

```

function Pi = elliptic3(u,m,c);
% -----
% Available at http://www.mathworks.com/matlabcentral/fileexchange/8805
% -----
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% -----
% ELLIPTIC3 evaluates incomplete elliptic integral of the third kind.
% Pi = ELLIPTIC3(U,M,C) where U is a phase in radians, 0<M<1 is
% the module and 0<C<1 is a parameter.
%
% ELLIPTIC3 uses Gauss-Legendre 10 points quadrature template
% described in [3] to determine the value of the Incomplete Elliptic
% Integral of the Third Kind (see [1, 2]).
%
% Pi(u,m,c) = int(1/((1 - c*sin(t)^2)*sqrt(1 - m*sin(t)^2)), t=0..u)
%
% Tables generating code ([1], pp. 625-626):
% [phi,alpha,c] = meshgrid(0:15:90, 0:15:90, 0:0.1:1);
% Pi = elliptic3(pi/180*phi, sin(pi/180*alpha).^2, c); % values of
integrals
%
% References:
% [1] M. Abramowitz and I.A. Stegun, "Handbook of Mathematical
% Functions" Dover Publications", 1965, Ch. 17.7.
% [2] D. F. Lawden, "Elliptic Functions and Applications"
% Springer-Verlag, vol. 80, 1989.
% [3] S. Zhang, J. Jin "Computation of Special Functions" (Wiley, 1996).
%
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if nargin<3, error('Not enough input arguments.');
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end
if ~isreal(u) | ~isreal(m) | ~isreal(c)
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    error('Input arguments must be real.')
end
if any(m < 0) | any(m > 1) | any(c < 0) | any(c > 1),
    error('M and C must be in the range [0, 1].');
end
if any(u > pi/2) | any(u < 0),
    error('U must be in the range [0, pi/2].');
end

[mm,nm] = size(m);
[mu,nu] = size(u);
if length(m)==1, m = m(ones(size(u))); end
if length(c)==1, c = c(ones(size(u))); end
if length(u)==1, u = u(ones(size(m))); end
if ~isequal(size(m), size(c), size(u)),
    error('U, M and C must be the same size.');
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end

Pi = zeros(size(u));
m = m(:).';    % make a row vector
u = u(:).';
c = c(:).';

I = find( u==pi/2 & m==1 | u==pi/2 & c==1 );

t = [ 0.9931285991850949, 0.9639719272779138,...    % Base points
      0.9122344282513259, 0.8391169718222188,...    % for Gauss-Legendre
      0.7463319064601508, 0.6360536807265150,...    % integration
      0.5108670019508271, 0.3737060887154195,...
      0.2277858511416451, 0.07652652113349734 ];
w = [ 0.01761400713915212, 0.04060142980038694,...    % Weights
      0.06267204833410907, 0.08327674157670475,...    % for Gauss-Legendre
      0.1019301198172404, 0.1181945319615184,...    % integration
      0.1316886384491766, 0.1420961093183820,...
      0.1491729864726037, 0.1527533871307258 ];

P = 0; i = 0;
while i < 10
    i = i + 1;
    c0 = u.*t(i)/2;
    P = P + w(i).*(g(u/2+c0,m,c) + g(u/2-c0,m,c));
end
P = u/2.*P;
Pi(:) = P;    % Incomplete elliptic integral of the third kind

% special values u==pi/2 & m==1 | u==pi/2 & c==1
Pi(I) = inf;
return;

function g = g(u,m,c)
% g = 1/((1 - c*sin(u)^2)*sqrt(1 - m*sin(u)^2));

sn2 = sin(u).^2;
g = 1./((1 - c.*sn2).*sqrt(1 - m.*sn2));
return;

```