A UNIFIED FRAMEWORK FOR IMPLEMENTATION, THE REVELATION PRINCIPLE, AND OPTIMAL APPROXIMATION

by

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Summary

This paper attempts to provide a unified framework for implementation to capture both private information (hidden type) and private action (hidden action) aspects. (We generalize the Mount-Reiter triangle to a "square".) In addition, it takes a different approach to study mechanism design. Given a goal, rather than dealing with full and/or weakly implementation, we search an optimal mechanism in the sense that it best approximates the given goal among all the conceivable mechanisms.

We extend privacy-preserving and the revelation principle to this more general setting. Based on the discussion of the properties of the space of incentive compatible mechanisms, the existence of an optimal incentive compatible mechanism is investigated. Stochastic mechanisms are also examined.

Introduction

Let us call implementation *social implementation* if the mechanism designer is not a player in the game (e.g., a social planner designs a resource allocation mechanism), *individual implementation* otherwise (e.g., a principal designs a mechanism in an agency problem). In addition, let us call implementation *complete implementation* if each outcome in the outcome space is verifiable, *incomplete implementation* otherwise. The literature on resource allocation mechanisms has focused primarily on complete social implementation, and the literature on agency problems has devoted to incomplete individual implementation. Complete individual implementation may be trivial, while incomplete social implementation is nontrivial, realistic and important—yet not attacked in the literature (to my best knowledge). The famous Mount-Reiter triangle is appropriate only for complete implementation, but not for incomplete implementation. This is because all outcomes are assumed enforceable in that framework, but clearly hidden actions can not be "assigned" according to an outcome function.

This paper attempts to give a unified framework for implementation to capture both private information and private action aspects. The key to accomplish this is by introducing a signal space. Signals are publicly observable and they depend on hidden actions and/or hidden types. Based on the messages that the agents report and the signals the agents and the designer see, an outcome function assigns an allocation in the enforceable part of the outcome space.

Given a goal, the standard approach is to study whether there exists a (or a desirable) mechanism which fully (or weakly) implements the goal. Frequently there is either no such a mechanism or no such a *desirable* mechanism. As a result, even for complete implementation we discovered many impossibility theorems. Conceivably we will get more impossibility results for incomplete implementation. To avoid more impossibility results and to be more realistic, this paper investigates a different approach: given a goal and a class of (desirable) mechanisms, rather than asking whether there exists a mechanism which fully (or weakly) implements the goal, we ask whether there exists an optimal mechanism in the sense which best approximates the goal in the given class of mechanisms. This approach generalizes the idea in Hurwicz-Marschak's work (1985) that studies the optimal approximation of a given goal in the class of discrete mechanisms.

Palfrey and Srivastava (1991) showed that almost any social goal could be implemented in undominated Nash strategies. Abreu and Sen (1991) showed essentially all social goals are virtually implementable in Nash equilibrium. However, all these results use integer games which are questioned by Jackson (1989). Jackson, Palfrey and Srivastava (1991) showed that the set of undominated Nash implementable social goals with bounded mechanisms is larger than that for Nash implementation. Abreu and Matsushima (1992) demonstrated that almost any social goal is virtually implementable in iteratively undominated strategies. Although they did not use integer games, they require each player to announce the entire preference profile——to report something (other players' preferences) he usually does not know in practice. Furthermore, all these results assume that there are three or more players.

In this paper, we first generalize the Mount-Reiter triangle to a "square" in section 1, then introduce implementation by approximation in section 2. The revelation principle is extended in section 3. Finally in section 4, based on the discussion of the properties of the space of incentive compatible mechanisms, the existence of an optimal incentive compatible mechanism is investigated.

1. A Unified Framework for Implementation

A. Notation

Let E denote a class of economies. A generic element of E is e that may describe preferences, initial endowments, technologies and knowledge. Let A be the set of allocations. A may consists of two parts: one publicly verifiable allocation set X, one nonpublicly verifiable allocation set D called *private action space*. Let S be the (publicly verifiable) *signal space*. A mapping q: $D \times E \longrightarrow S$ is called a *signal function*, which could be deterministic or stochastic (a density function $q(s|d, e)$), but it is common knowledge.

A tuple (E, D, X, S, q) is called *economic structure*, which is common knowledge. Depending on whether q is independent of hidden types e or not, we say that *there is no adverse selection* or *there* *is;* also depending on whether q is independent of hidden actions d or not, we say that *there is no moral hazard* or *there is.*

A mechanism is a pair (M, h), where M is the message space, h is an outcome function and $h: M \times S \longrightarrow X$, which is also common knowledge. Let H be the set of all conceivable mechanisms.

For social implementation, a social goal correspondence is a mapping $f: E \longrightarrow\!\!\!\!\rightarrow A$. For individual implementation, an individual goal function is a mapping $\pi: H \longrightarrow \mathbb{R}$. (In a meaningful individual implementation, the designer knows his own type and has priors about other persons' types, so for him the variable is mechanism instead of preference profile.)

Suppose there are n persons in a class of economies. Person i's type set is E_i (in which the preference relation is $R(e_i)$ on A if his type is e_i), his private action space is D_i, and his message space is M_i. Let $e = (e_1, \cdots, e_n)$, where e_i is person i's private information, and let $D = D_1 \times \cdots \times D_n$, $M = M_1 \times \cdots \times M_n$.

In "incomplete information" case, let $P_i(e_{-i}|e_i)$ be person i's prior about other persons' types, and $u_i: A \times E_i \longrightarrow \mathbb{R}$ be his utility function.

We will adopt the following conventions: $e/e_i' = (e_i', e_{-i}) = (e_1, \cdots, e_{i-1}, e_i', e_{i+1}, \cdots, e_n)$, and so on; the quantifier "for all" is sometimes omitted. The term "incomplete information" means that players do not know other players' types exactly but have priors about other players' types. The term "complete information" means that players have no priors about other players' types, it could be that either they know each other's types or they do not (see more discussions below).

B. Comments on Solution Concepts

Since players' payoffs in a game in this setting may be affected not only by joint strategies, but also by joint types, we can not use the standard Nash equilibrium concept. Instead we introduce a generalized solution concept called *rational expectations Nash equilibrium* (this name is suggested by Shomu Banejee), which is essentially the same as Nash equilibrium except that it treats strategic uncertainties and environmental uncertainties symmetrically. For the justification of the symmetric treatment of strategic uncertainties and environmental uncertainties, see Aumann (1987).

The typical solution concept adopted in our setting is Bayesian Nash equilibrium in the literature (by introducing priors). Here again the Bayesian strategy rules are not jointly observable, however, we also need rational expectations hypothesis as justification. In fact, the definition of Bayesian Nash equilibrium introduces new uncertainties (each player's conjecture about other players' strategy rules) into the model.

In the definition of Bayesian Nash equilibrium, the restriction—person i's belief about person j's strategy being a function from person j's type set to person j's action space—is too restrictive. This restriction to a large degree neglects the crucial feature of rationality—one person's best response depends on others' actions rather than on rules. As a result, the posterior information may invalidate the original equilibrium; that is, the equilibrium is not "durable" as the following example shows.

Example There are two players, player 1 has one type, player 2 has two possible types. Each player has two choices, the payoffs are determined by the joint actions and joint types:

		e_2		e_2'	
		U	D	U	D
e_1	L	(3,4)	(1,3)	(1,3)	(2,2)
	R	(2,2)	(2,1)	(3,0)	(4,3)

The Nash equilibria are (L, U) for (e_1, e_2), (R, D) for (e_1, e_2'). When person 1's prior about person 2' types is half-half, the Bayesian Nash equilibrium is $\sigma_1(e_1) = R$, $\sigma_2(e_2) = U$, $\sigma_2(e_2') = D$, and the outcomes are (R, U) for $(e_1. e_2)$, (R, D) for $(e_1. e_2')$. Clearly (R, U) is Pareto inferior to (L, U), and is not sustainable since ex-post player 1 knows that person 2 is type e_2 and hence switches his strategy to L, which leads to Nash equilibrium (L, U) for $(e_1. e_2)$.

The equilibrium concepts used in this paper are rational expectations Nash equilibrium (call it Nash equilibrium for short), strong (rational expectations) Nash equilibrium, undominated (rational expectations) Nash equilibrium and dominance equilibrium in "complete information" case, Bayesian Nash equilibrium in "incomplete information" case.

C. Definitions

<u>Definition</u> For "complete information" case, a (rational expectations) Nash equilibrium with respect to an economy e is a pair $(m^*, d^*) \in M \times D$ such that

(a) $(h(m^*, q(d^*, e)), d^*)\ R(e_i)\ (h(m^*/m_i, q(d^*/d_i, e)), d^*/d_i)$

for all m_i, all d_i, and all i, if q is deterministic; or

(b) $(\sum_s h(m^*,s)q(s|d^*,e),d^*)\ R(e_i)\ (\sum_s h(m^*/m_i,s)q(s|d^*/d_i,e),d^*/d_i)$

for all m_i, all d_i and all i, if q is stochastic.

<u>Definition</u> For "incomplete information" case, a Bayesian Nash equilibrium is a pair $(\sigma(\cdot),\ \alpha(\cdot))$, where $\sigma(\cdot) = (\sigma_1(\cdot),\ \cdots,\ \sigma_n(\cdot))$, $\sigma_i\colon E_i \longrightarrow M_i$, $\alpha(\cdot) = (\alpha_1(\cdot),\ \cdots,\ \alpha_n(\cdot))$, $\alpha_i\colon E_i \longrightarrow D_i$ such that $(\sigma(e_i),\ \alpha(e_i))$ solves

(a) $\max\limits_{m_i, d_i} \sum u_i((h(\sigma(e)/m_i,\ q(\alpha(e)/d_i,\ e)),\ \alpha(e)/d_i)|e_i)\ P_i(e_{-i}|e_i)$

for all i, all e_i, where $\sum$ denotes summation over E_{-i}, if q is deterministic; or

(b) $\max\limits_{m_i, d_i} \sum u_i((h(\sigma(e)/m_i,s),\ \alpha(e)/d_i)|e_i)q(s|\alpha(e)/d_i,e)\ P_i(e_{-i}|e_i)$

for all i, all e_i, where $\sum$ denotes summation over E_{-i} and S, if q is stochastic.

Let $(\mu,\ \lambda)$ be the *equilibrium strategy correspondence*, where $\mu\colon E \longrightarrow\!\!\!\!\rightarrow M$, $\lambda\colon E \longrightarrow\!\!\!\!\rightarrow D$, and $(\mu(e),\ \lambda(e))$ is the set of equilibrium strategy profiles with respect to e. A selection of $(\mu,\ \lambda)$ is called an equilibrium strategy selection. Let $\gamma\colon E \longrightarrow\!\!\!\!\rightarrow A$ be the *equilibrium outcome correspondence* (or *performance correspondence*), where

$$\gamma(e) = (h(\mu(e),\ q(\lambda(e),\ e)),\ \lambda(e))$$

if q is deterministic; or

$$\gamma(e) = (h(\mu(e),\ s),\ \lambda(e)) \quad \text{with probability } q(s|\lambda(e),e)$$

if q is stochastic. The equilibrium concept could be Nash, Bayesian Nash, dominance, undominated Nash or strong Nash.

Definition In complete information case, a mechanism (M, h) *weakly implements* a social goal correspondence f if $\mu(e) \neq \phi$ for all e, and

$$\gamma(e) \subset f(e) \text{ when } q \text{ is deterministic,}$$

or

$$\mathbf{E}\gamma(e) \subset f(e) \text{ when } q \text{ is stochastic}$$

for all e. If "$\subset$" is replaced by "=", then we say (M, h) *(fully) implements* f.

A mechanism (M, h) *implements* an individual goal fuction π: $H \longrightarrow \mathbb{R}$ if π attains the maximum (over H) at (M, h).

The definition for implementation in incomplete information case can be given similarly.

We may summarize the above discussions in the following square:

$$\begin{tikzcd}
E \arrow[r, two heads, "f"] \arrow[d, "\lambda"'] \arrow[dr, two heads, "\mu"] & D \times X \\
D \times E \arrow[r, two heads, "q"'] & S \times M \arrow[u, "h"']
\end{tikzcd}$$

D. Examples

(1) *A Principal-agent model with adverse selection and moral hazard* Suppose there is a principal and an agent. Let E be the set of (hidden) types of the agent, D be the set of (hidden) efforts of the agent, S be the set of outputs, M be the same as E, X be the set of allocations of outputs, q be the probability

distribution of output conditional on the effort and type of the agent, h be the contract that specifies how to divide the output between the agent and the principal conditional on the the output and the reported type of the agent, and π be the expected residual for the principal conditional on a mechanism. This is an incomplete individual implementation problem.

(2) *A class of production economies with externality* Suppose there are two firms, firm *i*'s output y_i is affected by firm *j*'s (hidden) input z_j, by firm *j*'s (hidden) type parameter a_j and by a random shock ε_i, and

$$y_1 = a_1 z_1 - b_1 z_1^2 - c_1 z_2 - d_1 a_2 z_1 + \varepsilon_1,$$
$$y_2 = a_2 z_2 - b_2 z_2^2 - c_2 z_1 - d_2 a_1 z_2 + \varepsilon_2.$$

Here, $E_i = \{(a_i, b_i, c_i, d_i)\}$, $D_i = \{z_i\}$, $X = S = \{(y_1, y_2)\}$, q = the probability distributuions of y_1 and y_2. Let f be the Pareto correspondence. This is an incomplete social implementation problem.

(3) *The class of classical economies* In this case, there is neither adverse selection nor moral hazard, and the 'square' reduces to the classical Mount-Reiter triangle:

$$\begin{tikzcd} E \arrow[rr, "f", two heads] \arrow[dr, "\mu"', two heads] & & A \\ & M \arrow[ur, "h"'] & \end{tikzcd}$$

This is a complete social implementation problem.

(4) *Direct revelation mechanism* (M, h) where $M = E$

(5) *Augmented revelation mechanism* (M, h) where $M \supset E$

E. Privacy Preserving

We close this section by discussing privacy preserving which plays a very important role in decentralization and (perhaps) in posterior implementability.

For complete information case, we follow the definition of privacy preserving by Mount and Reiter. Since the Mount-Reiter definition is not appliable to incomplete information case, we introduce a weaker version of their definition in this case.

<u>Definition</u> An equilibrium strategy correspondence (μ, λ) is *privacy preserving* if $\exists$ (μ^i, λ^i), $\mu^i\colon E_i \twoheadrightarrow M$, $\lambda^i\colon E_i \twoheadrightarrow D$, such that

$$(\mu(e), \lambda(e)) = \bigcap_{i=1}^{n} (\mu^i(e_i), \lambda^i(e_i))$$

for all e.

<u>Definition</u> An equilibrium strategy correspondence is *weakly privacy preserving* if every equilibrium strategy selection is privacy preserving.

<u>Remark</u> Privacy preserving implies weakly privacy preserving.

THEOREM 1.1 *(i) For complete information case, consider the following equilibrium concepts: Nash, dominance or undominated Nash. If there is no adverse selection, then every equilibrium*

strategy correspondence is privacy preserving.

(ii) For incomplete information case, every Bayesian Nash equilibrium strategy correspondence is weakly privacy preserving.

Proof. See the appendix.

Prescott and Townsend (1984) observed that "there do seem to be fundamental problems for the operation of competitive markets for economies or situations which suffer from adverse selection. We have not discovered a standard competitive equilibrium construct which would predict well in such situations."

2. Implementation by Approximation

Consider an economic structure (E, D, X, S, q), where $A = X \times D$ is a compact metric space with metric ρ, and q is deterministic. (The following discussion can be easily extended to the case that q is stochastic.) Equip X with the natural quotient metric topology. Then X is compact. Let $R(e_i)$ be person i's (if his type is e_i) regular (reflexive, transitive and complete) preference order on A. The upper contour set of $R(e_i)$ at x is $P(e_i)(x) = \{y \in A\colon y\ R(e_i)\ x\}$ (the strict upper contour set of $R(e_i)$ at x is $P'(e_i)(x) = \{y \in A\colon y\ R(e_i)\ x$ and not $x\ R(e_i)\ y\}$.

Given a message space M, all outcome functions form a function space $H = X^M$. For each outcome function $h \in H$, let $\gamma(h)$ be the corresponding equilibrium outcome correspondence. Given a social goal correspondence f, define

$$\alpha(f(e),\ \gamma(h)(e)) = \sup_{b\in\gamma(h)(e)}\ \inf_{a\in f(e)} \rho(a,\ b),$$

$$\delta(f(e),\ \gamma(h)(e)) = \max\{\alpha(f(e),\ \gamma(h)(e)),\ \alpha(\gamma(h)(e),\ f(e))\}$$

for all e (δ is the Hausdorff pseudo-metric in 2^A), and

$$\varepsilon_\alpha(f,\ h) = \sup_{e\in E} \alpha(f(e),\ \gamma(h)(e)),$$

$$\varepsilon_\delta(f,\ h) = \sup_{e\in E} \delta(f(e),\ \gamma(h)(e))$$

for each h. We call $\varepsilon_\alpha(f, h)$ ($\varepsilon_\delta(f, h)$) *α-deviation* (*δ-deviation*) of f with respect to h.

The following proposition characterizes the relationship between these indices and full/weakly implementation.

PROPOSITION 2.1 *(i) If h weakly implements f, then $\varepsilon_\alpha(f, h) = 0$. If $\varepsilon_\alpha(f, h) = 0$ and f is closed-valued, then h weakly implements f. (ii) If h implements f, then $\varepsilon_\delta(f, h) = 0$. If $\varepsilon_\delta(f, h) = 0$ and both f and h are closed-valued, then h implements f.*

Proof. See the appendix.

The following proposition deals with an important and widely used social goal correspondence—*the individually rational Pareto correspondence*. (The reader is referred to Maskin (1978) and Li (1991) for the notions.)

PROPOSITION 2.2 *Consider a class of pure exchange economies with a finite number of consumers and with A as the (compact) set of feasible allocations. If for all e_1, $P(e_1)$ is continuous, then the* *individually rational Pareto correspondence is nonempty-valued and closed-valued.*

Let the goal f be the individually rational Pareto correspondence. Consequently, if $\varepsilon_\alpha(f, h) = 0$, then h weakly implements f. If $\varepsilon_\delta(f, h) = 0$ and h is closed-valued, then h implements f.

Proof. See the appendix.

The following proposition takes strong Nash implementation of the individually rational Pareto correspondence as an example. Given a set X, we will use $|X|$ to denote the cardinality of X.

PROPOSITION 2.3 *Consider a class of pure exchange economies with two agents and with A as the (compact) set of feasible allocations.*

(i) (Maskin) An individually rational correspondence can be strongly Nash implemented if and only if it is the individually rational Pareto optimal correspondence.

(ii) If (M, h) strongly Nash implements the individually rational Pareto correspondence, then $|M| \geq |A|$.

(iii) If $|M| = |A|$, then h strongly Nash implements the individually rational Pareto correspondence f if and only if h is the unanimous rule.

(iv) Consequently, if $|M| < |A|$, then $\varepsilon_\delta(f, h) > 0$ for any closed-valued h; if $|M| = |A|$ and h is closed-valued, then $\varepsilon_\delta(f, h) > 0$ if and only if h is not the unanimous rule.

Proof. See the appendix.

Let us equip H with the weak topology; that is, a net $\{h_n\}$ converges to h if and only if $h_n(m) \longrightarrow h(m)$ for all $m \in M$.

PROPOSITION 2.4 *The outcome function space H is compact. Consequently, given a goal f and a message space M, if $\varepsilon_\alpha(f, \cdot)$ ($\varepsilon_\delta(f, \cdot)$) is lower semi-continuous, then there exists an optimal outcome function h^* which solves*

$$\begin{aligned} &\mathit{Min} \quad \varepsilon_\alpha(f, h) \\ &\mathit{S.t.} \quad h \in H \end{aligned}$$

(there exists an optimal outcome function h^* which solves

$$\begin{aligned} &\mathit{Min} \quad \varepsilon_\delta(f, h) \\ &\mathit{S.t.} \quad h \in H). \end{aligned}$$

Proof. Notice that the weak topology is the same as the product topology. Since X is compact, by Tychonoff's theorem, $H = X^M$ is compact.

Clearly H is not empty. By assumption, $\varepsilon_\alpha(f, \cdot)$ and $\varepsilon_\delta(f, \cdot)$ are lower semi-continuous. The existence results follow from Weierstrass's theorem. Q.E.D.

In general, define

$$\varepsilon_\delta(M)(f) = \inf \{\varepsilon_\delta(f, h): h \in X^M\},$$

$$\varepsilon_\delta(f) = \inf \{\varepsilon_\delta(M)(f): M \in \mathcal{M}\},$$

where $\mathcal{M}$ is the set of all conceivable message spaces. Clearly $\varepsilon_\delta(f)$ is the *lower bound of δ-deviation of f* with respect to the class of mechanisms $\{(M, h): h \in X^M, M \in \mathcal{M}\}$. In general $\varepsilon_\delta(f)$ can not be reached. But if $f(\cdot)$ is single-valued, we will show in Section 4 that the lower bound of δ-deviation of f with respect to the class of incentive compatible mechanisms could be attained under very general conditions.

So far we have considered deterministic mechanisms only. Given a class of deterministic mechanisms H, we call a probability measure $p \in \Delta(H)$ defined on H a *stochastic mechanism*. Define

$$\varepsilon_\delta(f, p) = \sum_{h \in H} \varepsilon_\delta(f, h)\ p(h).$$

By introducing stochastic mechanisms, we may enlarge the class of conceivable mechanisms hence lower the lower bound of deviations of a given goal. This issue will be discussed more in detail in the context of incentive compatible mechanisms.

In order to justify the use of incentive compatible mechanisms, we extend the revelation principle to our general setting in next section.

3. The Revelation Principle

The revelation principle is a very simple yet powerful tool in the applied implementation literature. Although there are many different versions of the same principle, the author is not aware of a generalized version incorporating all the aspects mentioned in this paper. The Dasgupta-Hammond-Maskin (1979) version does not incorporate incomplete implementation. The Myerson (1982) version deals with individual implementation.

We introduce a concept about the "separable" property of equilibrium correspondence, which turns out to be a sufficient condition for the revelation principle to hold.

<u>Definition</u> An equilibrium strategy correspondence is *coordinately separable* if every equilibrium selection (μ,λ) (we abuse notation) is of the form $((\mu_1, \lambda_1), \cdots, (\mu_n, \lambda_n))$, where μ_i: $E_i \longrightarrow M_i$, λ_i: $E_i \longrightarrow D_i$.

<u>Remark</u> Coordinate separability implies weakly privacy preserving.

<u>Remark</u> Both dominance equilibrium strategy correspondence and Bayesian Nash equilibrium strategy correspondence are coordinately separable. Nash equilibrium strategy correspondence is not coordinately separable in general, but it could be in some cases as the following example suggests.

Example There are two players. Each player has two types and two choices. The payoffs are determined by joint actions and joint types:

		e_2'			e_2''	
		U	D		U	D
e_1'	L	(4,3)	(5,2)	L	(4,0)	(5,4)
	R	(2,2)	(1,6)	R	(2,7)	(1,5)
		U	D		U	D
e_1''	L	(2,3)	(1,2)	L	(2,0)	(1,4)
	R	(4,8)	(5,6)	R	(4,7)	(5,8)

The Nash equilibrium strategy correspondence is λ with

$$\lambda(e_1', e_2') = \{(L, U)\},\ \lambda(e_1', e_2'') = \{(L, D)\},$$

$$\lambda(e_1'', e_2') = \{(R, U)\},\ \lambda(e_1'', e_2'') = \{(R, D)\}.$$

It is easy to see that $\lambda(e) = (\lambda_1(e_1), \lambda_2(e_2))$ for all e, where $\lambda_1(e_1') = \{L\}$, $\lambda_1(e_1'') = \{R\}$, $\lambda_2(e_2') = \{U\}$, $\lambda_2(e_2'') = \{D\}$.

THEOREM 3.1 *(The Revelation Principle) Consider the following equilibrium concepts: Nash, dominance, undominated Nash, strong Nash for complete information case; Bayesian Nash for incomplete information case. For any given mechanism, if the equilibrium correspondence is coordinately separable, then every equilibrium outcome can be truthfully implemented by a direct revelation mechanism.*

Proof. We prove the theorem for complete information case only. (The proof for incomplete information case is similar.) Consider Nash equilibrium first.

(i) *The case that q is deterministic.* Given a mechanism (M, h) and an economy e. Suppose there exists an equilibrium outcome a ($a \in A$) with respect to e, and there exists an equilibrium strategy selection (μ, λ) such that $(h(\mu(e), q(\lambda(e), e)), \lambda(e)) = a$. Define $g(e, s) = h(\mu(e), s)$ for all e and s. Clearly $(g(e, q(\lambda(e), e)), \lambda(e)) = a$. It remains to show that

$$(g(e, q(\lambda(e), e)), \lambda(e)) \; R(e_i) \; (g(e/e'_i, q((d_i, \lambda_{-i}(e_{-i})), e)), (d_i, \lambda_{-i}(e_{-i})))$$

for all i, all e'_i and all d_i.

Since (μ, λ) is coordinately separable,

$$(h((\mu_i(e_i),\mu_{-i}(e_{-i})),q((\lambda_i(e_i),\lambda_{-i}(e_{-i})),e)),(\lambda_i(e_i),\lambda_{-i}(e_{-i})) \; R(e_i) \; (h((m_i,\mu_{-i}(e_{-i})),q((d_i,\lambda_{-i}(e_{-i})),e)),(d_i,\lambda_{-i}(e_{-i}))$$

for all i, all m_i and all d_i. It follows that

$$(h((\mu_i(e_i),\mu_{-i}(e_{-i})),q((\lambda_i(e_i),\lambda_{-i}(e_{-i})),e)),(\lambda_i(e_i),\lambda_{-i}(e_{-i})) \; R(e_i) \; (h((u_i(e'_i),\mu_{-i}(e_{-i})),q((d_i,\lambda_{-i}(e_{-i})),e)),(d_i,\lambda_{-i}(e_{-i}))$$

for all i, all e'_i and all d_i. That is

$$(g(e,\ q(\lambda(e),\ e)),\ \lambda(e))$$
$$R(e_i)\quad (g(e/e_i',\ q((d_i,\ \lambda_{-i}(e_{-i})),\ e)),\ (d_i,\ \lambda_{-i}(e_{-i})))$$

for all i, all e_i' and all d_i.

(ii) *The case that q is stochastic.* Define $g(e,s) = h(\mu(e),\ s)$ for $e \in E$ and $s \in S$. Since $(\mu,\ \lambda)$ is coordinately separable, we have

$$(\sum_s h((\mu_i(e_i),\mu_{-i}(e_{-i})),s)q(s|(\lambda_i(e_i),\lambda_{-i}(e_{-i})),e),(\lambda_i(e_i),\lambda_{-i}(e_{-i}))$$
$$R(e_i)\quad (\sum_s h((m_i,\mu_{-i}(e_{-i})),s)q(s|(d_i,\lambda_{-i}(e_{-i})),e),(d_i,\lambda_{-i}(e_{-i}))$$

for all i, all m_i and all d_i. It follows that

$$(\sum_s h((\mu_i(e_i),\mu_{-i}(e_{-i})),s)q(s|(\lambda_i(e_i),\lambda_{-i}(e_{-i})),e),(\lambda_i(e_i),\lambda_{-i}(e_{-i}))$$
$$R(e_i)\quad (\sum_s h((\mu_i(e_i'),\mu_{-i}(e_{-i})),s)q(s|(d_i,\lambda_{-i}(e_{-i})),e),(d_i,\lambda_{-i}(e_{-i}))$$

for all i, all e_i' and all d_i. That is

$$(\sum_s g(e,\ s)q(s|\lambda(e),\ e),\ \lambda(e))$$
$$R(e_i)\quad (\sum_s g(e/e_i',\ s)q(s|(d_i,\ \lambda_{-i}(e_{-i}))),\ (d_i,\ \lambda_{-i}(e_{-i})))$$

for all i, all e_i' and all d_i.

This completes the proof of the theorem for Nash equilibrium.

Then for other solution concepts, the proofs of the theorem are straight forward. Let us consider strong Nash equilibrium for example. Following the above proof, now suppose $(\mu(e),\ \lambda(e))$ is a strong Nash equilibrium with respect to e in $(M,\ h)$, and there exists a coalition that could benefit by joint lie-telling in $(E,\ g)$, then clearly from the above proof, the same coalition can benefit from some deviation in $(M,\ h)$, a contradiction. Q.E.D.

COROLLARY 3.2 *(i) Every dominance equilibrium outcome can be truthfully implemented by a direct revelation mechanism in dominance strategies.*

(ii) Every Bayesian Nash equilibrium outcome can be truthfully

implemented by a direct revelation mechanism in Bayesian Nash strategies.

It should be noticed that for social implementation, even if (M, h) implements f, the direct revelation mechanism (E, g) constructed above may not weakly implement f, since untruthful equilibrium outcomes may not be in the range of f. But as Mookherjee and Reichelstein (1990) suggested if we augment the message space, undesired equilibria could be eliminated as the following theorem shows.

This leads to the augmented revelation principle which plays an important role in weakly implementation. The following theorem is a generalized version of Theorem 3.2 in Mookherjee and Reichelstein (1990) which deals with Bayesian Nash equilibrium and complete implementation only.

THEOREM 3.3 *(The Augmented Revelation Principle) Consider dominance equilibrium, Nash equilibrium for complete information case; Bayesian Nash equilibrium for incomplete information case. If f is weakly implemented by (M, h), and the equilibrium strategy correspondence (μ, λ) is coordinately separable, then f can be weakly implemented by an augmented revelation mechanism.*

Proof. Consider Nash equilibrium in complete information case. (The proofs for other cases are similar).

(i) *The case that q is deterministic.* Let (μ, λ) be an equilibrium strategy selection in (M, h). Define $\overline{M}_i = E_i \cup T_i$ with $T_i = M_i \setminus \mu_i(E_i)$. Define $\delta_i\colon \overline{M}_i \longrightarrow M_i$ by

$$\delta_i(\bar{m}_i) = \begin{cases} \mu_i(\bar{e}_i) & \text{if } \bar{m}_i = \bar{e}_i \text{ for } \bar{e}_i \in E_i \\ \bar{m}_i & \text{if } \bar{m}_i \in T_i \end{cases}$$

for all i and define g: $\bar{M} \times S \longrightarrow A$ by $g(\bar{m}, s) = h(\delta(\bar{m}), s)$, where $\delta = (\delta_1, \cdots, \delta_n)$.

Clearly $(g(e, q(\lambda(e), e)), \lambda(e)) = (h(\mu(e), q(\lambda(e), e)), \lambda(e)) \subseteq f(e)$ for all e and truth-telling is an equilibrium in $(\bar{M}, g)$ since (μ, λ) is coordinately separable. It remains to show that for all equilibrium strayegy selection (β, α) in $(\bar{M}, g)$ and all e, $(g(\beta(e), q(\alpha(e), e)), \alpha(e)) \subseteq f(e)$. Given (β, α), define $\mu*(e) = \delta(\beta(e))$, $\lambda*(e) = \alpha(e)$ for all e. It suffices to show that $(\mu*, \lambda*)$ is an equilibrium in (M, h) since (M, h) weakly implements f. Since (β, α) is an equilibrium in $(\bar{M}, g)$, we have for all i, all e, all $\bar{m}_i$ and all d_i,

$$\begin{aligned} &(h(\mu*(e), q(\lambda*(e), e)), \lambda*(e)) = (h(\delta\beta(e), q(\alpha(e), e)), \alpha(e)) \\ &R(e_i)\ (h(\delta(\bar{m}_i, \beta_{-i}(e)), q((d_i, \alpha_{-i}(e)), e)), (d_i, \alpha_{-i}(e))) \\ &= (h((\delta_i(\bar{m}_i), \mu*_{-i}(e)), q((d_i, \lambda*_{-i}(e)), e)), (d_i, \lambda*_{-i}(e))). \end{aligned}$$

The result follows from the fact that δ_i is onto M_i.

(ii) *The case that q is stochastic.* The proof is almost the same as above. Q.E.D.

COROLLARY 3.4 *(i) Every weakly dominance implementable social goal correspondence can be weakly implemented by an augmented revelation mechanism in dominance strategies.*

(ii) Every weakly Bayesian Nash implementable social goal correspondence can be weakly implemented by an augmented revelation mechanism in Bayesian Nash strategies.

4. Incentive Compatible Mechanisms

By the revelation principle, we can concentrate on the incentive compatible mechanisms. An important problem for the mechanism designer is whether there exists an optimal incentive compatible mechanism in the sense that it best approximates a given goal. The space of all incentive compatible mechanisms becomes the constraint set (or one of the constrain sets) of the designer. The existence and the properties of the optimal incentive compatible mechanisms crucially depend on the properties of the space of all incentive compatible mechanisms such as closedness, compactness and convexity.

There are some existence theorems regarding optimal incentive compatible mechanisms for incomplete individual implementation in the principal-agent literature (see Page (1991)), but the author is not aware of any existence theorem in a general implementation setting.

In this section, we will consider only complete information case in the class of classical economies. The results could be easily extended to incomplete information case. But it is very hard to extend the results to the class of economies with adverse selection and/or moral hazard since many topological properties of an equilibrium outcome correspondence are involved in this case.

A direct revelation mechanism in the class of classical economies (E, X) is a mapping g: $E \longrightarrow X$. A direct revelation mechanism is *incentive compatible* if truth-telling is a Nash equilibrium for each economy. Let G denote the space of all incentive compatible mechanisms. A *(finite) stochastic mechanism* is a probability measure over a finite set of deterministic direct revelation mechanisms.

Let X be a topological space. Equip the space of direct revelation mechanisms X^E with the weak topology.

We need to define some notions about the properties of preference order first.

<u>The First Axiom of Independence</u> Let R be a regular preference order on a deterministic outcome space X. We say that R satisfies the first axiom of independence if $x \; R \; y$ implies $\alpha x + (1 - \alpha)z \; R \; \alpha y + (1 - \alpha)z$ for all x, y, $z \in X$ and for all $\alpha \in [0,1]$.

<u>Remark</u> R satisfies the first axiom of independence if and only if $x \; R \; x'$ and $y \; R \; y'$ implies $\alpha x + (1 - \alpha)y \; R \; \alpha x' + (1 - \alpha)y'$ for all x, x', y, $y' \in X$ and for all $\alpha \in [0,1]$.

<u>The Second Axiom of Independence</u> Let R be a regular preference order on the space of probability measures over X $\Delta(X)$. We say that R satisfies the second axiom of independence if $p \; R \; q$ implies $\alpha p + (1 - \alpha)r \; R \; \alpha q + (1 - \alpha)r$ for all p, q, $r \in \Delta(X)$ and for all $\alpha \in [0,1]$.

<u>Remark</u> R satisfies the second axiom of independence if and only if $p_j \; R \; q_j$ for all $j = 1, \cdots, k$ implies $\sum_1^k \alpha_j p_j \; R \; \sum_1^k \alpha_j q_j$ for all p_j, $q_j \in \Delta(X)$, $j = 1, \cdots, k$, all $\alpha_j \in [0,1]$ with $\sum_1^k \alpha_j = 1$ and all positive integer k. The second axiom of independence is identical to the independence axiom in the expected utility

theory.

Let L denote the *lexicographic order* in $\mathbb{R}^k$; that is, for x, $y \in \mathbb{R}^k$, $x \; L \; y$ if $x_i \geq y_i$, where $i = \min \{j: x_j \neq y_j\}$.

The following proposition characterizes the class of preferences that satisfy the first axiom of independence or the second axiom of independence.

PROPOSITION 4.1 *(Hausner) (i) If* $X \subset \mathbb{R}^k$ *is nonempty and convex,* R *is a regular preference order on* X*, then* R *satisfies the first axiom of independence if and only if there exists a mapping* U*:* $X \longrightarrow \mathbb{R}^k$ *such that* $x \; R \; y$ *iff* $U(x) \; L \; U(y)$ *for all* $x, y \in X$.

(ii) If $|X| = k+1$, R *is a regular preference* order *on* $\Delta(X)$*, then* R *satisfies the second axiom of independence if and only if there exists a mapping* V*:* $\Delta(X) \longrightarrow \mathbb{R}^k$ *such that* $p \; R \; q$ *iff* $V(p) \; L \; V(q)$ *for all* $p, q \in \Delta(X)$.

Proof. See Hausner (1954) and Fishburn (1982).

Next we discuss the properties of the space of incentive compatible mechanisms.

PROPOSITION 4.2 *(i) If for all* i *and all* e_i*, the upper contour correspondence* $P(e_i)$ *is upper hemi-continuous in* X*, then* G *is closed.*

(ii) In *addition to the assumption in (i), assume* X *is compact. Then* G *is compact.*

(iii) If for all i *and* e_i*,* $R(e_i)$ *satisfies the first axiom of*

independence in X, then G is convex.

(iv) Assume for all i and all e_i, $R(e_i)$ satisfies the second axiom of independence in $\Delta(X)$. Let $g_1, g_2, \cdots, g_k \in G$, $p \in \Delta(\{g_1, \cdots, g_k\})$. Then p is an incentive compatible (finite stochastic) mechanism.

Proof. See the appendix.

Now we are ready to discuss the optimal approximation of a given goal by incentive compatible mechanisms.

PROPOSITION 4.3 *Assume for all i and e_i, $P(e_i)$ is upper hemi-continuous, X is compact. If an individual goal function π: $G \longrightarrow \mathbb{R}$ is upper semi-continuous, then there exists an optimal incentive compatible mechanism g^* which solves*

$$\begin{aligned} &Max \quad \pi(g) \\ &S.t. \quad g \in G. \end{aligned}$$

Proof. First G is not empty since a constant outcome function is incentive compatible. By part (ii) of Proposition 4.2 G is compact. The result follows from Weierstrass's Theorem.

Q.E.D.

THEOREM 4.4 *Assume for all i and e_i, $P(e_i)$ is upper hemi-continuous. Let X be a compact metric space with metric ρ, and f: $E \longrightarrow X$ be a social goal function. Define $\delta_e(g) = \rho(f(e), g(e))$* for $e \in E$ and $\varepsilon(g) = \sup\{\delta_e(g): e \in E\}$ for $g \in G$. *Then $\varepsilon(\cdot)$*

is lower semi-continuous. Hence there exists an optimal incentive compatible mechanism $g^* \in G$ which solves

$$\text{Min} \quad \varepsilon(g)$$
$$\text{S.T.} \quad g \in G.$$

Proof. By Proposition 4.3 it suffices to show that $\varepsilon(\cdot)$ is lower semi-continuous. Notice that $\delta_e(\cdot)$ is continuous for all e, it is enough to show that for all $a \in \mathbb{R}$,

$$\varepsilon^{-1}((a, \infty)) = \cup_{e\in E} \delta_e^{-1}((a, \infty))$$

since $\delta_e^{-1}((a, \infty))$ is open for all e hence $\cup_{e\in E} \delta_e^{-1}((a, \infty))$ is open in G.

For $g \in \cup_{e\in E} \delta_e^{-1}((a, \infty))$, $\delta_e(g) > a$ for some e. So $\varepsilon(g) > a$, $g \in \varepsilon^{-1}((a, \infty))$. Conversely if $g \in \varepsilon^{-1}((a, \infty))$, then $\varepsilon(g) > a$. It follows that there exists some e such that $\delta_e(g) > a$ (otherwise a will be a upper bound for $\{\delta_e(g)\colon e \in E\}$ which contradicts the fact that $\varepsilon(g)$ is the smallest upper bound for $\{\delta_e(g)\colon e \in E\}$). Hence $g \in \cup_{e\in E} \delta_e^{-1}((a, \infty))$. Q.E.D.

COROLLARY 4.5 *Assume for all i and all e_i, $P(e_i)$ is upper hemi-continuous. Let X be a compact metric space with metric ρ, and $f\colon E \longrightarrow X$ be a social goal function. Then the lower bound of δ-deviation of f with respect to the class of all deterministic incentive compatible mechanisms could be attained. Consequently, we cannot lower the lower bound of δ-deviation of f with respect to the class of all incentive compatible mechanisms by introducing stochastic mechanisms.*

Proof. Notice that from Theorem 4.4 $\varepsilon(g^*) = \varepsilon_\delta(f)$ with respect to G. Hence the lower bound of δ-deviation of f with respect to G is attained (at g^*). Consequently, the optimal stochastic mechanism p^* is degenerate and with a single support g^*. Q.E.D.

Appendix

Proof of Theorem 1.1. (i) Consider Nash equilibrium first. Given (μ, λ). Define

$$\mu^i(e_i) = \bigcup_{e_{-i} \in E_{-i}} \mu(e)$$

for all e and all i. Define $\lambda^i(e_i)$ similarly. Clearly

$$(\mu(e), \lambda(e)) \subseteq \bigcap_{i=1}^{n} (\mu^i(e_i), \lambda^i(e_i))$$

for all e. Conversely, for any e, suppose $(m, d) \in \bigcap_{i=1}^{n} (\mu^i(e_i), \lambda^i(e_i))$; i.e., $(m, d) \in (\mu^i(e_i), \lambda^i(e_i))$ for all i. Then for each i, $m \in \mu(e_i, \bar{e}_{-i})$ for some $\bar{e}_{-i}$, $d \in \lambda(e_i, \hat{e}_{-i})$ for some $\hat{e}_{-i}$, and since there is no adverse selection

$$(h(m, q(d)), d) \ R(e_i) \ (h(m/m'_i, q(d/d'_i)), d/d'_i)$$

for all m'_i and all d'_i. So $(m, d) \in (\mu(e), \lambda(e))$. Therefore

$$(\mu(e), \lambda(e)) = \bigcap_{i=1}^{n} (\mu^i(e_i), \lambda^i(e_i)) \quad \text{for all } e.$$

Now consider dominance and undominated Nash equilibrium. Since there is no adverse selection, one's dominant or undominated strategy has nothing to do with other person's types. The results follow from this fact.

(ii) Given (μ, λ). Let (σ, α) be a Bayesian Nash equilibrium strategy selection. Define $\sigma^i(e_i) = \sigma_i(e_i) \times M_{-i}$, $\alpha^i(e_i) = \alpha_i(e_i) \times D_{-i}$ for all i and all e. Clearly $(\sigma(e), \alpha(e)) = \bigcap_{i=1}^{n} (\sigma^i(e_i), \alpha^i(e_i))$.

Q.E.D.

Proof of Proposition 2.1. (i) If h weakly implements f, then $\gamma(h)(e) \subseteq f(e)$ for all e. It follows that for each $b \in \gamma(h)(e)$, $\inf \{\rho(a,b): a \in f(e)\} = 0$. Hence $\alpha(f(e), \gamma(h)(e)) = 0$, $\varepsilon_\alpha(f, h) = 0$. Conversely if g does not weakly implement f and f is closed-valued, then there exists some e such that $\gamma(h)(e) \notin f(e)$ and an open ball V around e with radius r such that $V \cap f(e) = \phi$. It follows that $\varepsilon_\alpha(f, h) \geq r > 0$, a contradiction.

(ii) The first part follows from the fact that $K = L$ implies $\delta(K, L) = 0$ for all $K, L \subseteq A$. The second part follows from the fact that $\delta(K, L) = 0$ if and only if closure (K) = closure (L) for all $K, L \subseteq$ A. Q.E.D.

Proof of Proposition 2.2. For nonemptiness of the individually rational Pareto correspondence, see Aliprantis, Brown and Burkinshaw (Theorem 1.5.3, 1990). Since all agents have continuous preferences, all agents' individually rational sets hence the intersection is closed. It suffices to show that the Pareto correspondence is closed-valued. Given an economy, suppose there is a net $\{a_n\}$ which converges to a and each a_n is Pareto optimal. We want to show that a is also Pareto optimal. Suppose there exists b such that $b \in P(e_i)(a)$ for all i and $b \in P'(e_i)(a)$ for some i. Since $P(e_i)(\cdot)$ is lower hemi-continuous for all i and there are a finite number of agents, there exists a subnet $\{b_k\}$ such that $b_k \in P(e_i)(a_k)$ for all k and all i and $b_k \longrightarrow b$. Since for some i, $b \in P'(e_i)(a)$ and $P'(e_i)(a)$ is open, $b_k \in P'(e_i)(a_k)$ for some large k. This contradicts to the fact that a_k is Pareto optimal. Q.E.D.

Proof of Proposition 2.3. Notice that the individual rationality concept in social choice setting is equivalent to the one in pure exchange economic setting when there are two agents. Then (i) follows from Maskin (1978), (ii) and (iii) follow from Li (1991), (iv) follows from (ii), (iii) and Theorem 2.2. Q.E.D.

Proof of Proposition 4.2. (i) Consider an economy e and $P(e_i)$ for agent i. Let $\{g_k\}$ be a net in G and $g_k \longrightarrow g$. Then $g_k(e) \in P(e_i)(g_k(e/e_i'))$ for all e_i'. Since $P(e_i)$ is upper hemi-continuous, we have $g(e) \in P(e_i)(g(e/e_i'))$ for all e_i'. Thus $g \in G$, G is closed.

(ii) By Proposition 2.4, X^E is compact. From (i), G is closed in X^E since $P(e_i)$ is upper hemi-continuous for all i and all e_i. Thus G is compact.

(iii) Consider an economy e and $R(e_i)$ for agent i. Let g_1, $g_2 \in G$, $\alpha \in [0,1]$, then $g_1(e)\ R(e_i)\ g_1(e/e_i')$, $g_2(e)\ R(e_i)\ g_2(e/e_i')$ for all e_i'. By the first axiom of independence, $\alpha g_1(e) + (1 - \alpha)g_2(e)\ R(e_i)\ \alpha g_1(e/e_i') + (1 - \alpha)g_2(e/e_i')$; that is, $(\alpha g_1 + (1 - \alpha)g_2)(e)\ R(e_i)\ (\alpha g_1 + (1 - \alpha)g_2)(e/e_i')$ for all e_i'. Thus $\alpha g_1 + (1 - \alpha)g_2 \in G$. So G is convex.

(iv) Consider an economy e and $R(e_i)$ for agent i. Let κ_j be a degenerate stochastic mechanism defined on $\Delta(\{g_1, \cdots, g_k\})$ with a single support g_j. Since $g_1, \cdots, g_k \in G$, $\kappa_j(e)\ R(e_i)\ \kappa_j(e/e_i')$ for all e_i' and for all $j = 1, \cdots, k$. It follows that $(p_1\kappa_1 + \cdots + p_k\kappa_k)(e)\ R(e_i)\ (p_1\kappa_1 + \cdots + p_k\kappa_k)(e/e_i')$ for all e_i' (where $p_j = p(g_j)$) by the second axiom of independence. Thus $p = p_1\kappa_1 + \cdots + p_k\kappa_k$ is incentive compatible. Q.E.D.

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