Theoretical Bias In Gravitational Wave Astronomy and the Parameterized Post-Einsteinian Framework

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Outline/Motivation

- Problem
  - observing the universe in gravitational waves using template based searches, how would we mischaracterize the universe if GR was *close to* but *not quite* the correct theory of gravity?
  - can we estimate the consistency of an observed event with GR, without appealing to a specific alternative theory?
- A suggested solution
  - a *parameterized post Einsteinian* (*ppE*) extension of a template bank for a given class of event *[N. Yunes & FP, PRD 80:122003, 2009]*
    - describe a *ppE* template for quasi-circular, non-spinning binary black hole coalescence
  - show results from an ongoing study with *N. Cornish, L. Sampson & N. Yunes* to test the efficacy of the ppE in addressing the above issues
- Conclusions

Gravitational Wave Detection

- What's the "problem" with gravitational wave astronomy (focusing on binary black hole coalescence)?
  - the reliance on matched filtering to identify and characterize source events
    - for expected merger events, the current generation (Advanced LIGO/Virgo/GEO) of detectors are not sensitive enough to have the instantaneous GW strain be visible above the noise level
      - nor, for template-based detections will the SNR of a likely event be high enough for stringent model-independent self-consistency checks
    - the event that is assumed to have been "observed" is one described by the member of a template family giving the largest SNR
    - clearly, there are many relevant and important issues associated with *modeling bias*
      - how well is a given source's waveform modeled, i.e. how accurately do we need to solve the Einstein equations for BH mergers?
      - have all plausible scenarios been considered , e.g. mergers with eccentricity?
    - here, however, we are more interested in a *fundamental bias* that may be introduced into interpretation of actual *detections* with general relativity (GR) based templates, if GR does not *exactly* describe the underlying event

Tests of General Relativity

- A couple of important pieces of evidence that tell us GR can't be far off in predicting BBH merger events
  - inferred GW emission form binary pulsar systems
  - strong evidence for black holes, or at least very compact, "dark" objects
- However, there are currently no tests/observations of the *dynamical strong-field regime* of GR, where the final stages of the merger will occur
- Why be "suspicious" of GR?
  - from one point of view we are not: rather, that there are regions in the universe where spacetime itself is undergoing collapse to a singularity is an astonishing prediction of GR, and we should demand that evidence for the observable signature of this (black holes!) pass a correspondingly high bar
  - on the other hand, that dark energy (and perhaps to a lesser extent dark matter) is a complete mystery today and may be a gravitational phenomena, is a warning that perhaps we should be more careful with our assumptions that general relativity accurately describes all sub-Planck scale gravitational phenomena

Potential for Bias in GW observation

- Many merger events are expected to have the inspiral (good evidence for GR description) and merger/ringdown (no evidence for a GR description) both contribute significantly to the SNR, suggesting pure GR template banks will still be able to make detections even if GR is wrong in the late stages, albeit with lower SNR
  - i.e., *all detectable events* look like GR events when filtered through a GR template bank, and given the above, unlikely that the most obvious sign of a problem with GR (no detections at all) will manifest
- *Can we test for deviations from/consistency with GR without explicitly constructing template banks for all conceivable alternatives?*
- *Are there more subtle warning signs?*
  - mergers are uncharacteristically more common in the more distant past?
  - an unusual distribution of inferred spins, masses (e.g. with Chern-Simons like modifications to rotating BHs?)

Proposed GW tests of GR

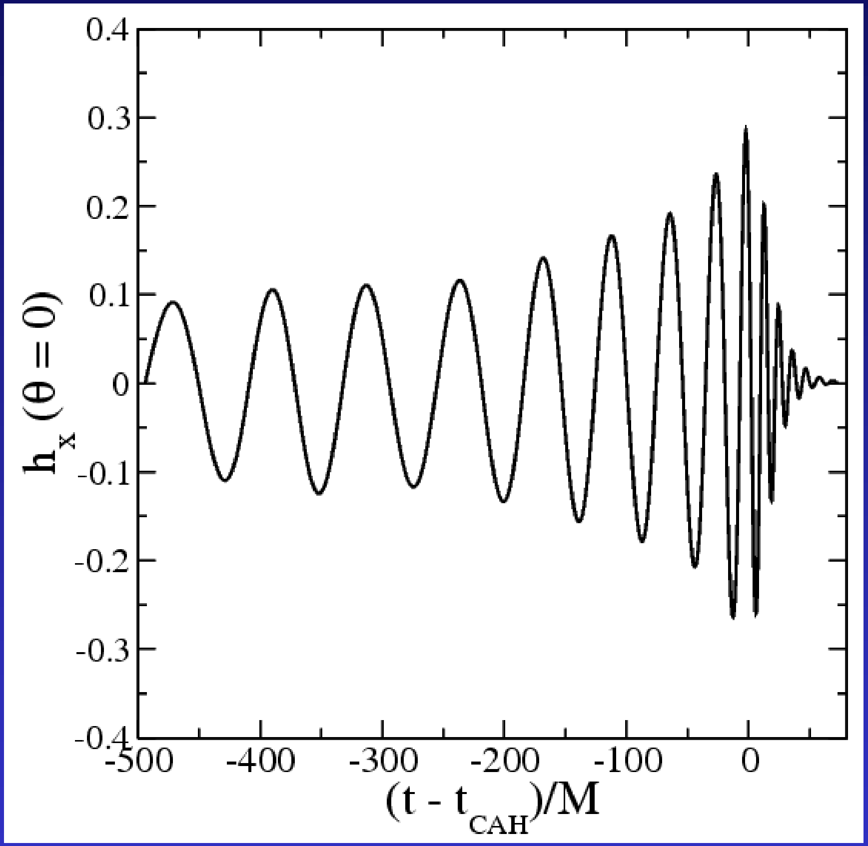
- classify as *intrinsic* (self-consistency checks with GR), e.g.
  - mapping the multipole structure of Kerr *[Ryan, PRD 56 (1997); Collins and Hughes, PRD 69 (2004)]*
  - black hole "spectroscopy" *[Dreyer et al., CQG 787 (2004); Berti et al. PRD 73 (2006)]*
  - the modified PN framework *[Arun et al., CQG 23 (2006)]*
  - various "unparameterized" tests, such as jacknife, coherent residuals, etc.
- vs. *extrinsic* (deviations from GR)
  - alternative theories, e.g. Brans-Dicke, Chern-Simons, massive graviton, scalar-tensor, etc.
  - any parameterized intrinsic test can be "inverted" to give information about a particular deviation, filtered through the assumptions of the particular test
  - the *ppE* approach is largely in this category, though makes no specific assumptions on a particular alternative theory, model deviation, etc. … such a *general* framework is essential to make headway in addressing the questions posed on the previous slide

The ppE framework

- Strategy
    - akin to the *parameterized post-Newtonian (ppN)* framework *[Schiff, Nordtvedt, Will, 60's & 70's]* to test for deviations from the Schwarzschild metric in the solar system, or the *parameterized post-Keplarian (ppK)* framework *[Damour & Taylor, 1992]*, to test GR in binary pulsar systems)
  - begin with a pure GR template bank for a given event, or class of events
  - introduce parameterized deformations of the templates, where each new parameter is ostensibly "well motivated", for .e.g. one or more of
    - consistent will all existing tests, yet can produce observable deviations in the dynamical, strong field regime
    - predicted by a reasonable alternative theory
    - characterizes a *plausible* strong-field correction, e.g. more rapid late time inspiral due to excitation of a new degree of freedom (scalar waves, different polarizations, etc).
  - use the ppE templates *post-detection* of an event with a GR template bank

An example ppE template family

- For initial studies, it will be most useful to have a simple, analytic PPE template bank
  - For the GR limit, restrict attention to equal mass, initially non-spinning, quasi circular binary black hole inspirals
  - Break the merger waveform into 3 regimes:
    - Inspiral : modeled via the post Newtonian expansion
    - Merger: smooth interpolation between inspiral and ringdown
    - Ringdown: dominant mode Quasi-normal ringdown



  - will construct templates in the frequency domain, employing the stationary phase approximation to compute the Fourier transform of the inspiral waveform
    - can do so to this order as the dominant wave frequency is monotonically increasing as the system evolves through these stages – of course, this restricts attention to deviations that do not "mix" frequency components between stages

An example ppE template family

$$\boxed{\tilde{h}(f) = \begin{cases} \tilde{h}_I^{GR}(f)\cdot\left(1+\alpha u^a\right)e^{i\beta u^b}, & f < f_{IM} \\ \gamma u^c e^{i(\delta+\varepsilon u)}, & f_{IM} < f < f_{MRD} \\ \zeta \dfrac{\tau}{1+4\pi^2\tau^2\kappa\left(f - f_{RD}\right)^d}, & f_{MRD} < f \end{cases}}$$

- Inspiral : $f<f_{IM}$
  - $h_I^{GR}(f)$ is the GR inspiral component, e.g. to leading order

$$\boxed{\tilde{h}_I^{GR}(f) \propto f^{-7/6}e^{i2\pi f t_0}}$$

  - $u=\pi\mathcal{M}f$, with $\mathcal{M}$ the chirp mass
  - a,b,α,β are ppE parameters

GR: $\alpha=0,\ \beta=0$

Brans-Dicke: $\alpha=0,\ \beta=\beta_{BD},\ b=-7/3$
(though β_{BD} is 0 for binary BH spacetime)

Massive graviton: $\alpha=0,\ \beta=\beta_{MG},\ b=-1$

Chern-Simons: $\alpha=\alpha_{CS},\ a=1,\ \beta=0$
(post-emission propagation)

Varying G: $\alpha,\beta \propto dG/dt,\ a=-8/3,\ b=-13/3$

Modified PN: $\alpha=0,\ \beta\neq 0,\ b=(k-5)/3,\ k \in I$

An example ppE template family

$$\tilde{h}(f) = \begin{cases} \tilde{h}_I^{GR}(f)\cdot\left(1+\alpha u^a\right)e^{i\beta u^b} , & f < f_{IM} \\ \gamma u^c e^{i(\delta+\varepsilon u)} , & f_{IM} < f < f_{MRD} \\ \zeta \dfrac{\tau}{1+4\pi^2\tau^2\kappa\left(f - f_{RD}\right)^d} , & f_{MRD} < f \end{cases}$$

- Transition between inspiral and merger : $f_{IM} < f < f_{MRD}$
  - c,ε are ppE parameters
  - γ,δ are continuity parameters
  - note: f_{IM} and f_{MRD} could also be considered ppE parameters, with the GR limits fixed by mathing (for example) to results from numerical solutions

GR: $c\sim -2/3,\ \varepsilon=1$ (matching to NR results)

Alternatives: no alternative theories have yet been explored in the fully non-linear merger/ringdown regime, so little to go on

An example ppE template family

$$\tilde{h}(f)=\begin{cases} \tilde{h}_I^{GR}(f)\cdot\left(1+\alpha u^a\right)e^{i\beta u^b} & , \quad f<f_{IM} \\ \gamma u^c e^{i(\delta+\varepsilon u)} & , \quad f_{IM}<f<f_{MRD} \\ \zeta \dfrac{\tau}{1+4\pi^2\tau^2\kappa\left(f-f_{RD}\right)^d} & , \quad f_{MRD}<f \end{cases}$$

- Ringdown : $f_{MRD}<f$
  - d, f_{RD}, τ, κ are ppE parameters
  - ζ is a continuity parameter

GR: $\kappa=1$, $d=2$; τ and f_{RD} are the dominant QNM decay constant and frequency respectively for the final BH (in GR, a Kerr black hole with a~0.7)

Scenarios that can be explored with ppE

	templates	GR	ppE
Theory	GR	**Business as usual**	***Quantify* the likelihood of GR being the underlying theory describing the event, within the class modeled by ppE**
	~~GR~~	**Understand the bias that could be introduced filtering non-GR events with a GR template**	**Measure deviations from GR characterized by non-GR ppE parameters**

A suggested use of ppE templates

- We are *not* suggesting the ppE templates should necessarily be used for detection for the following reasons
  - The GR limit of this sample family, and possibly any family that could be "easily" deformed, may not be good enough for high SNR detection or high accuracy parameter extraction
    - rather require that there be a "decent" overlap between the GR-ppE limit and the waveform used for actual detection
  - The more parameters that are blindly searched over in a matched filter search increases the likelihood of false-detections
  - The ppE waveforms are in a sense only well-motivated for small deviations from GR templates, and so at least initially it makes more sense to begin searches for deviations (consistency) from (with) GR from a set of GR-based detections

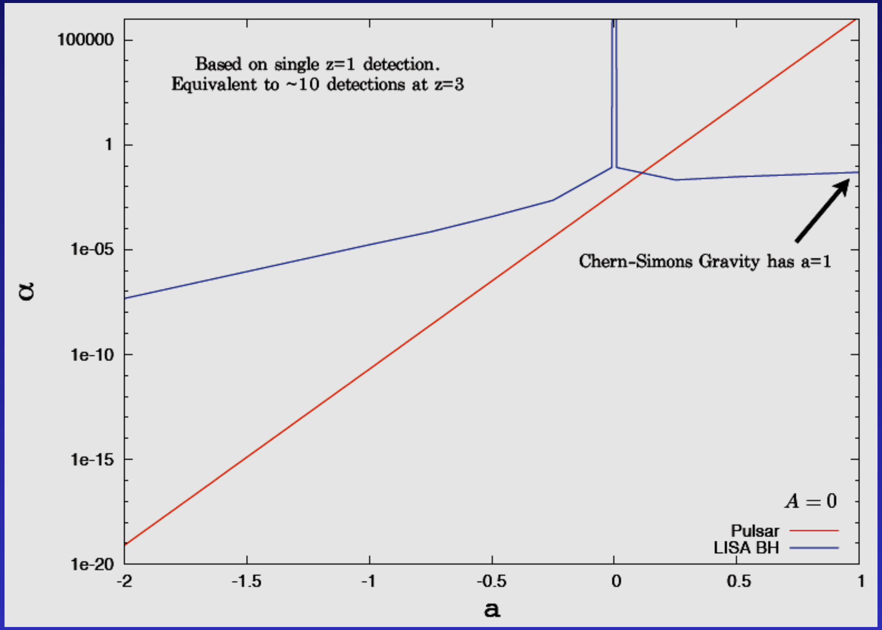
Results

- For a first study of the efficacy of the approach, we restrict to *inspiral only*, and use ppE waveforms as both the source and templates, with model advanced-LIGO/VIRGO/GEO & LISA noise curves.
  - extended ppE inspiral templates to include symmetric mass ratio η

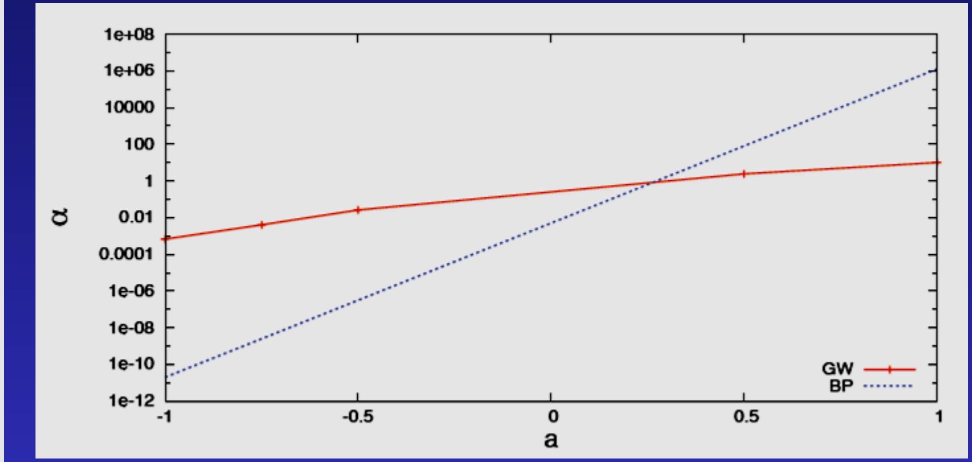
$$\boxed{\tilde{h}(f)=\tilde{h}_I^{GR}(f)\cdot\left(1+\alpha(4\eta)^A u^a\right)e^{i\beta(4\eta)^B u^b}}$$

- Unless otherwise noted, models were:
  - for LIGO : equal mass binary, total mass $8.7\,M_\odot$, with $SNR=20$
  - for LISA : $10^6\,M_\odot$ with $3 \times 10^6\,M_\odot$ unequal mass binary $SNR=880$ (z=1)
- Use a Bayesian approach where the posterior distribution functions are sampled with Markov Chain Monte Carlo (plus various "accelerants") *[Littenberg & Cornish, PRD 80 (2009), Cornish & Sampson]*
  - in all cases when showing plots of a subset of parameters, others have been marginalized over
- As a reference, will show how well GW observations can do vs. existing constraints from binary pulsar observations *[Yunes & Hughes, arXiv:1007.1995 (2010)]*

Constraining ppE parameters : a & α



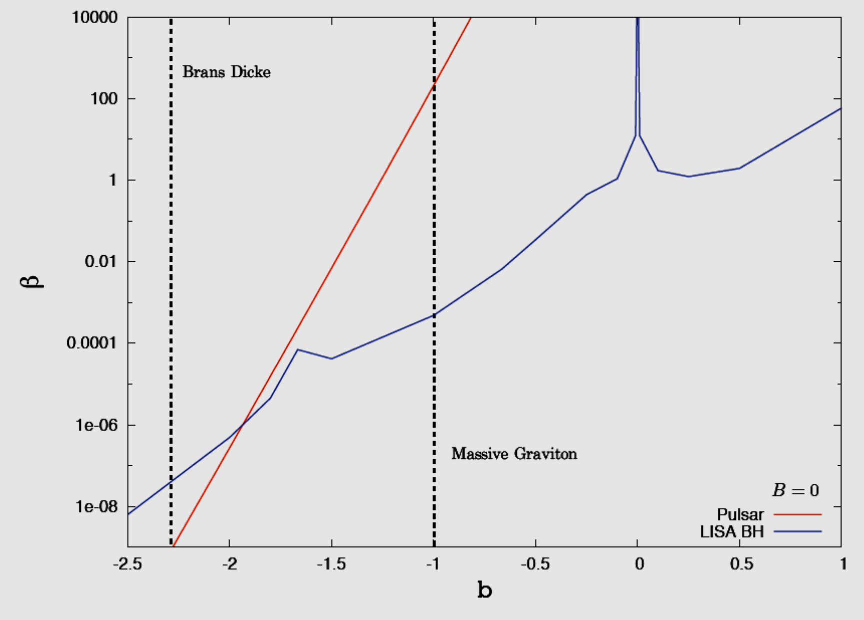
LISA



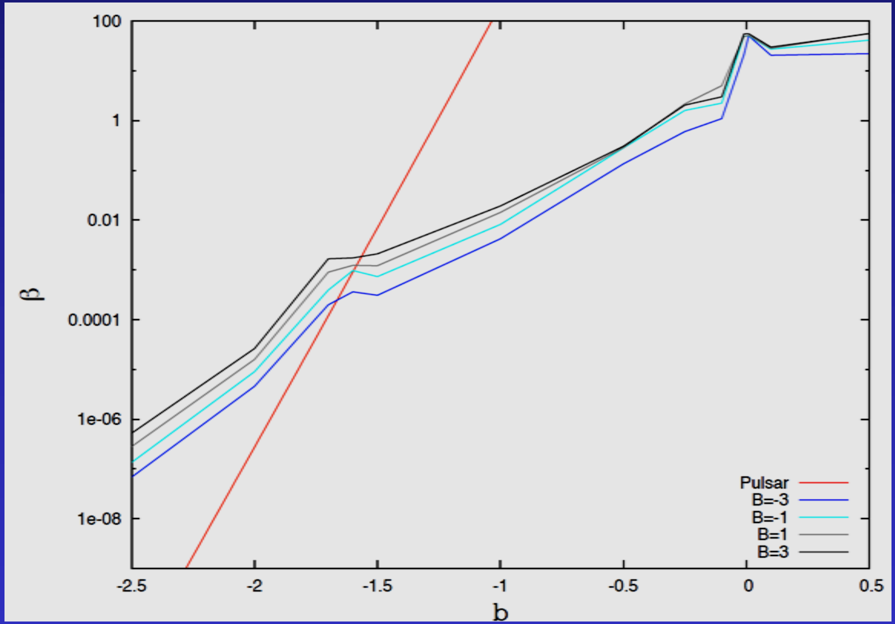
LIGO *(note opposite color scheme!)*

- negative (positive) exponents → weak (strong)-field deviations
- peaks of poorly/unconstrained values occur due to degeneracies (also happens for LIGO … degenerate point not sampled)

Constraining ppE parameters : b & β

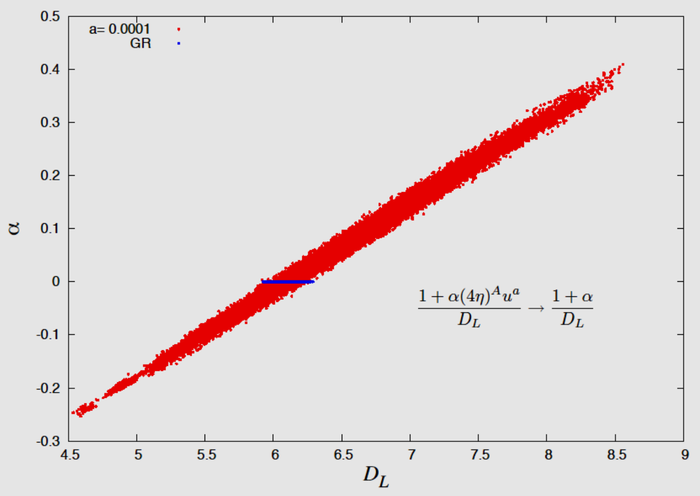


LISA

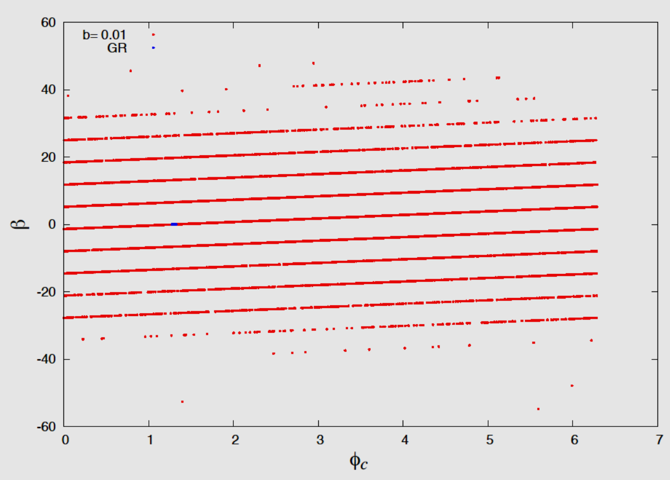


LIGO (for 3:1 mass ration binary)

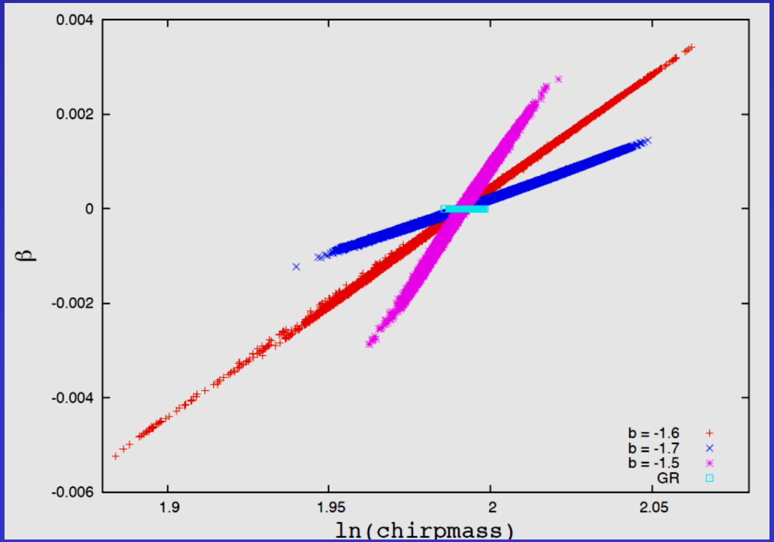
Parameter Correlations



peak at *a=0* is degeneracy between luminosity distance and effective α (LISA example)



peak at *b=0* is degeneracy between phase of coalescence and β (LISA example)

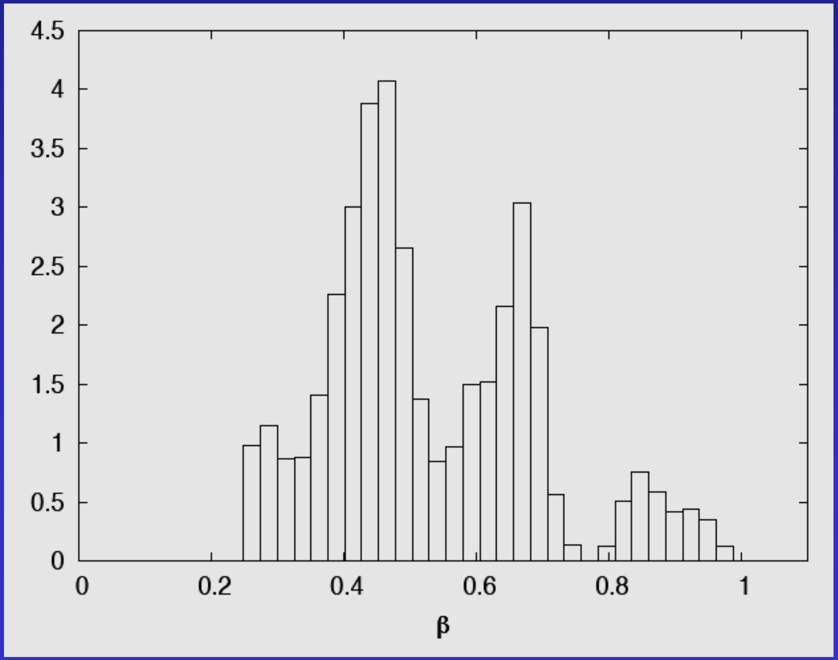


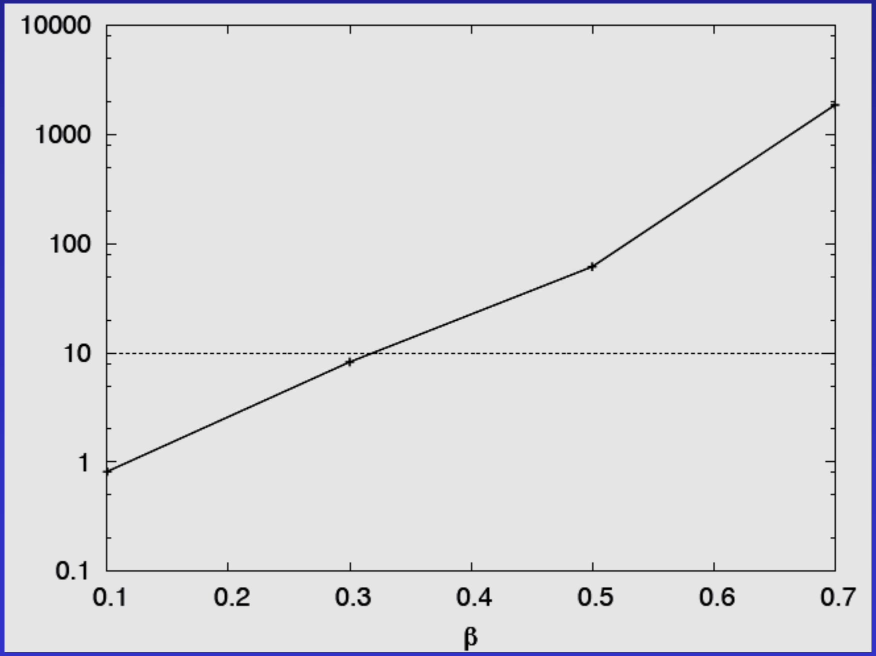
bump at *b=-5/3* (PN value) is a partial degeneracy between chirp mass and β (LIGO example)

Detecting deviations from GR: LIGO

- Left: sample histogram of β for source with $(a,\alpha)=(0,0);(b,\beta)=(.5,0.5); (A,B)=(0,0)$; clearly shows a non-GR modeled is favored

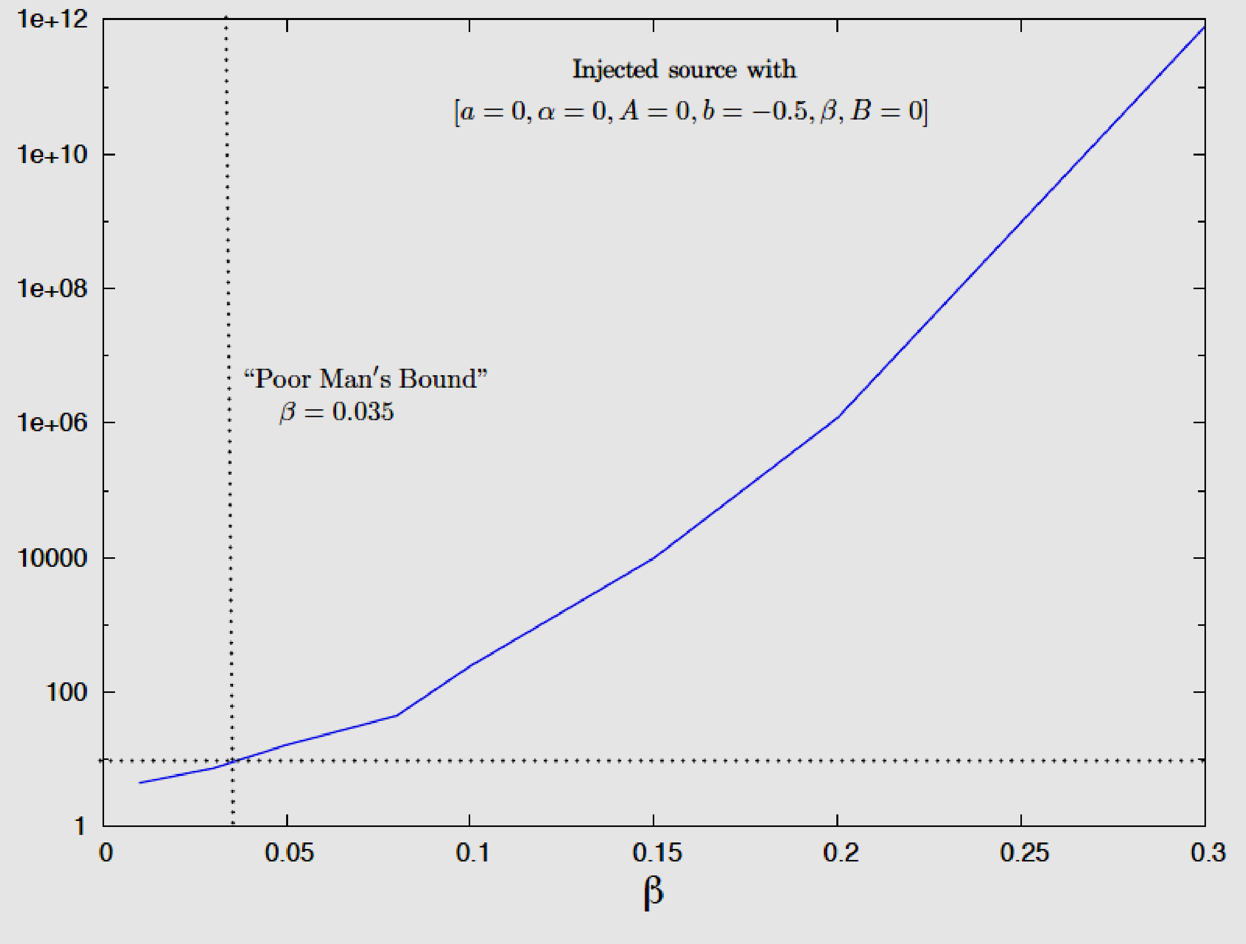
- Right: Bayes factor (odds ratio) for source with $(a,\alpha)=(0,0)$; $b=.5$ and several values of β; suggests $\beta >\sim .3$ can readily be observed





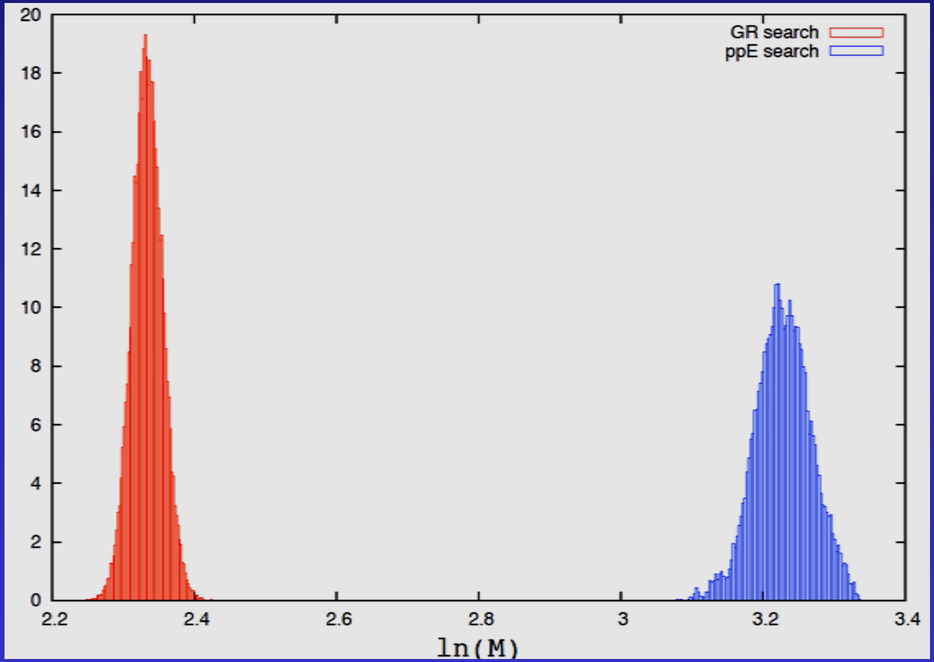
Detecting deviations from GR: LISA

- Bayes factor for source with *(A,B,a,α)=0; b=-0.5,* varying β



Parameter Bias

- non-GR source, detected with GR (red) and full ppE (blue) templates … extracting the chirp mass, GR templates are wrong by more than the uncertainty in the estimate:

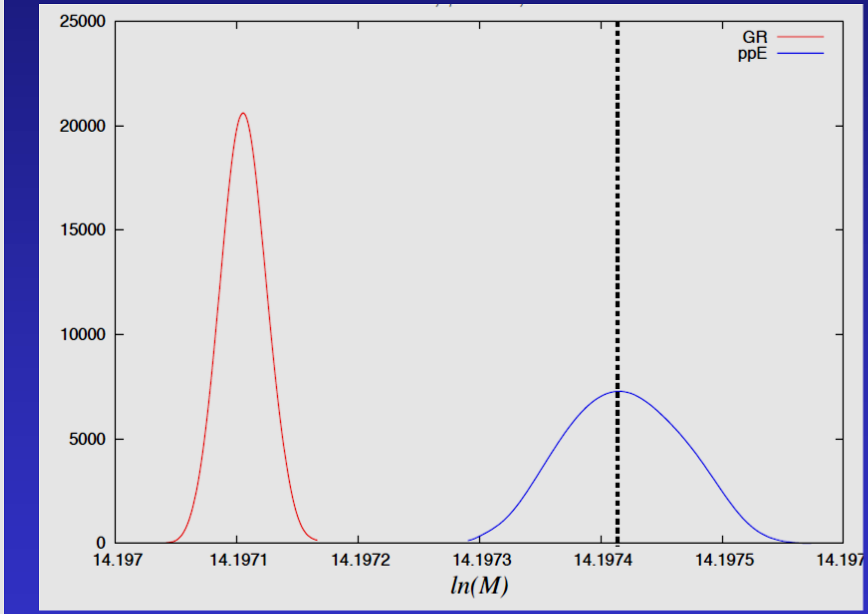


LIGO : mass ratio 2:1, injected $ln(M) = 3.3$

$(a,\alpha)=(0,0)$

$(b,\beta)=(-0.5,8.25)$

$(A,B)=(0,0,0.25)$



LISA : injected $ln(M) = 14.19741$

$(a,\alpha)=(0,0)$

$(b,\beta)=(0.5,0.3)$

$(A,B)=(0,0,0.0)$

Conclusions

- A “necessary evil” of the near-future of gravitational wave astrophysics is the reliance on matched filtering to go beyond detection
  - for binary compact object mergers, good evidence that much of the event will be adequately described by GR, though not so for the final merger/ringdown
  - if GR is not quite the correct theory describing black hole-like objects, or the last stages of merger, a GR-based template won’t catch this, in particular if (as expected with LIGO/VIRGO/GEO600) the typical SNR is low
  - moreover, it would be useful to be able to *quantify* the consistency of an event with GR, and place constraints on generic alternatives
- A parameterized post Einsteinian extension of waveforms is a framework that could address the above issues
  - early studies with mock data and signals are encouraging, providing quantitative answers to questions of fundamental bias, detecting deviations from GR, or conversely providing a likelihood that GR describes the event
  - many more directions to pursue
    - more thorough mapping out of ppE-space within existing observational priors
    - including the effects of merger/ringdown
    - extending the ppE family to include the full set of GR binary BH merger parameters
    - constructing ppE templates for other template-based events (e.g. BH-NS & NS-NS mergers)