

**DRYING OF GELATIN ASYMPTOTICALLY IN
PHOTOGRAPHIC FILM**

By

Chaocheng Huang

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Drying Of Gelatin Asymptotically In Photographic Film

Chaocheng Huang

*School of Mathematics, University of Minnesota
Minneapolis, MN 55455*

Abstract

In this paper we study a nonlinear parabolic equation with mixed nonlinear boundary condition: $\partial u/\partial t = \partial[\exp(-1/u)\partial(u/(1+u))/\partial x]/\partial x$. This equation is raised from industry of film development. We show that there exists a unique positive solution which converges to 0 asymptotically.

1 The Problem and Main Result

In a recent issue of SIAM News [2], Ross proposed a problem connected to drying of gelatin in photographic film. A photographic film usually contains several gelatin layers. When one develops an exposed film, water diffuses into gelatin layers. The gelatin first swells and then begins to dry. For more detailed description of film developing process, we refer to [1] and [4]. The drying process has been modeled by Ng and Ross [6] as follows. let x be the Lagrangian coordinate attached to the gelatin, t the time, and $u(x, t)$ the volume of fraction of water. Assume that the medium is pure gelatin and that the diffusion coefficient converges exponentially to 0 in dry gelatin. Then the volume $u(x, t)$ satisfies the equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(D(u) \frac{\partial}{\partial x} \left(\frac{u}{1+u} \right) \right), \quad 0 < x < L, \quad t > 0, \quad (1.1)$$

where

$$D(u) = D_0 \exp\left(-\frac{1}{u}\right),$$

and the initial and boundary conditions

$$u(x, 0) = u_o(x), \quad 0 \leq x \leq L, \quad (1.2)$$

$$\frac{\partial u}{\partial x} = 0, \quad x = 0, \quad t > 0, \quad (1.3)$$

$$D(u) \frac{\partial u}{\partial x} + ku = 0, \quad x = L, \quad t > 0, \quad (1.4)$$

where L is the thickness of the gelatin layer in its dry state, D_o , k are positive constants, and $u_o(x)$ is a positive function.

This is a parabolic problem which might be singular at some points (x_o, t_o) where $u(x_o, t_o) = 0$. We define a classical solution of problem (1.1)–(1.4) as a bounded function $u(x, t)$ in $\mathcal{Q} = \{(x, t) \in R^2 : 0 \leq x \leq L, t \geq 0\}$ for which all the derivatives appearing in equation (1.1) exist in the interior of \mathcal{Q} and can be continuously extended to the parabolic boundary of \mathcal{Q} except for the two lower corners $x = 0, t = 0$ and $x = L, t = 0$, and satisfy (1.1)–(1.4).

In [2] Ross raised the following question: Does the gelatin dry asymptotically? That is, does

$$\lim_{t \rightarrow +\infty} u(x, t) = 0?$$

In this work we prove:

Theorem 1.1 *For any smooth positive initial data $u_o(x)$, there exists a unique positive solution $u(x, t)$ of problem (1.1)–(1.4). Moreover the solution $u(x, t)$ converges to 0 uniformly as $t \rightarrow +\infty$.*

We also show that any non-negative classical solution can not equal zero in finite time. This fact combined with Theorem 1.1 means that the gelatin does not dry anywhere in finite time, but will dry asymptotically as $t \rightarrow +\infty$.

2 The Approximate problems

By rescaling, we may assume that $L = D_o = 1$. Throughout the paper we assume that $u_o(x)$ is smooth positive function in $0 \leq x \leq 1$. Since equation (1.1) might be degenerate whenever $u(x, t) = 0$ (we shall show later that in fact problem (1.1)–(1.4) is non-degenerate), and the initial and boundary conditions (1.2), (1.4) are incompatible at the two lower corners, we begin with introducing approximate problems which are obtained by modifying the initial condition (1.2) and the boundary condition (1.4) in the following way.

For any positive integer N such that $Nu_o(1) > 1$, let $\{\delta_n(x)\}_{n \geq N}$ be a sequence of smooth function in $0 \leq x \leq 1$ such that

$$0 \leq \delta_n(x) \leq 1, \quad \text{for } 0 \leq x \leq 1,$$

$$\left| \frac{d\delta_n(x)}{dx} \right| \leq \sup_{0 \leq x \leq 1} \left| \frac{du_o(x)}{dx} \right|, \quad \text{for } 0 \leq x \leq 1,$$

$$\frac{d\delta_n(0)}{dx} = -\frac{du_o(0)}{dx}, \quad \frac{d\delta_n(1)}{dx} = -\frac{du_o(1)}{dx},$$

$$\delta_n(x) \leq \delta_m(x), \quad \text{if } n > m,$$

$$\delta_n(x) \longrightarrow 0, \quad \text{as } n \longrightarrow +\infty, \quad \text{uniformly for } 0 \leq x \leq 1.$$

We then choose another sequence of positive smooth function $\epsilon_n(t)$ in R^1 such that

$$\epsilon_n(t) = \begin{cases} u_o(1) + \delta_n(1), & \text{if } t \leq 0, \\ \frac{1}{n}, & \text{if } t \geq \frac{1}{n}, \end{cases}$$

$$\frac{d\epsilon_n(t)}{dt} \leq 0, \quad \text{for all } t,$$

$$\epsilon_n(t) \leq \epsilon_m(t), \quad \text{if } n > m,$$

$$\epsilon_n(t) \longrightarrow (u_o(1) + \delta_n(1))\mathcal{X}_{(-\infty, 0]}, \quad \text{as } n \longrightarrow +\infty,$$

where $\mathcal{X}_{(-\infty, 0]}$ is the characteristic function of the interval $(-\infty, 0]$. The approximate problem for problem (1.1)–(1.4) is defined as equation (1.1) together with the boundary conditions (1.3),

$$D(u) \frac{\partial u}{\partial x} + k(u - \epsilon_n(t)) = 0, \quad x = 1, \quad t > 0, \quad (2.1)$$

and the initial condition

$$u(x, 0) = u_o(x) + \delta_n(x), \quad 0 \leq x \leq 1. \quad (2.2)$$

In the special case where the initial data $u_o(x)$ in the original model is constant, we choose $\delta_n(x) \equiv 0$, and hence the approximate problem is identical with problem (1.1)–(1.3), (2.1). We also point out that the functions $\delta_n(x)$ and $\epsilon_n(t)$ depend only on the initial data $u_o(x)$. For any two initial datum $u_o(x)$ and $\tilde{u}_o(x)$ such that $u_o(x) \leq \tilde{u}_o(x)$, we can choose $\{\delta_n(x), \epsilon_n(t)\}$ and $\{\tilde{\delta}_n(x), \tilde{\epsilon}_n(t)\}$, corresponding to $u_o(x)$ and $\tilde{u}_o(x)$ respectively such that $\epsilon_n(t) \leq \tilde{\epsilon}_n(t)$, $\delta_n(x) \leq \tilde{\delta}_n(x)$. We will use this fact later on.

The modified problem (1.1), (1.3), (2.1), (2.2) essentially is non-degenerate parabolic problem, and the initial and boundary conditions satisfy the compatibility condition. To study this approximate problem, we introduce some function spaces. Set $\mathcal{Q}_T = \{(x, t) \in R^2 : 0 < x < 1, 0 < t < T\}$, for any $T > 0$, and $\mathcal{Q} = \cup_{T>0} \mathcal{Q}_T$. Let $C^\alpha(\mathcal{Q}_T)$, for any $0 \leq \alpha < 1$, denote the Banach space of uniformly Hölder continuous functions $u(x, t)$ which has finite α -norm

$$\|u\|_{C^\alpha} = \sup_{\mathcal{Q}_T} |u| + \sup_{\mathcal{Q}_T} \frac{|u(x, t) - u(y, s)|}{d((x, t), (y, s))^\alpha},$$

where $d((x, t), (y, s)) = (|x - y|^2 + |t - s|)^{\frac{1}{2}}$ is the parabolic distance. Let $C^{2+\alpha}(\mathcal{Q}_T)$ denote the Banach space of all functions $u(x, t)$, for which the norm

$$\|u\|_{C^{2+\alpha}} = \sup_{\mathcal{Q}_T} |u| + \left\| \frac{\partial u}{\partial x} \right\|_{C^\alpha} + \left\| \frac{\partial u}{\partial t} \right\|_{C^\alpha} + \left\| \frac{\partial^2 u}{\partial x^2} \right\|_{C^\alpha}$$

is finite. Using the standard techniques for quasilinear parabolic equations, we can show existence of a solution of the approximate problem. In fact we have

Theorem 2.1 *For any positive integer n such that $1/n \leq \max_{0 \leq x \leq 1} u_o(x)$, there exists a unique classical solution $u_n(x, t)$ of approximate problem (1.1), (1.3), (2.1), (2.2) in \mathcal{Q} . The solution satisfies $1/n \leq u_n(x, t) \leq \max_{0 \leq x \leq 1} u_o(x)$. Moreover, if $u_o(x)$ is constant, then the solution $u_n(x, t)$ satisfies*

$$\epsilon_n(t) \leq u_n(x, t) \leq \max_{0 \leq x \leq 1} u_o(x), \quad \frac{\partial u_n}{\partial t} \leq 0, \quad -k \leq \exp\left(-\frac{1}{u_n}\right) \frac{\partial u_n}{\partial x} \leq 0.$$

Proof. Fix a $T > 0$. We choose $F_n(u)$ to be a smooth function such that $F_n(u) = u$ for $u \geq 1/n$, and $F_n(u) \geq 1/2n$ for $u < 1/n$. By a well-known result (cf. [5], Theorem V.7.4), there exists a solution $u_n(x, t) \in C^{2+\alpha}(\mathcal{Q}_T)$ of the equation

$$\frac{\partial u}{\partial t} = a(u) \frac{\partial^2 u}{\partial x^2} + b(u) \left(\frac{\partial u}{\partial x} \right)^2, \quad (2.3)$$

with initial and boundary conditions (1.3), (2.1), (2.2), where

$$a(u) = \frac{1}{(1 + F_n(u))^2} \exp\left(-\frac{1}{F_n(u)}\right),$$

$$b(u) = \frac{(1 + 2F_n(u))(1 - F_n(u))}{(1 + F_n(u))^3 F_n(u)^2} \exp\left(-\frac{1}{F_n(u)}\right)$$

are two bounded functions of u . Applying the maximum principle, we can derive that $1/n \leq u_n(x, t) \leq \max_{0 \leq x \leq 1} u_o(x)$. Hence this solution $u_n(x, t)$ actually is a solution of the approximate problem. Suppose now that $u_o(x)$ is constant. without loss of generality we may assume that $u_o(x) = 1$. In this case, since $u_n(x, 0) - \epsilon_n(0) = 0$ (recalling that $\delta_n(x) = 0$) and the fact that $u_n - \epsilon_n$ is a supersolution of (2.3), we get that $u_n(x, t) \geq \epsilon_n(t)$. Set $v = \exp(-1/u_n) \partial u_n / \partial x$. Then, by direct computation, we can see that $v(x, t)$ solves a non-degenerate linear parabolic equation and satisfies the following initial and boundary conditions

$$v(x, 0) = 0, \quad 0 \leq x \leq 1,$$

$$v(0, t) = 0, \quad t > 0,$$

$$-k \leq v(1, t) \leq 0, \quad t > 0.$$

It follows from the maximum principle that

$$-k \leq \exp\left(-\frac{1}{u_n}\right) \frac{\partial u_n}{\partial x} \leq 0.$$

To show that $\partial u_n / \partial t \leq 0$, we set $w = \partial u_n / \partial t$. Then $w(x, t)$ satisfies a parabolic equation with the initial and boundary conditions

$$\begin{aligned} w(x, 0) &= 0, \quad 0 \leq x \leq 1, \\ \frac{\partial w}{\partial x} &= 0, \quad x = 0, \quad t > 0, \\ \frac{\partial w}{\partial x} + \left[\frac{1}{u_n^2} \frac{\partial u_n}{\partial x} + k \exp\left(\frac{1}{u_n}\right) \right] w &= k \exp\left(\frac{1}{u_n}\right) \frac{d\epsilon_n(t)}{dt} \leq 0. \end{aligned}$$

The assertion $w \leq 0$ follows from the following comparison lemma:

Lemma 2.1 *Assume that $w(x, t)$ is a solution of following non-degenerate parabolic equation with the bounded coefficients:*

$$\begin{aligned} \frac{\partial w}{\partial t} &= a(x, t) \frac{\partial^2 w}{\partial x^2} + b(x, t) \frac{\partial w}{\partial x} + c(x, t)w, \quad \text{in } \mathcal{Q}_T, \\ w(x, 0) &\geq 0, \quad 0 \leq x \leq 1, \\ \frac{\partial w}{\partial x} &\leq 0, \quad x = 0, \quad 0 < t < T, \\ \frac{\partial w}{\partial x} + \beta(t)w &\geq 0, \quad x = 1, \quad 0 < t < T, \end{aligned}$$

for a bounded function β . Then $w(x, t) \geq 0$, for all $(x, t) \in \mathcal{Q}_T$.

Note that in the above lemma we do not impose the extra restriction $\beta \geq 0$.

Proof. The result is well-known if $\beta(t) \geq 0$. For general function β , we make substitution $v = we^{-Mx^2}$, where M is a constant such that $\sup_{t>0} |\beta(t)| \leq M$. Then the new function $v(x, t)$ solves

$$\frac{\partial v}{\partial t} = a(x, t) \frac{\partial^2 v}{\partial x^2} + \tilde{b}(x, t) \frac{\partial v}{\partial x} + \tilde{c}(x, t)v,$$

where \tilde{b} and \tilde{c} are still bounded in \mathcal{Q}_T , and satisfies the initial and bounded conditions

$$\begin{aligned} v(x, 0) &\geq 0, \quad 0 \leq x \leq 1, \\ \frac{\partial v}{\partial x} &\leq 0, \quad x = 0, \quad 0 < t < T, \end{aligned}$$

$$\frac{\partial v}{\partial x} + (2M + \beta)v \geq 0, \quad x = 1, \quad 0 < t < T.$$

Since $2M + \beta \geq 0$, it follows that $v(x, t) \geq 0$. Hence $w(x, t) \geq 0$.

Another direct consequence of the above lemma is that the solution of the approximate problem is unique in \mathcal{Q}_T , for any $T > 0$. Hence we can extend the local solution u_n to \mathcal{Q} to get a solution in \mathcal{Q} . The proof of Theorem 2.1 is complete.

Corollary 2.1 *Let u_n be the solution of the approximate problem. If $m \leq n$, then $u_n(x, t) \leq u_m(x, t)$, for all $(x, t) \in \mathcal{Q}$.*

Proof. Suppose for $n > m$. Since $\epsilon_n(t) \leq \epsilon_m(t)$, and $\delta_n(x) \leq \delta_m(x)$, we get that $u_m(x, 0) - u_n(x, 0) \geq 0$ and that

$$\exp\left(-\frac{1}{u_m}\right) \frac{\partial(u_m - u_n)}{\partial x} + \beta(t)(u_m - u_n) \geq 0, \quad x = 1, \quad t > 0,$$

where $\beta(t)$ is a bounded function. Applying Lemma 2.1 to $u_m - u_n$ we obtain $u_m(x, t) \geq u_n(x, t)$.

3 Proof of Theorem 1.1

From Theorem 2.1 and Corollary 2.1, we know that there exists a sequence of positive solutions $\{u_n(x, t)\}$ of approximate problem. We also know that $u_n(x, t)$ is decreasing along with the parameter n . Hence there exists a measurable function $u(x, t)$ such that

$$u_n(x, t) \searrow u(x, t), \quad \text{as } n \longrightarrow +\infty,$$

for all $(x, t) \in \mathcal{Q}$. Furthermore $0 \leq u(x, t) \leq \max_{0 \leq x \leq 1} u_o(x)$. From Theorem 2.1, we find that if $u(x, t) \geq \mu > 0$ in a compact subset G of \mathcal{Q} which excludes the two corner points $x = 0, t = 0$ and $x = 1, t = 0$, then the same is true for u_n . Hence in (2.3) the coefficient $a(u_n) \geq (1/4) \exp(-1/\mu) > 0$, for any n in G . Applying the local Schauder estimates for parabolic equations, we can derive that the $C^{2+\alpha}$ -norms of u_n are uniformly bounded in G . It follows that the limit function solves equation (1.1) in any compact set which does not contain the two lower corner points provided that $u(x, t) > 0$ in this set. Hence, to show existence and uniqueness, it suffices to show that $u(x, t) > 0$ everywhere in \mathcal{Q} . We start with studying the simple case in which the initial data $u_o(x)$ is constant.

Lemma 3.1 *Suppose that $u_o(x) \equiv u_o > 0$ is constant. Let $u_n(x, t)$ be the solution of approximate problem (1.1)–(1.3), (2.1). Then*

$$u_n(x, t) \longrightarrow \frac{1}{n}, \quad \text{as } t \longrightarrow +\infty,$$

uniformly in $0 \leq x \leq 1$.

Proof. Since $u_o(x)$ is constant, from Theorem 2.1 we know that $u_n(x, t)$ is decreasing in both t and x directions, for any $0 \leq x \leq 1$, $t \geq 0$. Hence the limit function $u(x, t)$ is also decreasing in both x and t . It follows that the limit

$$\lim_{t \rightarrow +\infty} u_n(x, t) = \phi_n(x)$$

exists, and the limit $\phi_n(x)$ is also decreasing. Integrating equation (1.1) with respect to space variable, we get that for any $0 < x < 1$,

$$\int_0^x u_t(y, t) dy = \frac{\exp(-1/u_n(x, t))}{(1 + u_n(x, t))^2} \frac{\partial u_n}{\partial x}(x, t). \quad (3.1)$$

Since $\partial u_n / \partial x \leq 0$, we integrate equation (3.1) again with respect to x over $(0, 1)$ to derive that

$$\begin{aligned} \frac{\partial}{\partial t} \int_0^1 \int_0^x u_n(y, t) dy dx &= \int_0^1 \frac{\exp(-1/u_n(x, t))}{(1 + u_n(x, t))^2} \frac{\partial u_n}{\partial x}(x, t) dx \\ &= \int_{u_n(0, t)}^{u_n(1, t)} \frac{1}{(1 + u)^2} e^{-1/u} du. \end{aligned} \quad (3.2)$$

It follows that for any $t > 0$,

$$\int_0^1 \int_0^x u_n(y, t) dy dx - \frac{u_o}{2} = \int_0^t \int_{u_n(0, s)}^{u_n(1, s)} \frac{1}{(1 + u)^2} e^{-1/u} du ds. \quad (3.3)$$

Since the left-hand side of the above equality is bounded for all $t > 0$ and $u_n(1, s) \leq u_n(0, s)$, the non-positive function

$$\int_{u_n(0, t)}^{u_n(1, t)} \frac{1}{(1 + u)^2} e^{-1/u} du$$

is integrable over $(0, +\infty)$. Therefore there exists a sequence $t_m \rightarrow +\infty$ such that $u_n(0, t_m) - u_n(1, t_m) \rightarrow 0$ as $t_m \rightarrow 0$. It follows that $\phi_n(1) = \phi_n(0)$. Since $\phi_n(x)$ is monotone, $\phi_n(x)$ must be a constant. We thus derive that there exists a constant $c \geq 0$ such that for all $0 \leq x \leq 1$,

$$u_n(x, t) \searrow c, \text{ as } t \rightarrow +\infty,$$

and the convergence is uniform for $0 \leq x \leq 1$ (by the monotonicity). We now show that $c = 1/n$. Indeed, from (3.1) and the boundary condition (2.1) we get

$$\frac{d}{dt} \int_0^1 u_n(y, t) dy = -k \frac{u_n(1, t) - \epsilon_n(t)}{(1 + u_n(1, t))^2}.$$

By integration over $(0, t)$, we get

$$\int_0^1 u_n(y, t) dy - u_o = -k \int_0^t \frac{u_n(1, s) - \epsilon_n(s)}{(1 + u_n(1, s))^2} ds. \quad (3.4)$$

Therefore $u_n(1, t) - \epsilon_n(t) \geq 0$ is integrable over $(0, +\infty)$. Since $u_n(1, t) - \epsilon_n(t)$ is also monotone, it follows that $u_n(1, t) - \epsilon_n(t) \rightarrow 0$ as $t \rightarrow +\infty$, and consequently that $c = 1/n$. Hence $u_n(x, t)$ uniformly converges to $1/n$.

The next result gives an estimate for the lower bounds of the average of the approximate solution. It will be used later on.

Lemma 3.2 *Suppose that $u_o(x) \equiv u_o > 0$ is constant. Then the solution $u_n(x, t)$ of approximate problem (1.1)–(1.3), (2.1) satisfies*

$$\int_0^1 u_n(x, t) dx \geq u_o e^{-kt}.$$

Proof. From (3.4) and the fact that $u_n(x, t)$ is decreasing in x , we get

$$u_n(1, t) \leq u_o - k \int_0^t \frac{u_n(1, t) - \epsilon_n(t)}{(1 + u_n(1, t))^2} dt.$$

Write

$$w(t) = \int_0^t \frac{u_n(1, t) - \epsilon_n(t)}{(1 + u_n(1, t))^2} dt.$$

Then $w(t)$ satisfies

$$\frac{dw}{dt} \leq u_n(1, t) \leq u_o - kw(t).$$

Hence we obtain

$$kw(t) \leq u_o(1 - e^{-kt}),$$

and from equality (3.4)

$$\int_0^1 u_n(x, t) dx \geq u_o e^{-kt}.$$

We now can prove our main result in the special case in which the initial data is constant.

Theorem 3.1 *Suppose that $u_o(x) \equiv u_o > 0$ is constant. Then all the assertions of Theorem 1.1 are true.*

Proof. From Lemma 3.1, we know that for any n ,

$$0 \leq u(x, t) \leq u_n(x, t) \rightarrow \frac{1}{n}, \quad \text{as } t \rightarrow +\infty,$$

and that the convergence is uniform in x . Hence

$$u(x, t) \rightarrow 0, \quad \text{uniformly as } t \rightarrow +\infty.$$

We now show that $u(x, t) > 0$ for any $(x, t) \in \mathcal{Q}$. If we let $n \rightarrow +\infty$ in (3.4), then for any $t > 0$,

$$\int_0^1 u(y, t) dy - u_\circ = -k \int_0^t \frac{u(1, s)}{(1 + u(1, s))^2} ds. \quad (3.5)$$

Since $u(x, t) \rightarrow 0$ as $t \rightarrow +\infty$, we get

$$k \int_0^{+\infty} \frac{u(1, s)}{(1 + u(1, s))^2} ds = u_\circ.$$

It follows

$$\int_0^1 u(y, t) dy = k \int_t^{+\infty} \frac{u(1, s)}{(1 + u(1, s))^2} ds.$$

If $u(1, t_\circ) = 0$ for some $t_\circ > 0$, then $u(1, t) = 0$ for all $t \geq t_\circ$ since $u(1, s)$ is decreasing in s . Hence the right-hand side of the above equality equals zero for $t = t_\circ$. But from Lemma 3.2 the left-hand side must be positive for all $t > 0$. We thus get a contradiction. Therefore $u(1, t) > 0$ for any $t > 0$. Now from the monotonicity of $u(x, t)$, for any fixed $T > 0$, $u_n(x, t) \geq u(1, T) > 0$ for $(x, t) \in \mathcal{Q}_T$. By the argument given in the first paragraph of this section, we conclude that $u(x, t)$ is a classical solution. The uniqueness follows from Lemma 2.1.

We are now in the position to prove our main result.

Proof of Theorem 1.1. Set

$$u_\circ^{min} = \min_{0 \leq x \leq 1} u_\circ(x), \quad u_\circ^{max} = \max_{0 \leq x \leq 1} u_\circ(x) + 1.$$

We compare the solution $u_n(x, t)$ with $u_n^{min}(x, t)$ and $u_n^{max}(x, t)$, where $u_n^{min}(x, t)$ is the solution of problem (1.1), (1.3), (2.1) and the initial condition $u_n^{min}(x, 0) = u_\circ^{min}$, and $u_n^{max}(x, t)$ is the solution of problem (1.1), (1.3), (2.1) and the initial condition $u_n^{max}(x, 0) = u_\circ^{max}$. Let ϵ_n^{min} , ϵ_n^{max} , ϵ_n , δ_n be the corresponding functions which have been chosen previously to modify the initial and boundary conditions. As we mentioned in the previous section, for large n these functions ϵ_n^{min} , ϵ_n^{max} , ϵ_n , δ_n can be chosen such that

$$\epsilon_n^{min}(t) \leq \epsilon_n(t) \leq \epsilon_n^{max}.$$

It follows from Lemma 2.1 that

$$u_n^{min}(x, t) \leq u_n(x, t) \leq u_n^{max}(x, t).$$

From Theorem 3.1, we know that the limit function $u^{min}(x, t) = \lim_{n \rightarrow +\infty} u_n^{min}(x, t)$ is positive everywhere. Hence $u(x, t) > 0$ for any $(x, t) \in \mathcal{Q}$. Consequently $u(x, t)$ is the unique classical solution of problem (1.1)–(1.4). Again from Theorem 3.1, we get that the limit function $u^{max}(x, t) = \lim_{n \rightarrow +\infty} u_n^{max}(x, t)$ converges to zero uniformly as $t \rightarrow +\infty$. Hence $u(x, t) \rightarrow 0$ uniformly as $t \rightarrow +\infty$.

Corollary 3.1 *There does not exist any non-negative solution of problem (1.1)–(1.4) which might be zero in finite time.*

Proof. Suppose that the assertion is not true. Then there exists a classical solution $\tilde{u}(x, t)$ which is equal to 0 at some points. Since $\tilde{u}(x, 0) = u_0(x) > 0$, there exists a $T > 0$ such that $\tilde{u}(x, t) > 0$ for $0 \leq t < T$ and $\lim_{t \rightarrow T} \tilde{u}(x_0, t) = 0$ for some $x_0 \in [0, 1]$. From Lemma 2.1, we get that $\tilde{u}(x, t) = u(x, t)$ for $0 \leq t < T$, where $u(x, t)$ is the positive solution asserted in Theorem 1.1. Hence $\lim_{t \rightarrow T} \tilde{u}(x_0, t) = u(x_0, T) > 0$, a contradiction.

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