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Hadrons in the Light-Front Field Theory

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Minnesota, Minneapolis- 2006

Introduction

1. Light-Front Formalism

2. Spin-0 Particles: Píon

3. Pion Pole Approximation

4. Covariance Restoration in the Light-Front

5. Conclusions

Light-Front Formalism

- Light-Front Coordinates

$x^+ = t + z \qquad x^+ = x^0 + x^3 \quad \Longrightarrow$ Time

$x^- = t - z \qquad x^- = x^0 - x^3 \quad \Longrightarrow$ **Position**

- **Four-Vector** $\Longrightarrow \quad x^\mu = (x^0, x^1, x^2, x^3) \; = \; (x^+, x^-, x_\perp)$
- **Metric Tensor**

$$g^{\mu\nu} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ and } g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- **Scalar product**

$$\begin{aligned} x \cdot y = x^\mu y_\mu &= x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 \\ &= \frac{1}{2}(x^+ y^- + x^- y^+) - \vec{x}_\perp \vec{y}_\perp \end{aligned}$$

$$\begin{aligned} p^+ &= p^0 + p^3 \\ p^- &= p^0 - p^3 \\ p^\perp &= (p^1, p^2) \end{aligned}$$

- **Dirac Matrix**

$\gamma^+ = \gamma^0 + \gamma^3 \quad \Longrightarrow$ **Electr. Current** $J^+ = J^0 + J^3$

$\gamma^- = \gamma^0 - \gamma^3 \quad \Longrightarrow$ **Electr. Current** $J^- = J^0 - J^3$

$\gamma^\perp = (\gamma^1, \gamma^2) \quad \Longrightarrow$ **Electr. Current** $J^\perp = (J^1, J^2)$

Fig. 1: Ligh-Front

- $p^\mu x_\mu = \frac{p^+x^- + p^-x^+}{2} - \vec{p_\perp}\vec{x_\perp}$
- $x^+, x^-, \vec{x_\perp} \quad \Longrightarrow p^+, p^-, \vec{p_\perp}$
- $p^- \quad \Longrightarrow$ **Light-Front Energy**
- $p^2 = p^+p^- - p_\perp^2 \Longrightarrow \quad p^- = \frac{p_\perp^2 + m^2}{p^+}$
- Bosons: $\quad \Longrightarrow \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$
- Fermions: $\quad \Longrightarrow \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

- Ref: Phys. Rept. 301, (1998) 299-486
  S. J. Brodsky, H.C. Pauli and S.S. Pinsky

Integration Light-Front

- **Example**

$$I_1^{cov} = \int d^4k \frac{1}{(k^2 - m^2 + \imath\epsilon)^3} = \frac{\pi^3}{2\imath m^2} \neq 0$$

$$I_1^{FL} = \int d^2k_\perp dk^+ dk^- \frac{1}{(k^+k^- - k_\perp^2 - m^2 + \imath\epsilon)^3} = 0 \text{ !!!}$$

- **Double Pole** $\Longrightarrow k^- = \frac{k_\perp^2 + m^2 - \imath\epsilon}{k^+}$

- Pole Dislocation Method $\Longrightarrow$ $p'^+ = p^+ + \delta$
- Boson Eletromagnetic Current

- Breit Frame $\Longrightarrow q^- = 0,\ q^+ \Longrightarrow 0_+,\ \vec{q_\perp} \neq 0$

- $J^+ = J^- + TRC$
- $J_\perp \propto q^+ \Rightarrow 0$

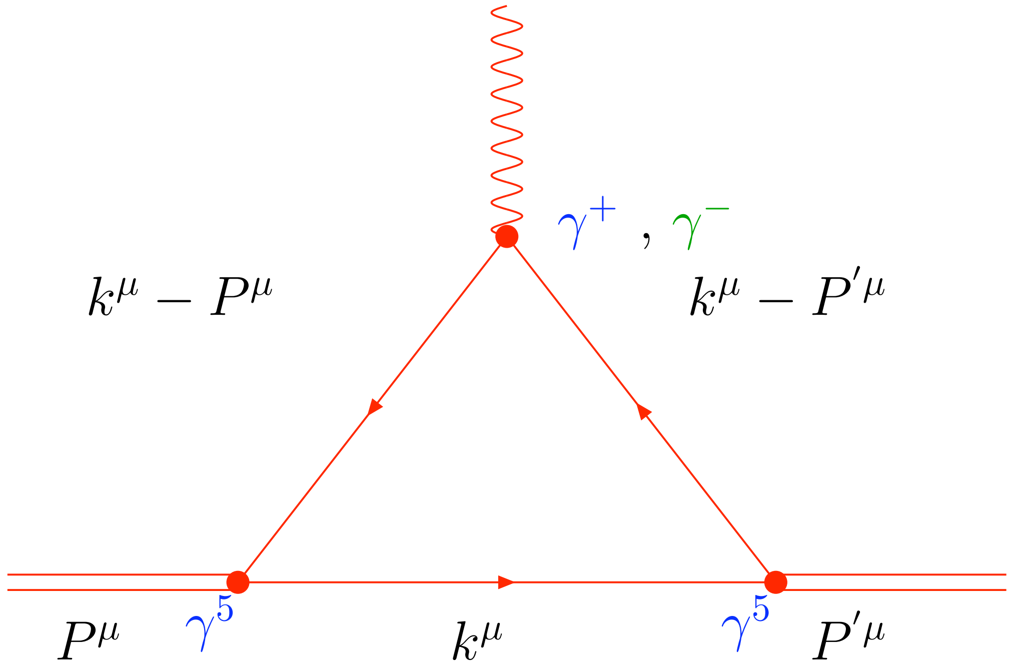
J.P.B.C. de Melo, J.H. Sales et T. Frederico
Nucl. Phys. B631, (1998) 574c-579c.

- Ward-Takahashi Identity $\Longrightarrow$ Pair Contribuition

H.W. Naus, J.P.B.C. de Melo et T. Frederico,

Few-Body Syst. **24**, 1998, 99-107

- Pion



- Frame

$$
\begin{aligned}
q^+ &= -q^- = \sqrt{-q^2}\sin\alpha \\
q_x &= \sqrt{-q^2}\cos\alpha,\ \ q_y = 0 \\
q^2 &= q^+q^- - (q_\perp)^2\ .
\end{aligned}
$$

- Breit Frame $(\alpha = 0)$ $\Longrightarrow q^+ \to 0\ , q^- = 0\ ;\ \vec{q} \neq 0$
- $J^+_\pi = J^0 + J^3 \Longrightarrow$ No Pair Term Contribuition
- $J^-_\pi = J^0 - J^3 \Longrightarrow$ Pair Term Contribuition
- J.P.B.C. de Melo, T. Frederico and H.L. Naus, Phy.Rev. **C59** (1999) 2278

Effective Lagrangian to Vertex $\pi \to q\bar{q}$

$$\mathcal{L}_I = -\imath \frac{m}{f_\pi} \vec{\pi} \cdot \overline{q} \gamma^5 \vec{\tau} q$$

- Electromagnetic Current

$$J^\mu = -\imath 2e \frac{m^2}{f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} Tr[S(k)\gamma^5 \times S(k-P')\gamma^\mu S(k-P)\gamma^5 \Lambda(k,P')\Lambda(k,P)]$$

$$S(p) = \frac{1}{\not p - m + \imath\epsilon}$$

- Symmetric Vertex Function

$$\Lambda(k,P) = \frac{N}{(k^2 - m_R^2 + \imath\epsilon)} + \frac{N}{((P-k)^2 - m_R^2 + \imath\epsilon)}$$

- Ref. Nucl. Phys. **A 707** (2002) 399-424

- Nonsymmetric Vertex Function

$$\Lambda(k,P) = \frac{N}{((P-k)^2 - m_R^2 + \imath\epsilon)}$$

- J.P.B.C. de Melo, T. Frederico and H.L. Naus, Phy.Rev. **C59** (1999) 2278

Electromagnetic Form Factor

$$j^{\mu} = e(P^{\mu} + P^{`\mu})F_{\pi}(q^2)$$

- "J^{+}" Component of the Electromagnetic Current
- Integration Intervals in k^-

I) $0 < k^+ < P^+$

II) $P^+ < k^+ < P'^+$

- Form Factor

$$F_{\pi}(q^2) = F_{\pi}^{(I)}(q^2,\alpha) + F_{\pi}^{(II)}(q^2,\alpha)$$

$$F_{\pi}^{(I)}(q^2,\alpha) = \frac{-\imath m^2 N_c}{(2\pi)^4 (P^+ + P'^+) f_{\pi}^2} \int_0^{P^+} \frac{d^2k_{\perp} dk^+ dk^{-2}\Pi(k,P,P')}{k^+(P^+ - k^+)(P'^+ - k^+)}$$

$$F_{\pi}^{(II)}(q^2,\alpha) = \frac{-\imath m^2 N_c}{(2\pi)^4 (P^+ + P'^+) f_{\pi}^2} \int_{P^+}^{P'^+} \frac{d^2k_{\perp} dk^+ dk^{-2}\Pi(k,P,P')}{k^+(P^+ - k^+)(P'^+ - k^+)}$$

$$\Pi(k,P,P^{'},) = \frac{Tr[\mathcal{O}^+]\Lambda(k,P)\Lambda(k,P^{'})}{(k^- - k^-_{on} + \imath\epsilon)(P^- - k^- - (P-k)^-_{on} + \frac{\imath\epsilon}{P^+ - k^+})} \times \frac{1}{(P'^- - k^- - (P'-k)^-_{on} + \imath\epsilon)}$$

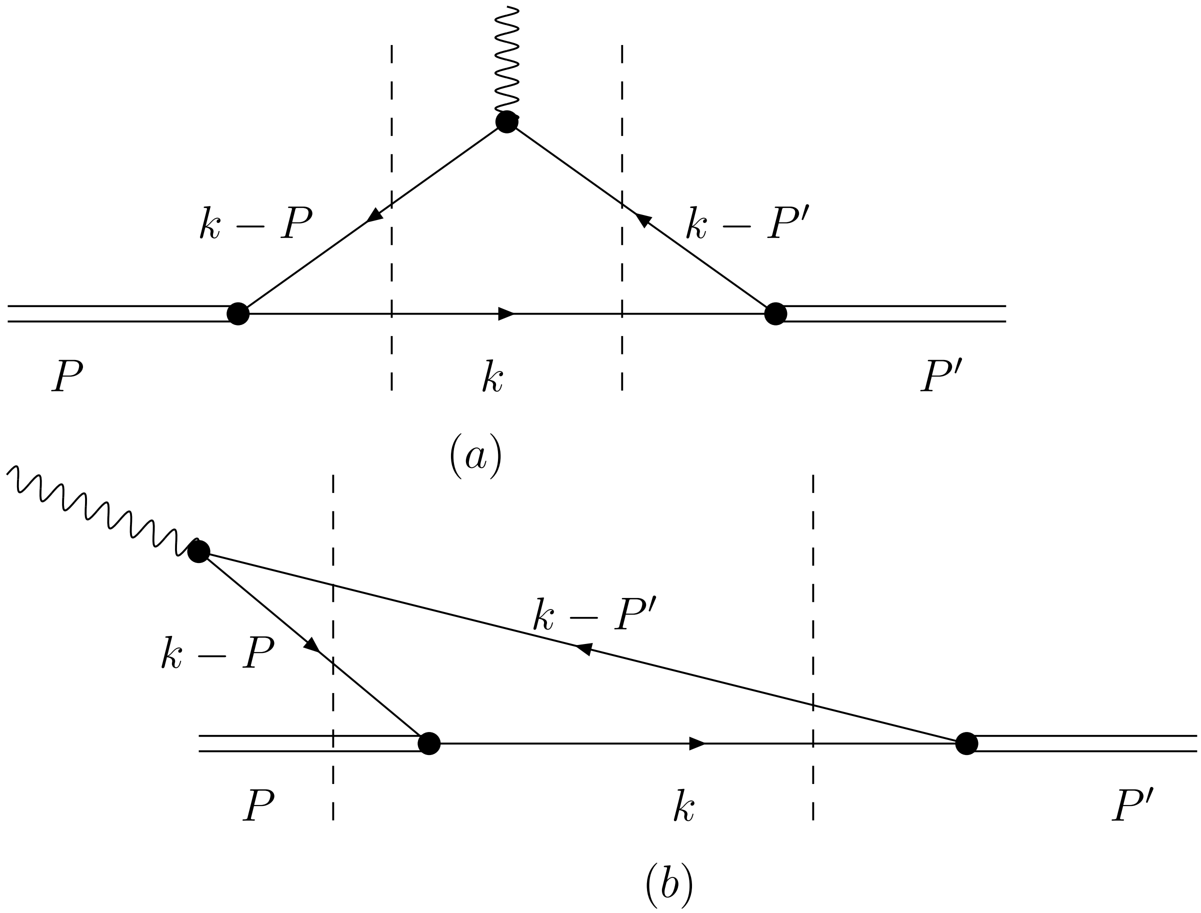


Fig. 2: Light-front time-ordered diagrams for the current: (a) $F_\pi^{(I)}$ and (b) $F_\pi^{(II)}$

- Dirac Propagator in the Light-Front

$$\frac{\not k + m}{k^2 - m^2 + \imath\epsilon} = \frac{\not k_{on} + m}{k^+(k^- - k^-_{on} + \frac{\imath\epsilon}{k^+})} + \frac{\gamma^+}{2k^+}$$

- Instantaneous Term Contribuition
- Wave Function in the Light-Front
- Bethe-Salpeter Amplitude

$$\Psi(k,P) = \frac{m}{f_\pi}\ \frac{\not k + m}{k^2 - m^2 + \imath\epsilon}\gamma^5\Lambda(k,P)\frac{\not k - \not P + m}{(k-P)^2 - m^2 + \imath\epsilon}$$

The Wave Function Appear after Elimination

i) Instantaneous Terms

ii) Factors with Gamma Matrix in the Numerador

iii) Factors k^+ and $(P^+ - k^+)$ in the Denominator

$$\begin{aligned}\Phi(k^+,\vec{k}_\perp;P^+,\vec{P}_\perp) &= \imath\,\mathcal{N}\int\frac{dk^-}{2\pi}\frac{1}{(k^- - k^-_{on} + \frac{\imath\epsilon}{k^+})} \\ &\times \frac{1}{(P^- - k^- - (P-k)^-_{on} + \frac{\imath\epsilon}{P^+ - k^+})} \\ &\times \left(\frac{1}{k^2 - m_R^2 + \imath\epsilon} + \frac{1}{(P-k)^2 - m_R^2 + \imath\epsilon}\right)\end{aligned}$$

$$\mathcal{N} = \sqrt{N_c}\, C \; \frac{m}{f_\pi}$$

- Integration k^-

$$\Phi(k^+,\vec{k}_\perp;P^+,\vec{P}_\perp) = \frac{P^+}{m_\pi^2 - M_0^2}\left[\frac{\mathcal{N}}{(1-x)(m_\pi^2-\mathcal{M}^2(m^2,m_R^2))} + \frac{\mathcal{N}}{x(m_\pi^2-\mathcal{M}^2(m_R^2,m^2))}\right]$$

- Where $x = k^+/P^+$, with $0 \leq x \leq 1$

$$\mathcal{M}^2(m_a^2,m_b^2) = \frac{k_\perp^2+m_a^2}{x} + \frac{(P-k)_\perp^2+m_b^2}{1-x} - P_\perp^2$$

- Free Mass $M_0^2 = \mathcal{M}^2(m^2,m^2)$
- Valence Electromagnetic Form Factor

$$F_\pi^{(WF)}(q^2,\alpha) = \frac{1}{2\pi^3(P'^+ + P^+)}\int_0^{P^+}\frac{d^2k_\perp dk^+\Phi(k^+,\vec{k}_\perp;P'^+,\frac{\vec{q}_\perp}{2})}{k^+(P^+-k^+)(P'^+-k^+)} \times \left[k_{on}^- P^+P'^+ + \frac{1}{2}\vec{k}_\perp\cdot\vec{q}_\perp(P'^+-P^+) - \frac{1}{4}k^+q_\perp^2\right] \times \Phi(k^+,\vec{k}_\perp;P^+,-\frac{\vec{q}_\perp}{2})$$

- $k_{on}^- = (k_\perp^2+m^2)/k^+$
- Normalization Constant $(C) \Longrightarrow F_\pi(0)=1$

- $F_\pi^{(WF)}$ for $q^2 = 0 \Longrightarrow$ Probability Density η.

No Dependence in α for the Components of the Valence $q\bar{q}$

- If $\eta < 1$, in this case the Pair Terms is VIP

- Transverse Probability

$$f(k_\perp) = \frac{1}{4\pi^3 m_\pi} \int_0^{2\pi} d\phi \int_0^{m_\pi} \frac{dk^+ M_0^2}{k^+(m_\pi - k^+)} \Phi^2(k^+, \vec{k}_\perp; m_\pi, \vec{0})$$

- **Integration of $f(k_\perp)$ under $\vec{k}_\perp$**

$$\eta = \int_0^\infty dk_\perp k_\perp f(k_\perp)$$

- $\eta = 0.77$ **Symmetric Vertex**

- $\eta = 1.0$ **NonSymmetric Vertex**

Pion Decay Constant

$$P_\mu < 0|A_i^\mu|\pi_j >= \imath\, m_\pi^2 f_\pi \delta_{ij}$$

$$A_i^\mu = \bar{q}\gamma^\mu\gamma^5\frac{\tau_i}{2}q$$

- We Have

$$\imath P^2 f_\pi \;\; = \;\; \frac{m}{f_\pi}N_c\int\frac{d^4k}{(2\pi)^4}Tr[\not{P}\gamma^5 S(k)\gamma^5 S(k-P)]\Lambda(k,P)$$

- k^- Integration $\Longrightarrow$ f_π by the use of the Valence Component

$$f_\pi = \frac{m\,\sqrt{N_c}}{4\pi^3}\int\frac{d^2k_\perp dk^+}{k^+(m_\pi - k^+)}\Phi(k^+,\vec{k}_\perp;m_\pi,\vec{0})$$

Model Parameters and Numerical Results

NonSymmetric Vertex

- Quark Mass m_q: $m = 0.220$ GeV
- Regulator Mass m_R : $m = 0.946$ GeV
- Fit for $f_\pi = 101.0$ MeV
- Pion Mass: 0.140 GeV
- Charge Radius

$\langle r^2 \rangle = 6\frac{\partial}{\partial q^2} F_\pi = 0.67 fm$

Symmetric Vertex

- Quark Mass m_q: $m = 0.220$ GeV
- Regulator Mass m_R : $m = 0.6$ GeV
- Fit for $f_\pi^{exp} = 92.4$ MeV
- Pion Mass: 0.140 GeV
- Charge Radius

$\langle r^2 \rangle = 6\frac{\partial}{\partial q^2} F_\pi = 0.74 fm$

10% Bigger than Experimental Value

($r_{exp} = 0.67 \pm 0.02$ fm)

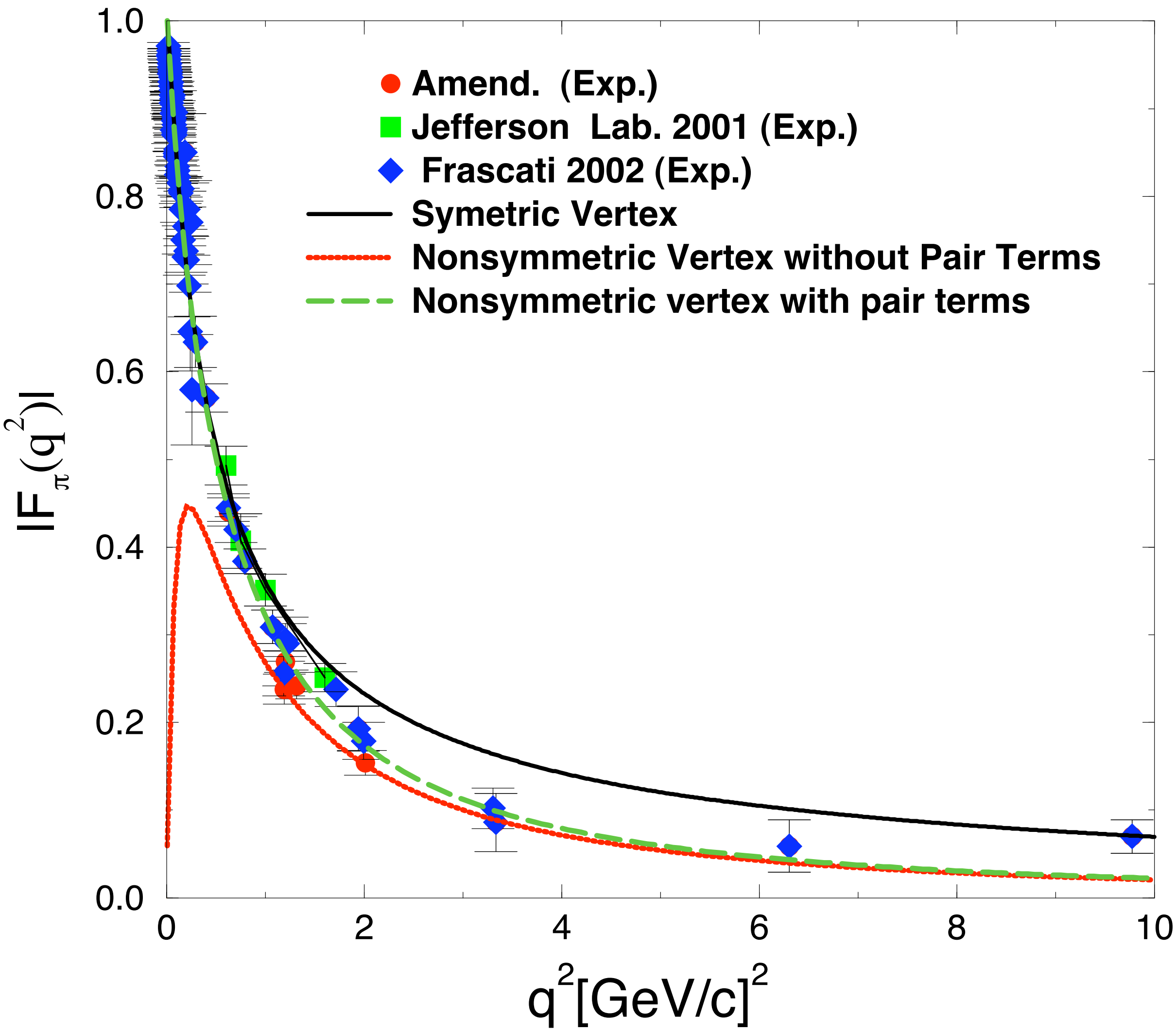


Fig. 3: Pion Form Factor

- J_π^- Without Pair Terms
- J_π^- With Pair Terms

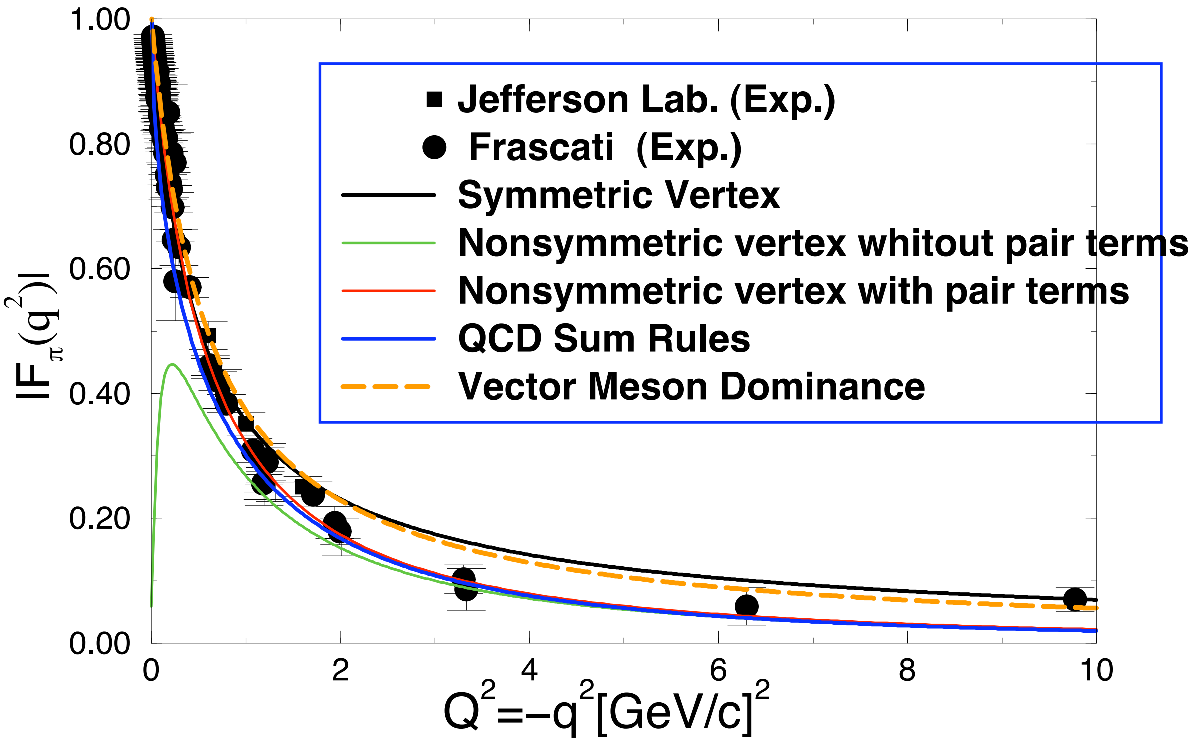


Fig. 4: Pion Form Factor

Ref. J. P. B. C. de Melo , AIP Conf.Proc.739:553-556,2005 and hep-ph/0507265

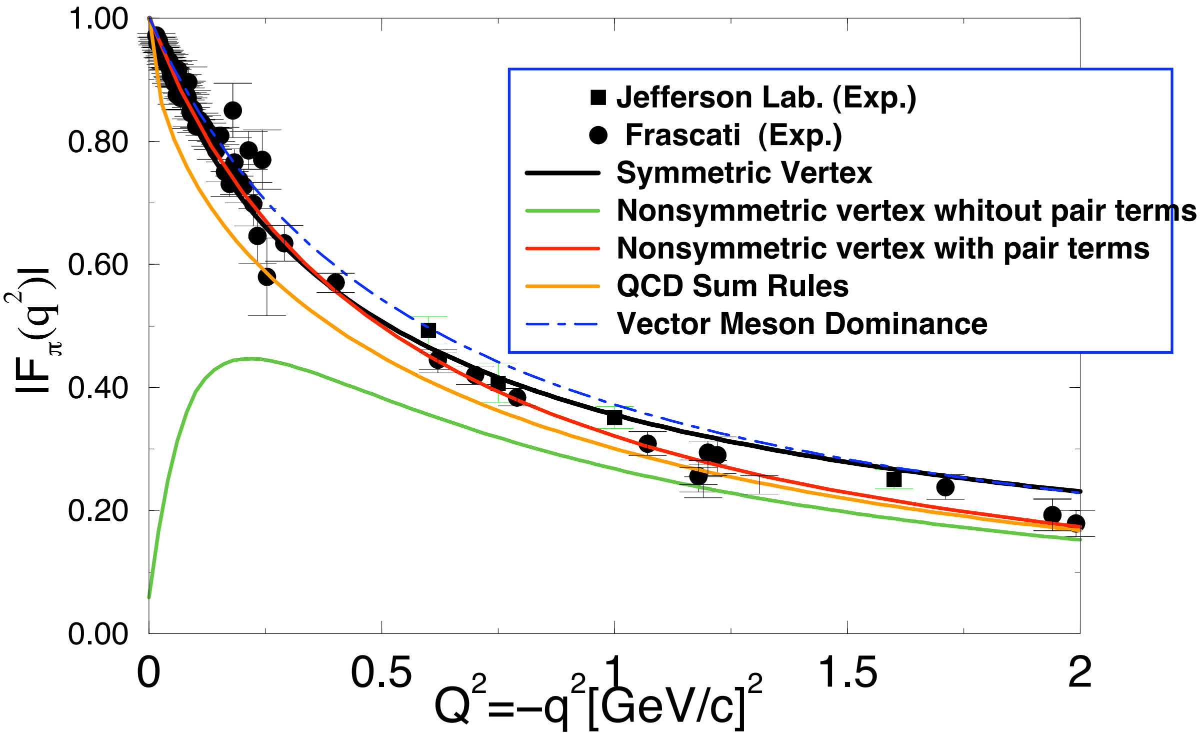


Fig. 5: Pion Form Factor at low energy

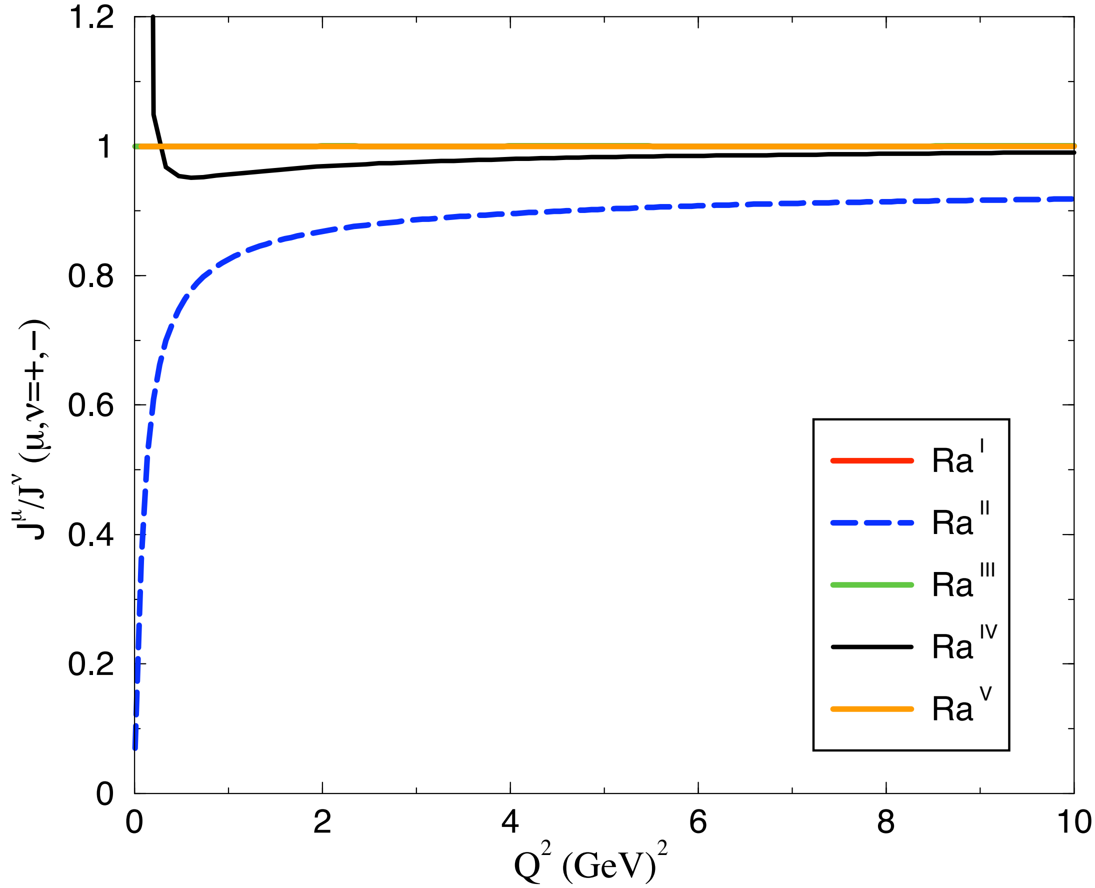


Fig. 6: Nonsymmetric Vertex.

$$Ra^I = \frac{J^+_{LF}}{J^+_{Cov}} \qquad Ra^{II} = \frac{J^-_{LF}}{J^-_{Cov}}$$

$$Ra^{III} = \frac{J^-_{LF}+J^{-(Pair)}_{LF}}{J^-_{Cov}} \qquad Ra^{IV} = \frac{J^-_{LF}}{J^+_{Cov}}$$

$$Ra^V = \frac{J^-_{LF}+J^{-(Pair)}_{LF}}{J^+_{Cov}}$$

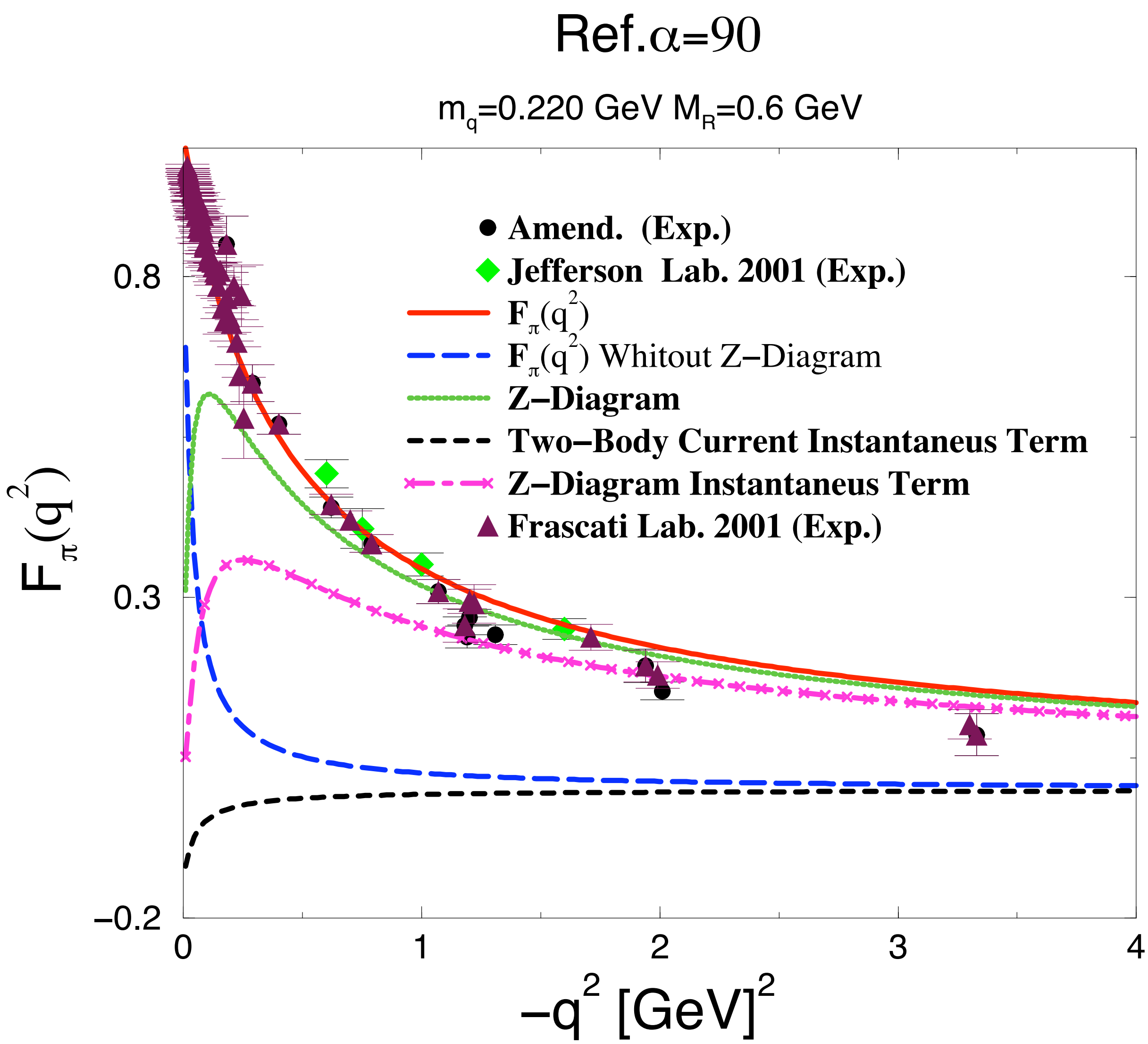


Fig. 7: Pion Form Factor

• Ref. de Melo, Frederico, Salmé and Pace
Nucl. Phys. A707 (2002) 399-424

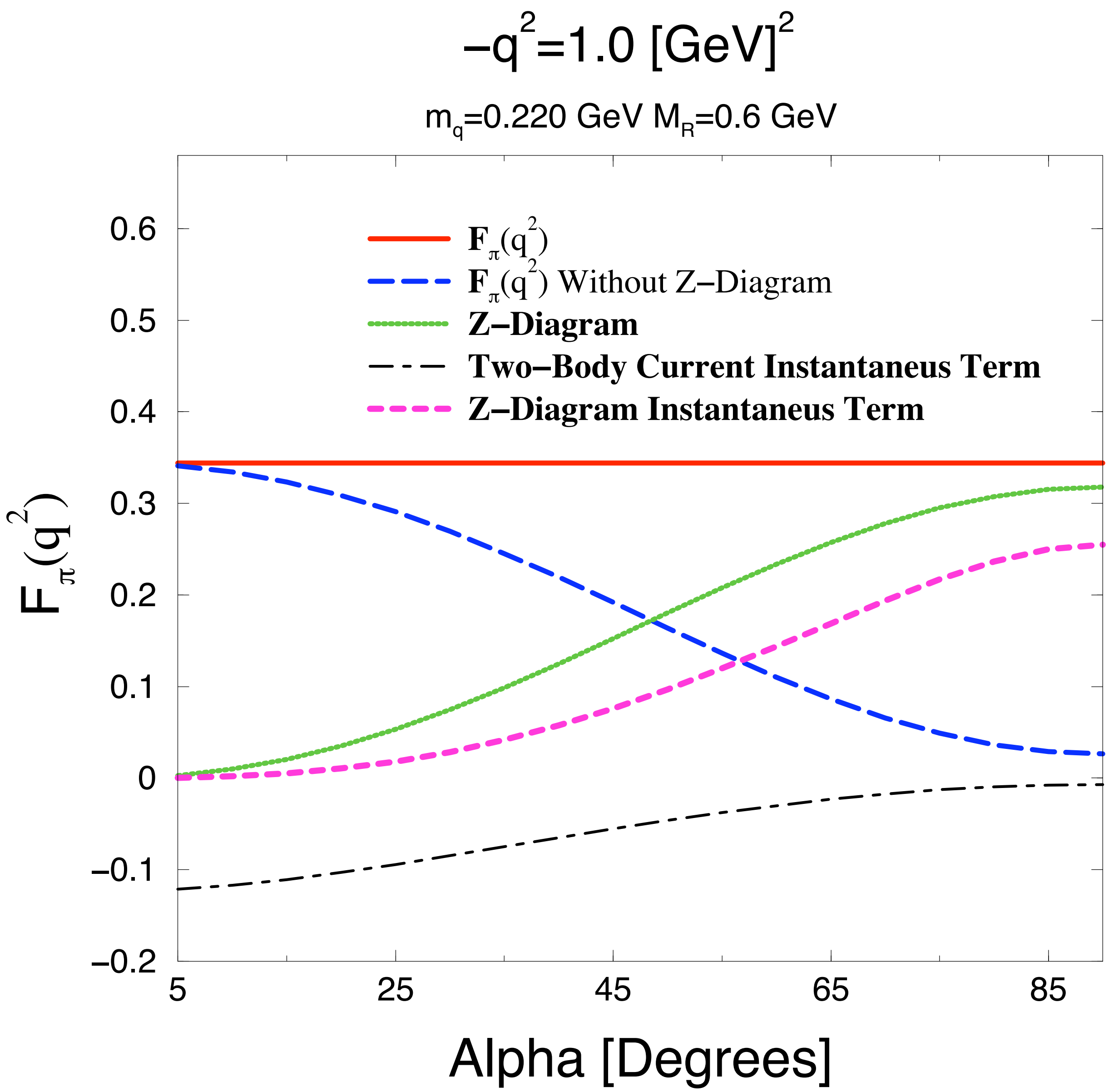


Fig. 8: Pion Form Factor vs *alpha*

• Ref. de Melo, Frederico, Salmé and Pace
Nucl. Phys. A707 (2002) 399-424

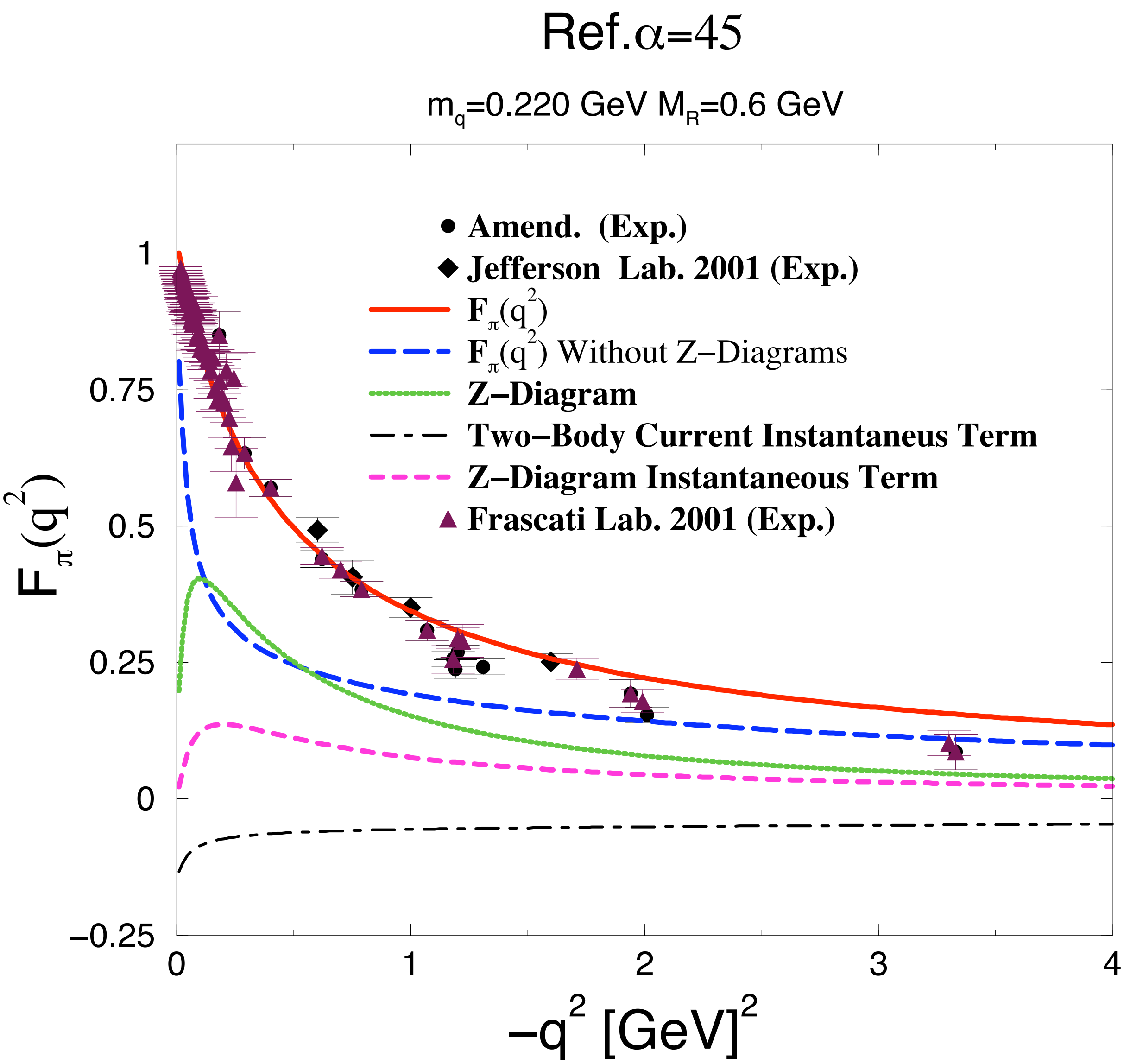


Fig. 9: Pion Form Factor with $alpha = 45$

• Ref. de Melo, Frederico, Salmé and Pace
Nucl. Phys. A707 (2002) 399-424

Pole Approximation

- Pole On-Shell Contribuition
- $k_1^- = \frac{k_\perp^2 + m^2}{k^+}$
- Pole Z-Diagramm Contribuition
- $k_3^- = (p'^- - \frac{(p-k)-\perp + m^2}{(p'^+ - k^+})$

I) $0 < k^+ < P^+$

II) $P^+ < k^+ < P'^+$

- Form Factor

$$F_\pi(q^2) = F_\pi^{(1)}(q^2, \alpha) \; + \; F_\pi^{(2)}(q^2, \alpha)$$

$$F_\pi^{(1)}(q^2, \alpha) = \frac{-\imath m^2 N_c}{(2\pi)^4 (P^+ + P'^+) f_\pi^2} \int_0^{P^+} \frac{d^2 k_\perp dk^+ dk^{-2} \Pi_1(k, P, P')}{k^+ (P^+ - k^+)(P'^+ - k^+)}$$

$$F_\pi^{(2)}(q^2, \alpha) = \frac{-\imath m^2 N_c}{(2\pi)^4 (P^+ + P'^+) f_\pi^2} \int_{P^+}^{P'^+} \frac{d^2 k_\perp dk^+ dk^{-2} \Pi_2(k, P, P')}{k^+ (P^+ - k^+)(P'^+ - k^+)}$$

$$\Pi_1(k, P, P^{'},) \quad = \frac{Tr[\mathcal{O}^+] \Lambda(k, P) \Lambda(k, P^{'})}{(P^- - k^- - (P-k)^-_{on} + \frac{\imath\epsilon}{P^+ - k^+})} \times \frac{1}{(P'^- - k^- - (P'-k)^-_{on} + \imath\epsilon)}$$

$$\Pi_2(k, P, P^{'},) \quad = \frac{Tr[\mathcal{O}^+] \Lambda(k, P) \Lambda(k, P^{'})}{(k^- - k^-_{on} + \imath\epsilon)(P^- - k^- - (P-k)^-_{on} + \frac{\imath\epsilon}{P^+ - k^+})}$$

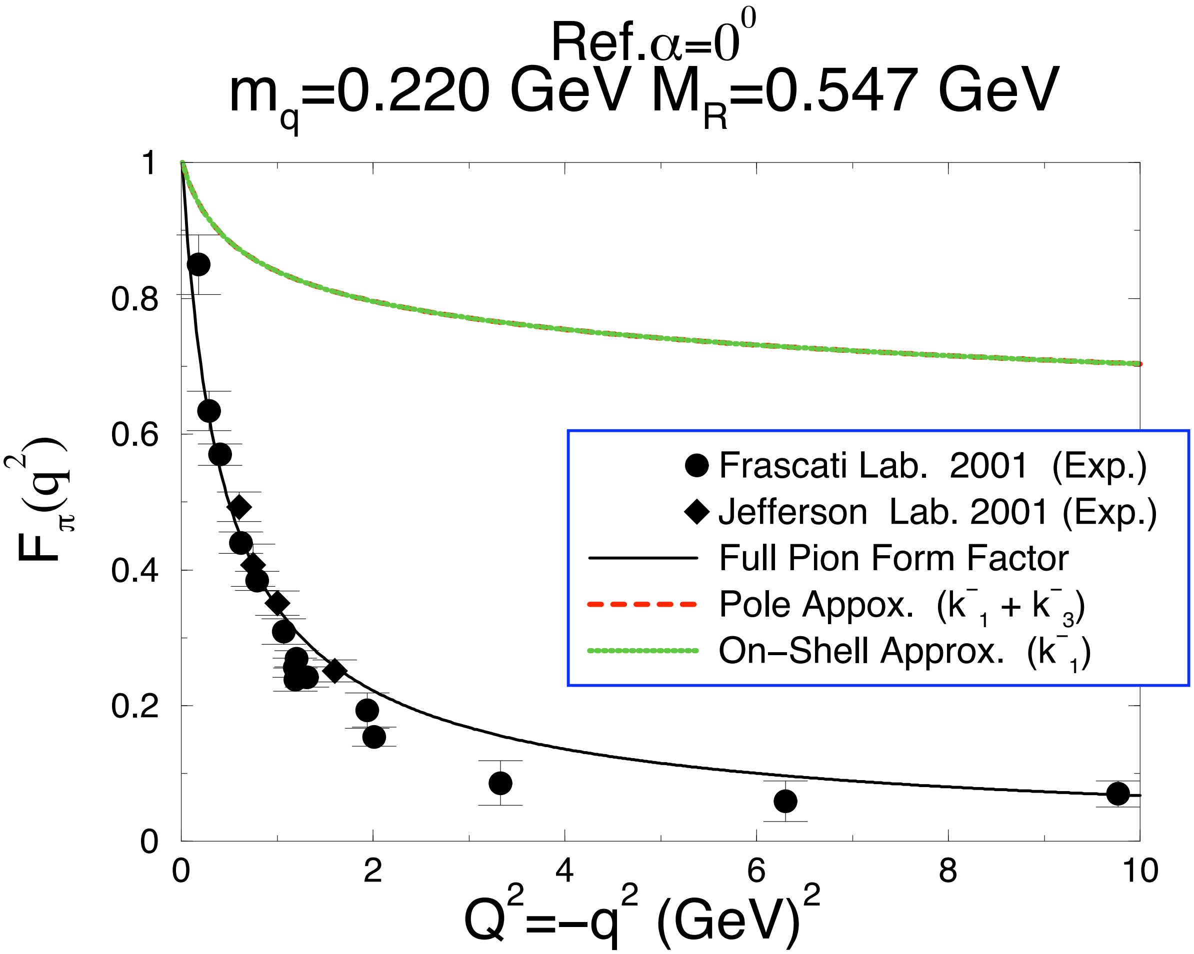


Fig. 10: Pion Pole Approximation

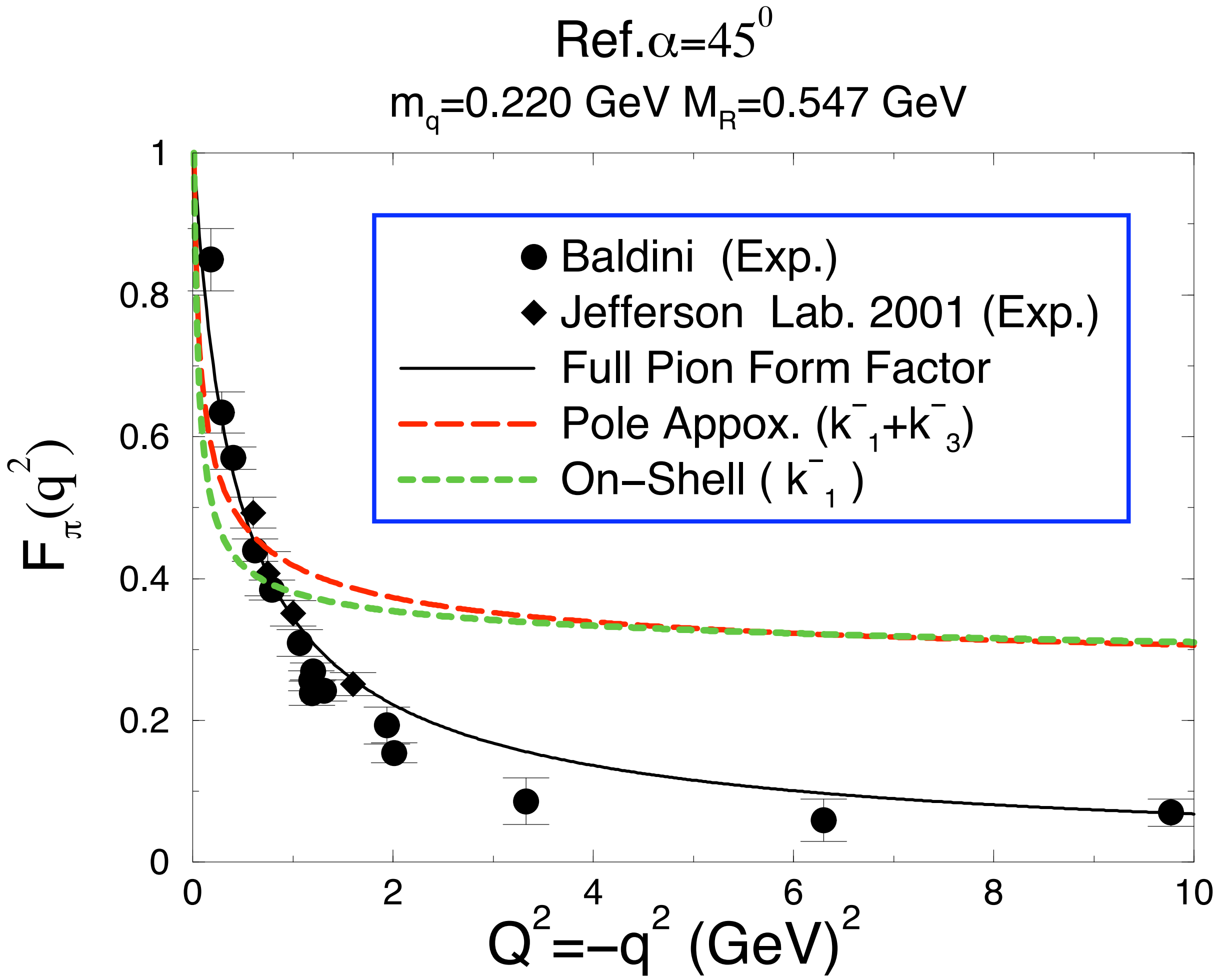


Fig. 11: Pion Pole Approximation

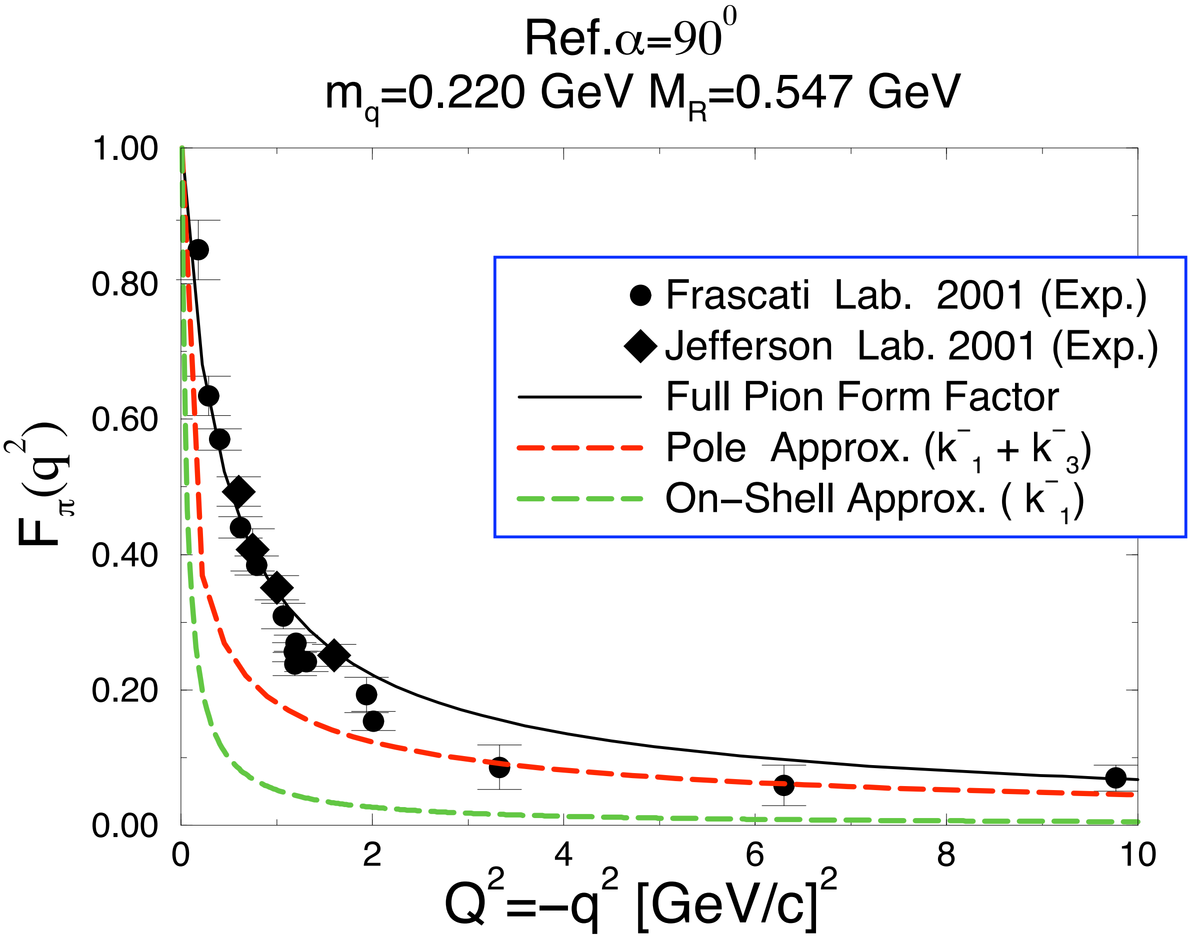


Fig. 12: Pion Pole Approximation

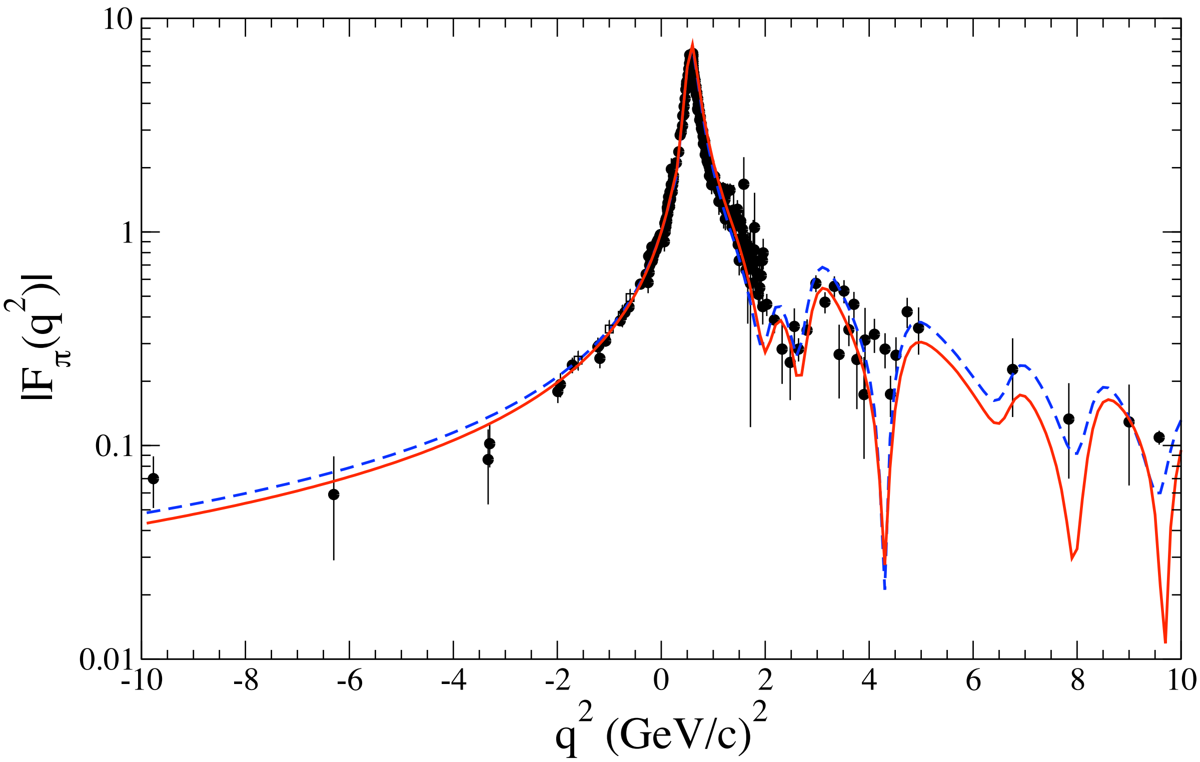


Fig. 13: Pion: Space and Time Like

• Ref. de Melo, Frederico, Salmé and Pace
Phy. Rev. D707 (2006) 074013

Conclusions

- Light-Front $\Longrightarrow \begin{cases} \textit{Bound States} \\ \textit{Covariance} \end{cases}$
- Rotational Invariance Broken $\Longrightarrow k^-$ Problematic
- Terms $\begin{cases} -\ \textit{Good} \\ -\ \textit{Bad} \end{cases}$
- Electromagnetic Current: " $+$ " , "- " and " $\perp$ "
- Particles $\begin{cases} -\ \textit{Bosons} \\ -\ \textit{Pseudoscalar} \\ -\ \textit{Vector} \end{cases}$
- Pairs Terms Contribuition $\Longrightarrow$ Full Covariance Restorate
- New Informations about Bound States $q\bar{q}$