Search, Occupational Choice and Learning

A THESIS

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL

OF THE UNIVERSITY OF MINNESOTA

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

Doctor of Philosophy

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August, 2015

Acknowledgements

I would like to express my foremost gratitude to my advisor V. V. Chari and to my co-advisor Larry Jones. Without their help, contribution and guidance my research would not be fruitful. I am forever grateful to them for taking my infant ideas seriously and guiding me on my way to mature those ideas. I would also like to express my deepest appreciation Christopher Phelan for his patience and invaluable in Growth & Development Workshops and during my oral exams. I owe special thanks to Aaron J. Sojourner, who served in my committee and provided the most valuable feedback and perspective. I would like to acknowledge the support I have received from the staff at Economics department. Caty Bach, Kara Kersteter and Wendy Williams made the wearisome bureaucracy one has to face in a big organization such as University of Minnesota disappear through their magic. I am also forever thankful to Simran Sahi for her support.

I am deeply indebted to my wife Zeycan Esra Büyükbaşaran, my daughter Duru Büyükbaşaran and my son Can Büyükbaşaran for their inspiration and support not only to my work but to all my life. Their love was the only thing that give me the strength and endurance to finish my Phd in spite of all health problems during my education.

I had the great luck to be house-mate with Murat Ali Çengelci and Mehmet Yiğit Gürdal during my first year of Phd, which would be the hardest year since my family was not with me. I am forever indebted to them and can not thank them enough for their support. The Turkish crowd of Minnesota harbored some of the most wonderful people I was fortunate enough to meet: Murat Ali Çengelci, Mehmet Yiğit Gürdal, Cüneyt Orman, Alperen Evrin, Tolga Umut Kuzubaş, Deniz Çiçek, Ayça Özdoğan, Ceyhun Elgin and Fatih Fazilet.

Finally, I would like to thank my father Musa Büyükbaşaran, my mother Nermin Büyükbaşaran and my sister Gül Büyükbaşaran. I would not be the person I am now without my parents' and sister's constant encouragement and support.

Dedication

To my everythings: my son and my daughter...

Abstract

This thesis examines the labor market effects of incomplete information about workers' own job-finding process and best occupations fitting to them. Search outcomes convey information about workers' job finding abilities and appropriate occupations suited to them, and workers use this information to infer their types. This learning process generates endogenous heterogeneity in occupational choices and workers beliefs. Our theory explains how unemployment can affect labor market decisions including the occupational choices. Characterization results in a simple value function with *reservation level of prior belief property* that is similar to reservation wage property. Some interesting facts about both micro and macro data are identified and our model's explanation of these facts is discussed. In particular, our characterization gives rational for why workers with less experience in searching have (1) longer unemployment duration and (2) higher probability of changing occupation by reemployment, and (3) why shifts in Beveridge curve may be observed. Theory can also be used to (4) explain the discouraged worker phenomenon.

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Chapter 1

Introduction

IF WORKERS have incomplete information about their job search and matching process such as their ability to find a job and the best occupation suitable for them, search outcomes brings important information. Even the same type of workers may have differences in search outcomes initially caused by luck, which bring differences in their beliefs about their abilities and their best-suited occupation. Differences in beliefs further affect their future search decisions and occupational choices, which eventually may have a huge effect on wage rate, employment rates and unemployment duration.

Consider two workers, who have the same skill set, same education etc. One worker is working at the same firm for ten years and never has been separated, the other worker has separated and had to search for a job every year for ten years trying different occupations. The second worker obviously is more experienced on searching for a job, and therefore has a better understanding about his/her job finding ability and about which occupation he can find a job more easily. He may be called as a 'bird in the air'. First worker on the other hand is less experienced on searching for a job. He may not have a good understanding in which occupation he can find a job more easily. He may

be called as a 'fish out of water'. Unfortunately, to the best of our knowledge there is not a data source which provides either a full search history or a belief history of workers for US economy. Nevertheless, the CPS 'Displaced Worker, Employee Tenure, and Occupational Mobility Supplement' (to be called as Displaced worker supplement from now on) includes data on displaced worker such as their previous and current occupations, job tenures and wage rates. In this paper, previous job tenure of displaced workers will be used as a proxy in order to identify these two different group of workers. Displaced workers with longer previous job tenures are assumed to be more likely in the first group, displaced workers with short previous job tenure are assumed to be more likely in the second group. Validity of this as a good proxy will be discussed further in the data motivation section. Our model in this paper will be a base to state actually that displaced workers with longer previous job tenure are more likely to be inexperienced workers in searching compared to same age displaced workers with shorter previous job tenure.

Data on Displaced worker supplement have the following stark features: First, compared to displaced workers with shorter previous job tenures, the displaced workers with longer tenures in previous job are more likely to have (1) longer unemployment durations, and (2) different reemployment occupations. Second, during the current recession, (3) the ratio of displaced workers with longer past tenure to displaced worker with shorter past tenure has been increased, which results in longer average unemployment duration.

Current aggregate data on the labor market suggests that the current recession is different from the previous periods. Firstly, there is a (4)higher number of discouraged workers during the current recession. The discrepancy between the U4 unemployment

rate and the U3 unemployment rate[1] has been increased from 0.2 (historical average) to 0.6 percent (average during current recession), which amounts to around 1.2 million discouraged worker throughout US. Secondly, during the current recession (5) average unemployment duration increases significantly. Finally, the (6) Beveridge curve, an empirical relation between vacancy and unemployment rates, does not seem to hold during the current recession whereas it was more pronounced in previous periods. After 2008, unemployment rate have been higher than the Beveridge curve suggests, which may be an indication of a shift.

This paper will attempt to give a possible explanation for these facts. In our model, workers are heterogeneous in their job finding ability and their suitability for certain occupations. Because workers have incomplete information about their types, they do not precisely know their job finding abilities and their best-suited occupations. They learn about their types from observing search outcomes. Firms, which are subject to free entry condition, will post vacancies at a cost and commit to pay a wage rate if a successful match occurs. Characterization of an equilibrium with these features gives some qualitative explanation in the line with the facts presented beforehand. Firstly, people with worse priors about their types (in the sense that belief is far away from the actual type) direct their search on a job in a less suitable occupation. Since job finding probabilities depend on actual type of agents and on suitability of occupation for the agent, search outcomes will give valuable information to the workers about directing themselves to more suitable occupations. Because this learning process takes time with possible occupation change, the people with less experience in searching stay

[1] U3 is the conventional unemployment rate whereas the U4 includes the discouraged workers to calculation. Beaure of Labor Statistics give definition of them as:

$$U3 = \frac{\#\text{ of unemployed}}{\#\text{ of Labor force}}, \quad U4 = \frac{\#\text{ of unemployed} + \#\text{ of Discouraged}}{\#\text{ of Labor force} + \#\text{ of Discouraged}}$$

unemployed longer and are more likely to change occupation. This learning process through search gives a rationale for the facts from the Displaced worker supplement. Secondly, if the amount of workers with worse priors in the unemployment pool increases, the average unemployment duration also increases and the observed unemployment-vacancy relation shifts as seen in the data. This gives a rationale for some aggregate data facts. Finally, with costly search extension to our main model, workers, whose prior get worse and worse because of the search outcomes, might decide not to search any longer but to stay out of the labor force becoming discouraged workers.

Our model is a directed search model in the spirit of Acemoglu and Shimer (1999). In Burdett and Vishwanath (1988) and Bikhchandani and Sharma (1996), workers also learn during the search process, but workers learn about unknown common distribution of matching process whereas in our paper workers learn about their own types. In Jovanovic (1979), a matched worker-firm pair draws an unobservable match quality from a known distribution. Through noisy signals which is correlated by match quality, both workers and firms learn about the actual match quality and decide whether to continue match. In all these papers, workers are homogeneous and learning is about an unobservable draw from a known distribution. In our paper, workers are heterogeneous in terms of their ability to find a job and the their best suited occupation. Moreover, unlike those papers workers learn about their own type by observing search outcomes.

Model in this paper is closely related to Gonzalez and Shi (2010) . Our model also is a competitive search model with different types of workers with different job finding probabilities. The productive technology of search effort is also very similar. However, in Gonzalez and Shi (2010) the workers do not have any occupational choice; all the jobs are homogeneous in terms of productivity and matching probability of same type of workers. Our model will assume that the jobs are differentiated, therefore they

can be interpreted as different occupations. Hence, different than Gonzalez and Shi (2010) our value function will have an endogenous *reservation prior belief property.* Our model is also related with Falk *et al.* (2006), which also uses different types of workers with different job finding probabilities and giving a motivation for discouraged worker phenomenon. However, they have homogeneous jobs also; therefore, they cannot address the occupational choices addressed in our paper. Moreover, they used a continuous time approach and random search with Nash bargaining whereas our model is a discrete time competitive search model.

In Papageorgiou (2010), both workers and jobs are heterogeneous. Jobs are divided among different occupations and workers divided into different unobservable types as in our paper. Workers also learn about their types through observing outcomes as in our paper. However, in Papageorgiou (2010) workers infer about their types observing noisy signal of their productivity during employment and decide on attempting to change occupation. Our model focus on informativeness of search outcomes rather than informativeness of productivity of a worker in certain occupation. Therefore the learning process is very different.

The paper is organized as follows. After this introduction, Chapter 2 discusses data motivation. Chapter 3 presents the model. Chapter 4 defines the equilibrium, characterizes it and gives an extension to model in order to explain discouraged worker notion. Chapter 5 concludes.

Chapter 2

Data Motivation

We have used two different data sources. First is CPS's Displaced worker supplement. Second is aggregate data obtained from the BLS.

Displaced worker supplement is conducted every two years in Januaries, which asks additional questions to the displaced workers . Displaced workers in this supplement are those who involuntarily separated from their jobs during the past three years before survey date by (i) mass layoff, (ii) plant closure or (iii) abolishment of their position[1] rather than because of individual job performance. Therefore supplement data has its own limitations: First, workers are surveyed just once, providing information on one post-displacement data, rather than about their full history of experiences over time. So it is not possible to obtain panel data from this survey. Second, it does not include all unemployment pool but only displaced workers; hence, voluntary quits and fires by case are not sampled. However, it also has one main advantage; it is a huge survey of around 150,000 individuals who are weighted to represent US workforce. We

[1] It means position is abolished and no new employee will take place of him after his/her separation.

have taken six supplements[2] to obtain data on displaced workers from 1999 to 2012, including their demographic information, total unemployment duration, previous and current occupations, previous and current job tenures,and previous and current wage rates etc.

As stated beforehand, we are trying to identify the workers who are more experienced in search. If search provides some valuable information about job finding ability and suitability of their skills to find a job in certain occupations, then people with more experience are more likely to realize their abilities and more likely to pick suitable occupation to search for. Because of that, experienced workers in searching would find a job more easily and their total unemployment duration would be lower compared to inexperienced one. Unfortunately, Displaced worker supplement does not include full search experience of the workers. Nonetheless, it includes previous job tenure of displaced workers, and this information will be used as a proxy for total search experience. It will be assumed that people who had to search for a job e.g. one years ago and now have to search again will be regarded as more likely to be more experienced in searching than people who had to search for a job ten years ago and now have to search again. Validity of the data motivation depends on whether this is a good proxy for overall search experience. Our model will not include imperfectness in recalling learning experience or unobservable aggregate/idiosyncratic fluctuations or trends in labor market. However, if people cannot recall their experience from far past perfectly or if the conditions of the current labor market has changed, farther the previous job search experience lesser the information content of it. Hence a worker who had search experience one year ago will remember or will be able to 'use' his/her experience from that search much better than a worker who had the same search experience ten years ago and

[2] Displaced worker supplements 2002, 2004,2006,2008, 2010 and 2012

Table 2.1: Basic Facts from micro data: the displaced workers with longer tenures in previous job are more likely to have (1) longer unemployment durations, and (2) different reemployment occupations

1	**Change of Occupation**		**2**	**Unemployment Duration**	
Previous tenure	*Change*	*No Change*	**Previous tenure**	*Short (<18 w)*	*Long (> 8 y)*
Short (≤8 y)	43%	57%	*Short (≤8 y)*	63%	37%
Long (> 8 y)	48%	52%	*Long(> 18 w)*	52%	48%

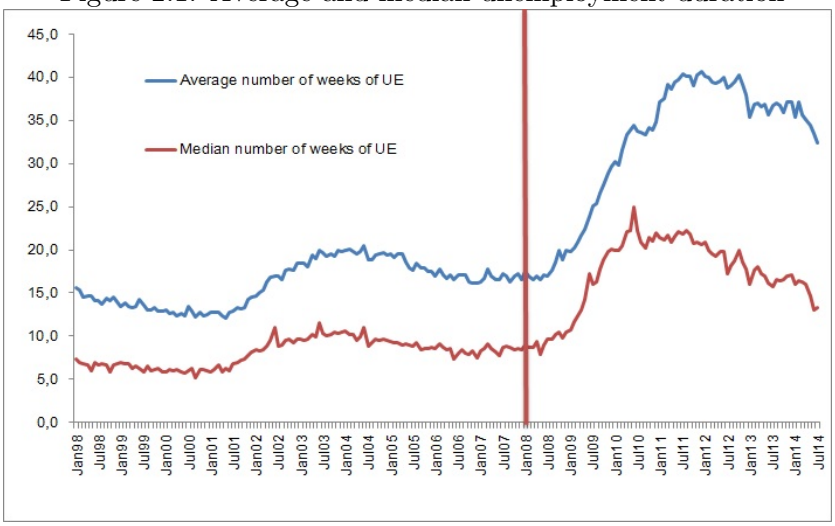
never searched again. If this is actually the case, the most current previous job search process will be more informative than the other previous experience and therefore the proxy mentioned here is actually a good one. Moreover, if anything else is constant, among the same age displaced workers, workers who have longer tenures on his previous job have a shorter time span for experimenting (i.e. off-the-job-searching) compared to displaced workers who have shorter previous job tenure on average. Of course, we do not have on-the-job search activities, intensity of the searches or other past search efforts, therefore our proxy is not the best one but currently the only available one to us for now.

Table 2.1[3] summarizes the main findings in this analysis. We will present only relevant statistics in this section; detailed data work can be seen in Appendix-Data section.

[3] For occupations two digit occupation codes of CPS classification has been chosen. Change in occupation means that if displaced worker is reemployed, his/her new occupation is different than his previous occupation in two digit codes. Long (short) tenure means that the displaced worker has worked for 9 years or more (8 years or less) in his/her previous job before displacement. Long (short) unemployment duration means that it takes more (strictly less) than 18 weeks to find a new job. Arranged displacements are disregarded, i.e. workers who did not stayed unemployed after displacement but immediately found job are taken as arranged displacement and were not included in the dataset.These summary statistics are for displaced workers who are male, white, aged between 35 and 45, having educational attainment of at least high school, and having same eligibility for unemployment insurance. There are around 3,100 observations in this group. Not all the workers have all relevant data like previous and/or current tenure duration. The table includes observations whenever relevant data exists. We have also looked at the other group of employees with respect to other demographic and educational characteristics. The statistics are not changing qualitatively. We have also done robustness check on the specification of long and short durations and its cut-off points. Again, the results does not change qualitatively. We tried to control for industries and occupations but small sample sizes directs us away from that approach

All figures in Table 2.1 are in terms of row percentage, e.g. in first mini-table, 43% implies that forty three percent of all workers with short previous tenure duration have changed their occupation upon reemployment and 57% of workers with short previous tenure duration have not changed occupation upon reemployment. Following observations from the Table 2.1 worth noting. First, from first mini-table of Table 2.1, upon re-employment new occupation of displaced workers with longer previous job tenure is more likely to be different than their previous occupation compared to displaced worker with shorter previous job tenure. 48% of displaced workers with long previous job tenure change occupation after re-employment whereas only 43% displaced workers with short previous job tenure change occupation after re-employment. Second, from second mini-table of Table 2.1, displaced workers with longer tenure in his previous job before displacement have longer unemployment durations. 48% of displaced workers with long previous job tenure have long unemployment durations whereas only 37% displaced workers with short previous job tenure have long unemployment durations. These two findings may appear to be a puzzle if one considers standard occupation specific human capital approach. Under standard human capital approach, it is assumed that if a person stayed longer in his previous occupation, he would obtain more occupation specific human capital. Therefore, one would expect a displaced worker with longer previous job tenure to find a job in his previous occupation once re-employed. Moreover, if everything else is same (including reservation wage, wealth and other types of human capital), the person with higher occupation specific human capital is expected to find a job easier and in shorter duration. Our hypothesis, namely search outcomes provide information about job finding ability and about suitability of a particular occupation to a particular worker, give a rationale to solve this puzzle between data facts and standard human capital approach.

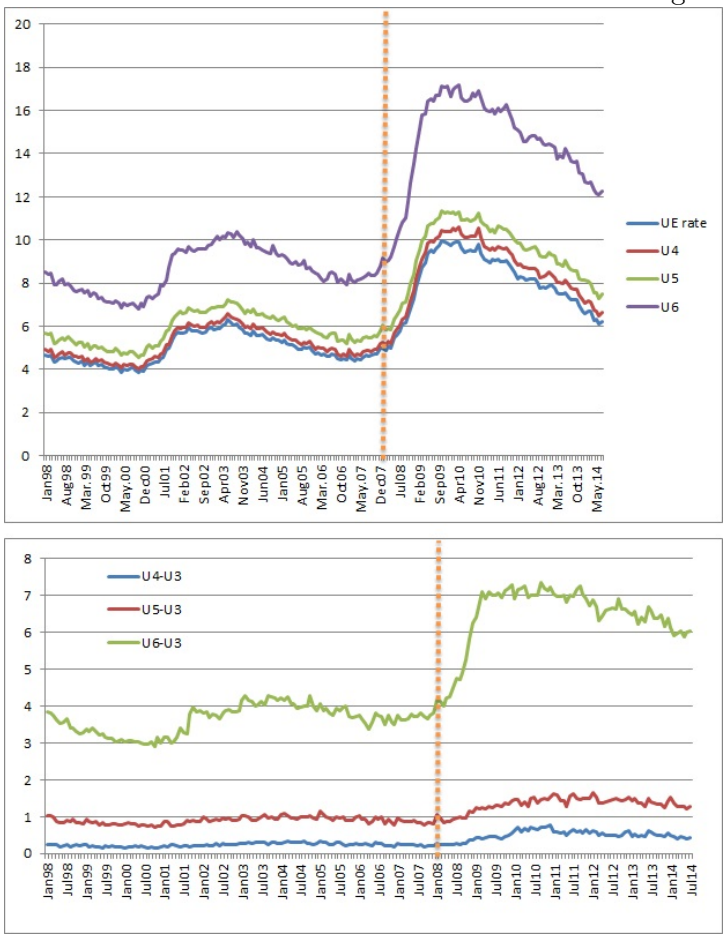
Figure 2.1: Average and median unemployment duration



Current aggregate data on labor market suggests that the current recession is different than the previous time periods. Firstly, there is a higher number of discouraged workers during current recession. Figure 2.2 shows U4 and U3 unemployment rates (see footnote 1). The discrepancy between U4 unemployment rate and U3 unemployment rate has been increased from 0.2 (historical average) to 0.6 percent (average during current recession), which amounts to around 1.2 million discouraged worker throughout US. Secondly, during the current recession average unemployment duration increases significantly. Figure 2.1 shows the average and median unemployment duration. Average unemployment duration has been increase from around 17 weeks of historical average to around 28 weeks during recession.

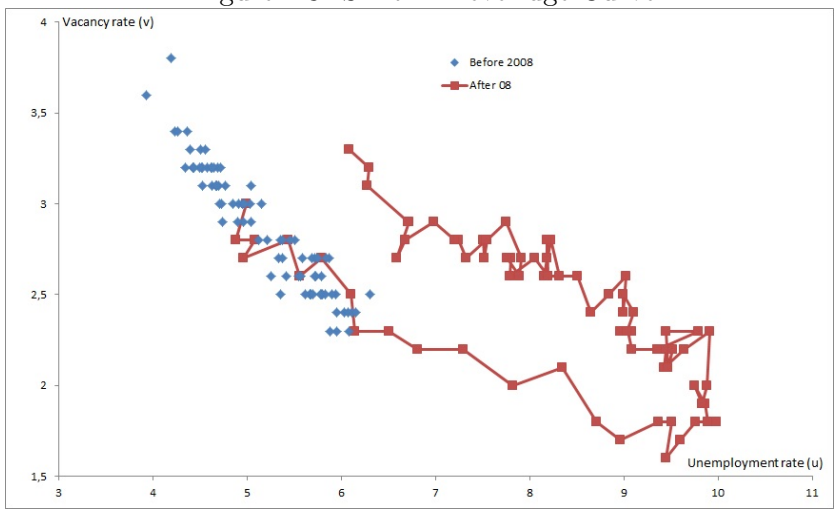
Finally, the empirical relation between vacancy rate and unemployment rate, namely Beveridge curve, does not seem to hold during current recession whereas it was more pronounced in previous periods. Figure 2.3 shows that unemployment rate is higher than the Beveridge curve suggests for the vacancy rate during current recession.

Figure 2.2: Different definitions of UE and Role of Discouraged Workers



In the next chapter we will provide a model which will be used to account for these facts. This model also will be a candidate to explain the puzzle between these data facts and standard human capital approach.

Figure 2.3: Shift in Beveridge Curve



Chapter 3

Model

3.1 Environment

There is a unit measure of infinitely-living workers which is divided to employed and unemployed (and out-of-labor force with the extension discussed in Chapter 4). Measure of firms in each occupation will be determined endogenously by free entry. All the agents are risk neutral and discount the future at a rate $r > 0$. Employed workers produces an amount of homogeneous goods according to their occupation until a separation or exit shock hits the worker. Unemployed worker searches for a job and receives a utility of $b > 0$ in each period which constitutes leisure benefit of being unemployed and/or unemployment benefit.

As in Gonzalez and Shi (2010), each worker has an unknown permanent ability of i, which is either high h or low l, and has an associated productivity parameter σ_h and σ_l respectively, where σ_h ,$\sigma_l \in (0,1)$ and $\sigma_h \geq \sigma_l$. Each new worker in the market has ability i with probability $p_i \in (0,1)$, where $p_h = 1 - p_l$. There are two different occupations j either good g or bad b and associated productivity parameters μ_g and

μ_b, where $\mu_g, \mu_b \in (0,1)$, $\mu_b = 1 - \mu_g$ and $\mu_g > \mu_b$. We will see that productivity of *employed* workers will *not* differ according to type of workers, on the other hand occupation g has better prospects for type h *unemployed* workers and occupation b has better prospects for type l *unemployed* workers in terms of finding a job. Ability and occupation determines a worker's search productivity as follows. First, a worker picks either occupation g or b and firms decide to open a vacancy position. Second, standard randomized matching occurs between unemployed workers and vacancy positions. So far, everything is very conventional as in directed search and matching model. Then following unconventional productivity assignment makes finding a job in distinct occupations different for distinct types of unemployed workers. Nature (which can see the type of the worker and assigns productivities accordingly) moves and assign a productivity to a matched worker according to his type and the occupation he chooses. If a worker is applying for occupation g, productivity of worker of type h will be $y_g > 0$ with probability $\sigma_h \mu_g$ and $y' < 0$ with probability $1 - \sigma_h \mu_g$ (otherwise); productivity of worker of type l will be $y_g > 0$ with probability $\sigma_l(1 - \mu_g$ and $y' < 0$ otherwise. In market b, a worker with type h will have a productivity of $y_b > 0$ with probability $\sigma_h \mu_b$) and $y' < 0$ otherwise; a worker of type l will have a productivity of $y_b > 0$ with probability $\sigma_l(1 - \mu_b)$ and $y' < 0$ otherwise. That is productivity of a (employed) worker is occupation specific and it is not depend type of the worker. Worker meets randomly drawn firm which offers a job in that occupation. Both firm and worker can see the productivity but not the type of the worker. Similar to Gonzalez and Shi (2010) we will call $\sigma\mu$ components as *productive units*. High ability workers are more likely to be productive than a low-ability worker in occupation h. Obviously, a firm will hire the worker only if worker has positive productivity. Note that every employed

worker in occupation g has a productivity y_g, and every employed worker in occupation l has a productivity y_l. I will also assume different separation shock for different occupations: an employed worker in occupation g has a probability $\delta_g > 0$ to separate and join unemployed pool and an employed worker in occupation b has a probability $\delta_b > 0$ to separate. Hence occupations g and b are differentiated by their prospects of job finding probability for different types of unemployed workers, by total product produced by each worker and by their separation probability. Note that in order to focus on learning aspects of only search outcomes of unemployed workers rather than of the job productivity signal of employed workers, this setup forces every employed worker in a certain occupation (even if workers are different types of workers) produces same amount of product. By this way we have clearly seperated informativeness of search outcomes from informativeness of on-the job productivity. This study will only focus on informativeness of search outcomes and should be seen as a complementary research rather than a substitutes to the studies (e.g. Javonovic 1979) about informativeness of on-the-job productivity. Nevertheless, please note that we have used similar concepts in different context, therefore there remains a possibility of confusing these concepts if one is more familiar with studies about informativeness of on-the-job productivity. We suggest that one should make himself clearly understand productive technology of search in our model to clearly perceive these concepts in our context.

As mentioned in Gonzalez and Shi (2010), this formulation of worker's ability to find a job can be interpreted in the context of worker and firm specific skill bundle of Lazear (2009). In this context, different firms or different occupations require different skill bundles and workers are heterogeneous in terms of their skill bundles. A firm reviews the worker in order to understand whether his/her skill bundle would fit the firm. In our study, we assume that high type workers have a higher probability to fit the firm

which offer a job with occupation g compared to low type workers. Likely, low types have a higher probability to fit a firm offering occupation b than to fit a firm offering an occupation g.

Learning will take place once an unemployed worker searches for a job. Since after a long history the worker would be able to learn his/her actual ability, we will assume an exit shock hits him to rule out this uninteresting case. Therefore, there is a probability of $\psi > 0$ that a person whether employed or unemployed dies.

We will use the search and matching approach as follows: The number of matches is given by a matching function $F : R^2 \to R$. We will use the index[1] x, rate of a match occurs, as the argument of all the following functions. $v(x)$ denotes total measure of vacancies created in the economy, whereas $u(x)$ denotes total measure of unemployed workers, just the sum of the measure of unemployment workers of each type.

A function $F(u(x), v(x))$ gives the number of matches in the economy. Therefore matching rate index x is

$$x = \frac{F\left(u\left(x\right), v\left(x\right)\right)}{u\left(x\right)}$$

Using matching function, ordinary definitions in competitive search model follows: the matching probability of a vacancy in economy is $F/v = x/\lambda(x)$, where $\lambda(x) \equiv v(x)/u(x)$ is the tightness in the labor market. As in Gonzalez and Shi (2010), we will assume the following standard assumptions for the matching function:

Assumption I: *(Regularity conditions of matching function)* Function F is such that (i) strictly increasing, strictly concave, and twice differentiable in each argument; (ii) F is linearly homogeneous; (iii) $F(1, 0) = 0$,$F(1, \infty) \geq 1/a_H$, and $x/\lambda(x) \leq 1$ for all

[1] Tightness, λ, can also equivalently be used as argument. There is not too much difference between use of λ or x. Although in literature λ is used more often, we have used x, because x has been used in derivations more often in this paper.

$x \leq 1/\sigma_h$.

Remark 1: Since $F(1,\lambda) = x$ this assumption implies that

$$\frac{\partial \lambda(x)}{\partial x} > \frac{\lambda(x)}{x} > 0, \qquad \frac{\partial^2 \lambda(x)}{\partial x^2} > 0 \quad \text{for all } x \in (0, 1/a_h) \tag{3.1}$$

moreover, $\frac{x}{\lambda(x)}$ is strictly decreasing in x.

By these specifications labor market is characterized by a wage level, $W(x,j)$, and a tightness, $\lambda(x)$. Every agent in the market takes $W(\cdot)$ and $\lambda(\ \cdot)$ as given. They will be determined in equilibrium. An unemployed worker's search decision in each period is to choose an occupation j and then he applies to lets say a central agency of matching. A firm set the wage menu $\{(w(x,j), \lambda(x)) : j \in \{g,b\}\}$ taken the equilibrium wage menu, $\{(W(x,j), \lambda(x)) : j \in \{g,b\}\}$ as given and commit to pay wage rate $w(x,j)$ if a productive match on occupation j occurs.

3.2 Value function of firms and free entry

Any firm can post a vacancy in economy after incurring a cost $c \in (0, y_b)$. If an occupation j is filled at a wage rate w, value of that filled occupation j to the firm discounted to the end of previous period is

$$(1+r)\, J_f^j(w) = y_j - w + (1-\psi)\,(1-\delta_j)\, J_f^j(w) \tag{3.2}$$

Match probability is $\frac{x}{\lambda(x)}$ and continuation value of the match is $(1-\psi)\, J_f^j\,(W\,(x,j))$. Therefore solving J_f^j from the equation above, value of opening vacancy is

$$J_v\,(x) = -c + \frac{x}{\lambda\,(x)} \frac{y_j - W\,(x,j)}{A_j} \tag{3.3}$$

where $A_j \equiv \frac{r+\psi}{1-\psi} + \delta_j$ is a constant for $j \in \{g, b\}$. Note that left hand side does not depend on j, since there should not be any arbitrage for the firm to open a vacancy in occupation g or occupation b due to free entry. Precisely, $J_v(x)$ and the number of vacancies,$v(x)$ satisfy $J_v(x) \leq 0$ and $v(x) \geq 0$, where the two inequalities hold with complementary slackness. Thus, if $v(x) > 0$, the wage rate is

$$W(x, j) = y_j - cA_j\lambda(x)/x \tag{3.4}$$

for $j \in \{g, b\}$. By remark 1 wage function has following properties: 1)$W'(x) < 0$ 2) $xW(x)$ is strictly concave.

3.3 Learning from search

Workers learn about their types by observing their search outcomes using Bayesian updating. We will refer to a workers conditional expectation about being a high type as P_h[2]. New born workers entering the market will have a belief of $P_h = p_h$, which is a common knowledge to every agent in the market.

Lets say that P_h is the prior belief about being a high type. Workers uses Bayesian updating over search outcomes. Updating will depend on in which particular occupation they try to find a job and on his/her search outcomes. Let $o = 1$ indicates that he has been productive in his current search (find the job) and $o = 0$ indicates that he has not been productive (fails to find the job). If a productive match occurs (worker finds a job) then posterior belief of being high type conditional on worker is searching in occupation j where the match rate in labor market is x is

[2]As well as P_h, conditional probability of being low type can also be used as belief.

$$\begin{aligned} P(h|x,j,o=1) &= \frac{P(o=1|x,j,\sigma_h)P_h}{P(o=1|x,j,h)P_h + P(o=1|x,j,l)P_l} \\ &= \frac{\sigma_h\mu_j x P_h}{\sigma_h\mu_j x P_h + \sigma_l\left(1-\mu_j\right)x P_l} = \frac{1}{1+\frac{\sigma_l(1-\mu_j)}{\sigma_h\mu_j}\frac{P_l}{P_h}} \end{aligned} \tag{3.5}$$

Posterior belief of being high type conditional on worker is searching in occupation j and has not been productive is

$$\begin{aligned} P(h|x,j,o=0) &= \frac{P(k=0|x,j,\sigma_h)P_h}{P(k=0|x,j,\sigma_h)P_h + P(k=0|x,j,\sigma_l)P_l} \\ &= \frac{\left(1-\sigma_h\mu_j x\right)P_h}{\left(1-\sigma_h\mu_j x P_h\right) + \left(1-\sigma_l(1-\mu_j)x\right)P_l} \\ &= \frac{1}{1+\frac{1-\sigma_l(1-\mu_j)x}{1-\sigma_h\mu_j x}\frac{P_l}{P_h}} \end{aligned} \tag{3.6}$$

It is informative to compare posterior beliefs $P(h|.)$ with prior beliefs P_h. For that the multiplier at the dominator just before $\frac{P_l}{P_h}$ is useful. If that multiplier is smaller than 1 than posterior is higher than prior and vice versa. Note that if search occurs in occupation h and $o=1$, then posterior of being high type is higher then prior since $\frac{\sigma_l(1-\mu_h)}{\sigma_h\mu_h} < 1$. If search occurs in occupation h and $o=0$, then posterior of being high type is smaller then its prior since $\frac{1-\sigma_l(1-\mu_h)x}{1-\sigma_h\mu_h x} > 1$. Moreover, increase on the belief of being high type with productive match does not depend on the rate x, whereas the decrease in beliefs with a non-productive match is higher for higher x's. Because, x does not affect the likelihood ratio of a match success between the two types, on the other hand, the fail in matching with a higher rate of job offers in occupation h give a

stronger signal of being type l.

If the search has occurred for occupation b, then a match will decrease, increase or not-change the belief of being high type depending on whether $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} > 1$, $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} < 1$ or $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} = 1$, respectively. Again, fail in finding a job will decrease, increase or not-change the belief of being high type depending on whether $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} < 1$, $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} > 1$ or $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} = 1$, respectively. Among this alternatives assuming either $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} > 1$ or $\frac{\sigma_l(1-\mu_b)}{\sigma_h\mu_b} = 1$ is meaningful in the sense that new information (e.g. being accepted by occupation b) does not take posterior away from correct type of worker on average. This observation is summarized in assumption II below. Moreover, as a special case no information content on the search outcome in occupation b will also be analyzed. The reason is that it may be more enlightening to characterize equilibrium with two different occupations; one with information content and one without the information content. In such a characterization, the workers' behavior towards information content of occupation, as well as interaction between information content of search and other labor market variables such as wage rate, unemployment rate and unemployment duration will be more clear.

Assumption II: *(Search give information about true type)* $\sigma_h \geq \sigma_l$, $\mu_g > \mu_b$ and $\sigma_l\left(1-\mu_b\right) \geq \sigma_h\mu_b$.

3.4 Workers Value Functions

Consider first a worker with belief P_h of being high type who is employed at wage w in occupation j in any period. Denote the worker's value function, discounted to the end of the previous period, as $J_e(P_h, w, j)$.After producing and obtaining the wage w, the

separation shock forces the worker into unemployment with probability δ_j depending on occupation and then, independently, the exit shock forces the worker out of the market with probability ψ. If the worker remains employed after these two shocks, the continuation value is $J_e(P_h, w, j)$. If the worker is separated from the job but remains in the market, the continuation value is denoted $V(P_h)$. If the worker is out of the market, the continuation value is 0. Thus Bellman equation for J_e

$$(1+r)\, J_e(P_h, w, j) = w + (1-\psi)\left[(1-\delta)\, J_e(P_h, w) + \delta_j V(P_h)\right]$$

This yields

$$J_e(P_h, w, j) = \frac{1}{A_j}\left[\frac{w}{1-\psi} + \delta_j V(P_h)\right] \tag{3.7}$$

where $A_j \equiv \frac{r+\psi}{1-\psi} + \delta_j$ is a constant for $j \in \{g, b\}$.

Now consider an unemployed worker who enters a period with belief P_h. If he chooses occupation j, expected probability of finding a job is $P(P_h, j, x)$ where

$$\begin{aligned} P(P_h, j, x) &= x\left[P_h \sigma_h \mu_j + (1-P_h)\sigma_l(1-\mu_j)\right] \\ &= x[C_j P_h + D_j] \end{aligned} \tag{3.8}$$

where $C_j = (\sigma_h + \sigma_l)\mu_j - \sigma_l$ and $D_j = \sigma_l(1-\mu_j)$ are positive constants.

His belief will be updated by $P(h|x, j, o=1)$ via equation (3.5) if he finds a job and by $P(h|x, j, o=0)$ via equation (3.6) if he/she fails. If he finds a job, his value function will be $\max\Big\{J_e\Big(P(h|x,j,o=1), W(x,j)\Big), V(P(h|x,j,o=1))\Big\}$; if he fails to find a job, the value function will be $V\Big(P(h|x,j,o=0)\Big)$. His expected return to apply occupation j excluding the unemployment benefit, is $(1-\psi)R(P_h, j; x)$, where

$$\begin{aligned} R(P_h, j; x) &= P(P_h, j; x) \max\Big\{ J_e\Big(P(h|x, j, o = 1), W(x, j)\Big), V(P(h|x, j, o = 1))\Big\} \\ &\quad + (1 - P(P_h, x, j)) V(P(h|x, j, o = 0)) \end{aligned}$$

Since we discounted value functions to the end of previous period, then

$$(1 + r) V(P_h) = b + (1 - \psi) \max_{j \in \{g,b\}} R(P_h, j; x) \tag{3.9}$$

As in Gonzalez and Shi (2010), we assume that the workers always accept the job offer in any occupation. In other words,

$$\begin{aligned} J_e\Big(P(h|x, j, o = 1), W(x, j), j\Big) &> V\Big(P(h|x, j, o = 1)\Big) \\ &\quad \text{for all } P_h \in [0, 1] \text{ and } j \in \{g, b\} \end{aligned} \tag{3.10}$$

Using definition of J_e from equation (3.7), this condition is equivalent to

$$W(x, j) > (r + \psi) V\Big(P(h|x, j, o = 1)\Big) \text{ for all } P_h \in [0, 1],\ x \in X \text{ and } j \in \{g, b\} \tag{3.11}$$

This condition is equivalent to condition (3.12) below. As X is bounded, this condition can be satisfied for sufficiently high productivity y_j and sufficiently small unemployment benefit b and cost of opening vacancy c. This is reasonable since equilibrium wage rate depends positively on productivity y_j which in turn make continuation value of employment higher whereas unemployment benefit b increases continuation value of staying unemployed.

Assumption III: *(*Reservation wage always met) Assume that productivity in the market $j \in \{g, b\}$ satisfy that

$$\frac{y_j - b}{c} > [A_j + \sigma_h x] \text{ for all } x \in X \tag{3.12}$$

There are two reasons for this assumption. First, it is already well known that the workers will prolong their unemployment duration if their reservation wage has not been met. However, in this paper, the focus is on the unemployment caused by learning process of workers. Best way to isolate this type of unemployment is closing the reservation wage channel. Second reason is a technical one: with the use of this assumption the value function will be identified further by Theorem 2.

Under Assumption II and Assumption III, we can rewrite expected return function using definition of J_e from equation (3.7)

$$\begin{aligned} R(P_h, j; x) &= x(C_j P_h + D_j)\left[\frac{1}{A_j}\frac{W(x,j)}{1-\psi} + \frac{\delta_j}{A_j} V\Big(P(h|x,j,o=1)\Big)\right] \\ &\quad + (1 - C_j P_h x - D_j x) V\Big(P(h|x,j,o=0)\Big) \end{aligned} \tag{3.13}$$

Finally, under Assumption II and III value function is

$$\begin{aligned} (1+r)V(P_h) &= b + (1-\psi) \\ &\quad \max_{j\in\{g,b\}} \left\{ x(C_j P_h + D_j)\left[\frac{1}{A_j}\frac{W(x,j)}{1-\psi} + \frac{\delta_j}{A_j} V\Big(P(h|x,j,o=1)\Big)\right] \right. \\ &\quad \left. + (1 - C_j P_h x - D_j x) V\Big(P(h|x,j,o=0)\Big) \right\} \end{aligned} \tag{3.14}$$

We use equation (3.14) to characterize equilibrium.

Chapter 4

Definition and characterization of equilibrium

4.1 Definition of equilibrium

Definition 1. *The stationary symmetric equilibrium with learning consists of value functions* $(J_e, V; J_f, J_v)$*, worker choices* (j)*, a wage function* $W(x, j)$ *and a sequence of beliefs such that*

1. *(a) The value functions* $(J_e, V; J_f, J_v)$ *satisfies* (3.7) (3.9); (3.2) (3.3)*, respectively*

   (b) Given the wage function, all workers with same belief P_h *use same optimal strategy* (j) $j = g(P_h) \in G_j(P_h)$ *which solves* (3.9)

   (c) Bayesian Update: A worker with belief P_h *and optimal strategy* (j) *as in (b) update his belief with* $P(h|x, j, o = 1)$ *as in (3.5) if he/she finds a job and with* $P(h|x, j, o = 0)$ *as in (3.6) if he/she fails to find a job.*

(d) *Free entry: Wage function $W(x, j)$ satisfies (3.4).*

(e) *Consistency: For labor market, the measure of all vacancies divided by the measure of unemployed workers is equal to $\lambda(x)$.*

After this general definition of the equilibrium, we will characterize equilibrium under Assumption I, II and III.

4.2 Some Characterization of the Equilibrium

Theorem 2. *(Existence Of Equilibrium) Under Assumptions I and III, there exists an equilibrium where all matches are accepted.*

Proof. Existence of value functions J_f and J_v are very standard and will be omitted here. One can check e.g. Rogerson et al. (2005) for the arguments. Existence of J_e depends on existence of V. For existence of V, it is almost immediate to check that the right-hand side of (3.9) satisfies the Blackwell sufficiency conditions. Using standard arguments in Stokey, Lucas, and Prescott (1989), one can show that a unique V exists, which is positive, bounded and continuous on M. Moreover, the correspondence of maximizers G_j is non-empty, closed and upper hemicontinuous. Sufficiency of Assumption III for all matches accepted is little detailed and will be addressed at the appendix. □

Although the existence result does not depend on Assumption II, it will help us to characterize the value function with a *reservation prior property.*

Theorem 3 (Reservation Belief Property)**.** *Under Assumptions I,II and III, there exists $P^* \in [0, 1]$ such that all unemployed workers with a belief $P_h < P^*$ choses to search in occupation b and all unemployed workers with a belief $P_h > P^*$ choses to search in occupation g. Value function $V(P_h)$ is (weakly) convex and strictly increasing on $[0, 1]$.*

Proof. Under Assumption II and III, value function is represented by (3.14). Since V is unique, first argument in max operator at the right hand side is strictly decreasing on [0,1] whereas second argument is strictly increasing first note that. I will just show that second argument is strictly increasing on [0,1] (proof of the other argument is very similar):

Let V be a weakly increasing function, $P_{ha} > P_{hb}$ where $P_{ha}, P_{hb} \in [0,1]$ and $g_i = g(P_{hi}) \in G(P_{hi})$ be particular optimum choices where $i \in \{a, b\}$. Then

$$
\begin{aligned}
& R(P_{ha}, g_a) - R(P_{hb}, g_b) \\
\geq\ & R(P_{ha}, g_b) - R(P_{hb}, g_b) \\
\geq\ & x\left(D_j + C_j P_{ha}\right) \frac{1}{A_h}\left[\frac{W\left(x,h\right)}{1-\psi} + \delta_h V(P(P_{ha}, g, 1))\right] \\
& + \left[1 - x\left(D_j + C_j P_{ha}\right)\right] V\left(P\left(P_{ha}, g, 0\right)\right) \\
& - \Bigg\{ x\left(D_j + C_j P_{hb}\right) \frac{1}{A_h}\left[\frac{W\left(x,h\right)}{1-\psi} + \delta_h V(P(P_{hb}, g, 1))\right] \\
& + \left[1 - x\left(D_j + C_j P_{hb}\right)\right] V\left(P\left(P_{hb}, x, 0\right)\right) \Bigg\} \\
=\ & x D_j\left(P_{ha} - P_{hb}\right) \frac{1}{A_h} \frac{W\left(x,h\right)}{1-\psi} + x\left(D_j + C_j P_{ha}\right) \frac{1}{A_h} \delta_h V(P(P_{ha}, g) \\
& - x\left(D_j + C_j P_{hb}\right) \frac{1}{A} \delta_h V(P(P_{hb}, g, 1)) \\
& + \left[1 - x\left(D_j + C_j P_{ha}\right)\right] V\left(P(P_{ha}, g, 0)\right) - \left[1 - x\left(D_j + C_j P_{hb}\right)\right] V\left(H\left(P_{hb}, x, 0\right)\right) \\
\geq\ & x D_j\left(P_{ha} - P_{hb}\right) \frac{1}{A_h} \frac{W\left(x,h\right)}{1-\psi} + x\left(D_j + C_j P_{ha}\right) \frac{1}{A_h} \delta_h V(P(P_{hb}, g, 1) \\
& - x\left(D_j + C_j P_{hb}\right) \frac{1}{A_h} \delta_h V(P(P_{hb}, g, 1)) \\
& + \left[1 - x\left(D_j + C_j P_{ha}\right)\right] V\left(P\left(P_b, g, 0\right)\right) - \left[1 - x\left(D_j + C_j P_{hb}\right)\right] V\left(P\left(P_{hb}, x, 0\right)\right) \\
=\ & x C\left(P_{ha} - P_{hb}\right) \left[\frac{W\left(x,h\right)}{A_h\left(1-\psi\right)} + \frac{\delta_h}{A_h} V(P(P_{hb}, g, 1) - V\left(P(P_{hb}, g, 0)\right)\right] \\
>\ & x C\left(P_{ha} - P_{hb}\right) \left[V(P(P_{hb}, g, 1)) - V\left(P(P_{hb}, g, 0)\right)\right] \\
\geq\ & 0
\end{aligned}
$$

First inequality uses the fact that g_a is the maximizer for P_{ha}. Second inequality uses $V\left(P(P_{ha}, g, 1)\right) \geq V\left(P(P_{hb}, g, 1)\right)$ and $V\left(P\left(P_{ha}, g_b, 0\right)\right) \geq V\left(P\left(P_{hb}, g_b, 0\right)\right)$. Strict inequaity uses Assumption III which is equivalent to $W\left(x, j\right) > (r + \psi) V((P(h|x, j, o = 1))$.Last inequality uses the fact that $P(P_{hb}, g, o = 1) > P\left(P_{hb}, g, o = 0\right)$ and V is weakly increasing function.

Since first component of max operator is decreasing and second operator is strictly increasing we have a reservation prior property (Note that we did not restrict P^* to be interior of $[0,1]$. For that a relevant bound on $y_g - y_b$ is sufficient).

Finally, proof of convexity of Value function is similar to proof in Nyarko (1994) . □

4.3 Steady state distributions

Denote $e_i^j(P_h)$ as measure of type-i workers employed in occupation-j with belief P_h and $u_i(P_h)$ as measure of unemployed type-i workers before labor market opens in a period for $i \in \{h,l\}$ and occupation-$j \in \{g,b\}$. Probability of new born to be type i is p_i. New workers enter with belief $p_0 = p_h$. Let $T(p_0)$ be tree of equilibrium beliefs generated from p_0. Then stationary distributions of workers over beliefs is $\left\{ \left(e_i^j(p), u_i(p) \right)_{i\in\{h,l\}}^{j\in\{g,b\}} : p \in T(p_0) \right\}$ Unemployed workers are in 3 groups: Newborns, UE was E in previous period, UE was UE in previous period,:

1. Newborns: Outflow and inflow from this group is ψp_i. Therefore this group is always stationary

$$u_i^(p_0) = \psi p_i \; p \in T(p_0) \; i \in h,l \tag{4.1}$$

2. Unemployed - was employed in previous period.

   - Outflow:All workers move out from this group.
   - Inflow: separate from jobs and survive
   - Occupation j: Belief is $\phi(p)$ for some $p \in T(p_0)$

$$u_i(\phi(p)) = \sum_j (1-\psi)\delta^j e_i^j(\phi(p)), \; p \in T(p_0) \; i \in H,L \tag{4.2}$$

3. Unemployed - was unemployed in previous period

   - Occupation j: Belief is $n(p) = P(p, x, j, o = 0)$ for some $p \in T(p_0)$. Everybody outflow. Inflow is who survives and fails to find a job

$$u_H^j(n(p)) = (1-\psi)[1-\sigma_h\mu_h x_{J(p)}]u_H(p),\ p \in T(p_0) \tag{4.3}$$

$$u_L^j(n(p)) = (1-\psi)[1-\sigma_L(1-\mu_h)x_{J(p)}]u_L(p),\ p \in T(p_0) \tag{4.4}$$

Employed workers are just in one group. For occupation j, belief is $p \in T(p_0)$. Outflow is worker who dies or separated. Inflow is worker who find a job among searchers:

$$[\psi+(1-\psi)\delta_j]e_H^j(\phi(p)) = [(1-\psi)\sigma_H\mu_j x_{J(p)}]u_H(p),\ p \in T(p_0) \tag{4.5}$$

$$\{\psi+(1-\psi)\delta_j\}e_L^j(\phi(p)) = [(1-\psi)\sigma_L(1-\mu_j)x_{J(p)}]u_L(p),\ p \in T(p_0) \tag{4.6}$$

The stationary distribution is determined by (4.1)(4.6) and with the requirement that the total measure of workers is one. Because the equilibrium is block recursive, optimal choices are independent of the distribution, and so (4.1)(4.6) are linear equations of the measures of workers. It is straightforward to solve for these equations by going through the nodes of the tree, starting at the root, p_0. Given the equilibrium tree of beliefs, $T(p_0)$, the stationary distribution of workers over such beliefs is unique.

In this unique stationary distribution we will call depth of any node as *age* since it takes a newborn that many 'time periods' to come to that node. Moreover, for an employed worker, we will call *tenure* as the difference between his *age* and depth of the node he is unemployed for the last time since he was working for that many 'time periods' in his current job.

4.4 Further Characterization of the equilibrium

Next two theorems will further characterize the equilibrium of the model as displaced workers with longer previous job tenure need longer time (on average) to find the job and they have higher probability to change occupations. However, we need a couple of propositions and corollaries in order to make the setup for the theorems and break down the proof of the final theorems much tractable. Next proposition simply states that search is informative. If you search, you learn better about your true type.

Proposition 4. *(Search provide information to workers about their types) Let any node τ on the tree T. Those workers who search for the job got beliefs no worse (in the sense that on average their beliefs are not farther away from their true types) than the workers who did not search and stay employed. In fact, if that node is a node of employment workers who search for the job (who got hit by external displacement shock δ) have strictly better beliefs (in the sense that on average their beliefs are not farther away from their true types) than the workers who stayed employed.*

Proof. Let any node τ at depth S on the tree T be given. Note that by external die probability ψ infinite life is probability zero event therefore depth is finite and it has a positive measure of workers. Law of large numbers hold at any node therefore it will not be referred on the rest of the proof.

By construction every worker on τ has the same belief, say P_h of being high type. If this node is a node of unemployment, then everybody will search for a job, therefore proposition vacuously hold.

Now, lets assume that that node is a node of employment. Let the distribution of worker type is given as y is the fraction of high type workers. Hence $1-y$ is the fraction of workers low type. I will just provide the proof that high types of workers who hit

displacement shock and search for a job have strictly better beliefs on average than the workers who stayed employed. For the low types proof is very similar and will be omitted here. Note that the workers who stayed employed does not change their beliefs of being high type, P_h. On the other hand a measure of $y\delta$ high type workers hit by displacement shock. Lets assume that $P_h > P^*$. For the case $P_h \leq P^*$ proof is very similar and will be omitted here. Idea of the proof is simple: average improvement of posterior beliefs of high types who search for a job in the occupation g and find a job is bigger than the average disimprovment of posterior beliefs of high types who search for a job in the occupation g and could not find a job. A fraction of $y\delta x_g^* \mu_g \sigma_H$ will find a job and their beliefs will be updated as

$$P(\sigma_h|x, g, o = 1) = \frac{\sigma_h \mu_g x^* P_h}{\sigma_h \mu_g x^* P_h + \sigma_l (1 - \mu_g) x^* P_l} \tag{4.7}$$

therefore on average posterior is

$$y\delta \frac{\sigma_h \mu_g x^* P_h}{P_h + \left(\frac{\sigma_l (1-\mu_g) x^*}{\sigma_h \mu_g x^*}\right) P_l} \tag{4.8}$$

A fraction of $y\delta(1 - x_g^* \mu_g \sigma_H)$ will not find a job and their beliefs will be updated as

$$P(\sigma_h|x_g, g, o = 0) = \frac{(1 - \sigma_h \mu_g x^*) P_h}{(1 - \sigma_h \mu_g x^*) P_h + (1 - \sigma_l (1 - \mu_g) x^*) P_l}$$

on average their posterior is

$$y\delta\frac{(1-\sigma_h\mu_g x^*)P_h}{P_h+\frac{(1-\sigma_l(1-\mu_g)x^*)}{(1-\sigma_h\mu_g x^*)}P_l}$$

Obviously, posterior on average is closer to 1 since

$$\frac{\sigma_l(1-\mu_g)x_g^*}{\sigma_h\mu_g x_g^*}<1<\frac{\left(1-\sigma_l(1-\mu_g)x_g^*\right)}{\left(1-\sigma_h\mu_g x_g^*\right)}$$

□

Corollary 5. *Take two same type workers at age $n+k$ which have the same history up until stage n, where n and k are positive integers. The worker who have longer unemployment spell during stage $n+1$ to $n+k$ has weakly better belief on average.*

Idea of the proof of the corollary is simple. Please note that by construction of the model, employment spells do not change beliefs of the type. Corollary simply follows from the fact that only change of the beliefs occurs by search and the previous proposition states that search on average improve the beliefs.

Corollary 6. *Take two same type workers at age $n+k$ which have the same history up until stage n, where n and k are positive integers. The worker who have longer unemployment spell during stage $n+1$ to $n+k$ is weakly more probable to be on the correct side of the threshold P^* (side of the threshold which is closer to his true type) than the worker who have shorter unemployment spell.*

This corollary follows immediately from the previous corollary and stated just for the theorem 12 below.

Corollary 7. *Take two same type workers at the stage $n + k$ which have the same history up until stage n, where n and k are positive integers. Assume worker 1 has a tenure of k period and the other worker has a tenure strictly shorter than k period. Worker with the shorter tenure has weakly better belief on average.*

Corollary 8. *Take two same type workers at the stage $n+k$ which have the same history up until stage n, where n and k are positive integers. Assume worker 1 has a tenure of k period and the other worker has a tenure strictly shorter than k period. Worker with the shorter tenure is weakly more probable to be on the correct side of the threshold P^* (side of the threshold which is closer to his true type) than the worker who have shorter unemployment spell. .*

This corollary follows immediately from the previous corollary and stated just for the theorem 12 below.

Next proposition states for the same type of workers the farther away a workers belief is from his true type, the longer he will search for a job on average.

Proposition 9. *(Farther away the belief, longer the ue duration) Compare two high [low] type unemployed workers such that worker 1 has better belief P_1 than worker 2 with belief P_2, i.e. $P_1 > P_2$ [i.e. $P_1 < P_2$]. Expected time to find a job is weakly longer for worker 2.*

Proof. I will prove here the case of two high type workers. Other case is very similar. If they had always searched for a job in the same occupation, their expected time to find a job would have been the same since these workers are same type. The problem is their belief of being high type might turned out to be so low that ($P_H < P^*$ they began to search in the not suitable occupation, occupation b. By construction it is obvious that if high type searches job in occupation b it takes longer him to find a job because

of matching probabilities. The case of $P_1 > P^* > P_2$ is trivial and will be omitted here. I will prove here the case $P_1 > P_2 > P^*$, the case $P^* > P_1 > P_2$ is very similar. For the case $P_1 > P_2 > P^*$, by induction I will show that for the same history Worker 1 is never the one whose beliefs drop below the P^* first. As the first step of induction it is obvious that if both workers have not find the job on occupation-g, their belief has relation

$$\frac{1}{1+\frac{1-\sigma_l(1-\mu_j)x}{1-\sigma_h\mu_j x}\frac{P_l}{P_1}} > \frac{1}{1+\frac{1-\sigma_l(1-\mu_j)x}{1-\sigma_h\mu_j x}\frac{P_l}{P_2}} \tag{4.9}$$

Assume that they could not find the job for k step in occupation-g and the prior beliefs are $P_{1k} > P_{2k}$. Now the posteriors, i.e. prior beliefs for the step $k+1$ are

$$\frac{1}{1+\frac{1-\sigma_l(1-\mu_j)x}{1-\sigma_h\mu_j x}\frac{P_l}{P_{1k}}} > \frac{1}{1+\frac{1-\sigma_l(1-\mu_j)x}{1-\sigma_h\mu_j x}\frac{P_l}{P_{2k}}}. \tag{4.10}$$

Therefore, the first worker is never the one whose beliefs to drop below the threshold first. □

Proposition 9 together with Corollary 7 proves the following theorem which states exactly the first fact stated in the abstract 1)longer tenured workers on their previous job stays unemployed longer.

Theorem 10. *Take two same type workers at the stage n+k which have the same history up until stage n, where n and k are positive integers. Assume worker 1 has a tenure of k period and worker 2 has a tenure strictly shorter than k period. If both workers are separated from their current job at stage n+k, worker with the shorter tenure stays unemployed weakly shorter on average.*

Proposition 11. *Take two same type workers with symmetric beliefs to the threshold, i.e. there is a* $\gamma \in (0, \min P^*, 1-P^*)$ *where worker 1 has belief of being high type of*

$P^* + \gamma$ whereas worker 2 has belief of being high type of $P^* - \gamma$. Worker with priors on the other side of threshold than his true type changes occupations they apply for more often on average than the worker with priors on the side of the threshold which is closer to his true type.

Proof. In appendix □

Proof of this proposition is in appendix but the idea of the proof is simple and helpful to understand the mechanism in the model. If a worker's belief is on the side which is closer to his true type, he will matched more often because by construction of the matching rates. On the other hand, if the belief is on the wrong side of threshold, more often he will not get a productive match and he will update beliefs towards to correct side of the threshold. After a while on average, worker with the worse beliefs will understand they are looking for the wrong occupation and change the type of the occupation they apply.

Now with Corollary 8 and Proposition 11 we can state the following theorem which is in terms with the second fact stated in the abstract 2)displaced workers with longer previous job tenure changes occupation more often.

Theorem 12. *Take two same type workers at the stage n+k which have the same history up until stage n, where n and k are positive integers. Assume worker 1 has a tenure of k period and worker 2 has a tenure strictly shorter than k period. If both workers are separated from their current job at stage n+k, worker with the longer tenure on his previous job changes occupation more often.*

Table 4.1: After 2008, more long tenured workers are displaced than historical average

	Previous tenure	
Time Period	*Short ($\leq$ 8 yrs)*	*Long ($\geq$ 9 yrs)*
Before 2008	81%	19%
After 2008	78%	22%
During 2001 crisis	81.3%	18.7%

4.5 Mismatch and Shifts in the Beveridge Curve

In this setup, it is easy to understand when a shift in Beveridge curve may occur. In stationary distribution, it is obvious that a relation between vacancy and unemployment rates as in Beveridge Curve exists in this setup (It is simply the property of random matching with a matching function as stated in Assumption I). If for some reason separation shocks hits longer tenured workers and shorter tenured workers asymmetrically such that there is a higher fraction of workers with longer tenure on their previous job in unemployed pool than in previous equilibrium, from theorem 10 we know that expected time of being unemployment will increase. This will happen endogenously in our setup without any change in matching process because now 'inexperienced in searching' or 'fish out of water' workers are looking for a job and it will take them longer to 'learn' labor market. In fact, the following table suggests that 2008 recession is such a process where a higher fraction of longer tenured workers are displaced, which was not the case for e.g. 2001 recession.

As can be seen in Table 4.1, after 2008 there is an increase of 3 percentage point of fraction of longer tenured workers in the total pool of displaced worker. During the previous recession, on the other hand, this fraction is very close to historical average. Table 4.1 is suggesting that 'inexperienced in search' workers might be a reason for the shift in the Beveridge Curve and gives one possible explanation for the fact 3) stated in

the abstract.

4.6 Discouraged workers

In the current setup there is no cost for search; therefore workers always search. However, if we add (utility) cost of searching to analysis, then some of the workers with very low beliefs may chose not to search on occupation b (given that prospects of working on occupation b is low enough compared to utility cost of search) but stay out of labor force. In the literature those type of workers are called as discouraged workers. Our setup with Assumption II is convenient for understanding this phenomena. Let k be the cost of search in terms of utility for the worker if he/she choose to search. Then a separated worker with belief P_H will have three choices in terms of occupational selection: not search, search in occupation b and search in occupation g.

His new value function will be

$$\begin{aligned}(1+r)\,V\left(P_H\right) &= \max\Bigg\{b, max_j\Bigg\{b-k+(1-\psi)\left(C_j x+D_j\right)\Bigg[\frac{1}{A_j}\frac{W\left(x,j\right)}{1-\psi} \\ &\quad +\frac{\delta_j}{A_j}V\Big(P(h|x,j,o=1)\Big)\Bigg]+\frac{\delta_j}{A_j}V(P_h|x,j,o=1)\Bigg\}\Bigg\}\end{aligned} \tag{4.11}$$

If we statethe following assumption,

Assumption IV:*(Search is too costly compared to prospect of finding a job or b is latent occupation)* In equilibrium if

$$k>(1-\psi)\left[C_b x\frac{1}{A_b}\frac{y_b-cA\lambda_b/x}{1-\psi}+\left(1-C_b x+\frac{\delta_b}{A_b}\right)b\right] \tag{4.12}$$

Under assumption IV, Workers will choose no-searching at all rather than searching for a job in occupation b. Derivation of this condition is in Appendix. Under (4.12), the value function will be

$$(1+r)\,V\left(P_h\right) = \max\left\{b, b-k+(1-\psi)\,R(P_h, x; g)\right\} \tag{4.13}$$

Theorem 13. *Let assumption I,II is satisfied. Under assumption IV, occupation b is latent in the sense that no workers apply a job in occupation b and no firms post vacancy in occupation b in equilibrium. There exists $P_h^d \in [0,1]$ such that all people with belief $P_h < P_h^d$ chose not to search (stay out of labor force) and all people with a belief $P_h > P_h^d$ choses to search in occupation g. Value function $V(\theta)$ is (weakly) convex on $[0,1]$, constant on $[0, P_h^d]$ and strictly increasing on $[P_h^d, 1]$.*

Proof. Proof is very similar to Theorem 2. Note that first component of max operator in (4.13) is constant, whereas second component is strictly increasing as stated in proof of Theorem 2. Therefore all the claims follow. Under assumption IV searching for a job in occupation b is dominated by staying unemployed (actually here staying out of labor force since in this setting if people ever choose to not search they will also choose not search in the future given that their belief do not change). □

Note that workers, who fail to find a job for enough successive number of periods, will update their beliefs so low that it is not beneficial for them to search for a job any longer. Therefore, actually they want to find a job but they are discouraged enough to bother search for a job. In this setting, these workers whose beliefs are in $[0, P_h^d]$ are *discouraged workers.*

Chapter 5

Conclusion

In this paper, we have used a competitive search model with learning through search to analyze equilibrium effects of workers' incomplete information on their types. Workers are heterogeneous in their abilities and jobs are differentiated in terms of their match rates for certain types. Characterization under certain assumptions resulted in a value function with reservation prior property. Once a worker's belief about his/her abilities are below a certain level he/she always search for an occupation with a lower productivity and a lower wage rate. However, if the worker's belief about his ability is higher than a certain level, he/she tries his chances by searching for a job with a higher productivity-higher wage rate occupation. It is discussed that our model can qualitatively explain certain peculiarities in the both micro and macro data.

References

Acemoglu, D., and R. Shimer (1999). Efficient Unemployment Insurance *Journal of Political Economy 107*, 893–928.

Burdett, K., and T. Vishwanath (1988). Declining ReservationWages and Learning *Review of Economic Studies 55*, 655–665.

Bikhchandani, S., and S. Sharma (1996) Optimal search with learning. *Journal of Economic Dynamics and Control 20*, 333–359.

Falk, A., D. Huffman and U. Sunde (2006). *o I Have What It Takes? Equilibrium Search with Type Uncertainty and Non-Participation.* IZA Working Paper DP 2531.

Gonzalez, F. M., and S. Shi (2010). An Equilibrium Theory of Learning, Search and Wages. *Econometrica 78*, 509-537.

Jovanovic, B. (1979). *A Cautionary Note on Using (March) CPS and PSID Data to Study Worker Mobility. Macroeconomic Dynamics 17*(1), 172-194.

Kambourov , G. and Manovskii, I. 2013. *Job Matching and the Theory of Turnover. Journal of Political Economy 117*, 914-940.

Lazear, E. P. (2009). Firm-Specific Human Capital: A Skill-Weights Approach. *Journal of Political Economy 117*, 914-940.

Nyarko, Y. (1994). On the Convexity of the Value Function in Bayesian Optimal Control Problems. *Economic Theory 4*, 303–309.

Rogerson, R., R. Shimer, and R. Wright (2005). Search theoretic models of the labor market: A survey. *Journal of Economic Literature XLIII*, 959–988.

Stokey, N., R. E. Lucas and E. Prescott (1989). Recursive Methods in Economic Dynamics. Cambridge, MA: Harvard University Press.

Appendix A

Appendices

A.1 Data analysis

Displaced worker supplement is conducted every two years in Januaries, which asks additional questions to the displaced workers. Technical documentation and related documents can be seen from http://www.census.gov/cps/methodology/techdocs.html . Related data can be downloaded from http://www.census.gov/cps/data/ .

Displaced workers in this supplement are those who involuntarily separated from their jobs during the past three years before survey date by (i) mass layoff, (ii) plant closure or (iii) abolishment of their position rather than because of individual job performance. Therefore supplement data has its own limitations: First, workers are surveyed just once, providing information on one post-displacement data, rather than about their full history of experiences over time. So it is not possible to obtain panel data from this survey. Second, it does not include all unemployment pool but only displaced workers; hence, voluntary quits and fires by case are not sampled. Third, not the Displaced

worker supplement but CPS March supplement data is criticized by Kambourov and Manovskii (2013) for the followings:

1. Coding error of coder
2. Imputing system of CPS
3. It measures mobility not in a year but for shorter period.

Most of these criticism by Kambourov and Manovskii (2013) is relevant to March CPS supplement and if you use panel and try to impute transitions from different groups of employment and unemployment pools. This study is using January supplement and it does not try to impute transitions or panel data. Kambourov and Manovskii (2013) critique is not directly related to this study. Nevertheless, it has been taken as a cautionary note.

Displaced worker supplement also has one main advantage; it is a huge survey of around 150,000 individuals who are weighted to represent US workforce. Moreover, it takes a specific group 'displaced workers' from the unemployment pool. At first this may appear as a disadvantage. However, note that self-selection bias problem of the data is less pronounced among displaced workers. Among the whole unemployment pool there would be workers who self-select their tenure structure and unemployment states. On the other hand, workers in Displaced worker supplement are those who involuntarily separated from their jobs. Therefore, I believe using this supplement decreases the self-selection issues relevant to every labor market data.

We have taken six supplements to obtain data on displaced workers from 1999 to 2012, including their demographic information, total unemployment duration, previous

and current occupations, previous and current job tenures,and previous and current wage rates etc. For occupations following two digit occupation codes of CPS classification has been chosen:

1. Management occupations
2. Business and financial operations occupations
3. Computer and mathematical science occupations
4. Architecture and engineering occupations
5. Life, physical, and social science occupations 6 Community and social service occupations
6. Legal occupations
7. Education, training, and library occupations
8. Arts, design, entertainment, sports, and media occupations
9. Healthcare practitioner and technical occupations
10. Healthcare support occupations
11. Protective service occupations
12. Food preparation and serving related occupations
13. Building and grounds cleaning and maintenance occupations
14. Personal care and service occupations
15. Sales and related occupations

16. Office and administrative support occupations
17. Farming, fishing, and forestry occupations
18. Construction and extraction occupations
19. Installation, maintenance, and repair occupations
20. Production occupations
21. Transportation and material moving occupations
22. Armed Forces

Change in occupation means that if displaced worker is reemployed, his/her new occupation is different than his previous occupation in two digit codes. Long tenure means that the displaced worker has worked for 8 years or more in his/her previous job before displacement. Short tenure means that he has worked 7 years or less in his/her previous job before displacement. Long (short) unemployment duration means that it takes more (strictly less) than 18 weeks to find a new job. Arranged displacements are disregarded, i.e. workers who did not stayed unemployed after displacement but immediately found job are taken as arranged displacement and did not included in the dataset. Summary statistics in Table 1 are for displaced workers who are male, white, aged between 35 and 45, having educational attainment of at least high school, and having same eligibility for unemployment insurance. There are around 3,100 observations in this group. Not all the workers have all relevant data like previous and/or current wage rates. The table includes observations whenever relevant data exists. We have also looked at the other group of employees with respect to other demographic and educational characteristics. The statistics are not changing qualitatively. We have also done robustness check on the

specification of long and short durations of tenures and its cut-off points. Results only change qualitatively if the cut-off points decreases to 1 year but there is small sample size (157 among 3100) under 1 year of tenure groups. We have also done robustness check on the specification of long and short durations of unemployment and its cut-off points. Again, the results do not change qualitatively. We tried to control for industries and occupations but small sample sizes directs us away from that approach.

A.2 Some Proofs and Derivation of Conditions

Assumption III states that parameters of the model are such that reservation wage of the workers are always met. In other words, workers will apply for the jobs only if they will accept the job offer if a match occurs. This assumption mainly done because we want to focus on unemployment process caused by learning process of the workers rather than the well known issue of unemployment caused by not meeting reservation wage of workers. On the other hand, it also drops one particular channel in learning models: apply for a job for just for the sake of experimentation. In other words workers apply for a job knowing that they will not accept the job if a match occurs but apply for use only the information content of whether a match occurs or not. This is more relevant learning models in other context, it may be less relevant for labor market as stated in Gonzalez and Shi (2010). For example, in practice it is unlikely that workers who target a management job apply for a job for experimentation in e.g. a barber's shop in order to understand whether they would get a management job or not. Condition in Assumption III can be derived by

$$J_e\Big(P(h|x,j,o=1),W\left(x,j\right),j\Big) \quad > \quad V\Big(P(h|x,j,o=1)\Big)$$
$$\text{for all } P_h \in [0,1] \text{ and } j \in \{g,b\} \qquad \text{(A.1)}$$

Using definition of J_e from equation (3.7), this condition is equivalent to

$$W\left(x,j\right) > (r+\psi)V\Big(P(h|x,j,o=1)\Big) \text{ for all } P_h \in [0,1],\ x \in X \text{ and } j \in \{g,b\} \qquad \text{(A.2)}$$

Since equilibrium wage is

$$W\left(x,j\right) = y_j - cA_j\lambda\left(x\right)/x \qquad \text{(A.3)}$$

and $V\Big(P(h|x,g,o=1) \geq V\Big(P(h|x,b,o=1)$, after some algebra condition in Assumption III follows.

Condition in Assumption IV is obtained by a similar method. In order to occupation b to be latent, note that from value function stated in equation (4.12), second argument for occupation b which is

$$\begin{aligned} b-k \quad + \quad & (1-\psi)\left(C_b x + D_b\right)\Bigg[\frac{1}{A_b}\frac{W\left(x,j\right)}{1-\psi} \\ & +\frac{\delta_j}{A_b}V\Big(P(h|x,b,o=1)\Big)\Bigg] + \frac{\delta_j}{A_j}V(P_h|x,b,o=1) \end{aligned} \qquad \text{(A.4)}$$

should be lower than pure unemployment benefit b. Using similar algebra as in Assumption III condition, this is equivalent to condition stated in Assumption IV.

Idea of the proof of Corollary 7 and 11

As in Figure A.1, the displaced worker with longer tenure on his previous job has shorter time period in his lifespan in order to search on average compared to the displaced worker with shorter tenure on his previous job. Therefore he is less experienced in searching and therefore has worse belief on average. This makes him more probable to search for a job for longer period and change occupation he is looking for a job more often, on average.

Figure A.1: Two same age displaced workers with different previous job tenure