# Exact three-body local correlations for excited states of the 1D Bose gas

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**Real work done by**

**Marton Kormos and Yang-Zhi Chou (Rice),**

**Some more recent work also with Aditya Shashi (Rice)**

Texas Higher Education Coordinating Board

# Outline

## Exact non-perturbative results for local correlations

- Motivation: why to evaluate  local correlations?
- 1D Bose gas : Lieb-Liniger model  and beyond
- Lieb-Liniger model as a limit of sinh-Gordon model
- Correlations at finite temperature and interactions
- Non-thermal excited states of  the 1D Bose gas
- Conclusions and outlook

# Local Correlations and Losses

Two-particle correlations determine two-particle losses

$$g_2 = \frac{\langle \psi^\dagger(0)\psi^\dagger(0)\psi(0)\psi(0)\rangle}{n^2}$$

Three-particle correlations determine three-particle losses

$$g_3 = \frac{\langle \psi^\dagger(0)\psi^\dagger(0)\psi^\dagger(0)\psi(0)\psi(0)\psi(0)\rangle}{n^3}$$

**Can be used to characterize states of matter**

# Local Correlations without Interactions

BEC: $\Psi(x_1, ..., x_N) = \prod_i \Psi_0(x_i)$

$$g_3 = \frac{\langle \psi^\dagger(0)\psi^\dagger(0)\psi^\dagger(0)\psi(0)\psi(0)\psi(0)\rangle}{n^3} = 1$$

Infinite temperature, different pairings are allowed:

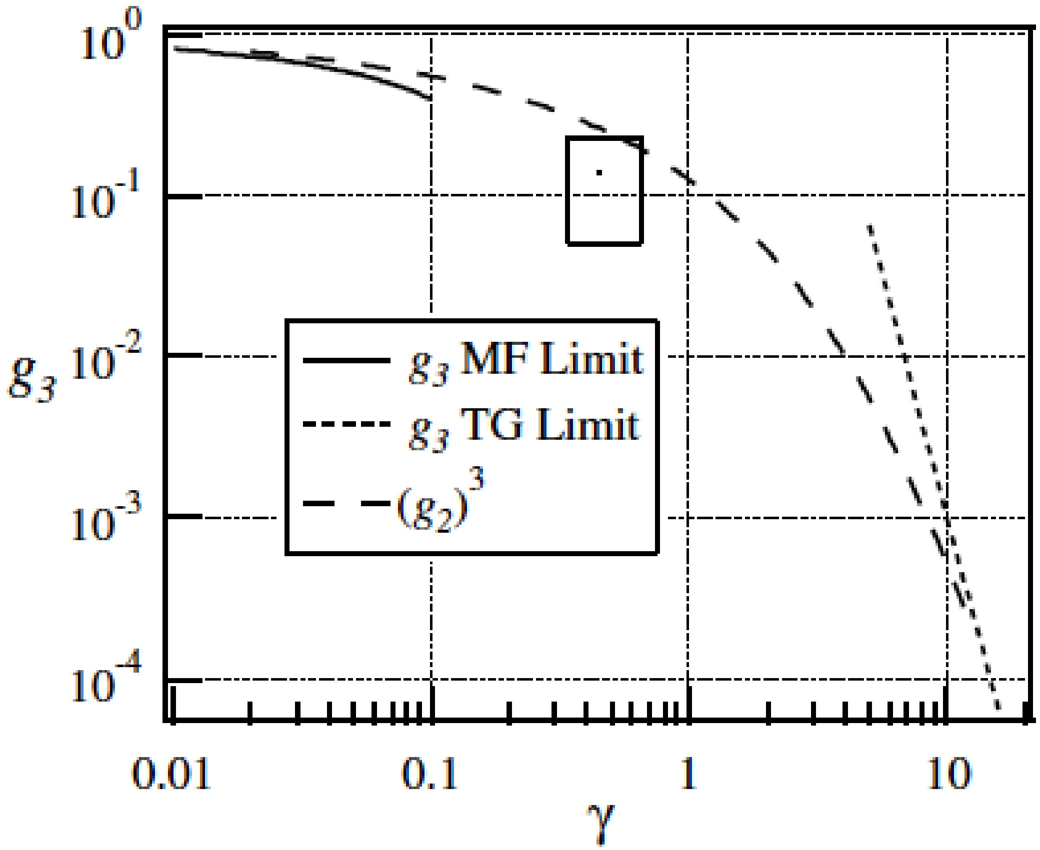
$$g_3 = \frac{\langle \sum \psi^\dagger_{p_1}\psi^\dagger_{p_2}\psi^\dagger_{p_3}\psi_{q_1}\psi_{q_2}\psi_{q_3}\rangle}{n^3} = 3! = 6$$

Different pairings of $p_i$ with $q_i$

Theory: Kagan, Svistunov, Shlyapnikov (1985)
Experiment with BEC: E.A.Burt et al (1997) : 7.4 (2.6)

# Interactions in 1D at Zero Temperature

B. Laburthe Tolra et al (2004)

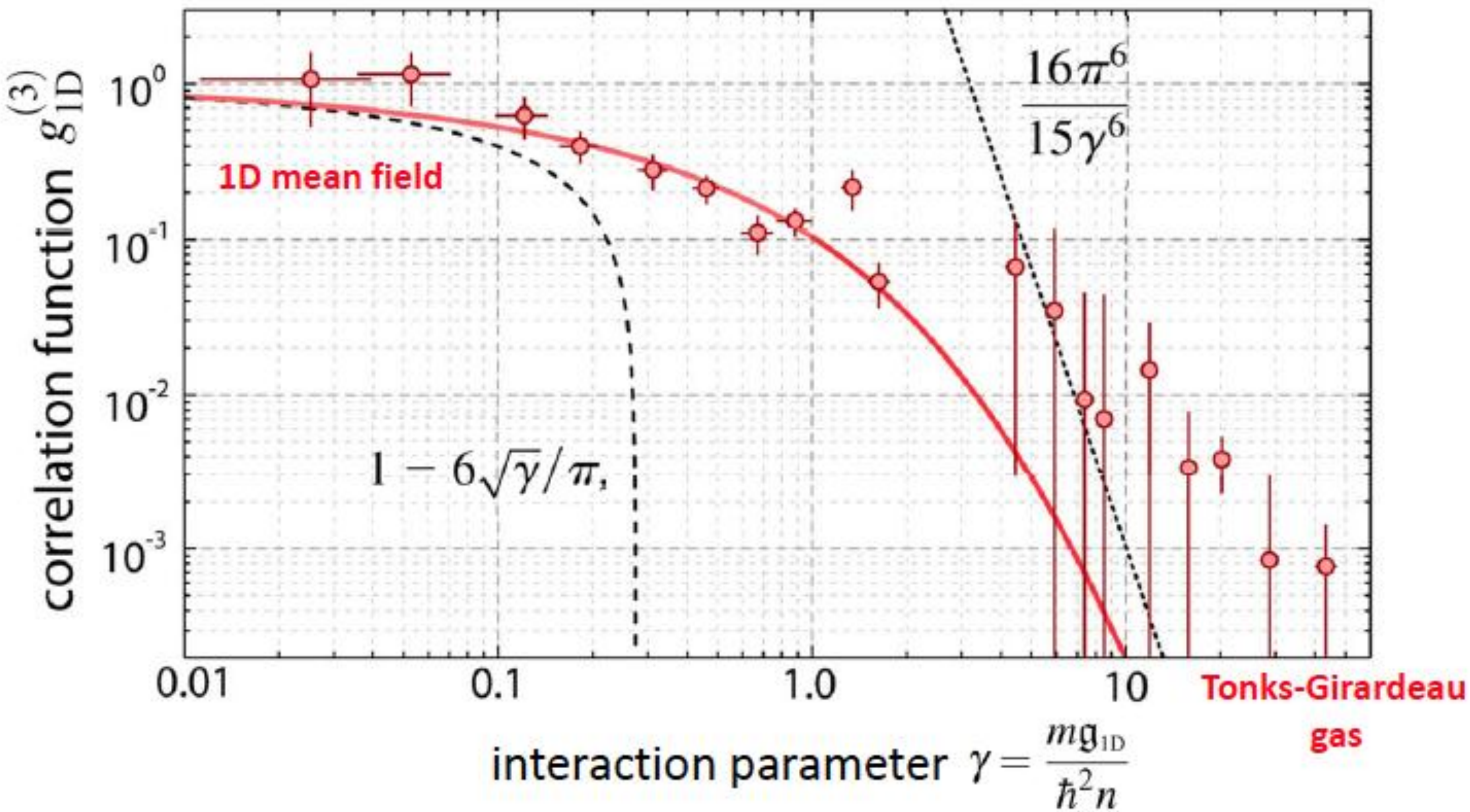



Repulsive interactions reduce the local correlations

Theory, asymptotic limits: Gangardt and Shlyapnikov (2003)

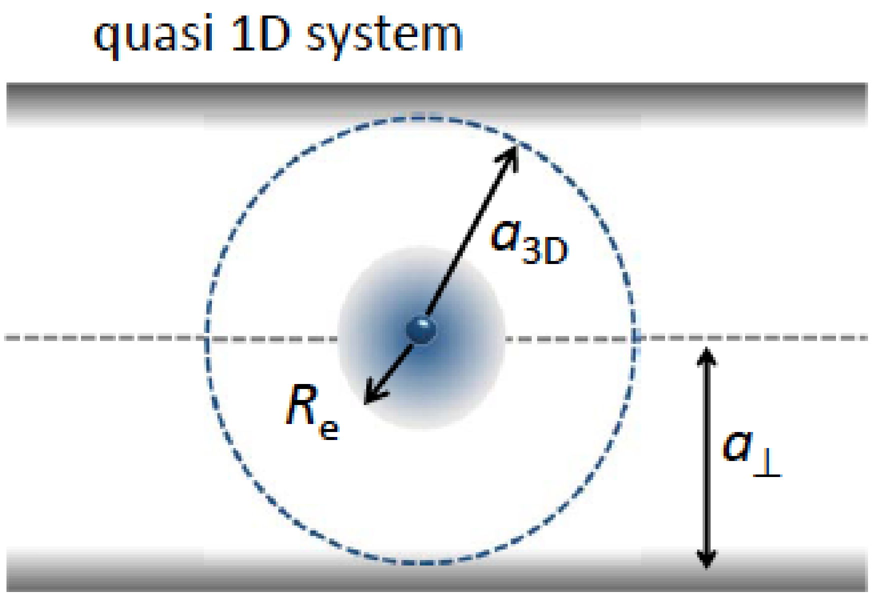
# Interactions in 1D at Zero Temperature

Very recent experiments, E. Haller et al, PRL 2011.




Exact zero T theory (red curve) : Cheianov,Smith,Zvonarev (2006)

# 1D Bose Gas: Lieb-Liniger model




Project out transverse motion for $a_{3D} \ll a_\perp;\ k_B T, \mu \ll \hbar\omega_\perp$

$$H_{LL} = \int_0^L dx\, \frac{\hbar^2}{2m} \left( \nabla\psi^\dagger(x)\nabla\psi(x) + c\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \right)$$

$$c = 2a_{3D}/a_\perp^2$$

Dimensionless interaction parameter $\gamma = c/n$

Theory: Olshanii (98); Experiment: Kinoshita, Wenger, Weiss (04)

# Beyond Lieb-Liniger: 3-p interactions

Integrate out transverse modes for $a_{3D} \ll a_\perp;\ k_B T, \mu \ll \hbar\omega_\perp$

Within second order perturbation theory:

$$H = H_{LL} + V$$

$$V = -\frac{2\hbar^2 \log 4/3}{m}\frac{a_{3D}^2}{a_\perp^2}\int_0^L dx \psi^\dagger(x)\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x)\psi(x)$$

Small integrability breaking terms!

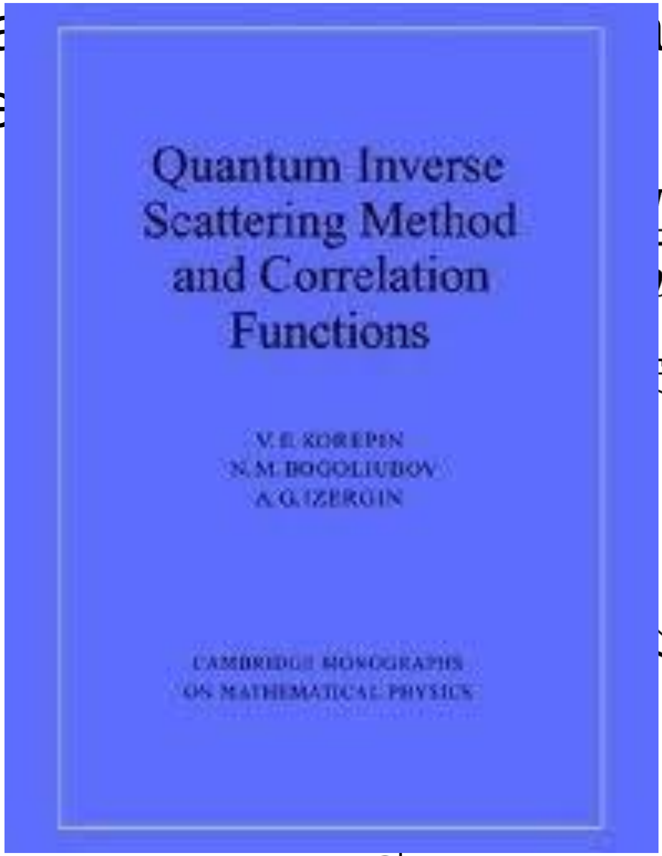
We evaluated their expectation values over exact eigenstates

3-p terms for arbitrary interactions:Mazets,Schumm,Schmiedmayer (10)
3-p terms for weakly interacting case: Muryshev et al (02)

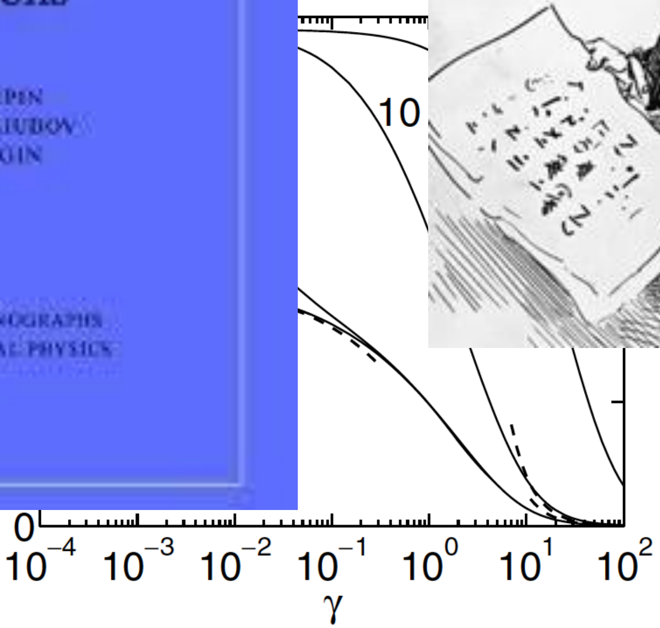
# $g_2$ in Lieb-Liniger model

Model is integrable, thermodynamics is known exactly,
but correlation functions are notoriously hard to evaluate:-(

To eva ... ates, one can use Hellman-Feynman theore ... heru



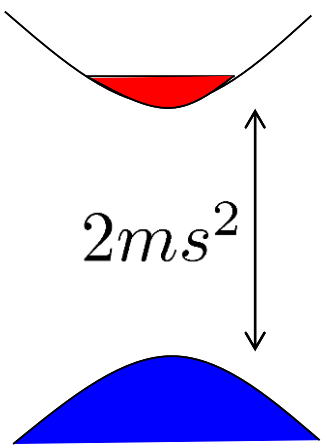




# Lieb-Liniger as a limit of sinh-Gordon

Any 1D model for spinless bosons reduces to LL at low densities (V. Gurarie, PRA 06): Rb, Cs, your favorite ultracold boson, …
Similarly: Bose-Hubbard, XXZ, q-bosons, sinh-Gordon model, …

$$L_{ShG} = \frac{1}{2}\left[\left(\frac{\partial\phi}{s\partial t}\right)^2 - (\nabla\phi)^2\right] - \frac{m^2}{g^2}\cosh(g\phi)$$




$$\phi(x,t) = \sqrt{\frac{\hbar^2}{2m}}\left[\psi(x,t)e^{-ims^2t/\hbar} + \psi^\dagger(x,t)e^{ims^2t/\hbar}\right]$$

To get LL model, expand cosh to fourth order, and take a double scaling non-relativistic limit at finite density:

$$g \to 0,\ s \to \infty,\quad gs = 4\sqrt{c}/\hbar$$

Kormos, Mussardo, Trombettoni (09,10)

# Why bother with sinh-Gordon?

$$L_{ShG} = \frac{1}{2}\left[\left(\frac{\partial\phi}{s\partial t}\right)^2 - (\nabla\phi)^2\right] - \frac{m^2}{g^2}\cosh\left(g\phi\right)$$

Also integrable, and relativistic invariance severely restricts the properties of the model, so many more things are known about it:-)

Many years of research on ShG + double scaling:

$$g_k = \sum_{l=k}^{\infty} \frac{1}{l!} \int \prod_{j=1}^{l} \frac{dq_j}{2\pi}\, \underbrace{f(q_j)}_{\text{quasimomentum occupation}}\ \gamma^k \underbrace{F_l^{(k)}(q_1,\ldots,q_l)}_{\text{diagonal formfactors}}$$

Several pages of calculations:

We resummed these series for $g_2$ and $g_3$, and obtained closed form analytical expressions

# Analytical answers for experts

$$g_2(\gamma,\tau) = \frac{2\gamma^2}{c^3}\int \frac{dp}{2\pi}\, f(p)\,\left[2\pi\rho(p)\,p^2 - h_1(p)\,p\right]$$

$$g_3(\gamma,\tau) = \frac{\gamma^3}{c^5}\int \frac{dp}{2\pi}\, f(p)\Bigg[(p^4 + c^2p^2)2\pi\rho(p) -$$

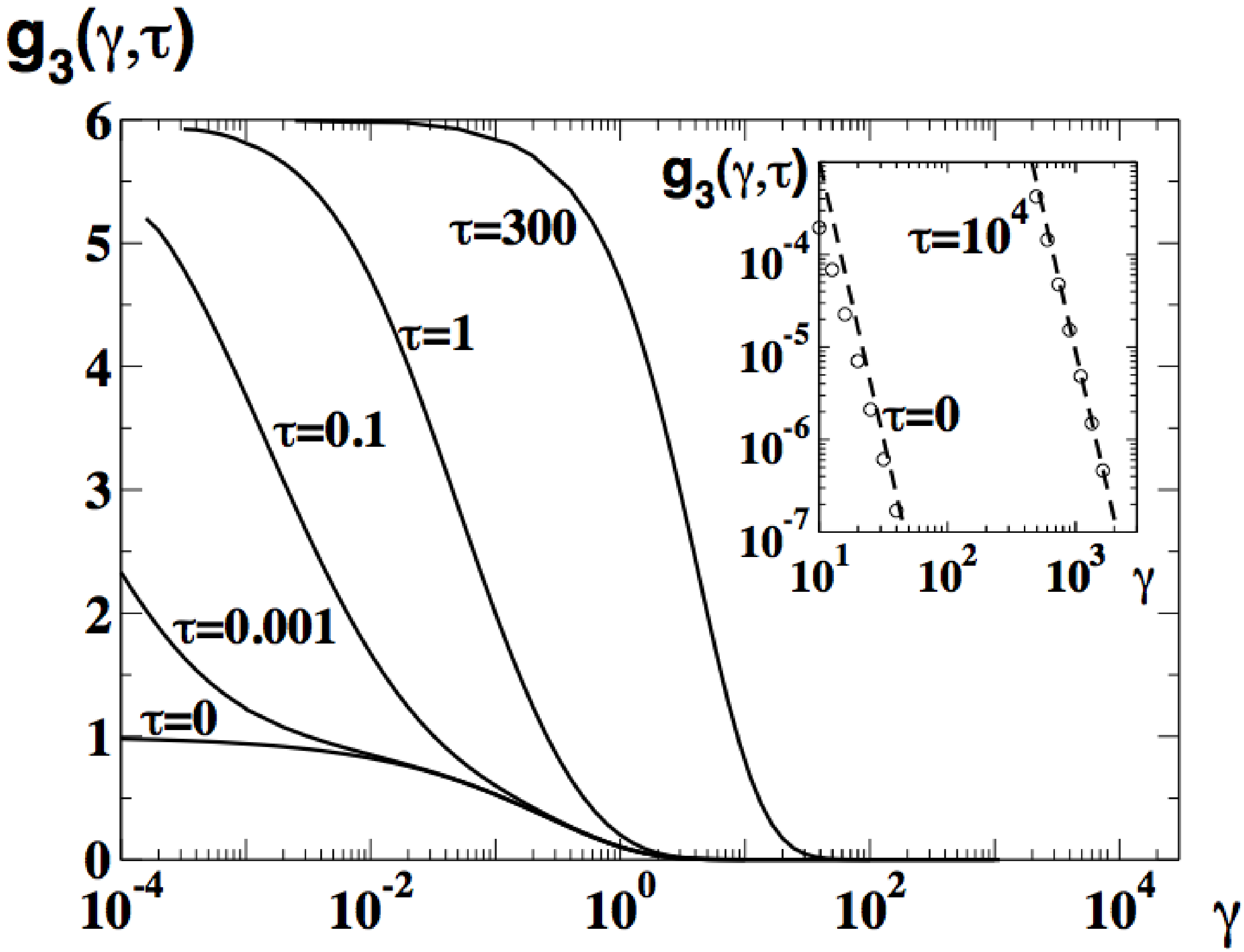
$$\left(4p^2 + (1+2/\gamma)c^2\right)p\,h_1(p) + 3p^2h_2(p)\Bigg]$$

where $$h_m(p) = p^m + \int \frac{dp'}{2\pi}\, f(p')\,\frac{2c}{c^2+(p-p')^2}\,h_m(p')$$

These equations can be simply solved/integrated numerically.
At finite temperatures, *f(p)* is obtained from Yang-Yang eqs.

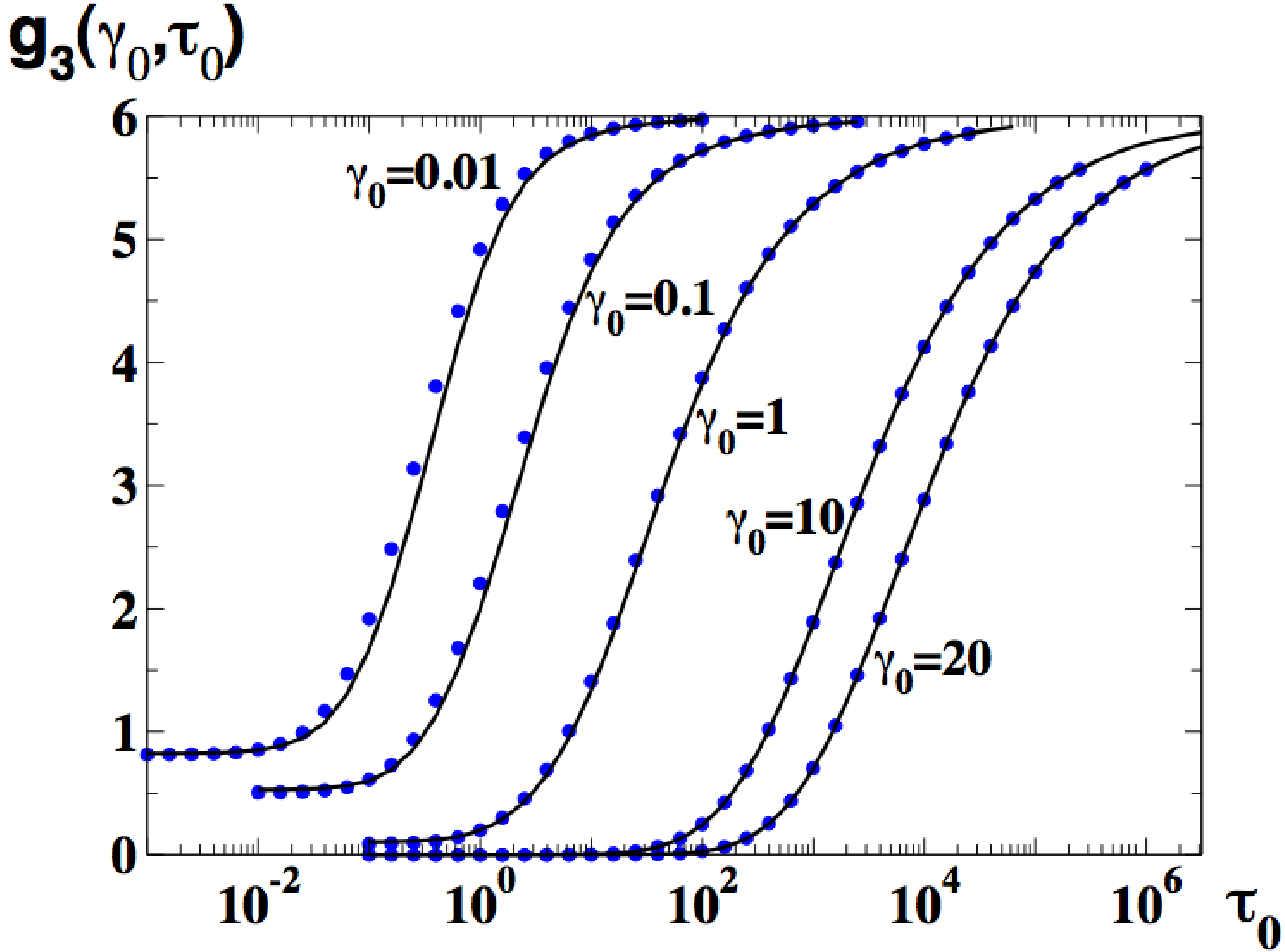
Has been also recently rederived by B.Pozsgay starting from XXZ,
And extended 4-point correlations (1108.6224), by linking $g_n$ to
“emptiness formation probability”

# $g_3$ at Finite Temperature and Interactions




$$\tau = \frac{T}{T_d}, k_B T_d = \frac{\hbar^2 n^2}{2m}$$

# $g_3$ in harmonic traps within LDA




Dots: $\overline{g_3}(\gamma_0, \tau_0) = \int dx \langle \psi^{\dagger 3}(x) \psi^3(x) \rangle / \left( \int dx\, n^3(x) \right)$
evaluated within local density approximation (LDA)

# Non-thermal states: crash course in BA

Finite size system, periodic boundary conditions

Free fermions

$$p_j L = 2\pi I_j$$

Lieb-Liniger model

$$L p_j + \sum_{k=1}^{N} \theta(p_j - p_k) = 2\pi I_j$$

Two-particle bosonic phase shift:

$$\pi - \theta(p) = \pi - 2\arctan(p/c)$$

Total energy $E = \sum p_j^2/(2m)$ defines quasienergy $\varepsilon(p)$

A set of integer $I_i$ characterizes the state; in thermodynamic limit, if the value $I \propto N$ is picked with probability $n(2\pi I/L)$, then

*0<n(p)<1* is an occupation number

$$< \psi_F^\dagger(x)\psi_F(0) >= \int \frac{dp}{2\pi} n(p) e^{ipx}$$

For thermal states,

$$n(p) = \frac{1}{\exp[p^2/(2m) - \mu]/(k_B T) + 1}$$

Need to recalculate $n(2\pi I/L) \to f(p)$

For thermal states,

$$f(p) = \frac{1}{\exp[\varepsilon(p) - \mu]/(k_B T) + 1}$$

Within Generalized Gibbs Ensemble:

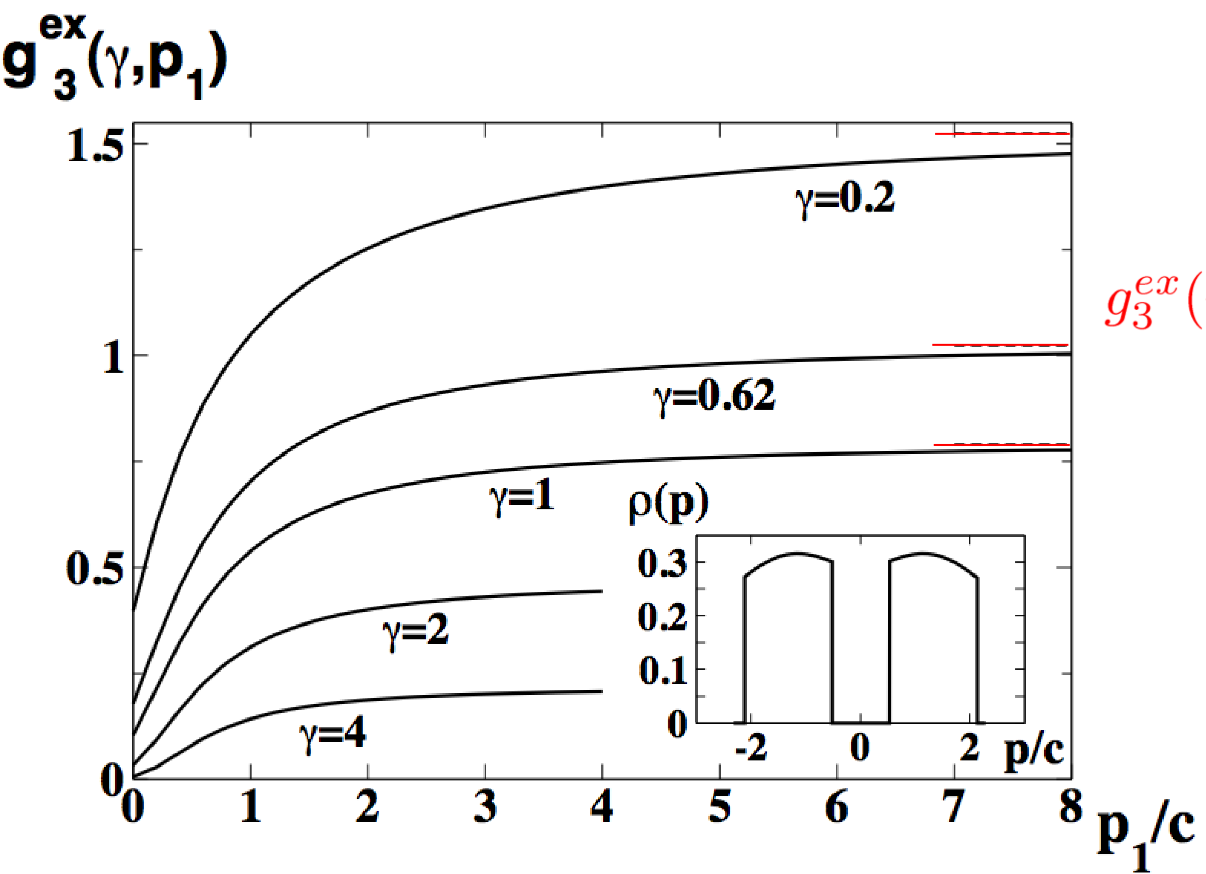
$$f(p) = \frac{1}{\exp\left[\sum \beta_j \varepsilon_j(p)\right] + 1}$$

Mossel&Caux, 1203.1305; M Kormos et al, 1204.3889

# Caricature state

Motivated by “quantum Newton’s cradle” experiment of Kinoshita, Wenger, Weiss (06)

$$f(p) = \theta(p^2 - p_1^2) - \theta(p^2 - p_2^2)$$




$$g_3^{ex}(\gamma, \infty) = [g_3(2\gamma) + 9 g_2(2\gamma)]/4$$

# Conclusions and Outlook

Exact results for local three-point correlations of 1D Bose gas at arbitrary interactions and temperatures, valid also for arbitrary non-thermal excited states.

Outlook:
Non-thermal states (after quenches?) described using arbitrary *f(p)*

Non-integrability $V \propto \int_0^L dx \psi^\dagger(x)\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x)\psi(x)$ as a small perturbation to build a non-perturbative kinetic theory in the basis of exact eigenstates, since matrix elements $\langle\alpha\,|V|\,\beta\rangle$ are also exactly known using XXZ to Lieb-Liniger mapping.