Investigating the Properties and Existence of a Macroscopic Fundamental Diagram for Arterial and Freeway Traffic Systems

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Master of Science

September, 2009

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ABSTRACT

A field experiment in Yokohama (Japan) revealed that a macroscopic fundamental diagram (MFD) linking space-mean flow, density and speed exists on a large urban area. Despite these and other recent findings for the existence of well defined MFDs for urban areas, these MFDs should not be universally expected. In this thesis, the properties of a well defined MFD are investigated and it is found out that the spatial distribution of density/occupancy in the network is one of the key components that affect the scatter of an MFD and its shape. An analytical derivation of the spatial distribution that considers correlation between adjacent links has also been proposed. The scatter of an MFD in terms of errors in probability density function of spatial occupancy and errors of individual links' fundamental diagram (FD) have also been investigated and an analytical estimation of the errors is provided. Using real data from detectors for an urban arterial and a freeway network we validate the proposed derivations and we show that an MFD is not well defined in freeway networks as hysteresis effects are present. The hysteresis phenomenon in network is investigated at different levels of the network and it is found out that the heterogeneity of the network and the hysteresis in the individual detectors should account for this phenomenon. The datasets in this paper consist of flow and occupancy measures from 500 fixed sensors in the Yokohama downtown area and 600 loop detectors in the Twin Cities Metropolitan Area Freeway network (Minnesota, USA).

Acknowledgements

I would like to first thank my advisor Nikolas Geroliminis, who made this thesis possible. His willingness to teach and help made my first year study in a new country easy and enjoyable. His guidance greatly increased my altitude not only from academical aspects but also from many other aspects.

I would also acknowledge the professors here in Minnesota, who taught me a lot during the time I was pursing my master degree. David Levinson extended my knowledge to broad areas of transportation and helped to learn ways of looking at problems. Gary Davis taught me a lot of advanced mathematics that is useful in the research. Henry Liu gave some insightful suggestions about the thesis, which helped to improve the quality of thesis. My study here also allowed me to have the opportunity to work with many amazing students. I have had some usefull discussions about my research with Anupam Srivastava, Hui Xiong and Indrajit Chatterjee, who provided some useful opinions. And my gratitude should also be extended to other students like Feili Hong, Carlos J Carrión and Heng Hu, who helped me a lot during the study. I would also thank my roommates who made it possible for me to just concentrate on my thesis.

I should especially thank my wife whose love kept on inspiring me during the last 6 years and who carefully helped me to improve the writing of thesis. I am grateful to my parents who make all of these possible and always encourage me to pursue my goals.

Dedication

To those who held me up over the years.

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Chapter 1

Introduction

1.1 Motivation

This thesis is to search for the properties of the macroscopic modeling of traffic flow under congested situation. The understanding of these properties should be the first step to macroscopic modeling-based alternative, such as pricing and control to mitigate congestion in urban areas and improve mobility in these areas.

Traffic congestion turns out to be a major problem for cities around the world as numerous vehicles run into the road networks and this problem becomes severer as time goes by. Let's take U.S as an example. The total travel delay experienced a 52.94% increase (from 2.72 billion hours to 4.16 billion hours) in 10 years (from 1997 to 2007). The 4.16 billion hours travel delay results in 2.81 billion gallons fuel waste and $87.2 billion congestion cost. During the same period, the public road mileage increased from 3.96 million miles to 4.05 million miles, only a 2% increase. At the mean while, the actual Vehicle Miles Traveled (VMT) increased by 18.73%. [1]

Although increasing infrastructures is an intuitive solution to traffic congestion, from the figures above, it is very obvious that new infrastructures can hardly catch up with the increasing rate of demand. Thus, a feasible solution to improve the mobility in urban area should turn to the utilization of existing infrastructures. Policies and strategies that aim to make full use of existing traffic infrastructures always require a model to describe the traffic. Current traffic model can be generally divided into two categories: microscopic and macroscopic.[2]

Microscopic models try to describe the behavior of individual vehicles and hope the characteristics resulting from the interactive between individual can replicate those in the real world. In these models the individual vehicle's behavior is usually a function of that of the leading vehicle.

Macroscopic models directly search for the relationship between some traffic variables such as traffic flow, traffic speed and density. Empirical studies have been made to search for the fundamental relationship "q=kv" (flow (q) equals density (k) multiplied by speed (v)), which can be illustrated by fundamental diagram [3][4]. And also theories like LWR [5][6] have been proposed to describe the traffic fundamental relationship.

Microscopic models for urban traffic have been developed for more than 50 years. As they become more sophisticated, the complexity and requirement for detailed data input also increases. Theoretically, with the help of modern computer and enough detailed data with enough accuracy, the most recent model could provide us satisfactory prediction. In practice, however, those models are quite fragile.

The first problem encountered by those models is the Origin-Destination (O-D) information's availability. The entries of O-D table with reasonable resolution for large cities could easily exceed the number of people in the city. And it is always a problem for traffic engineers to obtain a dynamic O-D tables which are required by dynamic traffic assignment models.

The second problem comes from the congested network, which we are most interested in. It has been found out that the output of congested networks are hyper-sensitive to the inputs.[7] This means that our prediction would experience dramatic inaccuracy even if there is only a small error in our inputs.

Furthermore, the movement of vehicles results from the navigation of drivers, which is unpredictable. This has been pointed out by Addison and Heydecker [8]. A "rationa" driver, which is always an assumption in many models, would evaluation the cost of several paths which he or she can take. This evaluation requires the decision of other "rational" drivers who are doing the same evaluation and require the same information. This leads us to a situation similar to "poker", which is unpredictable in nature[9].

It seems to us that the forecasting-based and detailed model for traffic system would encounter a lot of trouble when we apply them. An alternative is an observable and aggregated approach. There are similar aggregated approaches in physics that describe

a system by the average variables (such as pressure and temperature in Ideal Gas Law). Quite similar to our traffic system, the behavior of individual particles or molecules is hard to describe or predicate. The collective characteristics, however, are observable and relatively well-behaved. In traffic area, some works following this approach have been done, and in this thesis, it is hoped some contributions could be made to further enhance this approach.

1.2 Literature Review

This section is written to examine the past effort on the macroscopic modeling for traffic system. General approaches in macroscopic modeling will be illustrated and what are needed to be done will be pointed out in the end. Works following macroscopic modeling started from empirical studies. Data were got from field experiment and observations. Variables such as traffic flow, average traffic speed and traffic density that could characterize traffic systems were defined and the relationships between them were studied. Through analysis of data, it was found that maximum input flow of the central area of a city and the area of the city, the fraction devoted to roads and the capacity of the roads, which is expressed in vehicles per unit time per unit width of road[10]. Later Godfrey[11] brought up the idea of a "macroscopic fundamental diagram" (MFD) with optimum accumulation but didnt verify its existence due to the lack of sufficient data at that time.

A lot of previous work has been done to look for MFD patterns. From data in central London collected for many years, a linear-decreasing relationship between average speed and flow[12]. A similar relationship has also been found by Wardrop[13] and he further suggested that this relationship should depend on average street width and average intersection spacing.

Similarly, data from several cities in both United Kingdom and United States also showed a monotonically decreasing relationship between average speed and average flow. We should, however, notice that the data of these works all came from uncongested network and we cannot know relationship between variables when the traffic approaches very congested states, i.e. gridlock.

A useful theory that could help us to describe the rush hour in congested traffic
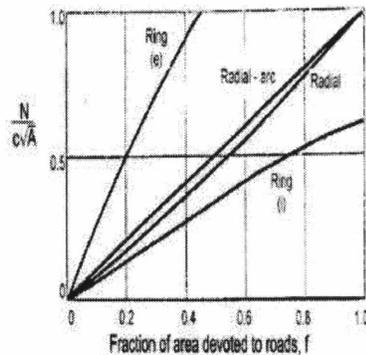
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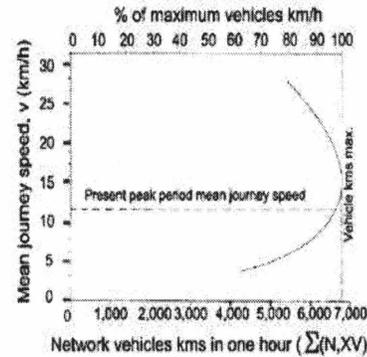
Figure 1.2: Relationship Between Average (Journey) Speed of Vehicles and Total Vehicle Mileage on Network(Godfrey 1969, Figure 2).

situation is two-fluid model[14]. It was then found that the average speed in an urban area is a function of the fraction of vehicles that are stopped at any given time. That is:

$$v = u_0(1 - f_s) = u_0 f_r \tag{1.1}$$

where $f_r = (1 - f_s)$ is the fraction of moving cars and u_0 is a parameter that depends on "desired speed" distribution function.

Extended from their hinetic theory of vehicular traffic[15], it was conjectured that:

$$v_r = v_m(1 - f_s)^n \tag{1.2}$$

where v_r is the average speed of the moving cars and v_m is the average maximum running speed. This model is in analogy with the Bose-Einstein condensation, in which molecular distribution function is divided into two parts at sufficiently low temperatures. For traffic problem, vehicles in the network (exclude parked ones) are split into two categories: moving vehicles and stopped ones.

If the concentration varies slowly over time and the fraction of the time stopped by a vehicle in the network is the same as the average fraction of stopped vehicles in the system over a same sufficient long period, the following relationship can be obtained:

$$T_r = T_m^{\frac{1}{n+1}} \cdot T^{\frac{n}{n+1}} \tag{1.3}$$
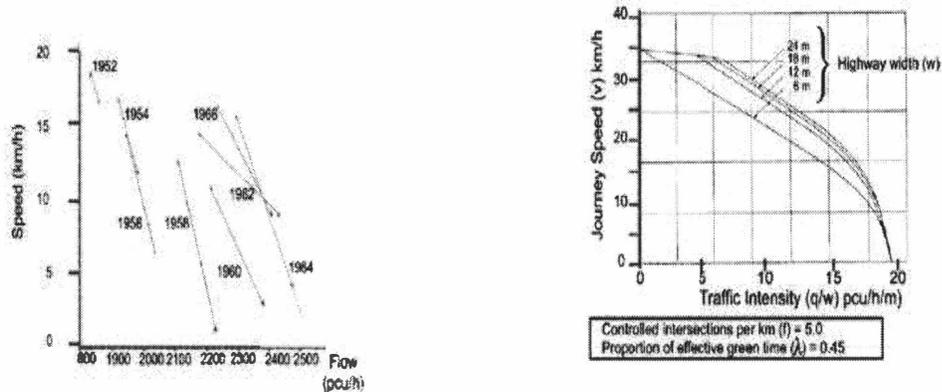
Figure 1.3: Speeds and Flows in Central London, 1952-1966, Peak and Off-Peak (Thomson 1967, Figure 11)

Figure 1.4: Effect of Roadway Width on Relation Between Average (Journey) Speed and Flow in Typical Case (Wardrop 1968, Figure 5).

Where T_r is the running time per unit distance and T_m is the average minimum trip time per unit distance.

This theory was tested later[16] and further extended to that the fraction of stopped vehicles was found to be a power function of the density. This idea allows a more realistic representation of crowded conditions in the steady state, but still insufficient for a macroscopic model describing rush hours dynamically.
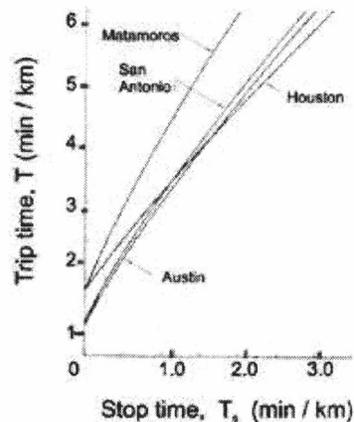
Figure 1.5: Trip Time vs. Stop Time Two-Fluid Model Trends (Herman and Ardekani 1984, Figure 6).

For an extensive literature review of macroscopic traffic models, the reader can refer to chapter 6 of Traffic Flow Theory Monograph written by James C. Willams[17].

More recently, Daganzo[9] proposed a framework which conjectured that if the congestion is evenly distributed among the network, the travel production P (i.e. the number of vehicle-miles traveled per unit time) can be expressed as a function of total accumulation of the network, n, when the system demand evolves slowly over time.

$$\sum P_i(n_i) \approx P \sum n_i \equiv P(n) \tag{1.4}$$

where P_i is the travel production for the individual link i with accumulation n_i. Unlike those previous models which can only describe monotonic curves between travel production and accumulation, the new proposed model has three different regimes, which are uncongested when there are few vehicles in the network, capacity when n in the range where maximum P is produced and congested when n is very large.

It was showed in the same reference that the rate at which vehicles leave a network (the trip completion rate) to the number of vehicles in the network (the accumulation) if the above conjecture is true. Based on the proposed model, it was showed that gridlock (i.e. severe congestion) can be prevented by controlling policies that require no demand data. The general idea of this schema is that the system functions in such a way that we need apply some control policies to avoid it running into certain regime that benefits nobody and cannot recover by itself.

The existence an MFD was found in a simulation study of rush hour in downtown San Francisco, even with very different demand distributions both in time and space[18]. The verification for the existence of MFD in congested neighborhood in real world has also been done using the data of detectors and GPS-equipped taxi in Yokohama, Japan recently[19]. Furthermore, a fixed relationship between the space-mean flows over the whole network and the trip completion rates has been also found in this literature.

With the micro-simulation of the San Francisco Business district and a field experiment in downtown Yokohama (Japan), these papers[18][19] showed:

- that urban neighborhoods approximately exhibit "Macroscopic Fundamental Diagram" (MFD) relating the number of vehicles (accumulation) in the neighborhood to the neighborhood's average speed (or flow)

- there is a robust linear relation between the neighborhood's average flow and its total outflow (rate vehicles reach their destinations)

Fixed detectors and floating vehicle probes are combined as sensors in the experiment in Yokohama, some of whose findings are shown in Figure 1.6 (time resolution is 5min).
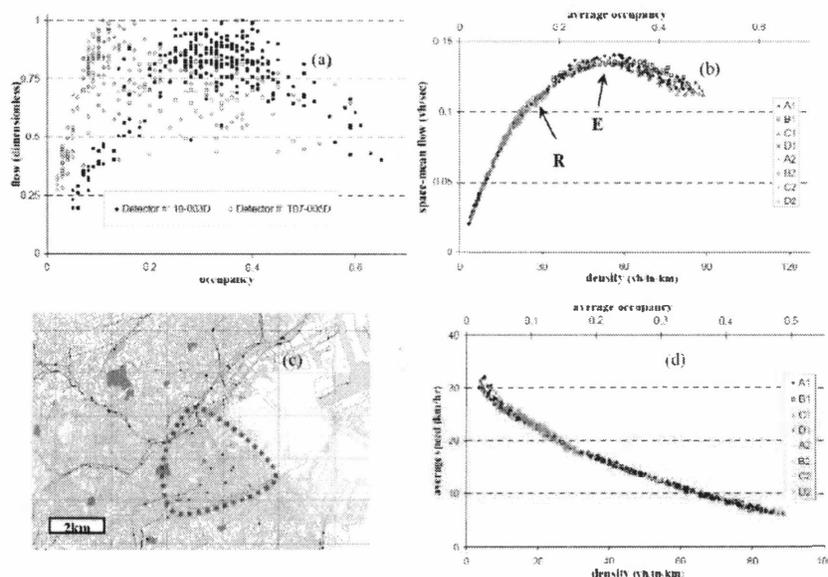
Figure 1.6: Loop detector data in Yokohama(Geroliminis and Daganzo, 2008a)

It was observed that when the somewhat chaotic scatter-plots of speed vs. density from individual fixed detectors were aggregated for a 10 km^2 region, the scatter nearly disappeared and points grouped neatly along a smoothly declining curve (compare fig. 1.6a with 1.6b and 1.6d).

The same references also showed that:

1. the MFD is a property of the network itself (infrastructure and control) but not of the demand, i.e. the MFD should have a well-defined maximum and remain invariant when the demand changes both with the time-of-day and across days (it may vary if the O-D pattern of demand changes significantly, though, e.g. due to an event or evacuation)

2. the space-mean flow, is maximum for the same value of density of vehicles or average speed, independent of the origin-destination tables

3. the average trip length for the study region is about constant with time, i.e. the total outflow vs. density curve is a scaled up version of the curve in fig. 1b

4. the MFD can be estimated accurately using existing monitoring technologies (e.g. detector data, GPS etc)

Later, Geroliminis and Levinson[20] applied the idea of an MFD to develop cordon pricing schemes for different states of the MFD as shown in fig. 1.6b State E is an equitable state (more users on the network) while state R is a more reliable state (speed is higher and less users in the network).

Despite these recent findings for the existence of well defined MFDs for urban areas, these MFDs should not be universally expected. To this end we define as Aggregated Traffic Relationship (ATR), a relationship between average traffic variables (like speed, density, occupancy or flow) for a specific network that not necessarily have a well defined shape to be called an MFD. In particular, networks with an uneven and inconsistent distribution of congestion may exhibit traffic states that are well below the upper bound of an ATR and much too scattered to line along an MFD. An inconsistent distribution of congestion is typical of freeway networks with multiple recurrent and non-recurrent bottlenecks, and of large urban areas with multiple congested sub-centers. Even networks that satisfy some of the regularity conditions pointed above may exhibit significant scatter on their MFDs because of a rapidly changing demand, e.g. during an emergency evacuation an MFD may not exist for an urban network which exhibits an MFD under normal conditions.

The effect of heterogeneity in ATRs or MFDs has been experimentally examined to some aspects with real data from a medium-size French city by Buisson and Ladier[21].The authors examined differences between the arterial and freeway network, impact of the distance between the loop detector and the traffic signal within the urban network, differences between normal and non-recurring conditions (during a strike of truck drivers). They showed that heterogeneity has a strong impact on the shape/scatter of an MFD, that may turn not to be an MFD in some cases, e.g. for a freeway network. In this thesis we investigate what are the properties that a network should satisfy, so that an MFD with low scatter exists. This can provide useful tools for developing macroscopic traffic management schemes.

Currently, there are several real examples around the world applying control policies that is similar to the strategy described above. In Zurich, a traffic light operating system was designed to control the traffic demand towards the central area based on the information from bus system. The general idea of this system is that if the average bus speed falls below a certain level, the traffic light is operated in such a way that the traffic flow towards the central area is restricted. In London, a congestion pricing scheme was introduced in 2003. The cordon covers an area of 22 km^2 in central London and the aim is to reduce the congestion in this area. Thus, the traffic system performance can be improved. The results of the schema are significant. The congestion in the charging zone has been reduced by about 26% during charging hour sand the reliability of buses has been largely improved[2].

1.3   Thesis Overview

Despite the theoretical progress in new macroscopic modeling in traffic system and some successful applications in the real world in agreement of its idea, more investigations into such models are still needed. One would wonder whether the homogeneity assumption (evenly distributed congestion) necessary for the existence of an MFD and what the congestion distribution looks like in a network that has a well-defined MFD. Furthermore, an appealing question would be whether there is a well-defined MFD in any large traffic network? According to our research, this is not always the case. There indeed is a well-defined MFD in arterial network in downtown Yokohama, Japan. For freeway network in Twin Cities, U.S., however, there appears hysteresis in the ART.

In first half part of thesis, the homogeneity assumption is first tested using the real data from downtown Yokohama, Japan, where gives us a well-defined MFD. Then a theoretical description for the occupancy probability density function (pdf) is proposed. Following this, the property of errors in the occupancy pdf is analyzed to provide a better understanding for the well-defined MFD.

In the second half part of the thesis, the data from freeway network in Twin Cities, U.S. are investigated in the same manner as that for the data from downtown Yokohama. The hysteresis phenomenon in ART is revealed and an initial investigation for it is made. Some possible explanations for the hysteresis phenomenon is provided based the

investigation above.

The thesis is organized as follows:

- Chapter 2 is a description of the study sites. The detailed information for the two study sites (i.e. arterial network in downtown Yokohama, Japan and freeway network in Twin Cities, U.S.) is provided.

- Chapter 3 mainly focuses on arterial network of downtown Yokohama. Homogeneity assumption is first examined. Then an analytical model for occupancy probability density function (pdf) is proposed. Finally, an analytical approach to estimate the errors in the occupancy pdf is proposed accompany with its application to arterial network.

- Chapter 4 deals with the data from freeway network of Twin Cities. Hysteresis phenomenon is observed and then analyzed in different scale of networks.

- Chapter 5 investigates the properties of MFD through comparison between arterial network and freeway network. Several possible factors for the hysteresis phenomenon are analyzed.

- Chapter 6 presents some conclusions and possible future works are suggested in the end.

Chapter 2

Study Sites Description

The test data in this work come from two different networks: Downtown Yokohama in Japan (arterial network) and Twin Cities in the U.S (freeway network).

2.1 Yokohama Arterial Network

Figure 2.1: Google map of downtown Yokohama

Yokohama is a fast developing Japanese prominent port city with a population of 3.6 million and a major commercial hub of the Greater Tokyo Area. Its road network includes streets of various types, with closely spaced signalized intersections ($\sim$100-300 m) at its center, and a few elevated freeways. Streets have 2 to 4 lanes in each direction.

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The speed limit is 50 km/hr on arterials. Major intersections are centrally controlled by multiphase traffic signals, with a cycle time that responds to traffic conditions: 110-120 sec long at night and 130-140 sec during the day. The part of downtown Yokohama examined in this paper is approximately a 10 km^2 triangle with corners at Yokohama Station, Motomachi-Chukagai Station and the Shin-Hodogaya Interchange. Yokohamas center is congested during peak hours with speeds considerably less than 50% of the average speed during the off-peak; average speeds less than 10 km/hr are observed for extended periods during morning and evening peak in weekdays in the arterial network. Figure2.2 shows a sketch of Downtown Yokohama (the perimeter of the study site is shown as a dashed line). Our data come from the fixed sensors distributed among the study site. There are 500 ultrasonic and loop detectors positioned midblock on arterial lanes of most major intersections in the study area and the detectors are located about 120 m upstream of the stop line in case of long links and about 50m in case of short links. They provided 5-min vehicle counts and occupancy measurements. For a more detailed description of the study site the reader could refer to Geroliminis and Daganzo[22].
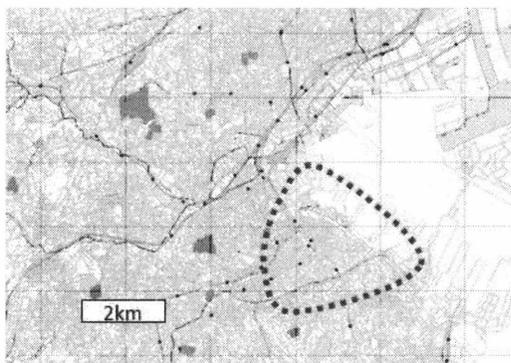
Figure 2.2: Study area of downtown Yokohama

2.2   Twin Cities Metropolitan Area Freeway Network

Traffic data on freeway system of the Twin Cities metro area are collected by the Minnesota Department of Transportation (Mn/DOT) from freeway loop detectors. An exhaustive system of over 4000 inductive loop detectors is maintained by Mn/DOT's Regional Transportation Management Center (RTMC). On the mainline, loop detectors
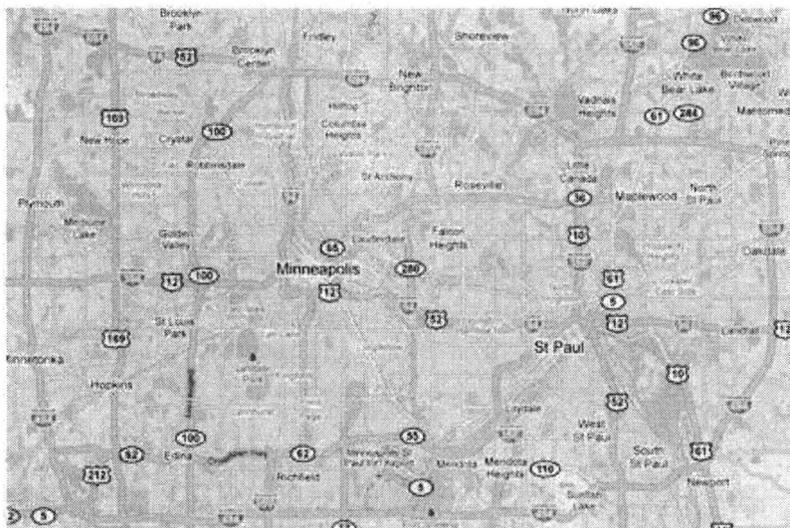
Figure 2.3: Google map of Twin Cities

are placed approximately every kilometer and there are also detectors on every entrance and exit ramps. Every 30 seconds the volume and occupancy data collected from the loop detectors is transferred to the RTMC over fiber optic lines. To be consistent with our data from arterial network, we use the data of every 5 minutes interval.

This paper focuses on the freeways that are in the central part of the Twin Cities metropolitan area. The detectors are chosen from a network that consists of several major freeways constrained within I-494 and I-694 ring. Those include I-94, I-394, TH-169, TH-100, and part of I-35W and I-35E that are inside the ring.
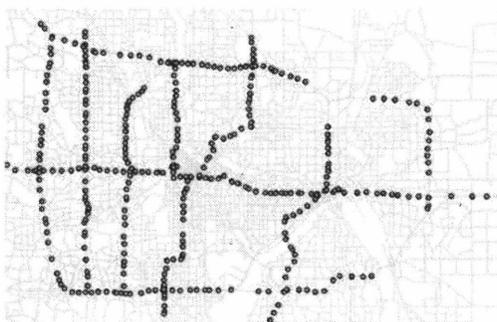
Figure 2.4: Twin Cities freeways of interest

Unlike arterial network, there is no signal system on freeway networks and speed limit is 65 mph (i.e. about 104 km/h) on most part of the freeways. During the peak hour, the average occupancy over the chosen network will reach about 20%. Figure 2.4 is the study area in Twin Cities and the detectors we are using. The area within the ring we choose is about 279 mi^2, or 744 km^2, which is much bigger than our previous site. This difference results from the differences between the natures of arterials and that of freeways.

Chapter 3

Properties of a Well-defined MFD for Arterial

A macroscopic fundamental diagram (MFD) linking space-mean flow, density and speed has been found existing in Yokohama, Japan through a field experiment[19]. The relation between space-mean flows on the whole network and the trip completion rates was also explored there, but not the properties of the MFD. In this chapter, we first examine the occupancy distributions using several groups of time intervals of the same length and of similar average occupancies within group. It is found out that the distributions within group are quite similar. These distributions, however, are not binomial. Then an analytical model for probability density function (pdf) for the distributions is proposed and found quite satisfactory. In the end of this chapter, we investigate the scatter of the MFD in terms of errors in pdf and errors of individual fundamental diagram (FD).

3.1 Occupancy distributions for a well-defined MFD

3.1.1 Data description

The new framework of MFD was proposed under the assumption that congestion is evenly distributed, i.e. the homogeneity assumption of MFD. Without testing the assumption, a well-defined MFD was found through the experimental facts from downtown Yokohama, Japan. The result in this section reveals that a well-defined MFD can exist

without satisfying the homogeneity assumption.

The data of a typical workday (12/14/2001) is chosen, which is consistent with previous study on Yokohama[19]. Our raw data is in form of volume and occupancy which come directly from loop detectors. Some obvious errors (e.g. data with occupancy over 100 or stay 0 for all the time) are eliminated before use of the data.
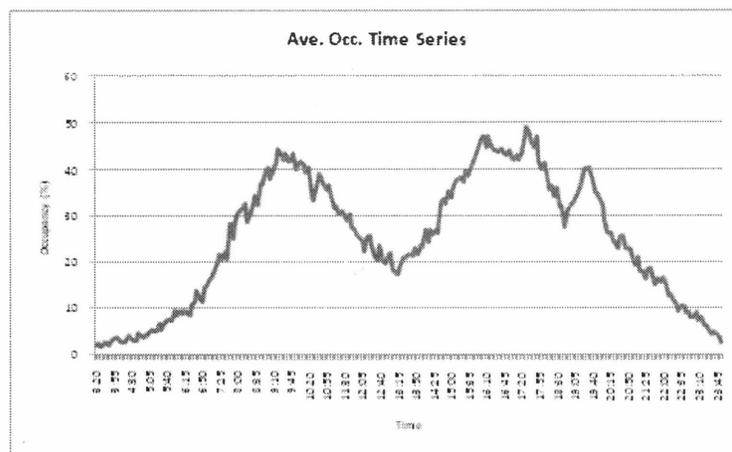
Figure 3.1: Time series of average occupancy

We can see from the time series plot of average occupancy for the network that the network is almost empty at the beginning of the day with average occupancy 0. There are two peak time during the day: morning peak and evening peak, when the average occupancy would reach as high as almost 50%.

Lets index the road lane section between two intersections in our study area by i, and let q_i and o_i donate the flow and occupancy measured by the detector during a time slice on this specific road lane section, link i. It is know that the density k at detector location can be given by o/s, where o is the occupancy and s is the effective vehicle length, which can be considered as a property of a specific network. What we are interested here is the relation between density and flow (k and q). If s is fixed for the network and we take the same time slice over the network, we can directly make use of our data ($volume/5\ min$ and $occupancy$) from the detectors.

To be representative, we choose average occupancy 15%, 25%, 35% and 45%, which are ranging from uncongested part to the most congested part of the network states,

to investigate the occupancy distribution. For each average occupancy level, we collect 5 time intervals whose differences in average occupancies are less than 1% (except for average 15%, where the maximum difference is 1.38%). The histograms for each average occupancy level are shown in figure 3.2.

(a)

(b)

(c)

(d)
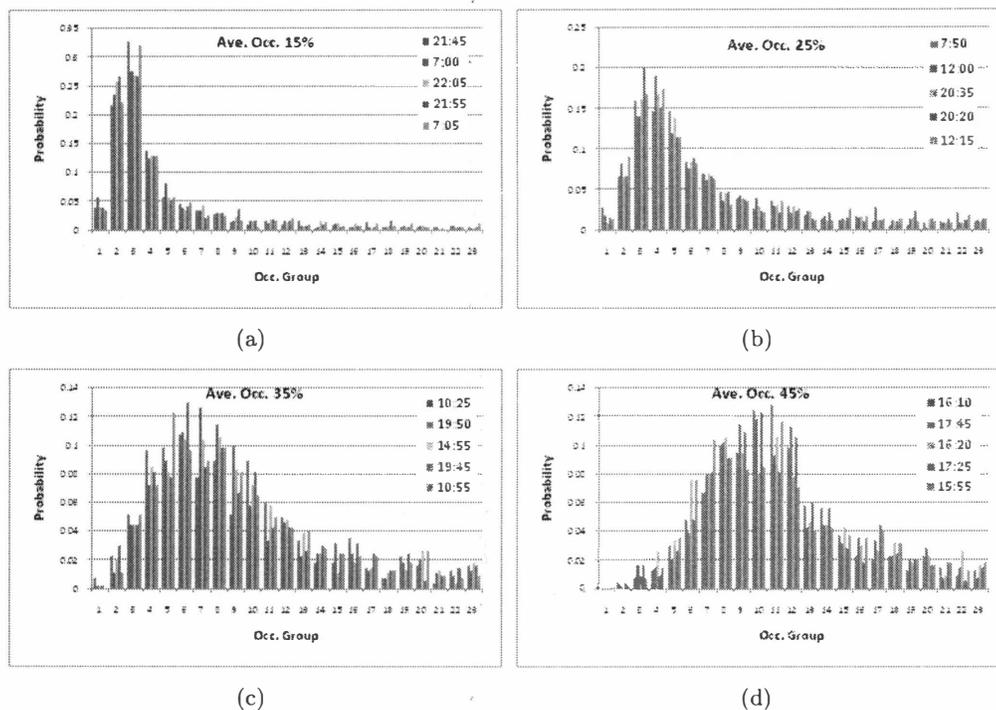
Figure 3.2: Histograms for different average occupancies

It is very clear from these graphs that the congestion is not evenly distributed. Some parts of the network are more congested than others. As more parts of the network become congested, the average occupancy level across the network increases. An interesting thing we notice from the graphs is that the distributions of individual occupancy with similar average occupancy are similar even if they come from different time intervals. We conjecture that the congestion distribution in a network with well-defined MFD would be the same when the average occupancy of the network is the same.

3.1.2 Statistical tools

Statistical tools are needed for our investigation into the property of the individual detector occupancy distributions. The two major statistical tests we are going to use in this thesis are Chi-square test and Mann-Whitney U test. A brief description of these two statistical tests will be presented below.

The kind of Chi-square test we are using in this thesis is called Pearsons chi-square test, which is first investigated by Karl Pearson. It is used to test whether the sample distribution of a categorical variable is different from a population with a hypothesized distribution or not and the frequency distribution of the events observed in a sample is approximately chi-square distribution when null hypothesis is true[23].

The χ^2 statistic can be obtained as:

$$\chi^2 = \sum \frac{(observed - expected)^2}{expected}$$

If there are r categories and c different samples, the theoretical distribution is approximate chi-square distribution with $(r - 1) \times (c - 1)$ degrees of freedom under null hypothesis. And the P-value is computed from the chi-square distribution.

Mann-Whitney U test is a non-parameter test used to see whether two samples are coming from the same distributions or not. The null hypothesis is that the two samples are from a single population and have equal probability distributions. This test is the same as performing a two-sample t test after ranking over the samples.

Samples are first ranked and a statistic U is calculated based the rank of observations from samples in the series. For large samples, U approximately follows normal distribution, thus, Z score can be calculated to obtain P-value from normal distribution.

3.1.3 Results

Statistical tools like Chi-square test and Mann-Whitney U test as we mentioned above can be used to evaluate our conjecture.

In Mann-Whitney U test, two samples can be compared at one time. The null hypothesis of the test is that the two samples are drawn from a single population. For each average occupancy level, we choose two different time intervals whose difference in average occupancy is mild (neither the biggest nor the smallest in the group). The results are shown in table 3.1.

Table 3.1: Mann-Whitney U test result for occ. distributions

Test	Time	Ave. Occ.	P-value	Conclusion
1	17:45	44.92963	0.9673	
	17:25	44.97963		
2	19:50	35.21296	0.9678	
	19:45	35.60556		not significantly different at =0.05
3	12:00	25.28148	0.7271	
	20:20	25.52407		
4	7:00	15.2037	0.8275	
	21:55	15.85926		

All the tests give very large P-values, which indicate the same conclusion that samples within one group are not significantly differently.

As we have 5 points in every group, to make a Mann-Whitney U test for each pair of them would require a large amount of work. Chi-square test can take several populations into account at the one time. In our settings, the null hypothesis of the test is that the distribution of the occupancy is the same in all times with same average occupancy.

The average link length and the jam density for the study site are estimated to be $l = 154m$ and $k_jam = 0.14veh/m$[19]. The maximum number of vehicles that a link would have is given by:

$$N = k_{jam} \times L = 0.14veh/m \times 154m = 21.56 \simeq 22veh$$

The possible outcomes of a link have vehicles range from 0 to 22, where there are 23 different outcomes in total. Thus, we can divide the total range of occupancy into 23 categories. Instead of probabilities, frequency of each group is used due to Chi-square test is designed for categorical variables that should be integer.

From the table above we can draw the same conclusion as we have got from Mann-Whitney U test. Up to this point, weve showed that evenly distributed congestion is not a necessary condition for a well-defined MFD and relaxed the homogeneity assumption in [9]. And we can conclude that individual occupancy distribution for different time intervals with similar average occupancy is similar in a well-defined MFD. We should,

Table 3.2: Chi-square test result for occ. distributions

Test #	degrees of freedom	Chi-square statistics	Critical Value $(p \leq 0.05)$	P-value	conclusion
1	88	84.32	110.9	0.591	
2	88	80.69	110.9	0.697	not significantly
3	88	96.77	110.9	0.245	different at =0.05
4	88	99.13	110.9	0.196	

however, still notice the small errors within the distribution of each average occupancy group. This will be investigated in section 3.3 when we are looking into errors in MFD.

3.2 An Analytical Distribution for Occupancy Distribution

We are further interested in what distribution the occupancy distribution would follow. If we assume that the number of vehicles in each link is uncorrelated over the network and the probability that a cell in a link has a car is the same, the number of vehicles in each link should follow a binomial distribution with parameter p, which should be equal to the occupancy of the link. We use the average occupancy as the success probability as our binomial parameter to test our occupancy distribution from the experiment.

In general, if the random variable X follows the binomial distribution with parameters N and p, we define $X \sim B(N, p)$. The probability of getting exactly x successes in N trials is given by the probability mass function:

$$Pr(X = x) = \mathcal{B}(x, N, p) = \binom{N}{x} p^x (1-p)^{(N-x)} \tag{3.1}$$

The number of trials for this experiment was estimated as the maximum number of vehicles that fit in one link, $N = L \cdot k_j am$, where L is the average length of a link and k_{jam} is the jam density of a link with units of $vehicles/km$. As explained before, for Yokohama these parameters have been estimated from [19] to be $k_{jam} = 0.14vhs/km$ and $L = 154m$, which leads to $N = 22vhs$ and the probability of success equal to the average occupancy, $p = \bar{o}$. It is like each link is divided in N cells and the probability that a cell is occupied by a vehicle is $p = \bar{o}$.

Unfortunately, it is found out that none of them follows binomial distribution by applying Chi-square test with expected counts giving by binomial distribution for each cell and the variances of individual detector occupancies from the experiment data are much higher than those of binomial distribution. This may be due to correlation between adjacent links. In a traffic network, when a downstream link becomes congested, the density of corresponding upstream links would be affected.

In an arterial network like that in downtown Yokohama, it is quite reasonable to assume the immediate upstream link can be affected downstream link because of the redundancy in the network. Mathematically speaking, we assume the probability of having a car in the cell of one link is decided by the occupancy of its downstream link. Furthermore, in case there are some strong turning movements or ending point like ramps, the probability again becomes the same as the average occupancy in the network. We present here a model to estimate the spatial distribution of occupancy by introducing the necessary notation.

Analysis of empirical data from the Yokohama experiment shows that the spatial distribution of occupancy does not follow a binomial distribution, as the spatial variance of occupancy is much higher than the one of a Binomial distribution with probability of success equal to the average occupancy, $p = \bar{o}$. (See figure 3.3)

As mentioned above, the main reason of this difference is that traffic congestion in a link has some correlation with successive links, while a binomial distribution assumes zero correlation from one link to another. To capture this spatial correlation and congestion propagation from one link to another we introduce a probability mixture model.

Consider K successive links in an arterial network and X_k is a random variable which shows the number of vehicles in link $k = 1, 2, , K$. Then, it is assumed that X_k follows a binomial distribution with probability of success $p_k = x_{k-1}/N$, where x_{k-1} is the number of vehicles in link $k - 1$.

$$Pr(X_k = x_k | X_{k-1} = x_{k-1}) = \mathcal{B}(x_k; N, x_{k-1}/N). \tag{3.2}$$

Using the law of total expectation for the unconditional probability $Pr(X_k = x_k)$ we get
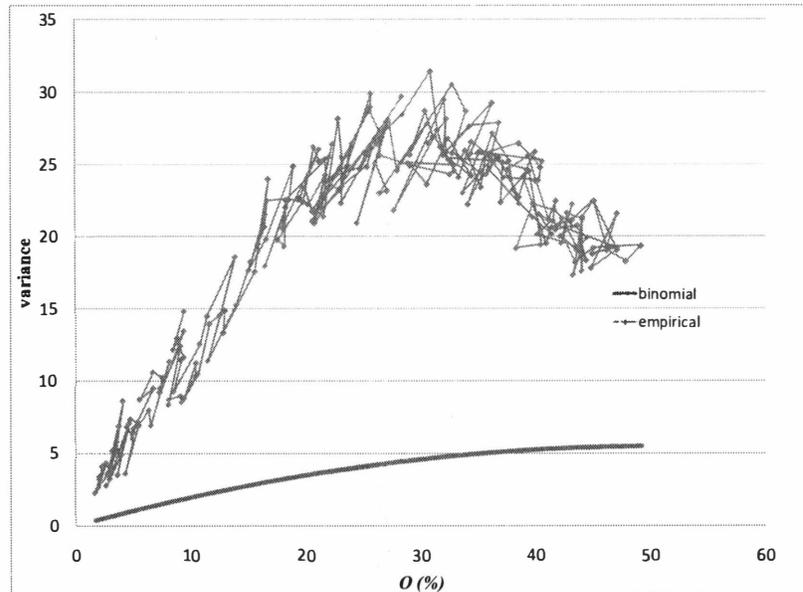
Figure 3.3: Variance of individual detector occupancy vs. average network occupancy: (a) empirical data; (b) assuming binomial distribution of congestion

$$Pr(X_k = x_k) = \sum_{i=0}^{N} \mathcal{B}(x_{k-1}; N, i/N) \cdot Pr(x_{k-1} = i). \tag{3.3}$$

Equation 3.3 provides the probability density function (pdf) of X_k if X_0 is known. It is not difficult to prove (tedious but straightforward) that the limit pdf of X_k as $k \to \infty$ if $X_0 \sim \mathcal{B}(N, p)$ is taking values 0 or N with

$$\lim_{k \to \infty} Pr(X_k = N) = p \ and \lim_{k \to \infty} Pr(X_k = 0) = 1 - p. \tag{3.4}$$

Figure 3.4 shows the probability density function of X_k for $k = 1, 2, \cdots, 10$ for $p = 7/22$.

Given that an arterial network is two-dimensional, each link has more than one successive links and receives flow usually from 1-3 links depending on the topology of the network. To define which is the k-th link in the network one can use a trajectory that travels and visits all links in the network only once. There are many different ways to do this, e.g. moving randomly or moving horizontally and then vertically etc. Because of different turning percentages in each link there is some probability that
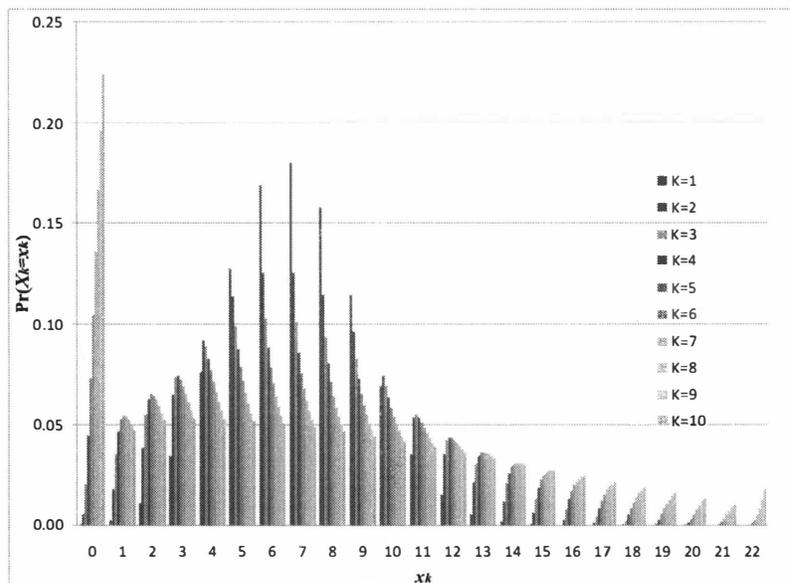
Figure 3.4: Pdf of proposed model

correlation between link k and $k-1$ is much smaller than this described by the model in equation 3.3, e.g. large turning movement or parking lot. For this reason we add one more parameter to the model, which is the probability π that links k and $k-1$ are independent and $Pr(X_k = x_k | X_{k-1} = x_{k-1}) = Pr(X_k = x_k) = B(x_k; N, \bar{o})$, $\bar{o}$ is a priori probability of success for a cell in the network. One should expect that probability π should be high for uncongested conditions (less spatial correlation).

To estimate now the spatial distribution of occupancy in the network we need to know what the probability that a specific link is of type X_k is, i.e. is correlated with $k-1$ upstream links. This is not difficult to estimate given the modification described above. For a specific link in the network there is probability $1 - \pi$ that this link is correlated with the previous one and probability π that it is of type X_0. Thus, the type of link follows a geometric distribution with probability of success $1 - \pi$, i.e. the number of failures until the first success.

$$Pr(X_k = X_i) = (1 - \pi)^i \pi. \tag{3.5}$$
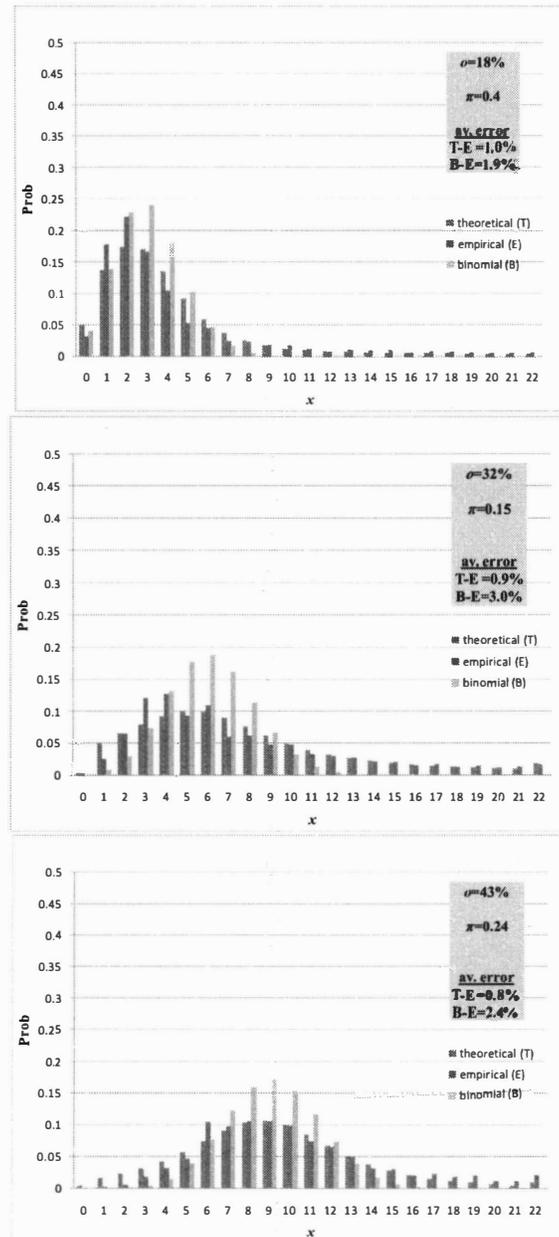
Figure 3.5: Distribution comparison between empirical data, proposed model and binomial distribution for O=18%, 32% and 43%.

Thus, using a miture of eq. 3.3 and 3.5 we can estimate which is the spatial probability density function of occupancy.

Figure 3.5 compares the proposed analytical model of spatial distribution of occupancy with the empirical data from Yokohama for different values of average occupancy, $\bar{o}$=0.18, 0.32 and 0.43, which is approximately equal with $\bar{o}$=4/22,7/22 and 9/22, respectively. Parameter has been calibrated to maximize goodness of fit (minimum average absolute error).

Table 3.3 summarizes the results of the analysis. It also includes the average speed in the network for illustrating the level of congestion. Notice that the theoretical model developed here provides a much better fit than the binomial distribution and a good estimator of spatial variance of occupancy, especially when the network becomes congested. For light conditions ($\bar{o} = 0.18$), the theoretical distribution cannot capture well the right tail of the empirical distribution. This can be explained by that congestion is generated in some pockets in the network and then starts to propagate. This could be captured if for the pdf of X_0, a distribution with more spread than the binomial was applied.

Table 3.3: Summary of results of the proposed analytical model; comparison with empirical data and binomial distribution

$\bar{o}$	$\bar{v}$	Variance			π	Aver. error	
	Km/hr	B	T	E		B - E	T - E
0.18	19	3.29	18.16	20.58	0.4	1.90%	1.40%
0.32	11	4.82	25.32	24.33	0.15	3.00%	1.10%
0.43	7	5.41	17.73	19.18	0.25	2.40%	0.80%

3.3 Analysis of errors for a Macroscopic Fundamental Diagram

3.3.1 Theoretical approach

This section provides an approximate estimation of the variance of the Macroscopic Fundamental Diagram in terms of $Q(O)$, where O is the average occupancy of all M detectors in the network during time interval t (comparable in size with one signal

cycle), and Q is the average flow of all detectors during the same time interval (interval t is omitted from the notation for simplicity purposes).

Further, let us define q^d and o^d the flow and occupancy of an individual detector d during a time interval. The relationship between q^d and o^d (expressed as fundamental diagram) exhibits high scatter, especially for congested conditions (see for example figure 1.6). If $q^d(o)$ represents an average (or best fit) fundamental diagram for the individual detector we can write $q^d = q^d(o^d) + \varepsilon^d(o^d)$, where $\varepsilon^d(o^d)$ is an error, which depends on occupancy o^d according to analysis for real data. If we now look at all individual detectors by omitting their label d (without any type of spatial aggregation or averaging) we may fit a fundamental diagram for all individual detectors $q = q(o) + \varepsilon(o)$. Denote $p(o)$ the continuous probability density function, which shows what fraction of detectors has occupancy o and we can write:

$$Q(o) \approx \int_0^{100\%} p(o) \cdot q(o)\, do \approx \sum_{i=0}^{N} \tilde{p}(o_i) \cdot \tilde{q}(o_i), \tag{3.6}$$

where $\tilde{p}(o_i)$ is the discrete version of the continuous probability density function $p(o)$ and $\tilde{q}(o_i)$ is the average flow of all detectors in occupancy group $i, i = 0, 1, \cdots, N$.

Now we will investigate the variance of $Q(O)$. But note that this error/variance can be high (even higher than the individual detector errors) if there is some type of significant correlation (not random errors) between some of the aforementioned variables. For example:

1. if individual detectors exhibit significantly different individual fundamental diagrams $q^d(o)$ in different parts of the network,

2. if congestion level is not homogeneously distributed among the network for different times of day with similar average occupancy O,

3. if $q^d(o)$ is not well defined and exhibits hysteresis phenomena (like in freeways).

Denote $V[\cdot]$ the variance of a random variable. The randomness of the aforementioned variables is related to the fact that every time the average occupancy in the network is O, $\tilde{p}(o_i)$ and $\tilde{q}(o_i)$ may have some randomness. If $\tilde{p}(o_i)$ and $\tilde{q}(o_i)$ have random errors $\varepsilon_{\tilde{p}}(o_i)$ and $\varepsilon_{\tilde{q}}(o_i)$ respectively, which are independent of each other. And the

random errors of their product is independent of all other products for different i's, then the variance of average network flow for a given network average occupancy O, $Q(O)$, is

$$V[Q(O)] \approx \sum_{i=0}^{N} V[\tilde{p}(o_i) \cdot \tilde{q}(o_i)]. \tag{3.7}$$

The estimation of the variance of the product of two random variables is not an easy task in statistical analysis. But, if these two variables are independent, the variance has a closed form expression[24].

$$V[XY] = E^2[X] \cdot V[Y] + E^2[Y] \cdot V[X] + V[X] \cdot V[Y] \tag{3.8}$$

As variable $\tilde{p}(o_i)$ expresses the fraction of links with occupancy close to o_i while $\tilde{q}(o_i)$ describes the average flow of the same group of links, there seems no dependence between them. For simplicity, we use $\bar{p}_i$ for $E[\tilde{p}p(o_i)]$ and $barq_i$ for $E[\tilde{q}(o_i)]$. Combining equations 3.7 and 3.8 we get:

$$V[Q(O)] \approx \sum_{i=0}^{N} \bar{p}_i^2 V[\varepsilon_{\tilde{q}}(o_i)] + \bar{q}_i^2 V[\varepsilon_{\tilde{p}}(o_i)] + V[\varepsilon_{\tilde{q}}(o_i)] V[\varepsilon_{\tilde{p}}(o_i)] \tag{3.9}$$

Also, if $\varepsilon_q(o_i)$ is the random error of the flow for all detectors for occupancy group i (close to o_i), using the Central limit theorem, a good approximation for the variance of their average $\tilde{q}(o_i)$ is $V[\varepsilon_{\tilde{q}}(o_i)] \approx V[\varepsilon_q(o_i)]/M\bar{p}_i$ this is the average of approximately $M\bar{p}_i$ detectors). After some manipulations equation 3.10 becomes:

$$V[Q(O)] \approx \sum_{i=0}^{N} \frac{1}{M} V[\varepsilon_q(o_i)] (\bar{p}_i + \frac{V[\varepsilon_{\tilde{p}}(o_i)]}{\bar{p}_i}) + \bar{q}_i^2 V[\varepsilon_{\tilde{p}}(o_i)]. \tag{3.10}$$

3.3.2 Results for the Yokohama experiment

In this section we use equations 3.6-3.10 to estimate the variance of the Macroscopic Fundamental diagram for the site of Yokohama. some of the assumptions of Section 3.3.1 may not be meet in the network of Yokohama, the application still illustrates how the method may work in a real-world application where the input data include some error.
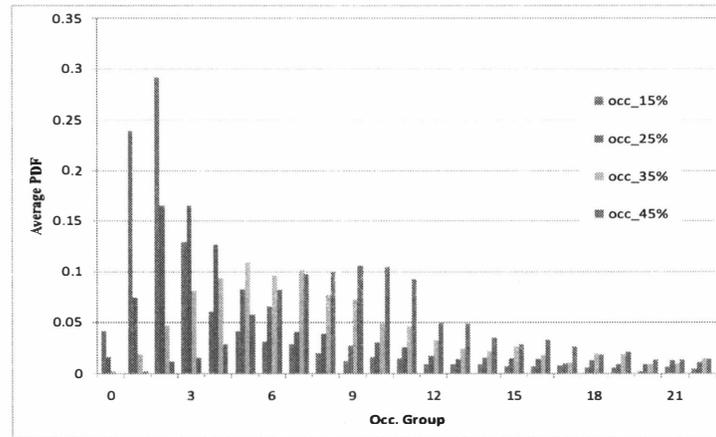
Figure 3.6: Average probability density function of detector occupancy, $\tilde{p}(o_i)$, for different values of average network occupancy O.
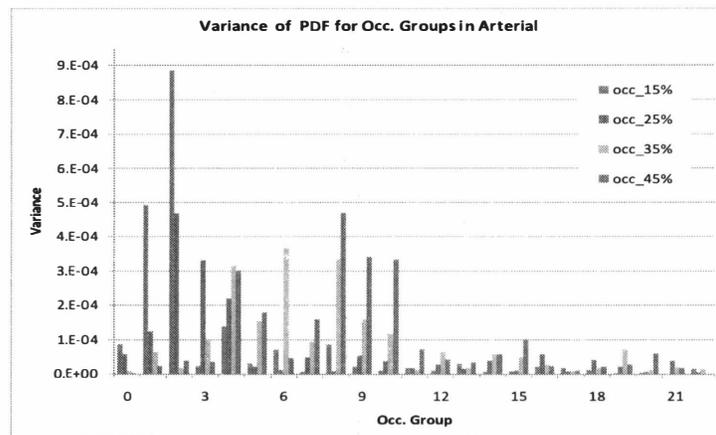

Figure 3.7: Variance of error of the probability density function, $V\left[\varepsilon_{\tilde{p}}(o_i)\right]$, for different values of average network occupancy O.

First, figures 3.6 and 3.7 summarize the estimated mean and variance of $\tilde{p}(o_i)$ for different occupancy groups o_i for $i = 0, 1, , N$ and for network average occupancy $O=15\%$, 25%, 35% and 45%. Note that the variance of the probability density function $V\left[\varepsilon_{\tilde{p}}(o_i)\right]$, is maximum for the occupancy group which is very close to the mode (the most frequent value) for the mean pdf $\tilde{p}(o_i)$, for example group 6 for $O=35\%$.


CV of Flow in FD for Arterial
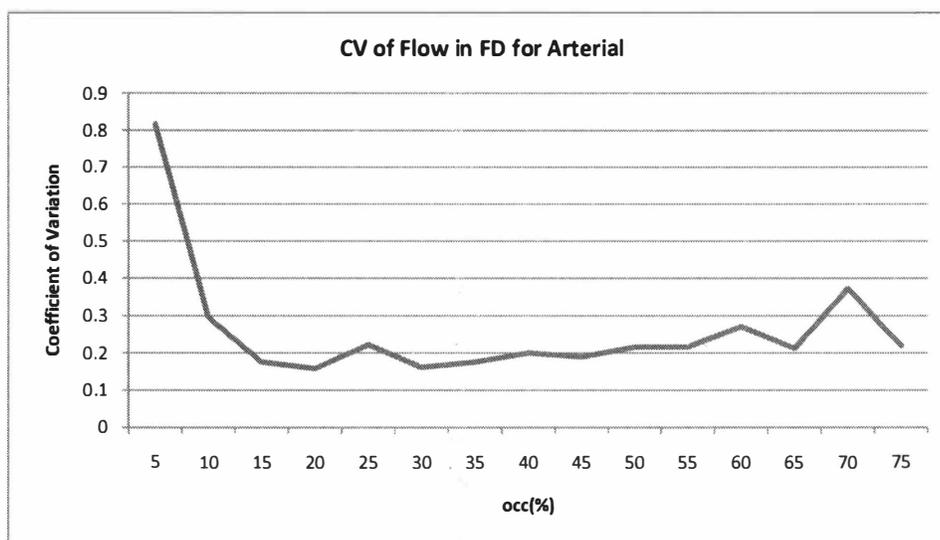
Figure 3.8: Coefficient of variation of individual detector flow, q, for different occupancy for a sample of detectors

Figure 3.8 shows the coefficient of variation, CV, (dimensionless quantity: standard deviation divided by the mean) of individual detector flow for different values of occupancies. Note that the coefficient of variation is almost constant at the value of 3, for $o < 5\%$. This is an interesting finding because by assuming the flow CV constant, one can derive a relationship between the level of spatial heterogeneity (as this expressed by the variance and mean of $\tilde{p}(o_i)$) and the variance of an MFD. This relationship can provide useful hints about the type of cities that exhibit a well defined MFD.

We did not categorize the CV for different network occupancy O levels, as we do not expect that this complication could improve the estimation and has no physical explanation (individual links are expected to produce similar q vs.o pairs independent of the aggregated level of congestion). This is not necessarily the case in freeways because of hysteresis phenomena [25].
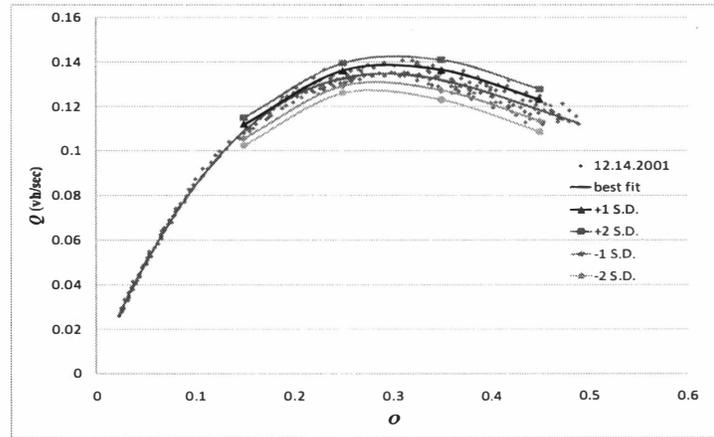
Figure 3.9: Yokohamas estimated 1- and 2-standard deviation bands for the MFD

Table 3.4: Variance of an MFD for different levels of congestion

$O(\%)$	15	25	35	45
$V\left[Q(O)\right] (vh/5min)^2$	0.9	1.01	1.35	1.45

Figure 3.9 shows the results of variance estimation for Yokohama. Individual dots are Q vs.O points for every 5 minute for Dec 14, 2001, one heavily congested weekday as taken by[19]. The lines show the 1- and 2-standard deviation bands arising from formula 3.10 on each side of a fitted curve. Table 3.4 summarizes the estimated variance. Our simplified theoretical approach captures well the variance increase as level of congestion increases. Note that the CV of the MFD is about 3-4% (The MFD for $O = 30\%$ is about $0.14vh/sec = 40vh/5min$. Variance is about 1.5, which means that $St.Dev. = 1.2vh/5min$. Thus, $CV = 1.2/40 = 4\%$ approximately). This value is 5-6 times smaller than the CV of an individual link, but much higher than it would have been estimated if the central limit theorem was applied for $M = 500$ detectors.

Chapter 4

MFD for Freeway in Twin Cities

In the previous chapter, we have investigated some properties of a well-defined MFD and provided some theoretical explanations. Base on the same idea we have for the arterial where there exists a well-defined MFD, we try to investigate the MFD of freeway for a typical workday. To have an general idea of what's going on in the freeway network, we first plot the average occupancy versus flow for three consecutive workdays. (see figure 4.1)
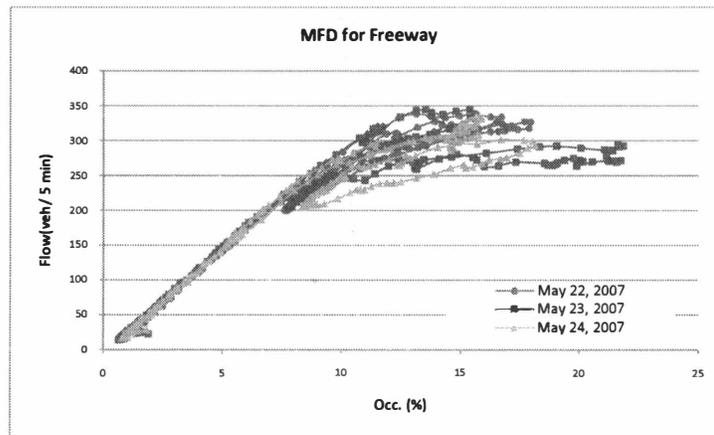

Figure 4.1: MFD of a large freeway network for 3 days

The freeway network is not as congested as arterial network. The highest average occupancy for the network is about 20%. On the other hand, the average flow is much

higher than that in the arterial network. See [19] for a MFD for arterial network.

4.1 Hysteresis Phenomenon in MFD for Freeway Network

In figure 4.1, note that for the uncongested part we still cannot distinguish one day from the others. For the congested part, however, there appears something like hysteresis (i.e. the evolutions of traffic states follow different routes). This phenomenon is quite interesting for us to investigate. In the remaining of this chapter, we choose May 22, 2007 to look at this hysteresis phenomenon in details and try to provide some possible explanations for it.
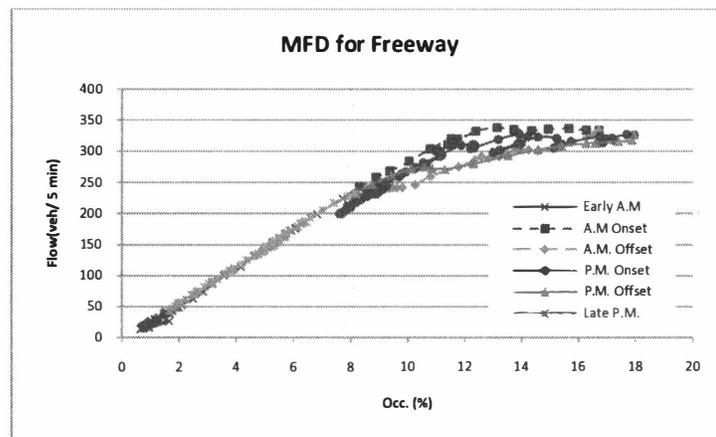

Figure 4.2: Different periods of an MFD for freeway
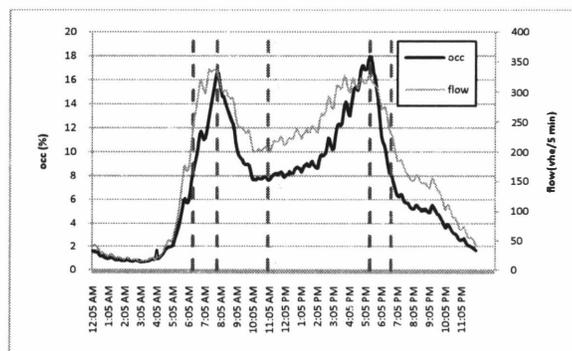
Figure 4.3: Time series of average occupancy and flow for freeway

We plot the MFD for May 22, 2007 in figure 4.2, using different symbols to represent different period of traffic states. Note how clearly the hysteresis phenomenon appears in front of us. The curves in the uncongested part are the same again. When it returns from the peak, however, it follows a route different from the route along which it reaches the peak, i.e. different routes for onset and the offset of the congestion. Furthermore, when it moves to the peak again, the route is again different from the route it moves towards the peak last time, i.e. different routes for morning onset and evening onset.

We should also notice that the routes for morning onset and evening onset is different but the routes for morning offset and evening offset is similar. For the onset of the morning congestion, the average occupancy of the whole network increases from the point where the whole network is almost empty. For the onset of the evening congestion, the system starts from the point where network average occupancy is about 8%. The difference is quite big for a freeway system whose maximum network average occupancy is no more than 20%. On the other hand, the difference in starting points in terms of network average occupancy between offset of the morning congestion and the offset of the evening congestion is only about 1%.

Also note in figure 4.3, the occupancy change rates for onset in the morning and onset in the evening are different and these state transitions follow different routes. On the other hand, the occupancy changing rates are quite similar for offsets in the morning and that in the evening where the transition routes are quite similar. Whether occupancy changing rate is a factor in deciding the route of state transition needs further investigation.

Variance of the individual detector occupancy should be another quantity describing the state of starting points of traffic evolution and we will examine this in later chapters. At this point, we can say the routes of the system evolution might be dependent on the starting points of the route. But further investigation is needed.

4.2 Occupancy Distributions for a Freeway MFD

We first conjecture that the difference in distributions might be a reason for the hysteresis phenomenon. The same approach is followed as we did for arterial network. We choose a 5% bin for freeways in a way that (i) each bin has a significant amount of data

points for statistical analysis and (*ii*) the bin size is small enough to determine different traffic behavior from between successive bins.

Occupancy from 0 to 100 is divided into 21 groups and pdfs are got for network average occupancy 5%, 10%, 12.3%, 15.2% and 17%, which are ranging from uncongested part to most congested part of the MFD for freeway, including the points where biggest hysteresis happens (i.e. 12.3%). For each level of network average occupancy, we have 4 time intervals as a group and the biggest average occupancy difference within group is less than 0.5% for all groups. Notice that this number is 1% for our arterial network due to the availability of the data, which means that the time intervals within each group for freeway should be closer in terms of average occupancy. Now we are going to see the occupancy distributions for each network average are similar or not.
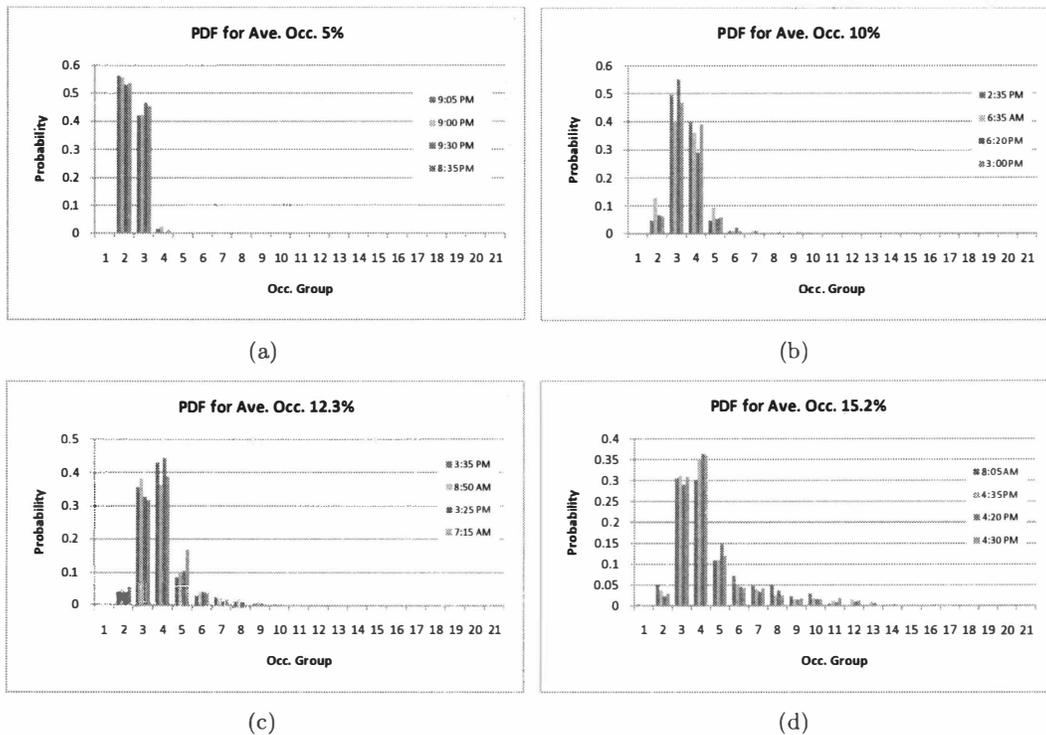

Figure 4.4: Individual occupancy distributions for freeway network

From figure 4.4 we can see that the differences of occupancy distributions for a given

network average distribution are larger than those in arterial network, since the errors within groups is larger even if our points are closer (remember the 0.5% difference versus 1% difference mentioned above).

Following the same procedure as we have in section 3.1, we first carry out Mann-Whitney U test between two time intervals with mild difference within each network average occupancy level. The results are shown in table 4.1.

Table 4.1: Mann-Whitney U test results

Test	Time	Ave. Occ.	P-value	Conclusion
1	9:05 PM	4.92	0.406	
	9:30 PM	4.999		
2	6:35 AM	10.037	0.394	
	3:00 PM	10.224		
3	3:35 PM	12.324	0.42	not significantly different at $\alpha = 0.05$
	3:25 PM	12.342		
4	8:05 AM	15.103	0.202	
	4:20 PM	15.236		
5	5:05 PM	16.851	0.545	
	5:10 PM	16.977		

Note the distributions are still not significantly different but the P-value is much smaller. Let's take the Chi-square test again for the freeway network. Note that the degrees of freedom are not the same for all the tests. This is because for freeways, the range of individual occupancies is not as wide as that in arterial network. Thus some of the categories would have zero observations. When we are doing Chi-square test, only categories with observations are kept. One more thing should be mentioned here is that the distributions for categorical variables have also something to do with the choice of bins. The results are shown in table 4.2.

We can see that the occupancy distributions are significantly different in 3 tests out of 5. Significant here means the difference is too big to happen just by chance. These results are somewhat different from what we have from the Mann-Whitney U test.

Lets first think about the difference between the two tests. In Mann-Whitney U test, we just compare two samples at one time. And when we were doing the test, we just choose the time intervals with modest difference in network average occupancy.

Table 4.2: Chi-square test results for freeway occupancy distribution

Test	degrees of freedom	Chi-square statistics	Critical Value $(p \leq 0.05)$	P-value	significantly different at $\alpha = 0.05$ level
1	6	12.3	12.59	0.056	no
2	27	98.43	40.11	0	yes
3	36	61.26	51	0.005	yes
4	39	67.26	51	0.003	yes
5	45	42.35	61.66	0.585	no

The results of the tests tell us that the differences are not big enough to claim that the distributions are significantly different. While we are carrying out the Chi-square test, we compare several distributions at one time. Even if only one of them is different from others, the result will be significant.

Then let's reconsider our hypothesis of the hysteresis phenomenon. We have claimed that the occupancy distribution might be a reason for hysteresis. Hysteresis here means that for different transition states of our traffic system (i.e. onset and offset of the congestion), the traffic flow is different when same network average occupancy is given. In other words, when the same network average occupancy is observed, different transition states may give us different values of flow.
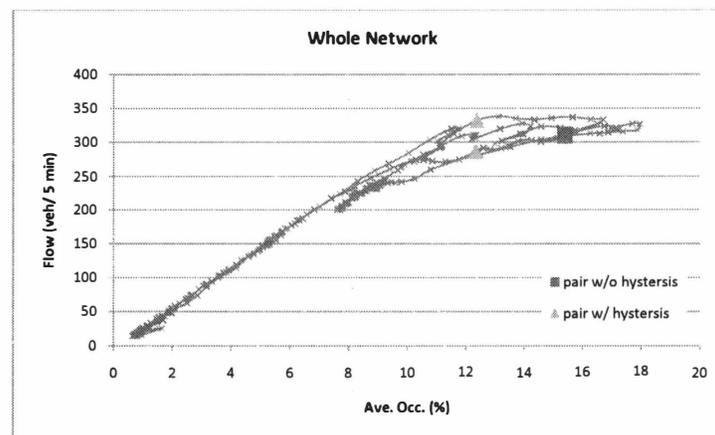

Figure 4.5: Two pairs of times with and without hystersis

We expect that, for the points in the freeway MFD with same network average occupancy but different network average flow, the corresponding individual link occupancy distribution should be different. To test our hypothesis, we choose 4 points from the freeway MFD and each two of them have the same network average occupancy. One pair of the points gives no hysteresis while the other pair of points gives different values of flow even with quite similar network average occupancies. The chosen pairs are shown in figure 4.5.

The results of the Mann-Whitney U Test are shown in table 4.3. Test 1 is for the pair without hysteresis and test 2 is for the pair with hysteresis.

Table 4.3: Mann-Whitney U test result for hysteresis

Test	Time	Ave. Occ.	P-value	Conclusion
1	4:25 PM	15.393	0.847	not significant
	5:45 PM	15.385		
2	7:15 AM	12.374	0.007	significant at =0.01
	8:50 AM	12.337		

Thus, we conclude that difference in individual detector occupancy distributions could be one reason for hysteresis phenomenon in freeway network. Given same network average occupancy, difference in individual detector occupancy distributions can result in difference in network average flows.

4.3 Hysteresis Phenomenon of MFDs for Different Scale Freeway Networks

4.3.1 A Compact Freeway Network

We are further interested whether the hysteresis phenomenon persists in a compact freeway network where congestion should be more homogeneous. A more congested part is chosen within the large freeway network we have just investigated. These freeway directions are towards downtown Minneapolis.

The chosen routes include part of I494, TH-169, I94 and I394 (see figure 4.6). There are about 150 detectors instead of more than 600 in the previous large freeway network.
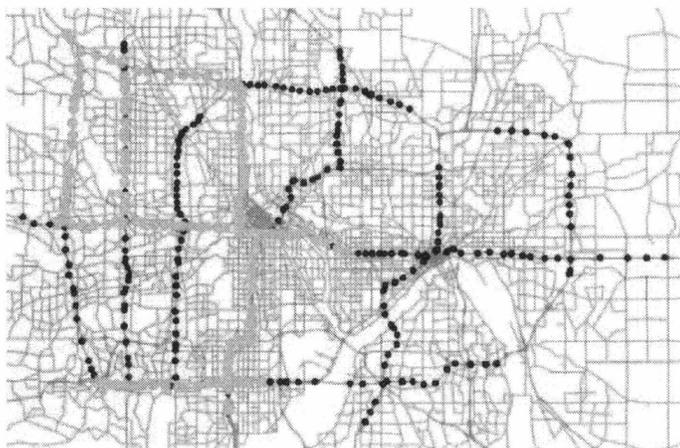
Figure 4.6: [A compact freeway network]A compact freeway network (bigger blue dots)

Hysteresis phenomenon is still observed in the MFD of the compact network (see figure 4.7). As our network only consists of the lanes towards downtown Minneapolis, we will mainly focus on the morning peak afterwards.

The maximum average flow and occupancy of the compact freeway network are both larger than those of the large freeway network (about 380 veh/ 5 min V.S. 330 veh/5 min and 20% V.S. 18%). This indicates that the compact freeway network is more congested.
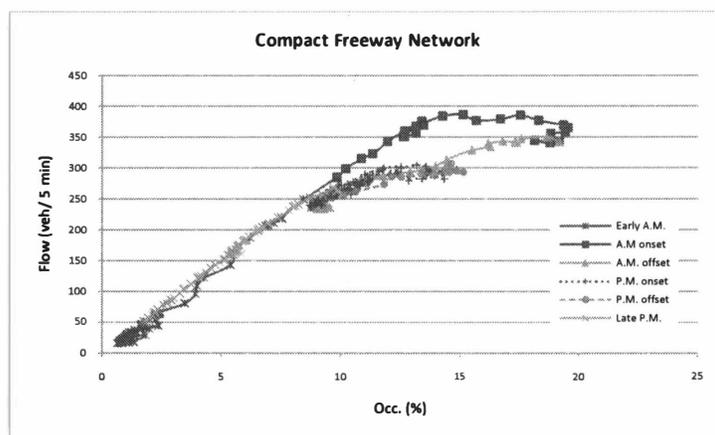

Figure 4.7: MFD of different time periods for a compact freeway network

Then, let's look at the distributions of the individual detector occupancy. With the same concerns as for the large freeway network, we choose time intervals with network average occupancy 5%, 10%, 14% (where hysteresis appears) and 19%. We can see the same pattern as we have observed for the large freeway network and in figure 4.8(c) we can see that the distributions are quite different where hysteresis appears.
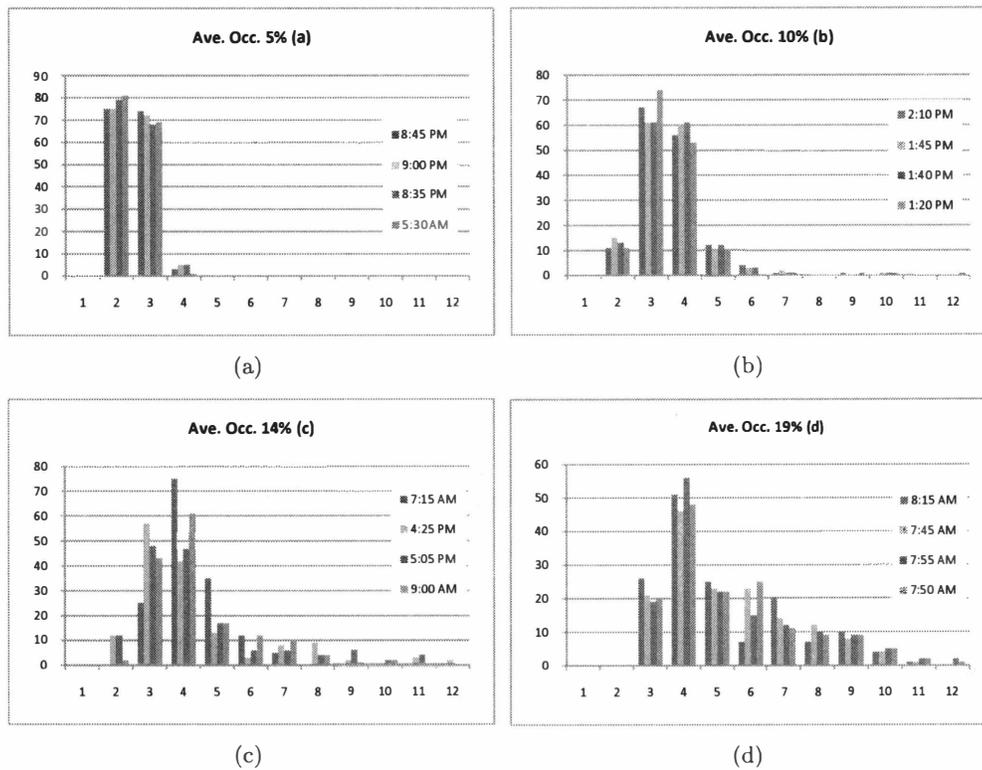
Figure 4.8: Individual detector occupancy distributions for a compact freeway network

4.3.2 A Freeway Route

Next, let's zoom in to a single section of the freeway with only 8 consecutive detectors. This section is along I-494 southbound between I94 and I394 towards downtown Minneapolis (see figure 4.9). The length is about 3.5 miles and there are two on ramps and two off ramps. Detectors S701 and S707 are right before off ramps and detectors S703
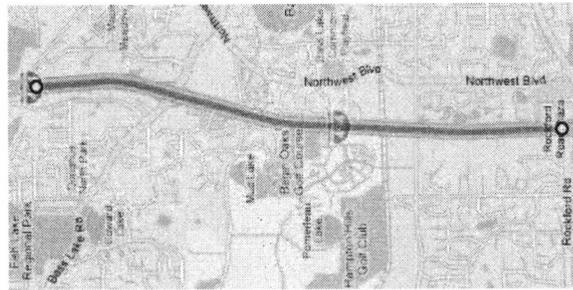
Figure 4.9: Google map of a freeway route

and S708 are right before on ramps. Detector S704 is just after an on ramp.

We only investigate its morning peak hour from 6am to 11am due to the same reason we have for the compact freeway network (direction towards downtown). Figure 4.10 is the MFD for this time period. The average occupancy is more than 30%, which indicates this section is very congested during the morning peak.
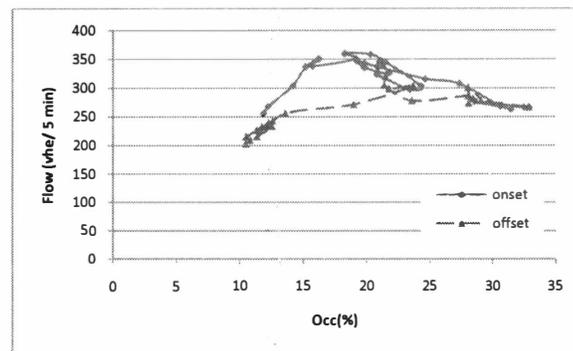
Figure 4.10: MFD of a freeway route in the morning

Hysteresis phenomenon is still very obvious in such a small section. We would also like to make a histogram for occupancy distribution of these detectors. In addition to this, as there are only 8 detectors and they are consecutive, we can plot a space and occupancy diagram for different time interval. If we choose time intervals with similar average occupancy but with different average flows (i.e. time interval pair gives hysteresis), we can see how individual detectors occupancy distribution affects average flows.

From figure 4.11, we can see the congestion is more evenly distributed at 6:30 AM

during onset of the congestion and the detectors near on ramp (S704 and S708) are more congested. At 9:05 AM during the offset of the congestion, the congestion is not as evenly distributed as it was during onset. Detector S707, which is just before the off ramp becomes the bottleneck, resulting in two nearest upstream detectors to be congested. At the same time, occupancies of other detectors are relatively low. This causes the flow at this time is relatively low, as some parts of the link is congested while the traffic on other parts is light, both of which situation yields low traffic flow. The same phenomenon can be found from the histogram plot (figure 4.12). In other words, the flow drops when the distribution of the congestion stretches out.
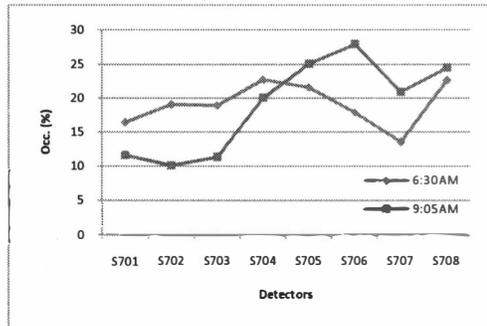

Figure 4.11: Space-occ. plot for freeway route
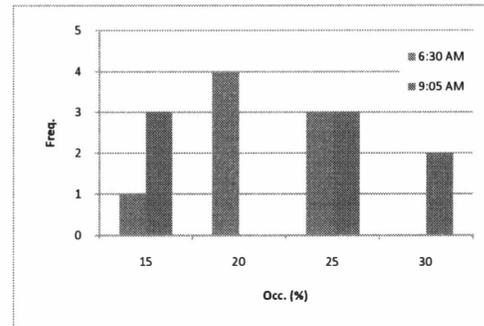
Figure 4.12: occ. distribution along the route at 2 different times

As hysteresis phenomenon persists from the whole large freeway network to compact part of the network and then to a congested single section, we suspect the hysteresis may also come from the hysteresis phenomenon existing in single detectors.

4.3.3 Individual Detectors

In this sub-section, we are going to examine the fundamental diagrams for two individual detectors. They are among the most 10 congested ones of the whole network, as we are interested in the congested part of the diagram. We want to see traffic states evolution during the onset and offset of congestion. Figure 4.13 and 4.14 demonstrates the FDs for two detectors with daily average occupancy more than 14% and the maximum occupancies go to around 50%.

What we have observed is that 1) for the uncongested parts both of them display

a linear positive relationship between flow and occupancy; 2) when occupancies go to certain points (about 11% for detector S8 and 16% for S196), the flows stop to increase or begin to drop; 3) the flows on onset parts in the diagrams are always larger than their offset counterparts (i.e. hysteresis exists in single detectors).
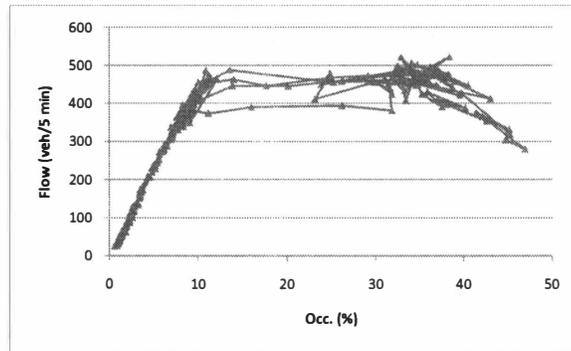
Figure 4.13: Fundamental diagram (FD) for detector S8
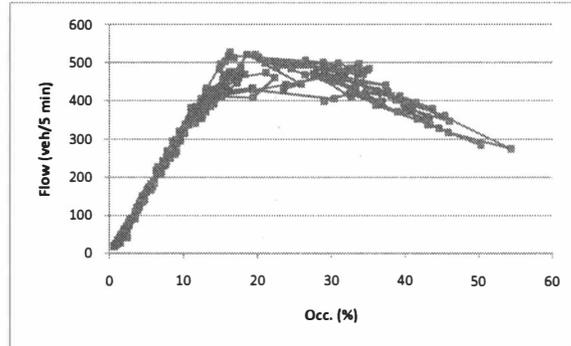
Figure 4.14: Fundamental diagram (FD) for detector S196

4.4 Discussion: Magnitude of the hysteresis phenomenon in different network scales

Here we define magnitude of hysteresis phenomenon as the biggest difference in flows for certain occupancy in fundamental diagram in one day. As our analysis in this chapter goes from the large freeway network to the single detectors in the network, the magnitude

of hysteresis phenomenon exhibits an increasing trend, i.e. the magnitude of hysteresis phenomenon is larger as the scale of network becomes smaller (see figure 4.15).
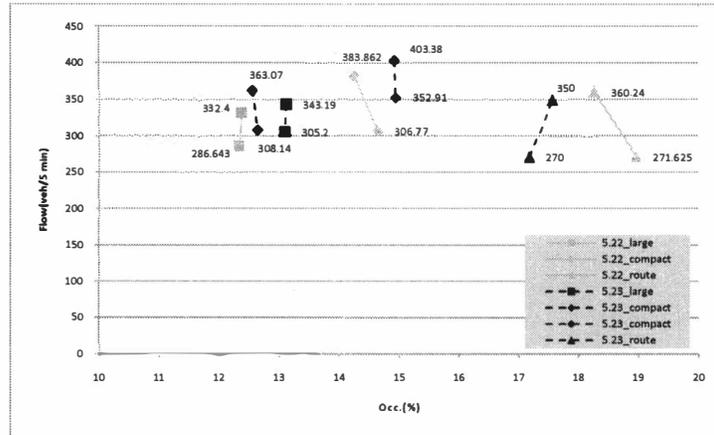
Figure 4.15: Magnitude of hysteresis for different network scales

The changing of the magnitude of hysteresis for different network scales might result from the inhomogeneity of the congestion over the network. As we've showed in the previous sub-section, individual detectors also exhibit hysteresis phenomenon, which happens only when the fundamental diagrams reach the congested part. For a large network, some of its detectors always stay in the uncongested part and contribute little to the hysteresis phenomenon. As we zoom in on more congested network region, the percentage of detectors whose fundamental diagrams reach the congest part and thus the hysteresis phenomenon is magnified. The collective affects of the individual detector hysteresis are then magnified.

Another interesting point to be noted in the figure is that as we move to the smaller size but more congested network, the occupancy which gives the maximum difference in flows also increases. We conjecture this should also result from the spatiol inhomogeneity in large network.

Chapter 5

Properties Investigation of MFDs

5.1 Spatial distribution of individual detector occupancy

Regarding to the hysteresis phenomenon, there are two aspects about the spatial distribution of individual detector occupancy over a network: 1) the distribution of the occupancy values for a certain time or for a network average occupancy value; 2) the locations of the individual detector occupancies over the network.

We have already done analysis about the first aspect in the previous chapters. In chapter 3, we've showed that for certain value of network average occupancy, the distributions of the individual detector occupancy values would be similar no matter what time periods it belongs to. In chapter 4, hysteresis phenomenon appears in the fundamental diagrams of freeway network no matter what scale of the network we investigate. When we plot the distribution of the individual detector occupancy values, we find out that the distributions tend to be different at the networks average occupancy values where we have hysteresis phenomenon. Thus, we conclude that difference in distribution of individual detector occupancy value should be a reason for hysteresis phenomenon.

If we i) assume that the difference between fundamental diagrams (FD) of individual detectors is just caused by random errors, or we say the fundamental diagrams are same for all the individual detectors over the network and ii) ignore the randomness in the network average value, we expect that a certain value of network average occupancy gives the same network average flow. In other words, the location differences of detector

average occupancies can be ignored if all the detectors have the same FDs. Furthermore, even if individual detectors have different FDs, but detectors with different FDs randomly distributed over the network instead of as a function the location, we can also expect that a certain value of network average occupancy gives the same network average flow. The latter one seems to be the situation of the arterial network in Yokohama. But as we don't have the digital map of Yokohama with detector location information, it is not easy to investigate this.
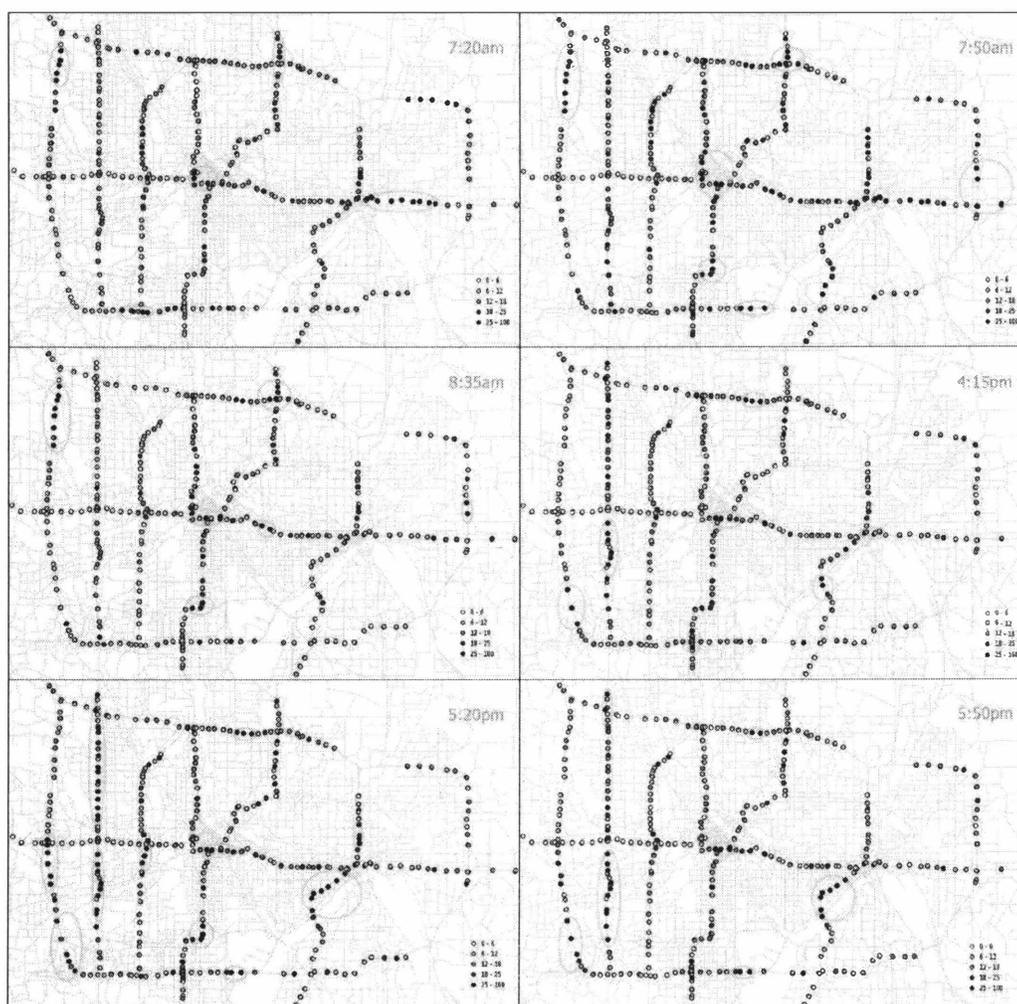

Figure 5.1: Contour plot of congestion in Twin Cities freeway network

For freeways, network exhibits hysteresis phenomenon. It has been already shown that the distributions of individual detector occupancy values are different when hysteresis phenomenon appears. But we do not know what the location distributions of different level of congestions are when there is hysteresis phenomenon or there is no hysteresis. Figure 5.1 is the contour plot of the congestion over Twin Cities' freeway network. Congestion levels are divided into 5 groups represented by different colors shown in the legend. The figures of 7:20 am and 8:35 am show the onset and offset of the congestion in the morning respectively. Both of the two times have network average occupancy about 13%, but the flows of them are different (338 veh/5 mins V.S. 290 veh/5 mins). 7:50 am is when the highest occupancy is observed in the morning. The figures of 4:15 pm and 5:50 pm are the onset and offset of the congestion in the evening respectively. This pair of time intervals not only has the similar network average occupancies but also similar network average flows. 5:20pm is the most congested time in the evening.

Compare the locations of congestion at 7:20 am and 8:35 am, the patterns are quite similar but congestion is more concentrated at 8:35 am. Given the network average occupancy is the same for both times, the occupancy value at 8:35 am at uncongested part should be lower than that at 7:20 am. This is observed in the figure as there are more light colored dots at 8:35 am. When it comes to evening pair where we have less hysteresis, the more similarities can be found between these two time intervals.

Another point worthy of notice is that the congested places in the morning are different from that in the evening. This suggests the MFD could have hysteresis or high scatter even if the distributions of individual occupancy values were similar at different times (unless the fundamental diagrams are the same for all detectors).

5.2 Individual detectors' fundamental diagrams

As we discussed above, whether the fundamental diagrams of individual detectors are the same or not is an important factor in deciding the shape of MFD for a network. In this sub-section, we will examine the fundamental diagrams of individual detectors from both arterial and freeway network.

For arterial network, it is not easy to study the fundamental diagram of individual

detectors as there are too much scatters there. But we can still see that the patterns for individual detectors are not all the same over the network (see figure 1.6(a) and figure 5.2).
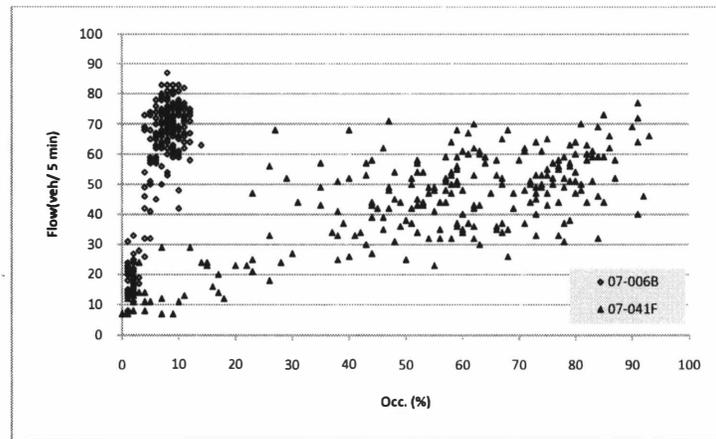
Figure 5.2: Fundamental Diagrams for individual detectors in arterial network

For freeway network, we choose one detector of most congested, medium congested and least congested respectively (figure 5.3). Without the interference of signal control, the fundamental diagrams for individual detectors in freeway network have much less scatters.
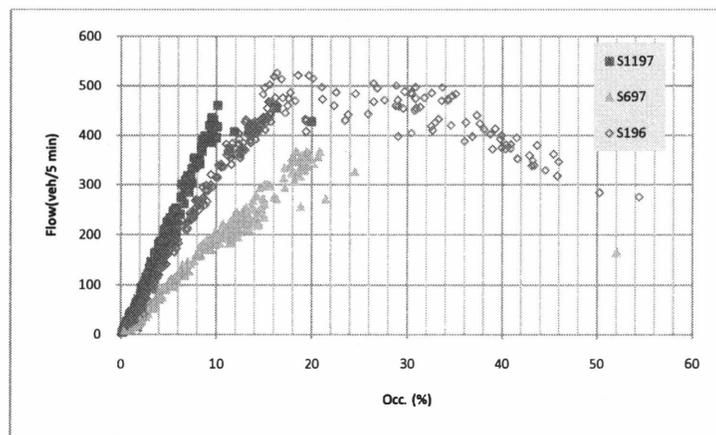

Figure 5.3: Fundamental Diagrams for individual detectors in freeway network

There are three fundamental diagrams in figure 5.3. The detector S1197 is the least congested one, detector S196 is the most congested one and the congestion level of detector S697 is in the middle. It is very clear that the fundamental diagrams are not all the same. But whether the detectors with different fundamental diagrams randomly distribute over the network or are location dependent needs further investigation.

5.3 Spatial variance of individual detector occupancy

Variance should be a quantity that can describe the distribution of individual detector occupancy in a simpler manner than probability density function does. In this section, we will examine the variance based on the data from compact freeway network same as 4.3.1 focusing on hysteresis phenomenon and then compare the result with its arterial counterpart.
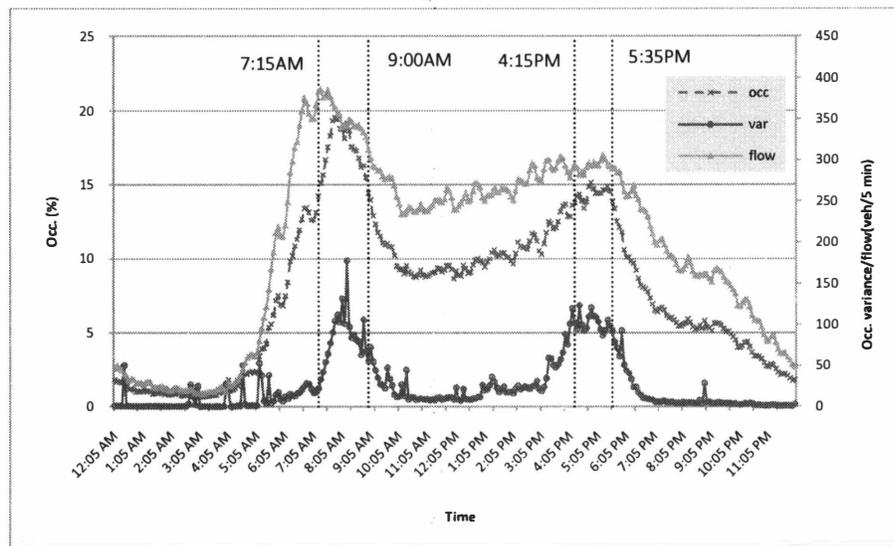
Figure 5.4: Occupancy, flow and occ. variance of a compact freeway network

Figure 5.4 shows two pairs of time intervals, one of which exhibits obvious hysteresis phenomenon while the other doesn't. Table 5.1 gives the exact data for the figure. Note the time intervals at 7:15 AM and 9:00 AM. They are on the onset of congestion and offset of the congestion respectively. Although they have similar network average

Table 5.1: Occupancy, flow and occ. variance in a compact freeway network

	7:15am	9:00am	4:15pm	5:35pm
occ(%)	14.26	14.44	13.77	13.9
flow (veh/5 min)	383.86	313.37	294.11	289.62
var	22.32	56.14	102.37	93.23

occupancy, the flows are quite different (about 18% drop). Meanwhile, the variance increases by about 154%, a large increase indicates that the congestion of the network is much less homogeneous at the second time interval.

If we look at the second pair of time intervals and do the same comparison, we will see how variance of individual detector occupancy relates to the hysteresis phenomenon. The change in variance is about 9%, which is much less than 154% for previous pair. Thus given the similar occupancy, the change flows is only about 2%.
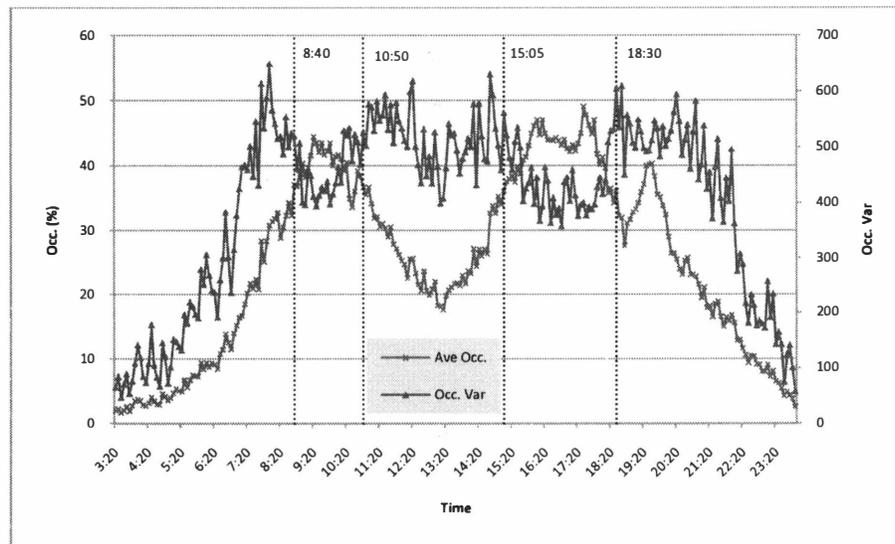

Figure 5.5: Occupancy and occ. variance of an arterial network

How about the variance when the average flows are similar in arterial network. Figure 5.5 is similar to figure 5.4 but is drawn from the data of arterial and without flow curve. As weve already shown that there is no hysteresis phenomenon in arterial network, we ignore flow here.

Table 5.2: Occ. and occ. variance of two pairs of time intervals in arterial network

	8:40	10:50	15:05	18:30
occ(%)	37.1	36.74	36.17	36.16
var	520.09	526.53	561	605

Similar to what we did for freeway network, we choose two pairs of time intervals, whose occupancies and corresponding variances are given in table 5.2. We can note that when the network average occupancies are the same, so are the corresponding variances. If we compare figure 5.4 and 5.5, there are two interesting things.

1. Both the average occupancy and occupancy variance of arterial network are generally higher than those of the freeway network.

2. In freeway network, the variance reaches its highest value around the peak of congestion. While in the arterial network, the variance is smaller at the peak of congestion than those at onset of congestion and offset of congestion.

For point 2, we conjecture this should have something to do with the network redundancy, which affect the propagation of congestion over the network.

This result of the analysis above coincides with the analysis in chapter 4 where we concludes that difference in distributions of individual detector occupancy would result in hysteresis phenomenon, which does not exist when the distributions of individual occupancy and the network average occupancy are the same. Furthermore, if we take a deeper look at the distributions of the individual detector occupancy (see fig. 4.8(c)), it can be seen that the distribution which stretches out would give lower network average flow and this is the same as where larger variance would give smaller network average flows as we have here in the first pair where exhibits obvious clear hysteresis phenomenon.

Note the second pair of our examples from freeway may contradict our argument as where occupancy variance is larger the flow is larger. As the difference in both occupancy variance and average flow is much smaller than the corresponding absolute value where clear hysteresis phenomenon is observed, we say they are the same in essence and the

difference comes only from randomness. To be more clear, we make a direct comparison between arterial network and freeway network occupancy distribution below.
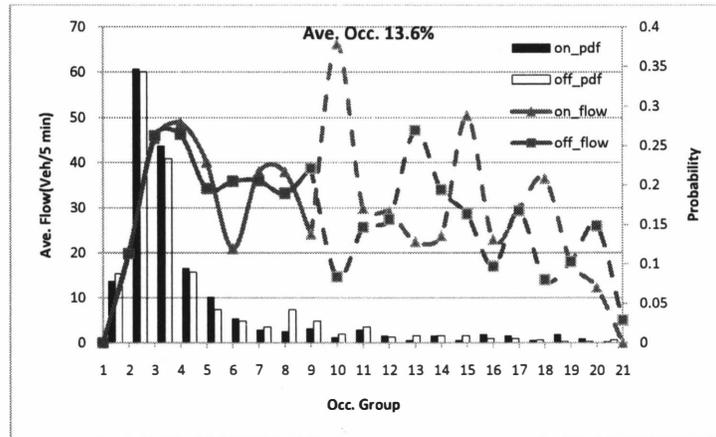
Figure 5.6: Average flow and occupancy pdf in arterial network
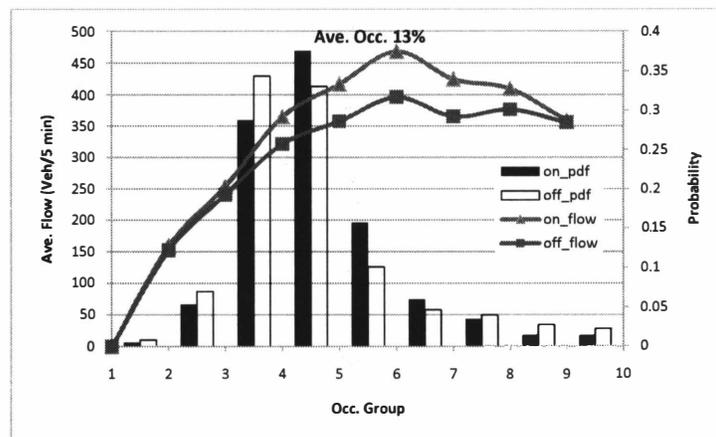
Figure 5.7: Average flow and occupancy pdf in freeway network

We choose two time intervals from both arterial network and freeway network with netwrok average occupancy about 13% (see figure 5.6 and 5.7). The pair in arterial network still doesn't exhibit hysteresis, but the pair in freeway network does. Espccially note that in figure 5.7, the average flows of onset of the congestion, when the distribution of occupancy stretches more, are lower than that of offset of congestion. And we can
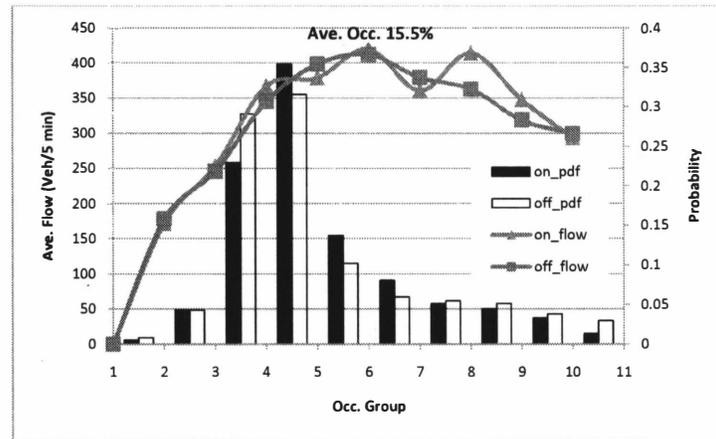
Figure 5.8: Average flow and occupancy pdf in freeway network at congestion peak

also show that when hysteresis phenomenon disappears in freeway network (e.g. at the peak time), the distributions of occupancy become similar (see figure 5.8).

5.4 Route comparison between arterial network and freeway network

Figure 5.9: Google map of a route in downtown Yokohama (route marked by blue)

For a route of freeway, we've already shown in 4.3.2 that there exists hysteresis

phenomenon (see figure 4.5) and the locations and distributions congestion are not the same for the pair of time intervals that exhibit hysteresis (see figure 4.11 and 4.12). Although hysteresis phenomenon is not observed in detectors over an arterial network (see figure 1.6(a) and figure 5.2), the high scatter make one difficult to study the fundamental diagram of these individual detectors. We are, however, able to see whether the congestion over a route in an arterial behave in the same way like freeway or not.

We choose an arterial section in downtown Yokohama and do the analysis in the same way as that in 4.3.2. A route about 1.1 miles from west to the east across downtown Yokohama is chosen (see figure 5.9) and the fundamental diagram is shown in figure 5.10 which shows less scatter than individual detector along the road and no hysteresis.
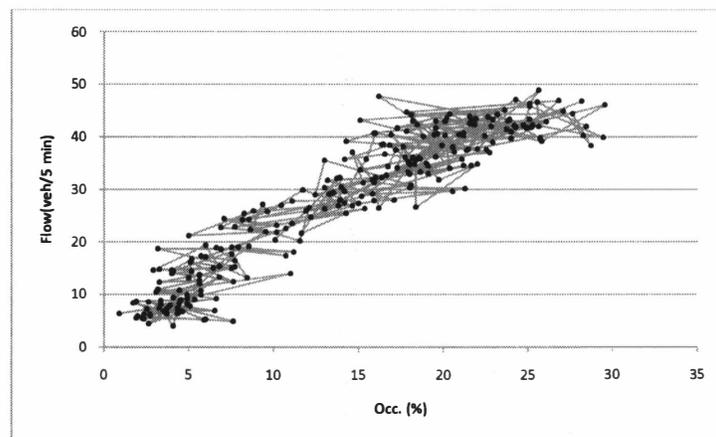
Figure 5.10: MFD for a route in downtown Yokohama

There are 12 intersections along the road controlled by a central signal system which responds to the traffic condition. As we have the data of 11 detectors along the road, we can draw a space-occupancy plot for time intervals with same average occupancy (see figure 5.11).

The average occupancies for the 3 time intervals are all 25.09% and as we can see from the figure that the congestion levels are quite similar for individual detectors, very different from what we have observed in freeway route (see figure 4.11).

As there are only a handful data points available and the occupancies are widely distributed in arterial network due to the signal control, we cannot compare the occupancy distribution here. But what we have can still provide us some hints about hysteresis
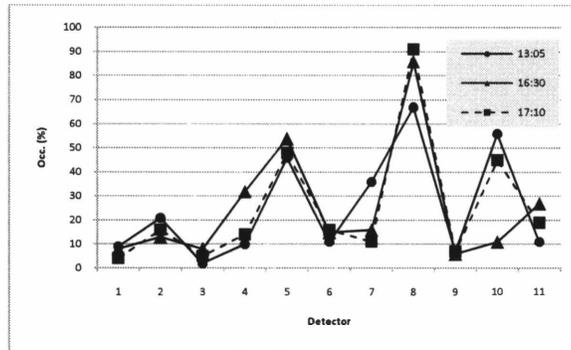
Figure 5.11: Space-Occ. plot for a route in arterial network

exists in freeway route. In arterial network, every time an average occupancy appears, the occupancies for a certain individual detector should around some values. That is the ratio between occupancy of individual detectors and average occupancy should be around some fixed value. In contrast to those in arterial network, the occupancies of individual detectors on freeway network can strongly negatively relate to its nearby detectors. For a given average occupancy, the occupancy of a certain detector can be very high or very low depending on the stage of traffic congestion transition.

This is also physically meaningful. For arterial network, it is very highly redundant. When one node becomes congested, drivers can easily switch to other route. Thus the congestion distributed quickly across the network and the congestion level of individual detectors is positively related to the congestion level of the network. When it comes to freeway network, the possible routes for a certain O-D pair are limited. It is very likely that the upstream get congested, so the occupancy of the upstream is high but the input flow from the upstream is low, which results in the low occupancy in the downstream section. Or the downstream gets congested but the input flow of upstream is small and the occupancy at upstream is low. Both of these two scenarios can result in same average occupancy as when the congestion is evenly distributed along the route, but much smaller flow. Although physically meaningful, this ratio hypothesis needs further scrutiny by data.

Chapter 6

Conclusion

This final chapter summarizes the results of this thesis and discusses some directions for future work.

6.1 Summary

In this thesis, we have investigated the properties of MFD by comparing two different networks, one arterial network with well-defined MFD and one freeway network that exhibits hysteresis phenomenon.

In chapter 3, we first test the homogeneity assumptions [9] in the arterial network where exists a well defined MFD. We find out that for a certain value of network average occupancy the individual detector occupancy distributions would be the same but with some small errors. This finding is confirmed by statistical tests followed. This result relaxed the homogeneity assumption where evenly distributed congestion across the network is assumed and pointed out the similarity of distribution of a given network average occupancy for a well-defined MFD.

Having found out that the distribution of individual detector occupancy is different from binomial distribution, a mixed distribution model is proposed to describe the distribution observed from real data. This model takes into account the correlation between links in a traffic network by assuming the probability of having a vehicle in a cell of one link depends on the number of vehicles in the upstream link. Factors like strong turning movement and ramps that make links uncorrelated are modeled by a

probability parameter that decides the chance the probability of the cell having a vehicle in one link depends on the network average occupancy instead of number of vehicles in the upstream link. The proposed model can describe the empirical distribution much better than binomial distribution, especially when the network becomes congested.

Following this, an analytical approach to estimate the errors is proposed and then illustrated by the application to the data from downtown Yokohama where a well defined MFD exists.

In chapter 4, data from freeway network that is quite different from arterial network is investigated. Unlike that in arterial network where there is a well-defined MFD, hysteresis phenomenon is observed in the fundamental diagram that describes the relationship between aggregated occupancy and flow. Statistical tests are carried out between distributions of individual detector occupancy at different time intervals but with same network average occupancy. It is found out that the distributions for the same network average occupancy would be different when hysteresis phenomenon appears.

To investigate the hysteresis phenomenon in freeway network, relationship between average occupancy and flow for different scales of network is studied, from large freeway network to compact freeway network, from a route of freeway to a single detector. Hysteresis phenomenon persists all the time. One interesting point we noticed is that the hysteresis tends to become more obvious as we zoom in. We conjecture it results from the heterogeneity of the network. Some parts of the network never reach the congested part and thus don't exhibit hysteresis. When we aggregate for the larger network, hysteresis becomes small.

In chapter 5, properties of MFD are further investigated. Spatial distributions of congestion in freeway network are first studied. It is found out that the congested areas in the morning are different from those in the afternoon. Although they are similar for onset and offset of the congestion, the congestion would be more concentrated when lower flow is observed for the same network average occupancy.

Then fundamental diagrams for individual detectors from both arterial network and freeway network are investigated. The result shows that not all the individual detectors have the same fundamental diagrams. But how detectors with different shapes of fundamental diagrams distributed among the network needs further study.

In section 5.3 we looked at variance of individual detector occupancy, which should

be an important indicator of the spatial distribution of congestion in a network. Two interesting things are observed there. 1) Both the average occupancy and occupancy variance of arterial network are generally higher than those of the freeway network. 2) In freeway network, the variance reaches its highest value around the peak of congestion. While in the arterial network, the variance is smaller at the peak of congestion than those at onset of congestion and offset of congestion. The relationship between occupancy and variance might play as a hint to detect the type of network that gives well-defined MFD.

Finally, a route from arterial network is analyzed and compared to a route from freeway network. The result of the comparison again suggests the heterogeneity as a factor of hysteresis phenomenon.

6.2   Future Work

Although many works about macroscopic modeling of traffic system have been done before this thesis and the properties of MFD were investigated here, we have to point out that this approach may not be the solution for every network. The application of macroscopic modeling based approaches to traffic congestion problem still needs more efforts.

First, more empirical studies of real world networks are needed. The studies of macroscopic fundamental diagrams are still sparse. It might be hard to investigate in the past due to the availability of data over a large network. As the application of advanced information technology to the transportation area increases, it would be easier for engineers to investigate the network scale properties of traffic system. As we can see from the example of Yokohama [19], the aggregated variables may have much well defined behavior than individual ones. These benefits still needs more scrutiny from real world, as we can see from the freeway example in this thesis that a well defined MFD does not always exist for any large network.

For a large network with areas of different traffic demand, we might explore a way to divide it into several smaller areas, each of which is more homogenous. We call this partitioning strategy. It is expected that homogenous network should have behave more regular than heterogeneous network, thus is easy for us to apply control strategies. Also, we may want to avoid too many regions because the route choice can play important

role when regions are too small and it can change and not be predicted easily.

The framework of this macroscopic modeling approach is only meaningful when control strategies can apply accordingly. Control strategies such as pricing and modifying signals can be used to improve the performance (ratio of vehicle-miles traveled and vehicle-hours traveled) of the system by keeping accumulations in their sweet-spots. Policies regarding to the allocate sources between different modes should be also studied to ensure the "win-win" situation. When applying the control strategies, some critical values are crucial in the application (e.g. the critical values are used to decide when to restrict the traffic flow into an area or when to encourage traffic towards some directions). This might be the critical average density, average occupancy or average flow speed. The application of control strategies may not be easy to apply. But a better understanding of the properties of aggregated traffic variables should provide some helps.

The monitor system is what the application of our approach depends on. Modern technology makes the real time monitor over a large network possible. For example, sensors such as loop detectors and GPS equipped vehicles can be used to collect real time data, which can be transferred over satellites or internet immediate. More researches are required to have better understanding of the properties of those information sources, so that engineers can estimate the traffic state more accurately.

Finally, topologies of different networks, which have been received relatively less attention, should be another direction for future study. Currently, we always assume the same link length or road width when modeling the network, which is not the case in the real world (e.g. figure 2.1 and 2.3). The geometries of the network can affect the fundamental diagram as showed by Wardrop[13]. Thus, how different topologies can affect the MFD deserves more researches.

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