Probing the early Universe and cosmology by detecting gravitational waves

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Content:

• Probing the primordial dark age by detecting relic GWs

• Astrophysical foregrounds

• Complementarity between various GW experiments
Disclosing the **primordial dark age of the Universe**

- **What is currently measured?**
  - $\rho_\gamma, \rho_m, \rho_b, (n_b - n_{\bar{b}})/s, \rho_\Lambda \cdots$
  - $(\Delta_R^2)_{k_0}, n_s, (d \log \Delta_R^2/d \log k)_{k_0}$

- **Particles as probes**
  - $\gamma \rightarrow$ free-streaming at $\sim 1\text{eV}$
  - $\nu \rightarrow$ streaming at $\sim 1\text{MeV}$
  - $h \rightarrow$ streaming since end of inflation
    - $\sim 10^7\text{TeV}$

\[ ds^2 = a^2 [-d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j] \]

**Very clean cosmological probes**

- **What can we probe by detecting primordial GWs?**
  - universe’s equation of state
  - reheating
  - phase transitions
  - cosmic strings
Characteristic intensity of relic gravitational waves

- **Two ways of describing the GW background:**

**Tensor power spectrum:** \[ \Delta_h^2(k, \tau) \equiv \frac{d<0|h_{ij}^2|0>}{d \log k} \propto k^3 |h_k(\tau)|^2 \]

**GW energy spectrum:** \[ \Omega_{GW}(k, \tau) \equiv \frac{1}{\rho_c(\tau)} \frac{d<0|\rho_{GW}(\tau)|0>}{d \log k} \propto \frac{k^2 \Delta_h^2(k, \tau)}{a^2(\tau) H^2(\tau)} \]
Phenomenological bounds

- **BBN bound**
  \[
  \int h_0^2 \Omega_{GW}(f) \, d\log f \leq 5.6 \times 10^{-6} (N_\nu - 3)
  \]
  [Copi, Schramm and Turner 97]

- **CMB bound**
  [Smith et al. 06]

- **COBE bound**
  \[
  h_0^2 \Omega_{GW}(f) \leq 7 \times 10^{-11} \left( \frac{H_0}{f} \right)^2
  \]
  \[
  H_0 \leq f \leq 10^{-16} \text{ Hz}
  \]

- **msec pulsars**
  \[
  h_0^2 \Omega_{GW}(f) \leq 4.8 \times 10^{-9} \left( \frac{f}{f_o} \right)^2
  \]
  \[
  f > f_o \equiv 4.4 \times 10^{-9} \text{ Hz}
  \]
  [Thorsett & Dewey 96; Janet et al. 06; Hobbs et al. 08]
Current and future GW detectors

[Abbott et al. (LIGO Scient. Coll. & Virgo) 09]

- S5-run data at design sensitivity provided the upper limit at 95% confidence:
  \[ h_0^2 \Omega_{GW} < 6.9 \times 10^{-6} \]
  assuming freq-independent spectrum.

- The search improved upon indirect limits from BBN and CMB at 100 Hz.

- **Current detectors taking data:** Enhanced LIGO/Virgo+

- **Adv. detectors:** Adv. LIGO/Virgo; Large Cryogenic Grav. Telescope; Einstein Telescope

- **Preliminary examples of GW detectors at very high frequency** (\( \sim \) MHz)

- **Space-based laser interferometer (LISA)** within the next 15 (?) years
Production of GWs from inflation: exiting and re-entering the Hubble radius

Introducing “canonical field” $\psi_k(\tau) = a h_k(\tau)$:

$$\psi''_k + \left[k^2 - U(\tau)\right] \psi_k = 0 \quad U(\tau) = \frac{a''}{a}$$

- If $k^2 \gg |U(\tau)| \Rightarrow \lambda_{\text{phys}} \ll H^{-1}$
  $\Rightarrow$ the mode is inside the Hubble radius

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$\psi_k \approx A_k$
Stochastic GW background from inflation

[Grishchuk 74; Starobinsky 79]

**Slow-roll inflation:**

- \( \Delta^2_h(k) \approx 8 \left( \frac{H}{2\pi M_{\text{pl}}} \right)^2 \quad \Delta^2_R(k) \approx \frac{1}{\epsilon} \left( \frac{H}{2\pi M_{\text{pl}}} \right)^2 \)
- \( r(k) \equiv \frac{\Delta^2_h(k)}{\Delta^2_R(k)} = 16\epsilon \)

\[ \epsilon \propto \frac{d \log V}{d \log N} \quad \eta \propto \frac{d \log V^{1/2}}{d \log N} \]

\[ \Delta^2_h(k; \tau) = \frac{T_h(k; \tau)}{\Delta^2_h(k; \tau_i)} \]

- \( \Omega_{GW}(f) \sim H^2 f^{-2} \) (re-entered during MD)
- \( \Omega_{GW}(f) \sim H^2 f^{n_t} \) (re-entered during RD)

\[ n_t = -r/8 \]

- **GWs carry information on two moments of cosmic history:** when \( k \) exit the Hubble radius and re-entered it

\( t_* \quad H^{-1} \quad \lambda \)

**Inflation:** \( H^{-1} \sim \text{const.} \)

RD: \( H^{-1} \propto a^2 \)

MD: \( H^{-1} \propto a^{3/2} \)
Predictions from slow-roll inflationary models

- In minimally tuned models, e.g., $V \sim \phi^n$, $\epsilon$ and $\eta$ don’t have zeros during the last 60 e folds
- Degree of fine-tuning: number of zeros in $\epsilon$ and $\eta$: number of extra accelerations, jerks, bumps when modes leave horizon during inflation

- CMB sensitive to long wavelengths that re-entered at low temperature (after BBN)
- GW IFOs sensitive to short wavelengths that re-entered at high temperature
Transfer function and sensitivity to inflationary models

[Boyle & Steinhardt 06]

- **Transfer function includes**
  - dark energy with time dependent eq of state
  - tensor anisotropic stress due to free-streaming of relativistic particle in early Universe

[Weinberg 03]

- *Convergence* effect if consistency relation, \( n_t = -r/8 \), holds
  [see also Ungarelli et al. 05; Smith et al. 06]
  [Efstathiou et al.; Kudoh et al.; Watanabe et al. 06]
Stochastic GW background induced by primordial scalar perturbations

[Matarrese et al. 93; Mollerach et al. 04]
[Ananda et al. 07]

- Independent on inflation
- Dependent on observed scalar spectrum and general relativity
- Model-independent lower limit on GW spectrum
- GWs produced from an early matter era, before RD and BBN
  [Baumann, Steinhardt, Takahashi & Ichiki 07]

[Assadulahi et al. 09; Jedamzik et al. 10]
Dependence of $\Omega_{\text{GW}}$ on equation of state at exit and re-entry

[Boyle & AB 08]

\[
\Omega_{\text{gw}}(f) = \left[ A_1 A_2^{\hat{\alpha}(f)} A_3^{\hat{n}_t(f)} \right] r \quad k/a_0 = 2\pi f
\]

\[
\Omega_{\text{gw}}(f) \propto f^{\hat{\alpha}(f)+\hat{n}_t(f)} \quad r \equiv \frac{\Delta^2_{R}(k_{\text{cmb}})}{\Delta^2_{R}(k_{\text{cmb}})} \quad \frac{k_{\text{cmb}}}{a_0} = 0.002 \text{ Mpc}
\]

- $\hat{\alpha}(f) \equiv 2\left(\frac{3\tilde{w}(f)-1}{3\tilde{w}(f)+1}\right)$
- $\hat{w}(f) \equiv \frac{1}{\ln(a_{\text{BBN}}/a_k)} \int_{a_k}^{a_{\text{BBN}}} \tilde{w}(a) \frac{da}{a}$

where $\tilde{w}(a) \equiv w(a) - \frac{8\pi G \zeta(a)}{H(a)}$ is the effective equation-of-state while

\[
w(a) = \frac{p(a)}{\rho(a)}
\]

is the ordinary equation-of-state
Dependence of $\Omega_{GW}$ on equation of state at exit and re-entry

[Boyle & AB 08]

$$\Omega_{gw}^f(f) = \left[ A_1 A_2 \hat{\alpha}(f) \hat{n}_t(f) \right] r \quad k/a_0 = 2\pi f$$

$$\Omega_{gw}^f(f) \propto f^{\hat{\alpha}(f)+\hat{n}_t(f)} \quad r \equiv \frac{\Delta^2_h(k_{cmb})}{\Delta^2_R(k_{cmb})} \quad \frac{k_{cmb}}{a_0} = 0.002 \text{ Mpc}$$

\[ \hat{n}_t(f) \equiv \frac{1}{\ln(k/k_{cmb})} \int_{k_{cmb}}^{k} n_t(k') \frac{dk'}{k'} \quad n_t(k) = 3 - 3 \left| \frac{1-w_{\text{exit}}(k)}{1+3 w_{\text{exit}}(k)} \right| \]

\[ \hat{n}_t(k) \equiv \frac{\log \Delta^2_h(k)}{d \log k} \] is the primordial tensor tilt
Constraining (or detect) the presence of a *stiff* energy component prior to BBN

\[ \rho \propto a^{-3(1+w)} \Rightarrow \text{the lower the } w, \text{ the slower it dilutes} \]

- **In standard picture** $w \leq 1/3$, but ... [Grishchuk 75; Peebles & Vilenkin 98; Sahni et al. 99]
Implications by combining CMB and BBN constraints

\[ \int_{f_{BBN}}^{f_{end}} \Omega_{gw}(f) \, \frac{df}{f} \leq 1.5 \times 10^{-5} \]

\[ \hat{n}_{t, \text{max}}(f) = - \ln \left[ \frac{r A_1 A_2 \hat{\alpha}(f) / \Omega_{\text{gw max}}(f)}{\ln[A_3]} \right] \]

\[ \hat{\alpha}(f) = 2 \left( \frac{3 \hat{w}(f) - 1}{3 \hat{w}(f) + 1} \right) \]

If CMB experiments detect a non-zero \( r \)
\[ \Rightarrow \text{upper bound on equation-of-state } w \]
during dark age!

\[ \hat{n}_t \approx 0 \Rightarrow \hat{w}(f_{end}) \lesssim \{0.54, 0.57, 0.6\} \]
\[ \hat{w}_t \approx 1/3 \Rightarrow \hat{n}(f_{end}) \lesssim \{0.36, 0.40, 0.43\} \]
Constraining early Universe evolution

[Abbott et al. (LIGO Scientific Collaboration & Virgo) 09]

\[ r = 0.1 \]

- For single-field inflation with potential

\[ V(\phi) = \frac{m^2 \phi^2}{2} \]

\[ r = 0.14 \text{ and } \hat{n}_t(100\text{Hz}) = -0.035 \]

\[ \Rightarrow \text{LIGO bound: } \hat{\omega}(100\text{Hz}) < 0.59 \]
Astrophysical GWBs due to comparable-mass binaries

10^{-6} \rightarrow 10^{-1}

\Omega_{GW} f (Hz)

10^{-1} \rightarrow 10^{1}

Sources per 10^{-8} Hz (i.e. 1/(3yr))

- Galactic background in principle subtractable because anisotropic
- Extra-galactic background due to WD-WD could be subtracted if \( f \gtrsim 50 \) mHz
- At high freq the dominant foreground sources are NS-NS, NS-BH and BH-BH [Cutler & Harms 06]
Astrophysical GWBs from cosmic supernovae

- Anisotropic mass-motion and $\nu$-emission in collapse of massive starts produce GWs
- At low frequencies anisotropic $\nu$-emission with luminosity $L_\nu$ and anisotropy $q(t)$ dominates $\Rightarrow h(t) = \frac{2G}{D} \int_{-\infty}^{t-D} dt' L_\nu(t') q(t')$, $f|\tilde{h}(f)| \sim 10^{-19} < q > \frac{10^{kpc}}{D} \frac{E_\nu}{3 \times 10^{53} \text{erg}}$

$q > = 0.45\%$; core collapse of rotating $15M_\odot$ star

[AB, Sigl, Raffelt, Janka & Mueller 05]
Astrophysical GWBs from Population II and III stars

[Marassi, Schneider & Ferrari 09]

- Pop II background extends up to high frequencies with peak $\sim 10^{-12} - 10^{-10}$ Hz

- Astrophysical stochastic backgrounds can be produced also by magnetars and NS-NS coalescences [see e.g., Regimbau & Mandic 10]
Conclusions

- Relic GWs at large and small wavelengths can carry information on otherwise unexplored physics between $\sim 10^2$ GeV and $\sim 10^{16}$ GeV.

- Current direct-detection experiments, such as LIGOs/Virgo, have approached the BBN bound and have started exploring interesting regions of parameter space.

- GW background from inflation contains several interesting physical features.

- Possibility of constraining (or detecting) the presence of a stiff energy component prior to BBN.

- If CMB, pulsars, LIGOs/Virgo detect a non-zero $r \Rightarrow$ upper bound on $w$ during dark age.

- In some frequency bands, cosmological signals compete with astrophysical ones.