A NOTE ON SHADOW PRICING WITH FIXED TAXES

by

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1. Introduction

Several studies have recently attempted to provide guidelines and
general principles for the calculation of shadow prices for benefit-cost
analysis, the most important being Little and Mirrlees (1969) and Dasgupta,
Marglin and Sen (1972). However, since these studies are deficient in
their specification of the economic model on which the advocated shadow
pricing procedures are based, the welfare basis of their recommendations
is frequently unclear, as are the underlying behavioral assumptions. The
recent paper by Dasgupta and Stiglitz (1974)--henceforth DS--has made an
important contribution to this literature by attempting to derive shadow
pricing rules from an explicit economic optimization model, thus throwing
light on the nature of the welfare calculus involved and giving a more
rigorous basis to recommendations on the use of shadow prices.

This note takes issue with the following three propositions asserted
in the concluding section of the DS paper (p. 30).

P.1. On the valuation of tradeables:

> The shadow price of a tradeable commodity is its inter-
> national price unless . . . there is a government budget-
> ary constraint; . . .

> When there is a government budgetary constraint the
> shadow price of a tradeable lies between the world price
> and its domestic price.

P.2. On the valuation of non-tradeables:

> . . . non-tradeables ought to be valued at their
> "foreign exchange" equivalent. That is the value
> of the foreign exchange that would be earned if
> one less unit of the given non-tradeable were pro-
> duced and the resources diverted to the production
> of tradeables.

P.3. On the applicability of the DS shadow pricing results:

> What is particularly useful about these results is
> that they show the project evaluator what to do even
> in a "second-best" (optimally chosen tariffs and
> taxes) or "third-best" (non-optimally chosen tariffs
> and taxes) situation.

In section 2 we show that P.1 is incorrect except in a special case and that, except for this case, tradeables should be valued at their international prices irrespective of whether there is a government budgetary constraint. Section 3 shows that P.2 is quite misleading in the form stated by DS, but that when properly interpreted this rule is not operationally useable. Section 4 explores the basis for P.3 and argues that this claim is not justified by the DS analysis since the latter ignores the existence of distortions due to non-optimal taxes and tariffs. An alternative analysis is presented from which it is shown that while P.3 is essentially correct in the case of tradeables it is false in the case of non-tradeables.[1]

Except where indicated we will, for brevity, adopt the same notation, basic model and assumptions as DS (pp. 5-9), to which the reader should now refer for definitions of symbols, etc. In particular it is assumed that the only "distortions" in the economy are those due to government tax policy but that from the point of view of government project evaluators all taxes and tariffs have to be taken as given and fixed.[2] The public

sector is presumed to be producing the same commodities as the private
sector, so the analysis is concerned only with the valuation of private
goods. Public goods are ignored. Note also that the international
prices of traded commodities are set at unity.

If public projects are given the shadow price vector s it is
assumed that designers, evaluators and managers of public projects will
attempt to maximize s·x, the value of public production at these prices,
with respect to the public production vector x, subject to $G(x) = 0$, the
implicit production function obtaining in the public sector.[3] Confining
ourselves to interior solutions, as DS implicitly do, this requires

$$\frac{\partial G/\partial x_i}{\partial G/\partial x_o} \equiv \frac{G_i}{G_o} = s_i \ , \qquad i, \ 0 \ \epsilon \ D \ ,$$

where the shadow price of the numeraire, commodity 0 (assumed to be traded,
domestically produced and consumed), is set at unity. The objective of our
exercise, then, is to find the value of G_i/G_o at the optimum and set shadow
prices equal to this ratio.

2. <u>Valuation of Tradeables When There is a Government Budgetary Constraint</u>

Suppose there is a government budgetary constraint meaning, as DS put
it, "a constraint on the government budget deficit in any given year" (p. 28).
We now consider its implications for public projects. When a public project
buys or sells a commodity this affects the government's budgetary deficit
in two ways. First is its effect through the financial balance sheet of
public projects, and second is its effect through the generation of tax
revenues. For example, suppose a project buys a tractor (a tradeable) on
the domestic market from an import agent. The effect of this on the

government's budget deficit is the domestic price paid <u>minus</u> the effect of the transaction on tax revenues--the tariff and any sales tax. The latter items constitute both an expenditure and a receipt for the public sector as a whole; at most these taxes are simply an intra-governmental transfer of funds and their payment by projects has no effect on the size of the government's budgetary deficit. In formulating the budgetary constraint actually facing public sector production we can make either of two assumptions:

A.1. the effects of project activity on the generation of tax revenues are recognized; or

A.2. these effects are ignored. Under A.1 the budget constraint would be formulated $\hat{q} \cdot x \geq b$, where $\hat{q}$ is the vector of domestic prices <u>net</u> of the effects that public use (production) of the various commodities has on the generation of tax revenues. In the case of non-tradeables this involves the general equilibrium effects of public sales or purchases of those commodities, but in the case of tradeables the matter is more straightforward. Since it has been assumed that the only "distortions" in the economy are those due to tax policy it must be true that, for tradeables k and 0, $\hat{q}_k/\hat{q}_o = 1$, the ratio of their international prices.

A.2 implies that the budgetary constraint would be formulated $q \cdot x \geq b$, where q is simply the vector of domestic prices actually paid and received by public projects.[4] Since this is the formulation that DS employ, it seems that A.2 is the case they have in mind, although this assumption is not made explicit by them. In practical circumstances this myopic kind of budgetary constraint may frequently be the kind actually faced by public projects, but this is not really "a constraint on the government budget

deficit," as DS state, at all. It is <u>a constraint on the financial deficit</u> <u>of public projects expressed in domestic market prices</u>, and this constraint may exist with or without the presence of a government budgetary constraint. The two are quite distinct.

Now consider the implications of A.1. We denote the dual variable associated with the constraint corresponding to A.1 by $\hat{\delta}$, append this new constraint to the basic Lagrangian,[5] and derive the first-order conditions for optimal public production of the tradeables k and 0. Again confining ourselves to an interior solution this gives

$$\frac{G_k}{G_0} = \frac{\lambda + \hat{\delta}\hat{q}_k}{\lambda + \hat{\delta}\hat{q}_0} , \qquad k, \, 0 \in T \cap D , \qquad (1)$$

We now recall that when the domestic market prices of goods k and 0 are distorted solely by tariffs and other taxes, $\hat{q}_k/\hat{q}_0 = 1$. Equation (1) now reduces immediately to $G_k/G_0 = 1$.

Thus in this case the optimal shadow prices of tradeables are their international prices irrespective of whether there is a government budgetary constraint or not. P.1 is therefore incorrect. The existence of a government budgetary constraint is <u>not</u> a sufficient reason for setting the shadow prices of tradeables at values other than their relative international prices. Two corrected versions of P.1 are that the relative shadow prices of tradeables lie between their world prices and their domestic prices:

P.1'. when there is a financial budgetary constraint on public projects expressed in domestic market prices; or

P.1". when there is a government budgetary constraint, and the domestic

prices of those goods are distorted for reasons other than government tax policy (e.g., import quotas, or monopoly power exercised by import agents with exclusive import licenses).

3. Valuation of Non-Tradeables

We now turn to the necessary conditions for optimal public production of the non-tradeable good n. We will assume for simplicity that there is no binding government budgetary constraint. Proceeding as before we obtain

$$\frac{G_n}{G_o} = \frac{\rho_n}{\lambda} , \qquad n \in N \cap D , \qquad (2)$$

where ρ_n is the Lagrangian multiplier corresponding to the constraint that total production must equal total use of commodity n, and λ is the shadow price of foreign exchange. DS interpret this result by rearranging to obtain

$$\rho_n = \lambda \frac{G_n}{G_o} , \qquad (3)$$

which then becomes the basis for P.2. Thus DS seem to have provided a justification for the "foreign exchange equivalent" method of valuing non-tradeables previously advocated by Little and Mirlees (1969).

There is a peculiarity about this result, however. Equation (2) is a characterization of the relationships holding at the optimal solution to the optimization problem we are considering and holds only at the optimal solution. Denoting the optimal public production vector by x*, equation (2) is a statement about $G_n(x^*)/G_o(x^*)$. But since the purpose of our exercise is to find this ratio it could hardly be used, as in equation (3), as a datum for computing the optimal value of ρ_n. If $G_n(x^*)/G_o(x^*)$ were known

ex ante there would be no need to calculate ρ_n nor, for that matter, for

any of the present analysis, since the desired shadow prices would already

be known. Undoubtedly DS are aware that equation (3) is correct only when

G_n/G_o is evaluated at the optimum, so perhaps P.2 is intended merely as a

characterization of the optimal solution rather than as an operational rule

for finding it. So interpreted P.2 would be a formally correct statement

about equation (2), but operationally empty.

As it stands, however, P.2 seems to mean that G_n/G_o is to be evaluated

at its currently observable (and presumably non-optimal) level. This is

wrong. If we denote the present non-optimal public production vector by x^o

then in general $G_n(x^*)/G_o(x^*) \neq G_n(x^o)/G_o(x^o)$. Unless public production is

already optimal, in which case the shadow pricing exercise is hardly neces-

sary, both P.2 and the Little-Mirrlees method are erroneous procedures for

the calculation of the shadow prices of non-tradeables. The central

problem with "foreign exchange equivalent" procedures is the rate of trans-

formation that is to be used in converting non-tradeables into tradeables.

The above argument shows that the correct rate to use is the rate obtaining

in the public sector at the optimum, not the distorted rates currently

obtaining in the public sector or the private sector, and the former cannot

be known without foreknowledge of the optimal shadow prices.

Clearly the correct interpretation of equation (2) requires us to

interpret the meaning of the Lagrangian multipliers ρ_n and λ, utilizing

the facts that at the optimum $\rho_n = \partial V/\partial x_n$ and $\lambda = \partial V/\partial B$, where V is the

objective function being maximized and B is the maximum acceptable depletion

of foreign exchange reserves.[6] We will not pursue this here, but it is

important to note the difference that this illustrates between the shadow

pricing problems posed by tradeables and non-tradeables. In the case of tradeables the optimal shadow prices (international prices) are independent of the precise form of the social welfare function being maximized and of the production relations obtaining in the domestic economy. But the optimal shadow prices of non-tradeables do not have this convenient property; they are dependent on both.

4. The Implications of Fixed Non-Optimal Taxes

So far the existence of market distortions in the private sector has not affected the nature of the results since, in effect, they have been ignored. This is disturbing since the existence of market distortions, implying that market prices are potentially misleading guides for decision-making in the public sector, is what motivates our interest in shadow prices. If project evaluators must take these distortions as given, can we be sure that shadow pricing rules whose adoption by both public and private sector producers would be socially optimal are still optimal when adopted only in the public sector? It is in fact a standard procedure in the literature on shadow pricing to ignore these "second-best" issues by assuming implicitly that it is legitimate to derive the necessary conditions for optimal production in the public sector while ignoring the non-achievement of the necessary conditions for socially optimal production in the private sector.[7]

For example, in the section of their paper where all taxes and tariffs are taken as fixed,[8] possibly at non-optimal levels, DS point out that this implies that the production vectors of private firms have also to be taken as fixed, also at non-optimal levels. The subsequent analysis then ignores the existence of these distortions. The non-achievement of the

necessary conditions for optimality in private sector production are not incorporated into the basic Lagrangian as constraints on the optimization problem facing public projects. But this entails the _implicit assumption_ that the particular non-optimal levels at which taxes and tariffs are fixed make no difference to the shadow pricing problem facing public projects, and yet this is precisely what P.3 claims as a _result_ of the DS analysis. This is claiming too much. Since the distortions concerned were left out of the model the result could hardly have been otherwise.

To analyze the implications of tax-induced distortions for shadow pricing in the public sector we will introduce them into the model explicitly as deviations of the production behavior of private firms from socially optimal behavior. Returning to our basic Lagrangian, consider the necessary conditions for the socially optimal behavior of private firms.[9] For the jth firm with the implicit production function $F^j(y^j) = 0$ we obtain, for tradeables k and 0,

$$\frac{F^j_k}{F^j_o} = 1 , \qquad j = 1, 2, \ldots , m; \; k, 0 \in T \cap D . \qquad (4)$$

For a non-tradeable, n, we have, proceeding similarly,

$$\frac{F^j_n}{F^j_o} = \frac{\rho_n}{\lambda} , \qquad j = 1, 2, \ldots , m; \; n \in N \cap D . \qquad (5)$$

Clearly, equations (4) and (5) imply exactly the same first-order conditions for socially optimal production in the private sector as were previously obtained for optimal production in the public sector. But suppose that profit maximizing firms in fact equate these marginal rates of

transformation (substitution) to <u>non-optimal</u> (tax-distorted) relative

domestic prices. For simplicity we will represent this as follows.[10]

In the production of tradeables,

$$\frac{F_v^j}{F_o^j} = g_v \, , \qquad\qquad j = 1, 2, \ldots , m; \; v, 0 \in T \cap D \, , \qquad (6)$$

and, in the production of non-tradeables,

$$\frac{F_w^j}{F_o^j} = g_w \frac{\rho_w}{\lambda} \, , \qquad j = 1, 2, \ldots , m; \; w \in N \cap D \, . \qquad (7)$$

Obviously, g_v and g_w equal unity when all taxes and tariffs are optimally

chosen, and are either greater or less than unity when this is not the case.

We now add the terms

$$\sum_v \sum_{j=1}^m \alpha_v^j (F_v^j / F_o^j - g_v)$$

and

$$\sum_w \sum_{j=1}^m \alpha_w^j (F_w^j / F_o^j - g_w \rho_w / \lambda)$$

to the basic Lagrangian of DS (p. 23), where α_v^j and α_w^j are Lagrangian

multipliers. We then differentiate with respect to the vector x as

before and obtain the first-order conditions for public production of

tradeables k and $0,$[11]

$$\frac{G_k}{G_o} = \frac{\lambda + \sum_w g_w \, \partial(\rho_w/\lambda)/\partial x_k \left(\sum_j \alpha_w^j \right)}{\lambda + \sum_w g_w \, \partial(\rho_w/\lambda)/\partial x_o \left(\sum_j \alpha_w^j \right)} \, , \qquad (8)$$

and the non-tradeable n

$$\frac{G_n}{G_o} = \frac{\lambda + \sum_w g_w \ \partial(\rho_w/\lambda)/\partial x_n \left(\sum_j \alpha_w^j\right)}{\lambda + \sum_w g_w \ \partial(\rho_w/\lambda)/\partial x_o \left(\sum_j \alpha_w^j\right)} \ . \tag{9}$$

We analyse first the case of tradeables. Consider the following term appearing in equation (8)

$$\frac{\partial(\rho_w/\lambda)}{\partial x_k} = \frac{1}{\lambda} \left(\frac{\partial \rho_w}{\partial x_k} - \frac{\rho_w}{\lambda} \frac{\partial \lambda}{\partial x_k}\right) \ . \tag{10}$$

We note that public production (use) of tradeables affects ρ_w and λ only through its contributions to (depletion of) foreign exchange earnings. If we define f^x to be the net contribution of public production to total foreign exchange earnings then, recalling that international prices are set at unity,

$$f^x = \sum_{k \in T} x_k$$

and it follows that

$$\frac{\partial \rho_w}{\partial x_k} = \frac{\partial \rho_w}{\partial f^x} \frac{\partial f^x}{\partial x_k} = \frac{\partial \rho_w}{\partial f^x} \ ,$$

and likewise $\partial \lambda/\partial x_k = \partial \lambda/\partial f^x$. Substituting into equation (10), this implies immediately that

$$\frac{\partial(\rho_w/\lambda)/\partial x_k}{\partial(\rho_w/\lambda)/\partial x_o} = 1 \ ,$$

which in turn implies that equation (8) reduces to our earlier result,

$G_k/G_o = 1$. Thus international prices should be used to value tradeables irrespective of whether domestic prices are distorted by non-optimal taxes and tariffs.[7] In the case of tradeables, P.3 is therefore correct.

We now turn to the case of non-tradeables. Proceeding similarly, consider the terms appearing in equation (9)

$$\frac{\partial(\rho_w/\lambda)}{\partial x_n} = \frac{1}{\lambda}\left(\frac{\partial\rho_w}{\partial x_n} - \frac{\rho_w}{\lambda}\frac{\partial\lambda}{\partial x_n}\right). \tag{11}$$

Here the analogy with tradeables ends, since it is not true that the public production of non-tradeables affects ρ_w and λ only through a single mechanism like the generation of foreign exchange. Furthermore, we have no reason for believing that the terms $\partial\rho_w/\partial x_n = \partial^2 V/\partial x_n\partial x_w$ and $\partial\lambda/\partial x_n = \partial^2 V/\partial x_n\partial B$ are zero. The social valuations of the non-tradeable good w and of foreign exchange will, in general, be affected by public production of the non-tradeable n.

How important these effects are likely to be is an open question. At this level of generality little can be said about this. The point here is that equation (9) does not reduce to equation (2). The correct shadow pricing rules for non-tradeables are indeed changed by the existence of non-optimally chosen domestic taxes. Equation (2) gives a correct shadow pricing rule, even when properly interpreted, only when there are no non-optimally chosen taxes on non-tradeables. Note that it is tax-induced distortions in domestic markets for non-tradeables, not tax or tariff-induced distortions in the domestic markets for tradeables, that cause the problem. Thus, while the claim embodied in P.3 has been verified for the shadow pricing of tradeables, we have shown it to be false for the shadow pricing of non-tradeables.

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FOOTNOTES

*This paper has benefited from correspondence with Partha Dasgupta and discussions with Walter P. Falcon, George Feiger, David Starrett and C. Peter Timmer. The author is responsible for all views and any errors it contains.

1. In this paper we follow DS in treating the distinction tradeable/ non-tradeable as synonymous with the distinction traded/non-traded. For a discussion of the important issue this raises see Sen (1972).

2. The first of these assumptions is made throughout the DS paper while the second corresponds to their section 5. The latter assumption will become crucial in our section 4.

3. Non-convexities in production sets are ruled out.

4. We assume, of course, that public projects actually pay the taxes and tariffs levied on the commodities concerned. Otherwise the results of this section follow trivially.

5. This is equation (2.12) of DS, p. 11.

6. See Warr (1974) for a detailed analysis of these derivatives.

7. The author should acknowledge that Warr (1973) also makes this assumption, although the basic results of that paper are unaffected by the necessary modification.

8. That is, section 5.

9. Again, non-convexities in production sets are ruled out.

10. It is a simple matter to allow for the existence of firm-specific distortions but this merely clutters the notation and does not change the nature of the results in any essential way.

11. Equations (8) and (9) assume that the distortions g_v and g_w and the Lagrangian multipliers α_v^j and α_w^j are not themselves affected by public production of tradeables and non-tradeables, respectively. See Negishi (1972).

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