Essays on Human Capital and Macroeconomic Policy

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Dedication

This dissertation is dedicated to my mother, Elena Dubovyk.

Abstract

This dissertation studies the role of endogenous human capital accumulation in evaluating tax and Social Security policies. There are number of proposals to reform Pay-as-You-Go (PAYG) Social Security system in the U.S. and the number of countries has implemented such reforms. I study the effect of macroeconomic policies on the saving behavior of agents and macroeconomic performance of the economy in the overlapping generations framework. I consider two economic environments with borrowing constraints: one with exogenous human capital and a second with human capital accumulation through time investment.

The second chapter considers the options for reforming the U.S. retirement system. Baseline environments are calibrated to the U.S. tax and Social Security system. Two alternative Social Security arrangements are analyzed: (a) voluntary and (b) mandatory retirement savings accounts. I find that the welfare ranking of these alternatives depends on the endogeneity of human capital investment. Both systems are welfare improving when compared to the baseline. However, the system with mandatory (voluntary) accounts leads to lower welfare gains in the endogenous (exogenous) human capital environment.

The benefits of system with voluntary retirement accounts are higher rates of return on contributions made to the system and higher level of aggregate savings. At the same time, the transition generations have to incur costs, in terms of reduced consumption or higher market hours, to implement the new system. By implementing reforms in both environments, I study behavior and welfare losses/gains of transition generations. The results of third chapter show that the level of compensation depends on the economic environment considered. The amount of resources that have to be transferred to transition generations is higher in the environment with endogenous human capital.

In 1990s, several pension reforms had been adopted to insure financial sustainability of Italian retirement system. The fourth chapter studies two main features of reforms: (i) adoption of notional defined contributions formula; (ii) price indexation of benefits as compared to wage indexation prior to 1992. The reforms decrease financial obligations of pension system. I study labor market decisions of and quantify the effect of the reforms on transition generations.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

There are number of proposals to reform Pay-as-You-Go (PAYG) Social Security system in the U.S. and the number of countries has implemented such reforms. Traditionally, these reforms have been studied in the overlapping generations framework with life-time labor productivity exogenously specified. The examples of these papers are Auerbach and Kotlikoff (1987), DeNardi et al. (1999), and Kotlikoff et al. (1999). On the other hand, an individual's schooling or labor force participation decisions determine the evolution of labor productivity over the life-cycle in the empirical labor literature. For example, Imai and Keane (2004) show the importance of human capital accumulation in estimating the intertemporal elasticity of labor supply. This dissertation studies the importance of human capital accumulation decisions in evaluating tax and social security policies.

To evaluate the role of human capital investment, I set up two baseline environments that differ by evolution of labor productivity over an agent's life-cycle. In the environment with exogenous human capital, the labor productivity is exogenously given. In the environment with endogenous human capital, an agent chooses the amount of time allocated to human capital accumulation. In both environments, labor supply is elastic, the retirement decision is exogenous, and negative asset holdings are not permitted.

In the second chapter, I evaluate two alternatives to the PAYG retirement system. The baseline environments incorporate the stylized version of the U.S. tax and social security system. Under the first alternative, the PAYG social security system is removed and the labor income tax is reduced for the portion used to finance social security benefits. Within this alternative, agents finance their consumption during the retirement years through their own savings. These savings are accumulated on the Voluntary Retirement Savings Accounts (RSA). The second retirement system does not have the PAYG system, and agents are required to contribute a fixed portion of their labor income toward tax-deferred retirement accounts. These accounts are called Mandatory Retirement Savings Accounts.

Under both alternatives, output and savings are higher than in the PAYG system. However, there are different welfare implications. Both systems are welfare improving when compared to the baseline. However, the system with Voluntary RSA leads to higher welfare gains in the environment with endogenous human capital investment. These welfare gains are measured by life-time consumption equivalents in comparison to the baseline. On the contrary, the arrangements with Mandatory RSA are the preferred ones in the environment with exogenous human capital. This difference is due to young individuals (i) switching time allocation towards human capital accumulation and (ii) being borrowing constrained under mandatory savings in the endogenous environment.

The benefits of fully-funded system are the higher rates of return on the contributions made to the system and the higher level of aggregate savings. At the same time, the transition generations have to incur the costs, in terms of reduced consumption or higher market hours, to implement the new system. How much the transition generations have to be compensated to gain their support for the retirement reform? Many authors propose to issue the compensation in terms of additional government debt and preserve the pension rights of the current working generations.

In the third chapter, I study the elimination of PAYG Social Security system in two economic environments that differ by the human capital production technology. The initial stationary equilibriums with PAYG Social Security system in the two environments coincide in terms of individual and aggregate variables due to the calibration strategy employed. Then, I eliminate PAYG system in both economic environments, introduce the system with voluntary retirement savings accounts and determine the agents' response to this reform. The welfare calculations show that the reform leads to welfare losses for the transition generations and the long-term welfare gains are smaller in the environment with the endogenous human capital. Therefore, the amount of resources that can be transferred from the future generations to the transition ones to compensate for welfare losses is smaller in the environment with the endogenous human capital.

The Italian pension system have underwent a number of reforms in 1990s. The system still maintains pay-as-you-go nature and is moving from a defined benefit (DB) to a defined contribution (DC) system. The reforms envision a very long transition and the new system will be fully phased in after 2030. The reforms also provide for a different treatment of generations based on the labor market seniority at the end of 1995. Many scholars argue that the reforms violate an intergenerational equity by placing most of the burden of transition on younger generations. The fourth chapter quantitatively evaluates the effect of the reforms on transition generations.

I develop a macroeconomic model with overlapping generations (OG) and endogenous human capital accumulation to study the reforms. The model is calibrated to match the main macroeconomic variables of the Italian economy before the introduction of the reforms. I model the institutional features of the Italian pension system before and after the reform. The pension reform introduced in 1992 changed the indexation of pensions to price inflation from wage growth, while the reform passed in 1995 introduced a DC system and provided rules for calculating the retirement benefits for transition generations. The workers who have entered the labor market before 1995 will have their benefits calculated based on two formulas: (1) *pro-rata* system for young workers with less than 18 years of seniority in 1995, and (2) *modified defined benefit* system for workers with more than 18 years of seniority.

The results of the fourth chapter show that the Italian reforms will reduce the pension expenditures and restore the financial sustainability of the system in the long-run. The change in the indexation rule alone reduces the pension expenditures only during the initial fifteen years after the introduction of the reforms, while the long-term reduction is achieved through transition to a DC retirement system. During the transition, the older workers with more than 18 years of seniority in 1995 are protected by the reforms, while young and future generations bear the cost of transition to a new system. The reforms induce the agents to increase investment into physical and human capital. In the new stationary equilibrium with a DC retirement system, the aggregate capital stock and labor supply are by 13% and 6.3% higher, respectively, as compared to pre-1992 levels. These increase in the factors of production results in the higher aggregate output and consumption. The future generations who will be born after year 2050 are only slightly worse off as compared to pre-1992 level, the welfare of these generations is only lower by 0.16% in terms of life-time consumption.

Chapter 2

Human Capital Investment and Retirement Savings Accounts

Labor productivity over an individual's life-cycle is modelled in a number of ways. One strand of literature calibrates the life-cycle profile to match the observed earnings profile. This approach is widely used in empirical macroeconomics literature. Example of these papers are Auerbach and Kotlikoff (1987), DeNardi et al. (1999), and Kotlikoff et al. (1999). In the empirical public finance literature, an individual's schooling or labor force participation decisions determine the evolution of labor productivity over the life-cycle. For example, Imai and Keane (2004) show the importance of human capital accumulation in estimating the intertemporal elasticity of labor supply. This chapter studies the importance of human capital accumulation decisions in evaluating tax and social security policies.

I analyze two retirement systems alternative to the Pay-As-You-Go (PAYG) social security system. Welfare ranking of these alternatives depends on the endogeneity of human capital accumulation. To evaluate the role of human capital investment, I set up two baseline environments that differ by evolution of labor productivity over an agent's life-cycle. In the environment with exogenous human capital, the labor productivity is exogenously given. In the environment with endogenous human capital, an agent chooses the amount of time allocated to human capital accumulation. In both environments, labor supply is elastic, the retirement decision is exogenous, and negative asset holdings are not permitted. These baseline environments incorporate the stylized version of the U.S. tax and social security system.

There is concern over the financial solvency of the current PAYG system due to demographic changes. A number of reforms have been proposed. These proposals stress the effect of reforms on savings and output. I evaluate two alternative retirement arrangements and consider the welfare effects as well. Under the first alternative, the PAYG social security system is removed and the labor income tax is reduced for the portion used to finance social security benefits. Within this alternative, agents finance their consumption during the retirement years through their own savings. These savings are accumulated on the Voluntary Retirement Savings Accounts (RSA). The second retirement system does not have the PAYG system, and agents are required to contribute a fixed portion of their labor income toward tax-deferred retirement accounts. These accounts are called Mandatory Retirement Savings Accounts.

Under both alternatives, output and savings are higher than in the baseline. However, there are different welfare implications. Both systems are welfare improving when compared to the baseline. However, the system with Voluntary RSA leads to higher welfare gains in the environment with endogenous human capital investment. These welfare gains are measured by life-time consumption equivalents in comparison to the baseline. On the contrary, the arrangements with Mandatory RSA are the preferred ones in the environment with exogenous human capital. This difference is due to young individuals (i) switching time allocation towards human capital accumulation and (ii) being borrowing constrained under mandatory savings in the endogenous environment.

This chapter builds on the quantitative tradition of evaluating tax and social security policies in an overlapping generations (OG) framework started by Auerbach and Kotlikoff (1987). I also incorporate the human capital investment technology proposed by Ben-Porath (1967). The papers closest to my work are Davies and Whalley (1991), Heckman et al. (1999), and Alvarez-Albelo (2004). This chapter differs from Davies and Whalley (1991) and Heckman et al. (1999) in two dimensions. First, I consider an elastic labor supply. As a result, time reallocation among different activities has important welfare implications. Second, I study different sets of tax and social security policies. In my models, time investment is an input into human capital accumulation while Alvarez-Albelo (2004) studies human capital enhancement through learning-by-doing, i.e., participation in the market production enhances the human capital from tomorrow on. One of the experiments in Alvarez-Albelo (2004) closely resembles the Voluntary RSA studied in this chapter. However, Alvarez-Albelo (2004) does not perform welfare analysis. The environment with exogenous human capital is motivated by and is comparable to DeNardi et al. (1999) and Kotlikoff et al. (1999).

The chapter is organized as follows. In section 2.1, I construct two baseline OG models with exogenous and endogenous human capital decisions. These models incorporate the stylized version of the U.S. tax and social security system. Section 2.2 discusses the alternative retirement arrangements and welfare implications of each. Section 2.3 discusses conclusions and extensions for the future research.

2.1 Baseline Overlapping Generations Models

To quantify the effects of different social security arrangements on savings and welfare, I consider two general equilibrium models with OG structure. Common features between these two economies are the finite and certain life-span of agents, the Cobb-Douglas production technology, and the set of government policies. The agents allocate their time endowment among leisure, market production, and time investment into human capital accumulation. These models differ by the evolution of the human capital profile over the agents' life-cycle and time allocation decisions. In the first model, the agents' time allocation decisions, in particular, the time investment into human capital enhancement, determine the evolution of the human capital and, consequently, the wage income profiles over the life-cycle. This model is called the model with endogenous human capital. The second model has an exogenous age-specific labor productivity profile and is called the economy with exogenous human capital. The traditional macroeconomics policy literature studies the model of this type.

I initially describe the model with endogenous human capital accumulation decision. Then, I explain the features of the model with exogenous human capital.

2.1.1 Model with Endogenous Human Capital

I start with a description of the demographic structure and preferences. The economy has overlapping generations of agents who live for J adult periods, with ages denoted by $j \in \Im \equiv \{0, ..., J-1\}$. The agents' life-spans are certain. In the first time period, the measure of newly born agents is normalized to 1. The population is constant, and the total population size is J.

A young agent born at a stationary equilibrium is endowed with initial levels of physical and human capitals, s_0 and h_0 respectively. Each period agents are endowed with one unit of time that can be allocated to leisure, production activities in the market sector, or investment into human capital accumulation. Let $\{c_j, l_{m,j}, l_{h,j}\}$ denote consumption, market hours, and investment hours, respectively, of an agent of age j (subscript). The preferences of a young agent are ordered by

$$\sum_{j=0}^{J-1} \beta^j u(c_j, 1 - l_{m,j} - l_{h,j}), \tag{2.1}$$

where β is a time preference parameter. Each agent chooses sequences of consumption, market hours, and investment hours to maximize the discounted value of life-time utility subject to its budget constraint,

$$(1 + \tau_c)\, c_j + s_{j+1} \leq (1 - \tau_l)\, w h_j l_{m,j} + (1 + (1 - \tau_k)\, r) s_j + d_j. \tag{2.2}$$

This constraint must be balanced at each age of the agent's life, i.e., for any $j \in \Im$. The agent's expenditures on consumption and savings in the form of physical capital, s_{j+1}, must be less or equal to the after-tax income.

The agent of age j receives labor income $w h_j l_{m,j}$, where w is the real wage per efficient unit of labor in terms of the consumption good. The agent's labor productivity at age j depends on the stock of human capital h_j, which is determined by the undepreciated human capital from the last period and the new human capital accumulation during the last period:

$$h_j = (1 - \delta_h) h_{j-1} + Q(h_{j-1}, l_{h,j-1}). \tag{2.3}$$

The creation of new human capital depends on its existing level and investment hours and is determined by the function $Q(h, l_h)$. The Q function is increasing in both arguments and has decreasing returns to scale. The agent's savings earn capital income at the real rate of return r. Agents are restricted to have strictly positive amount of savings at all ages

$$s_j \geq 0. \tag{2.4}$$

Agents pay taxes on consumption at rate τ_c, labor income at rate τ_l, and capital income net of depreciation at rate τ_k.

The government transfers to the agent of age j are denoted by d_j. These transfers consist of two components: a lump-sum transfer for agents of all ages, f_j, and social security benefits to retirees, b_j,

$$d_j = \begin{cases} f_j, & j = 0, \overline{J} - 1, \\ f_j + b_j, & j = \overline{J}, J - 1. \end{cases}$$

Agents are entitled to retirement benefits starting with age $\overline{J}$. The amount of social security benefits is the fraction of the average labor income during working periods. This fraction is called a replacement rate, ϕ. social security benefits are calculated as

$$b_j = \phi \frac{\sum_{i=0}^{\overline{J}-1} wh_i l_{m,i}}{\overline{J}}, j = \overline{J}, J-1.$$

The government's budget is balanced every period. The government levies taxes on consumption, labor income, and capital income and uses tax revenue to purchase a wasteful public good, G, and provide two types of transfers. Then, the government's budget constraint is

$$G + \sum_{j=0}^{J-1} f_j + \sum_{j=\overline{J}}^{J-1} b_j = \sum_{j=0}^{J-1} \left(\tau_c c_j + \tau_l w h_j l_{m,j} + \tau_k r s_j\right).$$

Firms hire capital, K, and labor, L, to produce output with a constant returns-to-scale production technology,

$$Y = AK^{\theta} L^{1-\theta},$$

where A is total factor productivity. The aggregate inputs are determined as

$$\begin{aligned} K &= \sum_{j=0}^{J-1} s_j, \\ L &= \sum_{j=0}^{J-1} h_j l_{m,j}. \end{aligned}$$

The aggregate feasibility constraint is

$$\sum_{j=0}^{J-1} c_j + G = AK^{\theta} L^{1-\theta} - \delta_k K. \tag{2.5}$$

Definition 1. *A competitive equilibrium is factor prices, (w, r); aggregate capital and labor supplies, (K, L); individual allocations, $\left(\{c_j, s_{j+1}, l_{m,j}, l_{h,j}, h_j\}_{j\in\Im}\right)$ for any generation born at the stationary equilibrium; and government policies, $(\tau_c, \tau_l, \tau_k, \phi, G, F)$, such that the following holds: (1) given factor prices and government policies, individual allocations, $\left(\{c_j, s_{j+1}, l_{m,j}, l_{h,j}, h_j\}_{j\in\Im}\right)$, maximize (2.1) subject to (2.2)-(2.4) for each generation; (2)*

factor inputs are paid the marginal products:

$$\begin{aligned} w &= (1-\theta) AK^{\theta} L^{-\theta}, \\ r &= \theta AK^{\theta-1} L^{1-\theta} - \delta_k; \end{aligned}$$

(3) government's budget is balanced every period; and (4) aggregate and individual allocations satisfy market clearing conditions.

2.1.2 Model with Exogenous Human Capital

This model is motivated by the macroeconomics literature that studies OG models with life-cycle labor productivity being exogenously given. To make comparisons to this literature, I modify the model from the previous subsection in the following way. The life-cycle profiles of human capital and investment hours are exogenously fixed at the level of the solution for the model with endogenous human capital under the baseline calibration.

The demographic structure, production technology, the set of government policies, and market clearing conditions of this economy are the same as the one in the model with endogenous human capital decisions. The difference between the two models is in the agents' decisions. Let $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$ be a fixed life-cycle profiles of investment hours and human capital stock. Introducing a fixed life-cycle profile of investment hours is equivalent to changing the time endowment over the life-cycle. Consequently, the time endowment for each agent is $\left(\left\{1-\bar{l}_{h,j}\right\}_{j\in\Im}\right)$. The preferences of a young agent are ordered by

$$\sum_{j=0}^{J-1} \beta^j u(c_j, 1-l_{m,j}-\bar{l}_{h,j}). \tag{2.6}$$

Each agent chooses a sequence of consumption and market hours to maximize a discounted value of life-time utility subject to the budget constraint,

$$(1+\tau_c) c_j + s_{j+1} \leq (1-\tau_l) w\overline{h}_j l_{m,j} + (1+(1-\tau_k) r) s_j + d_j. \tag{2.7}$$

This constraint must be balanced every period of an agent's life, i.e., for any $j \in \Im$. An agent's labor productivity over the life-cycle is predetermined by the profile of human capital. This human capital profile is frequently called the efficiency units profile.[1] The agents

[1] Examples are Rios-Rull (1996) and DeNardi et al. (1999).

are restricted to have positive physical capital asset holdings during all ages,

$$s_j \geq 0. \tag{2.8}$$

Taxes levied on the agents' income and expenditures and transfer system are the same as in the model with endogenous human capital accumulation. The government's budget constraint and market clearing conditions are as in the model with endogenous human capital accumulation.

Definition 2. *A competitive equilibrium is factor prices,* (w, r)*; aggregate capital and labor supplies,* (K, L)*; individual allocations,* $\left(\{c_j, s_{j+1}, l_{m,j}\}_{j\in\Im}\right)$ *for any generation; profiles of investment hours and human capital stock* $\left(\{\bar{l}_{h,j}, \overline{h}_j\}_{j\in\Im}\right)$*; and government policies,* $(\tau_c, \tau_l, \tau_k, \phi, G, F)$*, such that the following holds: (1) given factor prices, government policies, and life-cycle profiles* $\left(\{\bar{l}_{h,j}, \overline{h}_j\}_{j\in\Im}\right)$*, individual allocations,* $\left(\{c_j, s_{j+1}, l_{m,j}\}_{j\in\Im}\right)$*, maximize (2.6) subject to (2.7) and (2.8); (2) factor inputs are paid the marginal products:*

$$\begin{aligned} w &= (1-\theta) AK^{\theta} L^{-\theta}, \\ r &= \theta A K^{\theta-1} L^{1-\theta} - \delta_k; \end{aligned}$$

(3) government's budget is balanced every period; and (4) aggregate and individual allocations satisfy market clearing conditions.

2.1.3 Calibration of the Baseline OG Models

I calibrate the baseline economies to the U.S. tax and social security system. The calibration year is 2000. Parameters of demographics, preferences, and technology are the same between the economies with exogenous and endogenous human capital. The parameters of human capital production technology in the economy with endogenous human capital are calibrated to the life-cycle earnings profile. Appendix A.3 provides details on data sources and calculation procedures for all parameters. The parameter values are summarized in Table 2.1.

Parameters for both economies

The demographic structure of the economy is calibrated as follows. Agents enter the economy at age 20, retire at age 65, and die at age 80. Each model period corresponds to 5 years. Hence, the agents are working during the first nine model periods and are retired during the last three model periods. In this section, I report all parameters in annual terms and adjust these parameters accordingly in computations.

The time preference parameter β is calibrated to match the after-tax interest rate of 4.0 percent per year. I assume that the agents' flow utility functions are

$$u(c, 1 - l_m - l_h) = \log c + \alpha \log(1 - l_m - l_h),$$

where α is chosen to match average weekly hours of the population of ages between 20 and 64. Based on Census data, average hours for working age population is 29 hours per week.

The calibration of production technology is standard. Capital income share, θ, is set to 0.333. Depreciation of physical capital, δ_k, is calibrated to match the investment share in GDP. This investment share is equal to 16.9% of GDP in 2000. The resulting depreciation rate is 7.5%. This estimate of the depreciation rate is higher than the one commonly used in the literature. Stokey and Rebelo (1995) estimate the depreciation rate to be 6%. Rios-Rull (1996) calibration results in the rate of 5.4%.

Average effective tax rates are calibrated using the methodology of Mendoza et al. (1994) and are reported in Table 2.1. The share of government expenditures in output, g, is set to match the corresponding value in NIPA. In 2000, the government consumption expenditures are 14.44 percent of GDP. The replacement rate for social security benefits, ϕ, is calibrated to match the benefit payments from the Old-Age and Survivors Insurance (OASI) Fund. In the calibration year, OASI benefit payments are equal to 4.23 percent of GDP and the resulting replacement rate is $\phi = .195$.

Parameters of human capital production technology

I assume the following law of motion for human capital:

$$h_{j+1} = (1 - \delta_h) h_j + B h_j^{\psi_1} l_{h,j}^{\psi_2},$$

where the conditions $B, \psi_1, \psi_2 \geq 0$ and $\psi_1 + \psi_2 \leq 1$ guarantee the decreasing returns to scale. Hence, the life-cycle profile of time investment into human capital is time-independent.

I have to choose five parameters for the human capital production technology: initial stock of human capital, h_0; the depreciation rate of human capital, δ_h; productivity of human capital accumulation, B; weight of human capital stock in new accumulation, ψ_1; and weight of time investment, ψ_2. I calibrate these parameters to match the life-cycle earnings profile, which is constructed using 2000 decennial Census data. I divide the population of ages between 20 and 64 into nine age groups, $j \in \{0, ..., 8\}$. The size of the working age population is denoted by N_t. The measure of earnings is the hourly wage rate, denoted by e_j, $j \in \{0, ..., 8\}$. The average wage rate for the working population, $\overline{e}$, is \$17.24 per hour. This average rate for the working population is calculated using the size of each age group, $n_j(t)$:

$$\overline{e} = \frac{\sum_{j=0}^{8} n_j(t) e_j}{N_t}.$$

To express the wage earnings profile in units comparable to the model, I report the average hourly wage for an age group j as the ratio to the average hourly wage of the working population: $\varepsilon_j = e_j/\overline{e}$, $j = 0, ..., 8$.

Equivalently, the wage rate in the model is $w_t h_j$ and the average wage for the working population is

$$\overline{wh} = \frac{\sum_{j=0}^{8} w_t h_j}{\overline{J}}.$$

I choose parameters of the human capital production function to minimize the distance between the model and data wage hour profiles:

$$\min_{(h_0, \delta_h, B, \psi_1, \psi_2)} \sum_{j=0}^{8} \left(\frac{wh_j}{\overline{wh}} - \frac{e_j}{\overline{e}} \right)^2.$$

The chosen parameters are reported in Table 2.1.

2.1.4 Stationary Equilibrium in both Baseline Environments

Given calibrated parameters, I solve for a stationary equilibrium in the economy with endogenous human capital. The procedure for the numerical algorithm is described in Appendix A.2 . Equilibrium life-cycle profiles of investment hours and human capital are given in Figure 2.1. Time investment into human capital accumulation is the highest at the beginning of life and exhibits steady decline. I refer to this time investment as investment hours. Hours devoted to market production activities are called market hours. The sum of these two types of time usage are called total production hours. The life-cycle profile of investment hours and technology for human capital production determine the life-cycle labor productivity. Under the baseline calibration, an individual reaches a peak in labor productivity between ages 45 and 49.

Let $\left(\left\{l^*_{h,j}, h^*_j\right\}_{j\in\Im}\right)$ denote equilibrium life-cycle profiles of investment hours and human capital in the environment with endogenous human capital.

Proposition 3. *If* $\left\{\bar{l}_{h,j}\right\}_{j\in\Im} = \left\{l^*_{h,j}\right\}_{j\in\Im}$ *and* $\left\{\overline{h}_j\right\}_{j\in\Im} = \left\{h^*_j\right\}_{j\in\Im}$, *a stationary equilibrium in the environment with exogenous human capital is identical to the one in the environment with endogenous human capital under baseline calibrated parameters.*

Proof. The method of the proof is to compare equilibrium conditions in the two environments. These conditions are derived in the Section A.1 QED ■

Under the baseline calibration, the equilibria in the two environments are the same both on the aggregate and individual level. The values for various aggregate variables in the baseline environments are given in the second column of Table 2.3. With a calibrated after-tax interest rate of 4%, the resulting capital-to-output ratio is 2.44. In both environments, agents of working age devote on average 29 hours per week for market production activities.

2.2 Alternative Retirement Arrangements

I consider two alternative arrangements. First, I analyze an elimination of social security benefits with a corresponding reduction in labor income tax used to finance these benefits. This type of reform is analyzed by Kotlikoff et al. (1999) and DeNardi et al. (1999) in an OG model with life-cycle labor productivity exogenously specified. Due to the precautionary life-cycle saving motive, the agents choose to accumulate assets to finance their retirement. I call this arrangement as Voluntary Retirement Savings Accounts. For assets on this accounts, the return net of depreciation is subject to capital income taxation. Second, I consider a retirement arrangement without social security benefits and with Mandatory Retirement Savings Accounts. Many countries have introduced this retirement system, examples of which are Australia, Chile, and Mexico. Under this system, agents are required to contribute a fixed portion of their income to Mandatory RSA. Contributions to Mandatory RSA are tax-deferred. Within the environment with exogenous human capital, I compare a stationary equilibrium under the alternative retirement arrangements to the baseline one. The same comparison is conducted within the environment with endogenous human capital.

2.2.1 Arrangements with Voluntary Retirement Savings Accounts

Under this retirement arrangement, social security benefits are eliminated and the labor income tax is reduced to keep lump-sum transfers the same as in the baseline environment with endogenous human capital. In both environments, the labor income tax is reduced from 27% to 21.09%, whereas tax rates on consumption and capital income and government expenditures as a share of output are kept at the level of the baseline environments. The numerical algorithm of solving for a stationary equilibrium under this alternative is the same as in the baseline.

The welfare gains of replacing PAYG social security system with Voluntary Retirement Savings Accounts are reported in Table 2.2. I measure the welfare gains by life-time consumption equivalents. This measure determines the percentage increase in the agent's life-time consumption in the baseline environment needed to make her or him indifferent to the alternative retirement arrangements. Within the environment with exogenous human capital, an agent's life-time consumption in the baseline economy must be increased by 7.81% to make her or him indifferent to the policy considered. This measure takes into account the difference in labor supply between two economies. The welfare gains come from the reduction in the distortionary labor income tax.

Environment with exogenous human capital All comparisons in this section are between the economy with Voluntary RSA and the baseline one. To understand the welfare gains, we need to study the reasons behind the increase in the agents' consumption. An individual has three sources of income: labor and capital income and lump-sum transfers from the government. The lump-sum transfers are held the same between the two economies. Both labor and capital incomes are higher in the absence of the PAYG social security system. To analyze this income increase, let us consider the change in wage and interest rates. The factor prices are determined by the stocks of physical capital and labor supply. Because the agents save for their retirement on their own, the amount of savings under the Voluntary RSA is 16.31% higher. Labor supply is determined by the product of human capital stock and market hours. In the environment with exogenous human capital, the human capital stock is kept fixed and the agents devote more hours to market production, on average 1.62 hours per week more. The capital-to-labor ratio is higher. Consequently, the wage rate is higher by 3.69%, and the interest rate is slightly lower by 0.49 percentage points when the individuals provide for their retirement without public assistance. Even with a slightly lower interest rate, the capital income is higher due to the higher stock of savings, as reported in Figure 2.5.

The retirement arrangements affect the agents' time allocation throughout the life-cycle. Figure 2.3 shows time allocation among market production activities, time investment into human capital accumulation, and total production hours. The time endowment is normalized to 100 hours per week. The leisure consumed by the agents is the difference between the time endowment and total production hours. The difference in time allocation between the economy with Voluntary RSA and the baseline one is presented in Figure 2.4. In the environment with exogenous human capital, the agents are forced to devote part of their time to human capital accumulation. This exogenous profile of investment hours is the same between two economies as shown in the bottom panels of Figures 2.3 and 2.4.

The time allocation between two economies differs for three age groups: 20 to 34, 35 to 44, and 45 to 64 year olds. In the first age group, the agents work approximately the same amount in the two economies, because their labor productivity is not very high. Due to the higher after-tax wage rate, the agents still enjoy higher consumption under the Voluntary RSA. The agents of ages 35 to 44 work slightly less under the Voluntary RSA for three reasons. First, the after-tax wage income is higher. Second, the labor productivity is close to the peak one, as seen in Figure 2.5. Third, the consumption level is close to the desired life-time path. In the third age group, the agents work on average 4 hours per week more. These agents take advantage of high labor productivity and heavily save for retirement. As seen in Figure 2.5, the agents save more in the absence of the publicly provided social security benefits.

Environment with endogenous human capital Welfare gains from eliminating the social security system are lower in the environment with endogenous human capital. In this environment, agents have an additional margin of adjustment, human capital investment, as compared to the one with exogenous human capital. Therefore, the reduction in distortionary labor income tax leads to lower welfare gains.

An agent can take advantage of the higher after-tax wage rate in two ways: (i) supply more hours for market production and/or (ii) invest more into human capital. Due to the human capital technology, the young agents invest more into human capital and work less for the market production. Even though the young agents are borrowing constrained, the discounted life-time benefit from the human capital investment outweighs the forgone wage income. The comparison of the human capital profiles is in Figure 2.5. As can be seen in Figure 2.4, the agents of ages 50 to 64 work for the market production on average 6 hours more. There are two reasons for this. First, they want to take advantage of the high labor productivity. The productivity for the age group 60 to 64 is higher than that for the one 30 to 34. Second, the agents save to finance an increasing stream of consumption during retirement.

Table 2.3 reports changes in aggregate variables between the baseline economies and the ones under the Voluntary RSA. The interest rate in the environment with this alternative is lower than the one in the baseline environment. Kotlikoff et al. (1999) and DeNardi et al. (1999) report that the output is higher in the steady state under the new system by 12% and 8.7%, respectively, as compared to the initial steady state. I find that in the environment with exogenous and endogenous human capital the output is 8.16% and 11.81%, respectively, higher in the economies with the alternative retirement system as compared to the baseline. The total amount of savings under the new arrangements are higher by 16.31% and 16.91% in the environment with exogenous and endogenous human capital, respectively.

2.2.2 Arrangements with Mandatory Retirement Savings Accounts

Under this retirement arrangement, the agents are required to contribute a fixed fraction of labor income to Mandatory RSA. These contributions are tax-deferred, accumulation of assets on the Mandatory RSA is tax-exempt, and withdrawals are subject to labor income taxes.

Economy Description

I initially describe the economy with endogenous human capital and modifications for the economy with exogenous human capital are given at the end of section . The demographic structure is the same as in the baseline environment.

An agent of age j has two types of savings accounts: a voluntary one, $s_{1,j}$, and a mandatory one, $s_{2,j}$. An agent starts her/his life with zero assets, $s_{1,0} = 0$ and $s_{2,0} = 0$. Under this retirement arrangement, the agents are required to contribute a fraction ξ of wage income towards a tax-deferred retirement account.

An agent chooses a sequence of allocations, $\{c_j, s_{1,j+1}\}_{j\in\Im}$ and $\{l_{m,j}, l_{h,j}, h_j\}_{j\in\Im}$, to maximize the discounted value of life-time utility (2.1) subject to the following budget constraint:

$$(1+\tau_c)\,c_j + s_{1,j+1} \leq (1-\tau_l)\,(1-\xi)\,wh_jl_{m,j} + (1+(1-\tau_k)\,r)s_{1,j} + d_j, \tag{2.9}$$

borrowing constraints:

$$\begin{aligned} s_{1,j} &\geq 0, \\ s_{2,j} &\geq 0, \end{aligned}$$

and the law of motion for human capital technology (2.3).

The accumulation of assets in a mandatory savings account is

$$s_{2,j+1} = \begin{cases} \xi w h_j l_{m,j} + (1+r)\, s_{2,j}, & j = 0, \overline{J}-1, \\ (1+r)\, s_{2,j} - b, & j = \overline{J}, J-1. \end{cases}$$

The transfers to a household consist of two components: a lump-sum transfer for agents of all ages, f_j, and annuity payments from the mandatory savings account, b,

$$d_j = \begin{cases} f_j, & j = 0, \overline{J}-1, \\ f_j + (1-\tau_l)\, b, & j = \overline{J}, J-1. \end{cases}$$

During retirement, an agent receives a constant annuity payments from the Mandatory RSA, b. These annuity payments are calculated such that the mandatory savings account is exhausted by the end of agent's life, i.e., $s_{2,J} = 0$. The annuity payment depends on the accumulation of assets at the beginning of retirement, $s_{2,\overline{J}}$:

$$b^2 = \frac{(1+r)^{J-\overline{J}} s_{2,\overline{J}}}{[1 + (1+r) + (1+r)^2]}.$$

Asset accumulation on the mandatory savings account at the beginning of retirement is

$$s_{2,\overline{J}} = \sum_{j=0}^{\overline{J}-1} \xi w h_{\overline{J}-1-j} l_{m,\overline{J}-1-j} (1+r)^j .$$

The government's budget is balanced every period. The government makes the same lump-sum transfers to all living generations, f.

$$G + fJ = \sum_{j=0}^{J-1} [\tau_c c_j + \tau_l (1-\xi)\, w h_j l_{m,j} + \tau_k r s_{1,j}] + \sum_{j=\overline{J}}^{J-1} \tau_l b.$$

Production technology is as in the baseline environment with the capital stock determined by the accumulation of assets on two savings accounts:

$$K_t = \sum_{j=0}^{J-1} s_{1,j} + s_{2,j}.$$

And market clearing conditions are given by (2.5).

The model with exogenous human capital has the initial human capital stock and investment hours profile as in the baseline economy. The household's problem and retirement arrangements are as in the endogenous environment described above.

Calibration and Results

The labor income tax is calibrated to keep lump-sum transfers as the share of output the same between the baseline and Mandatory RSA arrangements in the environment with endogenous human capital. The labor income tax is reduced from 27% in the baseline to 26.05%. The rate of contributions to Mandatory RSA, ξ, is set at 9%. This contribution rate is motivated by the superannuation guarantee system in Australia. Within the australian system, the employers are required to contribute 9% of wage earnings towards an employee's private retirement account, see OECD (2005).

Welfare gains from introducing these retirement arrangements are reported in Table 2.4. Table 2.5 compares aggregate variables between the economies with alternative and baseline retirement arrangements. Figures 2.6 through 2.10 compare equilibrium individual allocations at two stationary equilibria: (i) Mandatory RSA and (ii) baseline. First, I discuss a stationary equilibrium within the environment with exogenous human capital. Then, I explain the role of the endogenous human capital in the second environment considered.

Environment with exogenous human capital The retirement system with Mandatory RSA results in the capital stock being higher by 36.89% as compared to the baseline economy. This retirement arrangement gives the highest welfare gains because the agents enjoy higher consumption and leisure. Since the wage rate is higher by 13.18% as compared to the baseline, an individual can afford to work less and still enjoy the higher consumption.

Environment with endogenous human capital Under this retirement arrangement, the labor income tax is only slightly lower in comparison to the baseline economy. The agents choose to take advantage of the higher wage rate by accumulating more human capital, since the time investment into human capital accumulation at the beginning of life leads to the highest return in accordance with the human capital technology. Due to this technology restriction, agents choose to invest substantially more time during ages 20 to 34 into schooling as compared to the baseline. This increase in investment hours is accompanied by a decrease in market hours. During this period their borrowing constraint binds and is tighter under Mandatory RSA.

Table 2.6 reports the welfare ranking for two alternative retirement arrangements and two environments that differ by human capital endogeneity. From this table, I conclude that the endogeneity of human capital is important in evaluating tax and social security policies.

2.3 Conclusion

This chapter considers three different retirement systems and analyzes the role of human capital endogeneity in evaluating these policies. I show that incorporating human capital investment into an overlapping generations model is important. Welfare ranking of two alternative retirement arrangements depends on the environment considered. Within the environment with exogenous human capital, a household enjoys the highest welfare when he or she is required to save a fixed fraction of labor income in the tax-deferred Mandatory RSA. These arrangements result in the highest level of savings and the highest wage rate. Hence, the households get the highest level of consumption while enjoying more leisure.

The ranking is different in the environment with endogenous human capital. The retirement arrangements with Voluntary RSA lead to the highest household's welfare. In comparison with the exogenous environment, an agent has an additional margin of adjustment, human capital investment. He or she prefers to accumulate more human capital under the alternative arrangements in order to take advantage of higher wage rates. Due to the human capital technology, time investment early in life earns the highest return. Because young agents are borrowing constrained and shift hours from market production into human capital accumulation, the agents prefer to save during middle ages. The arrangements with Voluntary RSA allow an agent to optimally and simultaneously choose the stocks of physical and human capitals. Hence, these arrangements result in the highest welfare. Under Mandatory RSA, young agents have tighter borrowing constraints when they are forced to save.

Table 2.1: Model Parameters.

Parameter	Expression	Value
Preferences and Technology		
Discount factor	β	0.966
Leisure preference parameter	α	1.88
Capital share	θ	0.333
Depreciation rate of physical capital	δ_k	0.075
Government Sector		
Tax rate on consumption	τ_c	0.05
Tax rate on labor income	τ_l	0.27
Tax rate on capital income	τ_k	0.40
Share of government expenditures	g	0.1444
Replacement rate for Social Security benefits	ϕ	0.195
Human Capital Technology		
Initial stock of human capital	h_0	0.25
Depreciation rate of human capital	δ_h	0.062
Productivity of HC accumulation	B	0.6
Weight of HC stock	ψ_1	0.43
Weight of time investment	ψ_2	0.43

All parameter values are given in annual terms. Since one model period corresponds to five years, the parameters are adjusted in computations accordingly.

Table 2.2: Welfare gains for Voluntary Retirement Savings Accounts.

	Welfare Gains, in %
Exogenous Human Capital	7.24
Endogenous Human Capital	6.53

Table 2.3: Aggregate variables: Baseline and Voluntary RSA.

	Value[a] in Baseline	Change in % from baseline	
		HC Exogenous	HC Endogenous
After-tax interest rate, %	4.00	-0.49[b]	-0.31[b]
Wage rate	41.65	3.69	2.25
Capital stock	38.80	16.31	16.91
Labor supply	1.27	4.31	9.34
Capital-labor ratio	30.53	11.51	6.92
Output	79.34	8.16	11.81
Capital-output ratio	2.44	7.54	4.56
Average market hours	29.00	4.56	3.88

[a]All variables are in annual terms.
[b]Change of interest rate from the baseline one is in percentage points.

Table 2.4: Welfare gains for Mandatory Retirement Savings Accounts.

	Welfare Gains, in %
Exogenous Human Capital	7.77
Endogenous Human Capital	4.77

Table 2.5: Aggregate variables: Baseline and Mandatory Retirement Savings Accounts.

	Value[a] in Baseline	Change in % from baseline	
		HC Exogenous	HC Endogenous
After-tax interest rate, %	4.00	-1.58[b]	-1.48[b]
Wage rate	41.65	13.18	12.19
Capital stock	38.80	36.89	41.20
Labor supply	1.27	-5.62	-0.03
Capital-labor ratio	30.53	45.04	41.24
Output	79.34	6.82	12.15
Capital-output ratio	2.44	28.15	25.90
Average market hours	29.00	-4.29	-5.91

[a]All variables are in annual terms.
[b]Change of interest rate from the baseline one is in percentage points.

Table 2.6: Welfare gains for Different Retirement Arrangements.

	Change in % from baseline	
	HC Exogenous	HC Endogenous
Voluntary RSA	7.24	6.53
Mandatory RSA	7.77	4.77

Welfare gains are measured by life-time consumption equivalents.

Table 2.7: National Accounts, Relative to GDP, 2000 (in percentage)

1	GROSS DOMESTIC PRODUCT (NIPA 1.1.5)	100.00
2	Private consumption	68.65
3	Personal consumption expenditures (NIPA 1.1.5)	68.65
4	Investment expenditures	16.91
5	Gross private domestic investment (NIPA 1.1.5)	17.68
6	Gross government investment (NIPA 3.1)	3.10
7	Net exports of goods and services (NIPA 1.1.5)	-3.87
8	Government consumption expenditures (NIPA 3.1)	14.44

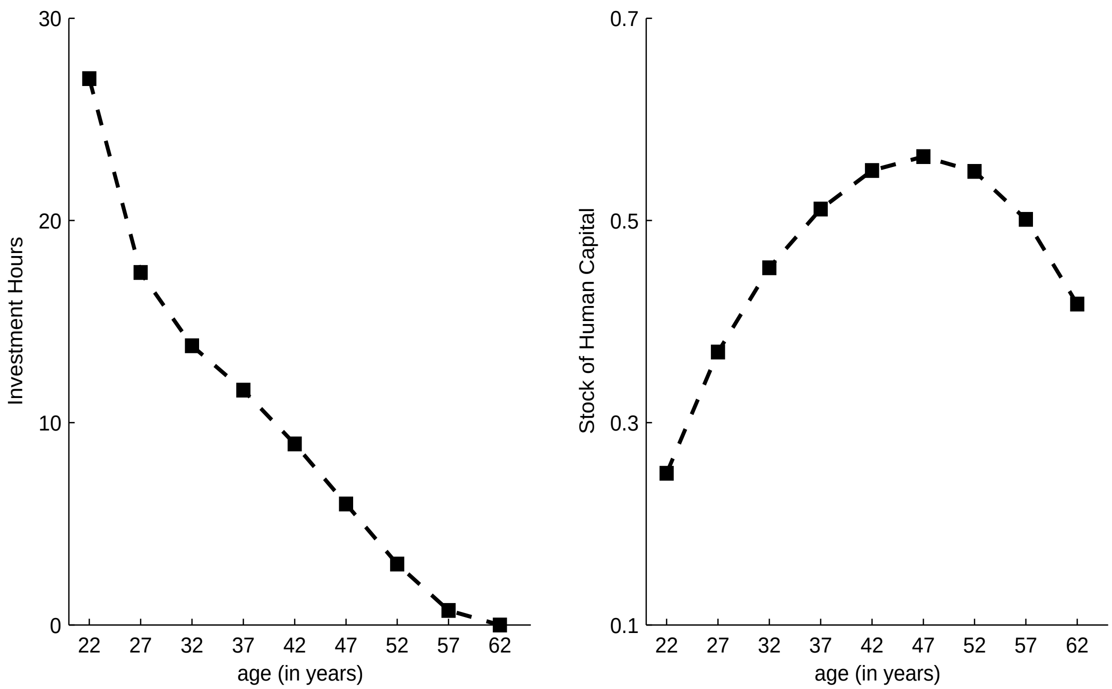
Sources are listed in the Appendix A.3 .

Table 2.8: Government Expenditure Programs, Relative to GDP, 2000 (in percentage).

1	Current expenditures (NIPA 3.1)	29.40
2	Government final consumption expenditure (NIPA 3.1)	14.44
3	Current transfer payments (NIPA 3.1)	10.82
4	Expenditures of OASI and DI fund (ASS 4.A3)	4.23
5	Other expenditures (NIPA 3.1)	4.15

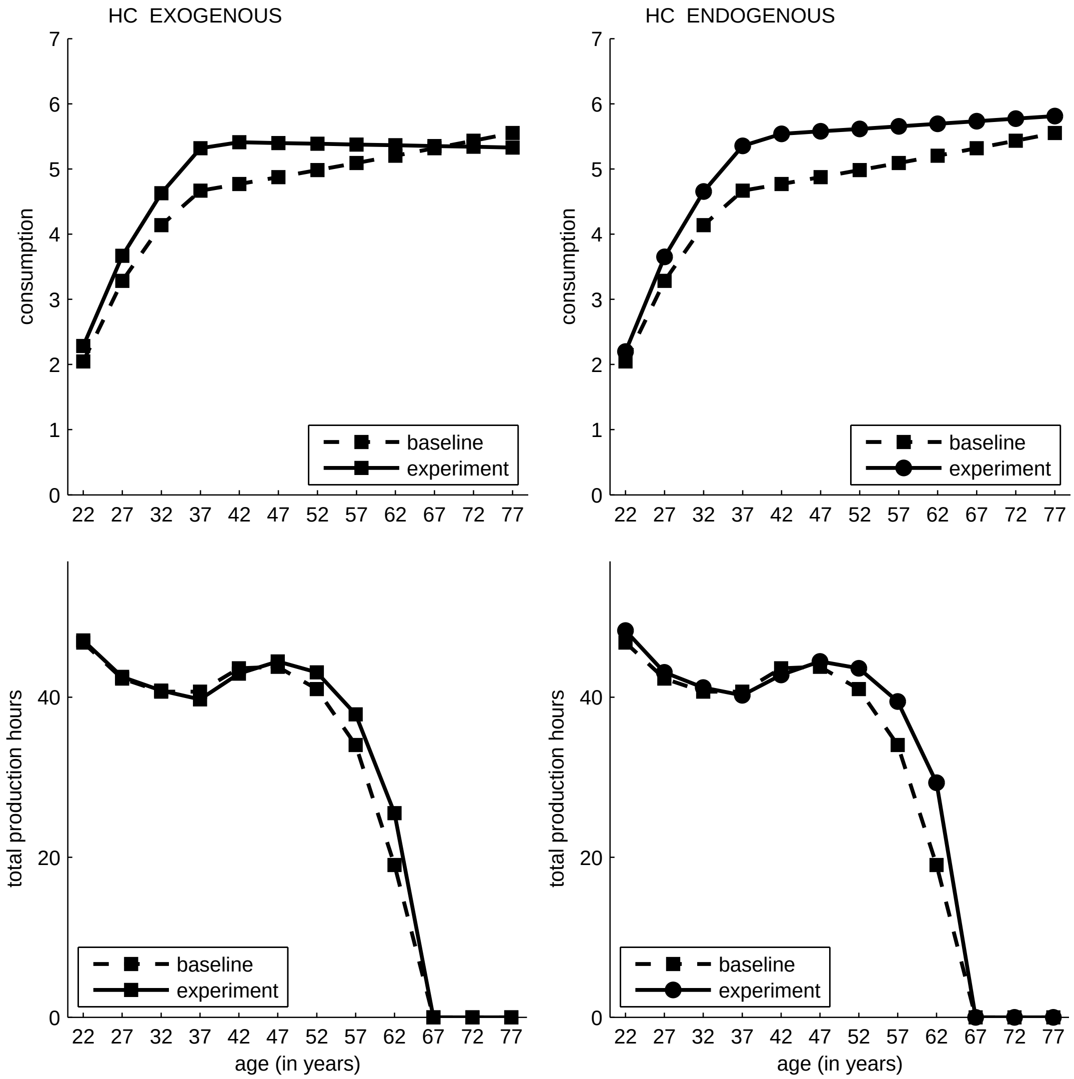
Sources are listed in the Appendix A.3 .

Figure 2.1: Life-cycle profiles of investment hours and human capital.



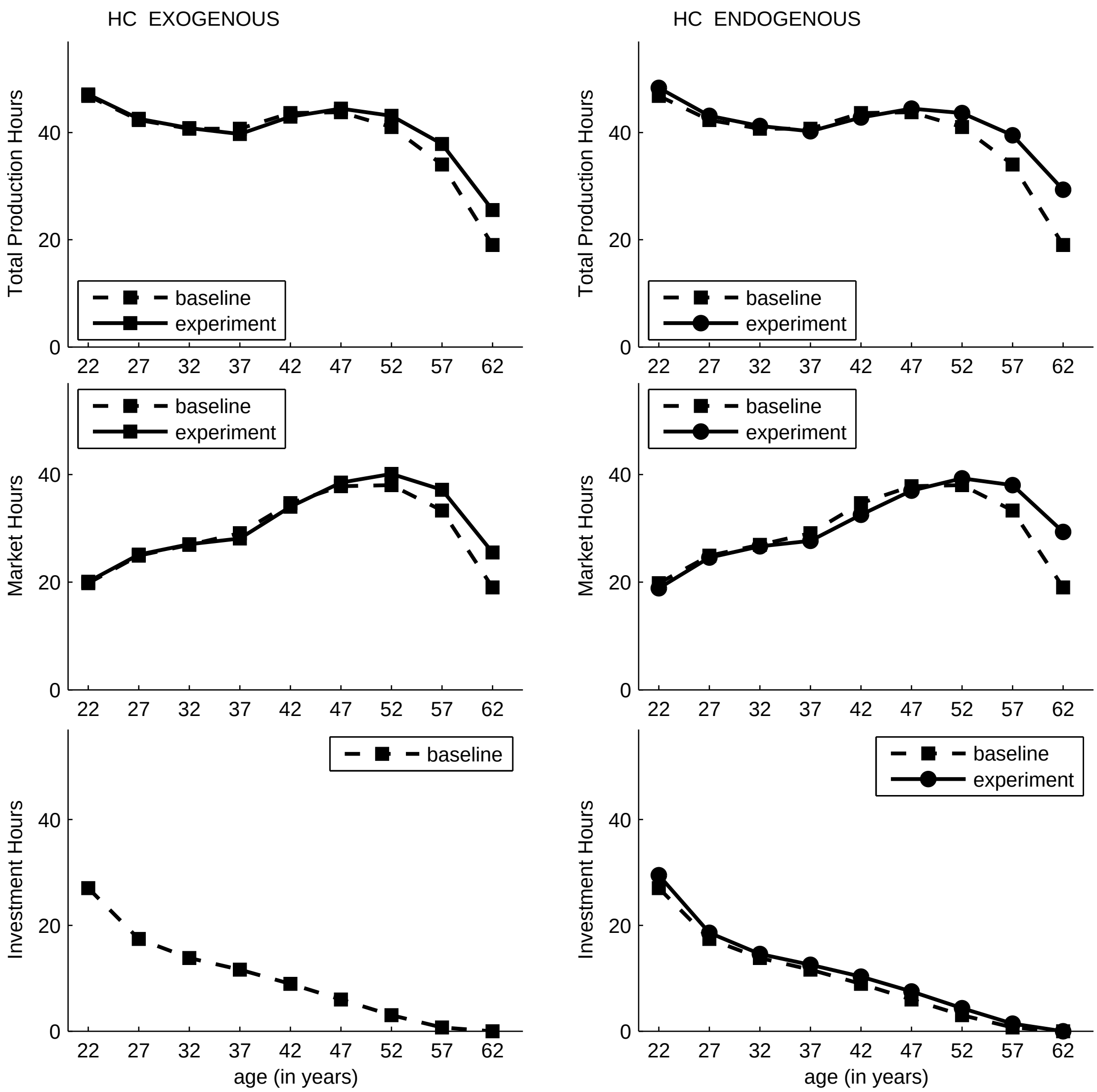
Left panel graphs the investment hours per week and right panel plots the stock of human capital in model units. These profiles are held fixed in the environment with exogenous human capital.

Figure 2.2: Baseline and Voluntary RSA environments: consumption and hours profiles.



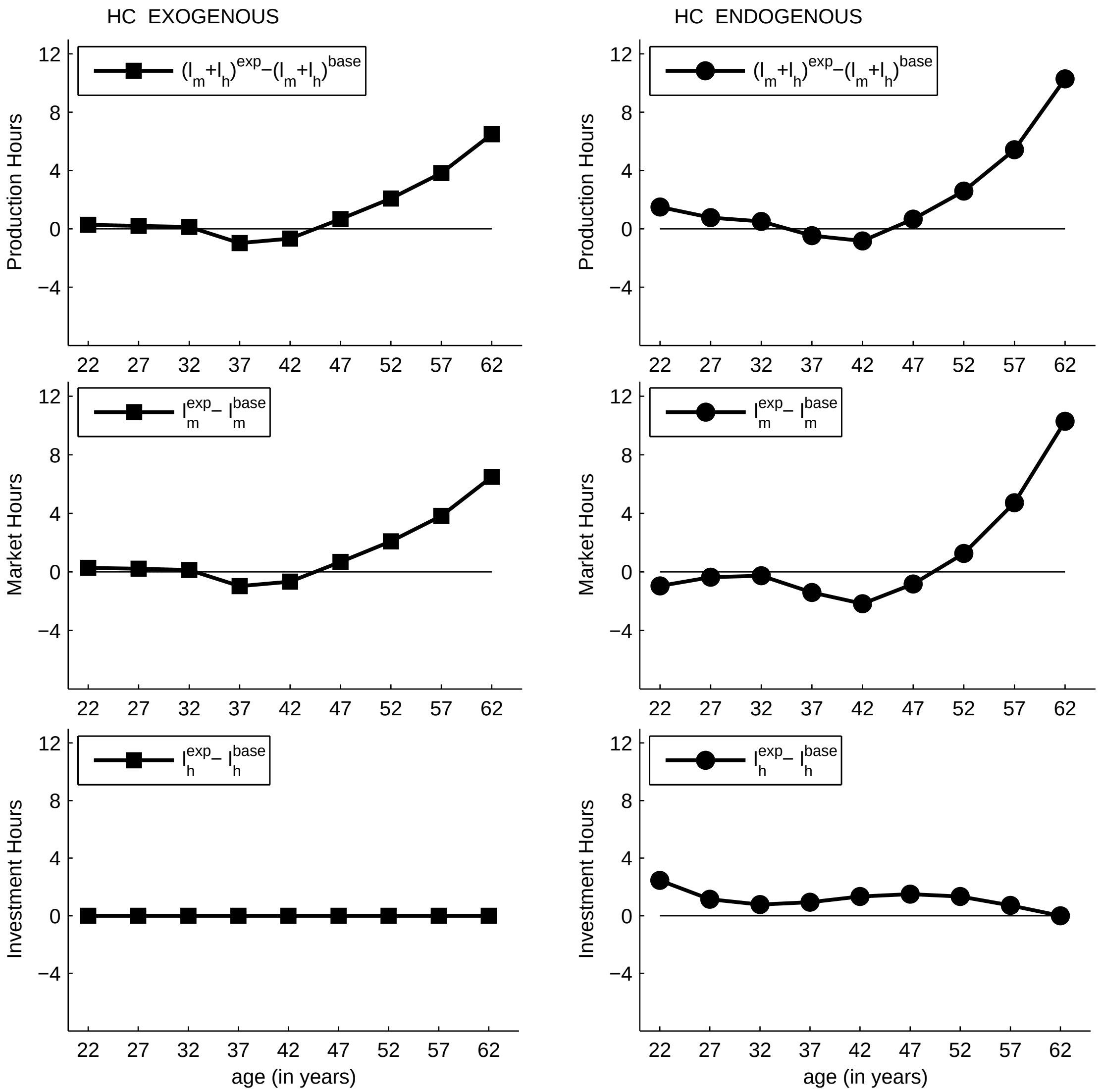
Figures 2.2 through 2.5 compare steady states in the environment with Voluntary Retirement Savings Accounts to the one in the baseline environment; variables in these equilibria are denoted as 'experiment' and 'baseline', respectively. Left panel: economies with exogenous human capital. Right panel: economies with endogenous human capital. Top panel: consumption. Bottom panel: total weekly production hours are hours for market production activities plus hours invested into human capital accumulation.

Figure 2.3: Life-cycle profiles of weekly hours.



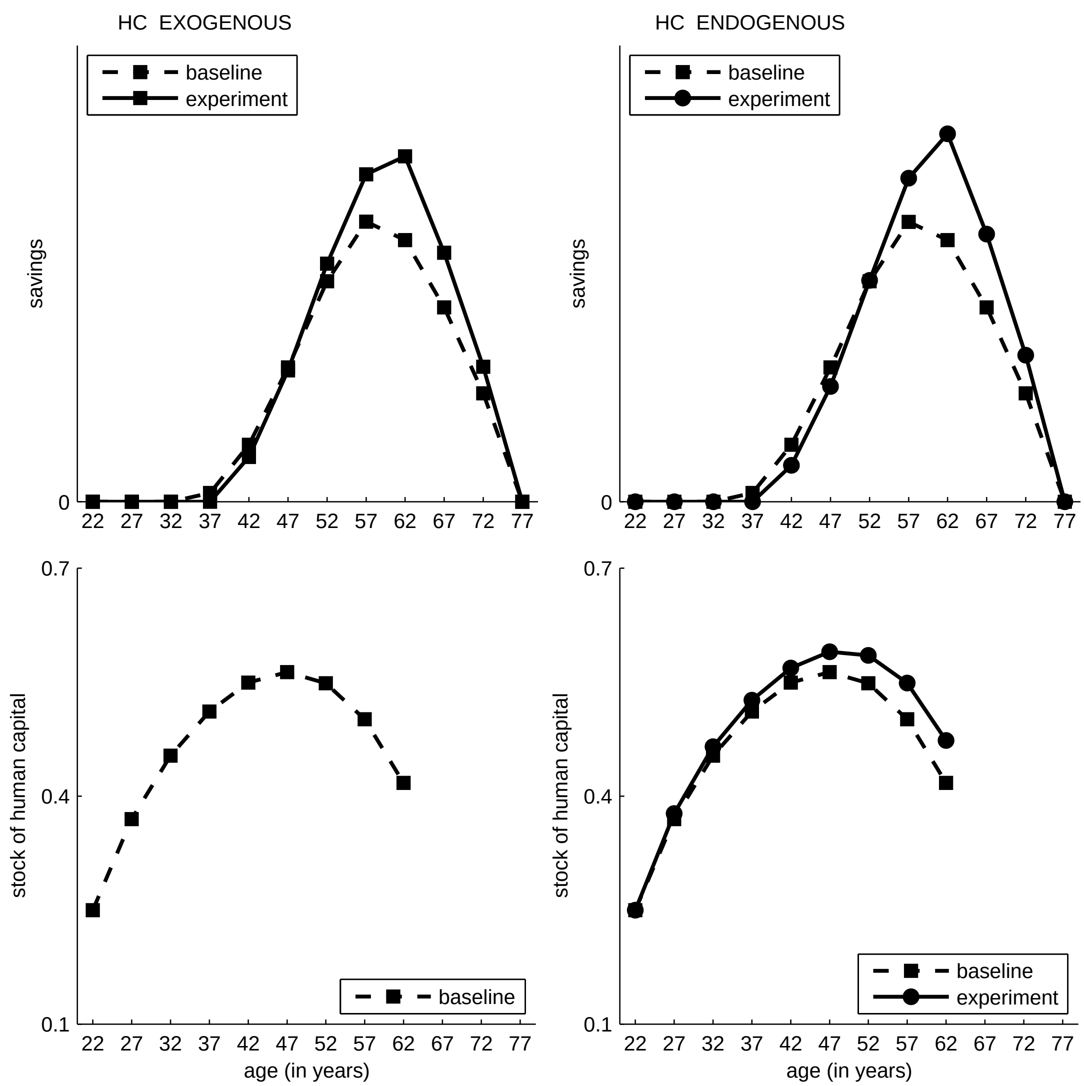
Top panel: total production hours. Middle panel: hours supplied for market production activities. Bottom panel: hours invested into human capital accumulation.

Figure 2.4: Difference in weekly hours between Voluntary RSA and baseline environments.



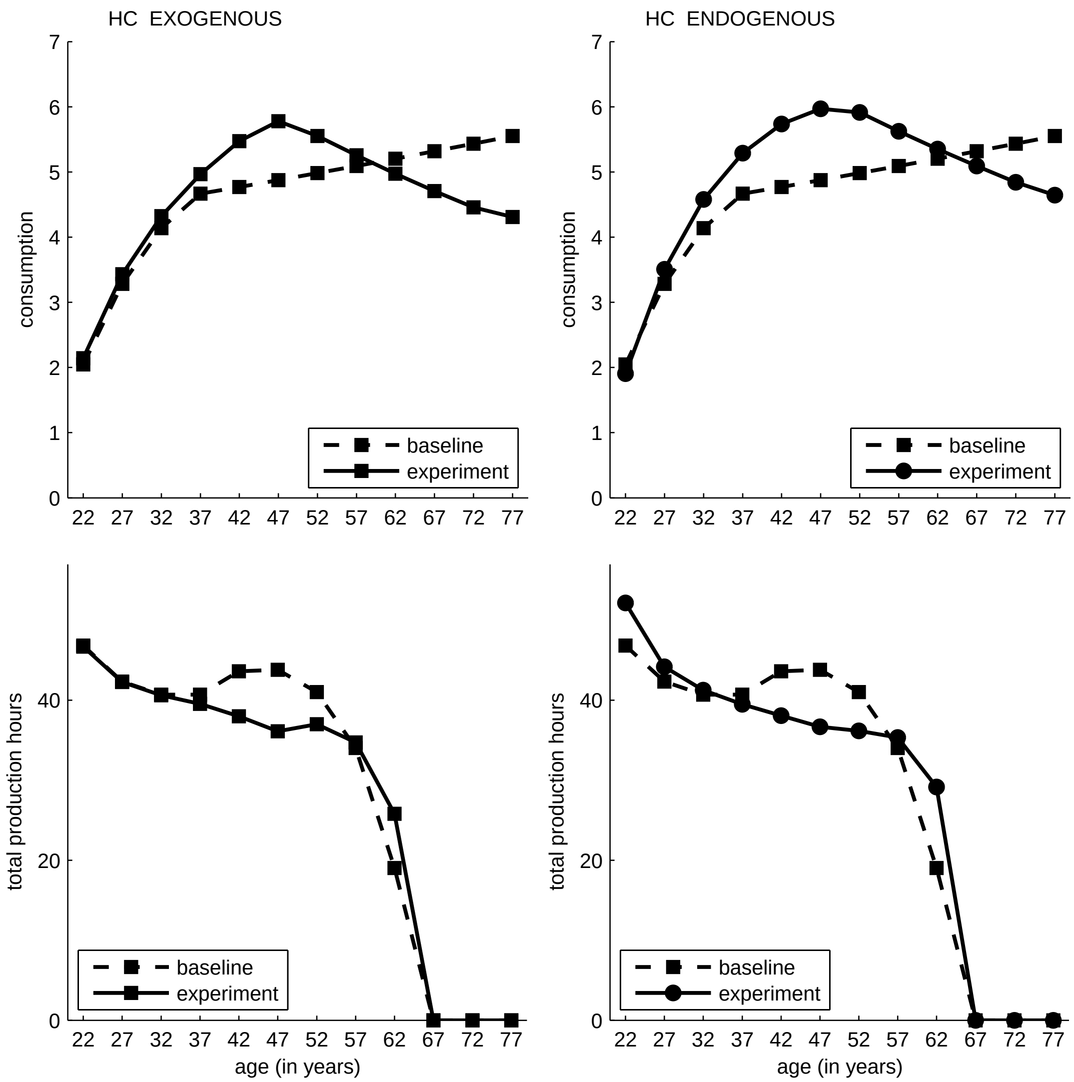
Top panel: total production hours. Middle panel: market hours. Bottom panel: investment hours.

Figure 2.5: Stocks of physical and human capital.



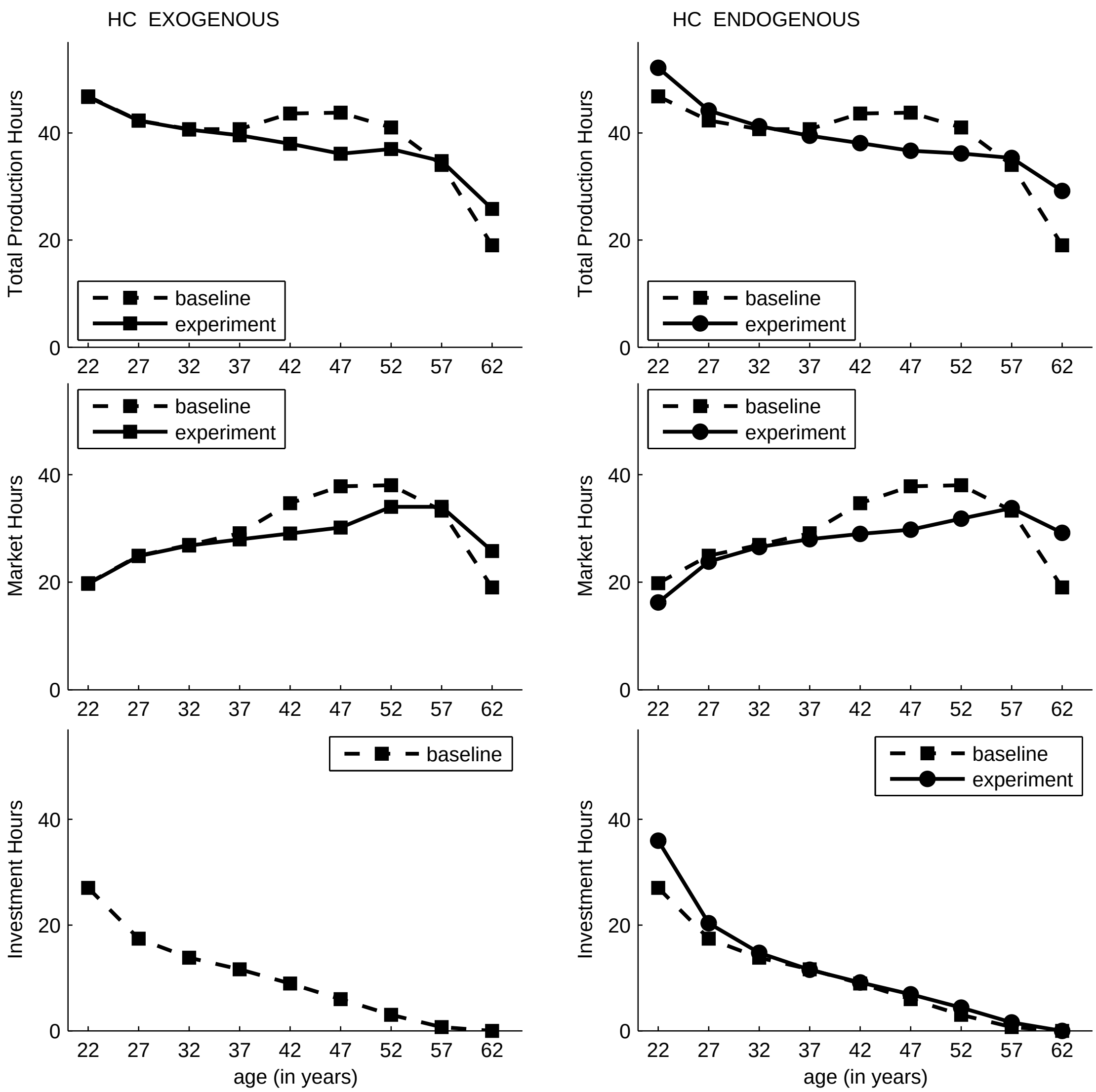
Top panel: stock of physical capital. Bottom panel: stock of human capital.

Figure 2.6: Baseline and Mandatory RSA environments: consumption and hours profiles.



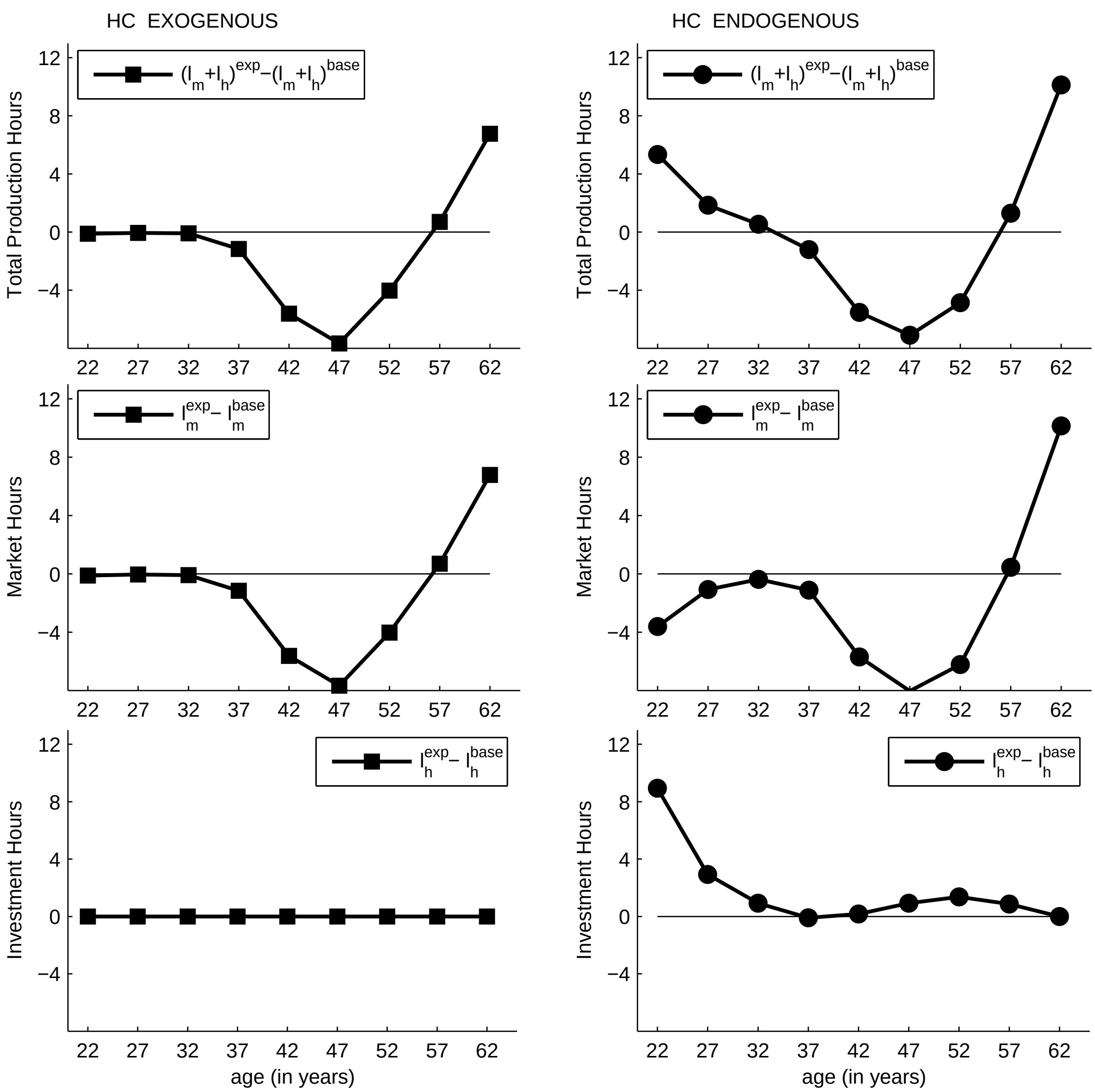
Figures 2.6 through 2.10 compare steady states in the environment with Mandatory Retirement Savings Accounts to the one in the baseline environment; variables in these equilibria are denoted as 'experiment' and 'baseline', respectively. Left panel: economies with exogenous human capital. Right panel: economies with endogenous human capital. Top panel: consumption. Bottom panel: total weekly production hours are hours for market production activities plus hours invested into human capital accumulation.

Figure 2.7: Baseline and Mandatory RSA environments: Life-cycle profiles of weekly hours.



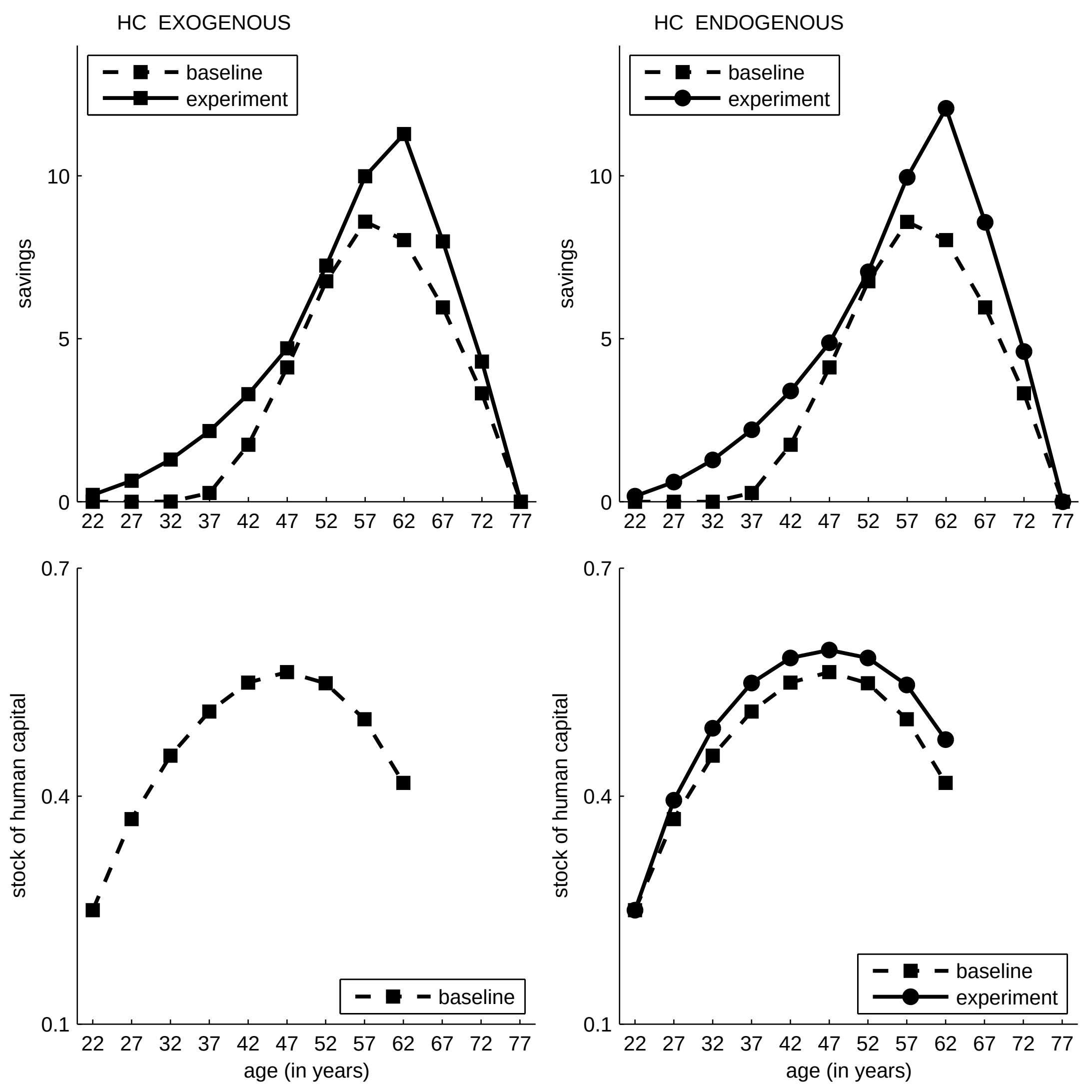
Top panel: total production hours. Middle panel: hours supplied for market production activities. Bottom panel: hours invested into human capital accumulation.

Figure 2.8: Difference in weekly hours between Mandatory RSA and baseline environments.



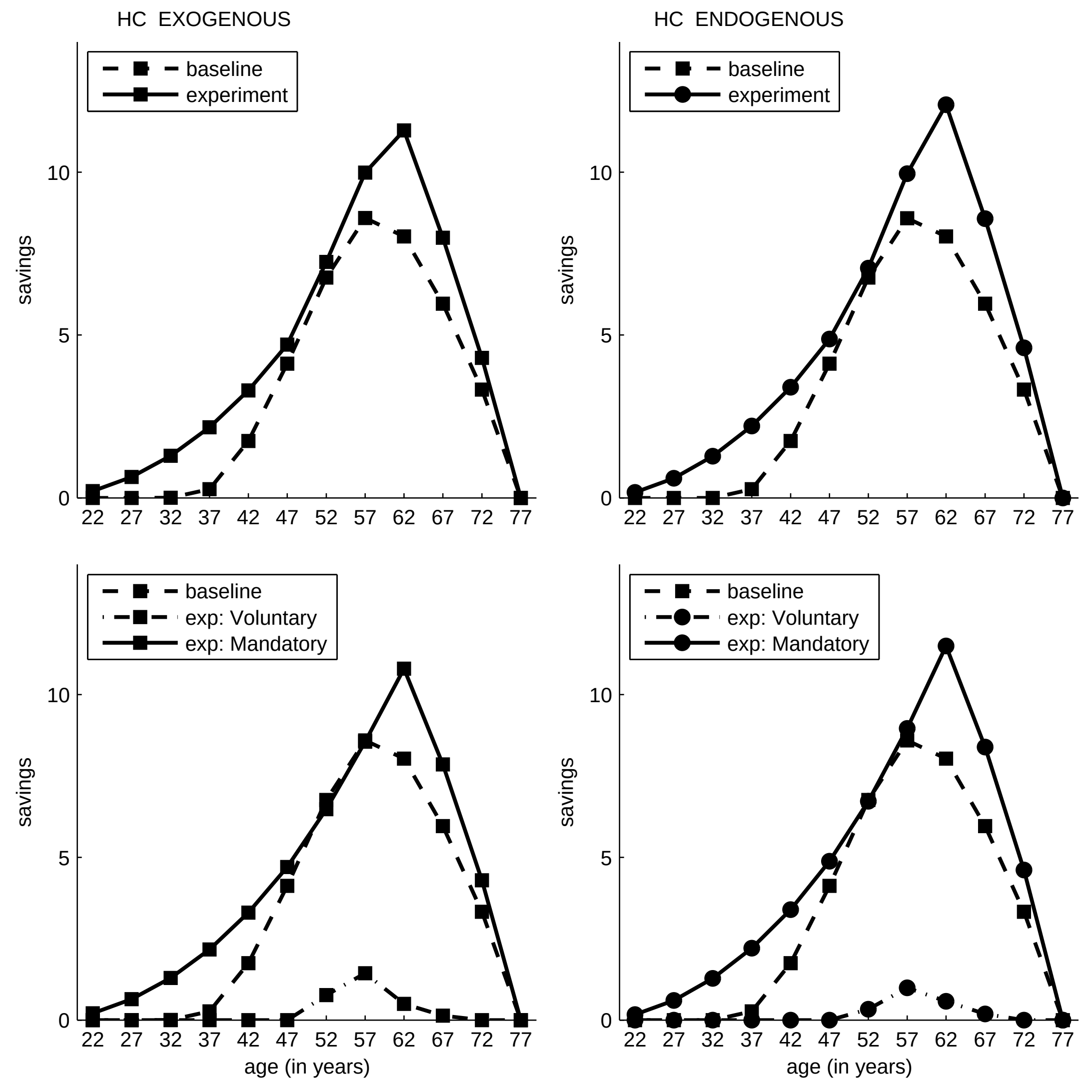
Top panel: total production hours. Middle panel: market hours. Bottom panel: investment hours.

Figure 2.9: Stocks of physical and human capital.



Top panel: stock of physical capital. Bottom panel: stock of human capital.

Figure 2.10: Mandatory RSA: decomposition of physical capital stock.



Top panel: total stock. Bottom panel: decomposition into assets held at voluntary and mandatory accounts.

Chapter 3

Feasibility of Welfare-Enhancing Social Security Reform in the Presence of Human Capital Investment

One of the proposals for reforming the pay-as-you-go (PAYG) retirement system is to introduce fully-funded system. The benefits of this system are the higher rates of return on the contributions made to the system and the higher level of aggregate savings. At the same time, the transition generations have to incur the costs, in terms of reduced consumption or higher market hours, to implement the new system. How much the transition generations have to be compensated to gain their support for the retirement reform? Many authors propose to issue the compensation in terms of additional government debt and preserve the pension rights of the current working generations.

Several previous studies have analyzed the transition from a PAYG to a fully funded Social Security system using overlapping generations framework and have confirmed the welfare gains in the long-run and substantial welfare losses during the transition. Kotlikoff, Smetters and Walliser (1999) consider various reforms to implement the transition to a fully funded system. They find that all reform proposals lead to the short-run welfare losses between two and four percent of life-time consumption. The reforms that neutralize or reverse the welfare losses for transition generations have been analyzed by Conesa and Garriga (2008) and Joines (2007). Conesa and Garriga (2008) can design a welfare-enhancing reform if the government has access to a large set of policy tools. In their paper, the reforms that benefit transition generations feature an age-dependent tax schedule with some cohorts subject to negative taxes. Joines (2007) limits the policy tools to the ones currently employed in the existing fiscal systems and rules out negative and age-dependent taxes. The welfare-enhancing reform is achieved through increase in the retirement benefits to transition generations. This increase in Social Security expenditures is financed through increase in consumption tax and government debt. All of the above papers consider the labor productivity during the life-cycle as exogenously given and abstract from the agents' investment decisions to enhance their human capital and labor productivity.

Fuster, Imrohoroglu and Imrohoroglu (2007) analyze a model with two-sided altruism and find a reform that is favored by the majority of transition generations. In their model, middle-aged generations invest into education of their children and older generations receive the transfers from the younger ones. In this chapter, I abstract from the altruism motive and consider the time investment that agents undertake to enhance their labor productivity during the working years.

In this chapter, I study the elimination of PAYG Social Security system in two economic environments that differ by the human capital production technology. In the environment with endogenous human capital, agents allocate their time endowment between leisure, market production and time investment into human capital. The human capital production technology is of Ben-Porath (1967) type. The labor productivity tomorrow depends on the undepreciated level of the human capital today and on the production of new human capital which depends on the time investment and the stock of current human capital. In the environment with exogenous human capital, the life-cycle labor productivity is exogenously given and the agents allocate time endowment between two activities, leisure and market production. This exogenous environment is similar to the one used by Joines (2007).

The initial stationary equilibriums with PAYG Social Security system in the two environments coincide in terms of individual and aggregate variables due to the calibration strategy employed. Then, I eliminate PAYG system in both economic environments and determine the agents' response to this reform. The welfare calculations show that the reform leads to welfare losses for the transition generations and the long-term welfare gains are smaller in the environment with the endogenous human capital. Therefore, the amount of resources that can be transferred from the future generations to the transition ones to compensate for welfare losses is smaller in the environment with the endogenous human capital.

The economic environment with overlapping generations to study the transition is described in section 3.1. The calibration of the model to macroeconomic and labor market observations is documented in section 3.2. The reform to implement the fully funded Social Security system is discussed in section 3.3. The results are discussed in section 3.4 and section 3.5 concludes the chapter.

3.1 Economic Environment

To quantify the effects of different social security arrangements on savings and welfare, I consider two general equilibrium models with OG structure. Common features between these two economic environments are the finite and certain life-span of agents, the Cobb-Douglas production technology, and the set of government policies. The agents allocate their time endowment among leisure, market production, and time investment into human capital accumulation. These environments differ by the evolution of the human capital profile over the agents' life-cycle and time allocation decisions. In the first environment, the agents' time allocation decisions, in particular, the time investment into human capital enhancement, determine the evolution of the human capital and, consequently, the wage income profiles over the life-cycle. This environment is called the environment with endogenous human capital. The second environment has an exogenous age-specific labor productivity profile and is called the economy with exogenous human capital. The traditional macroeconomics policy literature studies the model of this type.

I initially describe the environment with endogenous human capital accumulation decision. Then, I explain the features of the one with exogenous human capital.

3.1.1 Environment with Endogenous Human Capital

I start with a description of the demographic structure and preferences. The economy has overlapping generations of agents who live for J adult periods, with ages denoted by $j \in \Im \equiv \{0, ..., J-1\}$. The agents' life-spans are certain. In the first time period, the measure of newly born agents is normalized to 1. At each date t, a new cohort is born that is η percent larger than the previous cohort. The size of the cohort born at period t and of age j is denoted by n^t_{t+j}.

A young agent born at period t is endowed with initial levels of physical and human capitals, s^t_t and h^t_t respectively.[1] Each period agents are endowed with one unit of time that

[1]Notational convention for an agent's variables is as follows. The superscript denotes a period when an agent is born, and the subscript is a time period when an allocation takes place. Hence, an agent's age is given by the difference between the subscript and the superscript.

can be allocated to leisure, production activities in the market sector, or investment into human capital accumulation. Let $\left\{c^t_{t+j}, l^t_{m,t+j}, l^t_{h,t+j}\right\}$ denote consumption, market hours, and investment hours, respectively, of an agent born at period t (superscript) and at time period $t+j$ (subscript). The preferences of a young agent born at period t are ordered by

$$\sum_{j=0}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}), \tag{3.1}$$

where β is a time preference parameter. Each agent chooses sequences of consumption, market hours, and investment hours to maximize the discounted value of life-time utility subject to his or her budget constraint,

$$(1+\tau_{ct})\, c^t_{t+j} + s^t_{t+1+j} \leq (1-\tau_{lt})\, w_{t+j} h^t_{t+j} l^t_{m,t+j} + (1 + (1-\tau_{kt})\, r_{t+j}) s^t_{t+j} + d^t_{t+j}. \tag{3.2}$$

This constraint must be balanced at each age of the agent's life, i.e., for any $j \in \Im$. The agent's expenditures on consumption and savings in the form of physical capital, s^t_{t+1+j}, must be less or equal to the after-tax income.

The agent born at period t and of age j receives labor income $w_{t+j} h^t_{t+j} l^t_{m,t+j}$, where w_{t+j} is the real wage per efficient unit of labor in terms of the consumption good at period $t+j$. The agent's labor productivity at age j depends on the stock of human capital h^t_{t+j}, which is determined by the undepreciated human capital from the last period and the new human capital accumulation during the last period:

$$h^t_{t+j} = (1-\delta_h) h^t_{t+j-1} + Q(h^t_{t+j-1}, l^t_{h,t+j-1}). \tag{3.3}$$

The creation of new human capital depends on its existing level and investment hours and is determined by the function $Q(h, l_h)$. The Q function is increasing in both arguments and has decreasing returns to scale. The agent's savings earn capital income at the real rate of return r_{t+j}. Agents are restricted to have strictly positive amount of savings at all ages

$$s^t_{t+j} \geq 0. \tag{3.4}$$

Agents pay taxes on consumption at rate τ_{ct}, labor income at rate τ_{lt}, and capital income net of depreciation at rate τ_{kt}.

The government transfers to the agent born at t and of age j are denoted by d^t_{t+j}. These transfers consist of two components: a lump-sum transfer for agents of all ages, f^t_{t+j}, and social security benefits to retirees, b_{t+j}^{t},

$$d_{t+j}^{t} = \begin{cases} f_{t+j}^{t}, & j = 0, \overline{J}-1, \\ f_{t+j}^{t} + b_{t+j}^{t}, & j = \overline{J}, J-1. \end{cases}$$

Agents are entitled to retirement benefits starting with age $\overline{J}$. The amount of social security benefits is the fraction of the average labor income during working periods. This fraction for the individual born at period t is called a replacement rate, ϕ^{t}. The social security benefits of the individual born at period t are calculated as

$$b_{t+j}^{t} = \phi^{t} \frac{\sum_{i=0}^{\overline{J}-1} w_{t+i} h_{t+i}^{t} l_{m,t+i}^{t}}{\overline{J}}, j = \overline{J}, J-1. \tag{3.5}$$

The government's budget is balanced every period. The government levies taxes on consumption, labor income, and capital income and uses tax revenue to purchase a wasteful public good, G_t, and provide two types of transfers. Then, the government's budget constraint at period t is

$$G_t + \sum_{j=0}^{J-1} n_t^{t-j} f_t^{t-j} + \sum_{j=\overline{J}}^{J-1} n_t^{t-j} b_t^{t-j} = \sum_{j=0}^{J-1} n_t^{t-j} \left(\tau_{ct} c_t^{t-j} + \tau_{lt} w_t h_t^{t-j} l_{m,t}^{t-j} + \tau_{kt} r_t s_t^{t-j} \right).$$

At period t, firms hire capital, K_t, and labor, L_t, to produce output with a constant returns-to-scale production technology,

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta},$$

where A_t is total factor productivity. The aggregate inputs are determined as

$$\begin{aligned} K_t &= \sum_{j=0}^{J-1} n_t^{t-j} s_t^{t-j}, \\ L_t &= \sum_{j=0}^{J-1} n_t^{t-j} h_t^{t-j} l_{m,t}^{t-j}. \end{aligned}$$

The aggregate feasibility constraint at period t is

$$\sum_{j=0}^{J-1} n_t^{t-j} c_t^{t-j} + K_{t+1} + G_t = A_t K_t^{\theta} L_t^{1-\theta} + (1-\delta_k) K_t. \tag{3.6}$$

Definition 4. *A competitive equilibrium is factor prices,* (w_t, r_t)*; aggregate allocations,* (K_t, L_t)*; individual allocations,* $\left(\left\{c_{t+j}^t, s_{t+1+j}^t, l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j}^t\right\}_{j\in\Im}\right)$ *for any generation born at* $t \in [1, \infty)$*; and government policies,* $(\tau_{ct}, \tau_{lt}, \tau_{kt}, \phi, G_t, F_t)$*, for any period* $t \in [1, \infty)$ *such that the following holds: (1) given factor prices and government policies, individual allocations,* $\left(\left\{c_{t+j}^t, s_{t+1+j}^t, l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j}^t\right\}_{j\in\Im}\right)$*, maximize (3.1) subject to (3.2)-(3.4) for each generation* t*; (2) factor inputs are paid the marginal products for any period* t *:*

$$\begin{aligned} w_t &= (1-\theta) A_t K_t^{\theta} L_t^{-\theta}, \\ r_t &= \theta A_t K_t^{\theta-1} L_t^{1-\theta} - \delta_k; \end{aligned}$$

(3) government's budget is balanced every period; and (4) aggregate and individual allocations satisfy market clearing conditions for any period t*.*

3.1.2 Environment with Exogenous Human Capital

This model is motivated by the macroeconomics literature that studies OG models with life-cycle labor productivity being exogenously given. To make comparisons to this literature, I modify the model from the previous subsection in the following way. The life-cycle profiles of human capital and investment hours are exogenously fixed at the level of the solution for the model with endogenous human capital under the baseline calibration.

The demographic structure, production technology, the set of government policies, and market clearing conditions of this economy are the same as the one in the model with endogenous human capital decisions. The difference between the two models is in the agents' decisions. Let $\left(\{\bar{l}_{h,j}, \overline{h}_j\}_{j\in\Im}\right)$ be a fixed life-cycle profiles of investment hours and human capital stock. Introducing a fixed life-cycle profile of investment hours is equivalent to changing the time endowment over the life-cycle. Consequently, the time endowment for each agent is $\left(\{1-\bar{l}_{h,j}\}_{j\in\Im}\right)$. The preferences of a young agent born at period t are ordered by

$$\sum_{j=0}^{J-1} \beta^j u(c_{t+j}^t, 1 - l_{m,t+j}^t - \bar{l}_{h,j}). \tag{3.7}$$

Each agent chooses a sequence of consumption and market hours to maximize a discounted value of life-time utility subject to the budget constraint,

$$(1+\tau_{ct})\, c_{t+j}^t + s_{t+1+j}^t \leq (1-\tau_{lt})\, w_{t+j} \overline{h}_j l_{m,t+j}^t + (1 + (1-\tau_{kt})\, r_{t+j}) s_{t+j}^t + d_{t+j}^t. \tag{3.8}$$

This constraint must be balanced every period of an agent's life, i.e., for any $j \in \Im$. An agent's labor productivity over the life-cycle is predetermined by the profile of human capital. This human capital profile is frequently called the efficiency units profile.[2] The agents are restricted to have positive physical capital asset holdings during all ages,

$$s_{t+j}^{t} \geq 0. \tag{3.9}$$

Taxes levied on the agents' income and expenditures and transfer system are the same as in the model with endogenous human capital accumulation. The government's budget constraint and market clearing conditions are as in the model with endogenous human capital accumulation.

Definition 5. *A competitive equilibrium is factor prices,* (w_t, r_t)*; aggregate allocations,* (K_t, L_t)*; individual allocations,* $\left(\left\{c_{t+j}^{t}, s_{t+1+j}^{t}, l_{m,t+j}^{t}\right\}_{j\in\Im}\right)$ *for any generation born at* $t \in [1, \infty)$*; profiles of investment hours and human capital stock* $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$*; and government policies,* $(\tau_{ct}, \tau_{lt}, \tau_{kt}, \phi, G_t, F_t)$*, for any period* $t \in [1, \infty)$ *such that the following holds: (1) given factor prices, government policies, and profiles* $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$*, individual allocations,* $\left(\left\{c_{t+j}^{t}, s_{t+1+j}^{t}, l_{m,t+j}^{t}\right\}_{j\in\Im}\right)$*, maximize (3.7) subject to (3.8) and (3.9) for each generation t; (2) factor inputs are paid the marginal products for any period* t :

$$\begin{aligned} w_t &= (1-\theta) A_t K_t^{\theta} L_t^{-\theta}, \\ r_t &= \theta A_t K_t^{\theta-1} L_t^{1-\theta} - \delta_k; \end{aligned}$$

(3) government's budget is balanced every period; and (4) aggregate and individual allocations satisfy market clearing conditions for any period t*.*

3.2 Calibration of the economic environments

I calibrate the baseline economies to the U.S. tax and social security system. The calibration year is 2000. Parameters of demographics, preferences, and technology are the same between the economies with exogenous and endogenous human capital. The parameters of human capital production technology in the economy with endogenous human capital are calibrated to the life-cycle earnings profile. Appendix C provides details on data

[2] Examples are Rios-Rull (1996) and DeNardi et al. (1999).

sources and calculation procedures for all parameters. The parameter values are summarized in Table 3.2.

3.2.1 Parameters for both economies

The demographic structure of the economy is calibrated as follows. Agents enter the economy at age 20, retire at age 65, and die at age 80. Each model period corresponds to 5 years. Hence, the agents are working during the first nine model periods and are retired during the last three model periods. In this section, I report all parameters in annual terms and adjust these parameters accordingly in computations.

The time preference parameter β is calibrated to match the after-tax interest rate of 4.0 percent per year. I assume that the agents' flow utility functions are

$$u(c, 1 - l_m - l_h) = \log c + \alpha \log(1 - l_m - l_h),$$

where α is chosen to match average weekly hours of the population of ages between 20 and 64. Based on Census data, average hours for working age population is 29 hours per week.

The calibration of production technology is standard. Capital income share, θ, is set to 0.333. Depreciation of physical capital, δ_k, is calibrated to match the investment share in GDP. This investment share is equal to 16.9% of GDP in 2000. The resulting depreciation rate is 7.5%. This estimate of the depreciation rate is higher than the one commonly used in the literature. Stokey and Rebelo (1995) estimate the depreciation rate to be 6%. Rios-Rull (1996) calibration results in the rate of 5.4%.

Average effective tax rates are calibrated using the methodology of Mendoza et al. (1994) and are reported in Table 3.2. The share of government expenditures in output, g, is set to match the corresponding value in NIPA. In 2000, the government consumption expenditures are 14.44 percent of GDP. The replacement rate for social security benefits, ϕ, is calibrated to match the benefit payments from the Old-Age and Survivors Insurance (OASI) Fund. In the calibration year, OASI benefit payments are equal to 4.23 percent of GDP and the resulting replacement rate is $\phi = .195$.

3.2.2 Parameters of human capital production technology

I assume the following law of motion for human capital:

$$h_{j+1} = (1 - \delta_h)h_j + Bh_j^{\psi_1} l_{h,j}^{\psi_2},$$

where the conditions $B, \psi_1, \psi_2 \geq 0$ and $\psi_1+\psi_2 \leq 1$ guarantee the decreasing returns to scale. Hence, the life-cycle profile of time investment into human capital is time-independent.

I have to choose five parameters for the human capital production technology: initial stock of human capital, h_0; the depreciation rate of human capital, δ_h; productivity of human capital accumulation, B; weight of human capital stock in new accumulation, ψ_1; and weight of time investment, ψ_2. I calibrate these parameters to match the life-cycle earnings profile, which is constructed using 2000 decennial Census data. I divide the population of ages between 20 and 64 into nine age groups, $j \in \{0, ..., 8\}$. The size of the working age population is denoted by N_t. The measure of earnings is the hourly wage rate, denoted by e_j, $j \in \{0, ..., 8\}$. The average wage rate for the working population, $\overline{e}$, is \$17.24 per hour. This average rate for the working population is calculated using the size of each age group, $n_j(t)$:

$$\overline{e} = \frac{\sum_{j=0}^{8} n_j(t) e_j}{N_t}.$$

To express the wage earnings profile in units comparable to the model, I report the average hourly wage for an age group j as the ratio to the average hourly wage of the working population: $\varepsilon_j = e_j/\overline{e}$, $j = 0, ..., 8$.

Equivalently, the wage rate in the model is $w_t h_j$ and the average wage for the working population is

$$\overline{wh} = \frac{\sum_{j=0}^{8} w_t h_j}{\overline{J}}.$$

I choose parameters of the human capital production function to minimize the distance between the model and data wage hour profiles:

$$\min_{(h_0, \delta_h, B, \psi_1, \psi_2)} \sum_{j=0}^{8} \left(\frac{wh_j}{\overline{wh}} - \frac{e_j}{\overline{e}} \right)^2.$$

The chosen parameters are reported in Table 3.2.

3.2.3 Stationary Equilibrium in both Baseline Environments

Given calibrated parameters, I solve for a stationary equilibrium in the economy with endogenous human capital. The procedure for the numerical algorithm is described in Appendix 5.2. Equilibrium life-cycle profiles of investment hours and human capital are given in Figure 1. Time investment into human capital accumulation is the highest at the beginning of life and exhibits steady decline. I refer to this time investment as investment hours. Hours devoted to market production activities are called market hours. The sum of these two types of time usage are called total production hours. The life-cycle profile of investment hours and technology for human capital production determine the life-cycle labor productivity. Under the baseline calibration, an individual reaches a peak in labor productivity between ages 45 and 49.

Let $\left(\left\{l^*_{h,j}, h^*_j\right\}_{j\in\Im}\right)$ denote equilibrium life-cycle profiles of investment hours and human capital in the environment with endogenous human capital.

Proposition 6. *If $\left\{\bar{l}_{h,j}\right\}_{j\in\Im} = \left\{l^*_{h,j}\right\}_{j\in\Im}$ and $\left\{\bar{h}_j\right\}_{j\in\Im} = \left\{h^*_j\right\}_{j\in\Im}$, a stationary equilibrium in the environment with exogenous human capital is identical to the one in the environment with endogenous human capital under baseline calibrated parameters.*

Proof. The method of the proof is to compare equilibrium conditions in the two environments. These conditions are derived in the Appendix B.1 . QED ■

Under the baseline calibration, the equilibria in the two environments are the same both on the aggregate and individual level. The values for various aggregate variables in the baseline environments are given in Table 3.3. With a calibrated after-tax interest rate of 4%, the resulting capital-to-output ratio is 2.76. In both environments, agents of working age devote on average 29 hours per week for market production activities.

3.3 Reform towards fully funded system.

I consider the reform from PAYG Social Security system to the one with Voluntary Retirement Savings Accounts (VRSA). Under the new system, the agents voluntarily save for their retirement. Because the agents have time-consistent preferences, they save enough of resources to smooth consumption over the retirement years. The reform is introduced at period $t = 1$. The generations that enter the labor market at the period $t = 1$ and later have to save for their retirement on their own. The generations that have been working at the time of reform announcement will receive partial retirement benefits. The retirement benefits are calculated based on the average wage income throughout working years and is given by equation (3.5). The amount of benefits received by the generation born at period t is determined by the replacement rate ϕ^t.

The replacement rate for the generations alive at the time of reform announcement and for all future generations is reported in Figure 3.1. The generations that have completed their working lives under the PAYG system receive the retirement benefits at the level of 44% of average wage income. These generations have entered the labor market from the period -55 to the period -40. The generations that have entered the labor market from the period -35 to 0 receive the benefits based on the number of years worked under the initial retirement system. These generations will have to supplement their future retirement consumption with saving accounts.

3.4 Results

The results for elimination of the PAYG Social Security system are reported in Figures 3.2 to 3.5.

During transition, taxes on consumption and capital income are held fixed while the labor income tax is adjusted to balance the government's budget. Because the government's expenditures on the Social Security benefits are gradually eliminated, the labor income tax is reduced from 27% in the initial stationary equilibrium to 22.9% and 22.7% in the new stationary equilibrium in the environment with the exogenous HC and the endogenous HC, respectively.

Introduction of new retirement rules leads to the increase in aggregate capital, labor and output as shown in Figure 3.3. The aggregate capital increases by approximately the same amount in two economic environments, by 16% as compared to the initial stationary equilibrium. Due to higher time investment, the aggregate labor supply is 11.4% higher in the new stationary equilibrium as compared to the initial one in the environment with endogenous HC. The corresponding number for the environment with fixed life-cycle labor productivity is 7.9%. The increase in the aggregate output between the final and initial stationary equilibriums is 4.1% and 8.6% in the environment with exogenous and endogenous HC, respectively.

The welfare of transition and future generations is measured by compensating variations and is reported in Figure 3.5. Cohort -55 have completed their lives under the initial PAYG retirement system and their lifetime utility is taken as basis for the welfare calculations. The compensating variations are defined in the following way. Consider the life-cycle profiles for consumption and labor market hours for the generation born at the time period t, of age j and under a retirement system z, $\left(c_{t+j}^{t,z}, l_{m,t+j}^{t,z}, l_{h,t+j}^{t,z}\right)$. The compensating variation, $CV^{t,z}$, for cohort t under the system z is such that the lifetime utility under system z is equal to the one under PAYG system:

$$\sum_{j=0}^{J-1}\beta^j u(c_{t+j}^{t,PAYG}\frac{\left(100+CV^{t,z}\right)}{100}, 1-l_{m,t+j}^{t,PAYG}-l_{h,t+j}^{t,PAYG}) = \sum_{j=0}^{J-1}\beta^j u(c_{t+j}^{t,z}, 1-l_{m,t+j}^{t,z}-l_{h,t+j}^{t,z}).$$

The welfare losses for transition generations are as high as 4% of lifetime consumption in both economic environments. The number of generations with welfare losses is higher in the environment with endogenous human capital. The welfare gains at the new stationary equilibrium are smaller when the life-cycle labor productivity is endogenous, 0.21% of lifetime consumption versus of 0.92% in the environment with exogenous human capital.

3.5 Conclusion

This chapter studies the experience of transition generations during introduction of retirement system with voluntary retirement savings accounts. New system leads to higher savings and capital stock which is achieved through higher labor supply of transition generations.

Because the generations that are alive and have already entered the labor market at the time of the reform implementation experience welfare losses, the government needs to design a policy to compensate these generations and to gain the political support for the reform.

Since the level of welfare gains is smaller in the environment with endogenous human capital, the flexibility for compensating transition generations is limited in the environment with endogenous human capital. The next step is to determine the transfers to transition generations and consider different methods for financing these transfers.

Table 3.1: Population Structure during Transition.

GENERATIONS	TIME PERIODS													
period born	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$t=-11$	$\mathbf{c_0^{-11}}$													
$t=-10$	$\mathbf{c_0^{-10}}$	$\mathbf{c_1^{-10}}$												
$t=-9$	$\mathbf{c_0^{-9}}$	$\mathbf{c_1^{-9}}$	$\mathbf{c_2^{-9}}$											
$t=-8$	c_0^{-8}	$\mathbf{c_1^{-8}}$	$\mathbf{c_2^{-8}}$	$\mathbf{c_3^{-8}}$										
$t=-7$	c_0^{-7}	c_1^{-7}	$\mathbf{c_2^{-7}}$	$\mathbf{c_3^{-7}}$	$\mathbf{c_4^{-7}}$									
$t=-6$	c_0^{-6}	c_1^{-6}	c_2^{-6}	$\mathbf{c_3^{-6}}$	$\mathbf{c_4^{-6}}$	$\mathbf{c_5^{-6}}$								
$t=-5$	c_0^{-5}	c_1^{-5}	c_2^{-5}	c_3^{-5}	$\mathbf{c_4^{-5}}$	$\mathbf{c_5^{-5}}$	$\mathbf{c_6^{-5}}$							
$t=-4$	c_0^{-4}	c_1^{-4}	c_2^{-4}	c_3^{-4}	c_4^{-4}	$\mathbf{c_5^{-4}}$	$\mathbf{c_6^{-4}}$	$\mathbf{c_7^{-4}}$						
$t=-3$	c_0^{-5}	c_1^{-3}	c_2^{-5}	c_3^{-3}	c_4^{-3}	c_5^{-3}	$\mathbf{c_6^{-3}}$	$\mathbf{c_7^{-3}}$	$\mathbf{c_8^{-3}}$					
$t=-2$	c_0^{-2}	c_1^{-2}	c_2^{-2}	c_3^{-2}	c_4^{-2}	c_5^{-2}	c_6^{-2}	$\mathbf{c_7^{-2}}$	$\mathbf{c_8^{-2}}$	$\mathbf{c_9^{-2}}$				
$t=-1$	c_0^{-1}	c_1^{-1}	c_2^{-1}	c_3^{-1}	c_4^{-1}	c_5^{-1}	c_6^{-1}	c_7^{-1}	$\mathbf{c_8^{-1}}$	$\mathbf{c_9^{-1}}$	$\mathbf{c_{10}^{-1}}$			
$t=0$	c_0^{0}	c_1^{0}	c_2^{0}	c_3^{0}	c_4^{0}	c_5^{0}	c_6^{0}	c_7^{0}	c_8^{0}	$\mathbf{c_9^{0}}$	$\mathbf{c_{10}^{0}}$	$\mathbf{c_{11}^{0}}$		
$t=1$		c_1^{1}	c_2^{1}	c_3^{1}	c_4^{1}	c_5^{1}	c_6^{1}	c_7^{1}	c_8^{1}	c_9^{1}	$\mathbf{c_{10}^{1}}$	$\mathbf{c_{11}^{1}}$	$\mathbf{c_{12}^{1}}$	
$t=2$			c_2^{2}	c_3^{2}	c_4^{2}	c_5^{2}	c_6^{2}	c_7^{2}	c_8^{2}	c_9^{2}	c_{10}^{2}	$\mathbf{c_{11}^{2}}$	$\mathbf{c_{12}^{2}}$	$\mathbf{c_{13}^{2}}$

Table 3.2: Model Parameters.

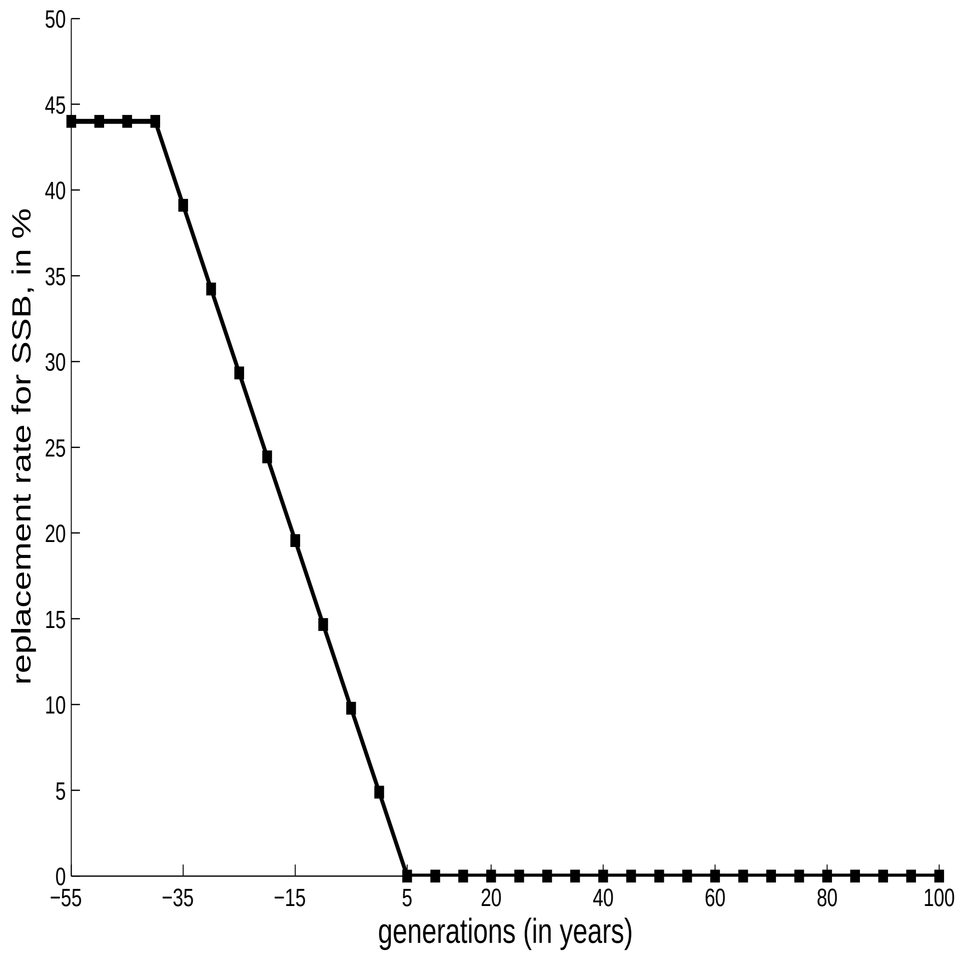
Parameter	Expression	Value
Preferences and Technology		
Discount factor	β	1.001
Leisure preference parameter	α	1.88
Capital share	θ	0.333
Depreciation rate of physical capital	δ_k	0.054
Rate of technological progress	γ	0.02
Rate of population growth	η	0.012
Government Sector		
Tax rate on consumption	τ_c	0.05
Tax rate on labor income	τ_l	0.27
Tax rate on capital income	τ_k	0.40
Share of government expenditures	g	0.1438
Replacement rate for Social Security benefits	ϕ	0.44
Human Capital Technology		
Initial stock of human capital	h_0	0.33
Depreciation rate of human capital	δ_h	0.057
Productivity of HC accumulation	B	0.56
Weight of HC stock	ψ_1	0.50
Weight of time investment	ψ_2	0.40

All parameter values are given in annual terms. Since one model period corresponds to five years, the parameters are adjusted in computations accordingly.
All parameters are given for initial stationary equilibrium.

Table 3.3: Calibration of initial stationary equilibrium with PAYG system.

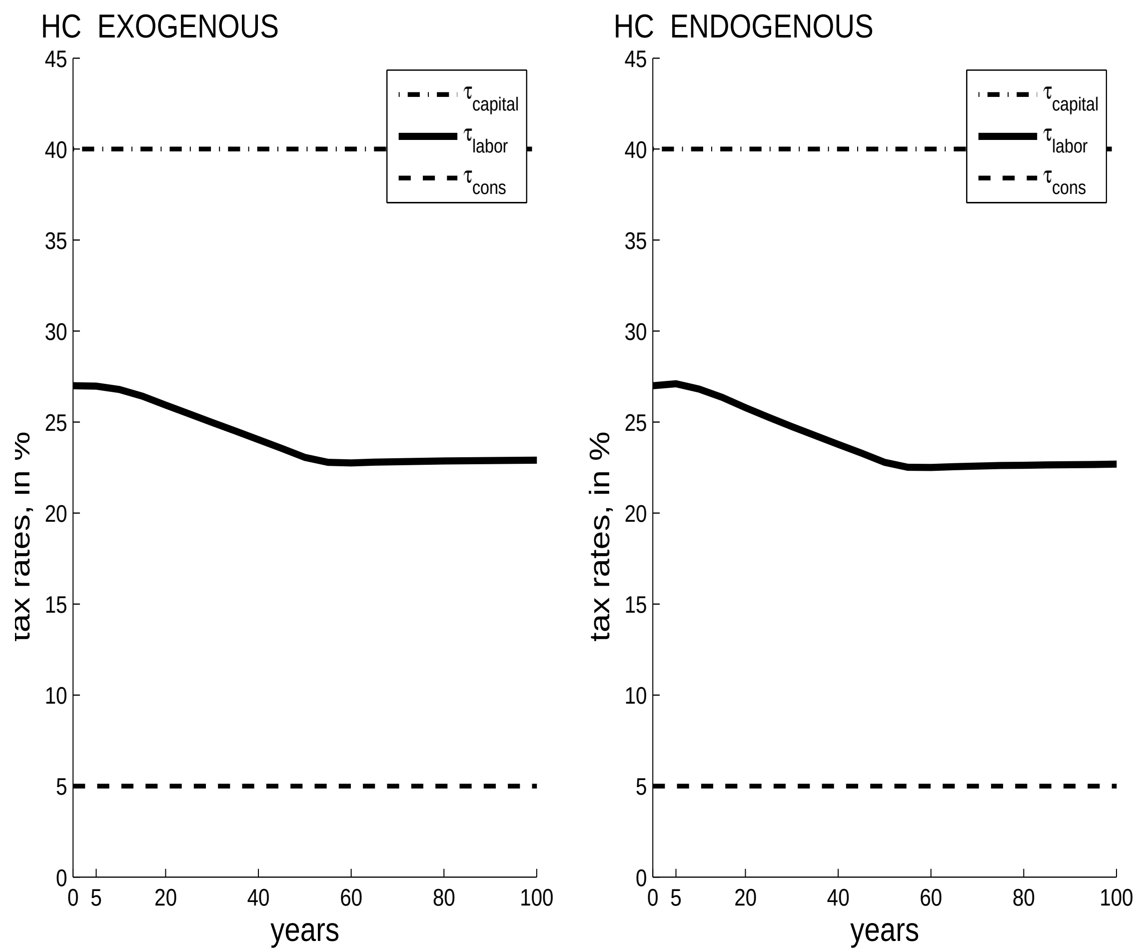
		Model with	
	Data	HC Exogenous	HC Endogenous
After-tax interest rate, %	4.00	4.00	4.00
Average weekly market hours	29.00	28.53	28.53
Capital-to-output ratio	2-3	2.76	2.76
Consumption-to-output, %	57.56	59.50	59.50
Investment-to-output, %	28.06	26.12	26.12
Government expenditures-to-output, %	14.38	14.38	14.38
Pension expenditures-to-output, %	4.25	3.32	3.32

Figure 3.1: Replacement rate for Social Security benefits.



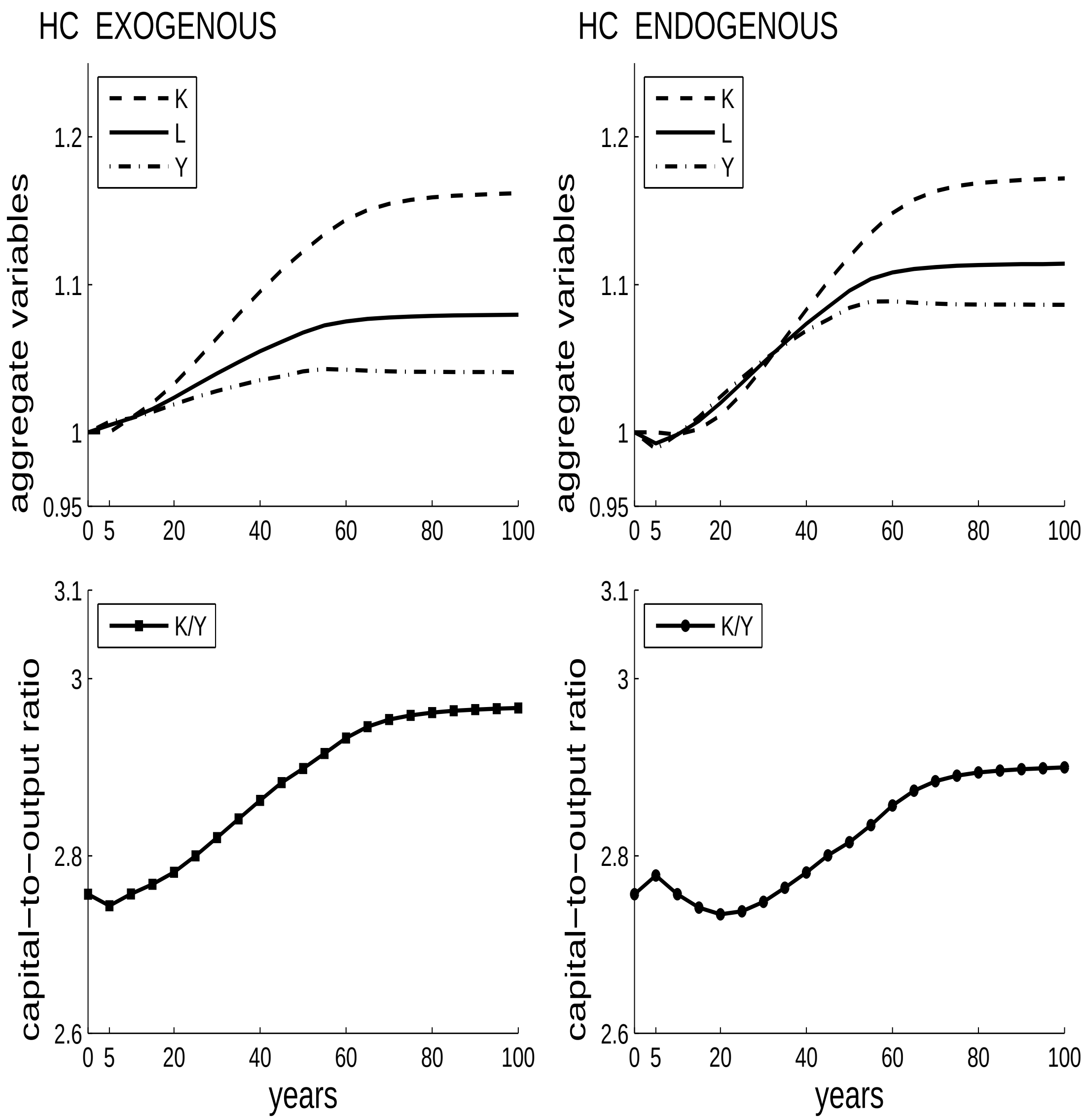
Replacement rate for Social Security benefits for retired generations at time of reform announcement is kept fixed. For working generations at time period 0, the replacement rate is reduced linearly. The generations who have entered the labor market at the period 0 and later have to save for their retirement on their own.

Figure 3.2: Tax rates during transition.



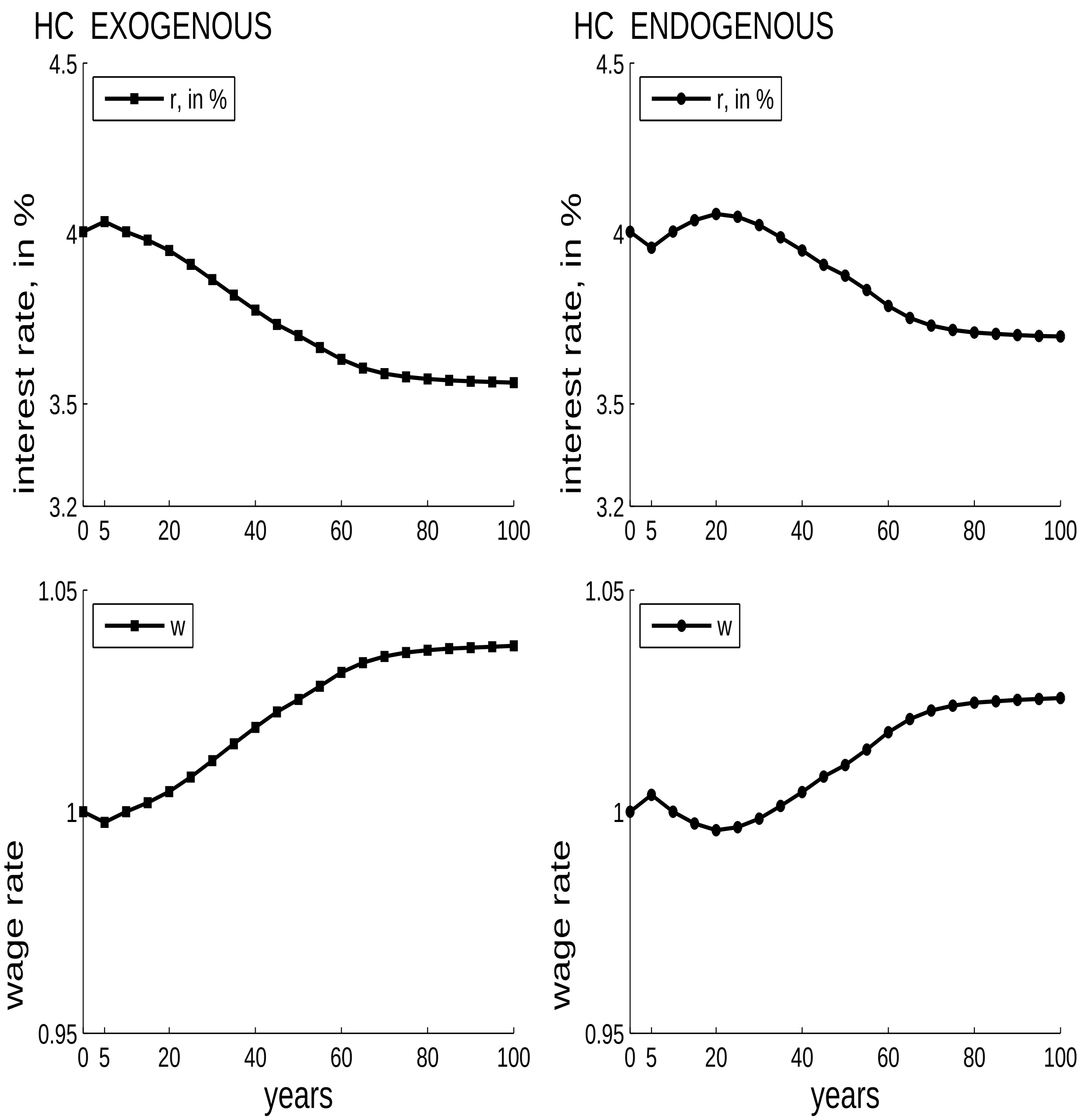
Taxes on consumption and capital income are fixed during the transition towards, the labor income tax is adjusted to balance the government's budget.

Figure 3.3: Aggregate variables.



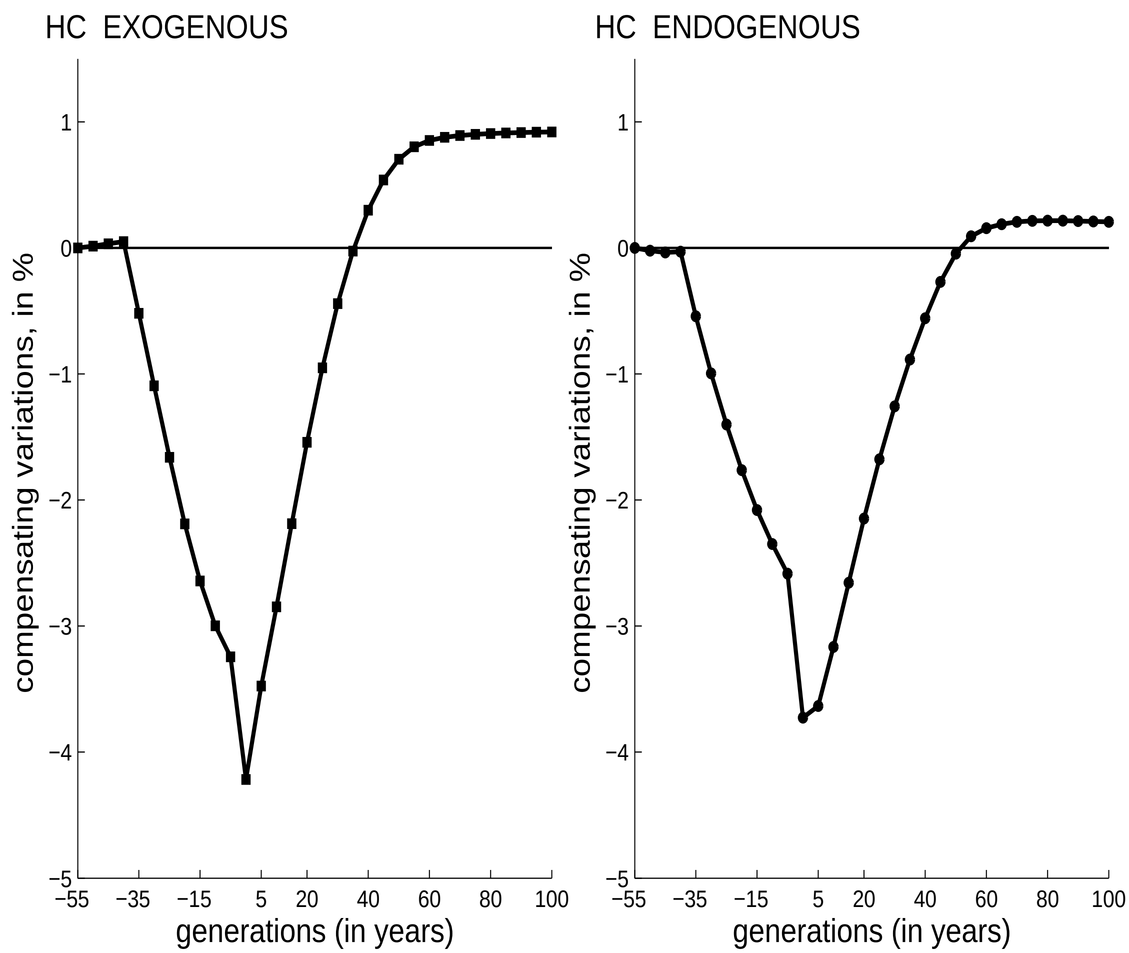
Top panel: aggregate variables - capital stock, labor supply, and output - are expressed as ratios towards the values at the initial stationary equilibrium with PAYG Social Security system.
Bottom panel: capital-to-output ratio.

Figure 3.4: Factor prices.



Top panel: after-tax interest rate was calibrated to 4% at the initial stationary equilibrium with PAYG Social Security system. The interest rate is given in annual terms.
Bottom panel: wage rate is expressed as the ratio of the one at the initial stationary equilibrium.

Figure 3.5: Household welfare.



Household welfare is given in terms of compensating variations which measures the amount of life-time consumption that an agent has to be given to be indifferent between a reform and the initial stationary equilibrium. The comparison is made with the welfare of the generation −60 which completed their lives in the initial stationary equilibrium with PAYG Social Security system.

Chapter 4

Macroeconomic and Welfare Implications of Italian Pension Reforms of 1990s

The Italian pension system have underwent a number of reforms in 1990s. The system still maintains pay-as-you-go nature and is moving from a defined benefit (DB) to a defined contribution (DC) system. The reforms envision a very long transition and the new system will be fully phased in after 2030. The reforms also provide for a different treatment of generations based on the labor market seniority at the end of 1995. Many scholars argue that the reforms violate an intergenerational equity by placing most of the burden of transition on younger generations, see for example Franco (2002). This chapter quantitatively evaluates the effect of the reforms on transition generations.

Following the quantitative tradition of evaluating tax and social security policies, see for example Auerbach and Kotlikoff (1987), DeNardi, Imrohoroglu and Sargent (1999), and Kotlikoff, Smetters and Walliser (1999), this chapter develops a macroeconomic model with overlapping generations (OG). The model is calibrated to match the main macroeconomic variables of the Italian economy before the introduction of the reforms. I model the institutional features of the Italian pension system before and after the reform. The pension reform introduced in 1992 changed the indexation of pensions to price inflation from wage growth, while the reform passed in 1995 introduced a DC system and provided rules for calculating the retirement benefits for transition generations. The workers who have entered the labor market before 1995 will have their benefits calculated based on two formulas: (1) *pro-rata* system for young workers with less than 18 years of seniority in 1995, and (2) *modified defined benefit* system for workers with more than 18 years of seniority (the details for these formulas are given in Sections 4.1 and 4.3.1).

Many economists have studied the impact of these reforms on individual decisions. Borella, Belloni and Fornero (2005) evaluate the changes in labor supply induced by the reforms, wealth accumulation and saving behavior are investigated by Attanasio and Brugiavini (2003) and Jappelli, Padula and Bottazzi (2003). To evaluate the effects of the Italian reforms on individual as well as aggregate macroeconomic variables, the agents in the model economy make consumption versus saving and leisure versus production activities decisions within the general equilibrium set-up. The time endowment is allocated between leisure, market production and human capital investment activities. The human capital investment technology is the one introduced by Ben-Porath (1967).

The results of this chapter show that the Italian reforms will reduce the pension expenditures and restore the financial sustainability of the system in the long-run. The change in the indexation rule alone reduces the pension expenditures only during the initial fifteen years after the introduction of the reforms, while the long-term reduction is achieved through transition to a DC retirement system. During the transition, the older workers with more than 18 years of seniority in 1995 are protected by the reforms, while young and future generations bear the cost of transition to a new system. The reforms induce the agents to increase investment into physical and human capital. In the new stationary equilibrium with a DC retirement system, the aggregate capital stock and labor supply are by 13% and 6.3% higher, respectively, as compared to pre-1992 levels. These increase in the factors of production results in the higher aggregate output and consumption. The future generations who will be born after year 2050 are only slightly worse off as compared to pre-1992 level, the welfare of these generations is only lower by 0.16% in terms of lifetime consumption.

The closest papers to my analysis are Borella and Coda-Moscarola (2006) and D'Amato and Galasso (2002). The distributive properties of the Italian reforms are studied in Borella and Coda-Moscarola (2006). This paper uses microsimulation approach and incorporates rich heterogeneity within cohorts to study the redistribution and fairness properties of the retirement system within and across generations. Since Borella and Coda-Moscarola (2006) model the change in retirement rules for different cohorts, they can analyze the change in the redistribution of the retirement system during the transition to a defined contribution system. At the same time, their paper abstracts from consumption, savings and labor supply decisons. Hence, Borella and Coda-Moscarola (2006) can not make predictions regarding macroeconomic characteristics of the Italian economy and pension system during the transition. D'Amato and Galasso (2002) use overlapping generations political economy model to study political sustainability of the new retirement system. At the same time, they limit the scope of their analysis to a steady state and consider the features of the DC retirement system that will be fully operational in Italy in 2050. In contrast to D'Amato and Galasso (2002), I model the transition process as well as convergence to a new stationary equilibrium. Therefore, this chapter can analyze welfare of all generations affected by the reforms.

The chapter is organized as follows. In section 4.1 , I describe the Italian pension system and the legislative basis for the reforms that took place in 1990s. Section 4.2 provides details of the economic environment to model the transition to a new retirement system. The calibration of the model is documented in section 4.3 . The results for simulating the transition to a new system are discussed in section 4.4 . Section 4.5 concludes the chapter.

4.1 Italian Pension Reforms of 1990s

The Italian pension system in characterized by pay-as-you-go nature. The system is highly fragmented and consists of over fifty different schemes. More than two thirds of public pension system is administered by *Istituto Nazionale Previdenza Sociale* (INPS) which covers the major part of private sector employees and self-employed workers. As for decomposition of pension expenditures by sectors in 2000, 62 % of expenditures were directed at the private sector employees, 24 % at public sector workers and 14 % at self-employed workers. Before 1992, the pension system was very generous and on average pension benefits constituted 80 % of wage income, according to OECD (2005). Prior to the reforms, retirement benefits were computed using a defined benefits (DB) formula. The DB formula depended on pensionable earnings computed over the last years' earnings, the number of working years and the annual accrual rate. The number of years over which the pensionable earnings have been averaged differed among the sectors: it was the last five years for private sector employees, the last one month for public sector workers, and the last ten years of taxable income for self-employed. The fragmentation of the system led to the excessive generosity of benefits. At the beginning of 1990s, Italy had one of the highest pension expenditures levels measured as fraction of GDP among OECD countries. There was a further concern to financial sustainability of the system due to aging population.

The Italian pension system has been heavily reformed through a number of legislative acts in 1990s. In particular, I consider three main reforms: Legislative Decree 503 of 1992, so called Amato reform; Law 335 of 1995, Dini reform; and Law 449 of 1997, Prodi reform.

Here, I discuss the elements of the reforms that are most relevant to my analysis. Amato reform has introduced indexation of benefits to inflation as compared to wage growth before. Secondly, Amato reform has harmonized the rules for computing pension benefits among the workers of different sectors.

The Dini reform has introduced the retirement system based on defined contributions (DC) principle while still preserving the pay-as-you-go nature of the funding. All people who have entered the labor market after December 31, 1995 will have their retirement benefits computed based on the amount of payroll taxes contributed to the system. The contributions to the system will be accrued at the rate of GDP growth and converted into an annuity at the time of retirement. This conversion coefficient is a function of retirement age and life expectancy. The Dini reform has also established the rules for transition generations. The people who have entered the labor market before December 31, 1995 are divided into two groups. The first group consists of people who had less than 18 years of experience at the end of 1995. The pension benefits of these workers is calculated on *pro-rata* basis and is composed of two parts. Labor income earned prior to 1995 constitutes the basis for the first part of benefits computed based on defined benefits formula. The contributions made to the retirement system after 1995 will be accounted for using Defined Contributions formula.

People with more than 18 years of experience in 1995 constitute the second group. Their retirement benefits are completely based on defined benefit formula but now the labor income is averaged over the longer period of time. These regime is called modified defined benefits (MDB).

The Prodi reform of 1997 have sped up the transition to a new system. In this chapter, I explicitly model a transition from a system with defined benefits to the one with defined contributions. The details for different generations and the benefit formulas are given in Section 4.3.1 The rules for computing the retirement benefits are specified on the webcite of INPS.

4.2 Economic Environment

Households. The economy is populated by overlapping generations. Time is discrete, indexed by $t = 0, 1, ...,$ and continues forever. The economy is populated by a continuum of individuals. At each date t, a new cohort is born that is η percent larger than the previous cohort. The agents live for J adult periods, with ages denoted by $j \in \Im \equiv \{0, ..., J-1\}$. The agents' life-spans are certain. The size of a generation born at a time period t and of age j is denoted by n^t_{t+j}. An individual's consumption allocation, c^t_{t+j}, is indexed by a superscript which denotes the date of birth t and by one subscript $t+j$ which denotes the time period at which allocation takes place.

Preferences of an agent born at period t are ordered by

$$\sum_{j=0}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}), \tag{4.1}$$

where β is a time preference parameter. Each agent chooses sequences of consumption, market hours, and investment hours, $\left\{c^t_{t+j}, l^t_{m,t+j}, l^t_{h,t+j}\right\}$, to maximize the discounted value of life-time utility subject to its budget constraint,

$$\begin{aligned} (1+\tau_{c,t})\, c^t_{t+j} + s^t_{t+1+j} \leq (1-\tau_{l,t})\,(1-\tau_{p,t})\, w_{t+j} h^t_{t+j} l^t_{m,t+j} \\ + (1 + (1-\tau_{k,t})\, r_{t+j}) s^t_{t+j} + d^t_{t+j}. \end{aligned} \tag{4.2}$$

This constraint must be balanced at each age of the agent's life, i.e., for any $j \in \Im$.

The market-clearing wage and the rate of return on physical capital at date $t+j$ are given by r_{t+j} and w_{t+j}, respectively. An agent's labor income depends on efficiency units of labor, $h^t_{t+j} l^t_{m,t+j}$. The agent's stock of human capital h^t_{t+j} is determined by the undepreciated human capital from the last period and the new human capital accumulation during the last period:

$$h^t_{t+j} = (1-\delta_h) h^t_{t+j-1} + Q(h^t_{t+j-1}, l^t_{h,t+j-1}). \tag{4.3}$$

The creation of new human capital depends on its existing level and investment hours and is determined by the function $Q(h, l_h)$. The Q function is increasing in both arguments and has decreasing returns to scale. The agent's savings earn capital income at the real rate of return r_{t+j}. Agents are restricted to have strictly positive amount of savings at all ages

$$s^t_{t+j} \geq 0. \tag{4.4}$$

Agents pay taxes on consumption at rate $\tau_{c,t}$ and capital income net of depreciation at rate $\tau_{k,t}$. Labor income is subject to ordinary labor income tax, $\tau_{l,t}$, and to payroll tax, $\tau_{p,t}$.

The government transfers to the agent born at t and of age j are denoted by d^t_{t+j}. These transfers consist of two components: a lump-sum transfer for agents of all ages, f^t_{t+j}, and social security benefits to retirees, b^t_{t+j},

$$d^t_{t+j} = \begin{cases} f^t_{t+j}, & j = 0, \overline{J}-1, \\ f^t_{t+j} + b^t_{t+j}, & j = \overline{J}, J-1. \end{cases}$$

The agents of the same cohort receive equal lump-sum transfers, f^t_{t+j}, independent of earnings' group. Agents are entitled to retirement benefits starting with age $\overline{J}$. The formula for calculation of Social Security benefits depends on the date when an agent entered the labor market. The detailed description of benefit formula is given in Section 4.3.1

Production technology. At period t, firms hire capital, K_t, and labor, L_t, to produce output with a constant returns-to-scale production technology,

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta},$$

where A_t is total factor productivity which is assumed to grow at a constant exogenously given rate γ. Factor markets are assumed to be competitive and factor inputs are paid the marginal products:

$$\begin{aligned} w_t &= (1-\theta) A_t K_t^{\theta} L_t^{-\theta}, \\ r_t &= \theta A_t K_t^{\theta-1} L_t^{1-\theta} - \delta_k, \end{aligned}$$

where δ_k is the depreciation rate of capital.

Government. The government operates through two different budgets, the social security and general budgets. Both budgets are balanced every period t. The retirement benefits, $b^{t,i}_{t+j}$, are financed through payroll taxes, $\tau_{p,t}$. The exogenously given stream of government consumption, G_t, and lump-sum transfers to households, f^t_{t+j}, are financed through consumption taxes, $\tau_{c,t}$, capital income taxes $\tau_{k,t}$, and labor income taxes, $\tau_{l,t}$.

$$\begin{aligned} G_t + \sum_{j=0}^{J-1} n_t^{t-j} f_t^{t-j} &= \sum_{j=0}^{J-1} n_t^{t-j} \left(\tau_{c,t} c_t^{t-j} + \tau_{k,t} r_t s_t^{t-j}\right) \\ &+ \sum_{j=0}^{\overline{J}-1} n_t^{t-j} \left(\tau_{l,t}(1-\tau_{p,t}) w_t h_t^{t-j} l_{m,t}^{t-j}\right). \end{aligned} \tag{4.5}$$

$$\sum_{j=\overline{J}}^{J-1} n_t^{t-j} b_t^{t-j} = \sum_{j=0}^{J-1} n_t^{t-j} \tau_{p,t} w_t h_t^{t-j} l_{m,t}^{t-j}, \tag{4.6}$$

Market arrangements. All markets are competitive. The aggregate inputs are determined as

$$K_t = \sum_{j=0}^{J-1} n_t^{t-j} s_t^{t-j},$$
$$L_t = \sum_{j=0}^{\overline{J}-1} n_t^{t-j} h_t^{t-j} l_{m,t}^{t-j}.$$

The aggregate feasibility constraint at period t is

$$\sum_{j=0}^{J-1} n_t^{t-j} c_t^{t-j,i} + K_{t+1} + G_t = A_t K_t^{\theta} L_t^{1-\theta} + (1-\delta_k) K_t. \tag{4.7}$$

Definition 7. *A competitive equilibrium is a sequence of prices and allocations such that: (i) given equilibrium prices and government policies, consumers maximize a discounted stream of utilities (4.1) subject to their constraints (4.2)-(4.4); (ii) firms maximize profits given prices; (iii) the social security and the general government budgets are balanced; (iv) the market clearing and feasibility conditions hold.*

4.3 Parameterization of the model

Since I model the transition from Defined Benefits to Defined Contributions retirement system in Italy, the calibration and computation strategies are as follows. The initial stationary equilibrium is calibrated to macroeconomic characteristics, tax and retirement system in Italy in 1992. Then I model the change in the rules for retirement benefits as prescribed by Italian laws. The parameter values are summarized in Table 4.4.

4.3.1 Demographics and timing of the reforms.

Agents enter the economy at age 20, retire at age 65, and die at age 80. Each model period corresponds to 5 years. Hence, the agents are working during the first nine model periods, $\overline{J} = 9$, are retired during the last three model periods and the life length is $J = 12$. The population grows at a constant rate which is set to 0.7 % annually. Population structure during transition is summarized in Table 4.1. The period from 1990 to 1994 is the initial stationary equilibrium with defined benefits (DB) retirement system, this period is labeled by 1992 in Table 4.1 and in all other results presented in tables and graphs. The first period in which reforms are being implemented is the one from 1995 to 1999, this period is labeled as 1997. The generation born in 1915 completes their lives under the initial stationary equilibrium with DB retirement system. The generation of 1915 is representative of the generations born from 1913 to 1917. To simplify notation, I relabel the time periods and generations as specified in Table 4.2. Hence, the flow of generations during transition using the model notation is described in Table 4.3.

The generations born from 1920 to 1930 have completed their working lives under the DB system. The retirement benefits of these generations are computed using DB formula but their benefits are indexed to prices as prescribed by Amato reform of 1992. Using the model notation, the retirement benefits for generations $t = -11, ..., -8$ are computed based on the average wage income during the last N_b periods of working life:

$$b^t = \frac{\overline{J}\phi^{1t}}{N_b} \sum_{j=\overline{J}-N_b}^{\overline{J}-1} w_{t+j} h_{t+j}^t l_{mt+j}^t, \tag{4.8}$$

where $\overline{J}$ accounts for number of periods an agent have been working,[1] ϕ^{1t} is the annual accrual rate.[2] To match the replacement rate for retirement benefits, I set ϕ^{1t} equal to 0.015. At the initial system, the benefits are calculated based on 5 calendar years, hence, N_b is set to one model period. The Social Security benefits are indexed during the retirement. At the pre-1992 system, the indexation is based on the wage growth rate, $\xi_t = \lambda_{wt}$. Starting with model period 1, the benefits are indexed to inflation, hence, $\xi_t = 1$ for $t = 1, 2, ...$

The retirement benefits for generations from 1935 to 1955 are computed using the Modified Defined Benefits (MDB) system. The benefits are based on the wage income received from 1997 up to retirement and are also indexed to inflation. The generations in this group had at least 20 years of seniority under the old system.[3,4] This formula is

[1] Since there is no labor market exit/entrance in my model, the number of years for Social Security contributions is equal to the length of working life.

[2] Annual accrual rate was equal to 2 percent for earnings up to a given threshold. For the earnings above the threshold, the annual accrual rate was gradually decreased. The non-linearity of the annual accrual rate represented the distributive feather of the system.

[3] The seniority is determined based on the number of years worked. Since the agents don't make decision of labor market entrance/exit, the seniority is determined by the number of working years.

[4] The provisions of Italian reform put the threshold at 18 years of seniority at the end of 1995. Since my model period corresponds to five calendar years, my threshold is at least 20 calendar years or 4 model periods of working prior to the time period $t = 1$.

the same as the one for DB system but the pensionable earnings are computed based on the wage income from the period $t = 1$ onwards. These are generations that were born at $t = (i - J), i = 5, ..., 9$. From a time period of reform announcement, a generation t will work for the number of periods $\tilde{J} = \overline{J} - (1 - t)$ and live for the number of periods $\hat{J} = J - (1 - t) = i - 1$. The Social Security benefits are calculated according to the formula

$$b^{1,(i-J)} = \frac{\overline{J}\phi^{1,(i-J)}}{\tilde{J}} \sum_{t=1}^{\tilde{J}} w_t h_t^{(i-J)} l_{mt}^{(i-J)}.$$

The generations born from 1960 to 1970 receive retirement benefits on pro-rata basis. These generations had less than 20 years of seniority at the time $t = 1$. The first part of benefits is based on wage income prior to the period $t = 1$ and is computed based on the MDB formula. The second part is computed based on contributions from $t = 1$ and later using the Defined Contributions formula. Using model notation, these are generations that were born at $t = (i - J), i = 10, 11, 12$. From a time period of reform announcement, a generation t will work for the number of periods $\tilde{J} = \overline{J} - (1 - t)$ and live for the number of periods $\hat{J} = J - (1 - t) = i - 1$. The first part of retirement benefits is calculated according to the MDB formula

$$b^{1,(i-J)} = \frac{\overline{J}\phi^{1,(i-J)}}{\overline{J} - \tilde{J}} \sum_{t=(i-J)}^{0} w_t h_t^{(i-J)} l_{mt}^{(i-J)}$$

and the second part is given by

$$b^{2,(i-J)} = \phi^{2,(i-J)} \sum_{t=1}^{\tilde{J}} \tau_{p,t} w_t h_t^{(i-J)} l_{mt}^{(i-J)} \prod_{z=t+1}^{\tilde{J}} (1 + \lambda_{Y,z}),$$

where λ_Y is the growth rate of output, ϕ^{2t} is the annuity rate. The annuity rate depends on the age of retirement, for the ages between 57 and 65. The annuity rates are published by *Istituto Nazionale Previdenza Sociale* (INPS) and is available at www.inps.it. In the calibration, I set $\phi^{2t} = 6.1\%$ as for 65 year-olds.

For all generations who enter the labor market at the time of reform announcement and later, the retirement benefits are calculated based on the Defined Contributions (DC) system. These are the generations that were born in 1975 and later. In the model, these generations are labeled by $t = 1, 2, ...$ Under the new system, the retirement age is equal to

65 calendar years or 9 period years. The Social Security benefits, $b^{2,t}$, are calculated based on Social Security contributions during the working life:

$$b^{2,t} = \left[\sum_{j=0}^{\overline{J}-1} \tau_{p,t+j} w_{t+j} h_{t+j}^{t} l_{mt+j}^{t} \prod_{z=j+1}^{\overline{J}-1} (1 + \lambda_{Y,t+z}) \right] \phi^{2t}, \tag{4.9}$$

where λ_Y is the growth rate of output, ϕ^{2t} is the annuity rate. The Social Security benefits are indexed to inflation at the new system. The Social Security benefits are calculated based on wage income during the whole working period, $N_b = \overline{J}$. The contributions to Social Security system is determined by payroll taxes, τ_p, and are compounded by the growth rate of output.

4.3.2 Parameters of utility and production functions

The time preference parameter β is calibrated to match the capital-to-output ratio at the level of 3. I assume that the agents' flow utility functions are

$$u(c, 1 - l_m - l_h) = \log c + \alpha \log(1 - l_m - l_h),$$

where α is chosen to match average weekly hours of the population of ages between 20 and 64.

The calibration of production technology is standard. Capital income share, θ, is set to 0.333. Depreciation of physical capital, δ_k, is calibrated to match the investment share in GDP. The resulting depreciation rate is 5.4%. This estimate of the depreciation rate is in line with the one commonly used in the literature. Stokey and Rebelo (1995) estimate the depreciation rate to be 6%. Rios-Rull (1996) calibration results in the rate of 5.4%.

Average effective tax rates are calibrated using the methodology of Mendoza, Razin and Tesar (1994) and are reported in Table 4.4. The share of government expenditures in output, g, is set to match the corresponding value in NIPA. The replacement rate for pension benefits, ϕ^1, is calibrated to match the replacement rate for pensions.

4.3.3 Parameters of human capital production technology

I assume the following law of motion for human capital:

$$h_{j+1} = (1 - \delta_h) h_j + B h_j^{\psi_1} l_{h,j}^{\psi_2},$$

where the conditions $B, \psi_1, \psi_2 \geq 0$ and $\psi_1+\psi_2 \leq 1$ guarantee the decreasing returns to scale. Hence, the life-cycle profile of time investment into human capital is time-independent.

I have to choose five parameters for the human capital production technology: initial stock of human capital, h_0; the depreciation rate of human capital, δ_h; productivity of human capital accumulation, B; weight of human capital stock in new accumulation, ψ_1; and weight of time investment, ψ_2. I calibrate these parameters to match the life-cycle earnings profile, which is constructed using data from Survey of Household Income and Wealth (SHIW) collected by the Bank of Italy. I divide the population of ages between 20 and 64 into nine age groups, $j \in \{0, ..., 8\}$. The size of the working age population is denoted by N_t. The measure of earnings is the hourly wage rate, denoted by e_j, $j \in \{0, ..., 8\}$. The average wage rate for the working population is denoted by $\overline{e}$. This average rate for the working population is calculated using the size of each age group, $n_j(t)$:

$$\overline{e} = \frac{\sum_{j=0}^{8} n_j(t) e_j}{N_t}.$$

To express the wage earnings profile in units comparable to the model, I report the average hourly wage for an age group j as the ratio to the average hourly wage of the working population: $\varepsilon_j = e_j/\overline{e}$, $j = 0, ..., 8$.

Equivalently, the wage rate in the model is $w_t h_j$ and the average wage for the working population is

$$\overline{wh} = \frac{\sum_{j=0}^{8} w_t h_j}{\overline{J}}.$$

I choose parameters of the human capital production function to minimize the distance between the model and data wage hour profiles:

$$\min_{(h_0, \delta_h, B, \psi_1, \psi_2)} \sum_{j=0}^{8} \left(\frac{wh_j}{\overline{wh}} - \frac{e_j}{\overline{e}} \right)^2.$$

The chosen parameters are reported in Table 4.4. The resulting life-cycle efficiency profile at the initial stationary equilibrium with the defined benefits system is plotted in Figure 4.1.

4.4 Results

People who entered the labor market after 1995 will have their retirement benefits computed using the DC formula. By 2040, the stock of all retirees will receive their benefits under the new system. This chapter shows that the Italian economy will reach a new stationary equilibrium by 2100, see evolution of wage and interest rates reported in Figure 4.4.

The simulations consider two reforms that took place during the 1990s. In the first reform, the indexation of retirement benefits is switched from wage growth to inflation. This reform affects all generations alive in 1995 including people who have already retired. The second reform is cumulative to the first one and considers the transition to a system with a defined contributions benefits. In this simulation, I model the change in the benefit formula depending on the labor market seniority in 1995 as prescribed by legislative acts of Dini reform. The details on the method for calculating the pension benefits for transition generations are given in section 4.3.1 In all graphs, these two reforms are labeled as 'Reform 1' and 'Reform 2', respectively.

Figure 4.2 shows the evolution of the government budget during the transition. The payroll tax used to finance the retirement benefits is fixed during the transition. The general government budget is balanced through adjustments in the labor income tax. Although the government maintains two budgets, one for the retirement system and one for the general government expenditures, the pension system can be subsidized from the general revenue. The switch to the indexation of the retirement benefits to inflation reduces the government expenditures by 10% in the first 10 years after the introduction of the reforms. Figure 4.2 confirms that the change in the indexation method is only effective in the short run. Since the Dini reform preserves the pension rights of the generations with more than 18 years of seniority at 1995, the government pension expenditures exceed the pre-reform level for the period from 2010 to 2024 when these generations enter the retirement. From year 2025, the pension expenditures are bellow the pre-reform level and eventually decrease by 24% in the new stationary equilibrium. Due to the labor supply decision responses, the revenues to the pension fund fluctuate around the pre-reform level until year 2024 and increase by 8.5% by year 2070. Therefore, the reforms are effective in restoring the financial balance of the pension system in the long-run.

Reduction in the pension benefits induces the agents to increase their savings. Hence, the aggregate capital stock will increase by 13% by year 2070 as shown in Figure 4.3. Before year 2025, the capital stock increases only by 2% which is due to the behavior of the cohorts that have already participated in the labor market at the time of the reforms implementation. Within the DC retirement system, the aggregate labor supply will be 6.3% higher than the one before 1992. These changes in the factors of production will lead to adjustments in the wage and interest rates. As reported in Figure 4.4, the wage rate will increase at the new stationary equilibrium, while the interest rate will decline by 28 percentage points.

Figure 4.5 reports household welfare as measured by compensating variations. The cohorts are shown by their year of birth. The cohort born in 1915 have completed their lives under the retirement system with the defined benefits. The welfare of all generations is compared to the life-time utility of the cohort born in 1915. The compensating variations measures in percentages by how much in terms of life-time consumption an agent has to be compensated in order to make him indifferent between the reforms and the initial stationary equilibrium with the DB system. Formally, let $c_{t+j}^{t,z}$, $l_{m,t+j}^{t,z}$, and $l_{h,t+j}^{t,z}$ be the consumption and labor market choices of cohort born at the time period t, of age j and under a retirement system z. The retirement systems considered are the pre-1992 system with defined benefits (DB), the Amato reform with the change in the indexation of benefits (Reform 1), and the Dini reform that have introduced the defined contributions system (Reform 2). The compensating variation associated with a switch from DB to system z, for cohort t, is calculated as the $CV^{t,z}$ such that the lifetime utility under system z and system DB are equal:

$$\sum_{j=0}^{J-1} \beta^j u(c_{t+j}^{t,DB} \frac{(100 + CV^{t,z})}{100}, 1 - l_{m,t+j}^{t,DB} - l_{h,t+j}^{t,DB}) = \sum_{j=0}^{J-1} \beta^j u(c_{t+j}^{t,z}, 1 - l_{m,t+j}^{t,z} - l_{h,t+j}^{t,z}).$$

In terms of welfare consequences, the cohorts can be divided into five groups. Firstly, the cohorts born from 1920 to 1935 have completed their working lives under the old system and are affected by the change in the indexation rules. Since these agents can't change their stock of savings, their welfare decreases by 1.1% on average. The second group consists of the cohorts who had more than 20 years of labor market experience in 1995. The pension benefits of these generation is computed using the MDB formula. The welfare of the generations born between 1940 and 1955 will increase by 0.7% on average. The generations whose pension benefits are calculated with the pro-rata system are in the third group. The welfare of these generations will decrease by 2.1%. The cohorts that have entered the labor market immediately after the implementation of the reforms are in the fourth group. The generations born from 1975 to 2000 are adversely affected by the macroeconomic adjustments that are taking place during the transition and the welfare of these cohorts is bellow negative 2%. The fifth group consists of the generations that are alive in the new stationary equilibrium, their welfare have decrease by only 0.16% as compared to the DB retirement system.

The explanation for the welfare experience of the different cohorts can also be provided through the present value ratio (PVR) which is the ratio between the present value of future pension benefits and the present value of contributions paid, both valued at the beginning of life. The agents' welfare and the PVR are both plotted in Figure 4.7. The PVR for the cohort born in 1915 is normalized to one. As we can see from the figure, the retirees at the time of the reform introduction, the cohorts from 1920 to 1935, have the drop in their PVR by as low as 11%. The generations born from 1940 to 1955 had their pension rights protected and their PVR increases to as high as 7.8%. Finally, the PVR for the generations born in 1960 and onwards drops by 24%. Clearly, the generations which had more than 18 years of seniority at the end of 1995 have been protected and have benefitted from the reforms. While the younger and future generations incur the cost of moving to a new retirement system.

Agents allocate their time endowment between leisure, market production and human capital investment activities. Figure 4.6 plots average weekly hours for working age population in a given year. Initially, there are a lot of changes in the hours of work as transition generations adjust their labor supply in response to the reform introduction. The average market hours decline by 1 hour per week by year 2022 and return back to a pre-reform level by year 2050. At the same time, the investment hours display an increasing trend during the transition and are on average 2 hours per week higher in the new stationary equilibrium with the DC retirement system as compared to the pre-reform level. Therefore, investment into human capital is an important channel for adjustments during the transition to the DC retirement system and leads to higher levels of aggregate labor supply and output at the new stationary equilibrium, see Figure 4.3.

Individual allocations are recorded in Tables 4.5 to 4.10. These tables show how different generations are affected by the rules of implementing a new retirement system. The generations who have already retired in 1995, for example, the generation born in 1930 in Table 4.5, are affected by the change in the indexation rule. These agents can not adjust their labor supply and experience the decrease in consumption due to the reduction in their pension benefits. The generations with more than 18 years of labor market experience in 1995 have been the most protected by the reforms. As can be seen from Table 4.6, the generation born in 1950 will increase their consumption with negligible adjustments in the labor supply behavior.

The generations who have already entered the labor market at the time of reform announcement but had very little experience have to change their labor market decisions. The generation born in 1970 has almost the same lifetime consumption profile as the generation which have completed their lives under the DB system. But this generation have to increase their savings to compensate for the reduction in the present value of their pension benefits. To achieve the higher stock of savings, these agents increase the investment into the human capital early in life and work more hours during the later stages of their career to take advantage of the higher labor productivity. The change in hours of work for this generation is documented in Table 4.7.

The generation born in 1985 will have their retirement benefits calculated under the new DC system. The consumption of this generation goes down early in life because they increase their human capital investment. These agents also postpone the savings to the later stages of life when they have the highest level of labor market earnings, see Table 4.8.

For generations born in the new stationary equilibrium with a DC pension system, such as the generation born in 2090 and reported in Table 4.10, the increase in the lifetime consumption is achieved through higher investment into the human capital early in life and through higher market hours during middle ages.

4.5 Conclusion

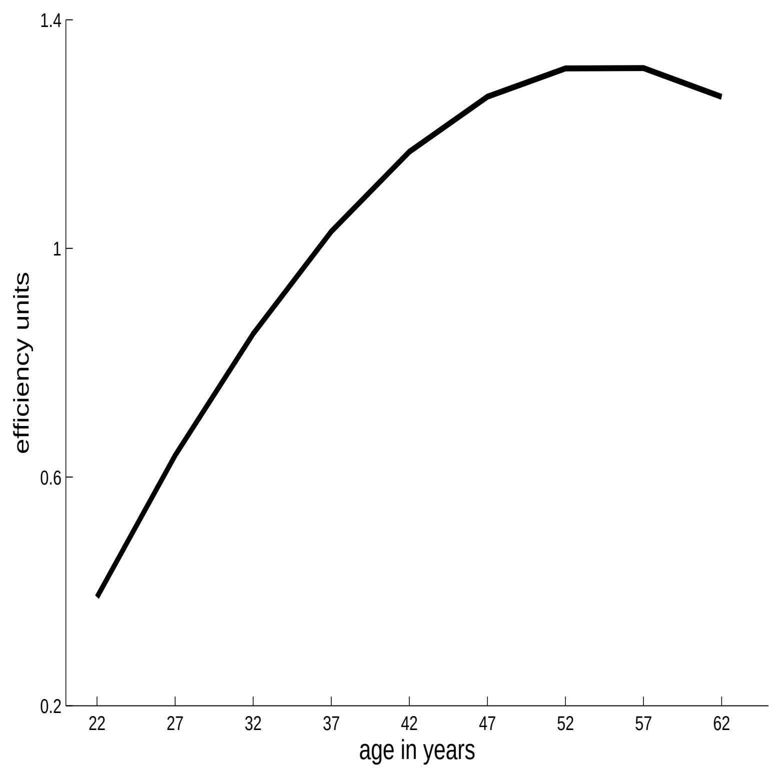
This chapter have used a general equilibrium overlapping generations model with endogeneous labor supply and human capital accumulation decisions to analyze the impact of the 1990s reforms in Italy on welfare of transition and future generations. The general equilibrium set-up also allows to predict the evolution of macroeconomic variables during the transition to a new retirement system with defined contributions benefits.

The analysis have shown that the introduced pension reforms will be effective in the long-run in reducing the pension expenditures and restoring the balance to the public retirement system. However, the implementation of the reforms has unequal treatment of transition generations. The agents with more than 18 years of labor market experience in 1995 have been protected by the reforms and experience welfare gains due to the changes in the retirement system. While younger and future generations bare the cost of moving to the DC retirement system.

This chapter have analyzed two major reforms that took place during the 1990s. The first reform implemented the indexation of pension benefits to prices as compared to wages before and the second reform introduced the defined contributions system. The simulations show that the change in the indexation rule decreases the pension expenditures only in the short-run while the introduction of the DC system leads to more sizeable reduction by year 2050.

The results of this chapter raise more general questions about government transfer programs. The introduction of the DC retirement system induces the agents to increase their human capital. This leads to higher time investment into human capital and lower consumption early in life. Hence, some form of transfers to young and borrowing-constrained agents will help to smooth their lifetime consumption and increase welfare.

Figure 4.1: Profile of efficiency units by age.



Profile of efficiency units is given for the initial stationary equilibrium with Defined Benefits retirement system. The profile represents the wage income during the working life scaled to the average wage income.

Table 4.1: Population Structure during Transition.

Generations	Time Periods													
year born	1992	1997	2002	2007	2012	2017	2022	2027	2032	2037	2042	2047	2052	2057
	Defined benefits, Pensione retributiva													
1915	$\mathbf{c^{15}_{92}}$													
1920	$\mathbf{c^{20}_{92}}$	$\mathbf{c^{20}_{97}}$												
1925	$\mathbf{c^{25}_{92}}$	$\mathbf{c^{25}_{97}}$	$\mathbf{c^{25}_{02}}$											
1930	c^{30}_{92}	$\mathbf{c^{30}_{97}}$	$\mathbf{c^{30}_{02}}$	$\mathbf{c^{30}_{07}}$										
	Modified defined benefits													
1935	c^{35}_{92}	c^{35}_{97}	$\mathbf{c^{35}_{02}}$	$\mathbf{c^{35}_{07}}$	$\mathbf{c^{35}_{12}}$									
1940	c^{40}_{92}	c^{40}_{97}	c^{40}_{02}	$\mathbf{c^{40}_{07}}$	$\mathbf{c^{40}_{12}}$	$\mathbf{c^{40}_{17}}$								
1945	c^{45}_{92}	c^{45}_{97}	c^{45}_{02}	c^{45}_{07}	$\mathbf{c^{45}_{12}}$	$\mathbf{c^{45}_{17}}$	$\mathbf{c^{45}_{22}}$							
1950	c^{50}_{92}	c^{50}_{97}	c^{50}_{02}	c^{50}_{07}	c^{50}_{12}	$\mathbf{c^{50}_{17}}$	$\mathbf{c^{50}_{22}}$	$\mathbf{c^{50}_{27}}$						
1955	c^{55}_{92}	c^{55}_{97}	c^{55}_{02}	c^{55}_{07}	c^{55}_{12}	c^{55}_{17}	$\mathbf{c^{55}_{22}}$	$\mathbf{c^{55}_{27}}$	$\mathbf{c^{55}_{32}}$					
	Combined system DB-DC, Pensione pro rata													
1960	c^{60}_{92}	c^{60}_{97}	c^{60}_{02}	c^{60}_{07}	c^{60}_{12}	c^{60}_{17}	c^{60}_{22}	$\mathbf{c^{60}_{27}}$	$\mathbf{c^{60}_{32}}$	$\mathbf{c^{60}_{37}}$				
1965	c^{65}_{92}	c^{65}_{97}	c^{65}_{02}	c^{65}_{07}	c^{65}_{12}	c^{65}_{17}	c^{65}_{22}	c^{65}_{27}	$\mathbf{c^{65}_{32}}$	$\mathbf{c^{65}_{37}}$	$\mathbf{c^{65}_{42}}$			
1970	c^{70}_{92}	c^{70}_{97}	c^{70}_{02}	c^{70}_{07}	c^{70}_{12}	c^{70}_{17}	c^{70}_{22}	c^{70}_{27}	c^{70}_{32}	$\mathbf{c^{70}_{37}}$	$\mathbf{c^{70}_{42}}$	$\mathbf{c^{70}_{47}}$		
	Defined contributions, Pensione contributiva													
1975		c^{75}_{97}	c^{75}_{02}	c^{75}_{07}	c^{75}_{12}	c^{75}_{17}	c^{75}_{22}	c^{75}_{27}	c^{75}_{32}	c^{75}_{37}	$\mathbf{c^{75}_{42}}$	$\mathbf{c^{75}_{47}}$	$\mathbf{c^{75}_{52}}$	
1980			c^{80}_{02}	c^{80}_{07}	c^{80}_{12}	c^{80}_{17}	c^{80}_{22}	c^{80}_{27}	c^{80}_{32}	c^{80}_{37}	c^{80}_{42}	$\mathbf{c^{80}_{47}}$	$\mathbf{c^{80}_{52}}$	$\mathbf{c^{80}_{57}}$

Table represents consumption of transition cohorts during the retirement reforms. Superscript for consumption allocation denotes the cohort and subscript stands for the time period when allocation takes place. 1992 is the year of initial stationary equilibrium with defined benefits retirement system. The reforms are implemented starting with 1997.

Table 4.2: Timing convention in the model.

Time Periods			Generations		
Time period	Year	Model period	Year born	Year model entrance	Model generation
1990-94	1992	0	1915	1935	-11
1995-99	1997	1	1920	1940	-10
2000-04	2002	2	1925	1945	-9
2005-09	2007	3	1930	1950	-8
2010-14	2012	4	1935	1955	-7
2015-19	2017	5	1940	1960	-6
2020-24	2022	6	1945	1965	-5
2025-29	2027	7	1950	1970	-4
2030-34	2032	8	1955	1975	-3
2035-39	2037	9	1960	1980	-2
2040-44	2042	10	1965	1985	-1
2045-49	2047	11	1970	1990	0
2050-54	2052	12	1975	1995	1
2055-59	2057	13	1980	2000	2
2060-64	2062	14	1985	2005	3

To simplify notation in the chapter, I relabel the periods as specified in the table. Timing for periods and generations continues to incorporate transition to a new stationary equilibrium.

Table 4.3: Model Structure of Generations during Transition.

GENERATIONS	TIME PERIODS													
period born	0	1	2	3	4	5	6	7	8	9	10	11	12	13
	Defined benefits, Pensione retributiva													
$t=-11$	$\mathbf{c_0^{-11}}$													
$t=-10$	$\mathbf{c_0^{-10}}$	$\mathbf{c_1^{-10}}$												
$t=-9$	$\mathbf{c_0^{-9}}$	$\mathbf{c_1^{-9}}$	$\mathbf{c_2^{-9}}$											
$t=-8$	c_0^{-8}	$\mathbf{c_1^{-8}}$	$\mathbf{c_2^{-8}}$	$\mathbf{c_3^{-8}}$										
	Modified defined benefits													
$t=-7$	c_0^{-7}	c_1^{-7}	$\mathbf{c_2^{-7}}$	$\mathbf{c_3^{-7}}$	$\mathbf{c_4^{-7}}$									
$t=-6$	c_0^{-6}	c_1^{-6}	c_2^{-6}	$\mathbf{c_3^{-6}}$	$\mathbf{c_4^{-6}}$	$\mathbf{c_5^{-6}}$								
$t=-5$	c_0^{-5}	c_1^{-5}	c_2^{-5}	c_3^{-5}	$\mathbf{c_4^{-5}}$	$\mathbf{c_5^{-5}}$	$\mathbf{c_6^{-5}}$							
$t=-4$	c_0^{-4}	c_1^{-4}	c_2^{-4}	c_3^{-4}	c_4^{-4}	$\mathbf{c_5^{-4}}$	$\mathbf{c_6^{-4}}$	$\mathbf{c_7^{-4}}$						
$t=-3$	c_0^{-3}	c_1^{-3}	c_2^{-3}	c_3^{-3}	c_4^{-3}	c_5^{-3}	$\mathbf{c_6^{-3}}$	$\mathbf{c_7^{-3}}$	$\mathbf{c_8^{-3}}$					
	Combined system DB-DC, Pensione pro rata													
$t=-2$	c_0^{-2}	c_1^{-2}	c_2^{-2}	c_3^{-2}	c_4^{-2}	c_5^{-2}	c_6^{-2}	$\mathbf{c_7^{-2}}$	$\mathbf{c_8^{-2}}$	$\mathbf{c_9^{-2}}$				
$t=-1$	c_0^{-1}	c_1^{-1}	c_2^{-1}	c_3^{-1}	c_4^{-1}	c_5^{-1}	c_6^{-1}	c_7^{-1}	$\mathbf{c_8^{-1}}$	$\mathbf{c_9^{-1}}$	$\mathbf{c_{10}^{-1}}$			
$t=0$	c_0^{0}	c_1^{0}	c_2^{0}	c_3^{0}	c_4^{0}	c_5^{0}	c_6^{0}	c_7^{0}	c_8^{0}	$\mathbf{c_9^{0}}$	$\mathbf{c_{10}^{0}}$	$\mathbf{c_{11}^{0}}$		
	Defined contributions, Pensione contributiva													
$t=1$		c_1^{1}	c_2^{1}	c_3^{1}	c_4^{1}	c_5^{1}	c_6^{1}	c_7^{1}	c_8^{1}	c_9^{1}	$\mathbf{c_{10}^{1}}$	$\mathbf{c_{11}^{1}}$	$\mathbf{c_{12}^{1}}$	
$t=2$			c_2^{2}	c_3^{2}	c_4^{2}	c_5^{2}	c_6^{2}	c_7^{2}	c_8^{2}	c_9^{2}	c_{10}^{2}	$\mathbf{c_{11}^{2}}$	$\mathbf{c_{12}^{2}}$	$\mathbf{c_{13}^{2}}$

This table presents model structure of generations during transition. I have relabeled the generations and time periods in accordance with convention specified in the previous table. Allocations in bold represent consumption during retirement.

Table 4.4: Model Parameters.

Parameter	Expression	Value
Preferences and Technology		
Discount factor	β	1.010
Leisure preference parameter	α	1.85
Capital share	θ	0.333
Depreciation rate of physical capital	δ_k	0.054
Rate of technological progress	γ	0.018
Rate of population growth	η	0.007
Government Sector		
Tax rate on consumption	τ_c	0.11
Tax rate on labor income	τ_l	0.15
Social Security tax rate	τ_{ss}	0.16
Tax rate on capital income	τ_k	0.25
Share of government expenditures	g	0.1438
Replacement rate for Social Security benefits	ϕ^1	0.015
Human Capital Technology		
Initial stock of human capital	h_0	0.45
Depreciation rate of human capital	δ_h	0.019
Productivity of HC accumulation	B	0.56
Weight of HC stock	ψ_1	0.43
Weight of time investment	ψ_2	0.40

All parameter values are given in annual terms. Since one model period corresponds to five years, the parameters are adjusted in computations accordingly.
All parameters are given for initial stationary equilibrium.

Table 4.5: Allocations for generation born in 1930.

					Change from generation 1915			
Age	c	l_m	l_h	s'	c	l_m	l_h	s'
22	0.51	7.91	61.76	0.00	0.00	0.00	0.00	0.00
27	1.47	20.37	32.35	0.00	0.00	0.00	0.00	0.00
32	2.52	26.14	20.81	0.13	0.00	0.00	0.00	0.00
37	3.26	37.05	13.39	1.98	0.00	0.00	0.00	0.00
42	4.21	42.42	8.21	5.69	0.00	0.00	0.00	0.00
47	5.45	43.90	4.51	11.03	0.00	0.00	0.00	0.00
52	7.04	41.87	1.98	17.19	0.00	0.00	0.00	0.00
57	9.10	36.03	0.50	22.39	0.00	0.00	0.00	0.00
62	11.77	25.34	0.00	23.53	0.00	0.00	0.00	0.00
67	14.52	0.00	0.00	21.53	-0.70	0.00	0.00	0.76
72	18.75	0.00	0.00	14.51	-0.92	0.00	0.00	0.82
77	24.15	0.00	0.00	0.00	-1.28	0.00	0.00	0.00

Consumption and savings variables are detrended towards the initial stationary equilibrium. Market production and human capital investment hours are weekly ones.

Table 4.6: Allocations for generation born in 1950.

					Change from generation 1915			
Age	c	l_m	l_h	s'	c	l_m	l_h	s'
22	0.51	7.91	61.76	0.00	0.00	0.00	0.00	0.00
27	1.47	20.37	32.35	0.00	0.00	0.00	0.00	0.00
32	2.52	26.14	20.81	0.13	0.00	0.00	0.00	0.00
37	3.26	37.05	13.39	1.98	0.00	0.00	0.00	0.00
42	4.21	42.42	8.21	5.69	0.00	0.00	0.00	0.00
47	5.56	43.62	4.50	10.99	0.12	-0.27	-0.01	-0.04
52	7.19	41.91	1.94	17.20	0.15	0.04	-0.05	0.01
57	9.25	35.89	0.48	22.26	0.15	-0.14	-0.02	-0.13
62	11.90	24.78	0.00	22.92	0.13	-0.56	0.00	-0.61
67	15.30	0.00	0.00	21.24	0.09	0.00	0.00	0.47
72	19.72	0.00	0.00	14.49	0.05	0.00	0.00	0.80
77	25.55	0.00	0.00	0.00	0.12	0.00	0.00	0.00

Consumption and savings variables are detrended towards the initial stationary equilibrium. Market production and human capital investment hours are weekly ones.

Table 4.7: Allocations for generation born in 1970.

					Change from generation 1915			
Age	c	l_m	l_h	s'	c	l_m	l_h	s'
22	0.51	7.91	61.76	0.00	0.00	0.00	0.00	0.00
27	1.47	19.93	33.78	0.00	-0.01	-0.44	1.43	0.00
32	2.57	25.80	21.54	0.11	0.05	-0.34	0.73	-0.02
37	3.31	36.85	14.01	2.00	0.05	-0.20	0.62	0.02
42	4.25	42.17	8.82	5.74	0.04	-0.25	0.60	0.05
47	5.47	43.79	5.05	11.13	0.02	-0.10	0.54	0.09
52	7.05	42.36	2.37	17.57	0.01	0.49	0.39	0.38
57	9.13	38.22	0.64	24.00	0.02	2.19	0.15	1.61
62	11.83	29.52	0.00	27.53	0.07	4.19	0.00	4.00
67	15.32	0.00	0.00	24.71	0.10	0.00	0.00	3.94
72	19.74	0.00	0.00	16.36	0.06	0.00	0.00	2.67
77	25.34	0.00	0.00	0.00	-0.09	0.00	0.00	0.00

Consumption and savings variables are detrended towards the initial stationary equilibrium. Market production and human capital investment hours are weekly ones.

Table 4.8: Allocations for generation born in 1985.

					Change from generation 1915			
Age	c	l_m	l_h	s'	c	l_m	l_h	s'
22	0.47	6.57	65.70	0.00	-0.04	-1.34	3.94	0.00
27	1.45	19.65	34.51	0.00	-0.02	-0.72	2.16	0.00
32	2.54	24.67	22.59	0.00	0.02	-1.47	1.78	-0.13
37	3.41	34.15	15.08	1.36	0.15	-2.89	1.69	-0.62
42	4.41	41.18	9.42	4.84	0.20	-1.25	1.21	-0.85
47	5.72	44.12	5.33	10.46	0.27	0.22	0.82	-0.57
52	7.40	43.68	2.45	17.68	0.36	1.81	0.47	0.49
57	9.54	39.49	0.68	24.81	0.44	3.46	0.18	2.42
62	12.25	30.93	0.00	28.88	0.48	5.60	0.00	5.35
67	15.68	0.00	0.00	25.55	0.47	0.00	0.00	4.78
72	20.05	0.00	0.00	16.74	0.38	0.00	0.00	3.05
77	25.62	0.00	0.00	0.00	0.19	0.00	0.00	0.00

Consumption and savings variables are detrended towards the initial stationary equilibrium. Market production and human capital investment hours are weekly ones.

Table 4.9: Allocations for generation born in 2030.

					Change from generation 1915			
Age	c	l_m	l_h	s'	c	l_m	l_h	s'
22	0.44	4.94	70.13	0.00	-0.06	-2.97	8.37	0.00
27	1.55	19.32	35.52	0.00	0.07	-1.06	3.17	0.00
32	2.77	24.67	22.67	0.00	0.25	-1.47	1.86	-0.13
37	3.69	34.85	14.81	1.62	0.43	-2.19	1.42	-0.36
42	4.71	41.47	9.27	5.42	0.50	-0.95	1.06	-0.27
47	6.01	44.17	5.26	11.31	0.56	0.28	0.75	0.27
52	7.67	43.54	2.43	18.60	0.63	1.67	0.45	1.41
57	9.78	39.39	0.67	25.72	0.68	3.36	0.17	3.33
62	12.47	30.79	0.00	29.68	0.71	5.46	0.00	6.16
67	15.91	0.00	0.00	26.10	0.70	0.00	0.00	5.33
72	20.29	0.00	0.00	17.03	0.62	0.00	0.00	3.35
77	25.88	0.00	0.00	0.00	0.45	0.00	0.00	0.00

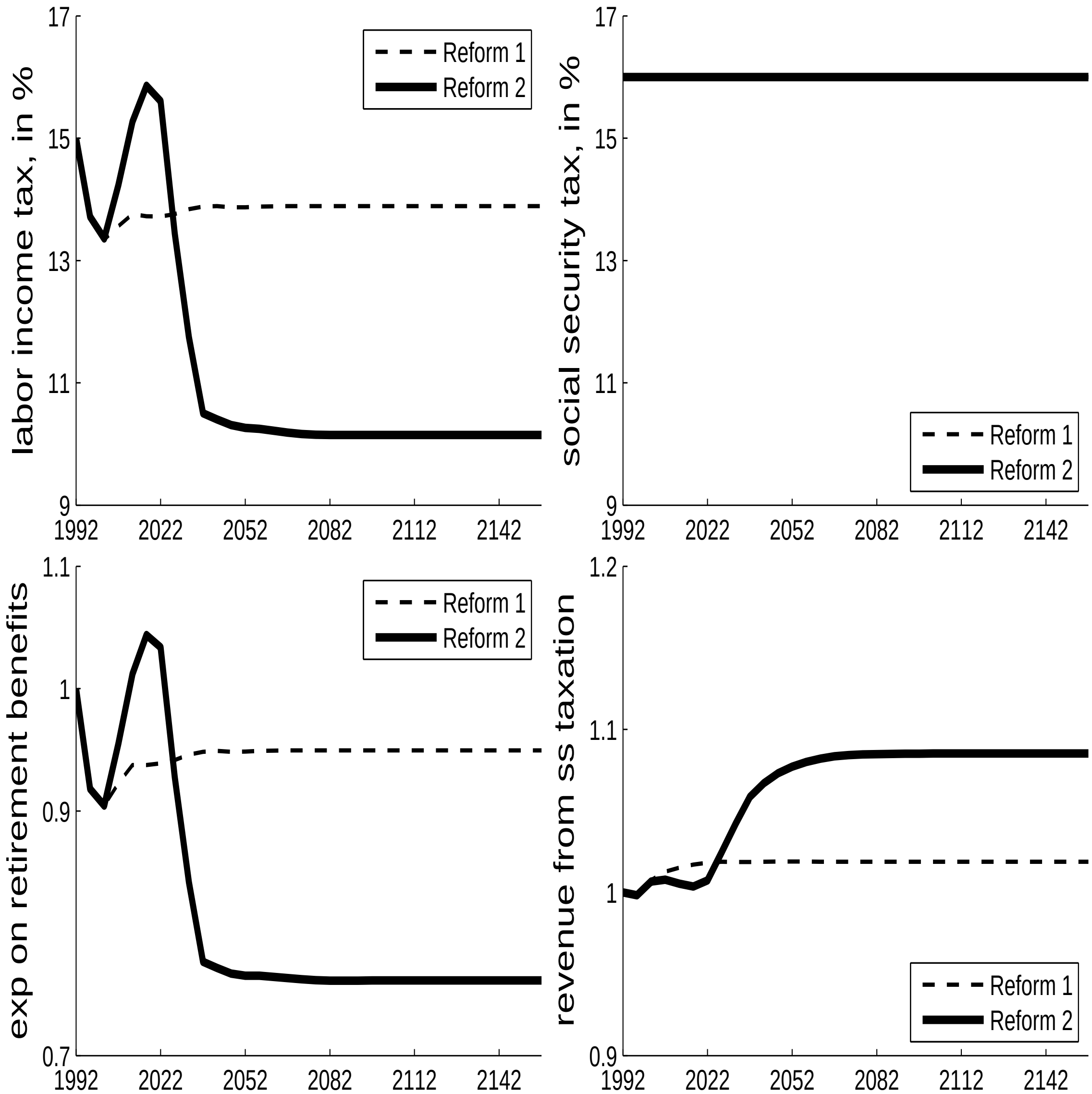
Consumption and savings variables are detrended towards the initial stationary equilibrium. Market production and human capital investment hours are weekly ones.

Table 4.10: Allocations for generation born in 2090.

					Change from generation 1915			
Age	c	l_m	l_h	s'	c	l_m	l_h	s'
22	0.45	4.95	70.09	0.00	-0.06	-2.96	8.33	0.00
27	1.55	19.32	35.50	0.00	0.08	-1.05	3.16	0.00
32	2.78	24.67	22.67	0.00	0.26	-1.46	1.85	-0.13
37	3.70	34.86	14.80	1.62	0.44	-2.18	1.41	-0.35
42	4.71	41.47	9.27	5.43	0.50	-0.95	1.06	-0.26
47	6.01	44.17	5.26	11.32	0.57	0.28	0.75	0.28
52	7.67	43.54	2.43	18.61	0.63	1.66	0.45	1.42
57	9.78	39.38	0.67	25.73	0.68	3.35	0.17	3.34
62	12.48	30.79	0.00	29.69	0.71	5.45	0.00	6.16
67	15.91	0.00	0.00	26.10	0.70	0.00	0.00	5.33
72	20.29	0.00	0.00	17.03	0.62	0.00	0.00	3.35
77	25.88	0.00	0.00	0.00	0.45	0.00	0.00	0.00

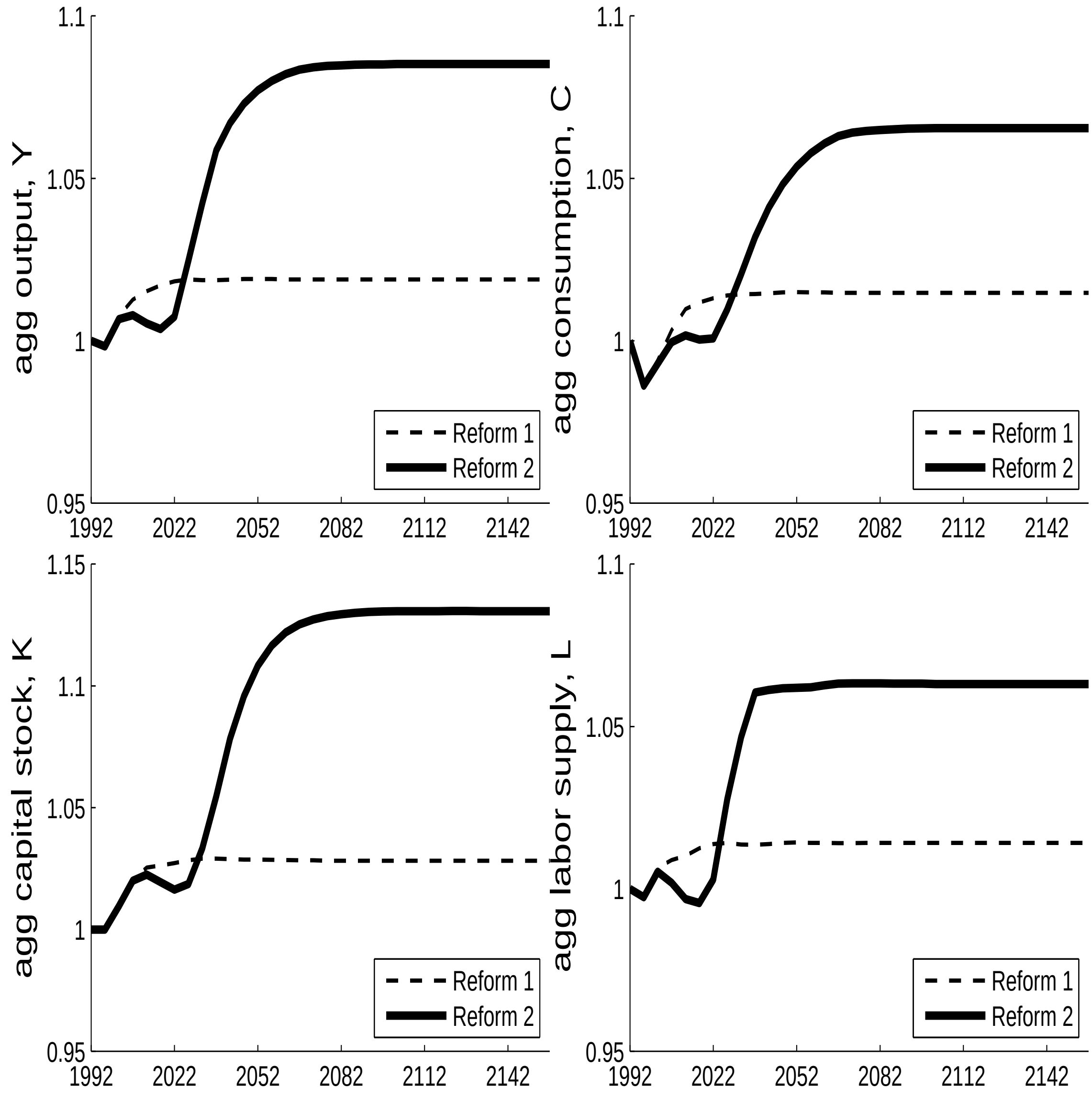
Consumption and savings variables are detrended towards the initial stationary equilibrium. Market production and human capital investment hours are weekly ones.

Figure 4.2: Budget for public pension system.



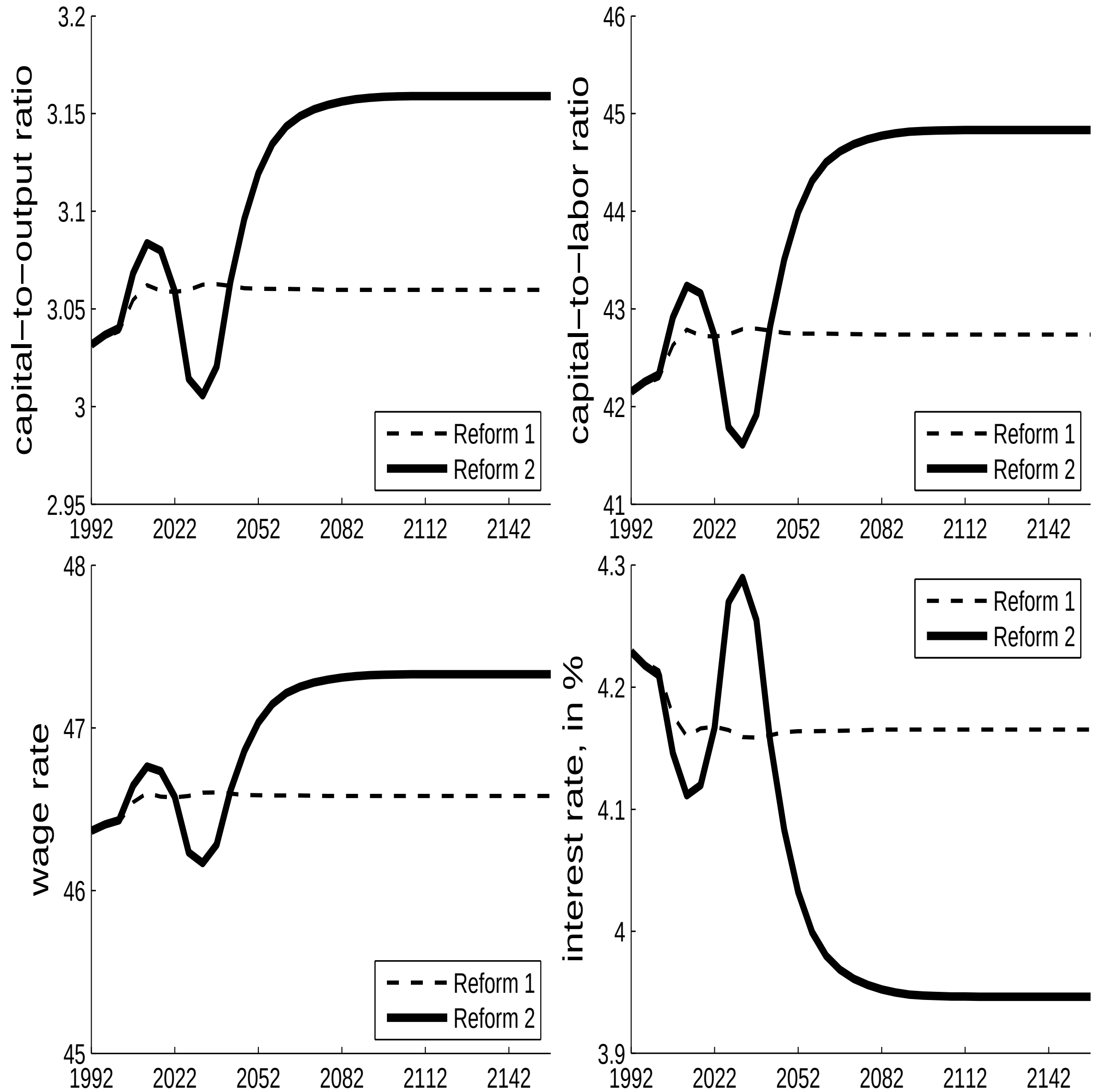
Tax on labor income is adjusted to keep the general government budget balanced during the transition while the social security tax is held fixed. Pension expenditures and revenue are detrended towards the stationary equilibrium and are expressed as ratios of the initial stationary equilibrium values. Year 1992 represents the initial stationary equilibrium with Defined Benefits retirement system. In Reform 1, indexation of retirement benefits is moved from real wages to prices, while Reform 2 also includes the shift towards Defined Contribution system.

Figure 4.3: Macroeconomic variables.



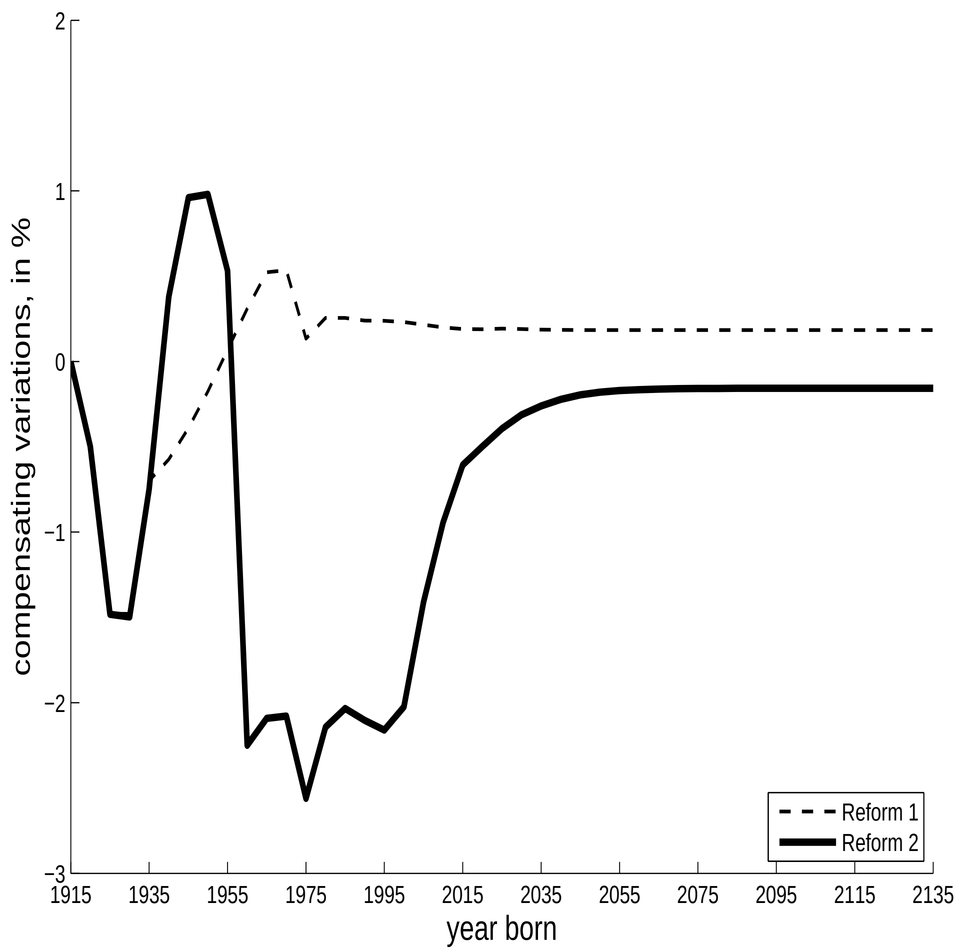
All macroeconomic variables are detrended towards the stationary equilibrium and are expressed as ratios of the initial stationary equilibrium values. Year 1992 represents the initial stationary equilibrium with Defined Benefits retirement system. In Reform 1, indexation of retirement benefits is moved from real wages to prices, while Reform 2 also includes the shift towards the Defined Contribution retirement system.

Figure 4.4: Macroeconomic variables and factor prices.



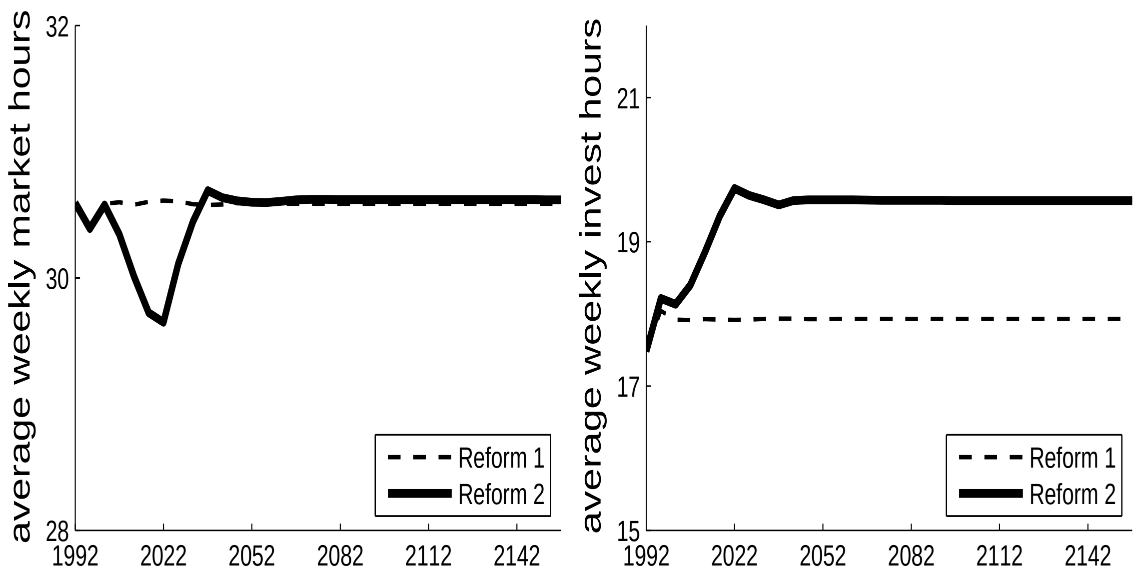
All macroeconomic variables are detrended towards the stationary equilibrium. Capital-to-labor ratio and wage rate are given in model units. Interest rate is expressed in annual terms. Year 1992 represents the initial stationary equilibrium with Defined Benefits retirement system. In Reform 1, indexation of retirement benefits is moved from real wages to prices, while Reform 2 also includes the shift towards the Defined Contribution retirement system.

Figure 4.5: Household welfare.



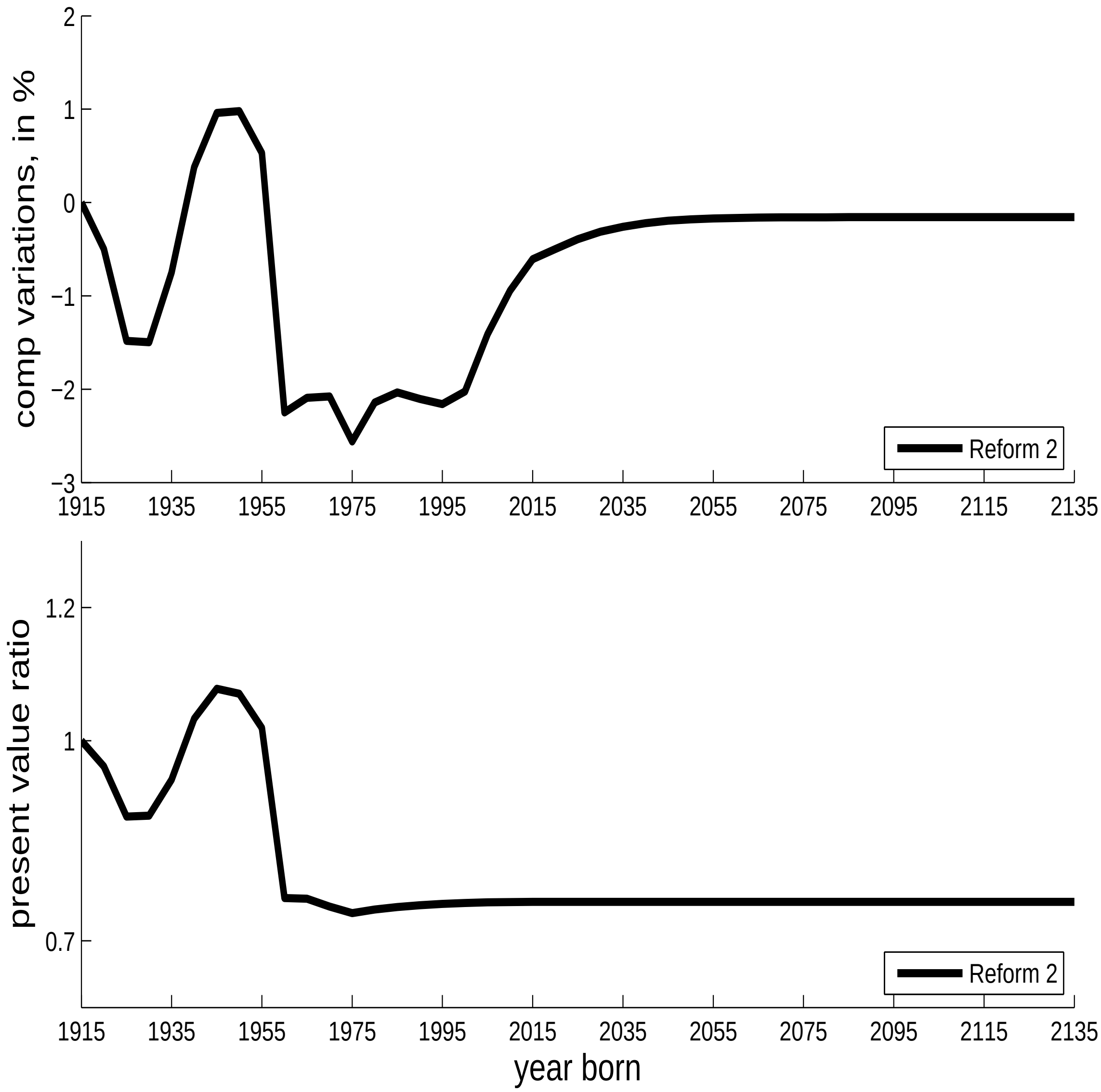
Household welfare is given in terms of compensating variations which measures the amount of life-time consumption that an agent has to be given to be indifferent between a reform and the initial stationary equilibrium. The generation born in 1915 has completed their lives under the initial stationary equilibrium with the Defined Benefits retirement system. The generation born in 1975 enters the labor market under the new rules of Defined Contributions retirement system. In Reform 1, indexation of retirement benefits is moved from real wages to prices, while Reform 2 also includes the shift towards the Defined Contribution retirement system.

Figure 4.6: Labor supply decisions.



Time endowment is allocated between leisure, market production and human capital investment activities. Average weekly hours are calculated for working age population in a given year. In Reform 1, indexation of retirement benefits is moved from real wages to prices, while Reform 2 also includes the shift towards the Defined Contribution retirement system.

Figure 4.7: Welfare and Present Value Ratio.



Household welfare is given in terms of compensating variations. Present value ratio (PVR) is the ratio between the present value of future pension benefits and the present value of contributions paid, both valued at the beginning of life. PVR for the generation born in 1915 is normalized to one. In Reform 1, indexation of retirement benefits is moved from real wages to prices, while Reform 2 also includes the shift towards the Defined Contribution retirement system.

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Chapter A

Appendix for Chapter 2

This appendix is for chapter on "Human Capital Investment and Retirement Savings Accounts". The appendix consists of three parts. In the first part, I describe a solution to a stationary equilibrium in the environments considered under different retirement arrangements. The second part provides a numerical algorithm to solve for a stationary equilibrium. Data sources and calibration procedures are discussed in the third part.

A.1 Solution for a Stationary Equilibrium

To compare different retirement arrangements, I solve for a stationary equilibrium. For each retirement system, I solve for a set of equations that must be satisfied in a stationary equilibrium. Within each retirement system, I provide details for the economies with endogenous and exogenous human capital accumulation.

A.1.1 Retirement arrangements with PAYG system

Environment with endogenous human capital An equilibrium in the environment with endogenous human capital must satisfy Definition 4. To derive equilibrium conditions, I start with a problem of a young agent. Given initial stocks of physical and human capital, (s_0, h_0), factor prices, and government policy, an individual chooses the allocations $\left(\{c_j, s_{j+1}, l_{m,j}, l_{h,j}, h_j\}_{j \in \Im}\right)$ to maximize (2.1) subject to (2.2)-(2.4).

In this appendix, a subscript denotes the period of an agent's life. Let λ_j, $j = 0, ..., J-1$, be multipliers on household's budget constraints (2.2) and μ_j, $j = 0, ..., J-1$, be multipliers on the law of motion of human capital (2.3). Assume that all non-negativity constraints are satisfied. The Lagrangian function for this problem is

$$
\begin{aligned}
& \pounds\left(\{c_j, s_{j+1}, l_{mj}, l_{hj}, h_{j+1}, \lambda_j, \mu_j\}_{j=0}^{J-1}\right) \\
= & \sum_{j=0}^{J-1} \beta^j u(c_j, 1-l_{mj}-l_{hj}) \\
& -\sum_{j=0}^{J-1} \lambda_j \left[(1+\tau_c)\, c_j + s_{j+1} - (1-\tau_l)\, w h_j l_{mj} - (1+(1-\tau_k)\, r) s_j - d_j\right] \\
& -\sum_{j=0}^{J-1} \mu_j \left[h_{j+1} - (1-\delta_h) h_j - G(h_j, l_{hj})\right].
\end{aligned}
$$

The following notation for partial derivatives is used:

$$
\begin{array}{cc}
u_{1j} = \frac{\partial u(c_j, 1-l_{mj}-l_{hj})}{\partial c_j}, & u_{2j} = \frac{\partial u(c_j, 1-l_{mj}-l_{hj})}{\partial\left(1-l_{mj}-l_{hj}\right)}, \\
G_{1j} = \frac{\partial G(h_j, l_{hj})}{\partial h_j}, & G_{2j} = \frac{\partial G(h_j, l_{hj})}{\partial l_{hj}}.
\end{array}
$$

First-order conditions with respect to an agent's choice variables are

$$
\begin{aligned}
\beta^j u_{1j} &= \lambda_j, && \text{(A.1)} \\
\lambda_j &= \lambda_{j+1}(1+(1-\tau_k)\, r), && \text{(A.2)} \\
\beta^j u_{2j} &= \lambda_j\, (1-\tau_l)\, w h_j, && \text{(A.3)} \\
\beta^j u_{2j} &= \mu_j G_{2j}, && \text{(A.4)} \\
\lambda_{j+1}\, (1-\tau_l)\, w l_{mj+1} &= \mu_j - \mu_{j+1}(1-\delta_h + G_{1j+1}), && \text{(A.5)} \\
(1+\tau_c)\, c_j + s_{j+1} &= (1-\tau_l)\, w h_j l_{mj} + (1+(1-\tau_k)\, r) s_j + d_j, && \text{(A.6)} \\
h_{j+1} &= (1-\delta_h) h_j + G(h_j, l_{hj}). && \text{(A.7)}
\end{aligned}
$$

I use (A.1) and (A.2) to get an intertemporal condition:

$$
\frac{u_{1j}}{\beta u_{1j+1}} = (1+(1-\tau_k)\, r). \tag{A.8}
$$

Using (A.1) and (A.3), I get a condition for an intratemporal trade-off between consumption and leisure:

$$
\frac{u_{2j}}{u_{1j}} = (1-\tau)\, w_j h_j, \tag{A.9}
$$

where $\tau = (\tau_l + \tau_c) / (1 + \tau_c)$ is a labor tax wedge.

The condition relating investment hours to other variables is derived as follows. Conditions (A.3) and (A.4) imply

$$\mu_j = \frac{\lambda_j (1 - \tau_l) w h_j}{G_{2j}}. \tag{A.10}$$

Combining (A.10), (A.5), and (A.2), I get an expression that implicitly determines the choice of investment hours:

$$l_{mj+1} = (1 + (1 - \tau_k) r) \frac{h_j}{G_{2j}} - \frac{h_{j+1}}{G_{2j+1}} (1 - \delta_h + G_{1j+1}). \tag{A.11}$$

Given the initial stocks of physical and human capital, s_0 and h_0, factor prices, r and w, and government sector variables, $(\tau_c, \tau_l, \tau_k, \phi, g, f)$, the household's choice variables can be found by solving a system of equations consisting of (A.8), (A.9), (A.11), (A.6), and (A.7) for the agent's life periods $j = 0, ..., J-1$. All retirees receive the same benefits, $b = \left(\phi / \overline{J}\right) \sum_{j=0}^{\overline{J}-1} w h_j l_{m,j}$, and agents of all ages get the lump-sum transfers, f.

Because I consider a stationary equilibrium, the market clearing conditions and government's budget constraint become:

$$\begin{aligned} K &= \sum_{j=0}^{J-1} s_j, \\ L &= \sum_{j=0}^{J-1} h_j l_{m,j}, &\text{(A.12)} \\ \sum_{j=0}^{J-1} c_j + \delta_k K &= (1 - g) A K^{\theta} L^{1-\theta}, &\text{(A.13)} \\ g A K^{\theta} L^{1-\theta} + J f + \left(J - \overline{J}\right) b &= \sum_{j=0}^{J-1} (\tau_c c_j + \tau_l w h_j l_{m,j} + \tau_k r s_j). \end{aligned}$$

Factor inputs are paid their marginal products:

$$w = (1 - \theta) A K^{\theta} L^{-\theta}, \tag{A.14}$$

$$r = \theta A K^{\theta - 1} L^{1-\theta} - \delta_k. \tag{A.15}$$

Environment with exogenous human capital Within this environment, profiles for investment hours and human capital stock, $\left(\{\bar{l}_{h,j}, \overline{h}_j\}_{j \in \Im}\right)$, are exogenous for an agent. A household also takes as given an initial endowment of physical capital, government policy variables, and factor prices. In the stationary equilibrium, the household's choice variables must satisfy (A.8), (A.9), and (A.6). The equations describing the government and production sectors and the market clearing conditions are the same as in the environment with endogenous human capital.

A.1.2 Retirement arrangements with Voluntary RSA

The PAYG social security system is eliminated by setting the replacement rate for the social security benefits, ϕ, equal to zero. The tax on labor income is recalibrated to keep the lump-sum transfers to households as the share of output the same as compared to the baseline retirement arrangements within the environment with endogenous human capital. Given these two changes in the policy parameters, the set of equations describing a stationary equilibrium is identical to the one under the baseline retirement arrangements.

A.1.3 Retirement arrangements with Mandatory RSA

Environment with endogenous human capital Because a household is forced to contribute a fraction of labor income toward the Mandatory RSA, the following variables in the household's problem change. Accumulation of physical capital takes place on two accounts: (i) one with voluntary contributions, $s_{1,j}$, and (ii) one with tax-deferred mandatory accounts, $s_{2,j}$. The endowment of assets on both savings accounts at the beginning of agents' lives is zero. The fraction of mandatory savings contributions is denoted by ξ. A household's budget constraint is

$$(1+\tau_c)\,c_j + s_{1,j+1} \leq (1-\tau_l)\,(1-\xi)\,wh_j l_{m,j} + (1+(1-\tau_k)\,r)s_{1,j} + d_j. \tag{A.16}$$

The amount of assets accumulated by households of different ages is

$$s_{2,j+1} = \begin{cases} \sum_{z=0}^{j} \xi w h_{j-z} l_{m,j-z}\,(1+r)^z\,, & j = 0, \overline{J}-1, \\ (1+r)\,s_{2,j} - b, & j = \overline{J}, J-1. \end{cases}$$

At the beginning of retirement, an agent has the following amount of assets on her or his tax-deferred Mandatory RSA:

$$s_{2,\overline{J}} = \sum_{j=0}^{\overline{J}-1} \xi w h_{\overline{J}-1-j} l_{m,\overline{J}-1-j}\,(1+r)^j\,.$$

Annuity payments from this savings account are the same for all retirees:

$$b = \left[\sum_{j=0}^{J-\overline{J}-1} (1+r)^j \right]^{-1} (1+r)^{J-\overline{J}} s_{2,\overline{J}},$$

and transfers to households become

$$d_j = \begin{cases} f, & j = 0, \overline{J}-1, \\ f + (1-\tau_l)\, b, & j = \overline{J}, J-1. \end{cases}$$

The labor wedge affecting an intratemporal condition is $\tau = (\tau_l + \tau_c + \xi - \tau_l \xi) / (1+\tau_c)$. Given initial stocks of physical and human capital, $s_{1,0}$, $s_{2,0}$, and h_0, factor prices, r and w, and government sector variables, $(\tau_c, \tau_l, \tau_k, \xi, \phi, g, f)$, the household's choice variables can be found by solving a system of equations consisting of (A.8), (A.9), (A.11), (A.16), and (A.7) for the agent's life periods $j = 0, ..., J-1$.

The market clearing condition for physical capital and the government's budget constraint are

$$\begin{aligned} K &= \sum_{j=0}^{J-1} s_{1,j} + s_{2,j}, \\ G + fJ &= \sum_{j=0}^{J-1} \left[\tau_c c_j + \tau_l (1-\xi)\, w h_j l_{m,j} + \tau_k r s_{1,j} \right] + \sum_{j=\overline{J}}^{J-1} \tau_l b. \end{aligned}$$

Market clearing conditions for the labor and the good markets are (A.12) and (A.13). The factor prices are determined by (A.14) and (A.15).

Environment with exogenous human capital Within this environment, profiles for investment hours and human capital stock, $\left(\left\{ \bar{l}_{h,j}, \overline{h}_j \right\}_{j \in \Im} \right)$, are exogenous for an agent. These profiles are kept the same under three retirement arrangements analyzed. The accumulation of assets on and the annuity payments from the Mandatory RSA are as in the environment with endogenous human capital. A household also takes as given an initial endowment of physical capital, government policy variables, and factor prices. In the stationary equilibrium, the household's choice variables must satisfy (A.8), (A.9), and (A.16). The equations describing the government and production sectors and the market clearing conditions are the same as in the environment with endogenous human capital.

A.2 Numerical algorithm

Environment with endogenous human capital I describe a numerical algorithm for finding a stationary equilibrium in the environment with endogenous human capital accumulation under retirement arrangements with PAYG system. Modifications of this algorithm for the economy with exogenous human capital and alternative retirement arrangements are discussed at the end of this subsection.

Since I solve for a stationary equilibrium, I omit the time subscript and consider cohorts alive at a period t. I also incorporate functional choices for the utility and human capital production functions. The values of parameters are given in Table 1. The life-span J is set at 12 periods, and the agents are working for the first $\overline{J} = 9$ periods. I solve for $3J - 1$ equilibrium variables using $3J - 1$ equations.

A.2.1 Initial guess for iteration procedure

I provide an initial guess for the following equilibrium variables:

- savings made by different cohorts at a given period[1], $\{s_{j+1}\}_{j=0}^{J-1}$;
- labor supply of different cohorts, $\{l_{mj}\}_{j=0}^{J-2}$;
- human capital stock of different cohorts[2], $\{h_j\}_{j=1}^{\overline{J}+1}$;
- aggregate capital stock, K;
- lump-sum transfers to cohorts of all ages, f.

A.2.2 Additional economy variables

Given the set of variables I iterate on, the rest of the economy variables are calculated as:

- during the last period of life, the agents are forced to retire, $l_{mJ} = 0$;
- human capital stock during the last period of life is $h_{J-1} = (1 - \delta_h)\, h_{J-2}$;
- aggregate labor supply is given by (A.12);

[1] The last element of the vector is savings of the oldest cohort at the last period of their lives, i.e., $s_J = 0$. The endowment of physical capital is $s_0 = 0$.

[2] Initial human capital stock is a given parameter, h_0.

- factor prices are determined by (A.14) and (A.15);
- social security benefits and transfers to households are

$$
\begin{aligned}
b &= \frac{\phi w}{\overline{J}} \sum_{j=0}^{\overline{J}-1} h_j l_{m,j}, \\
d_j &= \begin{cases} f, & j = 0, \overline{J} - 1 \\ f + b & j = \overline{J}, J - 1 \end{cases};
\end{aligned}
$$

- aggregate capital next period is

$$
K^{'} = \sum_{j=0}^{J-1} s_{j+1};
$$

- consumption of different cohorts, $\{c_j\}_{j=0}^{J-1}$, is determined by the household's budget constraints (A.6);
- aggregate consumption is $C = \sum_{j=0}^{J-1} c_j$;
- life-cycle profile for investment hours is determined by the human capital production technology:

$$
l_{h,j} = \left[\frac{h_{j+1} - (1 - \delta_h) h_j}{B (h_j)^{\psi_1}} \right]^{\frac{1}{\psi_2}}, j = 0, J - 1.
$$

A.2.3 System of equations to iterate on

The algorithm for imposing non-negativity constraints on savings consists of three steps: (1) solve the unconstrained problem; (2) for a cohort j with negative savings, replace an Euler equation for this cohort with the equation $s_{j+1} = 0$; and then (3) solve the modified system of equations. I solve the system of non-linear equations using a Newton-Raphson method as described in Press et al. (1986).

The system of equations for the unconstrained problem is

- Euler equations,

$$
\begin{aligned}
&c_{j+1} = \beta(1 + (1 - \tau_k) r) c_j, && j = 0, J - 2, \\
&s_{j+1} = 0, && j = J - 1;
\end{aligned}
$$

- given the labor tax wedge $\tau = (\tau_l + \tau_c)/(1+\tau_c)$, intratemporal conditions on labor supply are

$$\begin{array}{ll} \alpha c_j = (1-\tau) w h_j (1 - l_{m,j} - l_{h,j}), & j = 0, \overline{J}-1, \\ l_{m,j} = 0, & j = \overline{J}, J-2; \end{array}$$

- intertemporal condition on labor supply is

$$\begin{aligned} l_{m,j+1} B\psi_2 &= (1 + (1-\tau_k) r) (h_j)^{1-\psi_1} (l_{h,j})^{1-\psi_2} \\ &\quad - (h_{j+1})^{1-\psi_1} (l_{h,j+1})^{1-\psi_2} \left[1 - \delta_h + B\psi_1 (h_{j+1})^{\psi_1 - 1} (l_{h,j+1})^{\psi_2}\right], \\ j &= 0, \overline{J}; \end{aligned}$$

- market clearing condition for good market:

$$C + K' = (1-g) A K^{\theta} L^{1-\theta} + (1-\delta_k) K;$$

- government's budget constraint:

$$gAK^{\theta}L^{1-\theta} + \left(J - \overline{J}\right) b + Jf = \sum_{j=0}^{J-1} \left[\tau_c c_j + \tau_l w h_j l_{m,j} + \tau_k (r - \delta_k) s_j\right].$$

Environment with exogenous human capital The additional set of fixed parameters is the life-cycle profiles for investment hours and human capital stock, $\left(\{\bar{l}_{h,j}, \overline{h}_j\}_{j\in\Im}\right)$. The number of equilibrium variables to solve for is $2J+1$. These variables are $\left(\{s_{j+1}\}_{j=0}^{J-1}\right)$ and $\left(\{l_{mj}\}_{j=0}^{J-2}, K, f\right)$. The set of equations to iterate on is the same as in the environment with endogenous human capital excluding the intertemporal conditions on labor supply.

When I solve for a stationary equilibrium under alternative retirement arrangements, I modify the procedure using the set of equations characterizing each of these alternatives. The solution for a stationary equilibrium is derived in Section A.1

A.3 Data Sources and Calibration Procedure

Two main data sources are the Bureau of Economic Analysis (BEA), which publishes the national accounts tables, and the Integrated Public Use Microdata Series (IPUMS) provided by the University of Minnesota (www.ipums.org).

A.3.1 Details on NIPA accounts and government expenditures.

The rearrangement of NIPA accounts is specified in Table 2.7. Because I consider a closed economy, net exports are included into investment expenditures. The details on government expenditure programs are in Table 2.8.

A.3.2 Construction of wage earnings profile from decennial Census data.

To construct the life-cycle wage earnings profile, I use the Integrated Public Use Microdata Series (IPUMS) provided by the University of Minnesota (www.ipums.org). These series are based on the decennial Census surveys collected by the U.S. Bureau of the Census. I construct the wage earnings profile for 2000 and using a sample size of 1 percent of the population. The following variables are extracted from the IPUMS sample:

- PERWT: person's weight;
- AGE: person's age at last birthday;
- UHRSWORK: the number of hours per week that respondents usually worked if they worked during the previous calendar year;
- WKSWORK1: the number of weeks that the respondent worked during the preceding calendar year, including weeks with paid vacation and sick leave;
- INCWAGE: respondent's total pre-tax wage and salary income for the previous calendar year.
- CLASSWKR: class of worker.

In 2000, the top-code for INCWAGE is $175,000 (in contemporary dollars). Observations with INCWAGE above the top-code are assigned values of wage and salary income equal to the *state means* of values above $175,000. Hence, no adjustment for top-coded INCWAGE observations is needed.

I consider the age group 20 to 64 and restrict the sample to full-time full-year workers. These workers are individuals who worked at least 35 hours per week and 40 weeks a year during the last year. I further exclude workers reporting positive hours and zero wage income and self-employed people. The observations are divided into nine age groups:

Age group	00 =	Population 20 to 64 year old
	0 =	From 20 to 24 years
	1 =	From 25 to 29 years
	2 =	From 30 to 34 years
	3 =	From 35 to 39 years
	4 =	From 40 to 44 years
	5 =	From 45 to 49 years
	6 =	From 50 to 54 years
	7 =	From 55 to 59 years
	8 =	From 60 to 64 years

I denote each age group as A_j , where j is a group index and takes integer values from 0 to 9. For an individual i, hourly wage e_i is calculated as the ratio of the annual wage income and annual hours:

$$e_i = \frac{INCWAGE_i}{UHRSWORK_i \cdot WKSWORK1_i}.$$

Average hourly wage for an age group A_j is calculated using personal weights for observations:

$$e_j = \frac{\sum_{i \in A_j} e_i \cdot PERWT_i}{\sum_{i \in A_j} PERWT_i}.$$

To express the wage earnings profile in units comparable to the model, I report the ratio of the average hourly wage for an age group A_j to the average hourly wage of the working population: $\varepsilon_j = e_j / e_{00}$, $j = 0, ..., 8$.

To calculate average weekly hours for population ages 20 to 64, I use the variable $UHRSWORK$. Let $\overline{l_m}$ denote average weekly hours. These hours are then calculated as

$$\overline{l_m} = \frac{\sum_{i \in A_{00}} UHRSWORK_i \cdot PERWT_i}{\sum_{i \in A_{00}} PERWT_i}.$$

In 2000, a person in the age group 20 to 64 on average was working 29 hours per week.

Chapter B

Appendix for Chapter 3

The appendix describes the numerical algorithm for calculating the transition path of the economy in the chapter on "Feasibility of Welfare-Enhancing Social Security Reform in the Presence of Human Capital Investment". Initially, I calculate the stationary equilibrium in the economy with PAYG Social Security system. Then I derive the system of equations to calculate transition to the retirement system with voluntary RSA.

B.1 Stationary equilibrium in the baseline environment

Suppose that at the time period $t = 0$ the economy is at the stationary equilibrium with PAYG Social Security system. The variables to iterate on are $\left(K_1, L_0, \left(\sum_{j=0}^{J-1} n_0^{-j} f_0^{-j}\right)/Y_0\right)$ and $\left(\left\{h_0^{-j}\right\}_{j=1}^{\overline{J}-1}\right)$. The system of equations to solve for consists of $\left(3 + (\overline{J} - 1)\right)$ equations. The equations are (1) market clearing conditions for good market at time period $t = 0$, (2) market clearing condition for aggregate labor supply at $t = 0$, (3) government's budget constraint at $t = 0$, and (4) $\left(\overline{J} - 1\right)$ equations determining the human capital stock of generations alive at $t = 0$. The numerical algorithm for calculating the stationary equilibrium contains the following steps:

1. given initial guess for aggregate capital and labor and assumption for stationary equilibrium, calculate the time series $\{K_t, L_t\}_{t=-(J-1)}^{J-1}$. At the stationary equilibrium, the aggregate labor grows at the rate of population growth, η, and the aggregate capital grows at the rate $\eta\gamma^{1/(1-\theta)}$.

2. given aggregate capital and labor, calculate factor prices, $\{w_t, r_t\}_{t=-(J-1)}^{J-1}$:

$$w_t = (1-\theta) A_t K_t^{\theta} L_t^{-\theta},$$
$$r_t = \theta A_t K_t^{\theta-1} L_t^{1-\theta} - \delta_k;$$

3. given factor prices and government variables, calculate individual allocations for generations born at time periods $t = -(J-1), ..., J-1$,
$\left\{\left\{c_{t+j}^t, s_{t+j+1}^t, l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j}^t\right\}_{j=0}^{J-1}\right\}_{t=-(J-1)}^{J-1}$;

4. given government policy variables, calculate Social Security benefits for generations born at $t = -\overline{J}, ..., -(J-1)$:

$$b_0^{-j} = \phi^{-j} \frac{\sum_{i=0}^{\overline{J}-1} w_{-j+i} h_{-j+i}^{-j} l_{m,-j+i}^{-j}}{\overline{J}}, j = \overline{J}, ..., J-1;$$

5. calculate aggregate consumption and labor supply using individual allocations for period $t = 0$:

$$C_0 = \sum_{j=0}^{J-1} n_0^{-j} c_0^{-j},$$
$$L_0^s = \sum_{j=0}^{J-1} n_0^{-j} h_0^{-j} l_{m,0}^{-j};$$

6. calculate the system of equations to iterate on for period $t = 0$:

$$C_0 + K_1 = (1-g_0) A_0 K_0^{\theta} L_0^{1-\theta} + (1-\delta_k) K_0,$$
$$L_0 = L_0^s,$$
$$g_0 A_0 K_0^{\theta} L_0^{1-\theta} + \sum_{j=0}^{J-1} n_0^{-j} f_0^{-j} + \sum_{j=\overline{J}}^{J-1} n_0^{-j} b_0^{-j} = \sum_{j=0}^{J-1} n_0^{-j} \left(\tau_{c0} c_0^{-j} + \tau_{l0} w_0 h_0^{-j} l_{m,0}^{-j} + \tau_{k0} r_0 s_0^{-j}\right),$$

$$
\begin{aligned}
B\psi_1 l_{h,1}^j + B\psi_2 l_{m,1}^j = {} & R_1 \frac{(1-\tau_{l0})\, w_0}{(1-\tau_{l1})\, w_1} \left(h_0^j\right)^{1-\psi_1} \left(l_{h,0}^j\right)^{1-\psi_2} \\
& - \left(h_1^j\right)^{1-\psi_1} \left(l_{h,1}^j\right)^{1-\psi_2} (1-\delta_h)\,, \\
& j = 0, -1, ..., 2-\overline{J}.
\end{aligned}
$$

The step 3 of the above algorithm requires solving the agent's maximization problem with borrowing constraints. The solution of this problem is given in the next subsection.

B.1.1 Household's maximization problem with borrowing constraints

The agent's problem is to maximize (3.1) subject to constraints (3.2) to (3.4). The time-line for an agent's life is as follows: (i) during the periods $j = 0, ..., N_c - 1$, the agent is credit-constrained; (ii) working periods are $j = 0, ..., \overline{J} - 1$; (iii) retirement periods are $j = \overline{J}, ..., J - 1$.

By solving the agent's maximization problem using the Lagrangian method, I can derive the following set of equations. To simplify notation, I introduce the expression for after-tax interest rate and partial derivatives:

$$
\begin{aligned}
R_t &= 1 + (1-\tau_{kt})\, r_t, \\
G_{1j} &= \frac{\partial G(h_j, l_{hj})}{\partial h_j}, \\
G_{2j} &= \frac{\partial G(h_j, l_{hj})}{\partial l_{hj}}.
\end{aligned}
$$

For credit-constrained periods, $j = 0, ..., N_c - 1$, the expressions for consumption and market hours are:

$$
\begin{aligned}
c_{t+j}^t &= \frac{1}{(1+\alpha)\,(1+\tau_{c,t+j})} \left[(1-\tau_{l,t+j})\, w_{t+j} h_{t+j}^t \left(1 - l_{h,t+j}^t\right) + f_{t+j}\right], \\
l_{m,t+j}^t &= 1 - l_{h,t+j}^t - \frac{\alpha\,(1+\tau_{c,t+j})}{(1-\tau_{l,t+j})\, w_{t+j} h_{t+j}^t} c_{t+j}^t \\
&= \frac{1}{(1+\alpha)} \left(1 - l_{h,t+j}^t\right) - \frac{\alpha}{(1+\alpha)} \frac{f_{t+j}}{(1-\tau_{l,t+j})\, w_{t+j} h_{t+j}^t}.
\end{aligned}
$$

The investment hours have to satisfy the following equation:

$$l^t_{mt+j+1} = R_{t+j+1} \frac{(1-\tau_{l,t+j})\, w_{t+j}}{(1-\tau_{l,t+j+1})\, w_{t+j+1}} \frac{h^t_{t+j}}{G_{2j}} - \frac{h^t_{t+j+1}}{G_{2j+1}} (1-\delta_h + G_{1j+1}),$$
$$j = 0, ..., N_c - 1$$

Starting with period $j = N_c$, the agents can smooth consumption intertemporally and the intertemporal budget constraint is

$$\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod_{i=N_c+1}^{j} R_{1,t+i}} c^t_{t+j} \leq \sum_{j=N_c}^{\overline{J}-1} \frac{(1-\tau_{l,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j} l^t_{mt+j} + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}} + \sum_{j=\overline{J}}^{J-1} \frac{b^t}{\prod_{i=N_c+1}^{j} R_{t+i}}.$$

The Social Security benefits, b^t, are calculated based on average wage income during the working life:

$$b^t = \frac{\phi^t}{\overline{J}} \sum_{j=0}^{\overline{J}-1} w_{t+j} h^t_{t+j} l^t_{mt+j},$$

where $\overline{J}$ accounts for number of periods an agent have been working, ϕ^t is the replacement rate for retirement benefits.

Using the expression for retirement benefits, the intertemporal BC can be rewritten as:

$$\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} c^t_{t+j} \leq \sum_{j=0}^{N_c-1} \Phi^{1t} w_{t+j} h^t_{t+j} l^t_{mt+j} + \sum_{j=N_c}^{\overline{J}-1} \left[\frac{(1-\tau_{l,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} + \Phi^{1t} \right] w_{t+j} h^t_{t+j} l^t_{mt+j} + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}},$$

where $\Phi^{1t} = \frac{\phi^{1t}}{\overline{J}} \sum_{j=\overline{J}}^{J-1} \frac{1}{\prod_{i=N_c+1}^{j} R_{t+i}}$.

Plug-in the first-order conditions for credit-constrained and non-credit-constrained periods to find the expression for consumption during the first non-credit-constrained period, $c_{t+N_c}^t$:

$$\begin{aligned}
\left(1+\tau_{c,t+N_c}\right) c_{t+N_c}^t \tilde{\beta} \leq & \frac{\Phi^{1t}}{1+\alpha} \sum_{j=0}^{N_c-1} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right) \\
& + \sum_{j=N_c}^{\overline{J}-1} \Phi_{t+j}^{2t} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right) \\
& - \frac{\alpha \Phi^{1t}}{1+\alpha} \sum_{j=0}^{N_c-1} \frac{f_{t+j}}{1-\tau_{1l,t+j}} + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}}.
\end{aligned}$$

$$\Phi_{t+j}^{2t} = \frac{\left(1-\tau_{l,t+j}\right)}{\prod_{i=N_c+1}^{j} R_{t+i}} + \Phi^{1t}, j = N_c, ..., \overline{J}-1,$$

$$\Phi_{t+j}^{3t} = \left[\frac{\left(1-\tau_{l,t+j}\right)}{\prod_{i=N_c+1}^{j} R_{t+i}} + \Phi^{1t} \right] \frac{\left(\prod_{i=N_c+1}^{j} R_{t+i}\right)}{\left(1-\tau_{l,t+j}\right)}, j = N_c, ..., \overline{J}-1,$$

$$\tilde{\beta} = \sum_{j=N_c}^{J-1} \beta^{j-N_c} + \sum_{j=N_c}^{\overline{J}-1} \Phi_{t+j}^{3t} \alpha \beta^{j-N_c}.$$

Other household allocations during the non-credit-constrained periods are

$$c_{t+j}^t = \beta^{j-N_c} \left(\prod_{i=N_c+1}^{j} R_{t+i} \right) \frac{1+\tau_{c,t+N_c}}{1+\tau_{c,t+j}} c_{t+N_c}^t, j = N_c+1, ..., J-1,$$

$$l^t_{mt+j} = 1 - l^t_{ht+j} - \frac{\alpha\beta^{j-N_c}\left(1+\tau_{c,t+N_c}\right)c^t_{t+N_c}}{\left(1-\tau_{l,t+j}\right)w_{t+j}h^t_{t+j}}\left(\prod_{i=N_c+1}^{j} R_{t+i}\right), j = N_c, ..., \overline{J}-1,$$

$$l^t_{mt+j} = 0, j = \overline{J}, ..., J-1,$$

$$l^t_{mt+j+1} = R_{t+j+1}\frac{\left(1-\tau_{l,t+j}\right)w_{t+j}}{\left(1-\tau_{l,t+j+1}\right)w_{t+j+1}}\frac{h^t_{t+j}}{G_{2j}} - \frac{h^t_{t+j+1}}{G_{2j+1}}(1-\delta_h+G_{1j+1}),$$
$$j = N_c, ..., \overline{J}-2.$$

B.2 Numerical algorithm for the transition path of the economy

At time period $t=0$, the economy is at the initial stationary equilibrium with PAYG Social Security system. At $t=1$, the retirement reform is announced and the transition to the new system takes place during $t=1,...,T$. The generations born at $t=1$ and later will be subject to the rules of the new retirement system. The generations that are alive at the time of the reform announcement and worked partially during the old system were born at $t=1-j$, $j=1,...,J-1$. These generations can be divided into two groups. The first group consists of generations born at $t=1-j$, $j=\overline{J},...,J-1$. These are retirees and their retirement benefits are calculated using the PAYG system. The second group comprises of the working generations born at $t=1-j$, $j=1,...,\overline{J}-1$. At $t=1$, these agents learn about the retirement reform and solve their maximization problem given initial stocks of physical and human capitals and government policies. At the beginning of time period $t=1$, the life horizon of the agent born at $t=1-j$, $j=1,...,\overline{J}-1$ is $\hat{J}(t)=J-(1-t)$ and the number of working periods left is $\tilde{J}(t)=\overline{J}-(1-t)$. The generations from the second group that are making human capital accumulation decisions are born at $t=1-j$, $j=1,...,\overline{J}-2$.

To illustrate the decisions of the transition generations, I present the economy structure with population calibration for the initial stationary equilibrium. Table 3.1 illustrates consumption decisions when the life-span of the agents, J, is twelve model periods and the length of working life, $\overline{J}$, is nine periods. The generations born at $t=1$ and later will retire under the new retirement system. The generations born at $t=-8,-9,-10$ are retired at the time of reform announcement and their retirement benefits are calculated under the PAYG system. The generations born at $t=0,...,-7$ will reoptimize at the beginning of time period $t=1$. The generations born at $t=0,...,-6$ will make human capital accumulation decisions.

Variables to iterate on are $\left(\left\{K_{t+1}, L_t, \tau_{lt}, \left\{h_{t+j}^t\right\}_{j=1}^{\overline{J}-1}\right\}_{t=1}^{T}, \left\{\left\{h_j^t\right\}_{j=2}^{\tilde{J}(t)}\right\}_{t=0}^{3-\overline{J}}\right)$. The system of equations to solve for consists of $\left(3+\left(\overline{J}-1\right)\right)*T+N_h$ equations. Per each time period of transition, $t=1,...,T$, the equations are (1) market clearing conditions for good market, (2) market clearing condition for aggregate labor supply, (3) government's budget constraint, and (4) $\left(\overline{J}-1\right)$ equations determining the human capital stock of generations born at time period t. Plus, N_h equations for human capital stock of agents born prior to the reform announcement and alive during the transition. These generations were born at the time periods $t=1-j, j=1,...,\overline{J}-2$. Hence, the number of equations is given by

$$N_h=\sum_{t=0}^{3-\overline{J}}\left[\tilde{J}(t)-1\right].$$

Variables describing the initial conditions and government policies for the transition towards the new retirement system are:

- stock of aggregate physical capital, K_1;
- asset holdings for generations alive at $t=1$, $\left\{s_1^{1-j}\right\}_{j=0}^{J-1}$;
- human capital stock for generations alive at $t=1$, $\left\{h_1^{1-j}\right\}_{j=0}^{J-1}$;
- wage history for generations alive at the time of reform announcement, $\left\{\overline{w}^j\right\}_{j=0}^{J-2}$, where $\overline{w}$ is total wage income up to period $t=1$.
- endowment of physical and human capital for newly-born agents, $\left\{s_t^t, h_t^t\right\}_{t=1}^{T}$;
- replacement rate for social security benefits for all generations alive during the transition period, $\left\{\phi^j\right\}_{j=1-(J-1)}^{T}$;
- time path for government instruments, $\{(f_t, g_t, \tau_{ct}, \tau_{kt})\}_{t=1}^{T}$.

Given initial conditions, parameters and initial guess for variables to be iterated on, the numerical algorithm consists of the following steps:

1. given aggregate capital and labor, calculate factor prices, $\{w_t, r_t\}_{t=1}^{T}$:

$$\begin{aligned} w_t &= (1-\theta)\, A_t K_t^{\theta} L_t^{-\theta}, \\ r_t &= \theta A_t K_t^{\theta-1} L_t^{1-\theta} - \delta_k; \end{aligned}$$

2. given factor prices, government variables, and individual history variables, calculate individual allocations for generations born at time periods $t = -(J-2), ..., T$, $\left\{\left\{c^t_{t+j}, s^t_{t+j+1}, l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j}\right\}_{j=0}^{J-1}\right\}_{t=1}^{T}$ and $\left\{\left\{c^j_t, s^j_{t+1}, l^j_{m,t}, l^j_{h,t}, h^j_t\right\}_{j=-(J-2)}^{0}\right\}_{t=1}^{\hat{J}(j)}$;
3. given government policy variables, calculate Social Security benefits for generations born at $t = -(J-2), ..., T$:

$$b^{t-j}_t = \phi^{t-j} \frac{\sum_{i=0}^{\overline{J}-1} w_{t-j+i} h^{t-j}_{t-j+i} l^{t-j}_{m,t-j+i}}{\overline{J}}, j = \overline{J}, ..., J-1;$$

4. calculate aggregate consumption and labor supply using individual allocations for periods $t = 1, ..., T$:

$$C_t = \sum_{j=0}^{J-1} c^{t-j}_t,$$
$$L^s_t = \sum_{j=0}^{J-1} h^{t-j}_t l^{t-j}_{m,t};$$

5. calculate the system of equations to iterate on for periods $t = 1, ..., T$:

$$\begin{aligned} C_t + K_{t+1} &= (1-g_t) A_t K^\theta_t L^{1-\theta}_t + (1-\delta_k) K_t, \\ L_t &= L^s_t, \\ g_t A_t K^\theta_t L^{1-\theta}_t + J f_t + \sum_{j=\overline{J}}^{J-1} b^{t-j}_t &= \sum_{j=0}^{J-1} \left(\tau_{ct} c^{t-j}_t + \tau_{lt} w_t h^{t-j}_t l^{t-j}_{m,t} + \tau_{kt} r_t s^{t-j}_t\right), \end{aligned}$$

$$\begin{aligned} B\psi_1 l^t_{h,t+j+1} + B\psi_2 l^t_{m,t+j+1} &= R_{t+1} \frac{(1-\tau_{lt}) w_{t+j}}{(1-\tau_{lt+1}) w_{t+j+1}} \left(h^t_{t+j}\right)^{1-\psi_1} \left(l^t_{h,t+j}\right)^{1-\psi_2} \\ &\quad - \left(h^t_{t+j+1}\right)^{1-\psi_1} \left(l^t_{h,t+j+1}\right)^{1-\psi_2} (1-\delta_h), \\ &j = 0, ..., \overline{J}-2, t = 1, ..., T, \\ B\psi_1 l^j_{h,t+1} + B\psi_2 l^j_{m,t+1} &= R_{t+1} \frac{(1-\tau_{lt}) w_t}{(1-\tau_{lt+1}) w_{t+1}} \left(h^j_t\right)^{1-\psi_1} \left(l^j_{h,t}\right)^{1-\psi_2} \\ &\quad - \left(h^j_{t+1}\right)^{1-\psi_1} \left(l^j_{h,t+1}\right)^{1-\psi_2} (1-\delta_h), \\ &j = 0, ..., 3-\overline{J}, t = 1, ..., \tilde{J}(j)-1, \end{aligned}$$

The step 2 of the above algorithm requires to solve the household's problem with borrowing constraints. Next I describe the procedure to find allocations of generations alive during the transition periods when borrowing constraints are imposed during the first three periods of life or during the first N_c periods of life.

B.2.1 Maximization problem for transition generations

Because the Social Security reform has not been expected, all generations alive at the time of reform announcement will optimally allocate their time and resources given initial conditions and government instruments. Let's consider a problem of each generation separately. To simplify notation, define the gross after-tax return on physical capital for period t as

$$R_t = 1 + (1 - \tau_{kt})\, r_t.$$

Generation born at period $t = -(J-2)$. The oldest generation at the time of reform announcement, $j = -(J-2)$, will consume their savings and government transfers.

$$\begin{aligned}
c_1^{-(J-2)} &= \frac{1}{1+\tau_{c1}} \left(R_1 s_1^{-(J-2)} + f_1 + \phi^{-(J-2)} \frac{\overline{w}^{-(J-2)}}{\overline{J}} \right), \\
l_{m,1}^{-(J-2)} &= 0, \\
s_2^{-(J-2)} &= 0.
\end{aligned}$$

Generation born at period $t = -(J-3)$. Given initial assets, these agents allocate their resources between two periods.

$$\begin{aligned}
c_1^{-(J-3)} &= \frac{1}{(1+\tau_{c1})(1+\beta)} \left(R_1 s_1^{-(J-3)} + f_1 + \frac{f_2}{R_2} + \phi^{-(J-2)} \frac{\overline{w}^{-(J-3)}}{\overline{J}} \left(1 + \frac{1}{R_2}\right) \right), \\
c_2^{-(J-3)} &= \beta R_2 \frac{1+\tau_{c1}}{1+\tau_{c2}} c_1^{-(J-3)}, \\
l_{m,1}^{-(J-3)} &= l_{m,2}^{-(J-3)} = 0, \\
s_2^{-(J-3)} &= R_1 s_1^{-(J-3)} + f_1 + \phi^{-(J-3)} \frac{\overline{w}^{-(J-3)}}{\overline{J}} - (1+\tau_{c1})\, c_1^{-(J-3)}, \\
s_3^{-(J-3)} &= 0.
\end{aligned}$$

Generation born at period $t = -(J-4)$. Given initial assets, these agents allocate their resources between three periods.

$$
\begin{aligned}
c_1^{-(J-4)} &= \hat{\beta}\left(R_1 s_1^{-(J-4)} + f_1 + \frac{f_2}{R_2} + \frac{f_3}{R_2 R_3} + \phi^{-(J-4)}\frac{\overline{w}^{-(J-4)}}{\overline{J}}\left(1 + \frac{1}{R_2} + \frac{1}{R_2 R_3}\right)\right), \\
\hat{\beta} &= \frac{1}{(1+\tau_{c1})(1+\beta+\beta^2)}, \\
c_2^{-(J-4)} &= \beta R_2 \frac{1+\tau_{c1}}{1+\tau_{c2}} c_1^{-(J-4)}, \\
c_3^{-(J-4)} &= \beta^2 R_2 R_3 \frac{1+\tau_{c1}}{1+\tau_{c3}} c_1^{-(J-4)}, \\
l_{m,1}^{-(J-4)} &= l_{m,2}^{-(J-4)} = l_{m,3}^{-(J-4)} = 0, \\
s_2^{-(J-4)} &= R_1 s_1^{-(J-4)} + f_1 + \phi^{-(J-4)}\frac{\overline{w}^{-(J-4)}}{\overline{J}} - (1+\tau_{c1})\, c_1^{-(J-4)}, \\
s_3^{-(J-4)} &= R_2 s_2^{-(J-4)} + f_2 + \phi^{-(J-4)}\frac{\overline{w}^{-(J-4)}}{\overline{J}} - (1+\tau_{c2})\, c_2^{-(J-4)}, \\
s_4^{-(J-4)} &= 0.
\end{aligned}
$$

Generation born at period $t = -(J-5)$. Agents have to decide on labor supply during last period of working life and on resource allocation during the retirement. At pre-retirement period, the agents do not invest into human capital and $l_{h,1}^{-(J-5)} = 0$. Define the following variables to simplify the notation:

$$
\begin{aligned}
\prod_{t=2}^{1} R_t &= 1, \\
\Phi^{-(J-5)} &= \frac{\phi^{-(J-5)}}{\overline{J}} \sum_{i=1}^{J-\overline{J}} \left(\prod_{t=2}^{i+1} \frac{1}{R_t}\right), \\
\tilde{\beta} &= 1 + \frac{\Phi^{-(J-5)}}{1-\tau_{l,1}}, \\
d_t^{-(J-5)} &= \begin{cases} f_t, & t = 1, \\ f_t + \frac{\phi^{-(J-5)}}{\overline{J}}\left(\overline{w}^{-(J-5)} + w_1 h_1^{-(J-5)} l_{m,1}^{-(J-5)}\right), & t = 2, 3, 4. \end{cases}
\end{aligned}
$$

Then consumption at $t=1$ is

$$c_1^{-(J-5)}(1+\tau_{c1})\left(\sum_{i=0}^{3}\beta^i+\alpha\tilde{\beta}\right)=R_1 s_1^{-(J-5)}+\sum_{i=1}^{4}\frac{f_i}{\prod\limits_{t=2}^{i}R_t}+\Phi^{-(J-5)}\overline{w}^{-(J-5)}$$
$$+\left(1-\tau_{l1}+\Phi^{-(J-5)}\right)\left(1-l_{h,1}^{-(J-5)}\right)w_1 h_1^{-(J-5)}.$$

Other individual allocations for this generation are:

$$c_t^{-(J-5)}=\beta^{t-1}\left(\prod_{i=2}^{t}R_i\right)\frac{1+\tau_{c1}}{1+\tau_{ct}}c_1^{-(J-5)}, t=2,3,4,$$
$$l_{m,1}^{-(J-5)}=1-l_{h,1}^{-(J-5)}-\frac{\alpha(1+\tau_{c1})}{(1-\tau_{l1})w_1 h_1^{-(J-5)}}c_1^{-(J-5)},$$
$$l_{m,2}^{-(J-5)}=l_{m,3}^{-(J-5)}=l_{m,4}^{-(J-5)}=0,$$
$$l_{h,1}^{-(J-5)}=0$$
$$s_{t+1}^{-(J-5)}=(1-\tau_{lt})w_t h_t^{-(J-5)}l_{m,t}^{-(J-5)}+R_t s_t^{-(J-5)}+d_t^{-(J-5)}-(1+\tau_{ct})c_t^{-(J-5)},$$
$$t=1,2,3,$$
$$s_5^{-(J-5)}=0.$$

Generation born at period $t=-(J-i), i=6,...,10.$ From a time period of reform announcement, a generation t will work for the number of periods $\tilde{J}=\overline{J}-(1-t)=i-4$ and live for the number of periods $\hat{J}=J-(1-t)=i-1$. Then define the following variables to simplify the notation:

$$\Phi^{-(J-i)}=\frac{\phi^{-(J-i)}}{\overline{J}}\left(\prod_{t=2}^{\tilde{J}}\frac{1}{R_t}\right)\left[\sum_{z=1}^{J-\overline{J}}\left(\prod_{t=1}^{z}\frac{1}{R_{\tilde{J}+t}}\right)\right],$$
$$\prod_{z=2}^{1}R_z=1,$$
$$\hat{\beta}=\sum_{t=1}^{\hat{J}}\beta^{t-1}+\alpha\tilde{\beta},$$
$$\tilde{\beta}=\sum_{t=1}^{\tilde{J}}\left(\frac{(1-\tau_{l,t})}{\prod\limits_{z=2}^{t}R_z}+\Phi^{-(J-i)}\right)\frac{\beta^{t-1}\prod_{z=2}^{t}R_z}{1-\tau_{l,t}}.$$

Then consumption at $t=1$ is

$$c_1^{-(J-i)}\left(1+\tau_{c1}\right)\hat{\beta} = R_1 s_1^{-(J-i)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z} + \Phi^{-(J-i)}\overline{w}^{-(J-i)}$$
$$+ \sum_{t=1}^{\tilde{J}} \left(\frac{1-\tau_{lt}}{\prod\limits_{z=2}^{t} R_z} + \Phi^{-(J-i)} \right) \left(1-l_{h,t}^{-(J-i)}\right) w_t h_t^{-(J-i)}.$$

Other individual allocations for this generation are:

$$c_t^{-(J-i)} = \beta^{t-1}\left(\prod_{i=2}^{t} R_i\right)\frac{1+\tau_{c1}}{1+\tau_{ct}} c_1^{-(J-i)}, t=2,...,\hat{J},$$
$$l_{m,t}^{-(J-i)} = 1-l_{h,t}^{-(J-i)} - \frac{\alpha\left(1+\tau_{ct}\right)}{\left(1-\tau_{lt}\right) w_t h_t^{-(J-i)}} c_t^{-(J-i)}, t=1,...,\tilde{J},$$
$$l_{m,t}^{-(J-i)} = 0, t=\tilde{J}+1,...,\hat{J},$$
$$d_t^{-(J-i)} = \begin{cases} f_t, & t=1,...,\tilde{J}, \\ f_t + \frac{\phi^{-(J-i)}}{\overline{J}}\left(\overline{w}^{-(J-i)} + \sum\limits_{t=1}^{\tilde{J}} w_t h_t^{-(J-i)} l_{m,t}^{-(J-i)}\right), & t=\tilde{J}+1,...,\hat{J}, \end{cases}$$
$$s_{t+1}^{-(J-i)} = \left(1-\tau_{lt}\right) w_t h_t^{-(J-i)} l_{m,t}^{-(J-i)} + R_t s_t^{-(J-i)} + d_t^{-(J-i)} - \left(1+\tau_{ct}\right) c_t^{-(J-i)},$$
$$t=1,...,\hat{J}-1,$$
$$s_{\hat{J}+1}^{-(J-i)} = 0.$$

The conditions determining the life-cycle profile of human capital stock are

$$B\psi_1 l_{h,t+1}^{-(J-i)} + B\psi_2 l_{m,t+1}^{-(J-i)} = R_{t+1}\frac{\left(1-\tau_{lt}\right) w_t}{\left(1-\tau_{lt+1}\right) w_{t+1}}\left(h_t^{-(J-i)}\right)^{1-\psi_1}\left(l_{h,t}^{-(J-i)}\right)^{1-\psi_2}$$
$$-\left(h_{t+1}^{-(J-i)}\right)^{1-\psi_1}\left(l_{h,t+1}^{-(J-i)}\right)^{1-\psi_2}\left(1-\delta_h\right)$$
$$t=1,...,\tilde{J}+1.$$

Generation born at period $t=-1$**.** An agent born at period $t=-1$ was constrained during first two periods of his or her life during the initial steady state. Hence, the agent's physical asset holdings are equal to zero at the time of reform announcement, $s_1^{-1}=0$. Suppose that the agent is also credit-constrained during the period $t=1$. The agent works until period $t = 7$ and lives until period $t = 10$. The agent's allocations during time period $t = 1$ are:

$$c_1^{-1} = \frac{1}{(1+\tau_{c,1})(1+\alpha)}\left[(1-\tau_{l,1})\, w_1 h_1^{-1}\left(1-l_{h,1}^{-1}\right) + f_1\right],$$
$$l_{m,1}^{-1} = 1 - l_{h,1}^{-1} - \frac{\alpha(1+\tau_{c,1})}{(1-\tau_{l,1})\, w_1 h_1^{-1}} c_1^{-1},$$
$$s_2^{-1} = 0.$$

Starting with period $t = 2$, the agent can smooth consumption and consumption allocation at this period is found from the intertemporal budget constraint:

$$c_2^{-1}(1+\tau_{c,2})\,\hat{\beta} = \frac{\Phi^{-1} w_1 h_1^{-1}}{1+\alpha}\left(1-l_{h,1}^{-1}\right) + \sum_{t=2}^{7}\left(\frac{(1-\tau_{l,t})}{\prod\limits_{i=3}^{t} R_i} + \Phi^{-1}\right) w_t h_t^{-1}\left(1-l_{h,t}^{-1}\right)$$
$$- \frac{\Phi^{-1}\alpha}{1+\alpha}\frac{f_1}{(1-\tau_{l,1})} + \sum_{t=2}^{10}\frac{f_t}{\prod\limits_{i=3}^{t} R_i} + \Phi^{-1}\overline{w}^{-1},$$

where $\hat{\beta} = \sum_{t=2}^{10}\beta^{t-2} + \alpha\tilde{\beta}$,

$$\tilde{\beta} = \sum_{t=2}^{7}\left(\frac{(1-\tau_{l,t})}{\prod\limits_{i=3}^{t} R_i} + \Phi^{-1}\right)\frac{\beta^{t-2}\prod_{i=3}^{t} R_i}{1-\tau_{l,t}},$$
$$\Phi^{-1} = \frac{\phi^{-1}}{\overline{J}}\left(\prod_{t=3}^{7}\frac{1}{R_t}\right)\left[\sum_{z=1}^{J-\overline{J}}\left(\prod_{t=1}^{z}\frac{1}{R_{7+t}}\right)\right].$$

The consumption allocations and market hours in other periods are:

$$c_t^{-1} = \beta^{t-2}\left(\prod_{i=3}^{t} R_i\right)\frac{1+\tau_{c,2}}{1+\tau_{c,t}} c_2^{-1}, t = 3, ..., 10,$$
$$l_{m,t}^{-1} = 1 - l_{h,t}^{-1} - \alpha\beta^{t-2}\left(\prod_{i=3}^{t} R_i\right)\frac{(1+\tau_{c,2})}{(1-\tau_{l,t})\, w_t h_t^{-1}} c_2^{-1}, t = 2, ..., 7.$$

The Social Security benefits during the retirement are

$$b^{-1} = \frac{\phi^{-1}}{\overline{J}} \left(\overline{w}^{-1} + \sum_{t=1}^{7} w_t h_t^{-1} l_{m,t}^{-1} \right).$$

Government transfers to households are

$$d_t^{-1} = \begin{cases} f_t, & t = 1, ..., 7, \\ f_t + b^{-1}, & t = 8, ..., 10. \end{cases}$$

Then, savings are determined by the period budget constraints:

$$\begin{aligned} s_{t+1}^{-1} &= (1 - \tau_{lt})\, w_t h_t^{-1} l_{m,t}^{-1} + R_t s_t^{-1} + d_t^{-1} - (1 + \tau_{ct})\, c_t^{-1}, t = 2, ..., 9, \\ s_{11}^{-1} &= 0. \end{aligned}$$

The conditions determining the life-cycle profile of human capital stock are

$$\begin{aligned} \left(h_{t+1}^{-1}\right)^{1-\psi_1} \left(l_{h,t+1}^{-1}\right)^{1-\psi_2} (1 - \delta_h) = R_{t+1} & \frac{(1 - \tau_{lt})\, w_t}{(1 - \tau_{lt+1})\, w_{t+1}} \left(h_t^{-1}\right)^{1-\psi_1} \left(l_{h,t}^{-1}\right)^{1-\psi_2} \\ & - B\psi_1 l_{h,t+1}^{-1} - B\psi_2 l_{m,t+1}^{-1}, \\ & t = 1, ..., 8. \end{aligned}$$

Generation born at period $t = 0$. An agent born at period $t = 0$ was constrained during first period of his or her life during the initial steady state. Hence, the agent's physical asset holdings are equal to zero at the time of reform announcement, $s_1^0 = 0$. Suppose that the agent is also credit-constrained during the periods $t = 1, 2$. The agent works until period $t = 8$ and lives until period $t = 11$. The agent's allocations during time periods $t = 1, 2$ are:

$$\begin{aligned} c_t^0 &= \frac{1}{(1 + \tau_{c,t})(1 + \alpha)} \left[(1 - \tau_{l,t})\, w_t h_t^0 \left(1 - l_{h,t}^0\right) + f_t \right], \\ l_{m,t}^0 &= 1 - l_{h,t}^0 - \frac{\alpha (1 + \tau_{c,t})}{(1 - \tau_{l,t})\, w_t h_t^0} c_t^0, \\ s_{t+1}^0 &= 0. \end{aligned}$$

Starting with period $t = 3$, the agent can smooth consumption and consumption allocation at this period is found from the intertemporal budget constraint:

$$c_3^0 \left(1 + \tau_{c,3}\right) \hat{\beta} = \Phi^0 \sum_{t=1}^{2} \frac{w_t h_t^0}{1+\alpha} \left(1 - l_{h,t}^0\right) + \sum_{t=3}^{8} \left(\frac{(1 - \tau_{l,t})}{\prod\limits_{i=4}^{t} R_i} + \Phi^0 \right) w_t h_t^0 \left(1 - l_{h,t}^0\right)$$
$$- \Phi^0 \sum_{t=1}^{2} \frac{\alpha}{1+\alpha} \frac{f_t}{(1 - \tau_{l,t})} + \sum_{t=3}^{11} \frac{f_t}{\prod\limits_{i=4}^{t} R_i} + \Phi^0 \overline{w}^0,$$

where $\hat{\beta} = \sum_{t=3}^{11} \beta^{t-3} + \alpha \tilde{\beta}$,

$$\tilde{\beta} = \sum_{t=3}^{8} \left(\frac{(1 - \tau_{l,t})}{\prod\limits_{i=4}^{t} R_i} + \Phi^0 \right) \frac{\beta^{t-3} \prod_{i=4}^{t} R_i}{1 - \tau_{l,t}},$$

$$\Phi^0 = \frac{\phi^0}{\overline{J}} \left(\prod_{t=3}^{8} \frac{1}{R_t} \right) \left[\sum_{z=1}^{J-\overline{J}} \left(\prod_{t=1}^{z} \frac{1}{R_{8+t}} \right) \right].$$

The consumption allocations and market hours in other periods are:

$$c_t^0 = \beta^{t-3} \left(\prod_{i=4}^{t} R_i \right) \frac{1 + \tau_{c,3}}{1 + \tau_{c,t}} c_3^0, t = 4, ..., 11,$$

$$l_{m,t}^0 = 1 - l_{h,t}^0 - \alpha \beta^{t-3} \left(\prod_{i=4}^{t} R_i \right) \frac{(1 + \tau_{c,3})}{(1 - \tau_{l,t}) w_t h_t^0} c_3^0, t = 3, ..., 8.$$

The Social Security benefits during the retirement are

$$b^0 = \frac{\phi^0}{\overline{J}} \left(\overline{w}^0 + \sum_{t=1}^{8} w_t h_t^0 l_{m,t}^0 \right).$$

Government transfers to households are

$$d_t^0 = \begin{cases} f_t, & t = 1, ..., 8, \\ f_t + b^0, & t = 9, ..., 11. \end{cases}$$

Then, savings are determined by the period budget constraints:

$$s_{t+1}^0 = (1-\tau_{lt}) w_t h_t^0 l_{m,t}^0 + R_t s_t^0 + d_t^0 - (1+\tau_{ct}) c_t^0, t = 3, ..., 10,$$
$$s_{12}^0 = 0.$$

The conditions determining the life-cycle profile of human capital stock are

$$\begin{aligned}\left(h_{t+1}^{-1}\right)^{1-\psi_1}\left(l_{h,t+1}^{-1}\right)^{1-\psi_2}(1-\delta_h) &= R_{t+1}\frac{(1-\tau_{lt}) w_t}{(1-\tau_{lt+1}) w_{t+1}}\left(h_t^{-1}\right)^{1-\psi_1}\left(l_{h,t}^{-1}\right)^{1-\psi_2} \\ &- B\psi_1 l_{h,t+1}^{-1} - B\psi_2 l_{m,t+1}^{-1}, \\ &t = 1, ..., 8.\end{aligned}$$

Generations born at period $t = 1, ..., T$. Generations born at period $t = 1$ and later have to save for their retirement through Voluntary RSA. The government does not provide any assistance for the old age and replacement rate for Social Security benefits is equal to zero, $\phi^t = 0$, $t = 1, ..., T$.

For credit-constrained periods, $j = 0, 1, 2$, the expressions for consumption and market hours are:

$$\begin{aligned}c_{t+j}^t &= \frac{1}{(1+\alpha)(1+\tau_{c,t+j})}\left[(1-\tau_{l,t+j}) w_{t+j} h_{t+j}^t \left(1-l_{h,t+j}^t\right) + f_{t+j}\right], \\ l_{m,t+j}^t &= 1 - l_{h,t+j}^t - \frac{\alpha(1+\tau_{c,t+j})}{(1-\tau_{l,t+j}) w_{t+j} h_{t+j}^t} c_{t+j}^t \\ &= \frac{1}{(1+\alpha)}\left(1-l_{h,t+j}^t\right) - \frac{\alpha}{(1+\alpha)}\frac{f_{t+j}}{(1-\tau_{l,t+j}) w_{t+j} h_{t+j}^t}.\end{aligned}$$

Consumption allocation at the period $j = 3$ is

$$\begin{aligned}c_{t+3}^t (1+\tau_{c,t+3})\hat{\beta} &= \frac{\Phi^t}{(1+\alpha)}\sum_{j=0}^{2} w_{t+j} h_{t+j}^t \left(1-l_{h,t+j}^t\right) \\ &+ \sum_{j=3}^{8}\left[\frac{(1-\tau_{l,t+j})}{\prod_{i=4}^{j} R_{t+i}} + \Phi^t\right] w_{t+j} h_{t+j}^t \left(1-l_{h,t+j}^t\right) \\ &- \Phi^t \sum_{j=1}^{3}\frac{\alpha f_{t+j}}{(1+\alpha)(1-\tau_{l,t+j})} + \sum_{j=3}^{11}\frac{f_{t+j}}{\prod_{i=4}^{j} R_{t+i}},\end{aligned}$$

where $\hat{\beta} = \sum_{j=3}^{11} \beta^{j-3} + \alpha\tilde{\beta}$,

$$\tilde{\beta} = \sum_{j=3}^{8} \left[\frac{1-\tau_{l,t+j}}{\prod_{i=4}^{j} R_{t+i}} + \Phi^t \right] \frac{\beta^{j-3} \prod_{i=4}^{j} R_{t+i}}{1-\tau_{l,t+j}},$$

$$\Phi^t = \frac{\phi^t}{\overline{J}} \left(\prod_{j=3}^{8} \frac{1}{R_{t+j}} \right) \left[\sum_{z=1}^{J-\overline{J}} \left(\prod_{j=1}^{z} \frac{1}{R_{t+8+j}} \right) \right];$$

Consumption and market hours for the rest of life are given by

$$c_{t+j}^t = \beta^{j-3} \left(\prod_{i=4}^{j} R_{t+i} \right) c_{t+3}^t, j = 4, ..., 11,$$

$$l_{m,t+j}^t = 1 - l_{h,t+j}^t - \frac{\alpha\beta^{j-3} \left(\prod_{i=4}^{j} R_{t+i} \right) (1+\tau_{c,t+j})}{(1-\tau_{l,t+j})\, w_{t+j} h_{t+j}^t} c_{t+3}^t, j = 3, ..., 8.$$

Social security benefits and transfers to households are

$$b^t = \frac{\phi^t}{\overline{J}} \sum_{j=0}^{\overline{J}-1} w_{t+j} h_{t+j}^t l_{m,t+j}^t,$$

$$d_{t+j}^t = \begin{cases} f_{t+j}, & j = 0, \overline{J}-1 \\ f_{t+j} + b^t & j = \overline{J}, J-1 \end{cases};$$

The households do not save during the periods $j = 0, 1, 2$; the savings of older cohorts, $\{s_{j+1}\}_{j=3}^{J-1}$, is determined by the household's budget constraints:

$$s_{t+j+1}^t = (1-\tau_{l,t+j})\, w_{t+j} h_{t+j}^t l_{m,t+j}^t + R_{t+j} s_{t+j}^t + d_{t+j}^t - (1+\tau_{ct+j})\, c_{t+j}^t,$$
$$j = 3, ..., J-2,$$
$$s_{t+J}^t = 0.$$

The stock of human capital is determined by the intertemporal condition:

$$
\begin{aligned}
B\psi_1 l_{h,t+j+1}^t + B\psi_2 l_{m,t+j+1}^t = {} & R_{t+1} \frac{(1-\tau_{lt})\, w_{t+j}}{(1-\tau_{lt+1})\, w_{t+j+1}} \left(h_{t+j}^t\right)^{1-\psi_1} \left(l_{h,t+j}^t\right)^{1-\psi_2} \\
& - \left(h_{t+j+1}^t\right)^{1-\psi_1} \left(l_{h,t+j+1}^t\right)^{1-\psi_2} (1-\delta_h)\,, \\
j = {} & 0, ..., \overline{J} - 2.
\end{aligned}
$$

Chapter C

Appendix for Chapter 4

This appendix documents the system of equations used to solve for the transition from the Defined Benefits (DB) to the Defined Contribution (DC) system in the chapter on "Macroeconomic and Welfare Implications of Italian Pension Reforms of 1990s". The transition for Italian pension reforms is determined in the number of steps. First, I find the set of equations that characterize the initial stationary equilibrium with Defined Benefits system. Second, the maximization problem of transition generations is being solved. Thirdly, the equilibrium with DC retirement system is calculated.

As a first step, I solve for a stationary equilibrium in the economy with the Defined Benefits retirement system. The numerical algorithm to solve for a stationary equilibrium is described in Appendix B.1. The equations characterizing the agent's maximization problem are derived below in Section C.1 . The solution from the initial stationary equilibrium is used to define initial conditions for the transition path.

Secondly, I solve for the transition from the economy with DB retirement system to the one with DC retirement system. The numerical algorithm to solve for the transition path is described in detail in Appendix B.2 . The set of equations to determine allocations of transition generations is given in Section C.2 .

C.1 Initial Stationary Equilibrium: Defined Benefits System

Initially, I solve the maximization problem of the households. Given equilibrium prices and government policies, consumers maximize a discounted stream of utilities (4.1) subject to their constraints (4.2)-(4.4). Time-line for an agent's life is as follows: (i) during the periods $j = 0, ..., N_c - 1$, the agent is credit-constrained; (ii) working periods are

$j = 0, ..., \overline{J} - 1$; (iii) retirement periods are $j = \overline{J}, ..., J - 1$; (iv) the Social Security benefits are calculated based on the wage income in the periods $j = \overline{J} - N_b, ..., \overline{J} - 1$. Note that the retirement age is exogenous in the model.

Credit-constrained periods. During the credit-constrained periods, $j = 0, ..., N_c - 1$, the agents consume their labor income, do not save and determine the market hours based on intratemporal maximization. Since the borrowing constraints are binding, the level of consumption during these periods can be found from the household's budget constraint (4.2):

$$c_{t+j}^t = \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h_{t+j}^t l_{m,t+j}^t + \frac{f_{t+j}^t}{(1+\tau_{c,t+j})}. \tag{C.1}$$

Substitute (C.1) into the utility function and the household's maximization problem becomes the following with the decisions taken over the allocations $\left(l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j+1}^t\right)_{j=0}^{N_c-1}$:

$$\max \sum_{j=0}^{N_c-1} \beta^j u \left(\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h_{t+j}^t l_{m,t+j}^t + \frac{f_{t+j}^t}{(1+\tau_{c,t+j})}, 1 - l_{m,t+j}^t - l_{h,t+j}^t \right)$$

subject to

$$h_{t+j+1}^t = (1-\delta_h) h_{t+j}^t + Q(h_{t+j}^t, l_{h,t+j}^t), j = 0, ..., N_c - 1.$$

The Lagrangian function for these constrained maximization problem is

$$\begin{aligned}
&\pounds \left(\left\{ l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j+1}^t, \mu_{t+j} \right\}_{j=0}^{N_c-1} \right) \\
&= \sum_{j=0}^{N_c-1} \beta^j u \left(\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h_{t+j}^t l_{m,t+j}^t + \frac{f_{t+j}^t}{(1+\tau_{c,t+j})}, 1 - l_{m,t+j}^t - l_{h,t+j}^t \right) \\
&- \sum_{j=0}^{N_c-1} \mu_{t+j} \left(h_{t+j+1}^t - (1-\delta_h) h_{t+j}^t - Q(h_{t+j}^t, l_{h,t+j}^t) \right).
\end{aligned}$$

To simplify notation, I introduce the following notation for partial derivatives:

$$u_{1t+j} = \frac{\partial u\left(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}\right)}{\partial c^t_{t+j}}, u_{2t+j} = \frac{\partial u\left(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}\right)}{\partial\left(1 - l^t_{m,t+j} - l^t_{h,t+j}\right)},$$
$$Q_{1t+j} = \frac{\partial Q(h^t_{t+j}, l^t_{h,t+j})}{\partial h^t_{t+j}}, Q_{2t+j} = \frac{\partial Q(h^t_{t+j}, l^t_{h,t+j})}{\partial l^t_{h,t+j}}.$$

The tax wedge on labor income is given by

$$(1 - \tau_{t+j}) = \frac{(1 - \tau_{l,t+j})(1 - \tau_{p,t+j})}{(1 + \tau_{c,t+j})},$$
$$\tau_{t+j} = \frac{\tau_{c,t+j} + \tau_{l,t+j} + \tau_{p,t+j} - \tau_{l,t+j}\tau_{p,t+j}}{(1 + \tau_{c,t+j})}.$$

First-order condition with respect to market hours is

$$u_{1t+j}(1 - \tau_{t+j})\, w_{t+j} h^t_{t+j} = u_{2t+j}. \tag{C.2}$$

First-order condition with respect to investment hours is

$$\beta^j u_{2t+j} = \mu_{t+j} Q_{2t+j}. \tag{C.3}$$

First-order condition with respect to the stock of human capital is

$$\beta^{j+1} u_{1t+j+1}(1 - \tau_{t+j+1})\, w_{t+j+1} l^t_{m,t+j+1} = \mu_{t+j} - \mu_{t+j+1}(1 - \delta_h + Q_{1t+j+1}). \tag{C.4}$$

First-order condition with respect to the lagrangian multiplier on the law of motion for human capital is

$$h^t_{t+j+1} = (1 - \delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}).$$

Express μ_{t+j} from (C.2) and (C.3) as function of marginal utility of consumption and substitute into (C.4). Then, the set of equations describing the agents's decisions during the credit constrained periods, $j = 0, ..., N_c - 1$, is

$$\begin{aligned}
c^t_{t+j} &= \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h^t_{t+j} l^t_{m,t+j} + \frac{f^t_{t+j}}{(1+\tau_{c,t+j})},\\
u_{2t+j} &= u_{1t+j}(1-\tau_{t+j}) w_{t+j} h^t_{t+j},\\
l^t_{m,t+j+1} &= \frac{u_{1t+j}(1-\tau_{t+j}) w_{t+j}}{\beta u_{1t+j+1}(1-\tau_{t+j+1}) w_{t+j+1}} \frac{h^t_{t+j}}{Q_{2t+j}} - \frac{h^t_{t+j+1}}{Q_{2t+j+1}}(1-\delta_h + Q_{1t+j+1}),\\
h^t_{t+j+1} &= (1-\delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}).
\end{aligned}$$

Functional forms. I consider logarithmic utility function and Cobb-Douglas production function for human capital technology.

$$\begin{aligned}
u\left(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}\right) &= \log c^t_{t+j} + \alpha \log\left(1 - l^t_{m,t+j} - l^t_{h,t+j}\right),\\
Q\left(h^t_{t+j}, l^t_{h,t+j}\right) &= B\left(h^t_{t+j}\right)^{\psi_1}\left(l^t_{h,t+j}\right)^{\psi_2}.
\end{aligned}$$

Then partial derivatives become:

$$\begin{aligned}
u_{1t+j} &= \frac{1}{c^t_{t+j}},\\
u_{2t+j} &= \frac{\alpha}{1 - l^t_{m,t+j} - l^t_{h,t+j}},\\
Q_{1t+j} &= B\psi_1\left(h^t_{t+j}\right)^{\psi_1 - 1}\left(l^t_{h,t+j}\right)^{\psi_2},\\
Q_{2t+j} &= B\psi_2\left(h^t_{t+j}\right)^{\psi_1}\left(l^t_{h,t+j}\right)^{\psi_2 - 1}.
\end{aligned}$$

After substituting the partial derivatives and simplifying, the set of equations characterizing the household's problem becomes:

$$\begin{aligned}
c^t_{t+j} &= \frac{1}{1+\alpha}\left[(1-\tau_{,t+j}) w_{t+j} h^t_{t+j}\left(1 - l^t_{h,t+j}\right) + \frac{f_{t+j}}{(1+\tau_{c,t+j})}\right],\\
l^t_{m,t+j} &= \frac{1}{(1+\alpha)}\left(1 - l^t_{h,t+j}\right) - \frac{\alpha}{(1+\alpha)} \frac{f_{t+j}}{(1-\tau_{l,t+j})(1-\tau_{p,t+j}) w_{t+j} h^t_{t+j}},\\
B\psi_2 l^t_{m,t+j+1} &= \frac{c^t_{t+j+1}}{\beta c^t_{t+j}} \frac{(1-\tau_{t+j}) w_{t+j}}{(1-\tau_{t+j+1}) w_{t+j+1}}\left(h^t_{t+j}\right)^{1-\psi_1}\left(l^t_{h,t+j}\right)^{1-\psi_2}\\
&\quad - \left(h^t_{t+j+1}\right)^{1-\psi_1}\left(l^t_{h,t+j+1}\right)^{1-\psi_2}\left(1 - \delta_h + B\psi_1\left(h^t_{t+j+1}\right)^{\psi_1 - 1}\left(l^t_{h,t+j+1}\right)^{\psi_2}\right),\\
h^t_{t+j+1} &= (1-\delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}).
\end{aligned}$$

Periods with intertemporal substitution. During these periods, the agents can smooth consumption over life-cycle and the period budget constraints (4.2) can be combined into the intertemporal one. Hence, the maximization problem of an agent born at period t becomes:

$$\max_{\left(c^t_{t+j}, l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j+1}\right)_{j=N_c}^{J-1}} \sum_{j=N_c}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}) \tag{C.5}$$

subject to

$$\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} c^t_{t+j} \leq \sum_{j=N_c}^{\overline{J}-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j} l^t_{mt+j} + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}}$$
$$+ \sum_{j=\overline{J}}^{J-1} \frac{b^t_1 \prod_{z=\overline{J}+1}^{j} \xi_{t+z}}{\prod_{i=N_c+1}^{j} R_{t+i}},$$
$$h^t_{t+j+1} = (1-\delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}), j = N_c, ..., J-1,$$

where ξ_{t+j} is indexation rate for retirement benefits. The gross after-tax return on physical capital is denoted by $R_{t+j} = 1 + (1 - \tau_{k,t+j}) r_{t+j}$.

The Lagrangian function for the agent's maximization problem is

$$£\left(\left\{c^t_{t+j}\right\}_{j=N_c}^{J-1}, \left\{l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j+1}, \mu_{t+j}\right\}_{j=N_c}^{\overline{J}-1}, \lambda\right)$$
$$= \sum_{j=N_c}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j})$$
$$- \lambda \left[\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} c^t_{t+j} - \sum_{j=N_c}^{\overline{J}-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j} l^t_{mt+j}\right]$$
$$+ \lambda \left[\sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}} + \sum_{j=\overline{J}}^{J-1} \frac{b^t_1 \prod_{z=\overline{J}+1}^{j} \xi_{t+z}}{\prod_{i=N_c+1}^{j} R_{t+i}}\right]$$

$$- \sum_{j=N_c}^{\overline{J}-1} \mu_{t+j} \left(h^t_{t+j+1} - (1-\delta_h) h^t_{t+j} - Q(h^t_{t+j}, l^t_{h,t+j}) \right).$$

First-order conditions with respect to the arguments of Lagrangian function are

$$\beta^j u_{1t+j} = \lambda \frac{(1+\tau_{c,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}}, \tag{C.6}$$

$$\beta^j u_{2t+j} = \lambda \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j}, \tag{C.7}$$

$$\beta^j u_{2t+j} = \mu_{t+j} Q_{2t+j}, \tag{C.8}$$

$$\begin{aligned} \mu_{t+j} &= \mu_{t+j+1} \left[1 - \delta_h + Q_{1t+j+1}\right] \\ &+ \lambda \frac{(1-\tau_{l,t+j+1})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j+1} R_{t+i}} w_{t+j+1} l^t_{mt+j+1}, \end{aligned} \tag{C.9}$$

$$\begin{aligned} \sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} c^t_{t+j} &\le \sum_{j=N_c}^{\overline{J}-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j} l^t_{mt+j} \\ &+ \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} + \sum_{j=\overline{J}}^{J-1} \frac{b^t_1 \prod\limits_{z=\overline{J}+1}^{j} \xi_{t+z}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}}, \end{aligned} \tag{C.10}$$

$$h^t_{t+j+1} = (1-\delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}). \tag{C.11}$$

To find the path for consumption over the agent's life-cycle, express equations (C.6) for time periods $t+j+1$ and $t+j$. Then divide one equation by another to get an intertemporal condition:

$$\frac{\beta u_{1t+j+1}}{u_{1t+j}} = \frac{(1+\tau_{c,t+j+1})}{(1+\tau_{c,t+j})} \frac{1}{R_{t+j+1}}. \tag{C.12}$$

From equations (C.6) and (C.7), I can find an intratemporal condition for market hours:

$$\frac{u_{2t+j}}{u_{1t+j}} = (1-\tau_{t+j})\, w_{t+j} h^t_{t+j}. \tag{C.13}$$

From equations (C.7) and (C.8), I can find the following relationship between the Lagrange multipliers on the intertemporal budget constraint and the law of motion for human capital:

$$\mu_{t+j} = \lambda \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} \frac{w_{t+j} h_{t+j}^t}{Q_{2t+j}}. \tag{C.14}$$

I plug (C.14) into (C.9) to receive an intertemporal relationship determining the time investment into human capital:

$$l_{m,t+j+1}^t = \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1-\tau_{l,t+j+1})(1-\tau_{p,t+j})} R_{t+j+1} \frac{w_{t+j}}{w_{t+j+1}} \frac{h_{t+j}^t}{Q_{2t+j}} - \frac{h_{t+j+1}^t}{Q_{2t+j+1}} \left[1-\delta_h + Q_{1t+j+1}\right]. \tag{C.15}$$

For periods of life $j = N_c, ..., J-1$, the consumption is found by solving (C.12) and the stock of savings is determined by the period budget constraints (4.2). During the working periods $j = N_c, ..., \overline{J}-1$, the hours for market production and human capital investment are determined by (C.13) and (C.15), respectively. The stock of human capital is obtained from (C.11). Retirement benefits are calculated using formula (4.8).

I express the first-order conditions for consumption and market hours in terms of the consumption level at the period $j = N_c$. Then, I substitute the latter equations into the intertemporal budget constraint to find $c_{t+N_c}^t$. From equation (C.12) the consumption sequence is given by

$$c_{t+j}^t = \beta^{j-N_c} \left(\prod_{i=N_c+1}^{j} R_{t+i} \right) \frac{1+\tau_{c,t+N_c}}{1+\tau_{c,t+j}} c_{t+N_c}^t, j = N_c+1, ..., J-1. \tag{C.16}$$

From equations (C.13) and (C.16), the market hours are determined by

$$l_{mt+j}^t = 1 - l_{ht+j}^t - \frac{\alpha \beta^{j-N_c} (1+\tau_{c,t+N_c}) c_{t+N_c}^t}{(1-\tau_{l,t+j})(1-\tau_{p,t+j}) w_{t+j} h_{t+j}^t} \left(\prod_{i=N_c+1}^{j} R_{t+i} \right), j = N_c, ..., \overline{J}-1. \tag{C.17}$$

Due to the expression for retirement benefits (4.8), the intertemporal budget constraint

becomes:

$$\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} c_{t+j}^t \leq \sum_{j=N_c}^{\overline{J}-N_b-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h_{t+j}^t l_{mt+j}^t \tag{C.18}$$

$$+ \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \left[\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} + \Phi^{1t} \right] w_{t+j} h_{t+j}^t l_{mt+j}^t + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}},$$

where $\Phi^{1t} = \dfrac{\overline{J}\phi^{1t}}{N_b} \displaystyle\sum_{j=\overline{J}}^{J-1} \frac{\prod\limits_{z=\overline{J}+1}^{j} \xi_{t+z}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}}.$

Plug in the first-order conditions for consumption and market hours, (C.16) and (C.17), into the budget constraint (C.18):

$$(1+\tau_{c,t+N_c}) c_{t+N_c}^t \sum_{j=N_c}^{J-1} \beta^{j-N_c} \leq \sum_{j=N_c}^{\overline{J}-N_b-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right)$$

$$- \sum_{j=N_c}^{\overline{J}-N_b-1} \alpha\beta^{j-N_c} (1+\tau_{c,t+N_c}) c_{t+N_c}^t$$

$$+ \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \left[\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} + \Phi^{1t} \right] w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right)$$

$$- \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \Phi_{t+j}^{3t} \alpha\beta^{j-N_c} (1+\tau_{c,t+N_c}) c_{t+N_c}^t + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}},$$

To simplify notation, define the following variables:

$$\Phi_{t+j}^{2t} = \frac{\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)}{\prod_{i=N_c+1}^{j} R_{t+i}} + \Phi^{1t}, j = \overline{J} - N_b, ..., \overline{J} - 1,$$

$$\Phi_{t+j}^{3t} = 1 + \frac{\Phi^{1t}}{\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)} \left(\prod_{i=N_c+1}^{j} R_{t+i}\right), j = \overline{J} - N_b, ..., \overline{J} - 1.$$

Then the intertemporal budget constraint becomes:

$$\begin{aligned} \left(1+\tau_{c,t+N_c}\right) c_{t+N_c}^t \sum_{j=N_c}^{J-1} \beta^{j-N_c} \leq & \sum_{j=N_c}^{\overline{J}-N_b-1} \frac{\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)}{\prod_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right) \\ + \sum_{j=\overline{J}-N_b}^{\overline{J}-1} & \Phi_{t+j}^{2t} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right) + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}} \\ - \sum_{j=N_c}^{\overline{J}-N_b-1} & \alpha\beta^{j-N_c} \left(1+\tau_{c,t+N_c}\right) c_{t+N_c}^t - \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \Phi_{t+j}^{3t} \alpha\beta^{j-N_c} \left(1+\tau_{c,t+N_c}\right) c_{t+N_c}^t. \end{aligned} \tag{C.19}$$

Using the budget constraint (C.19), the consumption allocation during the first non credit-constraint period, $j = N_c$, can be found from the following expression:

$$\begin{aligned} \left(1+\tau_{c,t+N_c}\right) c_{t+N_c}^t \tilde{\beta} = & \sum_{j=N_c}^{\overline{J}-N_b-1} \frac{\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)}{\prod_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right) \\ & + \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \Phi_{t+j}^{2t} w_{t+j} h_{t+j}^t \left(1-l_{ht+j}^t\right) + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}}, \end{aligned} \tag{C.20}$$

where $\tilde{\beta} = \sum_{j=N_c}^{J-1} \beta^{j-N_c} + \sum_{j=N_c}^{\overline{J}-N_b-1} \alpha\beta^{j-N_c} + \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \Phi_{t+j}^{3t} \alpha\beta^{j-N_c}$.

Other household allocations during the non credit-constrained periods are

$$c_{t+j}^{t} = \beta^{j-N_c}\left(\prod_{i=N_c+1}^{j} R_{t+i}\right)\frac{1+\tau_{c,t+N_c}}{1+\tau_{c,t+j}}c_{t+N_c}^{t}, j = N_c+1,...,J-1,$$

$$l_{mt+j}^{t} = 1 - l_{ht+j}^{t} - \frac{\alpha\beta^{j-N_c}\left(1+\tau_{c,t+N_c}\right)c_{t+N_c}^{t}}{\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)w_{t+j}h_{t+j}^{t}}\left(\prod_{i=N_c+1}^{j} R_{t+i}\right),$$

$$j = N_c,...,\overline{J}-1,$$

$$l_{mt+j}^{t} = 0, j = \overline{J},...,J-1,$$

$$l_{mt+j+1}^{t} = \frac{R_{t+j+1}\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)}{\left(1-\tau_{l,t+j+1}\right)\left(1-\tau_{p,t+j+1}\right)}\frac{w_{t+j}}{w_{t+j+1}}\frac{h_{t+j}^{t}}{Q_{2t+j}} - \frac{h_{t+j+1}^{t}}{Q_{2t+j+1}}\left[1-\delta_h+Q_{1t+j+1}\right], j = N_c,...,\overline{J}-2.$$

C.2 Allocations of transition generations

At the time period $t = 1$, the reform is announced. First, I consider two changes to the pension system. First, the pension benefits are indexed to inflation starting with $t = 1$. Second, I model the transition from Defined Benefits (DB) to Defined Contribuions (DC) system.

The transition generations, i.e., ones that are alive at the time of reform announcement, can be divided into three groups. The first group consists of generations that completed their working career under the old system. These generations get the Social Security benefits based on the Defined Benefits system but their benefits are indexed to inflation now. These are generations that were born at $t = (i - J), i = 2,...,5$.

The generations that had at least 20 years of seniority under the old system constitute the second group.[1,2] Retirement benefits of these generations are computed based on the so-called Modified Defined Benefits formula. This formula is the same as the one for DB system but the pensionable earnings are computed based on the wage income from the period $t = 1$ onwards. These are generations that were born at $t = (i - J), i = 6,...,9$.

[1] The seniority is determined based on the number of years worked. Since the agents don't make decision of labor market entrance/exit, the seniority is determined by the number of working years.

[2] The provisions of italian reform put the threshold at 18 years of seniority on December 31, 1992. Hence, the cohort born in 1955 will have 18 years of seniority at the end of 1992. Based on my assumption, the cohort of 1955 have entered the model at 1975 and have continously worked until 1992. This cohort will have 20 years of seniority at the end of 1994. Hence, I difine the period from 1990 to 1994 as the period 0 of my model with the initial stationary equilibrium. According to the provisions of the Italian pension reform, the cohort of 1955 is the last one to whom the Modified Defined Benefits system applies.

The third group consists of generations that had less than 20 years of seniority at the time $t = 1$. The pension benefits of these generations are computed based on pro-rata system. The first part of benefits is based on wage income prior to the period $t = 1$ and is computed based on the MDB formula. The second part is computed based on contributions from $t = 1$ and later using the DC formula. These are generations that were born at $t = (i - J), i = 10, 11, 12$.

Generations born at periods $t = (i - J), i = 2, ..., 5$**.** From a time period of reform announcement, a generation t will live for the number of periods $\hat{J} = J - (1 - t) = i - 1$. Since the retirement is mandatory in the model, the market and investment hours are

$$l_{m,t}^{(i-J)} = 0, t = 1, ..., \hat{J},$$
$$l_{h,t}^{(i-J)} = 0, t = 1, ..., \hat{J}.$$

The retirement benefits are calculated based on the past wage history and on the indexation rule during the Defined Benefits system:

$$b_0^{1,(i-J)} = b^{1,(i-J)} \prod_{z=\overline{J}(i-J)}^{0} \xi_z$$

Given the stock of physical capital, $s_1^{(i-J)}$, the amount of retirement benefits, $b_1^{1,(i-J)}$, and the government policies, the retired households solve the following maximization problem:

$$\max_{\left(c_t^{(i-J)}, s_{t+1}^{(i-J)}\right)_{t=1}^{\hat{J}}} \sum_{t=1}^{\hat{J}} \beta^{t-1} u\left(c_t^{(i-J)}, 1\right)$$

subject to the budget constraints during the retirement periods $t = 1, ..., \hat{J}$:

$$(1 + \tau_{c,t})\, c_t^{(i-J)} + s_{t+1}^{(i-J)} \leq (1 + (1 - \tau_{k,t})\, r_t) s_t^{(i-J)} + f_t^{(i-J)} + b_{t-1}^{1,(i-J)} \xi_t.$$

The intertemporal budget constraint is

$$\sum_{t=1}^{\hat{J}} \frac{(1 + \tau_{c,t})\, c_t^{(i-J)}}{\prod_{z=2}^{t} R_z} \leq R_1 s_1^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod_{z=2}^{t} R_z} + \sum_{t=1}^{\hat{J}} \frac{b_0^{(i-J)} \prod_{z=1}^{t} \xi_z}{\prod_{z=2}^{t} R_z}. \tag{C.21}$$

The Lagrangian function for the household's problem is

$$\begin{aligned}
&\pounds\left(\left\{c_t^{(i-J)}\right\}_{t=1}^{\hat{J}},\lambda\right)\\
&=\sum_{t=1}^{\hat{J}}\beta^{t-1}u\left(c_t^{(i-J)},1\right)\\
&-\lambda\left[\sum_{t=1}^{\hat{J}}\frac{\left(1+\tau_{c,t}\right)c_t^{(i-J)}}{\prod\limits_{z=2}^{t}R_z}-R_1s_1^{(i-J)}-\sum_{t=1}^{\hat{J}}\frac{f_t}{\prod\limits_{z=2}^{t}R_z}-\sum_{t=1}^{\hat{J}}\frac{b_0^{(i-J)}\prod\limits_{z=1}^{t}\xi_z}{\prod\limits_{z=2}^{t}R_z}\right].
\end{aligned}$$

The first-order condition with respect to consumption is

$$\beta\frac{u_{ct+1}}{u_{ct}}=\frac{\left(1+\tau_{c,t+1}\right)}{\left(1+\tau_{c,t}\right)}\frac{1}{R_{t+1}}.$$

The first-order condition with respect to the lagrange multiplier results in the budget constraint (C.21).

Functional forms. With logarithmic utility function, the consumption allocation for periods $t=1,...,\hat{J}$ become

$$c_{t+1}^{(i-J)}=\beta\frac{\left(1+\tau_{c,t}\right)}{\left(1+\tau_{c,t+1}\right)}R_{t+1}c_t^{(i-J)}.$$

In terms of the consumption during $t=1$, the sequence of consumption for periods $t=2,...,\hat{J}$ is

$$c_t^{(i-J)}=\beta^{t-1}\frac{\left(1+\tau_{c,1}\right)}{\left(1+\tau_{c,t}\right)}\left(\prod_{z=2}^{t}R_z\right)c_1^{(i-J)}. \tag{C.22}$$

For periods $t=2,...,\hat{J}$, substitute expression (C.22) into (C.21) to find the consumption level at the time of reform announcement:

$$\left(1+\tau_{c,1}\right)c_1^{(i-J)}\sum_{t=1}^{\hat{J}}\beta^{t-1}=R_1s_1^{(i-J)}+\sum_{t=1}^{\hat{J}}\frac{f_t}{\prod\limits_{z=2}^{t}R_z}+\sum_{t=1}^{\hat{J}}\frac{b_0^{(i-J)}\prod\limits_{z=1}^{t}\xi_z}{\prod\limits_{z=2}^{t}R_z}. \tag{C.23}$$

Generations born at periods $t = (i - J), i = 6, ..., 9$. From a time period of reform announcement, a generation t will work for the number of periods $\tilde{J} = \overline{J} - (1 - t)$ and live for the number of periods $\hat{J} = J - (1 - t) = i - 1$.

The Social Security benefits are calculated according to the formula

$$b^{1,(i-J)} = \frac{\overline{J}\phi^{1,(i-J)}}{\tilde{J}} \sum_{t=1}^{\tilde{J}} w_t h_t^{(i-J)} l_{m,t}^{(i-J)}. \tag{C.24}$$

Given the initial stocks of physical and human capitals, $s_1^{(i-J)}$ and $h_1^{(i-J)}$, the intertemporal budget constraint is

$$\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod\limits_{z=2}^{t} R_z} c_t^{(i-J)} \leq R_1 s_1^{(i-J)} + \sum_{t=1}^{\tilde{J}} \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)} l_{m,t}^{(i-J)} \tag{C.25}$$
$$+ \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z} + b^{1,(i-J)} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod\limits_{z=\tilde{J}+1}^{t} \xi_z}{\prod\limits_{z=2}^{t} R_z}.$$

The Lagrangian function for the agent's maximization problem is

$$\begin{aligned}
&£\left(\left\{c_t^{(i-J)}\right\}_{t=1}^{\hat{J}}, \left\{l_{m,t}^{(i-J)}, l_{h,t}^{(i-J)}, h_{t+1}^{(i-J)}, \mu_t\right\}_{t=1}^{\tilde{J}}, \lambda\right) \\
&= \sum_{t=1}^{\hat{J}} \beta^{t-1} u\left(c_t^{(i-J)}, 1 - l_{m,t}^{(i-J)} - l_{h,t}^{(i-J)}\right) \\
&- \lambda \left[\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod\limits_{z=2}^{t} R_z} c_t^{(i-J)} - R_1 s_1^{(i-J)} - b^{1,(i-J)} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod\limits_{z=\tilde{J}+1}^{t} \xi_z}{\prod\limits_{z=2}^{t} R_z}\right] \\
&+ \lambda \left[\sum_{t=1}^{\tilde{J}} \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)} l_{m,t}^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z}\right] \\
&- \sum_{t=1}^{\hat{J}} \mu_t \left(h_{t+1}^{(i-J)} - (1-\delta_h) h_t^{(i-J)} - Q\left(h_t^{(i-J)}, l_{h,t}^{(i-J)}\right)\right).
\end{aligned}$$

First-order conditions with respect to the arguments of Lagrangian function are

$$\beta^{t-1}u_{1t} = \lambda \frac{(1+\tau_{c,t})}{\prod\limits_{z=2}^{t} R_z}, \tag{C.26}$$

$$\beta^{t-1}u_{2t} = \lambda \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)}, \tag{C.27}$$

$$\beta^{t-1}u_{2t} = \mu_t Q_{2t}, \tag{C.28}$$

$$\mu_t = \mu_{t+1}\left[1-\delta_h + Q_{1t+1}\right] + \lambda \frac{(1-\tau_{l,t+1})(1-\tau_{p,t+1})}{\prod\limits_{z=2}^{t+1} R_z} w_{t+1} h_{t+1}^{(i-J)}, \tag{C.29}$$

$$\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod\limits_{z=2}^{t} R_z} c_t^{(i-J)} \leq R_1 s_1^{(i-J)} + \sum_{t=1}^{\tilde{J}} \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)} l_{m,t}^{(i-J)} \tag{C.30}$$

$$+ \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z} + b^{1,(i-J)} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod\limits_{z=\tilde{J}+1}^{t} \xi_z}{\prod\limits_{z=2}^{t} R_z},$$

$$h_{t+1}^{(i-J)} = (1-\delta_h) h_t^{(i-J)} + Q\left(h_t^{(i-J)}, l_{h,t}^{-(J-i)}\right). \tag{C.31}$$

Based on (C.26), the sequence of consumption is determined by

$$\beta \frac{u_{1t+1}}{u_{1t}} = \frac{(1+\tau_{c,t+1})}{(1+\tau_{c,t})} \frac{1}{R_{t+1}}.$$

Using (C.26) and (C.27), the intratemporal condition for market hours is

$$\frac{u_{2t}}{u_{1t}} = (1-\tau_t)\, w_t h_t^{(i-J)}. \tag{C.32}$$

Since the retirement is mandatory, the market hours are equal to zero during these periods:

$$l_{m,t}^{(i-J)} = 0, t = \tilde{J}+1, ..., \hat{J}.$$

Also the agents do not invest into human capital during the last working period:

$$l_{h,t}^{(i-J)} = 0, t = \tilde{J}, ..., \hat{J}.$$

Based on conditions (C.27), (C.28) and (C.29), the intertemporal condition determining the investment hours for periods $t = 1, ..., \tilde{J} - 1$ is

$$l_{m,t+1}^{(i-J)} = \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{(1-\tau_{l,t+1})(1-\tau_{p,t+1})} R_{t+1} \frac{w_t}{w_{t+1}} \frac{h_t^{(i-J)}}{Q_{2t}} - \frac{h_{t+1}^{(i-J)}}{Q_{2t+1}} \left[1 - \delta_h + Q_{1t+1}\right].$$

For logarithmic utility function and Cobb-Douglas function for human capital accumulation, the system of equations describing the household's allocations is the following. In terms of the consumption during $t = 1$, the sequence of consumption for periods $t = 2, ..., \hat{J}$ is

$$c_t^{(i-J)} = \beta^{t-1} \frac{(1+\tau_{c,1})}{(1+\tau_{c,t})} \left(\prod_{z=2}^{t} R_z \right) c_1^{(i-J)}. \tag{C.33}$$

Based on equations (C.32) and (C.33), the market hours for $t = 2, ..., \tilde{J}$ are determined by

$$l_{m,t}^{(i-J)} = 1 - l_{h,t}^{(i-J)} - \frac{\alpha\beta^{t-1}(1+\tau_{c,1}) c_1^{(i-J)}}{(1-\tau_{l,t})(1-\tau_{p,t}) w_t h_t^{(i-J)}} \left(\prod_{z=2}^{t} R_z \right).$$

Retirement benefits type 1. First I consider, the case with retirement benefits computed using the Modifed Defined Benefits formula (C.24). The equations in this part are used to calculate transition for 'Reform 2'. Plug (C.24) into the intertemporal budget constraint (C.25):

$$\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod_{z=2}^{t} R_z} c_t^{(i-J)} = R_1 s_1^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod_{z=2}^{t} R_z}$$
$$+ \sum_{t=1}^{\tilde{J}} \left[\frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod_{z=2}^{t} R_z} + \Phi^{(i-J)} \right] w_t h_t^{(i-J)} l_{m,t}^{(i-J)},$$

$$\text{where } \Phi^{(i-J)} = \frac{\overline{J}\phi^{1,(i-J)}}{\tilde{J}} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod\limits_{z=\tilde{J}+1}^{t} \xi_z}{\prod\limits_{z=2}^{t} R_z}.$$

Plug expressions for consumption and market hours into the above budget constraint to receive equation for $c_1^{(i-J)}$:

$$\left(1+\tau_{c,1}\right) c_1^{(i-J)} \tilde{\beta} = R_1 s_1^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z}$$
$$+ \sum_{t=1}^{\tilde{J}} \left[\frac{\left(1-\tau_{l,t}\right)\left(1-\tau_{p,t}\right)}{\prod\limits_{z=2}^{t} R_z} + \Phi^{(i-J)} \right] w_t h_t^{(i-J)} \left(1 - l_{h,t}^{(i-J)}\right),$$

$$\text{where } \tilde{\beta} = \sum_{t=1}^{\hat{J}} \beta^{t-1} + \sum_{t=1}^{\tilde{J}} \alpha\beta^{t-1} \left[1 + \frac{\Phi^{(i-J)} \left(\prod\limits_{z=2}^{t} R_z \right)}{\left(1-\tau_{l,t}\right)\left(1-\tau_{p,t}\right)} \right].$$

Retirement benefits type 2. To calculate transition path, I need to start with the formula for the retirement benefits from the initial stationary eqilibrium given by (4.8). The equations in this part are used to calculate transition for 'Reform 1'. The relevant stages in life for the maximization problem are the following. The rest of life goes over the periods $t = 1, ..., \hat{J}$, the working periods are $t = 1, ..., \tilde{J}$, and the retirement benefits are calculated over the periods $t = \tilde{J} - N_b + 1, ..., \tilde{J}$. After using the formula for retirement benefits (4.8), the intertemporal budget constraint becomes

$$\sum_{t=1}^{\hat{J}} \frac{\left(1+\tau_{c,t}\right)}{\prod\limits_{z=2}^{t} R_z} c_t^{(i-J)} \leq R_1 s_1^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z}$$
$$+ \sum_{t=1}^{\tilde{J}} \frac{\left(1-\tau_{l,t}\right)\left(1-\tau_{p,t}\right)}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)} l_{m,t}^{(i-J)} + \sum_{t=\tilde{J}-N_b+1}^{\tilde{J}} \Phi^{(i-J)} w_t h_t^{(i-J)} l_{m,t}^{(i-J)},$$

$$\text{where } \Phi^{(i-J)} = \frac{\overline{J}\phi^{1,(i-J)}}{N_b} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod_{z=\tilde{J}+2}^{t} \xi_z}{\prod_{z=2}^{t} R_z}.$$

Plug expressions for consumption and market hours into the above budget constraint to receive equation for $c_1^{(i-J)}$:

$$\begin{aligned}(1+\tau_{c,1})\, c_1^{(i-J)} \tilde{\beta} = R_1 s_1^{(i-J)} &+ \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod_{z=2}^{t} R_z} \\ &+ \sum_{t=1}^{\tilde{J}} \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod_{z=2}^{t} R_z} w_t h_t^{(i-J)} \left(1 - l_{h,t}^{(i-J)}\right) \\ &+ \sum_{t=\tilde{J}-N_b+1}^{\tilde{J}} \Phi^{(i-J)} w_t h_t^{(i-J)} \left(1 - l_{h,t}^{(i-J)}\right),\end{aligned}$$

$$\begin{aligned}\text{where } \tilde{\beta} &= \sum_{t=1}^{\hat{J}} \beta^{t-1} + \sum_{t=1}^{\tilde{J}} \alpha\beta^{t-1} \\ &+ \sum_{t=\tilde{J}-N_b+1}^{\tilde{J}} \alpha\beta^{t-1} \frac{\Phi^{(i-J)} \left(\prod_{z=2}^{t} R_z\right)}{(1-\tau_{l,t})(1-\tau_{p,t})}.\end{aligned}$$

Generations born at periods $t = (i-J), i = 10, 11, 12$. From a time period of reform announcement, a generation t will work for the number of periods $\tilde{J} = \overline{J} - (1-t)$ and live for the number of periods $\hat{J} = J - (1-t) = i - 1$. Since these generations are subject to the pro-rata system, the first part of retirement benefits is calculated according to the MDB formula

$$b^{1,(i-J)} = \frac{\overline{J}\phi^{1,(i-J)}}{\overline{J} - \tilde{J}} \sum_{t=(i-J)}^{0} w_t h_t^{(i-J)} l_{mt}^{(i-J)} \tag{C.34}$$

and the second part is given by

$$b^{2,(i-J)} = \phi^{2,(i-J)} \sum_{t=1}^{\tilde{J}} \tau_{p,t} w_t h_t^{(i-J)} l_{mt}^{(i-J)} \prod_{z=t+1}^{\tilde{J}} (1 + \lambda_{Y,z}) . \tag{C.35}$$

Given the initial stocks of physical and human capitals, $s_1^{(i-J)}$ and $h_1^{(i-J)}$, the intertemporal budget constraint is

$$\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod\limits_{z=2}^{t} R_z} c_t^{(i-J)} \leq R_1 s_1^{(i-J)} + \sum_{t=1}^{\tilde{J}} \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)} l_{m,t}^{(i-J)} \tag{C.36}$$

$$+ \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z} + \left(b^{1,(i-J)} + b^{2,(i-J)}\right) \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod\limits_{z=\tilde{J}+1}^{t} \xi_z}{\prod\limits_{z=2}^{t} R_z} .$$

The Lagrange function is

$$\begin{aligned}
& \pounds \left(\left\{ c_t^{(i-J)} \right\}_{t=1}^{\hat{J}}, \left\{ l_{m,t}^{(i-J)}, l_{h,t}^{(i-J)}, h_{t+1}^{(i-J)}, \mu_t \right\}_{t=1}^{\tilde{J}}, \lambda \right) \\
&= \sum_{t=1}^{\hat{J}} \beta^{t-1} u \left(c_t^{(i-J)}, 1 - l_{m,t}^{(i-J)} - l_{h,t}^{(i-J)} \right) \\
&- \lambda \left[\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod\limits_{z=2}^{t} R_z} c_t^{(i-J)} - R_1 s_1^{(i-J)} - \left(b^{1,(i-J)} + b^{2,(i-J)}\right) \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod\limits_{z=\tilde{J}+1}^{t} \xi_z}{\prod\limits_{z=2}^{t} R_z} \right] \\
&+ \lambda \left[\sum_{t=1}^{\tilde{J}} \frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod\limits_{z=2}^{t} R_z} w_t h_t^{(i-J)} l_{m,t}^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod\limits_{z=2}^{t} R_z} \right] \\
&- \sum_{t=1}^{\hat{J}} \mu_t \left(h_{t+1}^{(i-J)} - (1-\delta_h) h_t^{(i-J)} - Q\left(h_t^{(i-J)}, l_{h,t}^{(i-J)} \right) \right) .
\end{aligned}$$

The first-order conditions are equations (C.26) to (C.29), (C.36) and (C.31).

For logarithmic utility function and Cobb-Douglas function for human capital accumulation, the system of equations describing the household's allocations is the following. In terms of the consumption during $t = 1$, the sequence of consumption for periods $t = 2, ..., \hat{J}$ is

$$c_t^{(i-J)} = \beta^{t-1} \frac{(1 + \tau_{c,1})}{(1 + \tau_{c,t})} \left(\prod_{z=2}^{t} R_z \right) c_1^{(i-J)}. \tag{C.37}$$

The market hours for $t = 2, ..., \tilde{J}$ are determined by

$$l_{m,t}^{(i-J)} = 1 - l_{h,t}^{(i-J)} - \alpha \beta^{t-1} \frac{(1 + \tau_{c,1}) \, c_1^{(i-J)}}{(1 - \tau_{l,t}) \, (1 - \tau_{p,t}) \, w_t h_t^{(i-J)}} \left(\prod_{z=2}^{t} R_z \right). \tag{C.38}$$

Since the retirement is mandatory, the market hours are equal to zero during these periods:

$$l_{m,t}^{(i-J)} = 0, t = \tilde{J} + 1, ..., \hat{J}.$$

Also the agents do not invest into human capital during the last working period:

$$l_{h,t}^{(i-J)} = 0, t = \tilde{J}, ..., \hat{J}.$$

The intertemporal condition determining the investment hours for periods $t = 1, ..., \tilde{J} - 1$ is

$$l_{m,t+1}^{(i-J)} = \frac{(1 - \tau_{l,t}) \, (1 - \tau_{p,t})}{(1 - \tau_{l,t+1}) \, (1 - \tau_{p,t+1})} R_{t+1} \frac{w_t}{w_{t+1}} \frac{h_t^{(i-J)}}{Q_{2t}} - \frac{h_{t+1}^{(i-J)}}{Q_{2t+1}} \left[1 - \delta_h + Q_{1t+1} \right].$$

Retirement benefits type 1. The retirement benefits for these generations are computed on the pro-rata basis. The first part of benefits is computed based on the wage income prior to the reform according to the formula (C.34) and is taken as exogenous variable by the households. The second part of benefits is determined by the defined contributions formula (C.35). The intertemporal budget constraint becomes

$$\sum_{t=1}^{\hat{J}} \frac{(1+\tau_{c,t})}{\prod_{z=2}^{t} R_z} c_t^{(i-J)} \leq R_1 s_1^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod_{z=2}^{t} R_z} + b^{1,(i-J)} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod_{z=\tilde{J}+1}^{t} \xi_z}{\prod_{z=2}^{t} R_z}$$

$$+ \sum_{t=1}^{\tilde{J}} \left[\frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod_{z=2}^{t} R_z} + \Phi_t^{1,(i-J)} \right] w_t h_t^{(i-J)} l_{m,t}^{(i-J)},$$

$$\Phi_t^{1,(i-J)} = \phi^{2,(i-J)} \tau_{p,t} \prod_{z=t+1}^{\tilde{J}} (1+\lambda_{Y,z}) \left[\sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod_{z=\tilde{J}+1}^{t} \xi_z}{\prod_{z=2}^{t} R_z} \right], t = 1, ..., \tilde{J}.$$

Plug into the budget constraint the expression for consumption and market hours, (C.37) and (C.38), respectively.

$$(1+\tau_{c,1})\, c_1^{(i-J)} \tilde{\beta} = R_1 s_1^{(i-J)} + \sum_{t=1}^{\hat{J}} \frac{f_t}{\prod_{z=2}^{t} R_z} + b^{1,(i-J)} \sum_{t=\tilde{J}+1}^{\hat{J}} \frac{\prod_{z=\tilde{J}+1}^{t} \xi_z}{\prod_{z=2}^{t} R_z}$$

$$+ \sum_{t=1}^{\tilde{J}} \left[\frac{(1-\tau_{l,t})(1-\tau_{p,t})}{\prod_{z=2}^{t} R_z} + \Phi_t^{1,(i-J)} \right] w_t h_t^{(i-J)} \left(1 - l_{h,t}^{(i-J)}\right),$$

$$\text{where } \tilde{\beta} = \sum_{t=1}^{\hat{J}} \beta^{t-1} + \sum_{t=1}^{\tilde{J}} \alpha \beta^{t-1} \left[1 + \frac{\Phi_t^{1,(i-J)} \left(\prod_{z=2}^{t} R_z \right)}{(1-\tau_{l,t})(1-\tau_{p,t})} \right].$$

Retirement benefits type 2. The same expressions as for generations born at $t = (i-J), i = 6, ..., 9$.

Generations born at periods $t = 1, ..., T$. For these generations the retirement benefits are calculated based on the defined contributions formula. Time-line for an agent's life is as follows: (i) during the periods $j = 0, ..., N_c - 1$, the agent is credit-constrained; (ii) working periods are $j = 0, ..., \overline{J}-1$; (iii) retirement periods are $j = \overline{J}, ..., J-1$; (iv) the Social Security benefits are calculated based on the wage income in the periods $j = \overline{J} - N_b, ..., \overline{J} - 1$.

Credit-constrained periods. During the credit-constrained periods, $j = 0, ..., N_c - 1$, the agents consume their labor income, do not save and determine the market hours based on intratemporal maximization. Since the borrowing constraints are binding, the level of consumption during these periods can be found from the household's budget constraint (4.2):

$$c^t_{t+j} = \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h^t_{t+j} l^t_{m,t+j} + \frac{f^t_{t+j}}{(1+\tau_{c,t+j})}. \tag{C.39}$$

The agents choose the sequence of market and investment hours and the level of human capital stock, $\left\{ l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j+1} \right\}_{j=0}^{N_c-1}$, to maximize their utility:

$$\max \sum_{j=0}^{N_c-1} \beta^j u \left(\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h^t_{t+j} l^t_{m,t+j} + \frac{f^t_{t+j}}{(1+\tau_{c,t+j})}, 1 - l^t_{m,t+j} - l^t_{h,t+j} \right)$$

subject to

$$h^t_{t+j+1} = (1-\delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}), j = 0, ..., N_c - 1.$$

The Lagrangian function for these constrained maximization problem is

$$\begin{aligned}
&£ \left(\{ l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j+1}, \mu_{t+j} \}_{j=0}^{N_c-1} \right) \\
&= \sum_{j=0}^{N_c-1} \beta^j u \left(\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1+\tau_{c,t+j})} w_{t+j} h^t_{t+j} l^t_{m,t+j} + \frac{f^t_{t+j}}{(1+\tau_{c,t+j})}, 1 - l^t_{m,t+j} - l^t_{h,t+j} \right) \\
&- \sum_{j=0}^{N_c-1} \mu_{t+j} \left(h^t_{t+j+1} - (1-\delta_h) h^t_{t+j} - Q(h^t_{t+j}, l^t_{h,t+j}) \right).
\end{aligned}$$

The first-order conditions with respect to choice variables are

$$u_{1t+j}\left(1-\tau_{t+j}\right)w_{t+j}h^t_{t+j}=u_{2t+j}, \tag{C.40}$$

$$\beta^j u_{2t+j}=\mu_{t+j}Q_{2t+j}, \tag{C.41}$$

$$\begin{aligned}\mu_{t+j}&=\mu_{t+j+1}\left(1-\delta_h+Q_{1t+j+1}\right)\\&+\beta^{j+1}u_{1t+j+1}\left(1-\tau_{t+j+1}\right)w_{t+j+1}l^t_{m,t+j+1},\end{aligned} \tag{C.42}$$

$$h^t_{t+j+1}=(1-\delta_h)h^t_{t+j}+Q(h^t_{t+j},l^t_{h,t+j}).$$

Express μ_{t+j} from (C.40) and (C.41) as function of marginal utility of consumption and substitute into (C.42). Then, the set of equations describing the agents's decisions during the credit constrained periods, $j=0,...,N_c-1$, is

$$\begin{aligned}c^t_{t+j}&=\left(1-\tau_{t+j}\right)w_{t+j}h^t_{t+j}l^t_{m,t+j}+\frac{f^t_{t+j}}{\left(1+\tau_{c,t+j}\right)},\\u_{2t+j}&=u_{1t+j}\left(1-\tau_{t+j}\right)w_{t+j}h^t_{t+j},\\l^t_{m,t+j+1}&=\frac{u_{1t+j}\left(1-\tau_{t+j}\right)w_{t+j}}{\beta u_{1t+j+1}\left(1-\tau_{t+j+1}\right)w_{t+j+1}}\frac{h^t_{t+j}}{Q_{2t+j}}-\frac{h^t_{t+j+1}}{Q_{2t+j+1}}\left(1-\delta_h+Q_{1t+j+1}\right),\\h^t_{t+j+1}&=(1-\delta_h)h^t_{t+j}+Q(h^t_{t+j},l^t_{h,t+j}).\end{aligned}$$

I consider logarithmic utility function and Cobb-Douglas production function for human capital technology. After substiting the partial derivatives and simplifying, the set of equations characterizing the household's problem becomes:

$$\begin{aligned}c^t_{t+j}&=\frac{1}{1+\alpha}\left[\left(1-\tau_{,t+j}\right)w_{t+j}h^t_{t+j}\left(1-l^t_{h,t+j}\right)+\frac{f_{t+j}}{\left(1+\tau_{c,t+j}\right)}\right],\\l^t_{m,t+j}&=\frac{1}{(1+\alpha)}\left(1-l^t_{h,t+j}\right)-\frac{\alpha}{(1+\alpha)}\frac{f_{t+j}}{\left(1-\tau_{l,t+j}\right)\left(1-\tau_{p,t+j}\right)w_{t+j}h^t_{t+j}},\\B\psi_2 l^t_{m,t+j+1}&=\frac{c^t_{t+j+1}}{\beta c^t_{t+j}}\frac{\left(1-\tau_{t+j}\right)w_{t+j}}{\left(1-\tau_{t+j+1}\right)w_{t+j+1}}\left(h^t_{t+j}\right)^{1-\psi_1}\left(l^t_{h,t+j}\right)^{1-\psi_2}\\&-\left(h^t_{t+j+1}\right)^{1-\psi_1}\left(l^t_{h,t+j+1}\right)^{1-\psi_2}\left(1-\delta_h+B\psi_1\left(h^t_{t+j+1}\right)^{\psi_1-1}\left(l^t_{h,t+j+1}\right)^{\psi_2}\right),\\h^t_{t+j+1}&=(1-\delta_h)h^t_{t+j}+Q(h^t_{t+j},l^t_{h,t+j}).\end{aligned}$$

Periods with intertemporal substitution. During these periods, the agents can smooth consumption over life-cycle and the period budget constraints (4.2) can be combined into the intertemporal one. Hence, the maximization problem of an agent born at period t

becomes:

$$\max_{\left(c^t_{t+j}, l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j+1}\right)^{J-1}_{j=N_c}} \sum_{j=N_c}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}) \tag{C.43}$$

subject to

$$\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} c^t_{t+j} \leq \sum_{j=N_c}^{\overline{J}-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j} l^t_{mt+j} + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} \tag{C.44}$$

$$+ \sum_{j=\overline{J}}^{J-1} \frac{b^t_2 \prod\limits_{z=\overline{J}+1}^{j} \xi_{t+z}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}},$$

$$h^t_{t+j+1} = (1-\delta_h) h^t_{t+j} + Q(h^t_{t+j}, l^t_{h,t+j}), j = N_c, ..., J-1, \tag{C.45}$$

where ξ_{t+j} is indexation rate for retirement benefits. The gross after-tax return on physical capital is denoted by $R_{t+j} = 1 + (1 - \tau_{k,t+j}) r_{t+j}$.

The Lagrangian function for the agent's maximization problem is

$$\begin{aligned}
&\pounds\left(\left\{c^t_{t+j}\right\}^{J-1}_{j=N_c}, \left\{l^t_{m,t+j}, l^t_{h,t+j}, h^t_{t+j+1}, \mu_{t+j}\right\}^{\overline{J}-1}_{j=N_c}, \lambda\right) \\
&= \sum_{j=N_c}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}) \\
&- \lambda \left[\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} c^t_{t+j} - \sum_{j=N_c}^{\overline{J}-1} \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} w_{t+j} h^t_{t+j} l^t_{mt+j}\right] \\
&+ \lambda \left[\sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}} + \sum_{j=\overline{J}}^{J-1} \frac{b^t_2 \prod\limits_{z=\overline{J}+1}^{j} \xi_{t+z}}{\prod\limits_{i=N_c+1}^{j} R_{t+i}}\right] \\
&- \sum_{j=N_c}^{\overline{J}-1} \mu_{t+j} \left(h^t_{t+j+1} - (1-\delta_h) h^t_{t+j} - Q(h^t_{t+j}, l^t_{h,t+j})\right).
\end{aligned}$$

After taking the first-order conditions of the Lagrangian function and simplification, I find the following set of equations to characterize the household's problem. For periods of life $j = N_c, ..., J-1$, the consumption is found by solving

$$\frac{\beta u_{1t+j+1}}{u_{1t+j}} = \frac{(1+\tau_{c,t+j+1})}{(1+\tau_{c,t+j})} \frac{1}{R_{t+j+1}}. \tag{C.46}$$

and the stock of savings is determined by the period budget constraints (4.2). During the working periods $j = N_c, ..., \overline{J}-1$, the hours for market production and human capital investment are respectively determined by

$$\frac{u_{2t+j}}{u_{1t+j}} = (1-\tau_{t+j})\, w_{t+j} h^t_{t+j}, \tag{C.47}$$

$$\begin{aligned} l^t_{m,t+j+1} = {} & \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{(1-\tau_{l,t+j+1})(1-\tau_{p,t+j+1})} R_{t+j+1} \frac{w_{t+j}}{w_{t+j+1}} \frac{h^t_{t+j}}{Q_{2t+j}} \\ & - \frac{h^t_{t+j+1}}{Q_{2t+j+1}} \left[1-\delta_h + Q_{1t+j+1}\right]. \end{aligned} \tag{C.48}$$

The stock of human capital is obtained from (C.45). Retirement benefits are calculated using the formula (4.9).

Using the functional forms chosen, the first-order conditions can be rewritten in the following way. I express the first-order conditions for consumption and market hours in terms of the consumption level at the period $j = N_c$. Then, I substitute the latter equations into the intertemporal budget constraint to find $c^t_{t+N_c}$. From equation (C.46) the consumption sequence is given by

$$c^t_{t+j} = \beta^{j-N_c} \left(\prod_{i=N_c+1}^{j} R_{t+i} \right) \frac{1+\tau_{c,t+N_c}}{1+\tau_{c,t+j}} c^t_{t+N_c}, j = N_c+1, ..., J-1. \tag{C.49}$$

From equations (C.47) and (C.49), the market hours are determined by

$$l^t_{mt+j} = 1 - l^t_{ht+j} - \frac{\alpha \beta^{j-N_c} (1+\tau_{c,t+N_c})\, c^t_{t+N_c}}{(1-\tau_{l,t+j})(1-\tau_{p,t+j})\, w_{t+j} h^t_{t+j}} \left(\prod_{i=N_c+1}^{j} R_{t+i} \right), j = N_c, ..., \overline{J}-1. \tag{C.50}$$

Due to the expression for retirement benefits (4.9), the intertemporal budget constraint

becomes:

$$\sum_{j=N_c}^{J-1} \frac{(1+\tau_{c,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} c_{t+j}^t \leq \sum_{j=0}^{N_c-1} \Phi_{t+j}^{4t} \tau_{p,t+j} w_{t+j} h_{t+j}^t l_{mt+j}^t \tag{C.51}$$

$$+ \sum_{j=N_c}^{\overline{J}-1} \left[\frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} + \Phi_{t+j}^{4t} \tau_{p,t+j} \right] w_{t+j} h_{t+j}^t l_{mt+j}^t$$

$$+ \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}},$$

$$\Phi_{t+j}^{4t} = \phi^{2t} \sum_{j=\overline{J}}^{J-1} \frac{(\xi_{t+j})^{j-\overline{J}}}{\prod_{i=N_c+1}^{j} R_{t+i}} \left(\prod_{z=j+1}^{\overline{J}-1} (1+\lambda_{Y,t+z}) \right).$$

Plug in the first-order conditions for consumption and market hours, (C.49) and (C.50), into the budget constraint (C.51):

$$(1+\tau_{c,t+N_c})\, c_{t+N_c}^t \tilde{\beta} = \sum_{j=0}^{N_c-1} \Phi_{t+j}^{4t} \tau_{p,t+j} w_{t+j} h_{t+j}^t l_{mt+j}^t +$$

$$+ \sum_{j=\overline{J}-N_b}^{\overline{J}-1} \Phi_{t+j}^{5t} w_{t+j} h_{t+j}^t \left(1 - l_{ht+j}^t\right) + \sum_{j=N_c}^{J-1} \frac{f_{t+j}}{\prod_{i=N_c+1}^{j} R_{t+i}},$$

where $\tilde{\beta} = \sum_{j=N_c}^{J-1} \beta^{j-N_c} + \sum_{j=N_c}^{\overline{J}-1} \alpha \beta^{j-N_c} \Phi_{t+j}^{6t},$

$$\Phi_{t+j}^{5t} = \frac{(1-\tau_{l,t+j})(1-\tau_{p,t+j})}{\prod_{i=N_c+1}^{j} R_{t+i}} + \Phi_{t+j}^{4t} \tau_{p,t+j},\ \Phi_{t+j}^{6t} = \left[1 + \frac{\Phi_{t+j}^{4t} \tau_{p,t+j} \left(\prod_{i=N_c+1}^{j} R_{t+i} \right)}{(1-\tau_{l,t+j})(1-\tau_{p,t+j})} \right]$$