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Abstract

Staggered price and staggered wage mechanisms are commonly viewed similar in generating persistent real effects of monetary shocks. In this paper, we distinguish these two mechanisms with individuals' optimizing behavior being explicitly taken into account. We show that, although the dynamic price and wage setting equations are alike, a key parameter governing persistence in these two equations is linked to the underlying preferences and technologies in very different ways. Consequently, the two mechanisms have quite different implications on persistence. While the staggered price mechanism by itself is incapable, the staggered wage mechanism has a much greater potential of generating persistence. (*JEL* E32, E32, E52)

The impact of monetary policy shocks on the magnitude and duration of business cycles has been an important and challenging issue that concerns economists and policy makers. Recent empirical studies such as Christiano, Eichenbaum, and Evans (1998) reveal that monetary shocks can have large and long-lasting effects on real activities. Yet, it is extremely difficult for economists to identify monetary transmission mechanisms that can generate these observed persistent real effects.[1]

In a seminal paper, Taylor (1980) proposes a staggered wage mechanism to help solve the persistence issue. In his model, nominal wage contracts are signed by firms and labor unions in a staggered fashion, that is, not all wage decisions in the economy are made at the same time, and each wage rate, after being set, is fixed for a short period of time (for example, a year). As summarized by Taylor (1998), there is much empirical evidence that price and nominal wage contracts are staggered. Taylor (1980) shows that this staggered wage setting mechanism can lead to endogenous wage inertia and thereby persistence in employment movements following a temporary shock. Taylor (1980) states the intuition behind this result as follows:

> Because of the staggering, some firms will have established their wage rates prior to the current negotiations, but others will establish their wage rates in future periods. Hence, when considering relative wages, firms and unions must look both forward and backward in time to see what other workers will be paid during their own contract period. In effect, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another ... contract formation in this model generates an inertia of wages which parallels the persistence of unemployment.

More recently, Chari, Kehoe, and McGrattan (CKM) (1996) carry Taylor's (1980) intuition to a general equilibrium environment, but find that a staggered *price* mechanism

[1] Although models with information lags and price stickiness are shown to be quite successful in generating output fluctuations driven by monetary shocks, the resulting effects are usually contemporaneous rather than persistent. See, for example, Lucas (1972), Lucas and Woodford (1993), Rotemberg (1996), and Yun (1996).

by itself cannot generate persistent real effects following monetary shocks, a puzzle given Taylor's (1980) insights. There are two interpretations of the CKM persistence puzzle. On one hand, CKM (1996) suggest that it is difficult to explain persistence based on a staggered price mechanism in a general equilibrium environment, and "we should look elsewhere for mechanisms to generate persistence." On the other hand, Taylor (1998) conjectures that, "the findings of Chari, Kehoe, and McGrattan (1998) may indicate that the monopolistic competition (stationary market power) model may not be sufficient as a microeconomic foundation." Behind these two arguments is a common perception that a staggered price mechanism and a staggered wage mechanism are embodied with the same implications on persistence: validating one mechanism is to validate the other, and refuting one is to refute the other as well.[2]

The purpose of this paper is to suggest a third interpretation of the CKM persistence puzzle. We find that a general equilibrium model along the lines of CKM (1996), incorporating a staggered wage rather than a staggered price mechanism, has a great potential of generating persistence. In other words, Taylor's (1980) original intuition stands up to a general equilibrium formalization, even when the underlying wage setting rule is based on the standard assumption of monopolistic competition. The microeconomic underpinning of our finding is the following. In a general equilibrium environment, the key parameter in the dynamic price and wage setting equations that governs persistence is a function of the underlying preferences and technologies of the economy. Although the two equations are apparently identical, this functional form, thereby the value of the parameter, differs fundamentally across the two mechanisms. In consequence, the two mechanisms have very different potentials in terms of generating real persistence following monetary shocks.

For the purpose of comparing these two mechanisms, we construct two stylized models in a symmetric way. Our first model features perfectly competitive goods markets, monopolistically competitive labor markets, and households endowed with differentiated labor

[2]This view is recently emphasized by Taylor (1998), who states that "the equations are essentially the same for wage setting and price setting."

skills setting nominal wages in a staggered fashion. Our second model, on the other hand, features perfectly competitive labor markets, monopolistically competitive goods markets, and firms producing differentiated products setting nominal prices in a staggered fashion. In the spirit of Taylor (1980), wage and price contracts in these models are assumed to be staggered, that is, not all wage (or price) decisions are made at the same time. Different from Taylor (1980) but in the spirit of CKM (1996), the wage and price setting rules are derived from households' and firms' optimizing decisions, and thus depend on underlying preferences and technologies of the economy. We show that the critical parameter governing persistence is the elasticity of relative wage (price) with respect to aggregate demand in the wage (price) equation. A larger value of this parameter corresponds to less persistence, because it implies a greater response of wage or price decisions to changes in aggregate demand conditions, thus, a faster adjustment of the aggregate wage or price index, and a quicker return of aggregate output back to its steady state following the initial impact effect. In the staggered wage mechanism, the value of this parameter is necessarily *less* than one, and decreases substantially with both the elasticity of substitution among differentiated labor skills in the production technology and the degree of relative risk aversion with respect to labor hours in households' preferences. The value of this parameter in the staggered price mechanism, in contrast, is necessarily *greater* than one, and increases with the degree of relative risk aversion in labor hours. In consequence, a staggered wage mechanism tends to generate persistent output response to monetary shocks but a staggered price mechanism does not.

The driving forces of these results can be best understood through comparing the optimal responses of households and firms to monetary shocks in the two models. In the staggered wage model, imperfectly competitive households choose nominal wages to balance their expected marginal dis-utility of labor hours and expected marginal utility of wage income during their contract period, taking into account the effect of their wage decisions on the demand for their labor services and thus their wage income as well. Since firms are price-takers, profit maximization requires that price equal marginal cost, which in turn is determined by the aggregate wage index. When a positive monetary shock occurs, aggregate wage index does not increase proportionally because of the staggering in wage setting, neither does the aggregate price level. Therefore, the real aggregate demand increases, raising both households' income and firms' demand for labor. For each household, the higher income reduces the marginal utility of income and the higher labor demand raises the marginal dis-utility of labor. Utility-maximization requires that households that can renew their contracts raise wage rates to re-balance their marginal utility of income and marginal dis-utility of labor. It turns out that the optimal percentage increase in relative wage rate is necessarily less than the percentage increase in aggregate demand, as long as households prefer smoothed labor hours and it is easy to substitute one type of labor for another in the production technology. This is true because, in our model economy, a higher relative wage reduces both demand for the corresponding type of labor (substitution effect) and associated wage income (income effect). These two effects both serve to restore the balance between marginal utility of income and of leisure. Thus the required increase in relative wage is small. In consequence, aggregate wage index rises slowly, and movements in aggregate output and employment, after their initial responses to the shock, are therefore also slow and persistent. The higher the elasticity of substitution between differentiated labor skills in production technology, and the larger the relative risk aversion with respect to labor hours in households' preferences, the smaller the optimal percentage change in wage rates, thus the more persistent the output and employment movements in response to a monetary shock. If the elasticity of substitution and the relative risk aversion are arbitrarily large, then the optimal percentage change in wage rates is arbitrarily close to zero, and movements in output and employment are arbitrarily close to random walks.

The microstructure underlying the staggered price mechanism is fundamentally different. There, imperfectly competitive firms choose prices to maximize expected profits during their contract periods, taking into account the effect of their price decisions on the demand for their outputs and thus their revenue as well. We show that the optimal pricing rule is a linear function of a firm's expected marginal production costs during the contract period.

That is, a higher price will be set if the firm is expecting higher marginal costs in the subsequent contract period. A positive monetary policy shock raises real aggregate demand because price level does not rise proportionally due to the staggering in price decisions. This increases demand for labor. The shock, on the other hand, renders the household more real income, who is thus willing to work less for each real wage. The outward shift of the labor demand curve and the inward shift of the labor supply curve both serve to drive up the real wage, thus the marginal production cost as well. It turns out that the equilibrium percentage increase in real wage exceeds the percentage increase in aggregate demand, as long as the household prefers smoothed labor hours. In other words, the marginal production cost rises more than the aggregate demand does. Profit-maximization requires that firms raise prices by a larger percentage whenever they have the chance to renew their contracts. In consequence, movements in aggregate output and employment, after their initial responses to the shock, are fast and transitory. Moreover, the larger the household's relative risk aversion in labor hours, the faster the change in marginal cost and aggregate price, and the less persistence of output movements in responding to monetary shocks.

In the literature, recent work focuses on the role of the staggered price mechanism in generating persistence in a general equilibrium environment. A leading example mentioned above is Chari, Kehoe and McGrattan (1996), who find that the staggered price mechanism by itself is not able to generate sufficient magnitude of persistence under empirically plausible parameter values, even when various features, such as convex demand curve, specific factor of production, and zero-income-effect utility function, are taken into account. This apparently puzzling finding, given Taylor's (1980) insights, has inspired a rapidly growing literature featured by adding other mechanisms to the staggered price contracts. Examples include Bergin and Feenstra (1998), who show that a staggered price mechanism, when combined with a non-CES production function and factor specificity, can generate more persistence only if the share of the fixed factor is sufficiently large; and Kiley (1997), who finds that there is no persistence unless the degree of increasing returns to scale at individual firms' level is implausibly large. In an important work, Blanchard (1983) constructs a model with multiple stages of production, in which firms at different stages set prices in a staggered fashion. He finds that this model is able to generate substantial amount of persistence through a "snake effect" along the production chain. More recently, Huang and Liu (1998) carry Blanchard's (1983) intuition to a general equilibrium setup by constructing a model with a chain-of-production structure, a perfectly competitive labor market, and a staggered price mechanism. They show that the "snake effect" does improve the model's ability of generating persistence, but the improvement is quantitatively small. Following the seminal work of Blanchard and Kiyotaki (1987), and Blanchard (1986), some attempts have been made to model staggered wage contracts in a dynamic general equilibrium setting (see, for example, Koenig (1997) and Erceg (1997)). Yet, little has been done to explore the microstructures that may distinguish the staggered wage from the staggered price mechanism. Therefore, it has not been made clear what economic forces are driving the persistence or lack thereof. The work presented here not only resolves the CKM persistence puzzle by clarifying the fundamental distinctions between these two mechanisms, but also illuminates such economic forces. It thus contributes to the literature on the transmission mechanisms of monetary policy.

The rest of the paper is organized as follows. Section I uses a simple partial equilibrium model to illustrate Taylor's (1980) original intuition, and contains a brief discussion on the CKM persistence puzzle. Section II and III presents two general equilibrium models with staggered wage and staggered price contracts, respectively, and elaborates, through analytical solutions, the distinctions of these two mechanisms in terms of their capabilities of generating persistence. Section IV further discusses the difference of the two mechanisms by examining a generalized version of each model in the previous two sections, with intertemporal links such as capital accumulation incorporated. Finally, Section V concludes the paper.

I. Taylor's Insights and the CKM Persistence Puzzle

In this section, we use a simplified version of Taylor (1980) to illustrate his intuition about the potential of such a model in generating persistence following a temporary shock. We then introduce the Chari, Kehoe and McGrattan (CKM) (1996) persistence puzzle to motivate our work in this paper.

A. *A Simple Model in the Spirit of Taylor (1980)*

As in Taylor (1980), prices are assumed to be set for $N > 1$ periods of time[3] and remain fixed during this "contract period." In each period, a fraction $1/N$ of firms can change their contract prices, and in doing so, they take into account of the prevailing price which, at any point of time, is an average of the N outstanding contract prices determined in the current and the last $N-1$ periods. Therefore, when setting new prices, firms would look at both the future and the past price decisions because these are part of the prevailing price. In a special case when $N = 2$, the price setting rule is fully described by the following equations:

$$p_t = \frac{1}{2}(x_t + x_{t-1}), \tag{1}$$

$$x_t = \frac{1}{2}(p_t + \mathrm{E}_t p_{t+1}) + \frac{\gamma}{2}(y_t + \mathrm{E}_t y_{t+1}) + \varepsilon_t, \tag{2}$$

where x_t denotes price (or wage) decision, p_t the prevailing price at date t, and y_t the current aggregate output. All variables are in log-terms, and ε_t is a shock to price setting. The system can be closed by assuming a static money demand equation $y_t = m_t - p_t$. We focus on the monetary shocks, and thus set $\varepsilon_t = 0$. The model can be reduced to a second order difference equation in x_t by substituting y_t and p_t using the money demand equation and equation (1), respectively. With an additional assumption that the money stock m_t follows a random walk process, a simple solution to this difference equation can be obtained, while the implied output dynamics is given by

$$y_t = a y_{t-1} + \frac{1+a}{2}(m_t - m_{t-1}), \tag{3}$$

where $a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}$. Since the autoregressive parameter a is a decreasing function of γ, a small value of γ corresponds to large output persistence. Taylor (1980, 1998) notes that the

[3] In Taylor's (1980) setup, there is no distinction between price setting and wage setting.

autoregressive output process arises because of the staggering in the price setting. Therefore, a model with staggered price (or wage) can potentially generate large amount of persistence, provided that the key parameter γ is small.

B. *The CKM Persistence Puzzle*

Chari, Kehoe, and McGrattan (CKM) (1996) carry Taylor's (1980) intuition to a general equilibrium business cycle model with staggered *price* contracts, and thereby linking the parameter γ to underlying preferences and technologies. However, perhaps surprisingly, they find that there is little persistence in output dynamics beyond the contract period, simply because the counterpart of γ in their model is too large for reasonable values of preference and technology parameters. And this result seems to be very robust.

The CKM (1996) study has stimulated much intellectual discussion, most of which focuses on combining various other mechanisms with the staggered price contracts in order to lower the value of γ. But it seems that a staggered price mechanism by itself cannot generate much persistence. Centering around this puzzling result, there are two strands of arguments. On one side, it is inferred that a staggered contract mechanism in the spirit of Taylor (1980) may not be able to explain persistence in a general equilibrium setup and people may have to look elsewhere for mechanisms that can do so. On the other side, it is conjectured that the conventional monopolistic competition framework may not be adequate for deriving the price setting equation.[4]

In this paper, we reassess the CKM persistence puzzle. We realize that a staggered *wage* mechanism, after all, may be quite different from a staggered *price* mechanism when individuals' optimizing behavior is explicitly taken into account. We find that, the difference does exist because the parameter γ is determined by different economic forces in models with these two mechanisms. The fine distinctions cannot possibly be uncovered unless the optimizing behavior of individual households and firms are explicitly modeled.

[4]As Taylor (1998) puts it, "the findings of Chari, Kehoe, and McGrattan (1998) may indicate that the monopolistic competition (stationary market power) model may not be sufficient as a microeconomic foundation."

In the following section, we explore the underlying microstructures of the staggered wage mechanism, and show that the value of γ can be small enough for empirically plausible values of preference and technology parameters. We thus argue that the mechanism has a great potential in generating large amount of persistence (even though the wage setting equation is based on the standard assumption of monopolistic competition).

II. A Model with Staggered Wage Contracts

We now describe a general equilibrium model with staggered wage contracts. Consider an economy populated by a large number of infinitely lived households endowed with differentiated labor services, and a large number of *identical* firms using each type of the labor services to produce a single consumption good. In each period t, the economy experiences a realization of shocks s_t, while the history of events up to date t is $s^t \equiv (s_0, \cdots, s_t)$ with probability $\pi(s^t)$. The initial realization s_0 is given.

The production technology for each firm is given by

$$Y(s^t) = L(s^t), \tag{4}$$

where the composite labor service is

$$L(s^t) = \left[\int_0^1 L(i, s^t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

with $L(i, s^t)$ being the labor services provided by household $i \in [0, 1]$. The parameter $\sigma > 1$ is the elasticity of substitution of labor services.

Firms behave competitively. Upon realization of s^t, they take output price $P(s^t)$ and wage rates $\{W(i, s^t)\}_{i\in(0,1)}$ as given, and choose output $Y(s^t)$ and labor services $\{L(i, s^t)\}_{i\in(0,1)}$ to maximize profits given by

$$\max \qquad P(s^t)Y(s^t) - \int_0^1 W(i, s^t)L(i, s^t)di,$$

subject to (4) and (5). The resulting labor demand functions are

$$L^d(i, s^t) = \left[\frac{W(i, s^t)}{\bar{W}(s^t)}\right]^{-\sigma} L(s^t), \tag{6}$$

where $i \in [0,1]$ and $\bar{W}(s^t) = \left[\int_0^1 W(i,s^t)^{1-\sigma}\, di\right]^{1/(1-\sigma)}$ is a wage index. Zero profit condition implies that

$$P(s^t) = \bar{W}(s^t) \tag{7}$$

Households are price-takers in the goods market, but behave as monopolistic competitors in the labor markets. They take the labor demand schedule as given and set wages in a staggered fashion. In particular, in each period t, there is a fraction $1/N$ of households that can choose new wages upon realization of s^t. Once a wage is set, it has to remain fixed for the subsequent N periods, as assumed in Taylor (1980). We sort the index of households so that those indexed by $i \in [0, 1/N]$ set new wages in period $t, t+N, t+2N, \cdots$, those indexed by $i \in (1/N, 2/N]$ set new wages in period $t+1, t+N+1, t+2N+1, \cdots$, and so on.

Household i has preferences represented by a utility function

$$U^i \equiv \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t|s_0)\{\log(C^*(i,s^t)) + V(L(i,s^t))\}, \tag{8}$$

where $C^*(i) = [bC(i)^\nu + (1-b)(M(i)/P)^\nu]^{1/\nu}$ is a CES composite of consumption and real money balances for household i, and $V(\cdot)$ is a strictly decreasing and strictly concave function. Upon realization of s^t, household i solves the utility maximization problem by choosing consumption $C(i,s^t)$, nominal money balances $M(i,s^t)$, and one-period nominal bonds $B(i,s^{t+1})$, taking goods price $P(s^t)$ and bond price $D(s^{t+1}|s^t)$ as given. If the household is a member of the cohort that can set new wages, it also chooses a nominal wage $W(i,s^t)$ for the subsequent N periods, taking the labor demand schedule (6) as given. The utility maximization is subject to the sequence of budget constraints

$$\begin{aligned} P(s^t)C(i,s^t) + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(i,s^{t+1}) + M(i,s^t) \leq \\ W(i,s^t)L^d(i,s^t) + \Pi(i,s^t) + B(i,s^t) + M(i,s^{t-1}) + T(i,s^t), t = 0,1,\ldots, \end{aligned} \tag{9}$$

and a borrowing constraint $B(i,s^t) \geq -\bar{B}$ for some large positive number $\bar{B}$, with initial conditions $M(i,s^{-1})$ and $B(i,s^0)$ being given. Here $B(i,s^{t+1})$ is a one-period nominal bond that costs $D(s^{t+1}|s^t)$ dollars in s^t and pays off one unit of currency in the next period if

s^{t+1} is realized, $\Pi(i, s^t)$ is household i's claim on firms' profits, and $T(i, s^t)$ is a nominal transfer to the household.

To close the description of the model, we need to specify the monetary policy. We assume that newly created money is equally distributed to all households via lump-sum transfers so that

$$\int_0^1 T(i, s^t)\, di = M(s^t) - M(s^{t-1}). \tag{10}$$

An equilibrium in this economy consists of a set of allocations $C(i, s^t)$, $M(i, s^t)$, $B(i, s^{t+1})$ for household $i \in [0, 1]$, and $Y(s^t)$ and $\{L(i, s^t)\}_{i\in[0,1]}$ for firms, together with prices $D(s^{t+1}|s^t)$, $P(s^t)$, $\bar{W}(s^t)$, and $\{W(i, s^t)\}_{i\in[0,1]}$ that satisfy the following conditions: (i)taking prices as given, firms' allocation solves their profit maximization problem; (ii)taking prices and all wages but his own as given, each household's allocation and wage solve its utility maximization problem; (iii)goods market, money market, and bond market clear; and (iv)money supply and transfer satisfy (10).

In what follows, we focus on a symmetric equilibrium in which all households in a given cohort make identical decisions. In this economy, there are complete contingent bond markets, and consumption and leisure time in each household's instantaneous utility function are additively separable. Consequently, in an equilibrium, consumption flows and real money balances are identical across all households.[5] Combining this observation with the market clearing conditions, we have $C(i, s^t) = C(s^t) = Y(s^t)$ and $M(i, s^t) = M(s^t)$ for all $i \in [0, 1]$. To simplify analysis, we impose a static money demand function for now and relax this assumption in section IV. In particular, here we assume

$$P(s^t)Y(s^t) = M(s^t). \tag{11}$$

In order to understand the mechanism by which staggered wage setting may help generate persistent output response following monetary shocks, we first consider a simple case without staggering, that is, the case with $N = 1$. The first order condition with respect to

[5] We assume, without loss of generality, that the initial distribution of wealth is identical across all households.

a household's wage decision implies

$$\frac{W(i,s^t)}{P(s^t)} = \frac{\sigma}{\sigma-1}\frac{-V_l(i,s^t)}{U_c(i,s^t)}, \tag{12}$$

where $-V_l(i,s^t)$ and $U_c(i,s^t)$ are the marginal dis-utility of labor and the marginal utility of consumption, respectively. Equation (12) reveals that the optimal real wage (or relative wage since $P = \bar{W}$ in equilibrium) is a constant "markup" over the marginal rate of substitution (MRS) between leisure time and consumption. If the marginal utility of leisure rises, the household increases its relative wage to reduce demand for its labor services; if the marginal utility of consumption is higher, then the household would like to lower wage in order to increase its labor income and hence consumption.[6] But with $N = 1$, all households make identical wage decisions in a symmetric equilibrium, and thus $W(i,s^t) = \bar{W}(s^t) = P(s^t)$ and $L(i,s^t) = L(s^t)$. In this case, the real wage is always constant and a monetary shock would only result in proportionally higher price level, leaving real variables unchanged.

However, if $N > 1$, when a cohort of households make their wage decision, the rest $N-1$ cohorts of households cannot set new wages. Thus, by raising its wage $W(i,s^t)$, the relative wage is also increased, and household i would face a reduced demand for its labor services and a lower wage income (since $\sigma > 1$). Before we turn to the N-period optimization condition for wage setting, we first develop a quantitative measure of the contemporaneous response of relative wage to a given aggregate demand shock, assuming that household i takes the wage index as given in making its wage decision, and that there is no forward or backward looking effects. The latter assumption is to be relaxed later. Notice that the wage decision equation (12) can be rewritten as follows

$$\frac{W(i,s^t)}{\bar{W}(s^t)} = \frac{\sigma}{b(\sigma-1)}\left\{-V_l\left[\left(\frac{W(i,s^t)}{\bar{W}(s^t)}\right)^{-\sigma} Y(s^t)\right]\right\} Y(s^t), \tag{13}$$

where we have used the fact that the utility in the composite consumption is logarithmic, and imposed the equilibrium conditions (6), (7), (11), and $C(i,s^t) = Y(s^t) = L(s^t)$ for all $i \in [0,1]$. Suppose now that there is a monetary shock that raises the money stock.

[6]Since the labor demand elasticity $\sigma > 1$, a lower wage $W(i,s^t)$ is associated with higher labor income.

Since the wage index does not rise proportionally due to the staggering in wage setting, the real aggregate demand $Y(s^t)$ rises. If household i's relative wage remains constant, this will raise the demand for labor services $L^d(i, s^t)$, resulting in a higher dis-utility of working. Thus, household i has to increase its relative wage in order to maintain the equality in (13). Clearly, the equilibrium relative wage is a fixed point of the function $f(x,Y) \equiv \frac{\sigma}{b(\sigma-1)}\{-V_l[x^{-\sigma}Y]\}Y$ with respect to $x \equiv W/\bar{W}$. That is, the equilibrium relative wage x^* is a solution to $f(x^*,Y) = x^*$.

We now calculate how much the relative wage has to be raised in response to a given demand shock in order to maintain the equality in (13). Total differentiation of this equation gives us $dx = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial Y}dY$, with x and $f(x,Y)$ defined above. This in turn implies that the elasticity of relative wage with respect to aggregate output is given by

$$\epsilon_{x,Y} \equiv \frac{dx}{dY}\frac{Y}{x} = \frac{1+\xi}{1+\sigma\xi}, \tag{14}$$

where $\sigma > 1$ is the elasticity of substitution of different types of labor services, and $\xi \equiv \frac{V_{ll}L(i)}{V_l} > 0$ measures the relative risk aversion with respect to labor hours in the utility function. Since $\epsilon_{x,Y} < 1$, a one percentage change in aggregate output results in less than one percentage change in relative wage. Additionally, $\epsilon_{x,Y}$ is decreasing in both ξ and σ. The intuition of this result is very simple. Given $\sigma > 1$, a stronger incentive for a household to smooth its labor hours (a higher ξ) leads to less incentive to adjust its relative wage. On the other hand, given $\xi > 0$, a larger value of σ implies a greater reduction in labor demand for a given change in relative wage, and thus, in response to an aggregate demand shock, less wage increase is necessary in order to maintain the equality in (13).

We now turn to analyze the intertemporal forward- and backward-looking behavior of the households when they set their wages. The first order condition with respect to wage decision for $N \geq 1$ is given by

$$W(i,s^t) = \frac{\sigma}{\sigma-1}\frac{\sum_{\tau=t}^{t+N-1}\sum_{s^\tau}\beta^{\tau-t}\pi(s^\tau|s^t)(-V_l(i,s^\tau))L^d(i,s^\tau)}{\sum_{\tau=t}^{t+N-1}\sum_{s^\tau}\beta^{\tau-t}\pi(s^\tau|s^t)[U_c(i,s^\tau)/P(s^\tau)]L^d(i,s^\tau)}.$$

Thus, the optimal wage for household i is a constant "markup" over the ratio of two items. The first is a weighted average of the future N-period marginal utility of leisure, and the second is a weighted average of the future N-period marginal utility of income, where the weights are both given by the normalized demand for household i's labor services. Clearly, if $N = 1$, this equation reduces to (12).

In order to gain further insights in analyzing the wage decision rule, it is helpful to log-linearize this equation around a steady state, and impose $\beta = 1$ to obtain a linearized version of the wage setting rule

$$w_t = \sum_{s=1}^{N-1} b_s w_{t-s} + E_t \sum_{s=1}^{N-1} b_s w_{t+s} + \frac{\gamma}{N-1} E_t \sum_{s=0}^{N-1} y_{t+s}, \tag{15}$$

where lower-case variables denote log-deviations of the corresponding upper-case variables from their steady state values, and the date-event argument of each variable s^t is replaced by a subscript t to save notations. In this equation, the weights on lagged and forward wages are given by $b_j = \frac{N-j}{N(N-1)}$, and the coefficient in front of output is given by

$$\gamma = \frac{1+\bar{\xi}}{1+\sigma\bar{\xi}}, \tag{16}$$

where $\bar{\xi}$ is the relative risk aversion with respect to labor hours in the utility function, evaluated at steady state labor hours. Thus, γ is the steady state counterpart of $\epsilon_{x,Y}$, the elasticity of relative wage with respect to aggregate output.

Equation (15) is apparently identical to Taylor's (1980) structural equation, except that the parameter γ in his paper is a structural parameter, while here it is a parameter determined by the underlying preferences and technologies. It is clear from this equation that when a household sets a new wage, he looks at both the wages set in the past $N-1$ periods and those expected to be set in the future $N-1$ periods. Since b_s is declining in s, the household assigns lower weights to those wages set either in the further past or in the further future. This backward and forward looking behavior means that a household try to keep in line with the peer groups when it decides on its own wages, as emphasized in Taylor (1980).

More importantly, the household who can adjust wage takes into account of changes in aggregate demand during the contract period. The parameter γ measures how much his wage would respond to aggregate demand conditions. The smaller the γ, the smaller the wage response to aggregate demand shocks, the slower the wage adjustment, and the more persistent movements of output following monetary shocks. Equation (16) reveals that the value of γ depends on both the elasticity of substitution among differentiated labor skills and the steady state relative risk aversion in labor hours. Given our assumptions that $\bar{\xi} > 0$ and $\sigma > 1$, γ is necessarily less than one, and it is substantially decreasing with both σ and $\bar{\xi}$. This is the reason why the staggered wage mechanism has a great potential in delivering persistence.[7]

The role of γ in helping generate persistence is best illustrated by obtaining explicit solutions to the equilibrium dynamics. We focus on the case with $N = 2$, so that equation (15) can be simplified as[8]

$$w_t = \frac{1}{2}w_{t-1} + \frac{1}{2}E_t w_{t+1} + \gamma(y_t + E_t y_{t+1}). \tag{17}$$

We use the log-linearized money demand equation

$$p_t + y_t = m_t,$$

and the zero profit condition $p_t = \bar{w}_t = (1/2)(w_t + w_{t-1})$ to get a second order difference equation in w_t

$$E_t w_{t+1} - \frac{2(1+\gamma)}{1-\gamma}w_t + w_{t-1} = -\frac{2\gamma}{1-\gamma}E_t(m_t + m_{t+1}).$$

With an additional assumption that m_t follows a random walk process, the solution to this difference equation is

$$w_t = aw_{t-1} + (1-a)m_{t-1},$$

where

$$a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}. \tag{18}$$

[7]The wage decision rule (15) also reveals that the effect of γ on persistence can be reinforced by the number of cohorts. A larger N tends to dampen wage response to changes in current and future aggregate outputs.

[8]Notice the similarity of this equation to the price setting rule in Taylor's (1980) simple model as described by (1) and (2).

Finally, with straightforward substitutions we get the output dynamics

$$y_t = ay_{t-1} + \frac{1}{2}(1+a)(m_t - m_{t-1}). \tag{19}$$

From (18) and (19), we see that when $\gamma = 1$, $a = 0$, and there is no persistence. If $\gamma = 0$, then $a = 1$, and output follows a random walk process. A smaller value of γ corresponds to higher persistence in output movements. From equations (16) and (18), it is clear that, for given $\bar{\xi} > 0$, the easier to substitute among labor skills (larger σ), the slower the wage adjustment in response to aggregate demand changes (smaller γ), and the higher persistence (larger a). On the other hand, for given $\sigma > 1$, the more smoothed pattern in labor hours (larger $\bar{\xi}$), the smaller the wage adjustment for given shocks (smaller γ), and thus the larger the persistence (larger a). The dependence of the persistence parameter a on σ and ξ is illustrated in Table 1[9]

Table 1. Values of the Persistence Parameter a for Different Combinations of σ and $\bar{\xi}$

	$\sigma = 2$	$\sigma = 5$	$\sigma = 10$	$\sigma = 20$	$\sigma = 50$	$\sigma = 100$
$\xi = 0.1$	0.02	0.08	0.15	0.25	0.40	0.52
$\xi = 0.5$	0.07	0.21	0.33	0.46	0.61	0.71
$\xi = 1$	0.10	0.27	0.40	0.53	0.67	0.75

To summarize this section, we have analyzed a model with staggered wage contracts in the spirit of Taylor (1980), and discovered the underlying determinants of Taylor's structural parameters. A crucial parameter that determines whether there is persistent real effects following a monetary shock is the elasticity of relative wage with respect to aggregate output. This elasticity is less than unity, and it is inversely related to both the elasticity of

[9]The parameter σ is difficult to calibrate so we take an agnostic approach by experimenting with a wide range of values of σ. Koenig (1997) calibrates σ based on the average markup of union wages over non-union wages in the U.S. over the period from 1950 to 1980 and obtains $\sigma = 20$, and Kim (1998) obtains an estimated value of $\sigma = 12$ using maximum likelihood method, while smaller estimated values of σ are reported based on more aggregated levels of data for labor skills. To get a sense of the $\bar{\xi}$ value, consider a log-utility for leisure, that is, $V(L) = \eta \log(1-L)$, so that $\xi = L/(1-L)$. In this case, $\bar{\xi} = 0.5$ is our benchmark choice, corresponding to a steady state fraction of hours devoted to market activity being 1/3.

substitution among differentiated labor skills in the production function and the degree of relative risk aversion with respect to labor hours in the utility function. If it is relatively easy to substitute among different labor skills and households prefer relatively smoothed labor hours, then, in response to a given aggregate demand shock, those households that can renew wage contracts choose not to adjust their wage rates very much, thus the change in output is slow and persistent.

In the next section, we present a model with staggered price contracts. We show that, although the dynamic price setting equation is apparently identical to the dynamic wage setting equation, it is unlikely for this model to generate persistence because the counterpart of γ is linked to the fundamentals of the economy in very different ways.

III. A Model with Staggered Price Contracts

We have thus far established that a general equilibrium model with staggered wage contracts in the spirit of Taylor (1980) can potentially deliver persistence, and the degree of persistence increases substantially with both the labor elasticity of substitution and individuals' desire to smooth their consumption of leisure time.

In this section, we construct a model with staggered price setting behavior on firms' side. The purpose is to illustrate the difference between this staggered price mechanism and the staggered wage mechanism considered above. We find that, when agents' optimizing behavior is explicitly modeled, these two mechanisms are fundamentally different in their roles of generating persistence. This result dispels the common perception that staggered price contracts work in identically ways as staggered wage contracts in terms of generating persistence.

The model is a simplified version of Chari, Kehoe, and McGrattan (1996). Specifically, consider an economy populated by a continuum of firms indexed by $j \in [0,1]$ using homogeneous labor services to produce differentiated goods. There is a representative household endowed with labor and consuming a composite of all types of goods produced by firms.

The representative household's utility function is given by

$$U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t|s_0) \log(C^*(s^t)) + V(L(s^t)),$$

where $C^*(s^t) = \left[bC(s^t)^\nu + (1-b)(M(s^t)/\bar{P}(s^t))^\nu\right]^{1/\nu}$ is a constant elasticity of substitution (CES) composite of consumption and real money balances, with $\bar{P}(s^t)$ being a price index, and $V(\cdot)$ is strictly decreasing and strictly concave. The consumption is a composite of all types of goods produced. That is,

$$C(s^t) = \left[\int_0^1 Y(j,s^t)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \equiv Y(s^t), \tag{20}$$

where $Y(j,s^t)$ is the output of firm $j \in [0,1]$, and $\theta > 1$ is the elasticity of substitution among all types of differentiated goods. Upon realization of s^t, the household solves the utility maximization problem by choosing consumption of each type of goods $Y(j,s^t)$, nominal money balances $M(s^t)$, and one-period nominal bonds $B(s^{t+1})$, taking goods prices $\{P(j,s^t)\}_{j\in[0,1]}$, competitive nominal wage rate $W(s^t)$, and bond price $D(s^{t+1}|s^t)$ as given. The utility maximization is subject to (20) and a sequence of budget constraints

$$\begin{aligned}\int_0^1 P(j,s^t)Y(j,s^t)dj + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(s^{t+1}) + M(s^t) \quad &\leq \\ W(s^t)L(s^t) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t), \qquad & t = 0,1,\ldots.\end{aligned}$$

The first order conditions imply that the demand function for good j is given by

$$Y^d(j,s^t) = \left(\frac{P(j,s^t)}{\bar{P}(s^t)}\right)^{-\theta} Y(s^t), \tag{21}$$

where $\bar{P}(s^t) = \left(\int_0^1 P(j,s^t)^{1-\theta}dj\right)^{\frac{1}{1-\theta}}$ is the price index. In addition, the optimal labor supply decision is

$$\frac{-V_l(s^t)}{U_c(s^t)} = \frac{W(s^t)}{\bar{P}(s^t)}. \tag{22}$$

Firms are price-takers in the labor market, but behave as monopolistic competitors in the product markets. Each firm takes the goods demand schedule (21) as given and sets price in a staggered fashion. All firms are divided into N cohorts based on their timing of price setting. If a firm j is in the cohort that gets the chance to adjust prices, it solves the N-period profit maximization problem

$$\mathrm{Max}_{\mathrm{P(j,s^t)}} \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} \mathrm{D}(s^\tau|s^t) \left[\mathrm{P}(j,s^t) - \mathrm{W}(s^\tau)\right] \mathrm{Y}^d(j,s^\tau),$$

subject to (21). The solution gives us the pricing rule

$$P(j,s^t) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} D(s^\tau|s^t)\bar{P}(s^\tau)^\theta W(s^\tau)Y(s^\tau)}{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} D(s^\tau|s^t)\bar{P}(s^\tau)^\theta Y(s^\tau)}. \tag{23}$$

Finally, the monetary policy and the money demand equation are assumed to be the same as in the staggered wage model, and an equilibrium can be defined analogously.

The pricing equation (23) states that firm j's optimal price is a markup over a weighted average of the future N-period marginal costs, where the marginal cost in each period is given by the nominal wage rate $W(s^\tau)$. The weights are simply the discounted value of the quantity demanded for good j in each of the N periods.[10] In the case with $N = 1$, the price equation reduces to a constant mark-up over the marginal cost. Since the price will be fixed for N periods once it is set, the relevant marginal cost is the weighted average of the marginal costs for all these periods. If the expected average marginal cost is high, a higher price will be set. If for some period $\tau \geq t$ the producer expects a higher demand, it assigns a higher weight on the marginal cost in that period when the pricing decision is made in period t. Therefore, when a firm sets price, it looks at both the change in marginal costs over the subsequent N periods and the effects of its pricing decision on current and future demand for its product. If marginal cost rises, the firm has to increase price to maintain the markup. However, a rise in its price relative to other firms would cause a reduction in the demand for its product, which in turn results in a loss of revenue (since the demand

[10] To see this, note that the demand schedule for goods (21) gives us $P(j,s^t)^\theta Y^d(j,s^t) = \bar{P}(s^t)^\theta Y(s^t)$. This relation along with the fact that the $P(j,s^\tau) = P(j,s^t)$ for all $\tau = t, t+1, ..., t+N-1$ implies that we can replace the term $\bar{P}(s^\tau)^\theta Y(s^\tau)$ in the pricing equation with $P(j,s^t)^\theta Y^d(j,s^\tau)$. Since this term appears on both the denominator and the numerator, the term $P(j,s^t)^\theta$ can be factored out and cancelled. Thus the weight becomes $D(s^\tau|s^t)Y^d(j,s^\tau)$, the discounted value of the quantity demanded.

elasticity $\theta > 1$). If the marginal cost consideration dominates the relative demand effect, the firm will raise price.

In what follows, we show that, with a staggered price setting mechanism, marginal cost consideration is the dominant force. Following a positive aggregate demand shock, marginal cost rises more than the change in aggregate output, and firms are forced to increase their prices by a large amount whenever they get the chance to do so. Thus, prices rise quickly and aggregate output returns to its steady state value when all firms finish adjusting prices. Consequently, there is no persistence.

To see why this happens, we calculate a measure of the sensitivity of real wage (which is the real marginal cost in this model) to changes in aggregate output. The equilibrium real wage clears the competitive labor market. The labor supply equation (22) can be rewritten as

$$\frac{W(s^t)}{\bar{P}(s^t)} = \left(\frac{1}{b}\right) \left[-V_l(L(s^t))\right] Y\left(s^t\right), \tag{24}$$

given the logarithmic form of the utility in consumption. The labor demand function in this model is given by

$$L(s^t) = \int_0^1 L(j, s^t)dj = \int_0^1 Y^d(j, s^t)dj = \left[\int_0^1 \left(\frac{P(j, s^t)}{\bar{P}(s^t)}\right)^{-\theta} dj\right] Y(s^t) \equiv G(s^t)Y(s^t), \tag{25}$$

where the second equality uses the production function, the third equality uses the output demand function, and the last equality is a definition of the term $G(s^t)$.

Figure 1 illustrates the labor market equilibrium. In both the labor supply and demand equations, the term $Y(s^t)$ is a shift variable representing the aggregate demand. In Figure 1, a change in aggregate output from Y_0 to Y_1 leads to a shift in both the labor supply and demand curves. The labor supply equation (24) shows that, for given labor demand, a one percentage increase in aggregate output Y causes an equal percentage increase in real wage (from point **A** to **B** in the diagram). The labor demand equation (25) shows that an increase in aggregate output causes a one-for-one increase in labor demand, shifting the labor demand curve to the right, and further pushing up the real wage via moving along the new labor supply schedule (from point **B** to **C**). The magnitude of the increase in real wage due to the latter effect can be calculated by totally differentiating the labor supply equation (24) with respect to real wage and labor. This yields an elasticity of real wage with respect to labor that equals $\xi \equiv V_{ll}L/V_l > 0$, the degree of relative risk aversion with respect to labor hours. The total effect of a higher aggregate output on real wage (from **A** to **C**) is then the sum of these two effects, that is

$$\epsilon_{w,Y} \equiv \frac{\partial(W/\bar{P})}{\partial Y} \frac{Y}{(W/\bar{P})} = 1 + \xi, \tag{26}$$

where $\epsilon_{w,Y}$ is the elasticity of real wage with respect to aggregate output, and it is necessarily greater than one unless $\xi = 0$, in which case the consumer is risk-neutral in labor hours.

This calculation suggests that, as aggregate demand rises following an increase in money stock, there is an upward shift of the labor supply curve due to income effect, which tends to raise the real wage for any given labor hours; meanwhile, since those firms that cannot adjust prices have to employ more workers in order to meet the higher demand for their products, there is an outward shift of the labor demand curve, which further drives up the real wage due to the substitution effect (as marginal utility of leisure increases). As shown in Figure 1, these two effects reinforce each other, raising the real wage, and hence the real marginal cost of production. It is clear that the magnitude of such an increase in real marginal cost exceeds the change in aggregate output, forcing firms to raise prices whenever they get the chance to do so.

The above mechanism differs from the mechanism in the staggered wage model. Given a one percentage change in aggregate output, the response of real wage and real marginal cost in this model is larger than one percent, while the response of relative wage in the staggered wage model is less than one percent, and can be even smaller for larger yet plausible values of labor elasticity of substitution and risk aversion with respect to labor. However, much of the confusion between the staggered wage mechanism and the staggered price mechanism arises because of the similarity of the linearized decision rules. The log-linearized version of the pricing equation is

$$p_t = \sum_{s=1}^{N-1} b_s p_{t-s} + E_t \sum_{s=1}^{N-1} b_s p_{t+s} + \frac{\gamma}{N-1} E_t \sum_{s=0}^{N-1} y_{t+s},$$

which is apparently identical to the linearized wage decision rule (15) in the staggered wage model, with w_t being replaced by p_t everywhere. The coefficients b_s are the same as in the wage staggering case, but the value of γ is different. Here,

$$\gamma = 1 + \bar{\xi},$$

where $\bar{\xi}$ is steady state value of the relative risk aversion parameter with respect to labor in the utility function. Although the intertemporal backward and forward looking effects work in the same way as in the staggered wage model, the staggered price model cannot while the staggered wage model can generate persistence, simply because of the different underlying determinants of the parameter γ. This is again best illustrated by obtaining an explicit solution to the output dynamics in the special case with $N = 2$ and $\beta = 1$:

$$y_t = a y_{t-1} + \frac{1}{2}(1 + a)(m_t - m_{t-1}),$$

where

$$a = \frac{1 - \sqrt{\gamma}}{1 + \sqrt{\gamma}}.$$

Since $\gamma > 1$, the value of a is necessarily negative, and there is no persistence in output dynamics.

We conclude this section by pointing out that staggered price setting and staggered wage setting are two fundamentally different mechanisms in transmitting monetary shocks. This difference lies at the microstructures of the models, and it cannot be revealed without explicitly studying agents' optimizing behavior.

In the next section, we consider a more general case where capital accumulation is incorporated and interest rate sensitive money demand equations are derived from households' optimizing behavior. It is shown that the qualitative results obtained so far stand.

IV. Models with Intertemporal Links

In the previous two sections, we have established that staggered wage contracts and staggered price contracts are two fundamentally different mechanisms in transmitting and propagating monetary shocks. Although the dynamic equations characterizing these two types of contracts are alike, the microstructures underlying the key parameter γ are quite different, leading to different predictions about how prices and real output respond to monetary shocks. While a staggered wage mechanism tends to generate persistent output response to monetary shocks, a staggered price mechanism does not.

For the purpose of exposition, we have abstracted in the last two sections from intertemporal links such as interest rate sensitive money demand and capital accumulation. In this section, we add in these intertemporal links to the models and show that the basic findings stand up to these generalizations. The models and calibration strategies are described in the Appendix. With intertemporal links added, analytical solutions to the models are difficult to obtain. Thus we resort to numerical methods by first log-linearizing the equilibrium conditions around a steady state, then solving the linearized system.

In what follows, we report the impulse response of aggregate output to a given monetary policy shock. The money supply process is specified as

$$M_t^s = \mu_t M_{t-1}^s,$$

$$\ln \mu_t = \rho \ln \mu_{t-1} + \varepsilon_t, \tag{27}$$

where $0 < \rho < 1$, and ε_t has an i.i.d. normal distribution with zero mean and finite variance. In calculating the impulse response, we choose the magnitude of innovation in the money growth rate (the ε_t term) such that money stock increases by 1% one year after the shock. The dynamic response of output is then expressed relative to the initial response.

Figure 2 plots the relative impulse response of output to a monetary policy shock in the model with staggered price setting. Output initially rises, but after a year, it returns to below the steady state. This result holds true for different degrees of asynchronization in price adjustment (that is, for different values of N). It confirms our findings in Section III, and is consistent with Chari, Kehoe, and McGrattan (1996).

Figures 3-5 show the dynamic response of output in the model with staggered wage contracts, for $N = 2, 4, 12$, respectively. Since it is difficult to calibrate σ, the parameter measuring substitutability among differentiated labor skills, we plot the impulse response for a plausible range[11] of σ. Specifically, we choose $\sigma \in \{2, 5, 10, 20\}$. In Section II, we find that a larger value of σ implies a smaller response of relative wages to aggregate demand shock and therefore slower adjustment of aggregate wage index, which corresponds to higher persistence in real output. Here we find the same is true when intertemporal links are added to the model. For $N = 4$, at the end of the first year following the monetary shock, output falls to 14% of the initial response if $\sigma = 2$, 34% if $\sigma = 5$, 54% if $\sigma = 10$, and 76% if $\sigma = 20$. In addition, the persistence is an increasing function of N, that is, a higher degree of asynchronization in wage setting corresponds to more persistence. For example, given $\sigma = 10$, the relative response of output one year after the monetary shock increases from 48% for $N = 2$ to 54% for $N = 4$ and 58% for $N = 12$. This result is also consistent with Taylor's (1980) original finding as well as our analytical solution in Section II.

We conclude this section by noting that the basic insights elaborated by the analytical solutions in the previous two sections stand up to the generalization of adding to the model capital accumulation and interest rate sensitive money demand. Compared to the model with staggered price setting, the model with staggered wage setting has much larger potential, both qualitatively and quantitatively, in generating persistent output movements following monetary policy shocks. Additionally, in the staggered wage model, persistence is an increasing function of both the elasticity of substitution among differentiated labor

[11] As noted in Section II Koenig (1997) obtains a value of $\sigma = 20$ based on the average markup of union workers' wage over non-union workers' wage in the United States during the period from 1950 to 1980, and Kim (1998) finds an estimate of $\sigma = 12$ using maximum likelihood method, while the estimated values of σ are smaller in the labor demand and income inequality literature because of higher level of aggregation in the data of labor skills (for example, Katz and Murphy (1992)).

skills and the degree of asychronization in wage adjustment.[12]

V. Conclusion

The seminal work of Taylor (1980) illustrates the potential of staggered nominal contracts in solving the persistence issue. Since Taylor (1980), it has been commonly viewed that staggered price and staggered wage contracts are two similar mechanisms, both capable of generating persistence. The recent influential work of Chari, Kehoe and McGrattan (CKM) (1996), however, shows that, with a general equilibrium formalization a staggered price mechanism by itself cannot generate real persistence, a puzzle in light of Taylor's (1980) insights. Revolved around this puzzle, much has been written focusing on adding other features to a staggered price mechanism, in the hope of generating more persistence. But little has been done to ask whether the two mechanisms are indeed embodied with the same implications on persistence in a general equilibrium environment.

In this paper, we have taken up this question and provided a resolution to the CKM persistence puzzle. Our main finding is that staggered price and staggered wage mechanisms,

[12] An important implication of the staggered wage model (with capital) is that real wage is negatively correlated with employment because price level is flexible while nominal wages are sticky. Empirical evidence on the cyclical properties of real wage is mixed. While some studies find that real wage is acyclical or weakly procyclical, Bernanke and Carey (1996) find that, using data for 22 countries during the Great Depression, nominal wages adjusted quite slowly to falling prices, resulting in rising real wages amid the dramatic reduction in employment and output. As noted by Friedman and Schwartz (1963), monetary shocks played an important role during the Great Depression. Therefore, Bernanke and Carey's (1996) finding suggests that real wage is countercyclical in response to monetary shocks. Additionally, Wouter J. den Haan (1996) finds that, using the postwar data, labor hours and real wages are negatively correlated in the short run but positively correlated in the long run, which is consistent with a model in which demand shocks dominate in the short run while supply shocks dominate in the long-run. Finally, a plot of the U.S. data during the period from 1980-1984 also suggests that real wage is high when output is low, and it is well-known that there is a major monetary contraction during that period.

after all, are embodied with very different implications on persistence when individuals' optimizing behavior is explicitly taken into account. The microeconomic underpinning of this result is that, although the dynamic price and wage setting equations are apparently identical, the key parameter that governs persistence in these two equations is linked to preferences and technologies in very different ways and thereby results in different persistence implications of the two mechanisms. While the staggered price model by itself is incapable, the staggered wage model has a much greater potential of generating persistence; and this difference cannot possibly be uncovered unless the optimizing behavior of individual households and firms are explicitly modeled.

APPENDIX

This appendix contains detailed descriptions of the models with capital accumulation. Here, we focus on the staggered wage model since the two models are sufficiently similar and the staggered price model is a special version of Chari, Kehoe and McGrattan (1996).

A. *Model Description and Definition of Equilibrium*

There is a representative firm who has access to a Cobb-Douglas production function:

$$F(K(s^t), L(s^t)) = K(s^t)^{\alpha} L(s^t)^{1-\alpha},$$

where $K(s^t)$ is the capital stock at state s^t, and $L(s^t)$ is the composite labor service given by equation (5) in Section II. Since the firm is a price taker, its profit maximization requires price equal to marginal cost, that is

$$P\left(s^t\right) = \tilde{\alpha}\bar{W}(s^t)^{1-\alpha}[P(s^t)r(s^t)]^{\alpha},$$

where $\tilde{\alpha}$ is an unimportant constant and $r\left(s^t\right)$ is the capital rental rate.

There are a continuum of households endowed with differentiated labor skills indexed by $i \in [0,1]$, each maximizing utility (8), subject to the sequence of budget constraints

$$P(s^t)C(i,s^t) + P(s^t)I(i,s^t)\left[1+\phi\left(\frac{I(i,s^t)}{K(i,s^{t-1})}\right)\right] + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(i,s^{t+1}) + M(i,s^t) \leq$$
$$W(i,s^t)L^d(i,s^t) + P(s^t)r(s^t)K(i,s^{t-1}) + \Pi(i,s^t) + B(i,s^t) + M(i,s^{t-1}) + T(i,s^t), \quad t = 0,1,\ldots,$$

where $L^d(i, s^t)$ is the labor demand function given by (6) and $I(i, s^t)$ is the investment of household i. The law of motion of capital stock is given by

$$K(i, s^t) = (1 - \delta)K(i, s^{t-1}) + I(i, s^t), \tag{28}$$

and the term $\phi(I/K)I$ represents capital adjustment cost.

Defining $q(i, s^t) = I(i, s^t)/K(i, s^{t-1})$ and the effective cost of capital $H(q) = 1 + \phi(q) + q\phi'(q)$. The first order conditions for the household problem are given by

$$\begin{aligned}
U_c(i, s^t) &= \lambda(i, s^t)\bar{P}(s^t), && (29) \\
U_m(i, s^t)/\bar{P}(s^t) &= \lambda(i, s^t) - \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\lambda(i, s^{t+1}), && (30) \\
Q(s^{t+1}|s^t) &= \beta\pi(s^{t+1}|s^t)\lambda(i, s^{t+1})/\lambda(i, s^t), && (31) \\
U_c(i, s^t)H(q(i, s^t)) &= \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t)U_c(i, s^{t+1})\{r(s^{t+1}) + \\
&\quad (1 - \delta)H(q(i, s^{t+1})) + q(i, s^{t+1})^2\phi'(q(i, s^{t+1}))\}, && (32)
\end{aligned}$$

$$\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} \beta^{\tau-t}\pi(s^\tau|s^t)V_L(L(i, s^\tau))\frac{\partial L^d(i, s^\tau)}{\partial W(i, s^t)} = \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} \beta^{\tau-t}\pi(s^\tau|s^t)\lambda(i, s^\tau)L^d(i, s^\tau)(1 - \sigma), \tag{33}$$

where $\lambda(i, s^t)$ is the Lagrangian multiplier associated with the budget constraint.

Equations (29) -(32) are standard first order conditions with respect to the household's choice of consumption, money balance, bond holding, and capital investment, respectively. Equation (33) corresponds to the wage setting rule. The left-hand side of this equation is the expected discounted marginal utility gain by increasing wage and thus reducing labor hours for the subsequent N periods, and the right-hand side is the expected present value of the loss in utility due to unemployed hours and hence reduced labor income. The wage is set in such a way that the gain and the loss are equal on the margin. Since there are complete contingent asset markets, each household's consumption and money balance decisions only depend on initial distributions of wealth. Without loss of generality, we assume that the initial holdings of wealth are identical across households. This assumption, along with the assumption that consumption and leisure are additively separable in the utility function, the equilibrium consumption and money balances are identical across households for each date t and realization of event s^t, and thus $\lambda(i, s^t) = \lambda(s^t)$ for all $i \in [0, 1]$ from equation (29).

An equilibrium for this economy is a set of allocations $\{C(i, s^t), I(i, s^t), M(i, s^t), B(i, s^{t+1})\}$ for household $i \in [0, 1]$, a set of allocations for the representative firm $\{Y(s^t), K(s^t)\}$, a demand function $\{l^d(i, s^t)\}$ for household $i \in [0, 1]$, together with a set of prices $D(s^\tau | s^t)$ for $\tau = t, \cdots, t + N - 1$, $P(s^t)$, $\bar{W}(s^t)$, and $W(i, s^t)$ for $i \in [0, 1]$ that satisfy the following conditions:

- Taking prices and all wages but his own as given, each household's allocation and wage solve its utility-maximization problem;
- Taking prices as given, the representative firm's allocation solves its profit-maximization problem;
- Capital market and goods market clear;
- Money supply process and transfers satisfy (10) and (27).

We are interested in a symmetric equilibrium in which all households in the same cohort make identical decisions. The equilibrium conditions can be reduced to a system of three equations, including the wage setting equation (33), an Euler equation for capital, and an Euler equation for money. Given the Markov money supply process (27), a stationary equilibrium for this economy consists of stationary decision rules which are functions of the state of the economy. The state at t must record wages set in the previous $N - 1$ periods in addition to the beginning-of-period capital stock and the growth rate of money supply. At any date t, before any decisions are made, there are $N - 1$ prevailing wage rates, which are set at period $t - 1$ back through period $t - N + 1$. Since households in the same cohort set the same wage, the wage rates in the state vector only depend on the period at which they are set, not on individual households' indexes. We denote by $W(s^t)$ the wage set in date-event s^t for periods t through $t + N - 1$, by $W(s^{t-1})$ the wage set in date-event s^{t-1} for periods $t - 1$ through $t + N - 2$, and so on. Since money supply is growing over time, the price and wage rates are non-stationary. We normalize all prices and wages by dividing them by the money stock. The state of this economy in date-event s^t is

$$\left[\frac{W(s^{t-1})}{M(s^t)}, \dots, \frac{W(s^{t-N+1})}{M(s^t)}, k(s^{t-1}), \mu(s^t)\right].$$

The decision variables for period t are aggregate consumption, $C(s^t)$; aggregate capital stock $K(s^t)$; and the normalized wage $W(s^t)/M(s^t)$ of the cohort of households that are setting their wages at period t for current and future periods.

B. *Calibration of Parameters*

In both models, the utility function is assumed to take the form

$$U(C, M/\bar{P}, L) = \log\left[bC^\nu + (1-b)(M/\bar{P})^\nu\right]^{1/\nu} + \eta\log(1-L).$$

The production function is

$$F(K, L) = K^\alpha L^{1-\alpha}.$$

The capital accumulation rule is

$$K(s^t) = I(s^t) + (1-\delta)K(s^{t-1}),$$

where $I(s^t)$ denotes investment, the relative price of which is $1 + \phi\left(\frac{I(s^t)}{K(s^{t-1})}\right)$, with the adjustment cost function given by

$$\phi\left(\frac{I}{K}\right) = \frac{\psi}{2}\left(\frac{I}{K}\right)^2.$$

Finally, money stock grows at an exogenous rate of $\mu(s^t)$, which follows the process

$$\log\mu(s^t) = \rho\log(\mu(s^{t-1})) + \varepsilon_t,$$

where ε_t is i.i.d normally distributed with mean zero and finite variance.

Parameters to be calibrated include preference parameters β, b, ν, η in both models, technology parameters σ in the staggered wage model, θ in the staggered price model, and α in both models, capital accumulation parameters δ and ψ, and finally monetary policy parameter ρ. The calibrated values of these parameters are presented in Table 2.

In Table 2, the preference parameters b and ν are obtained based on regression of the models' implied money demand equation

$$\log \frac{M(s^t)}{P(s^t)} = -\frac{1}{1-\nu}\log\left(\frac{b}{1-b}\right) + \log C(s^t) - \frac{1}{1-\nu}\log\left(\frac{R(s^t)-1}{R(s^t)}\right),$$

where $R(s^t) = \left(\sum_{s^{t+1}} D(s^{t+1}|s^t)\right)^{-1}$ is the gross nominal interest rate. The regression as performed in Chari, et al (1996) implies that $\nu = -1.56$, and $b = 0.98$ for quarterly U.S. data ranging from 1960:1 to 1995:4.[13]

The subjective discount factor β is chosen based on standard business cycle literature (for example, Chari, Christiano and Kehoe, 1994). Following Chari, et al. (1996), we choose α, δ, and η so that the model predicts an annualized capital-output ratio of 2.65 and an investment-output ratio of 0.23, and a share of time allocated to market activity of 1/3. We set $\theta = 10$, corresponding to a markup of 11%. The adjustment cost parameter ψ is selected so that the initial impulse response of investment to a monetary shock in the models is about 3.23 times as large as that of aggregate output. The parameter ρ is obtained from regression of the money growth process. Notice that the values of β, δ, and ρ have to be adjusted if we change the frequency of time interval. In particular, we set $\beta = 0.96^{1/N}$, $\delta = 1 - 0.92^{1/N}$, and $\rho = 0.57^{4/N}$ to reflect such adjustment. If $N = 4$, agents make decisions on a quarterly basis, but if $N = 12$, decision making is on a monthly basis. Finally, since there is no consensus on the values of σ, we choose a plausible range of this parameter.

[13] In our numerical simulations, we adjust the value of b whenever we change the period frequency (for example, from quarterly to monthly frequency). This is because consumption is a flow variable while the nominal money balance is a stock.

Table 2. Benchmark Parameters

Preferences:	$\nu = -1.56$,
$U(C, M/\bar{P}, L) = \log\left[bC^{\nu} + (1-b)(M/\bar{P})^{\nu}\right]^{1/\nu} + \eta\log(1-L)$	η and b adjusted
Technologies: $Y = K^{\alpha}L^{1-\alpha}$	$\alpha = 0.33$
Staggered wage model: $L = \left[\int L(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$	$\sigma \in \{2, 5, 10, 20\}$
Staggered price model: $Y = \left[\int Y(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$	$\theta = 10$
Capital Accumulation:	$\delta = 1 - 0.92^{1/N}$
$K_t = I_t + (1-\delta)K_{t-1}$, $\phi(I_t/K_{t-1}) = \psi(I_t/K_{t-1})^2/2$	ψ adjusted
Money Growth: $\log\mu(s^t) = \rho\log(\mu(s^{t-1})) + \varepsilon_t$	$\rho = 0.57^{4/N}$
Subjective discount factor	$\beta = 0.96^{1/N}$
Frequency of Price or Wage Adjustment	$N \in \{2, 4, 12\}$

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Figure 1. Real Wage Response to Aggregate Demand Shock

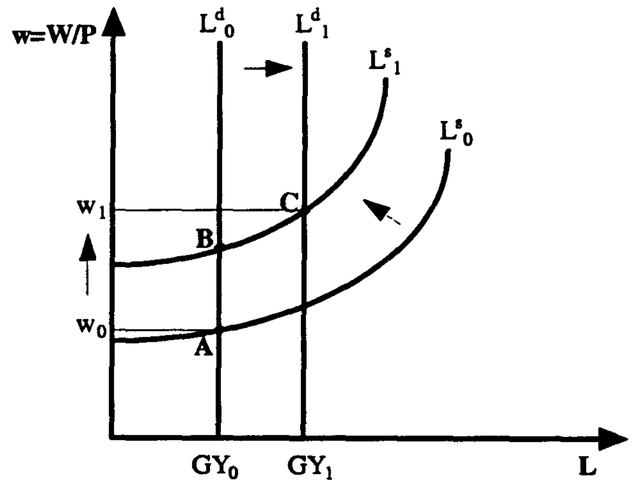

Figure 2: Impulse Response of Output in the Staggered Price Model

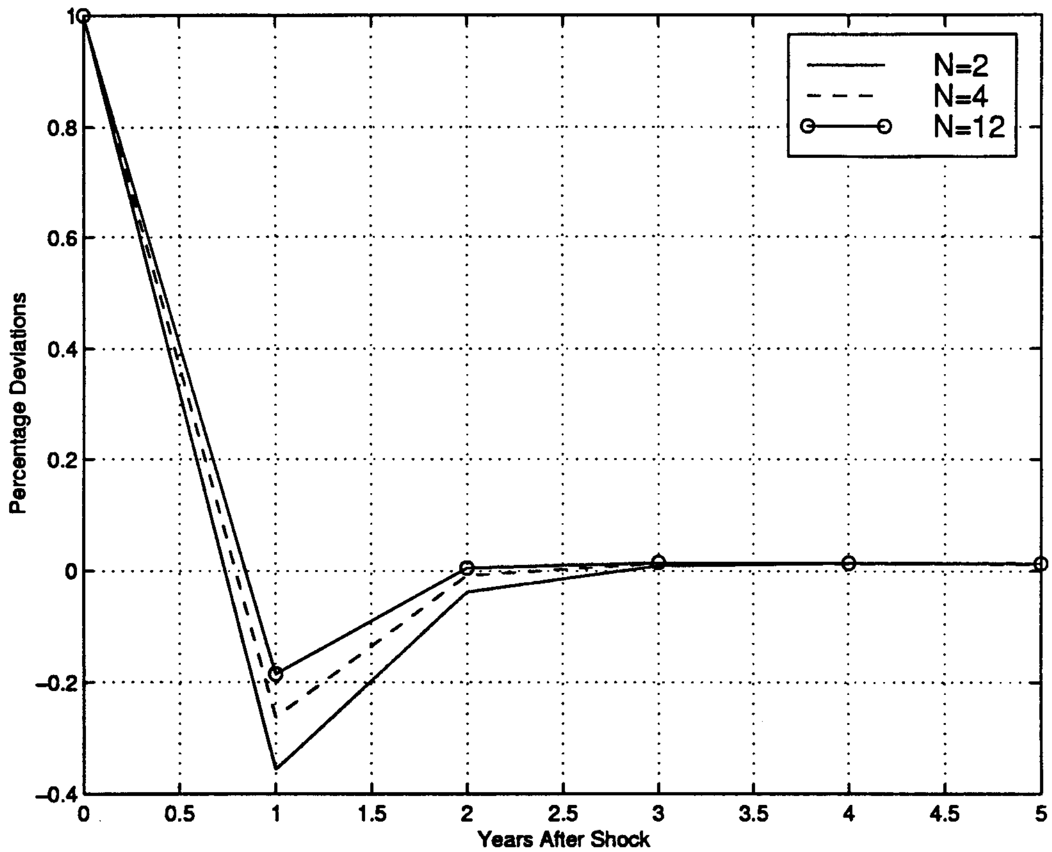

Figure 3: Impulse Response of Output in the Staggered Wage Model: $N = 2$

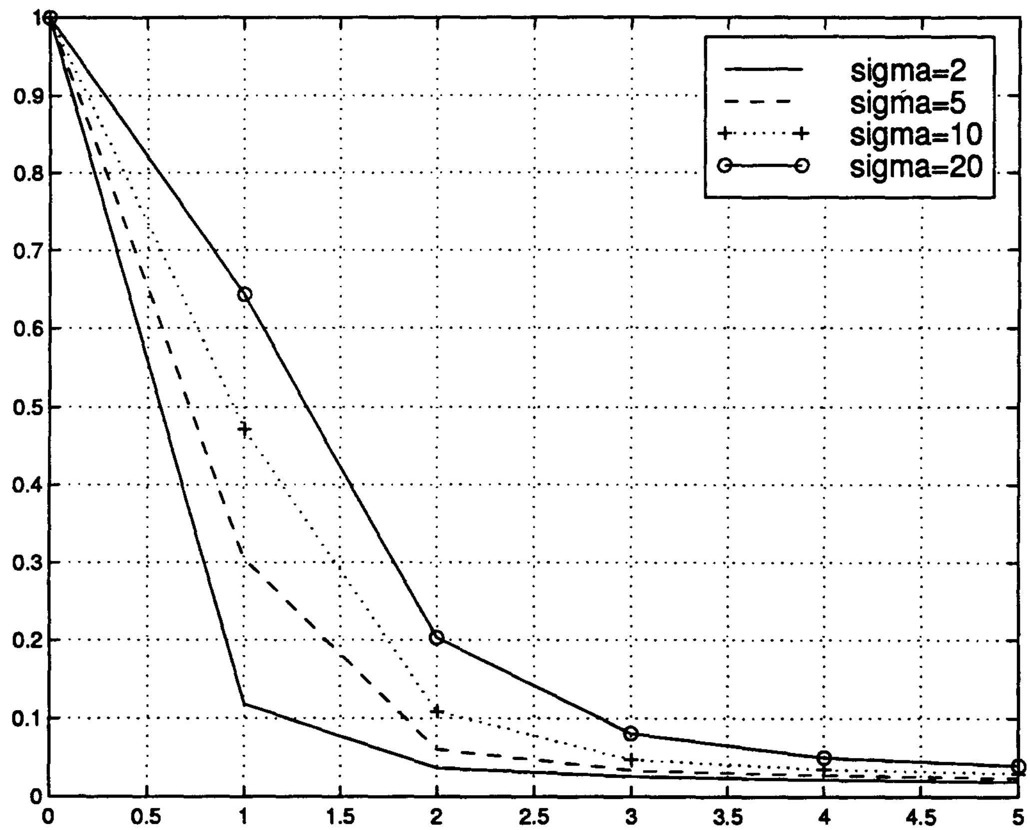

Figure 4: Impulse Response of Output in the Staggered Wage Model: $N = 4$

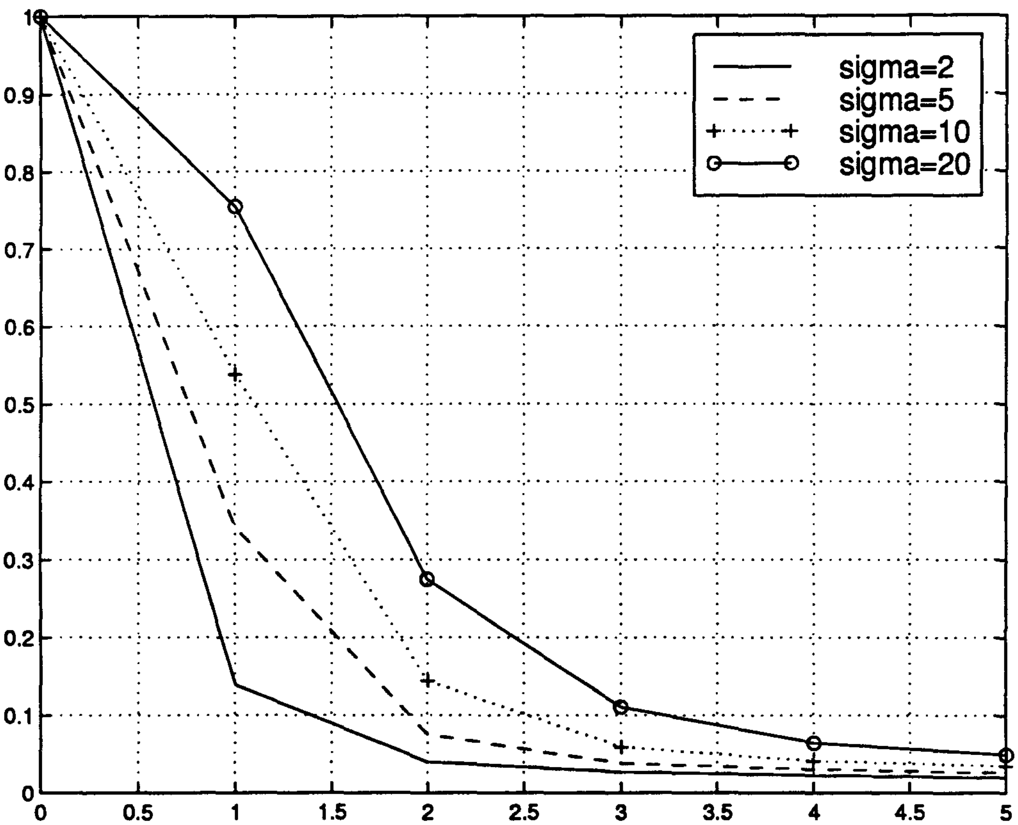

Figure 5: Impulse Response of Output in the Staggered Wage Model: $N = 12$