# Technical Report

Department of Computer Science
and Engineering
University of Minnesota
4-192 Keller Hall
200 Union Street SE
Minneapolis, MN 55455-0159 USA

## TR 13-030

## Equilibria of Prices and Investments in Two-sided Internet Content Delivery Market

Pengkui Luo, Zhi-Li Zhang, and Andrew Odlyzko

September 24, 2013

# Equilibria of Prices and Investments in Two-sided Internet Content Delivery Market

Pengkui Luo*, Zhi-Li Zhang*, Andrew Odlyzko†
*{pluo,zhzhang}@cs.umn.edu, †odlyzko@umn.edu

*Abstract*—The Internet content delivery market has been positioned for tremendous growth. To capture the essentials of pricing structures in this market, we develop an economic model incorporating prices with provider investments in the demand function, and obtain equilibrium prices and investments under both monopoly and competitive settings for transport providers. We also investigate network neutrality and price discrimination issues in this market.

## I. Introduction

One of the main functions of Internet is delivering content such as web pages, video streams, or other services. Content is usually produced by Content Providers (usually in forms of websites like YouTube or Yahoo), delivered by Transport Providers (from Tier-1 providers like AT&T to "eyeball providers" like Qwest), and consumed by end users. We abbreviate the Content Provider and the Transport Provider as *CP* and *TP* in the rest of the paper.

End users usually have to pay their TP for accessing the Internet. This charge can be either flat-rate or volume-based. Whether or not end users pay CP for the content itself depends on the agreements between end users and CPs. Some big content providers like Google tend to waive it, and they have the ability to get revenues solely from advertisement clicks (e.g. Google Ads) by end users. To attract more users and thus more ad clicks, a CP needs to invest on maintenance and upgrades. This is also how the investment issue comes from.

TPs act like a platform connecting CPs and end users, delivering the content produced by CPs to end consumers. There are two charging regimes here: one is *one-sided market*, where TPs only charge end users for accessing the Internet; the other is *two-sided market*, where TPs charge not only end users but also CPs. Current the content delivery market is one-sided, but there is an increasing number of advocates by TPs for the two-sided regime.

This paper aims to analyze the pricing structure of the content delivery network as a two-sided market. There are two groups of competitions we want to capture in general: one is the *investment competition* among different CPs - investing more would likely yield richer contents and therefore attract more users clicks; the other competition resides among TPs who make decisions on the investment and pricing.

## II. Related Work

Our work is largely based on Musacchio's paper [1]. Musacchio develops a theoretical model to compare two pricing alternatives: the one-sided pricing (which they call "neutral") and the two-sided pricing (which they call "non-neutral") where TPs are allowed to charge not only end users but also CPs. Our work differs from theirs in several aspects. First, we adopt their theoretical model which incorporate investments and price with demands but we offer a simplified version which still captures the essentials. To ease the comparison, we even adopt their notations. Second, disagreeing with their definition of "Network Neutrality", we interpret it as "no price discrimination across CPs by TPs". Third, we assume the two-sided pricing regime at the first place without analyzing the possible transit from currently one-sided market. Fourth, we consider the price $p$ that TPs charge end users as "market-determined" as opposed to a decision variable that TPs can arbitrarily charge.

There are vast literatures on two-sided markets. Katz [2] is perhaps among the first studying network externality and competition issues. Rechet [3] analyzes platform competitions. Armstrong [4] analyzes competitions in three different types of two-sided markets and studies three determinants that affect equilibrium prices. What Armstrong talks about is general two-sided markets which act as platforms connecting two groups of agents, without giving out concrete forms of demand functions. But our paper models the demand function explicitly in order to appropriately capture the essentials of supply and demand in the Internet content provisioning market. Another important difference is his paper only studies the competitions among different platforms, while in our model competitions reside not only among TPs but also different CPs. Economides [5] provides a two-sided market analysis on the same issue as ours, with emphasis on net neutrality. Varian [6] studied "free riding" issue. Armstrong [7] studied price discrimination.

The rest of the paper is organized as follows: in Section III we investigate the *monopoly* case where there is only one transport provider. In Section IV we increase the number of TPs to $N$ and study the competition among them.

## III. Monopoly Situation

We start from the case where there is only one *monopoly* TP, who charges the $m^{th}$ CP price $q_m (m \in \{1, 2, ..., M\})$ per unit traffic volume flowing through it, and charges end users $p$ per unit volume too. We assume that the number of advertisement clicks on a CP is proportional to the amount of traffic volume going through it, and CP $m$ gains revenue $a_m$ per unit volume. We denotes the investments of CP $m$ and the
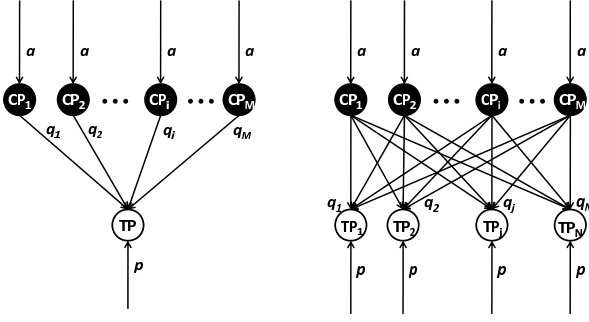
Fig. 1. Pricing Structures (left: monopoly; right: general)

TP as $c_m$ and $t$ respectively. All terminologies are shown in Table I.

Now we model the demand function $B_m$ for CP $m$, i.e. how much traffic volume through CP $m$ is determined by investments and prices. We know that increasing price $p$ discourages users from consuming the content, while investing more by either CP or TP attracts more volume. So $B_m$ increases with $c_m$ or $t$ and decreases with $p$. We also need a concave form of function. We model the demand function as follows, which is a simplified version of Musacchio [1]:

$$B_m = c_m^{v_m} t^w e^{-p/\theta}, (0 < w + v_m < 1) \qquad (1)$$

The utility functions of the TP and CPs can be formulated as its revenue less its investment:

$$\Pi_{C_m} = B_m(a_m - q_m) - \beta_m c_m \qquad (2)$$

$$\Pi_T = \sum_{m=1}^{M} B_m(p + q_m) - \alpha t \qquad (3)$$

Investment $c_m$ is decided by CP $m$. Price $q_m$ and investment $t$ are decided by the TP. They are all decision variables. As for the price $p$, we can either see it as a decision variable determined by the TP too, or to treat it as a variable predetermined by the "market" based on some optimization objectives such as maximizing the total demand or the total social welfare.

Given the model above, we can obtain the equilibrium prices $p^*$, $q_m^*$ and investment levels $c_m^*$, $t^*$, in the sense that the first-order conditions (FOCs) on utilities with respect to these four decision variables are satisfied (Because the chosen demand function is concave, FOCs are sufficient and necessary). Here we adopt a two-stage procedure: in the first stage, each CP simultaneously chooses its investment $c_m$ so as to maximize its own utility $\Pi_{C_m}$; in the second stage, having observed CPs' responses, the TP chooses its investment $t$, charges $q_m$ and possibly $p$ aiming to maximize its utility $\Pi_T$.

### A. Equilibria when $p$ is Decided by the TP

If the price $p$ is decided by the TP, then the TP has three decision variables: investment $t$, charge $p$ to users, charge $q_m$ to the $m^{th}$ CP.

TABLE I
TERMINOLOGIES

| | |
|---|---|
| $M$ | Number of CPs |
| $N$ | Number of TPs ($N = 1$ when monopoly) |
| $a$ | Advertising revenue of a CP per unit volume |
| $\theta, \alpha, \beta$ | Parameters |
| $v, w$ | ($0 < w + v < 1$) |
| $p$ | Price that users pay to its TP per unit volume |
| $c_m$ | Investment of the $m^{th}$ CP ($m \in [1, M]$) |
| $t_n$ | Investment of the $n^{th}$ TP ($n \in [1, N]$). |
| $t$ | Reduced to $t$ in the monopoly case. |
| $q_{mn}$ | Price that CP $m$ pays to TP $n$ per unit volume. |
| $q_m$ | Reduced to $q_m$ when monopoly; Reduced to |
| $q_n$ | $q_n$ in the general situation for simplicity |
| $B_{mn}$ | Units of volume for CP $m$ through TP $n$. |
| $B_m$ | Reduced to $B_m$ in the monopoly case |
| $B$ | Aggregated units of volume in the market |
| $\Pi_{C_m}$ | Utility of Content Provider $m$ |
| $\Pi_{T_n}$ | Utility of Transport Provider $n$. |
| $\Pi_T$ | Reduced to $\Pi_T$ in the monopoly case |

Let us make further simplification by assuming $v_1 = v_2 = \ldots = v_m = v$ and $\beta_1 = \beta_2 = \ldots = \beta_m = \beta$. By FOC $\partial \Pi_{C_m}(c_m^*)/\partial c_m = 0$ we have

$$c_m^*(p, q_m, t) = \left[ v\beta^{-1}(a - q_m)t^w e^{-p/\theta} \right]^{\frac{1}{1-v}} \qquad (4)$$

Plugging it into (3), we have $\Pi_{T_n}^{\sharp}(p, q_m, t)$. Now we have finished the first stage for CPs.

In the second stage for the TP, the *Nash Equilibrium* is reached when FOCs with respect to all decision variables are satisfied:

$$\begin{cases} \partial \Pi_{T_n}^{\sharp}(t^*, p^*, q_m^*)/\partial t = 0 \\ \partial \Pi_{T_n}^{\sharp}(t^*, p^*, q_m^*)/\partial p = 0 \\ \partial \Pi_{T_n}^{\sharp}(t^*, p^*, q_m^*)/\partial q_m = 0 \end{cases} \qquad (5)$$

which yields the single solution $t^*$, $p^*$ and $q_m^*$.

### B. Equilibria when $p$ is Market-Determined

A market-determined $p$ means that $p$ is determined by the whole market or some governmental regulators, as opposed to becoming a decision variable of the TP. Comparing with the previous subsection, the first stage is exactly the same and the form of $c_m^*(p, q_m, t)$ is exactly equation (4), but the decision variables for the TP are only $t$ and $q_m$ and therefore the TP's best response is represented as functions of $p$.

By solving FOC-equations $\partial \Pi_T^{\sharp}(p, q_m^*, t^*)/\partial q_m = 0$ and $\partial \Pi_T^{\sharp}(p, q_m^*, t^*)/\partial t = 0$ we can get $q_m^*(p)$ and $t^*(p)$ as the best responses. Particularly $q_m^*$ has nothing to do with $m$:

$$q_m^*(p) = (1 - v)a - pv \qquad (6)$$

Plugging $q_m^*(p)$ and $t^*(p)$ to (4), we can also obtain $c_m^*(p)$, and then $\Pi_{C_m}^*(p)$, $\Pi_T^*(p)$, $B^*(p)$.

Now the best responses of both CPs and the TP are represented in terms of $p$. We hereby discuss how the $p$ should be chosen and how this choice would affect the utilities of all parties.

We assume that its optimal point $p^*$ is achieved when the total demand $B^*$ is maximized. By $\partial B^*(p^*)/\partial p = 0$ we have:

$$p^* = \theta(w + v) - a \qquad (7)$$

Therefore we can plug $p^*$ back to $c_m^*(p)$, $t^*(p)$, $\Pi_{C_m}^*(p)$, $\Pi_T^*(p)$ and $B^*(p)$ to get the final result. We are particularly interested in $q_m^*$

$$q_m^* = a - \theta v(w + v) \tag{8}$$

The most important insight we can get here is that the TP charges different CPs the same price $q_m$ in its best response, i.e. the *Network Neutrality* is achieved naturally.

## IV. **Multiple Transport Providers**

In the previous section where there is only one transport provider, we derived the equilibrium price and investments. Now we are looking into more general case where there are $N$ transport providers. We are particular interested in what the additional TPs would bring in and whether it would change the Network Neutrality conclusion gained in the monopoly situation.

From (6) in the monopoly case we see that although price discrimination across different CPs by the TP is not prohibited at the first place, the result of demand maximization turns out that TP tends not to cast such discrimination. Therefore in the case of multiple TPs, we generalize this result in our setting, i.e. we assume a transport provider does not incur price discrimination on different content providers. That said, the $n^{th}$ transport provider charges every content provider the same per-click price $q_n$. Of course, different transport provider can have different $q_n$. That is how the "price competition" comes from. The rest part of the modeling is very similar to the Monopoly case.

$$B_{mn} = c_m^v t_n^w e^{-p/\theta} \tag{9}$$

$$\Pi_{T_n} = \sum_{m=1}^{M} B_{mn}(p + q_n) - \alpha t_n \tag{10}$$

$$\Pi_{C_m} = \sum_{n=1}^{N} B_{mn}(a - q_n) - \beta c_m \tag{11}$$

### A. **Equilibria when $p$ is Market-Determined**

FOC $\partial \Pi_{C_m}(c_m^*)/\partial c_m = 0$ yields the best responses for CPs $c_m^*(p, q_1, ..., q_N, t_1, ..., t_N)$. Plugging this $c_m^*$ into $\Pi_{T_n}$ yields $\Pi_{T_n}^\sharp(p, q_1, ..., q_N, t_1, ..., t_N)$. Then we can solve the FOC equations for TPs:

$$\begin{cases} \partial \Pi_{T_n}^\sharp(q_n^*, t_n^*)/\partial q_n = 0 \\ \partial \Pi_{T_n}^\sharp(q_n^*, t_n^*)/\partial t_n = 0 \end{cases} \tag{12}$$

The two equations above may have multiple solutions. By assuming $q_1^* = ... = q_N^*$ we obtain the single best response $q_n^*(p)$ and $t_n^*(p)$. Similar as the Monopoly case, the $q_n^*(p)$ has nothing to do with $n$ either.

$$q_n^*(p) = \frac{aN(1-v) - pv}{N(1-v) + v} \tag{13}$$

Plugging them back to $c_m^*(p, q_1, ..., q_N, t_1, ..., t_N)$ and formulas (9)(10)(11) yields best responses $c_m^*(p)$, $B^*(p)$, $\Pi_{T_n}^*(p)$ and $\Pi_{C_m}^*(p)$, all in terms of $p$.

Finally, let us optimize over $p$ for two different objectives: the total market demand $B^*(p)$, and TPs's utilities $\Pi_{T_n}$. To maximize the total demand $B^*(p)$, by FOC $\partial B^*(p^*)/\partial p = 0$ we have

$$p_{(1)}^* = \theta(w + v) - a \tag{14}$$

But to maximize utilities, then by FOC $\partial \Pi_{T_n}^*(p)/\partial p = 0$ or the $\partial \Pi_{C_m}^*(p)/\partial p = 0$ we have

$$p_{(2)}^* = \theta - a \tag{15}$$

Since $0 < w + v < 1$, we always have

$$p_{(1)}^* < p_{(2)}^* \tag{16}$$

which means the demand and the utilities cannot be maximized together, and the $p_{(2)}^*$ that maximizes utilities is always larger than the $p_{(1)}^*$ that maximizes demands.

If we increase $p^*$ from $p_{(1)}^*$ to $p_{(2)}^*$, we get increased utilities for both TP and CP, at the expense of a decrease of total demand $B$. During this process, $q_n^*$ decreases while $t_n^*$ and $c_m^*$ increase, which is reasonable.

## V. NUMERICAL ANALYSIS

We hereby use some numerical examples to show the total demand $B$ changes with the price $p$. We use different set of price $q_m$ to illustrate the relationship. For simplicity we assume there are only two transport providers. All parameters are set as the Table II. Figure 2 clearly show that the set of decision variable which maximizes the utility $\Pi_T^*$ or $\Pi_C^*$ does not maximize demand $B^*$ either.

## REFERENCES

[1] J. Musacchio, G. Schwartz, and J. Walrand, "A Two-Sided Market Analysis of Provider Investment Incentives with an Application to the Net-Neutrality Issue," *Review of Network Economics*, vol. 8, no. 1, pp. 22–39, March 2009.
[2] M. L. Katz and C. Shapiro, "Network Externalities, Competition, and Compatibility," *American Economic Review*, vol. 75, no. 3, pp. 424–40, June 1985.
[3] J.-C. Rochet and J. Tirole, "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, vol. 1, no. 4, pp. 990–1029, 06 2003.
[4] M. Armstrong, "Competition in Two-Sided Markets," *RAND Journal of Economics*, vol. 37, no. 3, pp. 668–691, Autumn 2006.
[5] N. Economides and J. Tag, "Net Neutrality on the Internet: A Two-sided Market Analysis," NET Institute, Working Papers 07-14, Sep. 2007.
[6] H. R. Varian, "System Reliability and Free Riding," in *in Economics of Information Security, Kluwer*. Kluwer Academic Publishers, 2004, pp. 1–15.
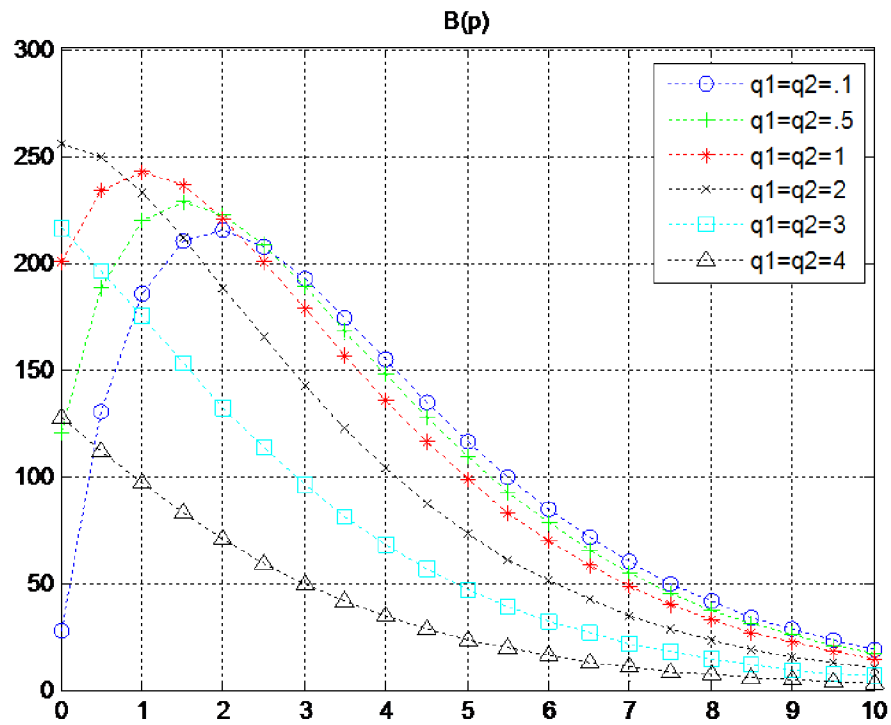[7] M. Armstrong, "Recent Developments in the Economics of Price Discrimination," EconWPA, Industrial Organization 0511004, Nov. 2005.

Fig. 2. Numerical Example