A Theory of Demand for Gambles

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Abstract

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Although gambling is primarily an economic activity, no single theory of the demand for gambles has gained wide-spread acceptance among economists. This paper proposes a simple model of the demand for gambling that is based on the standard economic assumptions that (1) resources are scarce and (2) consumer's utility increases with income at a decreasing rate. This model has the advantages that (1) it is based solely on changes in income, (2) is potentially applicable to most consumers, (3) preserves the assumption of diminishing marginal utility of income, (4) is consistent with the insurance-buying gambler, and (5) has intuitive appeal.

Keywords: gambling, demand for gambles, expected utility theory

JEL category: D81, D11

Gambling is a ubiquitous human activity. In recent years, it has become the basis of a large and growing industry in the U.S, much of it government sponsored. The growth of this industry has coincided with the increasing awareness that gambling is also a significant public health concern. This concern, in turn, has generated renewed interest in understanding why people gamble. However, despite the fact that gambling is primarily an economic activity, economists do not possess a well-accepted theory for this behavior (Machina, 1987, 1989; Sauer, 1998; Starmer, 2000).

This paper presents a new theory of the demand for gambles that is based on the concept that a gamble lets the consumer obtain "something for nothing." As such, it is based on two fundamental concepts in economics: (1) additional income increases utility but at a diminishing rate, and (2) economic resources are scarce, therefore, for the typical consumer, additional income is normally costly to obtain. Thus, the salient feature of a gamble to a consumer is not merely that it represents a vehicle for gaining additional income, but for gaining additional income without working for it.

In the next section, the literature is reviewed. Then, the new model is presented, followed by a discussion of its behavioral implications.

Literature Review

The most oft-cited theory of gambling is based on the assumption that the gambler's utility as a function of unearned income or wealth is increasing at an increasing rate. This theory, however, has generated some discomfort among economists. First, many economists are reluctant to set aside the intuitively-appealing and theoretically-necessary notion of diminishing marginal utility just to explain gambling behavior. Second, as it is generally applied, this theory has little predictive capability. That is, it is based on a circularity of reasoning that if a person gambles, she must exhibit a utility function that is increasing with successive marginal gains in income or wealth. But, the conventional procedure for determining a consumer's utility function (von Neumann and Morgenstern, 1946) relies on first determining whether a consumer would be willing to accept a gamble. Third, many consumers both gamble and purchase insurance at the same time, which under the this theory would require both concavity and non-concavity of the utility function simultaneously. Indeed, explaining this paradoxical behavior has become a crucial test for any acceptable model of gambling behavior.

The most famous and parsimonious explanation of the insurance-purchasing gambler was suggested by Friedman and Savage (1948), who postulated that a generally concave utility function might contain segments that are locally convex. This model (and its main variant, Markowitz, 1952) have obvious mathematical appeal, but they beg the question of why utility would be increasing with some portions of income, and not with others. That is, it solves the mathematical problem, but fails to provide an intuition for why people gamble. Furthermore, Bailey et al. (1980) argued that non-concavity actually could not in principle explain gambling because the option of saving and borrowing would dominate when the consumer's rate of time preference differs from the interest rate. While Hartley and Farrell (2002) dispute this argument, there is still the issue of whether consumers actually regard saving and borrowing as substitutes for gambling, or more generally, if they do, what proportion of gambling behavior can be explained by this model.

Many authors have concentrated on finding a mathematical solution to gambling-insurance paradox that preserves the assumption of concavity. These models are based primarily on the identification of specific market imperfections or indivisibilities that can generate non-concavity (Eben, 1979; Ng, 1975; Kim, 1973; Hakansson, 1970; Flemming, 1969). For example, Dobbs (1988) suggests that as income/wealth increases, consumers switch from one concave utility function based on work to another that is based on leisure, and because workers cannot readily vary their choice of working hours, this indivisibility generates a non-concavity that can explain the purchase of both lotteries and insurance. Dobbs suggests that his model is most specifically applicable to explaining the demand for lotteries with very small wagers[1] and very large payoffs.

Another class of models postulates that gambling has some direct consumption value overlaid on its monetary implications (Conlisk, 1993; Simon, 1998). These models suggest that the utility of gambling derives from some value function (i.e., gambling is fun) that is independent of its implications for actual expected changes in income or wealth. Although these models are no doubt important in explaining the recent growth of gambling in the U.S., especially among the elderly, they suffer from the lack of a variable that can independently measure the direct consumption value of gambling. Moreover, not all gambling is fun, and this model does not explain gambling in the absence of this consumption value.

Still another important explanation is that the gambler's subjective probability is different than either the objective probability or probability that is implicit in the wager/payoff ratio. For example, a gambler may consider himself "lucky," or may have better or inside information, or

[1]To be clear, the term "wager" refers to the bet itself, or the amount that could be lost.

think that he does, or have better skill at understanding and playing the gambling game than his opponents. This class of motivations for gambling is based on the presumption that the gamble is, or is perceived to be, unfair for some consumers, but this explanation does not require a formal model to understand.

The present paper takes the position that, while the last explanation is important, economists are most interested in explaining behavior related to fair gambles. Thus, the present paper addresses the demand for fair gambles. It also takes the position that, while a direct consumption value of the gambling experience clearly adds to our understanding of the demand for gambling, it cannot represent the basic model since it is not rooted in the desire for an income gain. That is, the value of gambling still derives from the underlying possibility of an actual income gain because without that, any entertainment or "dream" value of gambling would be short-lived or substantially diluted. This position is consistent with empirical findings that the demand for gambling increases with the size of the expected monetary reward (Kearney, 2004).

Furthermore, the paper takes the position that, while indivisibilities and market imperfections may also contribute to understanding the demand for gambling in certain circumstances, these models apply only to the situations specified and lack the generality that is necessary to explain common gambling behavior. Also, since the impetus for some of these models is to establish a mathematical argument for a non-concave utility function, they may lack the intuition necessary to be convincing explanations. For example, Friedman and Savage (1948) specify a non-concave segment in the utility function but do not explain how or why it originates.

Finally, economic theory generally presumes consumer sovereignty and economic rationality. This means that, while each consumer's behavior can be seen as maximizing utility, each consumer defines the determinants of his or her own utility function. In a similar way, each consumer defines the perspective or context with which utility maximization is achieved. While some consumers may adopt one perspective or context, others may adopt another. As has been demonstrated convincingly with the empirical evidence underlying prospect theory, perspective or context can have important implications for economic behavior (Kahneman and Tversky, 1981).

Thus, this paper suggests that, while existing models may identify a number of useful reinforcements to the basic demand for gambling, there exists a still more fundamental understanding of this phenomenon that has so-far escaped identification. That is, nowhere in the literature is there a model that simultaneously (1) motivates the demand for gambling primarily based on expected changes in income or wealth,[2] (2) is potentially applicable to almost all consumers, (3) preserves the assumption of concavity, (4) explains why consumers sometimes gamble and purchase insurance at the same time, and (5) has compelling intuitive appeal.

Model

The appeal of gambling is not merely the prospect of obtaining additional income, but the prospect of obtaining "something for nothing." This home-spun motivation has a direct translation into economic theory. It suggests that the context of gambling is the labor market, and that for many, gambling is principally a way to obtain additional income without having to

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[2]At this point, the paper drops the reference to "wealth" as the alternative argument in the utility function. It should be assumed that any future reference to "income" could just as readily be a reference to "wealth."

work for it. Thus, the expected benefit from gambling is not merely the chance of obtaining additional income, but the chance of obtaining additional income for which the consumer does not need to work.

The behavioral intuition of the decision to accept a gamble or lottery can be explained using the standard labor supply model. The consumer-worker derives utility from income, y leisure, l. At wage rate w, he faces a labor market constraint on his earnings based on the total amount of time he has available for both work and leisure. The total time available is normalized to unity, so the individual's problem can be written as:

(1) $\max u(y, l)$ subject to $y = w(1 - l), 0 \leq l \leq 1$.

The utility function is assumed to be continuous, twice differentiable, and strictly concave, so that, u_l, $u_y > 0$, u_{ll}, $u_{yy} < 0$ and $u_{ll}u_{yy} - u^2_{ly} > 0$. For example, a standard Cobb-Douglas utility function could represent this relationship. The consumer maximizes utility at (y^*, l^*).

Figure 1 illustrates this individual's problem and solution. It is clear that, as the worker gives up leisure from (0,1), utility from earnings increases until the point (y^*, l^*) and decreases thereafter. Thus, the consumer maximizes utility by working until (y^*, l^*) is achieved.

The relationship between utility and earnings alone is shown in Figure 2 as $u^e(y, l)$, indicating that utility from working to obtain income is maximized at y^* level of earnings. If it is assumed that the consumer has maximized utility by working and has y^* in earnings, then in the next step, it would be possible to consider whether the consumer would spend some of these earnings on a gamble. If so, it would be necessary to establish the relationship between utility, and gains and losses of *unearned* income from y^*. This relationship is represented by $u^u(y, l \mid l = l^*)$ in Figure 2. This utility function exhibits the standard concave functional form necessary to explain the purchase of fair insurance and is derived from the original utility function simply by holding leisure constant and allowing income to vary. Note that this would be the appropriate functional basis for evaluating the expected utility gain from purchasing fair insurance against a loss of income from y^* because the loss of income that is being insured against here does not also entail a gain in leisure time.

The decision to gamble depends on whether the consumer-worker considers the context of the gamble the gain in utility measured from a reference point on $u^u(y, l \mid l=l^*)$ or the gain in utility from a reference point on $u^e(y, l)$. That is, it depends on whether the gain in utility is simply the gain from additional goods and services that can be purchased with the additional income *that is simply given to the consumer*, or the gain from the additional goods and services that can be purchased with additional income *for which it was not necessary for the consumer-worker to sacrifice leisure to obtain*, respectively. If the reference is u^u, then the consumer-worker would not gamble, but if the reference is u^e, then the consumer-worker might exhibit a demand for gambling.

For example, assume that the consumer is at (y^*, l^*) and is confronted with a fair gamble where he would win y^w in additional unearned income with a 50 percent probability and lose y^w with that same probability. This gamble could represent a fair coin-toss game, where the consumer bets y^w on heads and the payoff is $2y^w$ if heads appears, otherwise 0. These gains and losses are illustrated in Figure 2. Expected utility of unearned income with the gamble is

$$(2) \qquad E(u_g) = \mathrm{pr}_{\mathrm{win}}\, u^u(y^* + y^w, l \mid l=l^*) + (1 - \mathrm{pr}_{\mathrm{win}})u^u(y^* - y^w, l \mid l=l^*)$$

$$= \tfrac{1}{2}\, u^u(y^* + y^w, l \mid l=l^*) + \tfrac{1}{2}\, u^u(y^* - y^w, l \mid l=l^*),$$

and without the gamble is

(3) $$E(u_n) = u^u(y^*, l \mid l{=}l^*)$$

$$= \tfrac{1}{2}\, u^u(y^*, l \mid l{=}l^*) + \tfrac{1}{2}\, u^u(y^*, l \mid l{=}l^*).$$

The net gain in utility if he makes the gamble is, therefore,

(4) $$E(u_g) - E(u_n) = \tfrac{1}{2}\,[u^u(y^* + y^w, l \mid l{=}l^*) - u^u(y^*, l \mid l{=}l^*)]$$

$$+ \tfrac{1}{2}\,[u^u(y^* - y^w, l \mid l{=}l^*) - u^u(y^*, l \mid l{=}l^*)].$$

Since the loss of utility from making the wager is evaluated on a steeper portion of u^u than is the payoff gain, the net utility gain is negative and the utility-maximizing consumer-worker would not make this gamble.

On the other hand, if the consumer-worker were to consider as the context of the gamble the alternative, but standard, means for obtaining greater income in an economy characterized by scare resources, then he would compare the gain in utility through winning unearned income in a gamble against the reduction of utility that would be required to obtain the same income gain through working. If a utility gain existed, then the net-utility-maximizing consumer-worker would accept the gamble.

Here, expected utility with the gamble is again represented by equation (3)

(3) $$E(u_g) = \tfrac{1}{2}\, u^u(y^* + y^w, l \mid l{=}l^*) + \tfrac{1}{2}\, u^u(y^* - y^w, l \mid l{=}l^*),$$

but without the gamble the context becomes

(5) $$E(u_w) = \tfrac{1}{2}\, u^e(y^* + y^w, l^* - l^w) + \tfrac{1}{2}\, u^u(y^*),$$

where l^w is the leisure foregone to obtain an additional y^w at that wage rate, or the additional amount of work that is normally required to obtain y^w in additional income in an economy characterized by scarcity. Note that the context for the loss of y^w remains the utility of y^* because the consumer/worker understands that he has already earned y^* and cannot gain any additional leisure if he loses income in a gamble. The net gain in utility from the gamble is, therefore,

$$(6) \qquad E(u_g) - E(u_w) = \tfrac{1}{2}\,[u^u(y^* + y^w, l \mid l{=}l^*) - u^e(y^* + y^w, l^* - l^w)] + \tfrac{1}{2}\,[u^u(y^* - y^w, l \mid l{=}l^*) - u^u(y^*)].$$

In equation (6), the first term on the right hand side is the expected gain from receiving a payoff for which the consumer is not required to work, and the second term is the expected loss from making the wager. In Figure 2, the expected gain exceeds the expected loss, therefore, the net utility gain is positive and the wager would be made.

Discussion

This model has a number of appealing features. First, it is based on the expected utility of changes in income from gambling, rather than on any conceptualization of the direct utility value of the gambling experience. Second, it is based on circumstances that could apply to most people–those who must work for a living–rather than situations requiring specific indivisibilities or market imperfections that may be relatively uncommon.

Third, it preserves the intuitively-appealing and theoretically-necessary concavity of the utility function for unearned income, consistent with the idea that a consumer values *being given* an additional dollar of income more if he were poor than if he were rich. Fourth, it is consistent with the often observed phenomenon that consumers gamble and purchase insurance at the same time. This is because the net gain from insurance is measured from the u^u function alone, while the net gain from gambling incorporates both functions. Finally, it is consistent with the folk wisdom that an individual gambles because he wants to obtain "something for nothing," which implicitly suggests a labor market context for the gambling winnings. Thus, the model has a certain basic intuition.

The model in this paper is also consistent with a fundamental asymmetry related to the timing of the wager and the payoff. In order to make a wager, the consumer generally must already have earned income. As a result, the consumer does not choose to forego both income and work in order to make the wager, but must forego only income, implying that the consumer must evaluate the expected utility cost of the wager according to the utility function for "unearned income." In contrast, the payoff has not (yet) been earned. Therefore, there is an ambiguity regarding whether the consumer views the context of the utility gain (in unearned income from the payoff) from the reference point of the starting level of utility, or from the utility level after working for an equivalent gain in income–the normal context for gaining income. This ambiguity means that one consumer might evaluate the expected gain from the payoff according to the utility function for "unearned income" alone, and choose not to gamble; while another might evaluate the expected gain in "unearned utility" from the payoff compared with the utility from earning the same amount of income in the labor market, and choose to gamble.[3]

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[3]Another possible specification of the alternative to gambling is simply *earning* the income gain under both states, or

(5') $$E(u_{w'}) = u^e(y^* + y^w, l^* - l^w) = ½\, u^e(y^* + y^w, l^* - l^w) + ½\, u^e(y^* + y^w, l^* - l^w).$$

Therefore, the decision to purchase insurance would depend on whether the following expression is positive or not:

(6') $$E(u_g) - E(u_{w'}) = ½\,[u^u(y^* + y^w, l \mid l{=}l^*) - u^e(y^* + y^w, l^* - l^w)] + ½\,[u^u(y^* - y^w, l \mid l{=}l^*) - u^e(y^* + y^w, l^* - l^w)].$$

Although the timing asymmetry argues against this specification, it is possible that consumers who are oriented toward the labor market context would be thinking in these terms. If so, this

This theory has an important predictive component. For those who are oriented toward the labor market perspective, the theory suggests that the demand for gambles will tend to be greater among those for whom additional income is more costly to obtain in terms of leisure forgone, that is, for lower wage workers. This is because, the cost (in terms of leisure forgone) of obtaining a certain amount of additional income by working for it is greater for low wage workers than for high wage workers, therefore, the leisure savings are greater for low wage workers if the additional income is obtained through a gamble. This prediction is consistent with studies that have found that low income households spend a greater proportion of their income on state lotteries than do middle or high income households (Clotfelter et al., 1999; Kearney, 2004).

The theory further predicts that, again for those who have a labor market perspective, those who dislike their jobs or who value their leisure time highly would be more likely to gamble than those for whom income and leisure are closer substitutes. As a result, we might expect that those who hold blue collar jobs that are physically demanding to be more likely to gamble than those white collar workers with relatively pleasant jobs, wages held constant. Or, it might suggest that those who value leisure time and dislike work would be more likely to view the gamble from a labor market perspective and, therefore, be more likely to gamble than those who are relatively indifferent about income and leisure.

The labor market context can be generalized to represent costs of gaining additional

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specification would increase the demand for gambles compared with the specification using $E(u_w)$, since the reference utility level for the wager would be at a lower level implying a smaller expected loss of utility from making the wager. For some gambles, however, the wager itself may represent an expected utility gain. If so, the consumer would gain by losing, which is not reasonable and represents another argument favoring the original specification.

income in terms other than the leisure forgone. For example, this model is consistent with the prediction that those for whom obtaining additional income is especially difficult–for example, those who are discriminated against in the labor market, or those who have difficulty finding employment–would be more likely to gamble. This prediction is consistent with studies that have found that minorities and individuals with previous convictions are more likely to gamble (e.g., Welte et al., 2004).

The theory also predicts that the demand for fair gambles with large payoffs and small wagers will be greater than the demand for fair gambles where payoffs and wagers are similar or equal, all other things held constant. This is because, for those who adopt a labor market context, the utility gain from the payoff increases with the size of the payoff, and the utility loss diminishes with as the wager becomes smaller. This is consistent with studies that have found that the demand for gambles increases with the size of the monetary reward (Kearney, 2004) and that have found that lotteries and raffles are by far the most popular forms of gambling (Welte et al., 2004).

The new theory also avoids the technical inconsistency within the conventional model that was identified by Rabin (2000) and Rabin and Thaler (2001). These researchers note that, because conventional theory is derived from a single continuous utility function for unearned income, if a consumer were to reject a certain low wager/low payoff gamble (such as a $10 wager for a 50 percent chance at a $21 payoff), the same gambler must also reject another higher wager/high payoff gamble (such as a $100 wager for a 50 percent chance at, say, a $1 million payoff) in order for the conventional theory to hold. Since rejecting the latter gamble appears to be unreasonable, this generates an inconsistency in conventional theory. Under the new theory, the decision to gamble would depend on 2 utility functions instead of 1, thus it would be possible under the new theory for the consumer reasonably to reject the former gamble and accept the latter one. That is, the new theory would capture not only the utility of these payoffs, but also how easy it would be to earn an additional $21 compared with how difficult it would be to earn an additional $1 million. As a result, there is no similar inconsistency within the new theory.

It should be noted, however, that this theory is not intended to explain why some people become pathological or habitual gamblers. Nevertheless, it does address the issue of the initial attraction to gambling and suggests why habitual gamblers might get started down their path.

In summary, no single theory exists to explain the demand for fair gambles generally. Existing theories either focus on special circumstances–indivisibilities or market imperfections–or suggest explanations that are primarily designed to resolve the issue of the insurance-buying gambler mathematically. The most promising existing model (Conlisk, 1993) can only explain a portion of present demand. The new model of the demand for gambling presented in this paper represents a more universal explanation and is consistent with empirical studies that describe those who are more likely to gamble. It is based on the basic economic assumption of scarce resources, preserves the concept of diminishing marginal utility of income, can serve as the foundation for other theories, and perhaps most importantly, has compelling intuitive appeal.

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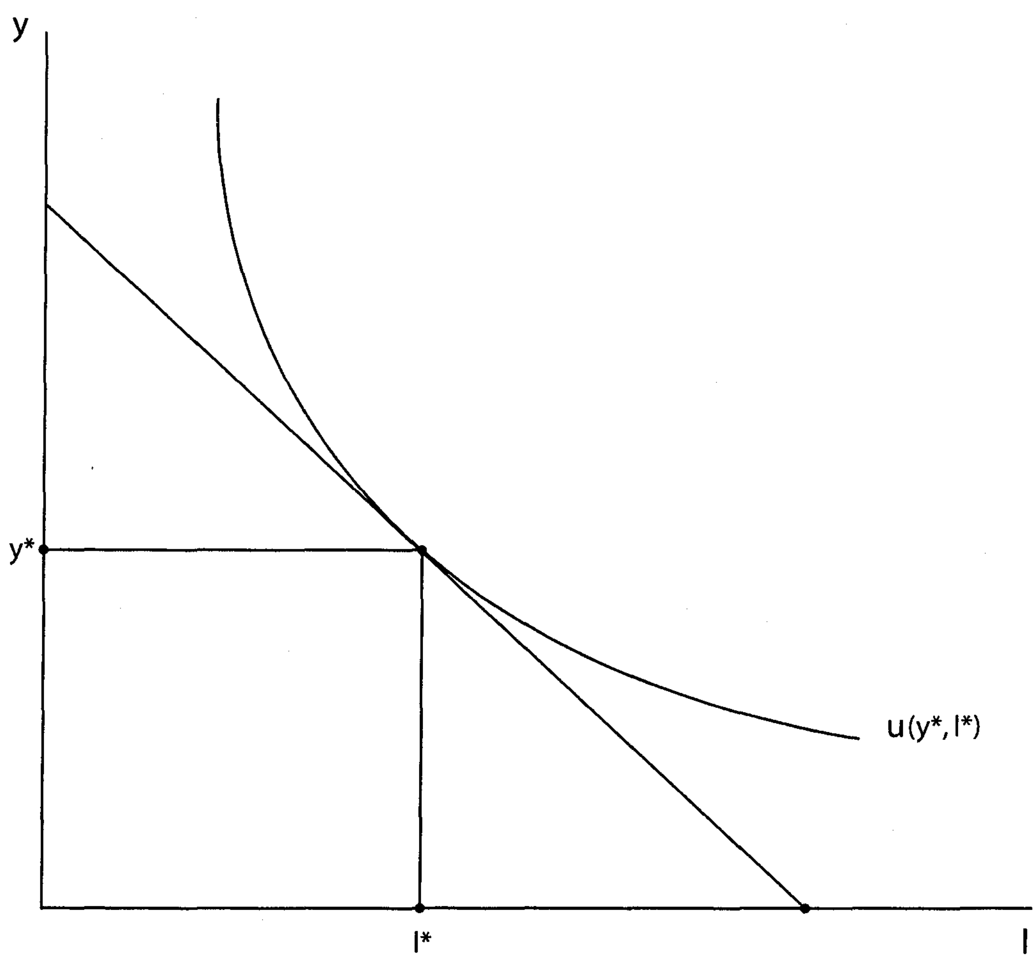

Figure 1
The Consumer-Worker's Optimum

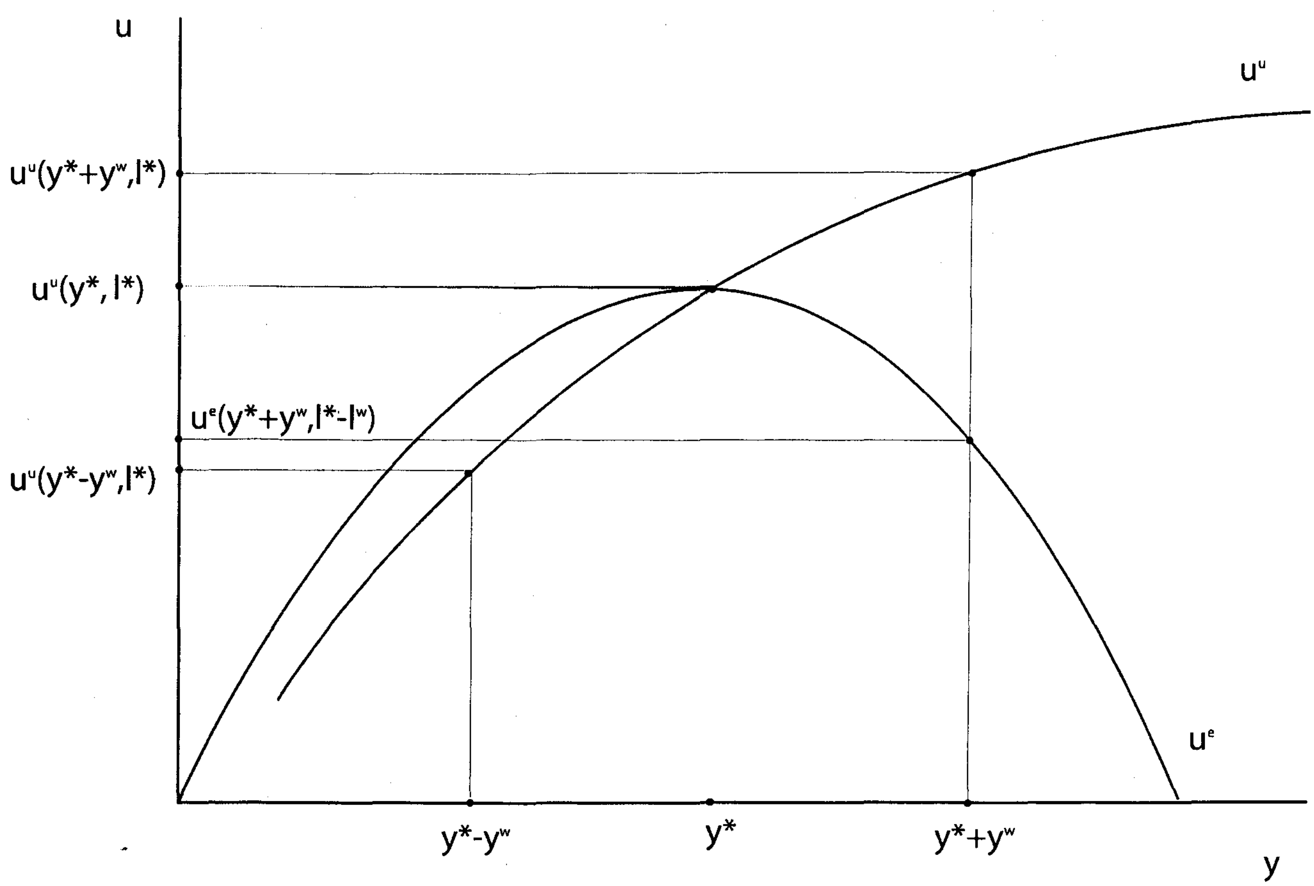

Figure 2
Net Gain From a Fair Gamble