated from their point of view. The social planning problem becomes then a dynamic game, where each planner chooses the climate policy today taking as given the policies that future ones will pursue in the future. I restrict attention to time consistent Markov perfect equilibrium.

For recursive structures, the solution to this game can be characterized as a policy fixed point using standard recursive methods. Each period, the current planner takes as given the policy rules of future planners, $\varphi = \left\{\varphi^k\left(k,S\right),\varphi^S\left(k,S\right)\right\}$, and chooses a pair of functions $\left\{k' = g^k(k, S; \varphi); S' = g^S(k, S; \varphi)\right\}$ in order to solve

$$\max_{k', S'} u(c) - v(z) - x\left(S'\right) + \delta\beta W\left(k', S'\right)$$

$$st\ c + k' = k \tag{3.5}$$

$$S' = (1-\gamma) S + k - k' - z \tag{3.6}$$

$$W(k, S) = u(\varphi^c(k, S)) - v(\varphi^z(k, S)) - x(\varphi^s(k, S)) + \beta W(\varphi^k(k, S), \varphi^s(k, S)) \tag{3.7}$$

where $\varphi^c(k, S) = k - \varphi^k(k, S)$ and $\varphi^z(k, S) = (1-\gamma) S + \varphi^c(k, S) - \varphi^S(k, S)$. The function W is an indirect utility function that satisfies the functional equation (3.7). Note that this equation reflects that future planners follow the policy rule φ. It also incorporates that the current planner has a long run discount factor β that differs from the short run one, $\delta\beta$. The Markovian assumption is reflected in the policy functions being time independent and only a function of the current stock of fuel and carbon.

The maximization problem can be thought of as a mapping function from the space of policy functions into itself. One can feed a policy function φ and the problem delivers a policy function g. Solving for the equilibrium in this context involves looking for a policy fixed point of this mapping function.

Definition 9 *The policy functions* $\{g, \varphi\}$ *constitute a* time consistent Markov Perfect equilibrium *if*

$$\begin{aligned} g^k(k, S) &= \varphi^k(k, S)\ \forall (k, S) \\ g^S(k, S) &= \varphi^S(k, S)\ \forall (k, S) \end{aligned}$$

The *time consistent social allocation* is the path of consumption, fossil fuel, sequestration and carbon level, $\{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^{\infty}$, given by $k_t^* = g^k\left(k_{t-1}^*, S_{t-1}^*\right)$ and $S_t^* = g^S\left(k_{t-1}^*, S_{t-1}^*\right)$ for every period t, with initial k_0^* and S_0^* given and where c_t^* and z_t^* are recovered from (3.5) and (3.6).

The characterization of the time consistent allocation supposes the derivation of two generalized Euler equations. These equations differ from the standard ones in that they include the derivatives of the policy functions. Assuming differentiability of the value function and the policy functions, if a solution exists and is interior, the first order conditions for the planner's problem are

$$u'\left(g^{c}\left(k,S\right)\right)-v'\left(g^{z}\left(k,S\right)\right)=\delta\beta W_{1}(g^{k}\left(k,S\right),g^{S}\left(k,S\right)) \tag{3.8}$$

$$v'\left(g^{z}\left(k,S\right)\right)=x'\left(g^{s}\left(k,S\right)\right)-\delta\beta W_{2}(g^{k}\left(k,S\right),g^{S}\left(k,S\right)) \tag{3.9}$$

where $g^{c}\left(k,S\right)=k-g^{k}\left(k,S\right)$ and $g^{z}\left(k,S\right)=\left(1-\gamma\right)S-g^{s}\left(k,S\right)+g^{c}\left(k,S\right)$. For ease of exposition, I suppress the arguments of the policy functions in what follows so that, for example, g_1^s is the shortcut for $\partial g^{s}(k,S)/\partial k$. Using (3.7) , the corresponding derivatives of the function W are given by

$$\begin{aligned} W_1(k,S) &= u'(g^c)[1-g_1^k]-v'(g^z)[-g_1^s+1-g_1^k]-x'(g^s)g_1^s \\ &\quad +\beta[W_1(g^k,g^s)g_1^k+W_2(g^k,g^s)g_1^s] \end{aligned} \tag{3.10}$$

$$\begin{aligned} W_2(k,S) &= -u'(g^c)g_2^k-v'(g^z)[1-\gamma-g_2^s-g_2^k]-x'(g^s)g_2^s \\ &\quad +\beta[W_1(g^s,g^s)g_2^k+W_2(g^k,g^s)g_2^s] \end{aligned} \tag{3.11}$$

Further, the derivatives of the value function can be substituted away using (3.8)-(3.9).

$$\begin{aligned} W_1\left(k,S\right) &= \left[u'\left(k-g^k\right)-v'\left(g^z\right)\right]\left[1-g_1^k(1-\frac{1}{\delta})\right]+\left[v'\left(g^z\right)-x'\left(g^S\right)\right]g_1^S(1-\frac{1}{\delta}) \\ W_2\left(k,S\right) &= \left[v'\left(g^z\right)-u'\left(k-g^k\right)\right]g_2^k(1-\frac{1}{\delta})-v'\left(g^z\right)(1-\gamma)+\left[v'\left(g^z\right)-x'\left(g^S\right)\right]g_2^S(1-\frac{1}{\delta}) \end{aligned}$$

Finally, update these equations one period ahead and plug them back into the first order conditions to get the following *Generalized Euler Equations*, rewritten in sequential form for ease of reading

$$\begin{aligned} u'(c_t)-v'(z_t) &= \delta\beta[u'(c_{t+1})-v'(z_{t+1})][1+g_1^k(k_{t+1},S_{t+1})(\frac{1}{\delta}-1)] \\ &\quad -\ \delta\beta[v'(z_{t+1})-x'(S_{t+2})]g_1^s(k_{t+1},S_{t+1})(\frac{1}{\delta}-1) \end{aligned} \tag{3.12}$$

$$\begin{aligned} x'(S_{t+1})-v'(z_t) &= -\ \delta\beta v'(z_{t+1})(1-\gamma)+\delta\beta[u'(c_{t+1})-v'(z_{t+1})]g_2^k(k_{t+1},S_{t+1})(\frac{1}{\delta}-1) \\ &\quad -\delta\beta[v'(z_{t+1})-x'(S_{t+2})]g_2^s(k_{t+1},S_{t+1})(\frac{1}{\delta}-1) \end{aligned} \tag{3.13}$$

These are the main behavioral equations in the planner's problem. They constitute a system of functional equations in the policy rules $g^{k}\left(k,S\right)$ and $g^{s}\left(k,S\right)$. Equation (3.12) is the Euler equation for this economy and it rules the rate at which fossil fuels are depleted. Equation (3.13) states that the marginal cost of removing carbon emissions through sequestration has to be equalized to the social value of having one extra unit less of carbon forever after. Note the presence of the derivatives of the policy functions. When $\delta = 1$, these derivatives disappear by the envelope theorem. They show up here to reflect that the current planner decides how much fossil fuel and carbon to pass on to the next one based on what he expects future planners will do. In turn, what future planners do depends on how much fuel and carbon they inherit. This is the essence of the dynamic game between planners. When deciding allocations, the planner considers the trade-off between the benefits of eating up fuels reserves and delaying mitigation activities (left hand side of equations (3.12) and (3.13), respectively) against the value of bequeathing an economy with more fossil fuels and less carbon to the next generation (the right hand side of the equations).

It is useful to define the social (shadow) cost of carbon since it plays a crucial role in the characterization of carbon policies. Formally, it corresponds to the Lagrange multiplier, λ_t^*, on the carbon cycle constraint (3.6) and it can be obtained by iterating on equation (3.13).

$$\lambda_t^* = \sum_{j=1}^{\infty} [\delta\beta\,(1-\gamma)]^{j-1} x'(S_{t+j}^*) + \theta_t^* \tag{3.14}$$

where

$$\theta_t^* = \sum_{j=1}^{\infty} (\delta\beta)^j (1-\gamma)^{j-1} (\frac{1}{\delta} - 1) \left[[u'(c_{t+j}^*) - v'(z_{t+j}^*)] g_2^k + [v'(z_{t+j}^*) - x'(S_{t+j}^*)] g_2^s \right]$$

Typically, the social cost of carbon is equal to discounted sum of marginal damages associated to an increase in the level of carbon in the atmosphere. This is given by the first summand in equation (3.3). In a model with imperfect altruism, however, the social cost of carbon includes an extra term θ_t^*. The reason is that a marginal increase in the stock of carbon bequeathed to the next generation affects their marginal decision on how much to pollute and how much to sequester thereafter, which is taken as given by the current generation. This adds an extra kick on the overall effect (besides the climate damages) of increasing the level of carbon today by a marginal unit. The additional term θ_t^*, which contains the derivatives of the policy functions, recovers this extra effect.

Since it will proved to be useful to characterize the carbon policies, I will denote ω_t^*

the social cost of carbon expressed in terms of consumption utils. That is,

$$\omega_t^* \equiv \frac{\lambda_t^*}{u'(c_t^*)} \tag{3.15}$$

The next section proposes a market economy to study the decentralization of the time consistent social allocation.

3.4 Recursive Competitive Equilibrium

Consider the following market economy. A representative firm operates a linear technology to produce the single consumption good in the economy. A continuum of mass one households derive utility from consumption and incur in a disutility cost when they provide sequestration services z. Households dislike carbon in the atmosphere but they are small to individually control its level. Therefore, the market outcome does not coincide with the planner's one and there is room for policy intervention. Households face two forms of taxation: a carbon tax τ^k on fossil fuel extraction and a sequestration subsidy τ^z. Households own the stock of fossil fuels and sell it to firms which use it as an input for production. Households consume, provide sequestration services and save subject to the following budget constraint

$$c = \tau^z(K,S)z + q(K,S)(1-\tau^k(K,S))(k-k') + T \tag{3.16}$$

where $q(K,S)$ is the price of fossil fuels in terms of the consumption good and T is a lump sum rebate from the government. The economy starts with $\{k_0, S_0\}$ given. Note that the price of fuel and the tax policy are functions of the aggregate states: the aggregate stock of fossil fuels, K, and the aggregate level of carbon, S. Throughout this section, I use small letters to denote individual variables and capital letters to denote economy wide variables. The recursive competitive equilibrium requires three states variables for the consumers: one for the individual's stock of fossil fuel, one for the average level of fossil fuel and one for the aggregate level of carbon. These last ones are needed to predict future policies and prices. I restrict attention to a Markov recursive competitive equilibrium.

Taking as given the policy rules of the future generations, $\hat{\varphi} \equiv \{\hat{\varphi}^k(k,K,S), \hat{\varphi}^z(k,K,S)\}$, the problem of the current generation is to choose $k' = \hat{g}^k(k,K,S;\hat{\varphi})$ and $z' = \hat{g}^z(k,K,S;\hat{\varphi})$

in order to solve the following problem

$$\max_{k',z} u(c) - v(z) - x(S') + \delta\beta V(k', K', S')$$

$$st\ c = \tau^z(K,S)z + q(K,S)(1-\tau^k(K,S))(k-k') + T \tag{3.17}$$

$$K' = h^k(K,S) \tag{3.18}$$

$$S' = h^s(K,S) \tag{3.19}$$

$$V(k,K,S) = u(\hat{\varphi}^c(k,K,S)) - v(\hat{\varphi}^z(k,K,S)) - x(S') + \beta V(\hat{\varphi}^k(k,K,S), K', S') \tag{3.20}$$

where $\hat{\varphi}^c(k,K,S) = \tau^z(K,S)\hat{\varphi}^z(k,K,S) + q(K,S)(1-\tau^k(K,S))(k - \hat{\varphi}^k(k,K,S)) + T$.

Consumers make their decisions taking as given the price of fossil fuel $q(K,S)$, the government policies $\{\tau^k(K,S), \tau^z(K,S), T\}$, the law of motion for the aggregate level of fuel, $h^k(K,S)$ and of the carbon level, $h^s(K,S)$. The current generation takes as given also the decision rules of the future generations $\hat{\varphi}$.

Definition 10 *A Markov recursive competitive equilibrium with a tax scheme $\{\tau^k(K,S)$, $\tau^z(K,S)$, $T\}$ is a price function $q(K,S)$, decision rules $\{\hat{g}^k(k,K,S), \hat{g}^z(k,K,S)\}$, a value function $V(k,K,S)$ and a law of motion for fossil fuel $h^k(K,S)$ and for carbon $h^s(K,S)$ such that: (i) given $V(k,K,S)$ and taxes, the functions $\hat{\varphi}$ solve the consumer's problem, (ii) given $\hat{\varphi}$, $V(k,K,S)$ satisfies the functional equation (3.20), (iii) firms maximize profits, $q(K,S) = 1$, (iv) The budget constraint of the government is satisfied, (v) Aggregate consistency holds so that $h^k(K,S) = \hat{\varphi}^k(K,K,S)$ and $h^s(K,S) = (1-\gamma)S + K - \hat{\varphi}^k(K,K,S) - \hat{\varphi}^z(K,K,S)$.*

I characterize next the competitive equilibrium with taxes. Assuming differentiability of the value function and the policy functions, if a solution exists and is interior, the first order conditions for the current generation's problem are given by:

$$k' \ : \ -u'[\hat{g}^c(k,K,S)]q(K,S)(1-\tau^k(K,S)) + \delta\beta V_1(\hat{g}^k(k,K,S), K', S') = 0 \tag{3.21}$$

$$z \ : \ u'[\hat{g}^c(k,K,S)]\tau^z(K,S) - v'(\hat{g}^z(k,K,S)) = 0 \tag{3.22}$$

For ease of exposition, I suppress the arguments of the functions in what follows. Using (3.20), the derivative of the value function is given by

$$V_1(k,K,S) = \ u'(\hat{g}^c)[\tau^z\hat{g}_1^z + q(1-\tau^k)(1-\hat{g}_1^k)] - v'(\hat{g}^z)\hat{g}_1^z + \beta V_1(\hat{g}^k, K', S')\hat{g}_1^k \tag{3.23}$$

we can use (3.21) and (3.22) to replace away the derivatives of the value function in the equation to get

$$V_1(k,K,S) = u'(\hat{g}^c)[q(1-\tau^k)(1+\hat{g}_1^k(\frac{1}{\delta}-1))] \tag{3.24}$$

Finally, update the expressions for the derivatives of the value function one period ahead and plug them back into the first order conditions to get the following Generalized Euler Equations, which I write in sequential form for ease of reading.

$$\frac{1-\tau_{t+1}^k}{1-\tau_t^k} = \frac{u'(c_t)}{\delta\beta u'(c_{t+1})}\frac{1}{[1+\hat{g}_1^k(k_{t+1},K_{t+1},C_{t+1})(\frac{1}{\delta}-1)]} \tag{3.25}$$

$$\tau_t^z = \frac{v'(z_t)}{u'(c_t)} \tag{3.26}$$

The household's problem is characterized by two relevant margins: the intratemporal decision between consumption and sequestration and the intertemporal choice regarding how much oil to bequeath to the next generation. The intratemporal condition (3.26) is standard and says that the current generation will decide on consumption and sequestration in order to equalize marginal utilities to the ratio of prices. It is clear that, absent any tax policy, there are no incentives to do sequestration. Once the government sets a price for this missing market, households are willing to offer these services since it is a source of income that allows them to consume. The Euler equation (3.25) rules the rate at which fossil fuels are eaten up over time. Note the presence of the derivative of the policy function in this equation. If the current generation depletes an extra unit of fossil fuel today, it gains the marginal utility of consumption net of taxes. However, future generations will inherit less fossil fuel and this has a long-lasting effect in the whole path of fossil fuel extraction. In particular, future generation will modify their fossil fuel extraction by $\hat{g}_1^k$. This marginal change in the path of extraction is valued δ by the current generation but $\beta\delta$ by the future one. The last term between parenthesis adjusts for this effect.

3.5 Time Consistent Carbon Policies

Characterizing carbon policies with a time varying social discount rate is complicated since it renders the objective of the social planning problem dynamically inconsistent, thereby making the notion of efficiency itself become problematic. I first assume that society can successfully resolve the inconsistency problem by committing itself to future policies and I solve for the social efficient allocation under commitment. I then assume instead that the political arrangement does not contemplate any commitment device. For this case, I abandon the notion of efficiency and solve for the Markov Perfect (time-consistent) social allocation. I refer to it as the constrained efficient allocation since it is the best outcome society can seek constrained by the fact that current generations can not commit future ones to continue with a specific climate policy.

3.5.1 No Policy Intervention

I will briefly discuss the laissez-faire (or business-as-usual) equilibrium as a benchmark case. That is, the decentralized equilibrium outcome without any policy intervention. The laissez-faire economy collapses to a simple version of a cake-eating problem. The only relevant decision households face is how much fossil fuel to hand on to the next generation. No sequestration takes place since households derive disutility from it and get no compensation. The Euler equation reduces to

$$\beta\delta u'(c_{t+1})[1+\hat{g}^k(k_{t+1},K_{t+1},C_{t+1})(\frac{1}{\delta}-1)] = u'(c_t) \qquad (3.27)$$

which is a version of the Hotelling rule from the perspective of the consumers. By comparing (3.27) with (3.12), it follows that the rate of depletion of fossil fuels is not optimal in the laissez faire economy. There are three sources of divergence. First, individuals do not take into account that, associated to the utility gains and costs derived from moving consumption across generations, there are gains and costs from the environmental damages this consumption generates. Because of that, a second source of non-optimality of the laissez faire economy is that the marginal propensity to keep fossil fuel underground for future generations, $\hat{g}^k(k_{t+1},K_{t+1},C_{t+1})$ will typically not coincide with the planner's one, $g^k(K_{t+1},C_{t+1})$. Moreover, saving fossil fuel for future generations will affect their marginal decision on how much to sequester, which in turn will affect the level of carbon by $\hat{g}^z(k_{t+1},K_{t+1},C_{t+1})$. However, individuals are small to take this effect into account and this is the third source of divergence between the business-as-usual market economy and the social planning outcome. The next proposition summarizes this result.

Proposition 11 ***(Laissez faire)*** *The competitive equilibrium allocation with no policy intervention is not optimal.*

3.5.2 Standard Policies

A second enlightening benchmark case is the one with perfect altruism. That is, consider an economy in which $\delta = 1$. Every generation discounts the future at a constant discount factor β and the inconsistency problem vanishes. The generalized Euler equations that characterize the optimal allocation collapse then to the following set of standard optimality conditions, where the terms containing the derivatives of the policy function just disappear by means of the Envelope theorem.

$$v'(z_t^*) = \sum_{j=1}^{\infty} [\beta (1-\gamma)]^{j-1} x'(S_{t+j}^*) \tag{3.28}$$

$$\beta[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*) \tag{3.29}$$

The right hand side of equation (3.28) represents the social cost of carbon, which measures the marginal damage of increasing carbon emissions in an extra unit. The optimality condition simply states that the marginal cost of removing carbon emissions through sequestration must be equalized to its social value, which is the accumulated benefits from having one less unit of carbon in the atmosphere forever after. Equation (3.29) is the Euler condition for this economy. It rules the rate of depletion of fossil fuels so that the marginal utility of consumption, net of environmental damages, is equalized across generations.

Since it is useful to characterize the carbon policies, I will denote μ_t^* the social cost of carbon expressed in terms of utils. That is

$$\mu_t^* \equiv \frac{\sum_{j=1}^{\infty} [\beta (1-\gamma)]^{j-1} x'(S_{t+j}^*)}{u'(c_t^*)} \tag{3.30}$$

The following proposition states that, in an economy without a time inconsistency problem, the optimal allocation can be decentralized through standard carbon taxes.

Proposition 12 ***(Standard Policies)*** *Suppose that* $\delta = 1$. *If* $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ *solves the social planning problem, then it solves the competitive equilibrium with taxes* $\{\tau_t^k, \tau_t^z, T_t\}_{t=0}^{\infty}$

defined by

$$\tau_t^k = \tau_t^z = \mu_t^*$$

for every period t and all proceeds from taxation rebated/financed lump-sum through T_t.

The proposition highlights a basic principle in optimal taxation of externalities. A Pigouvian tax/subsidy equal to the value of the externality is enough to make the private individuals internalize it. I refer to it as "standard policies" since they regulate the net emissions of carbon (emissions net of sequestration). That is, a regulator typically does not care about how the optimal level of carbon is reached, if by reducing emissions or by offsetting them through sequestration. In order to reach the social optimum, it is enough to control the net amount of emissions.

The goal of the next two sections is to study the extent to which standard policies are useful to deal with economies that display time inconsistency problems.

3.5.3 Optimal Policies with Commitment

One way to deal with time inconsistency problems is for society to develop ways to commit itself to future actions (such as through laws or a constitution). I will not model these commitment technologies explicitly, but will simply assume for now that they exist. If the current generation can commit future ones to follow a specific climate policy, it will prescribe them to follow the allocation that arises from solving the social planning problem with a constant social discount factor equal to β. Note that, with $\delta < 1$, it implies that futures generations are asked to be more patient towards the future than what the current generation actually is. In particular, future generations will be called to follow the decision rules $\{\tilde{g}^k, \tilde{g}^s\}$ that solve

$$V(k,S) = \max_{k',S'} u(c) - v(z) - x(S') + \beta V(k', S')$$

$$st \; c + k' = k \tag{3.31}$$

$$S' = (1-\gamma) S + k - k' - z \tag{3.32}$$

Given $\{\tilde{g}^k, \tilde{g}^s\}$ and the value function V, the current generations choose policies $\{g^k, g^s\}$ in order to solve

$$\max_{k',S'} u(c) - v(z) - x(S') + \delta\beta V(k', S')$$

$$st \; c + k' = k \tag{3.33}$$

$$S' = (1-\gamma) S + k - k' - z \tag{3.34}$$

$$V(k,S) = u\left(k - \tilde{g}^k(k,S)\right) - v\left(\tilde{g}^z(k,S)\right) - x\left(\tilde{g}^s(k,S)\right) + \beta V\left(\tilde{g}^k(k,S), \tilde{g}^s(k,S)\right) \tag{3.35}$$

where $\tilde{g}^z(k,S) = (1-\gamma) S + k - \tilde{g}^k(k,S) - \tilde{g}^s(k,S)$.

The *optimal allocation with commitment* is the path for consumption, fossil fuel, sequestration and carbon level, $\{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^{\infty}$, that satisfies $k_t^* = \tilde{g}^k\left(k_{t-1}^*, S_{t-1}^*\right)$ and $S_t^* = \tilde{g}^s\left(k_{t-1}^*, S_{t-1}^*\right)$ for every period $t \geq 1$, with $k_1 = g^k(k_0^*, S_0^*)$ and $S_1^* = g^s(k_0^*, S_0^*)$ in the current period 0 and initial k_0^* and S_0^* given. In turn, c_t^* and z_t^* are recovered from (3.33) and (3.34).

The following proposition states the first main result of this paper: when society can successfully resolve the inconsistency problem by means of a commitment technology (such as a Climate bill), standard policies coupled with a subsidy on oil reserves are enough to induce future generation to follow the climate plan. However, an initial period of sophisticated policies are required to induce the present generation to abide by it as well.

Proposition 13 ***(A Climate Bill)*** *If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the social planning problem with commitment, then it solves the competitive equilibrium with standard policies coupled with a subsidy on the stock of oil τ_t^s, for $t > 2$ given by*

$$\tau_t^k = \tau_t^z = \mu_t^* \;\; ; \;\; \tau_t^s = [\frac{1}{\delta + \hat{g}_{1,t}^k(1-\delta)} - 1](1-\mu_t^*)$$

and an initial period of sophisticated policies.

The social allocation with commitment aligns future generations preferences over the stream of consumption, sequestration and carbon level with the preferences of the current one. This requires future generations to discount future tradeoffs less than they would have wished, absent any commitment. The subsidy on the stock of oil is then required to induce future generations to be more patient towards subsequent ones than what the current generation is towards them. Once the time inconsistency problem is resolved through the introduction of the subsidy, standard policies are required to reach the optimal amount of emissions to control the externality. Carbon taxes provide incentives for eating up fuel resources at the optimal rate and a subsidy on sequestration induce individuals to devote the right amount of effort to pursue carbon offsets projects. It is important to note that a subsidy on the stock of oil would be optimal, even absent a climate externality.

3.5.4 Constrained Optimal Policies

If the political system is such that no commitment technology is available then, the social planning problem becomes one in which each planner decides how much fossil fuel and carbon to hand on to the next generation under the constraint that policies from tomorrow on must be taken as given. I refer to this allocation as the 'Constrained optimal' since it is the best outcome society can seek constrained by the fact that current generations can not commit future ones to continue with a specific climate policy.

The following proposition states the second main result of this paper: when no commitment mechanisms are available, sophisticated policies are required to implement the constrained optimal level of carbon. Standard carbon policies are insufficient.

Proposition 14 ***(Time Consistent Climate Policies)*** *If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the contained efficient social planning problem, then it solves the competitive equilibrium with sophisticated policies given by a carbon subsidy on sequestration*

$$\tau_1^z = \omega_t^*$$

and a carbon tax that follows the following rule

$$1 - \tau_{t+1}^k = [\frac{1 - \omega_{t+1}^*}{1 - \omega_t^*} - \frac{[\omega_{t+1}^* - \frac{x'(S_{t+2}^*)}{u'(c_{t+1}^*)}]g_1^s(\frac{1}{\delta} - 1)}{(1 - \omega_t^*)[1 + g_1^k(\frac{1}{\delta} - 1)]}](1 - \tau_t^k)$$

with τ_0^k given.

The subsidy for removing a ton of carbon from the atmosphere is not equal to the tax for creating a ton of carbon, even though creating a unit less of carbon or sequestering a unit more have both the same effect on the amount of the externality. The reason is that they do not have the same effect on the overall welfare. The reason is that a unit less of carbon that the current generation passes on to the next one has an extra effect on the overall stream of utilities, which is given by the future generations' marginal propensities to consume and to sequester out of carbon. In the same way, when the current generation passes an extra unit of oil underground to the next one, this has an effect on the overall stream of utilities given by the future generations' marginal propensity to consume and to sequester out of oil. But note that, in addition, this also has an extra effect on welfare given by the future generations' marginal propensity to consume or sequester out of carbon since keeping oil underground implies both, bequeathing more oil *and* less carbon to future generations. The overall effect on society's welfare of controlling the level of carbon through less extraction of oil or though more sequestration is not symmetrical and the tax policy reflects it. In particular, note that the subsidy rate formula contains the marginal propensities to consume and to sequester out of carbon (both embedded in the formula for ω_t^*) while the tax rate on emissions contains as well the marginal propensities to consume and to sequester out of oil.

I rely on numerical methods to further characterize the constrained optimal allocation and climate policy. The solution method applied in the computed example economy may prove to be interesting in itself since it is an application of endogenous grid methods to a problem with two state variables, two control variables and hyperbolic discounting. I include a discussion of the algorithm in the appendix.

3.6 An Example Economy

This section presents an example economy. The objective of this numerical exercise is to illustrate the main features of the optimal policies with and without commitment as well as of the path for sequestration, consumption and the stocks of carbon and fossil fuels that solve the social planning problem. It was computed assuming the following functional forms and parameters: $\delta = 0.9,\ \beta = 0.96,\ U(c, z, S) = log(c) - \phi_z \frac{z^2}{2} - \phi_s \frac{S^2}{2}$, $\phi_z = 500,\ \phi_s = 1, \gamma = 0.001,\ k \in [0, 1],\ S \in [0, 1], k_0 = 1,\ S_0 = .3$.

Figure 3.1: Social Planner's allocation with and w/o Commitment

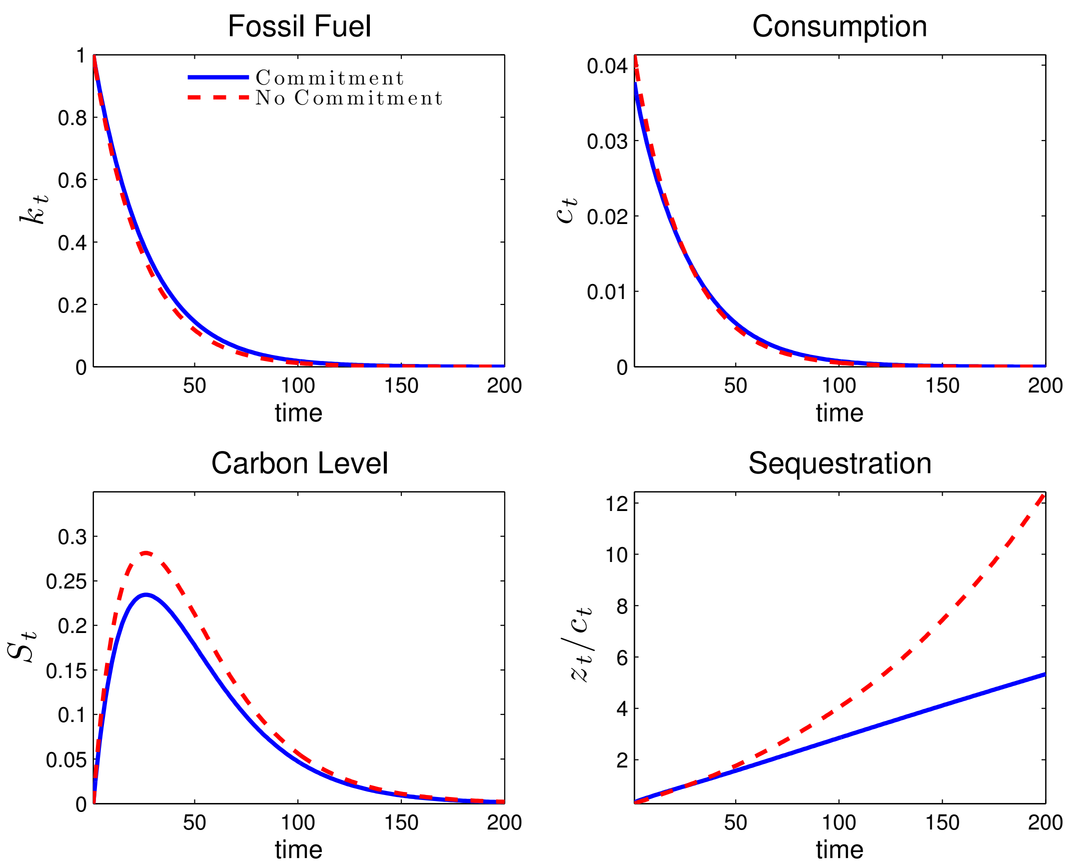
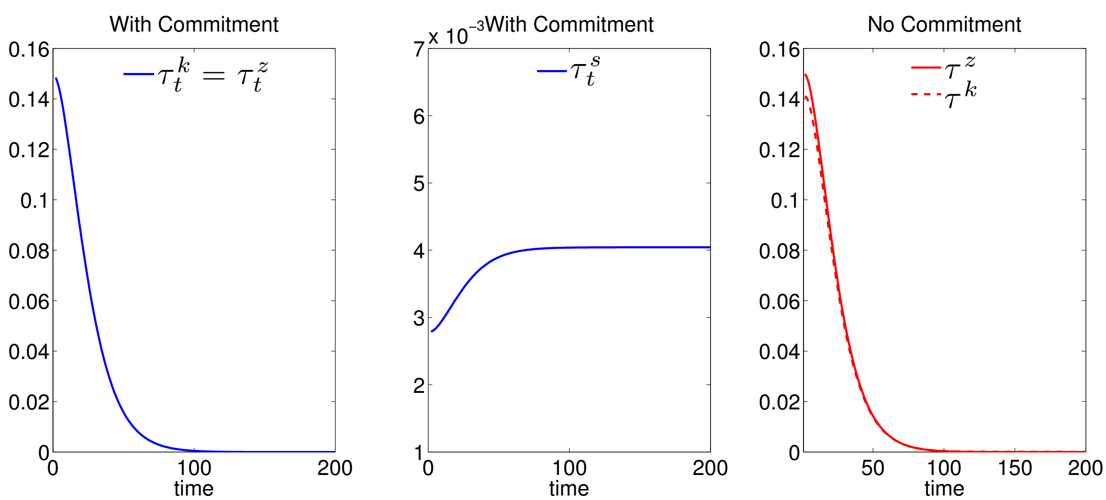


Figure 3.1 shows the time path for consumption, sequestration and the stocks of carbon and fossil fuels. The blue line corresponds to the allocation with commitment and the red dotted line to the constrained optimal one. The commitment solution displays lower extraction and relatively more sequestration in early periods. The current generation extracts relatively more than generations to come (there is a jump downwards, which is hardly seeable from the picture). Carbon in the atmosphere is overall lower when there is commitment given that carbon accumulates less in early periods. As a

consequence, future generations are expected to sequester less than in the no commitment solution. Note that, in the allocation without commitment, future generation the burden of climate policies on future generations is big. They inherit little oil and too much carbon which forces them to sequester much more carbon than the amount they are actually releasing to the atmosphere. Figure 3.2 illustrates the main results of this paper. When society can commit to a climate policy, this must include standard carbon taxes and a subsidy on the stock of oil reserves. When no commitment technologies are available, sophisticated policies are required. In particular, the carbon tax for creating a ton of carbon is different from the carbon subsidy for removing it.

Figure 3.2: Carbon Policies with and w/o Commitment



3.7 Conclusions

This paper considers the optimal taxation of carbon emissions in an economy with imperfect intergenerational altruism. By this I mean that the current generations discount tradeoffs in the near future more than those which happen in the distant future. As a result, a time inconsistency problem arises. I show that standard carbon policies, such as price controls on the net emissions of carbon, are often insufficient to achieve the social optimum: More sophisticated policies are necessary. These sophisticated policies consist on carbon taxes and subsidies which are *not* equal to the value of the climate externality. Moreover, the carbon tax for creating a ton of carbon in the atmosphere is in general not equal to the carbon subsidy for removing it.

Chapter 4

Concluding Remarks

The long lifetime of the carbon emissions that we release while burning fossil fuels makes the problem of climate change be not only about an externality but also about intergenerational equity. Much has been discussed in the literature regarding the implications that this has for the choice of the social discount factor in climate change models. I make two main points in this dissertation:

1. It is a normative question what intergenerational approach the policy maker adopts to study climate change. A welfare function that places direct weights on the current as well as future generations allows to track any point along the Pareto frontier between the current and future generations. Moreover, the efficient allocations associated with strictly positive direct weights on unborn generations imply a social discount rate lower than that of the individuals living in the society. Whether we get to see the policy maker as such a social architect or as a representative government that places Pareto weights only in the generation it represents, so that the social and private discount rates coincide, is a normative discussion.

2. Addressing climate change not only as an externality but also as an intergenerational problem has implications for the optimal design of carbon policies, which have been much less discussed in the literature. I argue that standard carbon policies, such as price or quantity controls on the net emissions of carbon, are often insufficient to achieve the social optimum: More sophisticated policies are necessary. This is the focus and the main contribution of my dissertation.

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Appendix A

Main Proofs

Proof of Proposition 2. The proof consists on showing that the equilibrium conditions do not coincide with the optimality conditions in the planning problem. Let $\tau_t^k = \tau_t^z = 0 \, \forall t$. Then, firms' optimization implies that $w_t = 0$ and the equilibrium condition (2.22) becomes $v'(z_t) = 0$, while optimal sequestration satisfies (2.12). In addition, the equilibrium intertemporal condition is (2.36) which does not coincide with (2.14). In the cap and trade economy, if there is no caps then $\hat{\tau}_t = \hat{\tau}_t^z = 0$. The equilibrium condition (2.34) becomes $v'(z_t) = 0$, while optimal sequestration satisfies (2.12). In addition, the equilibrium intertemporal condition is (2.36) which does not coincide with (2.14). ■

Proof of Proposition 3. The proof consists on showing that all conditions for an equilibrium are satisfied by the efficient allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when policy instruments are set optimally. I will refer to the optimal allocation and the *star allocation* indistinctly. The first part of the proof shows that all conditions for a competitive equilibrium with taxes are satisfied. It is useful to rewrite taxes in terms of allocations (instead of shadow prices) as follows

$$\tau_t^k = \tau_t^z = \frac{v'(z_t^*)}{u'(c_t^*)}$$

The intertemporal condition (2.23), evaluated at the star allocation, is

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1-\tau_t^k}{1-\tau_{t+1}^k}$$

Plug the optimal tax rate to get

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1 - \frac{v'(z_t^*)}{u'(c_t^*)}}{1 - \frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}}$$

Rearranging terms we get

$$\beta[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*)$$

which is satisfied by the optimal allocation since it coincides with (2.14), given that $\hat{\beta} = \beta$. Consider now the equilibrium condition (2.22), together with the optimality condition for firms, evaluated at the star allocation

$$\frac{u'\left(c_t^*\right)}{v'\left(z_t^*\right)} = \frac{1}{\tau_t^z}$$

This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (2.20) is already satisfied by the efficient allocation. Given the sequence of taxes $\left\{\tau_t^k, \tau_t^z\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government (2.17) is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers $b_{t+1} = R_t b_t$ for all t, which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial $b_0 = 0$. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (2.21). This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied by the optimal allocation.

The second part of the proof shows that, when caps on carbon emissions are set optimally, all conditions for a competitive equilibrium with a cap and trade system are satisfied by the optimal allocation. Since there is no cap on sequestration, $\hat{\tau}_t^z = 0$. The intratemporal condition (2.34) together with the condition on prices from the firm's problem implies that the equilibrium price of permits on net emissions is given by

$$\frac{v'\left(z_t\right)}{u'\left(c_t\right)} = \frac{\hat{\tau}_t}{p_t}$$

Plug it into the intertemporal condition (2.35) to get

$$\frac{\beta u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = \frac{1 - \frac{v'(z_t)}{u'(c_t)}}{1 - \frac{v'(z_{t+1})}{u'(c_{t+1})}}$$

Rearranging terms and using we get

$$\beta[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t) \tag{A.1}$$

Given $\{\theta_t\}_{t=0}^{\infty}$, the equilibrium sequence $\{c_t, z_t\}_{t=0}^{\infty}$ must also satisfy

$$\theta_t = c_t - z_t \tag{A.2}$$

Conditions (A.5) and (A.2) become a system of two equations with two unknowns: c_t, z_t. Using (2.14), the definition of caps given in the Proposition and the condition $\hat{\beta} = \beta$, the system (A.5) and (A.2) is satisfied by the optimal allocation. The market clearing condition for fossil fuel (2.20) is already satisfied by the efficient allocation. The budget constraint of the consumer delivers $b_{t+1} = R_t b_t$ for all t, which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial $b_0 = 0$. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (2.33). This completes the proof that all conditions for a competitive equilibrium with a cap and trade system are satisfied by the optimal allocation. ■

Proof of Proposition 4. It is sufficient to show that at least one of the conditions for an equilibrium is violated by the optimal allocation. In the tax economy, the intratemporal condition (2.22), together with the condition on prices from the firm's problem and the restriction on taxes, can be rewritten as

$$\frac{v'(z_t)}{u'(c_t)} = \tau_t^k$$

Plug it into the intertemporal condition (2.23) to get

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \frac{v'(z_t)}{u'(c_t)}}{1 - \frac{v'(z_{t+1})}{u'(c_{t+1})}}$$

Rearranging terms we get that the competitive allocation satisfies

$$\beta[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t) \tag{A.3}$$

But this condition is violated by the star allocation. The optimal allocation satisfies

$$\hat{\beta}[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t) \tag{A.4}$$

for $\hat{\beta} > \beta$. It follows that the optimal allocation violates condition (2.23) for an equilibrium. Hence, the optimal allocation does not solve the competitive equilibrium with taxes. Standard carbon taxes alone are not optimal. In the cap and trade economy, the intratemporal condition (2.34) together with the condition on prices from the firms' problem imply that the equilibrium price of permits is given by

$$\frac{v'(z_t)}{u'(c_t)} = \hat{\tau}_t$$

Plug it into the intertemporal condition (2.35) to get

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \frac{v'(z_t)}{u'(c_t)}}{1 - \frac{v'(z_{t+1})}{u'(c_{t+1})}}$$

Rearranging terms we get that the competitive allocation satisfies

$$\beta[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t) \tag{A.5}$$

But this condition is violated by the star allocation. The optimal allocation satisfies

$$\hat{\beta}[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t) \tag{A.6}$$

for $\hat{\beta} > \beta$. It follows that the optimal allocation violates condition (2.35) for an equilibrium. Hence, the optimal allocation does not solve the competitive equilibrium with a standard cap and trade. A cap and trade system with caps on the net emissions on carbon alone is not optimal. ■

Proof of Proposition 5. The proof consists on showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when taxes and subsidies are set optimally. It is useful to rewrite taxes in terms of allocations (instead of shadow prices) in the following way

$$1 - \tau_t^k = \left(\frac{\hat{\beta}}{\beta}\right)^t \left[1 - \frac{v'(z_t^*)}{u'(c_t^*)}\right]$$

The intertemporal condition (2.23), evaluated at the star allocation, is

$$\frac{\beta u'(c_{t+1}^*)}{u'(c_t^*)} = \frac{1 - \tau_t^k}{1 - \tau_{t+1}^k}$$

Plug the optimal tax rate to get

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{\left(\frac{\hat{\beta}}{\beta}\right)^t \left[1 - \frac{v'(z_t^*)}{u'(c_t^*)}\right]}{\left(\frac{\hat{\beta}}{\beta}\right)^{t+1} \left[1 - \frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}\right]}$$

Rearranging terms we get

$$\hat{\beta}[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*)$$

which is satisfied by the efficient allocation since it coincides with (2.14). Consider now the equilibrium condition (2.22) evaluated at the star allocation

$$\frac{u'\left(c_t^*\right)}{v'\left(z_t^*\right)} = \frac{1}{\tau_t^z}$$

This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (2.20) is already satisfied by the efficient allocation. Given the sequence of taxes $\left\{\tau_t^k, \tau_t^z\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government (2.17) is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers $b_{t+1} = R_t b_t$ for all t, which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial $b_0 = 0$. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (2.21). This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied. ■

Proof of Proposition 6. The proof consists on showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when taxes and subsidies are set optimally. The intertemporal condition for this tax economy, evaluated at the star allocation, is

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1 - \tau_t^k}{1 - \tau_{t+1}^k + \tau_{t+1}^s}$$

Plug the optimal tax rate to get

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1 - \frac{v'(z_t^*)}{u'(c_t^*)}}{1 - \frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)} + \left(\frac{\hat{\beta}}{\beta} - 1\right) \left[1 - \frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}\right]}$$

Rearranging terms we get

$$\hat{\beta}[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*)$$

which is satisfied by the efficient allocation since it coincides with (2.14). Consider now the equilibrium condition (2.22) evaluated at the star allocation

$$\frac{u'(c_t^*)}{v'(z_t^*)} = \frac{1}{\tau_t^z}$$

This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (2.20) is already satisfied by the efficient allocation. Given the sequence of taxes $\{\tau_t^k, \tau_t^z, \tau_t^s\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government $p_t\tau_t^k(k_t - k_{t+1}) - p_t\tau_t^z z_t^d - p_t\tau_t^s k_t = T_t$ is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers $b_{t+1} = R_t b_t$ for all t, which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial $b_0 = 0$. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (2.21). This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied. ■

Proof of Proposition 7. The proof is by construction. The constraints (2.24) and (2.25), together with the market clearing condition for the caps, eq. (2.31) and (2.32), imply that the competitive allocation coincides with the optimal. Equilibrium permit prices are then recovered from the equilibrium conditions (2.34) and (2.35).■

Proof of Proposition 8. The proof is by construction. The constraints (2.40) and (2.41), together with the market clearing condition for the cap, eq. (2.31) and for sequestration (2.30), imply that the competitive allocation coincides with the optimal. Equilibrium prices for carbon permits and for sequestration are then recovered from the equilibrium conditions (2.34) and (2.35).■

Proof of Proposition 11. The proof consists on showing that the equilibrium conditions do not coincide with the optimality conditions in the planning problem. Let $\tau_t^k = \tau_t^z = 0\ \forall t$. Then, the equilibrium condition (3.26) becomes $v'(z_t) = 0$, while optimal sequestration satisfies (3.13). In addition, the equilibrium intertemporal condition is (3.25) which does not coincide with (3.13). ■

Proof of Proposition 12. The proof consists on showing that all conditions for an equilibrium are satisfied at the planner's allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$. It is useful to rewrite taxes in terms of allocations (instead of shadow prices) as follows

$$\tau_t^k = \tau_t^z = \frac{v'(z_t^*)}{u'(c_t^*)}$$

The intertemporal condition (3.21) with $\delta = 1$ and evaluated at the star allocation, is

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1-\tau_t^k}{1-\tau_{t+1}^k}$$

Plug the optimal tax rate to get

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1-\frac{v'(z_t^*)}{u'(c_t^*)}}{1-\frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}}$$

Rearranging terms we get

$$\beta[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*)$$

which is satisfied by the optimal allocation since it coincides with (3.29). Consider now the equilibrium condition (3.22) evaluated at the star allocation

$$\frac{u'\left(c_t^*\right)}{v'\left(z_t^*\right)} = \frac{1}{\tau_t^z}$$

This condition holds by definition of the optimal taxes. Given the sequence of taxes $\left\{\tau_t^k, \tau_t^z\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government is satisfied. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm which delivers $q_t = 1$ for every period t. This completes the proof that all conditions for a competitive equilibrium with standard policies are satisfied by the optimal allocation. ■

Proof of Proposition 13. The optimal allocation with commitment satisfies the following set of equations for periods t=1,2,...

$$\frac{\beta u'(c_{t+1}^*)}{u'(c_t^*)} = \frac{1-\mu_t^*}{1-\mu_{t+1}^*} \tag{A.7}$$

$$\frac{v(z_t^*)}{u'(c_t^*)} = \mu_t^* \tag{A.8}$$

together with

$$\frac{\delta\beta u'(c_1^*)}{u'(c_0^*)} = \frac{1-\mu_0^*}{1-\mu_1^*} \tag{A.9}$$

$$\frac{v(z_0^*)}{u'(c_0^*)} = \mu_0^* \tag{A.10}$$

for the current period $t = 0$. The proof consists on showing that, given the climate policy, all conditions for an equilibrium are satisfied at the optimal allocation with commitment $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$. Define the sophisticated policies in the initial periods to be $\tau_1^k = 1 - \frac{1-\mu_1^*}{1+\hat{g}_1^k(1/\delta-1)}$, $\tau_1^z = \mu_1^*$; $\tau_0^k = \tau_0^z = \mu_0^*$ and $\tau_2^s = [\frac{1}{\delta[1+\hat{g}_{1,t+1}^k(1/\delta-1)][1+\hat{g}_{1,t+2}^k(1/\delta-1)]} - 1](1-\mu_2^*)$. Note that τ_2^s is a subsidy for delta low enough. Plug these policies into the competitive equilibrium conditions (3.21)-(3.22) in period 0 to get

$$\frac{\delta\beta u'(c_1)}{u'(c_0)} = \frac{1-\mu_0^*}{\frac{1-\mu_1^*}{1+\hat{g}_1^k(1/\delta-1)}[1+\hat{g}_1^k(1/\delta-1)]} \tag{A.11}$$

$$\frac{v(z_0^*)}{u'(c_0^*)} = \mu_0^* \tag{A.12}$$

which, given (A.9)-(A.10), are satisfied at the optimal allocation. From period 1 on, the competitive equilibrium conditions (3.21) with the subsidy τ_t^s is given by

$$\frac{1-\tau_{t+1}^k+\tau_{t+1}^s}{1-\tau_t^k} = \frac{u'(c_t)}{\delta\beta u'(c_{t+1})}\frac{1}{[1+\hat{g}_1^k(k_{t+1},K_{t+1},C_{t+1})(\frac{1}{\delta}-1)]} \tag{A.13}$$

Plug the policies proposed for t=1 and get

$$\frac{1-\mu_2^*}{1-\mu_1^*}\frac{[1+\hat{g}_1^k(k_1,K_1,C_1)(1/\delta-1)][1+\hat{g}_1^k(k_2,K_2,C_2)(\frac{1}{\delta}-1)]}{\delta[1+\hat{g}_1^k(k_2,K_2,C_2)(1/\delta-1)][1+\hat{g}_1^k(k_1,K_1,C_1)(1/\delta-1)]} = \frac{u'(c_1)}{\delta\beta u'(c_2)} \tag{A.14}$$

where all the derivatives of the policy function cancel out. Evaluated at the optimal allocation, this equilibrium condition coincides with (A.7) and so it is satisfied by the optimal allocation. Finally, plug the policies proposed for $t \geq 2$ and get

$$\frac{(1-\mu_{t+1}^*)}{(1-\mu_t^*)[\delta(1+\hat{g}_{1,t}^k(1/\delta-1)]} = \frac{u'(c_t)}{\delta\beta u'(c_{t+1})}\frac{1}{[1+\hat{g}_1^k(k_{t+1},K_{t+1},C_{t+1})(\frac{1}{\delta}-1)]} \tag{A.15}$$

which again coincides with (A.7) when evaluated at the optimal allocation. It is easy to see that, at the given policies, the equilibrium condition (3.22) coincides with (A.7) when evaluated at the optimal allocation. Given the sequence of taxes $\{\tau_t^k, \tau_t^z\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government is satisfied. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm which delivers $q_t = 1$ for every period t. This completes the proof that all conditions for a competitive equilibrium with a Climate Bill are satisfied by the optimal allocation. ■

Proof of Proposition 14. The proof consists on showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when taxes and subsidies are set optimally. Plug the optimal tax rate into the intertemporal condition (3.25), evaluated at the star allocation, to get

$$\frac{u'(c_t^*)}{\delta\beta u'(c_{t+1}^*)} = \frac{1-\omega_{t+1}^*}{1-\omega_t^*}\hat{g}_1^k(\frac{1}{\delta}-1) - \frac{[\omega_{t+1}^* - \frac{x'(S_{t+2}^*)}{u'(c_{t+1}^*)}]g_1^s(\frac{1}{\delta}-1)\hat{g}_1^k(\frac{1}{\delta}-1)}{(1-\omega_t^*)[1+g_1^k(\frac{1}{\delta}-1)]}$$

From (3.12), the optimal allocation satisfies

$$\frac{u'(c_t^*)}{\delta\beta u'(c_{t+1}^*)} = \frac{1-\omega_{t+1}^*}{1-\omega_t^*}g_1^k(\frac{1}{\delta}-1) - \frac{[\omega_{t+1}^* - \frac{x'(S_{t+2}^*)}{u'(c_{t+1}^*)}]g_1^s(\frac{1}{\delta}-1)}{(1-\omega_t^*)}$$

At the optimal allocation, $g_1^k(K_{t+1}^*, C_{t+1}^*) = \hat{g}_1^k(K_{t+1}^*, K_{t+1}^*, C_{t+1}^*)$ and both equations coincide. Therefore, the optimal allocation satisfies the equilibrium condition (3.25). Next, we need to show that the intratemporal condition (3.26) is also satisfied by the optimal allocation. To show this, iterate on the optimality condition (3.13) to get

$$\frac{v'(z_t^*)}{u'(c_t^*)} = \omega_t^*$$

Plug the optimal subsidy on sequestration into equation (3.26) and both equations coincide. Given the sequence of taxes $\left\{\tau_t^k, \tau_t^z\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government is satisfied. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm which delivers $q_t = 1$ for every period t. This completes the proof that all conditions for a competitive equilibrium with time consistent climate policies are satisfied by the optimal allocation. ■

Appendix B

Algorithm

The solution method applied to provide the computational examples may prove to be interesting in itself since it is an application of endogenous grid methods to a problem with two state variables, two control variables and hyperbolic discounting.[1] I assume continuity and differentiability of the policy functions. The solution to the planner's problem involves solving equations (3.12) and (3.13), which contain four partial derivatives of the policy functions. This implies that there are two Euler equations but six unknows: the two policy functions $g^k(k,S)$ and $g^s(k,S)$ and the four partial derivatives $g_1^k(k,S), g_2^k(k,S), g_1^S(k,S), g_2^S(k,S)$. Although I have not proved that the equilibrium is unique, the algorithm converges always to the same solution[2] . The steps involved in the algorithm are as follows:

1. Define a grid on current fossil fuell $k \equiv \{k_1, k_2, k_3, ...\}$ and on tomorrow's carbon level $S \equiv \{S'_1, S'_2, S'_3, ...\}$.

2. Use the first order condition with respect to fossil fuel to get the value $k'(k, S')$ that satisfies the optimality condition with respect to k

$$u'(k,k') = \beta[V_1(k',S') - V_2(k',S')] - x'(S') \tag{B.1}$$

   Use the envelope conditions to compute the partial derivatives of the value function.

[1] A similar algorithm can be found in [25].
[2] Convergence depends on the value of δ and on the number of grid points. For some low values of δ and for fine grids, the algorithm fails to converge. This problem is also found in [24]. They do not apply endogenous grid methods but their algorithm also exploits the Euler equation.

3. The optimal values for $k'(k,S')$ are given by

$$\min\{k'(k,S'),k\}$$

   The non negativity constraint is satisfied by assuming Inada conditions.

4. The values for sequestration $z(k,S')$ are recovered from the first order condition with respect to carbon

$$v'(z(k,S')) = x'(S') - \beta V_2(k',S')$$

5. Use the equation for the carbon cycle to recover the current state S

$$S = S' - (k - k'(k,S')) + z(k,S')$$

6. Use interpolation to retrieve the new policy functions $\{k'(k,S),S'(k,S)\}$ on the endogenous grid obtained for S.

7. Iterate until convergence of the policy functions