THE EFFECT OF INSIDER TRADING BY A DOMINANT TRADER IN A SIMPLE SECURITIES MARKET MODEL

by

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Abstract

This paper studies the effects of strategic behavior by an informed trader who is large relative to the remaining traders in a simple securities market model. The strategic interaction among market participants is modelled as a price-setting game in which the dominant trader sets price, taking into account the reactions of uninformed traders who behave competitively. The equilibrium outcomes of this game are compared with those which emerge when all traders act as price-takers and/or all information is public. Besides characterizing the informational efficiency of prices under alternative assumptions and describing the dual advantages conferred by superior information and the ability to move first and set prices, these comparisons help to determine whether observations on equilibrium prices and quantities traded suffice to reveal the existence of strategic insider trading in an otherwise competitive market.
1. Introduction

That insider trading is an important phenomenon in securities markets emerges from numerous studies, including those by Homel and Stein [1979], Finnerty [1976], Jaffe [1974], and Pratt and DeVere [1970]. Furthermore, there exists empirical evidence to suggest that informed traders may influence the price of securities with their trades or offers to trade. For example, the studies by Dean, Mayers, and Raab [1977], Grier and Albin [1973], Kraus and Stoll [1982], and Raab [1976] show that large blocks of stock and secondary offerings trade at prices significantly different from trades executed only minutes earlier. Moreover, the view that emerges from these studies is that such price changes reflect new information revealed through the offer to trade. In a different context, the belief that informed traders can affect security prices forms the basis for various regulatory measures designed to control insider trading. As an example, Krynasowski [1979] notes that sudden jumps in the price of a particular security have, on occasion, been interpreted as signalling the existence of insider trading and led to trading suspensions in that security, presumably in an effort to publicize the "monopoly" information.

Motivated by the empirical findings cited above, this paper analyzes the effect of strategic behavior by an informed trader who is large relative to the market. The strategic interaction among market participants takes a particularly simple form: conditional on observing a noisy signal about future payoffs, an informed trader sets the price of a particular security, taking into account the reactions of uninformed traders who behave competitively. As a description of actual behavior in securities markets, the price-setting assumption may be appropriate for an insider who trades in large blocks or who stands ready to buy and sell sizable quantities at a given price (as, for example, during merger or takeover bids). At the same time that it allows a simple
departure from price-taking behavior for informed traders, the assumption that
the insider moves first and sets prices provides a price formation mechanism
which is consistent with uninformed traders using observations on price to com-
pute their demands and to make inferences about the inside information.

The equilibrium concept used to derive the optimal strategies of the in-
formed and uninformed traders for the price-setting game differentiates this
paper from some recent papers dealing with the interaction between information
and market power. One such paper is by Kihlstrom and Postlwaite [1983] who
show that if a monopolist can pre-commit to a given strategy prior to receiving
an informative signal about the future state, then he may be better off, ex
ante, by ignoring his information. This is because using his information will
cause it to be revealed through the equilibrium price and thereby eliminate ex
ante risk sharing opportunities for both the monopolist and the competitive
trader. These authors also describe the role of random strategies as an optimal
form of concealment which occurs when the monopolist pre-commits to basing his
pricing decisions on a less informative signal than the one he actually receives.

Second, using a framework similar to the one in this paper, Grinblatt and Ross [1983]
examine the effect of market power possessed by a large informed risk-neutral
trader with a linear demand schedule. However, while they characterize the best
linear demand schedule for the monopolist, they do not demonstrate that demand
functions which are linear in the monopolist's information and the current price
of the asset constitute an optimal strategy. Furthermore, they assume that the
monopolist will not deviate from the actions prescribed by his decision rule
once he knows the value of his signal and the price of the asset. In fact, one
possible deviation for the monopolist which always guarantees non-negative ex
post profits is to buy (sell) arbitrarily large quantities of the asset whenever
the expected value of the future payoff, conditional on the monopolist's signal
exceeds (falls short of) the current price of the asset. By contrast, this paper uses an equilibrium concept which is subgame perfect. Consequently, it tries to analyze the effects of strategic trading by an informed trader in an otherwise competitive market when pre-commitment on the part of this trader cannot be enforced.

Section 3 characterizes the equilibrium outcomes with an insider who behaves strategically, setting prices in a simple two-period, two asset economy with a continuum of uniformed traders. To separate out the effects of superior information and the ability to move first and set prices, this section also describes the equilibrium under alternative assumptions.

First, the rational expectations equilibrium of Grossman and Stiglitz [1976] with informed and uninformed traders is used as a natural benchmark against which to compare the effects of non-competitive behavior by the insider. Provided each uninformed trader is unsure about the initial position or endowments of the remaining traders, it turns out that the price set by the insider as well as the rational expectations equilibrium price when all traders behave competitively conveys only statistical information about the noisy signal. This occurs because changes in supply become confounded with changes in information, preventing the full revelation of information. However, compared to the case when all traders act as price-takers, the optimal price-setting rule implies that price will respond less to any given piece of news received by the insider. In effect, the insider increases his expected utility (or expected profit) by making price less correlated with his information.

To examine further the role of informational advantage, Section 3 also describes the equilibrium outcomes when all information is public but a large trader (who may be interpreted as a specialist or market maker) sets prices. Elimination of the informational monopoly implies that all profit opportunities
due to information are exhausted by the trades of the competitive fringe. Without a private source of information, the dominant trader merely behaves as a textbook monopolist (monopsonist) facing an exogenous demand (supply) curve. Alternatively, differential information confers greater market power on the dominant trader because the demands of uninformed traders are jointly determined with the equilibrium price. By optimally setting price as a function of his information, the informed monopolist can alter the beliefs of uninformed traders and thereby increase his profits.

The possibility that superior information alone confers market power on individual traders has also been recognized by various authors. Commenting on the literature dealing with information revelation and aggregation in securities markets—exemplified by the work of Grossman [1976, 1978], Grossman and Stiglitz [1976], Danthine [1978], Bray [1981], Verrechia [1982]—Hellwig [1980] points out that the practice of modelling informed traders as price-takers leads to a "schizophrenia" problem. Although these traders are aware of the covariance between their own information and price, they do not attempt to "manipulate the price or the information content of price". His solution is to consider a limit economy where the equilibrium price still aggregates all available information but the effect of each individual trader becomes negligible. A recent paper by Kyle [1984] proposes a different solution: he considers a model with "noise" traders and a large number of informed and uninformed speculators who take into account the effect of their trades on equilibrium price.

Characterizing a Nash equilibrium in which each trader takes as given the strategies of the remaining traders and conditions on his private information and the equilibrium price when choosing his demand function, Kyle [1984] shows that some of the undesirable consequences of the competitive rational expectations equilibrium disappear: even in the absence of noise trading, prices reveal only
half the precision of each trader's information. When noise trading exists, the incentives for costly information acquisition are greater because price incorporates a smaller amount of the private information compared to the competitive case. Finally, a well-defined equilibrium exists even if speculators are risk-neutral. Similar results emerge from my price-setting game with inside information. The reason is that the equilibrium outcomes of the price-setting game also describe those of a quantity-setting game similar to Kyle's where the insider chooses asset demands based on an observation of the price, his private information and the value of his initial endowment. However, even under the second interpretation, there exist differences between Kyle's [1984] framework and mine. First, uninformed traders behave as price-takers in my model: it is only the insider who exploits the joint advantages of his informational monopoly and his size relative to the market. Furthermore, all traders are rational. In other words, there are no "noise" traders in my framework who purchase a random, inelastic quantity regardless of their losses. Finally, this paper addresses some issues not considered by Kyle [1984] and others. In particular, it tries to determine whether observations on equilibrium prices and quantities traded suffice to identify the existence of strategic insider trading in an otherwise competitive market. For example, it shows that contrary to some commonly held beliefs, insider trading does not necessarily lead to "excess" variability of equilibrium prices: by taking into account uninformed traders' reactions, the insider who behaves strategically may reduce the variance in price compared to the competitive outcome.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 presents the equilibrium under different information structure and assumptions about the behavior of the large trader. Equilibrium outcomes are
analyzed in Section 4. Section 5 discusses whether such market outcomes can be identified from observable series on prices and quantities traded.
2. **The Model**

Consider a simple two period economy: in the first period, individuals can trade claims to two assets, a risky asset which has an uncertain payoff of $r$ and a riskless asset which yields one unit with certainty. The market at date one in which these claims are traded is comprised of a continuum of uninformed traders and an informed trader or insider who receives a noisy signal $y = r + v$ about the future payoff $r$. The two types of traders are also differentiated with respect to size: while uninformed traders are non-atomic, the insider is large relative to the market in the sense that he is of positive measure, normalized here at one. Furthermore, each uninformed trader is assumed to be risk-averse with a strictly positive coefficient of absolute risk aversion equal to $a_u$. The insider may or may not be risk-averse, depending on the context. Hence, $a_I \geq 0$.

The initial wealth of each trader is determined by his initial position or endowment of the two assets. Uninformed traders, who are identical in preferences and beliefs, also possess the same number of shares of the riskless asset, denoted by $B_u^i = B$. However, they differ with respect to their ownership of the risky asset: a fraction $m$ has a large initial position in the risky asset, denoted by $e_u^1 = e^1_y$ while the remainder $1-m$ own only $e_u^i = e^2_u$ shares with $e^1_u > e^2_u, 0 < m < 1$. Similarly, the insider's initial wealth is determined by his endowment of the two assets, denoted by $B_I^i$ and $e_I$, respectively. The initial wealth of each trader fluctuates randomly because his initial holdings of the risky asset fluctuate randomly, i.e., $e_u^1, e_u^2$ and $e_I$ are realizations from random variables. On the other hand, the values of $B$ and $B_I$ are fixed and known by all traders.

In period one, all traders evaluate the uncertainty in their environment with the same prior beliefs, which specify that the random variables $r, v, e_I$
and \( e^1_u \) and \( e^2_u \) are independently and normally distributed with means \( r^m \), 0, \( e^1_m \), \( e^2_m \) and variances \( \sigma^2_r \), \( \sigma^2_v \), \( \sigma^2_i \), \( \sigma^2_{1u} \) and \( \sigma^2_{2u} \). Furthermore, the structure of these beliefs is common knowledge.

Here \( 1/\sigma^2_v \) measures the precision of the insider's signal, \( \sigma^2_r \) the riskiness of investment opportunities, \( \sigma^2_i \) the variability of the insider's initial wealth and \( m \) the distribution of wealth among uninformed traders. Finally,

\[
e^m_u = m e^1_u + (1-m) e^2_u
\]

and

\[
\sigma^2_u = m \sigma^2_{1u} + (1-m) \sigma^2_{2u}
\]

determine distribution of uninformed traders' aggregate holdings of the risky asset, defined by the normal random variable

\[
e_u = \int_0^m e^1_u \, d\mu(i) + \int_m^1 e^2_u \, d\mu(i)
\]

\[
= m e^1_u + (1-m) e^2_u.
\]

In this definition, \( \mu(i) \) denotes the Lebesgue measure on the unit interval.

The information possessed by each trader in this economy depends on both public and private sources of information. Prior to trading, the insider receives a piece of news \( y \) about the future payoff \( r \). Each trader also observes the number of shares of the risky asset he is endowed with in period one. Further, all traders learn the total number of shares available for trade in the risky asset. From this information, the insider also infers uninformed traders' aggregate endowment, i.e., knowledge of the total \( e = e^1_I + m e^1_u + (1-m) e^2_u \) together with \( e^1_I \) allows him to infer the current realization of \( e_u \). On the other hand, any given uninformed trader observes only \( e - m e^1_u = e^1_I + (1-m) e^2_u \),
\( i, j = 1, 2, \ i \neq j \) and is unable to infer the value of \( e_i \) or \( e_u \).
3. **Equilibrium**

The large trader of this model derives market power from his informational advantage as well as from his size relative to the market. This section describes the securities market equilibrium which emerges when the insider optimally exploits both sources of market power. As a benchmark with which to compare the consequences of strategic behavior with inside information, it also describes the equilibrium derived under the assumption that information is private but the informed trader nevertheless behaves competitively. However, even if the large trader loses his informational advantage, his size relative to the remaining trader still enables him to affect equilibrium outcomes. Consequently, a second benchmark is provided by the dominant trader equilibrium with public information.

The way in which the insider makes use of his dual advantages is simple: conditional on observing $y$, $e_I$, and $e$ or $e_u$, the informed trader sets a price $p$ for the risky asset to maximize the expected utility of next period's wealth $W_I = rX^I + B^I$ and agrees to fulfill all desired trades at this price. Consequently, his position in the market $X_I$ is determined by the constraint

$$X_I = e_I + e^1_u - X^1_u + e^2_u - X^2_u$$

where $e^1_u - X^1_u$ and $e^2_u - X^2_u$ denote the excess demands of "wealthy" and "poor" uninformed traders. On the other hand, each uninformed trader behaves competitively. Taking as given the price set by insiders, he maximizes his expected utility of wealth $W^I_u = rX^I_u + B^I_u$ by choosing how many shares $X^I_u$ and $B^I_u$ of the risky and riskless asset to hold. Both types of traders are permitted to engage in short sales of the assets. In period two, the payoff on the risky asset is realized and consumption takes place.

When setting a price, the insider recognizes the effect of his decision on the demands of the uninformed traders and takes their reaction into account.
While the insider observes the total endowment \( e_u \) of the uninformed traders, he must nevertheless conjecture a set of demand functions \( \chi^i_u(p) \) and \( B^i_u(p) \) in order to determine their net trades at any price \( p \). In this way, he also determines the value of his own position for different values of the price. Similarly, when computing their demands, uninformed traders use observations on price to update their prior beliefs about the uncertain payoff \( r \). To do this, they must conjecture a price-setting rule \( p(y,e_I,e_u) \) which specifies the price charged by the insider for any realization of \( y, e_I \) and \( e_u \).

Defining \( p: \mathbb{R}^3 \rightarrow \mathbb{R} \), \( \chi^i_u: \mathbb{R} \rightarrow \mathbb{R} \) and \( B^i_u: \mathbb{R} \rightarrow \mathbb{R} \) as the strategies for the insider and each uninformed trader, respectively, the strategies \( p^*(y,e_I,e_u) \), \( \chi^i_u(p) \) and \( B^i_u(p) \) constitute an equilibrium if each is a best response to the other. More formally, \((p^*(y,e_I,e_u), \chi^i_u(p), B^i_u(p)) \) for all \( i \in [0,1] \) is an equilibrium if

\[
(3.1) \quad \chi^i_u(p), B^i_u(p) \in \text{argmax } E^i_u[-\exp(-a_u(r\chi^i_u + B^i_u)) | e_u, e, p, p^*(y,e_I,e_u)]
\]

subject to \( px^i_u + B^i_u \leq pe^i_u + B_u \)

\[
(3.2) \quad p(y,e_I,e_u) \in \text{argmax } E^i_1[-\exp(-a_1(rX^i_1 + B^i_1)) | y,e_I,e, \chi^i_u(p), B^i_u(p)]
\]

subject to \( px^i_I + B^i_I \leq pe^i_I + B_I \)

\[
X^i_I = e^i_I + \int_0^1 (e_u^i - \chi^i_u(p)) du(i).
\]

Notice that in (3.1), each uninformed trader's expectation about the future payoff \( r \) is conditional on observations of his individual endowment \( e_u^i \), the aggregate \( e \), price \( p \) and the conjectured equilibrium strategy \( p^*(y,e_I,e_u) \) for the insider. Similarly, because the insider always holds the residual position in the market as shown by the second constraint in (3.2), his pricing
decision depends on his conjecture about the uninformed traders' demands. The equilibrium pricing rule and asset demands are such that each trader's conjecture about other traders' behavior is confirmed.

Notice that in this definition of equilibrium, the dominant trader makes his pricing decision after observing the current realization of the signal and endowments. Consequently, unlike the papers by Kihlstrom and Postlewaite [1983], and Grinblatt and Ross [1982], issues of pre-commitment do not arise. Alternatively, the equilibrium of this paper is subgame perfect, with the postulated strategy constituting a Nash equilibrium for subgames defined for any realizations of \( y, e_I \) and \( e_u \).

Second, note that the large trader also serves as an actual price-setter in this market, satisfying the demands of uninformed traders in order to keep price at the value defined by \( p = p(y, e_I, e_u) \). An alternative way of defining equilibrium is to assume that, conditional on information available in period one, all traders choose excess demand schedules showing the quantity demanded as a function of the current price and other relevant variables and submit these schedules to a fictitious auctioneer. The equilibrium price is then chosen by the auctioneer to make the value of these excess demands equal to zero. If the large trader chooses an excess demand schedule which is perfectly elastic at the price defined in (3.2), then the two interpretations of equilibrium lead to identical outcomes. In this case, the insider effectively sets price by trading only at the price defined by \( p(y, e_I, e_u) \). However, under either interpretation, the set of possible strategies available to the traders is very large. Considering again the case of the price-setting insider, one alternative to the strategy of satisfying all excess demands of uninformed traders at the price \( p(y, e_I, e_u) \) is to charge a price subject to a limit on short sales, i.e., choose a price \( p \) to satisfy (3.2) with the additional constraint that \( x_I \geq -B \).
Another possible strategy is to price successive blocks of stock. Despite the potential interest, finding and characterizing the optimal strategy in such general strategy spaces is a difficult task and not attempted in this paper.

In what follows, I further restrict the strategies in (3.1) and (3.2) to be linear in the relevant variables. Given the extensive literature which has analyzed linear rational expectations equilibria in similar models, such a restriction does not seem to be a serious shortcoming in this context as well. This implies a price-setting rule linear in the informed traders' information and the values of initial endowments $e_I$ and $e_u$. Similarly, the demand schedules of uninformed traders are restricted to be linear in price. Since uninformed traders are identical in all respects except for the value of their initial endowments, their equilibrium demand functions $x^{i*}_u(p)$ are conjectured to be identical. In this case, the functions $B^{i*}_u(p)$, $i \in [0,1]$ showing uninformed traders' demand for the riskless asset will differ across traders only because of their dependence on the value of $e^i_u$. Under these conditions, I can prove

Proposition 3.1: The set of coefficients $(\pi_0, \pi_1, \pi_2, \pi_3, \alpha_0, \alpha_1)$ describe a linear price-setting equilibrium with inside information such that

$$p^*(y,e^I,e_u) = \pi_0 + \pi_1 y + \pi_2 e^I + \pi_3 e_u$$

$$x^{i*}_u(p) = \alpha_0 + \alpha_1 p$$

and

$$B^{i*}_u(p) = p e^i_u - \alpha_0 - \alpha_1 p, \ i \in [0,1]$$

and

$$\alpha_0 = \frac{r m_d - \pi_1 \sigma_r^2 p^m}{a_u \sigma_r^2 (D_1 - \pi_2 \sigma_r^2)}$$

$$\alpha_1 = \frac{\pi_1 \sigma_r^2 - D_1}{a_u \sigma_r^2 (D_1 - \pi_2 \sigma_r^2)}$$
\[ \pi_0 = \left[ -\frac{\alpha_0}{\alpha_1} (1 - D_2) + \frac{\sigma_r^2}{(\sigma_r^2 + \sigma_Y^2)(D_2 + 1)} \right] \]

\[ \pi_1 = \frac{\sigma_r^2}{(\sigma_r^2 + \sigma_Y^2)(D_2 + 1)} \]

\[ \pi_2 = -\frac{a_1a_1^2}{(\sigma_r^2 + \sigma_Y^2)(D_2 + 1)} \]

\[ \pi_3 = \frac{D_2}{\alpha_1(D_2 + 1)} \]

and

\[ D_1 = \pi_1^2(\sigma_r^2 + \sigma_Y^2) + \pi_2^2 + \pi_3^2 \]

\[ D_2 = 1 - \frac{a_1a_1^2}{(\sigma_r^2 + \sigma_Y^2)} \]

\[ p^m = \pi_0 + \pi_1r^m + \pi_2e_1^m + \pi_3e_u^m. \]

Furthermore,

\[ 0 < \pi_1 < \frac{\sigma_r^2}{2(\sigma_r^2 + \sigma_Y^2)}, \quad \frac{-a_1a_1^2}{2(\sigma_r^2 + \sigma_Y^2)} < \pi_2 < 0 \quad \text{and} \quad \pi_3 < 0. \]

Proof: See Appendix I.

To identify the consequences of informed trading by a dominant trader, I also prove

**Corollary 3:** The set of coefficients \((n_0, n_1, n_2, n_3, \nu_0, \nu_1, \nu_2)\) describe a price-setting equilibrium with public information such that

\[ p(y,e_1,e_u) = \eta_0 + \eta_1y + \eta_2e_1 + \eta_3e_u, \]

\[ x_u^i(y,p) = \nu_0 + \nu_1y + \nu_2p \quad \text{and} \quad B_u^i(y,p) = pe_u^i + B_u^i - px_u^i(y,p) \quad i \in [0,1] \]
Proof: Follows directly from the proof of Proposition 3.1 by noting that observing price yields no additional information to the competitive fringe when \( y \) is public information.

The assumption that the insider sets prices by taking into account the reactions of uninformed traders may appear as an extreme description of behavior in securities markets. However, it turns out that the outcomes implied by Proposition 3.1 or its corollary are also consistent with a quantity-setting game in which the large trader chooses a linear demand function, taking into account his effect on the equilibrium price and using observations on the price \( p \), his private information \( y \) and the value of his endowment \( e_I \). The uninformed traders behave competitively and choose asset demands as a linear function of price. The equilibrium price is determined from the market-clearing condition which sets the sum of the excess demands of informed and uninformed traders equal to zero. Unlike the price-setting game where the large trader moves first and announces a price at which he is willing to fulfill all trades, the quantity-setting game is a simultaneous move game where all traders choose...
asset demand functions.

To see how the large trader takes into account his effect on the equilibrium price, notice that the price may be expressed as a function of \( X_I \) by using the market-clearing condition

\[
e_I - X_I(p, y, e_I) + \int_0^1 [e_u^i - X_u^i(p)]d\mu(i) = 0
\]

and conjecturing that \( X_u^i(p) = \alpha_0 + \alpha_1 p \) with \( \alpha_1 \neq 0 \), i.e.,

\[
(3.3) \quad p = -\frac{\alpha_0}{\alpha_1} + \frac{1}{\alpha_1} (e_I + e_u) - \frac{1}{\alpha_1} X_I.
\]

It is easy to show that the coefficients in the price-setting rule of Proposition 3.1 or its corollary also describe the equilibrium price for the simultaneous move game when the large trader chooses linear asset demands \( X_I \) and \( B_I \) to maximize his expected utility of end-of-period wealth \( W_I \) subject to his budget constraint and given condition (3.3). As before, the asset demands of the uninformed traders, \( X_u^i(p) \) and \( B_u^i(p) \), solve (3.1). The two different games yield identical equilibrium outcomes due to the existence of a single large trader who behaves non-competitively. In the absence of any restrictions on short sales, choosing asset demands \( X_I \) and \( B_I \) to satisfy (3.3) and the single period budget constraint is equivalent to choosing price such the \( X_I \) and \( B_I \) are determined from

\[
pX_I + B_I \leq pe_I + B_I
\]

and

\[
X_I = e_I + \int_0^1 [e_u^i - X_u^i(p)]d\mu(i).
\]

Alternatively, the equilibrium outcomes of the price-setting game will not be identical to those of the quantity-setting game when there exist several large
traders who behave as imperfect competitors. However, for the purposes of this paper, the ability to interpret the equilibrium outcomes of Proposition 3.1 and its corollary in terms of the quantity-setting game has one advantage: that is, I can use the competitive rational expectations equilibrium as a natural benchmark for analyzing the consequences of non-competitive behavior by the insider.

In this case, the informed trader bases his decisions on his private information but otherwise behaves competitively. Furthermore, all traders move simultaneously and, taking prices as given, choose the quantities they wish to trade at these prices. However, in order to do this, they must conjecture a pricing function which describes the statistical relationship between price, on the one hand, and the signal \( y \) and endowments \( e_I + e_u \), on the other.

Defining the functions \( p: R^2 \to R \), \( X^i: R^2 \to R \), \( B^i: R^2 \to R \), \( X^i_U: R \to R \), and \( B^i_U: R \to R \), the pricing function \( p^*(y,e_I + e_u) \) and asset demands \( x^i_U(p), \ B^i_U(p), \ i \in [0,1], \ X^i(p,y) \) and \( B^i(p,y) \) constitute an equilibrium if

\[
(3.4) \quad \begin{align*}
X^i_U(p), \ B^i_U(p) & \in \text{argmax} \ E_i[-\exp(-a_u(rX^i_U + B^i_U))] \text{ s.t. } X^i_U(p), B^i_U(p) \\
\text{subject to } \ pX^i_U + B^i_U & \leq pe^i U + B^i_U
\end{align*}
\]

\[
(3.5) \quad \begin{align*}
X^i(p,y), \ B^i(p,y) & \in \text{argmax} \ E_i[-\exp(-a_l(rX^i + B^i))] \text{ s.t. } X^i(p,y), B^i(p,y) \\
\text{subject to } \ pX^i + B^i & \leq pe^i + B^i
\end{align*}
\]

\[
(3.6) \quad \begin{align*}
X^i + \int_0^1 X^i_U(p)du(i) & \leq e_I + \int_0^1 e^i_Udu(i) \\
B^i + \int_0^1 B^i_U(p)du(i) & \leq B_I + \int_0^1 B^i_Udu(i).
\end{align*}
\]

The first part of this definition merely describes how each uninformed trader maximizes the expected value of next period's wealth, conditional on the value of his endowment and the aggregate \( e \) as well as on observations of the
current price and his conjecture about the equilibrium pricing function 
\( p(y, e_I + e_u) \). The analogous problem for the informed trader is presented in (3.5). The next proposition characterizes the pricing function which is linear in the insider's signal and the initial endowments such that for any realization of \( y \), \( e \) and \( e_u \), markets clear according to Condition (3.6).

**Proposition 3.2:** The set of coefficients \( (\delta_0, \delta_1, \delta_2, \gamma_0, \gamma_1, \beta_0, \beta_1, \beta_2) \) describe a linear rational expectations equilibrium with inside information such that

\[
\begin{align*}
    p^*(y, e_I + e_u) &= \delta_0 + \delta_1 y + \delta_2 (e_I + e_u) \\
    x^*_u(p) &= \gamma_0 + \gamma_1 p, \quad B^*_u(p) = p e^*_u + B^*_u - p x^*_u(p), \quad i \in [0, 1] \\
    X^*_I(p,y) &= \beta_0 + \beta_1 y + \beta_2 p \quad \text{and} \quad B^*_I(p,y) = p e^*_I + B^*_I - p X^*_I(p,y)
\end{align*}
\]

and

\[
\begin{align*}
    \beta_0 &= \frac{r_m}{a_I \sigma_v^2} \quad \beta_1 = \frac{1}{a_I \sigma_v^2} \\
    \beta_2 &= \frac{(\sigma_r^2 + \sigma_v^2)}{a_u \sigma_r \sigma_v^2} \\
    \gamma_0 &= \frac{r_m D_3 - \delta_1 \sigma_r^2 \rho_m}{a_u \sigma_r (D_3 - \delta_1 \sigma_r^2)} \\
    \gamma_1 &= \frac{\delta_1 \sigma_r^2 - D_3}{a_u \sigma_r (D_3 - \delta_1 \sigma_r^2)} \\
    \delta_0 &= \frac{- (\beta_0 + \gamma_0)}{\beta_2 + \gamma_1} \\
    \delta_1 &= \frac{2}{\sigma_r (\sigma_r^2 + \sigma_v^2) D_4}
\end{align*}
\]
\[ \delta_2 = \frac{-a_I \sigma_r^2 \sigma_v^2}{(\sigma_r^2 + \sigma_v^2)D_4} \]

and

\[ D_3 = \delta_1^2(\sigma_r^2 + \sigma_v^2) + \delta_2^2(\sigma_r^2 + \sigma_u^2) \]

\[ D_4 = 1 - \frac{\gamma_1 a_I \sigma_r^2 \sigma_v^2}{2(\sigma_r^2 + \sigma_v^2)} \]

\[ p^m = \delta_0 + \delta_1 p^m + \delta_2 (e^m_1 + e^m_u). \]

Furthermore,

\[ 0 < \delta_1 < \frac{\sigma_r^2}{\sigma_r^2 + \sigma_v^2} \quad \text{and} \quad \frac{-a_I \sigma_r^2 \sigma_v^2}{2(\sigma_r^2 + \sigma_v^2)} < \delta_2 < 0. \]

**Proof:** See Appendix I.

**Corollary 3.2:** The set of coefficients \((\xi_0, \xi_1, \xi_2, \nu_0, \nu_1, \nu_2, \phi_0, \phi_1, \phi_2)\) describe a rational expectations equilibrium with public information such that

\[ p(y, e_1 + e_u) = \xi_0 + \xi_1 y + \xi_2 (e_1 + e_u) \]

\[ x^i_u(y, p) = \nu_0 + \nu_1 y + \nu_2 p, \quad \delta^i_u(y, p) = pe^i_u + \delta^i_u - px^i_u(y, p) \quad i \in [0, 1] \]

and

\[ x^i_1(y, p) = \phi_0 + \phi_1 y + \phi_2 p \quad \text{and} \quad B^i_1(y, p) = pe^i_1 + B^i_1 - px^i_1(y, p) \]

and

\[ \phi_0 = \frac{\sigma_v^2 p^m}{a_I(\sigma_r^2 + \sigma_v^2)} \quad \phi_1 = \frac{1}{a_I \sigma_v^2} \quad \phi_2 = \frac{-a_I \sigma_r^2 \sigma_v^2}{a_I(\sigma_r^2 + \sigma_v^2)} \]
\[
\xi_0 = \frac{\sigma_v^2 \rho_m}{\sigma_v^2 + \sigma_r^2} \\
\xi_1 = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_v^2} \\
\xi_2 = \frac{-a_u \sigma_v^2 \sigma_r \alpha_1}{(\sigma_r^2 + \sigma_v^2)(\alpha_1 + a_u)}
\]

and \( \nu_0, \nu_1 \) and \( \nu_2 \) are as in Corollary 3.1.

\textbf{Proof:} Directly follows the proof of Proposition 3.2 by noting that price yields no additional information to traders when \( y \) is public.
4. **Equilibrium Outcomes**

Given the simplicity of the equilibrium strategies, the main interest for the results of Proposition 3.1 lies in the predicted equilibrium outcomes. In particular, one would like to know how the existence of an insider with market power affects the information content of price and the responsiveness of price to new information compared to the case when all traders behave competitively. Alternatively, it is of interest to determine whether strategic insider trading leads to a loss of informational efficiency by making the market "noisier" from the point of view of each uninformed trader. Yet the complexity of the expressions in Propositions 3.1 and 3.2—which arises because the coefficients of equilibrium price are jointly determined with the coefficients of uninformed traders' asset demands—precludes a direct analysis. However, some special cases yield determinate results.

The first case occurs when there is no uncertainty about the initial wealth of the different types of traders. Denoting the equilibrium prices of Propositions 3.1 and 3.2 as $p_1$ and $p_2$, the next example describes the relevant equilibria when $e_1$, $e_u^1$ and $e_u^2$ are fixed and known by all traders.

**Example 4.1:** $(\sigma_u^2 = \sigma_i^2 = 0)$

$$p_1 = \frac{\sigma_{vr}^m}{2(\sigma_r^2 + \sigma_v^2)} + \frac{\sigma_r^2}{2(\sigma_r^2 + \sigma_v^2)} y$$

whereas

$$p_2 = \frac{\sigma_{vr}^m}{\sigma_v^2 + \sigma_r^2} + \frac{\sigma_v^2}{\sigma_r^2 + \sigma_v^2} y$$

This example illustrates the well-known result by Grossman [1976], and Grossman and Stiglitz [1978] that the competitive rational expectations equilib-
rium price fully reveals all information in the market. Yet, under the conditions of Example 4.1, this occurs even if the insider attempts to optimally exploit his market power. In both cases, price varies one-to-one with information, the coefficients \( \pi_0 \) and \( \delta_1 \) reflecting the informativeness of the signal \( y \). The only difference between the equilibrium outcomes of Propositions 3.1 and 3.2 are that \( \pi_0 = 1/2\delta_0 \) and \( \pi_1 = 1/2\delta_1 \): the insider who recognizes his market power prevents price from being pushed up against himself with his information. In the absence of opportunities for re-trading, the informed trader can exploit his market power to earn positive profits even when the equilibrium price perfectly reveals his information.

On the other hand, introducing even a small amount of noise in the form of uncertainty about initial wealth alters these conclusions. To illustrate the consequences of non-competitive behavior in this situation, it is useful to consider the cases when the insider has perfect information or is risk-neutral. For, not only are the equilibrium outcomes under these assumptions easy to characterize, but they are of interest in their right. In particular, the assumption of risk neutrality seems appropriate for an informed trader with a large initial position.

In Examples 4.2(a) and 4.3(a), \( P_1 \) and \( P_2 \) again denote the equilibrium prices of Propositions 3.1 and 3.2. However, for future reference, Examples 4.2(b) and 4.3(b) describe the equilibria of Corollaries 3.1 and 3.2 where the signal \( y \) is publicly observed. In this case, the relevant prices are denoted by \( \tilde{P}_1 \) and \( \tilde{P}_2 \).

**Example 4.2:** \( (\sigma_y^2 = 0) \)

(a) \( y \) is private information

\[
P_1 = \frac{-\sigma_0}{2\alpha_1} + \frac{1}{2} y + \frac{1}{2\alpha_1} e_u
\]
and

\[ p_2 = y \]

(b) \( y \) is public information

\[ p_1 = y \]

and

\[ p_2 = y. \]

**Example 4.3: \( a_1 = 0 \)**

(a) \( y \) is private information

\[ p_1 = \frac{1}{2} \left[ \frac{\sigma_{Vr}^m}{\sigma_r^2 + \sigma_v^2} - \frac{\alpha_0}{\alpha_1} \right] + \frac{\sigma_r^2}{2(\sigma_r^2 + \sigma_v^2)} y + \frac{1}{2\alpha_1} e_u \]

and

\[ p_2 = \frac{\sigma_{Vr}^m}{\sigma_r^2 + \sigma_v^2} + \frac{\sigma_r^2}{\sigma_r^2 + \sigma_v^2} y \]

(b) \( y \) is public information

\[ p_1 = \frac{\sigma_{Vr}^m}{\sigma_r^2 + \sigma_v^2} + \frac{\sigma_r^2}{\sigma_r^2 + \sigma_v^2} y - \frac{a_u \sigma_v^2 \sigma_r^2}{(\sigma_r^2 + \sigma_v^2)} e_u \]

and

\[ p_2 = \frac{\sigma_{Vr}^m}{\sigma_v^2 + \sigma_r^2} + \frac{\sigma_r^2}{\sigma_v^2 + \sigma_r^2} y. \]

Recall from Proposition 3.2 that the informed trader who behaves competitively reacts to his signal with a weight reflecting his risk aversion and the precision of his signal, i.e.,
When either $a_1 = 0$ or $v = 0$, his reaction to his information becomes so strong that the effect of random supplies on price disappears ($\delta_2 = 0$) and price becomes fully revealing. This is demonstrated by the expressions for $P_2$ in Examples 4.2(a) and 4.3(b). In both cases, price varies one-to-one with the signal $y$. As in the case with fixed endowments, any gains accruing to superior information are eliminated in equilibrium. Further, the competitive outcome achieves informational efficiency because the price of the risky asset equals the expected value of the future payoff $r$, conditional on observing the noisy signal, i.e.,

$$P_2 = E\{r|y\} = \frac{\sigma_r y}{\sigma_r} + \frac{\sigma_y y}{\sigma_y}.$$ 

When $\sigma_y = 0$, this expression simplifies to the value for $P_2$ given in Example 4.3(a), namely $y$.

On the other hand, if the insider can behave strategically and set prices, equilibrium outcomes differ markedly from those emerging with competitive behavior. As in Example 4.1, only half as much information is incorporated in price, i.e., $\pi_1 = \frac{1}{2} \delta_1$. But, more important, price also fluctuates with the aggregate endowment of uninformed traders, i.e.,

$$\pi_3 = \frac{1}{2a_1} \neq 0 \text{ provided } a_1 \neq -\infty.$$ 

This last condition is always satisfied in equilibrium because uninformed traders are assumed to be risk averse. If, in addition, there is some heterogeneity among uninformed traders so that $m \neq 1$, price fails to be fully revealing from the point of view of each such trader who observes the quantity $e_1 + me_u$ or $e_1 + (1-m)e_u$ but not $e_1$ or $e_u$ separately.
Furthermore, because the price of the risky asset is no longer an unbiased predictor of future payoffs, the insider always earns positive profits in equilibrium. Using the expressions for price $p$ and $e_1 - x_1$ implied by Examples 4.2 and 4.3 expected profits are given by

$$
(4.2) \quad [E(r|y) - p][e_1 - x_1] = \left[ \frac{\sigma_r^2 + \sigma_y^2}{\sigma_r^2 + \sigma_y^2} - \pi_0 - \pi_1 y - \pi_3 e_u \right] \cdot \left[ e_u - \alpha_0 - \alpha_1 p \right]
$$

$$
= \frac{1}{4} \left[ \frac{\sigma_r^2 + \sigma_y^2}{\sigma_r^2 + \sigma_y^2} + \frac{\alpha_0 - e_u}{\alpha_1} \right] \cdot \left[ e_u - \alpha_0 - \alpha_1 \frac{\sigma_r^2 + \sigma_y^2}{\sigma_r^2 + \sigma_y^2} \right]
$$

$$
= \frac{-\alpha_1}{4} \left[ E(r|y) + (\alpha_0 - e_u)/\alpha_1 \right]^2 > 0.
$$

since $\alpha_1 < 0$. Notice that expected profits reflect the information possessed by the insider. Consequently, unlike the informed trader who behaves competitively, the price-setting insider can benefit from his informational monopoly as long as uninformed traders desire to share risks through the market but are unsure about the quantity of shares held by the different types of traders. On the other hand, with competitive behavior, $p = E(r|y)$ so that expected profits are always zero.

The above examples can also be used to examine the role of informational advantage. For this purpose, consider a risk-neutral monopolist who has no private source of information. From Corollary 3.1 and Example 4.2(b), his expected profits can be expressed as

$$
(4.3) \quad \left[ E(r|y) - p \right] \cdot [e_1 - x_1] = [E(r|y) + \frac{e_u}{2\mu_1}] \cdot [e_u - \mu_0 - \mu_1 y - \mu_2 p] =
$$

$$
= -\frac{e_u^2}{4\mu_1^2}
$$

$$
= \frac{e_u^2 a_u \sigma_r^2 \sigma_y^2}{4(\sigma_r^2 + \sigma_y^2)} > 0.
$$
Comparing (4.3) with (4.2) shows that when information is public, any profit opportunities due to information are exhausted by the trades of the competitive fringe. Hence, (4.3) is positive but independent of the signal \( y \). Since \( \nu_2 \) determines uninformed traders' elasticity of demand for the risky asset, this expression also shows that the large trader benefits from his ability to set price under the same conditions as a textbook monopolist facing an exogenous demand curve. This occurs when the demand function of competitive traders is a negative function of price.

Alternatively, Example 4.3(b) shows that the actions of a risk-neutral monopolist can prevent price from being an unbiased predictor of future payoffs, even when all information is public. However, this example also shows that the existence of such a trader does not affect the efficiency with which information is incorporated into price: in both Examples 4.2(b) and 4.3(b), the coefficient of \( y \) in the expression for the price set by the large trader, \( p_1 \), is equal to coefficient of \( y \) in the competitive price \( p_2 \). But this is just a special case of the results in Corollaries 3.1 and 3.2 that with public information, the coefficient of \( y \) merely reflects the informativeness of this signal, regardless of how the large trader behaves.

Table 4.1 provides a more general comparison of the equilibrium outcomes implied by Propositions 3.1 and 3.2. Given the simultaneous determination of the coefficients of equilibrium prices, on the one hand, and the coefficients of traders' asset demands, on the other, such a comparison cannot be achieved analytically. Consequently, I numerically compute the relevant equilibria for given values of the exogenous parameters \( a_u, a_I, c_r, c_v, a_u \) and \( a_I \), as described in Appendix II. This determines the coefficients \( \pi_0, \pi_1, \pi_2 \) and \( \pi_3 \) characterizing the equilibrium price in Proposition 3.1. The coefficients \( \delta_0, \delta_1 \) and \( \delta_2 \) of Proposition 3.2 are similarly determined. The expressions
in columns four and five of Table 4.1 are then evaluated by using these coefficients as

\[ \sigma_1^2 = \text{var}(p^1) = \pi_1^2(\sigma_r^2 + \sigma_v^2) + \pi_2^2\sigma_1 + \pi_3^2\sigma_u^2 \]

(4.4)

\[ \sigma_2^2 = \text{var}(p^2) = \delta_1^2(\sigma_r^2 + \sigma_v^2) + \delta_2^2(\sigma_I^2 + \sigma_u^2) \]

and

\[ \tau_1 = \frac{\pi_2^2(\sigma_r^2 + \sigma_v^2)}{\sigma_1^2} \]

(4.5)

\[ \tau_2 = \frac{\delta_2^2(\sigma_r^2 + \sigma_v^2)}{\sigma_2^2} \]

| Table 4.1 |
|------------------|------------------|------------------|------------------|------------------|------------------|
| \( \pi_1 - \delta_1 \) | \( \pi_2 - \delta_2 \) | \( \pi_3 - \delta_2 \) | \( \sigma_1^2 - \sigma_2^2 \) | \( \tau_1 - \tau_2 \) |
| \( \sigma_1^2 \) | < 0 | > 0 | < 0 | > 0 | < 0 |
| \( \sigma_2^2 \) | > 0 | < 0 | < 0 | > 0 | > 0 |
| \( \sigma_r^2 \) | > 0 | < 0 | < 0 | > 0 | < 0 |
| \( \sigma_v^2 \) | < 0 | > 0 | < 0 | > 0 | < 0 |
| \( \sigma_u^2 \) | < 0 | > 0 | < 0 | > 0 | < 0 |
| \( \sigma_I^2 \) | > 0 | ? | ? | < 0 |< 0 |

Table 4.1 compares the magnitudes of \( \pi_1 \) versus \( \delta_1 \), \( \delta_2 \) versus \( \delta_2 \), etc.,
as the values of the exogenous parameters are individually varied. For example, the entries of \(<0\) in column one of Table 4.1 show that, regardless of which parameter is increased, the coefficient of the signal \(y\) in the price-setting rule, \(\pi_1\), is always less than the corresponding coefficient in the competitively determined price, \(\delta_1\). Consequently, the first column replicates the results of Examples 4.1, 4.2(a) and 4.2(b) and shows that price responds less to new information when the insider can behave strategically. In this case, the insider profits from his informational advantage by reducing the covariance of price with his information.

The second and third columns compare the responsiveness of price under the two equilibria to changes in the initial endowments of the insider and the uninformed traders, respectively. Recall that \(\pi_2, \delta_2\) and \(\pi_3\) are all negative. Consequently, columns two and three show that, compared to the competitively outcome, the price set by the insider fluctuates more with changes in the endowments of uninformed traders than with changes in his own endowment \(e_I\). In other words, for most parameter values, \(|\pi_3| > |\delta_2|\) while \(|\pi_2| < |\delta_2|\): prices fall less with increases in \(e_I\) in the latter case because the insider accounts for the fact that a higher price increases the value of his initial endowments and consequently his initial wealth.

The fourth column compares the absolute variability of price under the two equilibria: for most parameter values, the variance of price is greater in the price-setting equilibrium. This result formalizes the intuition that strategic trading may lead to "excess volatility" in asset prices and cause the market to be "noisier" from the point of view of small uninformed traders. However, Table 4.1 also shows that large variations in price may not prove very useful for indicating the presence of an insider who manipulates price. For notice from the second and sixth entries of columns four that \(\sigma_1^2\) may be less than \(\sigma_2^2\) if the informed trader is very risk-averse (large \(a_1\)) or suffers large vari-
ability in his initial wealth (large $\sigma^2_I$). In any case, the last column of Table 4.1 shows that if the information content of price is measured by the proportion of variance in price attributed to the signal $y$, then strategic insider trading always causes equilibrium prices to be a worse indicator of the information in the market compared to the competitive outcome.

Finally, for the sake of completeness, Tables 4.2 and 4.3 show comparative statics results for the individual equilibria of Propositions 3.1 and 3.2. As an example, notice that an entry of (-) in the first row, first column of Table 4.2 implies a fall in $\pi_1$ as $\sigma^2_u$ increases. Under this interpretation, the first columns of both tables show that the responsiveness of price to new information decreases as the market becomes noisier ($\sigma^2_v$ increases) or the informed trader becomes more risk-averse ($a^2_I$ increases). On the other hand, greater variability in the payoffs on the risky asset (large $\sigma^2_r$) or increases in the risk aversion of uninformed traders (large $a_v$) have the opposite effect. Notice also that an increase in the risk aversion of the informed trader does not necessarily increase the overall variability of price when the insider behaves strategically. This is a reflection of the results in Table 4.1. Another interesting result is that greater risk aversion on the part of uninformed traders does not reduce the informativeness of the competitive price, i.e., $\tau_2$ does not change when $a_u$ increases. With strategic insider trading, a rise in this parameter increases the overall variability of price, causing $\tau_1$ to fall. This occurs because the coefficients on $e^I$ and $e^u$ increase in absolute value more than the coefficient on the signal. When uninformed traders are very risk-averse, their asset demands will not fluctuate very much with price. Taking this reaction into account, the insider will set price to more fully reflect changes in endowments and consequently ensure that his own position in the market, $x_I$, varies more with information rather than endowments.
Table 4.2
Comparative Statics for Propositions 2.1

|     | $\pi_1$ | $|\pi_2|$ | $|\pi_3|$ | $\sigma_1^2$ | $\tau_1$ | $|a_1|$ |
|-----|---------|-----------|-----------|--------------|----------|--------|
| $\sigma_u^2$ | -       | -         | -         | +            | -        | +      |
| $\sigma_I^2$ | -       | -         | -         | +            | -        | +      |
| $\sigma_r^2$ | +       | +         | +         | +            | ?        | -      |
| $\sigma_v^2$ | -       | +         | -         | -            | -        | +      |
| $a_u$    | +       | +         | +         | +            | -        | -      |
| $a_I$    | -       | +         | ?         | ?            | -        | +      |

Table 4.3
Comparative Statics for Propositions 2.2

|     | $\delta_1$ | $|\delta_2|$ | $\sigma_2^2$ | $\tau_2$ | $|\gamma_1|$ |
|-----|------------|--------------|--------------|----------|------------|
| $\sigma_u^2$ | -         | -            | +           | -        | +          |
| $\sigma_I^2$ | -         | -            | +           | -        | +          |
| $\sigma_r^2$ | +         | +            | +           | +        | -          |
| $\sigma_v^2$ | -         | +            | -           | -        | +          |
| $a_u$    | +         | +            | +           | unchanged| -          |
| $a_I$    | -         | +            | +           | -        | +          |
5. Some Empirical Implications

Summarizing, the previous sections described the equilibria in a simple securities market model under different information structures, and behavioral assumptions for individual traders. In particular, this analysis emphasized the consequences of strategic insider trading for the "noisiness" or informational efficiency of securities markets. The analysis of these sections may also be used to answer a question with potential implications for policy: that is, is it possible to distinguish the different equilibria and to detect the existence of an insider who behaves strategically using observations on prices and quantities traded?

Section 4 showed that focusing on such measures as the volatility of share prices may not be useful for this purpose because an informed trader who tries to manipulate prices may actually reduce the variance of overall prices. On the other hand, Examples 4.1-4.3 suggest that zero restrictions on equilibrium prices and cross-equation restrictions between the demand schedules of uninformed traders and prices may help to distinguish the different equilibrium configurations. For example, when information is perfect ($\sigma^2 = 0$), $p \neq E(r|y)$ only in the presence of an insider who behaves strategically. In this situation, price also depends on the initial wealth of uninformed traders'. However, regardless of the information structure, the equilibrium with strategic trading will differ from the competitive outcome in that the price will depend on the initial holdings of uninformed traders as well as on the signal $y$.

On the other hand, cross-equation restrictions between prices and uninformed traders' demands may be used to distinguish whether the information is "inside information" or publicly known. Using the results of Example 4.3, Proposition 3.1 and its corollary, I obtain, with inside information, that
\[ p_1 = \pi_0 + \pi_1 y + \pi_2 e_u \quad \text{where} \quad \pi_2 = \frac{1}{2\alpha_1} \]

(5.1)

\[ \chi_{u}(y,p) = \sigma_0 + \alpha_1 p. \]

When \( y \) is publicly known but a large trader sets prices,

\[ \tilde{p}_1 = \eta_0 + \eta_1 y + \eta_2 e_u \quad \text{where} \quad \eta_2 = \frac{1}{\nu_2} \]

(5.2)

\[ \tilde{\chi}_{u}(y,p) = \nu_0 + \nu_1 y + \nu_2 p. \]

Hence, the cross-equation restriction between \( \pi_2 \) and \( \alpha_1 \), on the one hand, and \( \eta_2 \) and \( \nu_2 \), on the other, may serve to identify the nature of information by market participants. Under the more likely situation that the large trader is risk-neutral (\( \alpha_1 = 0 \)), Example 4.3 shows that price will be an unbiased predictor of future payoffs only if the large trader takes prices as given. If the signal \( y \) is publicly known, it will be efficiently incorporated into prices even in the presence of a dominant trader who sets prices.

More generally, the existence of a "wealth effect" distinguishes the equilibria where the large trader sets prices from those in which he behaves competitively. Adding and subtracting the term \( \frac{e_I}{\alpha_1(2 - \alpha_1\alpha_1\sigma_r^2 + \sigma_v^2)} \) to the expression for equilibrium price in Proposition 3.1 and re-arranging terms yields

\[ p(y,e_{I},e_{u}) = \pi_0 + \pi_1 y + \pi_2^{+} e_{I} + \pi_3(e_{I} + e_{u}) \]

(5.3)

where

\[ \pi_2^{+} = \frac{1}{\alpha_1(2 - \alpha_1\alpha_1\sigma_r^2 + \sigma_v^2)}. \]

With \( \alpha_1 < 0 \), \( \pi_2^{+} > 0 \), while \( \pi_3 < 0 \), \( e_I \) enters with a positive coefficient because a higher price makes the initial holdings of the insider more valuable. With competitive behavior, \( e_I \) and \( e_u \) always enter with the same coefficient,
i.e., from Proposition 3.2

\[(p,y,e_1 + e_u) = \delta_0 + \delta_1 y + \delta_2(e_1 + e_u).\]

As a concluding remark, it should be pointed out that the above discussion is highly suggestive. Any serious empirical implementation would consider the institutional framework as well as the nature of the observable series.
Appendix I

Proof of Proposition 3.1: (i) Each uninformed trader conjectures that the equilibrium pricing function is given by $p(y, e_I, e_u) = \pi_0 + \pi_1 y + \pi_2 e_I + \pi_3 e_u$. Given this conjecture, the solution of each uninformed trader's problem can be obtained as a special case of Hellwig [1980], pp. 483, equations (6a)-(6e). This implies the demand function $X^i_u(p) = \alpha_0 + \alpha_1 p$ for all $i \in [0,1]$, with $\alpha_0$ and $\alpha_1$ as given in the text. The solution for $B^i_u(p)$ is obtained as a residual from the budget constraint in (3.1). With constant absolute risk aversion utility functions, the asset demands of individual traders do not depend on initial wealth or endowments.

(ii) Similarly, the insider conjectures that $X^i_u(p) = \alpha_0 + \alpha_1 p$ for all $i \in [0,1]$. Given this conjecture about the demand functions of the uninformed, he uses the two constraints

(I.1) $pX_I + B_I = p\bar{e}_I + \bar{B}_I$
(I.2) $X_I = e_I + e_u - \alpha_0 - \alpha_1 p$

to express $E_I[-\exp(-\alpha_1 W_I)]$ solely in terms of $p$. This yields the problem

$$\max_p E(r|y,e_I,e_u)(e_I + e_u - \alpha_0 - \alpha_1 p) + p\bar{e}_I + B_I - p(e_I + e_u - \alpha_0 - \alpha_1 p)$$
$$- \alpha_1^2 \text{Var}(r|y,e_I,e_u)(e_I + e_u - \alpha_0 - \alpha_1 p)^2$$

(I.3) $$p = [\alpha_0(1 - \alpha_1) \text{Var}(r|y,e_I,e_u)) - \alpha_1 \text{Var}(r|y,e_I,e_u)e_I +$$
$$(1 - \alpha_1 \text{Var}(r|y,e_I,e_u))e_u + \alpha_1 E(r|y,e_I,e_u)]/(2 - \alpha_1 \text{Var}(r|y,e_I,e_u))$$. Substituting for

(I.4) $E(r|y,e_I,e_u) = r^m + \sigma_r^2 (y - r^m)/(\sigma_r^2 + \sigma_y^2)$
and

\[(I.5) \quad \text{Var}(r|y,e_I,e_u) = \frac{\sigma_r^2\sigma_v^2}{(\sigma_r^2 + \sigma_v^2)} \]

yields the solution for \(\pi_0\), \(\pi_1\), \(\pi_2\) and \(\pi_3\) given in the text and further shows that all traders' conjectures are confirmed in equilibrium. The price of the riskless asset is normalized to one and the equilibrium holdings of the riskless asset \(B_I\) for the informed trader are obtained as a residual from the budget constraint (I.1) once \(X_I\) is known.

(iii) To complete the proof, I need to show that a solution exists for the system of non-linear equations defining \(\pi_0\), \(\pi_1\), \(\pi_2\) and \(\pi_3\). Since \(\pi_1\), \(\pi_2\) and \(\pi_3\) are independent of \(\pi_0\) and \(\pi_2 = -a_1\sigma_v^2\pi_1\), only a solution for \(\pi_1\) and \(\pi_3\) needs to be found. With constant absolute risk aversion utility functions, the risky asset is not a Giffen good (see Grossman [1978]), i.e., \(\alpha_1 < 0\). Further, provided \(\sigma_r \neq 0\), \(\alpha_1 > -\infty\). Hence, whenever \(\sigma_r \neq 0\),

\[0 \leq \pi_1 \leq \frac{\sigma_r^2}{2(\sigma_v^2 + \sigma_r^2)} - \frac{a_1\sigma_v^2\sigma_r^2}{2(\sigma_v^2 + \sigma_r^2)} + \frac{1}{2\alpha_1} \leq \pi_3 \leq 0.\]

To show that a solution exists for \(\pi_1\) and \(\pi_3\), define the mappings

\[T_0^2(\pi_1,\pi_3) = \left[ -\frac{a_1\sigma_v^2\sigma_r^2}{2(\sigma_v^2 + \sigma_r^2)} + \frac{1}{2\alpha_1} \right] T_0^1(\pi_1,\pi_3).\]

Also define

\[T_1(\cdot,\cdot) = 0 \quad \text{if} \quad T_0^1(\cdot,\cdot) < 0\]
and

\[ T_1^2(\cdot, \cdot) = T_0^2(\cdot, \cdot) \]

otherwise

\[ T_1^2(\cdot, \cdot) = T_0^2(\cdot, \cdot). \]

Then, \( T(\pi_1, \pi_3) = (T_1^1(\pi_1, \pi_3), T_1^2(\pi_1, \pi_3)) \) is a continuous mapping which maps the compact, convex set

\[ S = [0, \frac{\sigma_r^2}{2(\sigma_r^2 + \sigma_v^2)}] \times \left[ \frac{-a_I \sigma_v^2 \sigma_r^2}{2(\sigma_r^2 + \sigma_v^2)} + \frac{1}{2a_I}, 0 \right] \]

into itself. Therefore, by Brouwer's fixed point theorem, \( T(\pi_1, \pi_3) \) has a fixed point \( \pi_1^* \) and \( \pi_3^* \), with \( \pi_2^* \) and \( \pi_0^* \) defined from \( \pi_1^* \) and \( \pi_3^* \). When \( a_I = 0 \) or \( \sigma_v^2 = 0 \), \( \pi_1^* \) and \( \pi_3^* \) clearly fall on the boundary of \( S \), with \( \pi_2^* = 0 \). Otherwise, it can be shown by means of a contradiction that the point is strictly interior to \( S \), as claimed.

Proof of Proposition 2.2: (i) In this case, both types of traders conjecture a pricing function

\[ p(y, e_I + e_u) = \delta_0 + \delta_1 y + \delta_2 (e_u + e_I). \]

Given this conjecture, the coefficients in the asset demands for both types of traders, i.e., \( \beta_0, \beta_1, \beta_2 \) and \( \gamma_0 \) and \( \gamma_1 \) as defined in the text, are again found as special cases of Hellwig [1980], pp. 483, equations (6a)-(6e). Then, \( \delta_0, \)
\( \delta_1 \) and \( \delta_2 \) are determined from the market-clearing condition which specifies that

\[
\beta_0 + \beta_1 y + \beta_2 p + \gamma_0 + \gamma_1 p = e_I + e_u.
\]

In this case, it is easy to show that

\[
0 < \delta_1 < \sigma_r^2/(\sigma_r^2 + \sigma_v^2)
\]

and

\[-a_1 \sigma_v^2 < \delta_1 < 0\]

where \( \delta_0, \delta_1 \) and \( \delta_2 \) are defined in the statement of Proposition 2.2.

(ii) The existence of a solution for this set of equations follows exactly as in part (iii) of the proof of Proposition 3.1. \( \delta_0 \) is determined from the solution for \( \delta_1 \) and \( \delta_2 \).
This appendix describes the algorithm for numerically computing the equilibria of Propositions 3.1 and 3.2, given a set of values for the exogenous parameters $\sigma_r^2$, $\sigma_v^2$, $\sigma_l^2$, $\sigma_u^2$, $a_I$ and $a_u$. For both cases, the coefficients of uninformed traders' demand functions must be jointly determined with the coefficients of the equilibrium pricing function.

(i) For Proposition 3.2, the algorithm for jointly computing $\gamma_0$, $\gamma_1$ and $\delta_0$, $\delta_1$, $\delta_2$ is simple: given a feasible initial value for $\delta_1$, say $\delta^1_1$, $\delta^1_2 = -a_I\sigma_v^2\delta^1_1$ and $\gamma^1_1$ is determined from $\delta^1_1$, $\delta^1_2$ and the remaining exogenous parameters as

$$\gamma^1_1 = \frac{\delta^1_1\sigma_r^2 - \delta^1_2(\sigma_v^2 + \sigma_r^2) - \delta^2_2(\sigma_l^2 + \sigma_u^2)}{a_u\sigma_l^2(\delta^1_1\sigma_v^2 + \delta^2_2(\sigma_l^2 + \sigma_u^2))}. $$

Once $\gamma^1_1$ is known, $\delta^2_1$ is determined from

$$\delta^2_1 = \frac{\sigma_r^2}{(\sigma_r^2 + \sigma_v^2)(1 - \gamma^1_1a_I\sigma_v^2/(\sigma_r^2 + \sigma_v^2)).}$$

Continuing in this way, an equilibrium is found for a given set of parameters when $|\gamma^{i+1}_1 - \gamma^i_1| < \epsilon$ where $\epsilon$ is chosen a priori.

(ii) For Proposition 2.1, this algorithm includes an additional step. Again, assume an initial feasible $\pi^1$, compute $\pi^1_2 = -a_I\sigma_v^2\pi^1_1$. Now

$$\pi^1_3 = -(a_I\sigma_v^2 + (\sigma_r^2 + \sigma_v^2)/\sigma_r|a_I|)\pi^1_1$$

depends on $a_I$ and $\pi_1$ but $a_I$ depends on $\pi_1$, $\pi_2$ and $\pi_3$. Hence, substituting for $\pi_3$ in the expression for $a_I$ yields a fifth degree polynomial

$$a_a^5 + ba_a^4 + ca_a^3 + da_a^2 + ea_a + \sigma_u^2 = 0.$$
Given the values $\pi_1, \pi_2$ and the exogenous parameters, a numerical root finding routine yields the roots of the above polynomial. The real negative root, which always exists, is chosen as the value of $\alpha_1$. Then,

$$\pi_1^2 = \frac{\sigma_r^2}{(\sigma_r^2 + \sigma_v^2)(2 - \alpha_1 a_I^2 \sigma_r^2 \sigma_v^2 / (\sigma_r^2 + \sigma_v^2))}.$$  

Continuing the above until $|\pi_{1i+1} - \pi_{1i}| < \epsilon$ yields the solution.
References


Palfrey, Thomas [1982], "Uncertainty Resolution, Private Information Aggregation and the Competitive Limit", Carnegie-Mellon University, working paper.


Footnotes

1. I thank Rick Green, Steven Matthews and Tom Palfrey for helpful comments. Special thanks goes to Robert Miller and Robert Townsend for their encouragement and advice. Remaining errors are my own.

2. These include Blume and Easley [1983], Novshek and Sonnenschein [1982] and Palfrey [1982].

3. The price of the riskless asset is normalized at one, yielding an implicit risk-free interest rate of zero.

4. The insider's holdings of the riskless asset $B_I$ are determined from the budget constraint in (3.2).

5. A similar result is obtained by Grinblatt and Ross [1982].