EQUILIBRIUM MODELS OF TACIT COLLUSION IN OLIGOPOLY EXPERIMENTS WITH PRICE-SETTING FIRMS

by

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Abstract

In a noncooperative oligopoly with price-setting firms, tacit collusion is present when prices and profits exceed the "competitive" levels determined by a Nash equilibrium in prices. Such tacit collusion has been observed in oligopoly experiments with product differentiation. This paper discusses several equilibrium models of tacitly collusive behavior, including a "consistent conjectures equilibrium" which was recently proposed by Bresnahan (1980). The payoff parameters for some previously reported oligopoly experiments are used to compute the price predictions for alternative equilibrium theories. The consitent conjectures equilibrium provides the best explanation of the pattern of reported price averages.

Equilibrium Models of Tacit Collusion in Oligopoly

Experiments with Price-Setting Firms*

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Considering economists' success in explaining and predicting behavior in large competitive markets, the lack of a widely accepted theory of tacit collusion in oligopoly markets is disconcerting. John Hause (1977, p. 73) has remarked that: "One of the ongoing embarrassments of economic theory (and there are several) is the absence of a persuasive model that links the number of firms and their relative size with the expected degree of competition in an industry." There is, of course, no shortage of oligopoly models. Indeed, one of the most frustrating aspects of teaching microeconomic theory is dealing with the multiplicity of equilibrium concepts found in the oligopoly chapters of most textbooks. Jack Hirshleifer (1980, p. 407) puts things into a positive perspective: "As can be seen from the wide range of solution concepts, the theory of oligopoly is one of the most unsettled (and therefore one of the most exciting) areas of analytical economics."

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The most commonly used solution concept in the recent industrial organization literature is the Nash noncooperative equilibrium.[1] This concept, first formally analyzed by John Nash (1950), has a long history. Augustin Cournot's (1838) oligopoly solution is a Nash equilibrium for a noncooperative game in normal form in which firms' strategies are output quantities. Joseph Bertrand's (1883) solution is a Nash equilibrium in which strategies are prices. Despite the early and recently enhanced popularity of the Nash noncooperative approach, it is my impression that most economists firmly believe that a significant amount of cooperation is likely in tight oligopoly situations with a small number of competitors. The Nash noncooperative assumption is nevertheless used frequently, perhaps because it is operational, computationally convenient, and useful for providing limiting results when all collusive efforts fail.

Consider an oligopoly situation with price-setting firms and product differentiation in the sense that the firm with the lowest price does not necessarily capture the entire market. If direct communication among firms is prohibited, then any collusion must be tacit. The presence of tacit collusion in such situations is indicated by the presence of prices and profits which exceed "competitive" levels determined by a Nash equilibrium in prices.[2] Such tacit collusion has been observed in duopoly experiments reported by F. Trenery Dolbear, Lester Lave, G. Bowman, A. Lieberman, Edward Prescott, Frederick Rueter, and Roger Sherman (1968). These authors also performed experiments with four or more subjects, and in these experiments the Nash equilibrium in prices does provide a good explanation of the average levels of prices chosen by the subjects. A satisfactory equilibrium model of tacit collusion should be able to explain both the degree of observed tacit collusion in duopoly situations and the absence of tacit collusion with sufficiently large numbers of firms. One purpose of this paper is to suggest an equilibrium model of tacit collusion which provides a more or less satisfactory prediction of the average prices reported by Dolbear *et al*.

I. Introduction

A number of well known economists, including Edward Chamberlin (1933), have asserted that joint profit maximization is a reasonable assumption for an industry with a small number of firms, even if they cannot communicate directly. Many game theoretic analyses of collusion are also based on cooperative solution concepts; see Andrew Schotter and Gerhard Schwödiauer (1980) for a survey of this work. There is, however, considerable empirical and experimental evidence which suggests that *perfect* collusion is not likely for markets in which direct communication is not allowed. There may, in fact, be market situations which result in perfect collusion, but a satisfactory *theory* of tacit collusion should indicate which market situations result in perfect collusion and which do not.

A second approach to the modelling of tacit collusion, i.e. collusion without communication, is to show that what looks like collusive behavior at a point in time is really noncooperative, maximizing behavior in a regularly repeated game (a "supergame"). This approach is based on the observation that although one firm may profit by a surprise price cut, other firms are likely to react in future periods. A careful analysis of particular supergames can be extremely difficult and complex, even if ad hoc restrictions are placed on the possible forms of firms' strategies or "reaction functions". (The Schotter and Schwödiauer (1980) survey covers the literature on supergames.) One promising approach to the analysis of collusion in a dynamic, noncooperative game is the recent work of Edward Green and Robert Porter (1980). Following the advice of George Stigler (1964), Green and Porter model the effects of imperfect information on the degree of successful collusion. Their work is a stochastic generalization of the "balanced temptation equilibrium" proposal by James Friedman (1977).

A third approach to the analysis of collusion relies on the notion of a "conjectural variation". For example, the first firm in a duopoly must consider the effect of a change in its price p_1 on the other firm's price p_2. There is a Bertrand conjecture, $dp_1/dp_2 = 0$, which yields a Nash noncooperative equilibrium. In contrast, the conjecture, $dp_1/dp_2 = 1$, results in joint profit maximization in a symmetric model. There is a multiplicity of intermediate conjectures and a corresponding multiplicity of equilibria in most oligopoly models. Another problem with the conjectural variation approach is that firms are affected by rivals' reactions but seem to ignore the transitory profits which result from opportunistic behavior. Richard Cyert and Morris DeGroot (1970) avoid this problem by considering a duopoly model with alternating choices in which each firm considers the profits which accrue both before and after the next firm's reaction. Thomas Marschak and Reinhard Selten (1978) argue that the conjectural variation approach which ignores transitory profits is a reasonable simplification in the analysis of supergames when 1) a change in a player's action results in an adjustment cost, and 2) time periods are short so reactions can occur quickly. Michael Spence (1978) discusses the effects of imperfect information which can delay conjectured reactions.

Given the difficulties of formally modelling tacit collusion, it is not surprising that there are many heuristic models of tacit collusion which often involve discussions of sophisticated communication procedures such as price signaling, price leadership, focal point pricing, private discounts, etc. These models are often realistic and suggestive, but they possess all of the disadvantages of any approach which is not based on an explicit equilibrium analysis in the neoclassical tradition.

Two equilibrium models which can explain tacit collusion are considered in this paper: the Cournot equilibrium in output quantities and a "consistent conjectures equilibrium".[3] Each equilibrium concept is characterized by a vector of firms' outputs and prices which is stable in the sense that no one firm can profit by altering its price given the conjectured price reactions of other firms. The Cournot conjecture in this context with price-setting firms is that one firm's price deviation will cause all other firms to simultaneously alter their prices in a way that restores their outputs to the Cournot equilibrium levels. In contrast, the "consistent" conjecture is that one firm's exogenous price deviation will cause each rival to adjust its price in a way that maximizes its profit given the price reactions of other firms. These conjectured price reactions are consistent with individual profit maximization. This consistency concept is defined precisely in section IV.

This paper's approach to modelling tacit collusion is based on the classical conjectural variation approach, and each of the equilibrium concepts considered provides a way of avoiding the indeterminacy problem which has plagued this approach. These equilibrium concepts will be discussed in the context of a linear/quadratic model which is introduced in section II. The perfect collusion and Nash equilibria are derived and compared in section III. Conjectural variations are discussed in section IV, and the consistent conjectures equilibrium is presented in section V. The quadratic oligopoly model is not only convenient for illustrating differences in alternative equilibrium concepts; this model is also of interest because it has been used to compute the payoffs for the laboratory oligopoly experiments conducted by Dolbear et al. The results of these experiments are discussed in section VI, and the final section contains a summary.

II. The Oligopoly Market

Martin Shubik (1959) has persuasively argued that what economists need in order to understand particular oligopoly markets is game theory with an institutional face. It would be desirable from a practical point of view to have empirically validated solutions for dynamic models with elaborate institutional features, but it is important from a scientific point of view to have simple equilibrium solutions which explain empirical or experimental data in simple, "well-behaved" contexts. The oligopoly model to be considered in this paper is as well-behaved as any: Demand functions are continuous in all relevant variables, there are no capacity constraints, and profit functions are quadratic. These assumptions rule out discontinuities which are known to cause severe computational and existence problems for noncooperative solutions.[4]

Consider an oligopoly with n firms. The prices and outputs are denoted by p_i and x_i, i = 1, ... n, respectively. Firms have constant average costs denoted by c_i, i = 1, ... n. The products are heterogeneous, and firms' inverse demand functions are parameterized:

$$p_i = A - B\Sigma_j x_j - bx_i \ , \ i = 1, \ldots n. \tag{1}$$

where A, B, and b are positive parameters and $A > c_i$ for i = 1, ... n. The "Σ_j" notation in (1) denotes a summation from j = 1 to j = n.

The product differentiation is assumed to be symmetric in the sense that the demand parameters in (1) are not subscripted. Some of the earliest criticisms of monopolistic competition focused on the vagueness of this type of symmetric product differentiation. An explicit model with congestion effects which generates the symmetric inverse demand functions in (1) is given in Charles Holt (1980c). In this congestion model, each firm's output is the number of consumers purchasing a standard service. If a firm increases its output, then the resulting congestion causes that firm's product to be less valuable to existing consumers, so the firm will have to lower its price in order to maintain a high output. The parameter b in (1) determines this tradeoff between congestion and price for each firm.[5] Of course, the demand function in (1) can result from some other model in which product differentiation is not due to congestion effects.[6]

It may be more natural for some people to think of demand in a heterogeneous product case as a function which determines each firm's output given the prices charged by all firms. In fact, the payoffs for the experiments to be discussed in section VI are determined with demand functions parameterized:

$$x_i = \alpha + \beta\Sigma_j p_j - \gamma p_i \ , \quad i = 1, \ldots n, \tag{2}$$

where $\alpha > 0$ and $\gamma > \beta > 0$.

Of course, it is possible to express the demand functions in (2) in the inverse form of (1). One way to do this is to sum (2) from $i = 1$ to $i = n$, solve for $\Sigma_j p_j$ as a function of $\Sigma_j x_j$, and substitute this solution into (2). The result can be expressed as (1) with

$$A = \frac{\alpha}{\gamma - n\beta} \ , \tag{3}$$

$$B = \frac{\beta}{\gamma(\gamma - n\beta)} \ , \tag{4}$$

$$b = \frac{1}{\gamma} \ . \tag{5}$$

Similarly, it is straightforward to rewrite the inverse demand functions in (1) as:

$$x_i = \frac{1}{b(b + nB)} \left[Ab + B\Sigma_j p_j - (b + nB)p_i \right] \tag{6}$$

for i = 1, ... n. This is in the ordinary form (2) with

$$\alpha = \frac{A}{b + nB}, \tag{7}$$

$$\beta = \frac{B}{b(b + nB)}, \tag{8}$$

$$\gamma = \frac{1}{b}. \tag{9}$$

It is important to note that the parameters in the ordinary demand functions will depend on n if the parameters in the inverse demand functions are independent of n, and vice versa. The nature of the dependence of the demand parameters on n is critical if one is interested in the comparative statics of changes in n. Therefore, an important issue one must face in modelling imperfect competition is whether the ordinary demand functions or the inverse demand functions are independent of n. This choice should be based on a careful consideration of the nature of the consumer preferences and market frictions which result in positive, finite cross elasticities of demand.

III. Perfectly Collusive and Nash Noncooperative Equilibria

For a price-setting oligopoly with demand functions in (6), firms' profits are:

$$\pi_i = \frac{(p_i - c_i)}{b(b + nB)} \left[Ab + B\Sigma_j p_j - (b + nB)p_i \right] \qquad (10)$$

for i = 1, ... n. It will be useful for later comparisons to compute the joint profit maximizing price when costs are identical: $c_i = c$ for i = 1, ... n. The perfect collusion price for this symmetric case is denoted by $\hat{p}$, and it is straightforward to show that

$$\hat{p} = c + \frac{A - c}{2} . \qquad (11)$$

All firms charge the same price in the symmetric cost case, and the profits per firm are maximized when this common price equals $\hat{p}$ as shown in figure 1.

A Nash equilibrium <u>in prices</u> is characterized by a price p_i^* for each firm (i = 1, ... n) such that p_i^* maximizes the ith firm's profit given the other firms' equilibrium prices. Setting the derivative of (10) with respect to p_i equal to zero, one can show that the equilibrium prices satisfy:

$$\frac{1}{b(b + nB)} \left[Ab + B\Sigma_j p_j - (2b + 2nB - B)p_i + (b + nB - B)c_i \right] = 0 \qquad (12)$$

for i = 1, ... n. When all firms have the same cost parameter, i.e. $c_i = c$ for i = 1, ... n, then there is a common equilibrium price p^* such that $p_i^* = p^*$ for i = 1, ... n. It follows from (12) that[7]

$$p^* = c + \frac{b(A - c)}{2b + (n - 1)B} . \qquad (13)$$

As $b \to 0$, product differentiation vanishes and the equilibrium price converges to the common marginal cost c. When $n = 1$, p^* equals $\hat{p}$ which is the monopoly price, and $p^* \to c$ as $n \to \infty$.

Next, consider the case in which firms choose quantities. If the inverse demand functions are given in (1) as before, then the ith firm's profit is $x_i(p_i - c_i)$ where p_i is given in (1). It is straightforward to show that the Nash equilibrium quantities must satisfy:

$$A - B\Sigma_j x_j - bx_i - (B + b)x_i - c_i = 0 \ . \tag{14}$$

When $c_i = c$ for $i = 1, \ldots n$, there is a common equilibrium output per firm which will be denoted by x^{**}. It follows from (14) that[8]

$$x^{**} = \frac{A - c}{2b + (n + 1)B} \ . \tag{15}$$

Then the inverse demand functions in (1) imply that the common equilibrium price (p^{**}) which corresponds to the equilibrium output in (15) is:

$$p^{**} = c + \frac{(A - c)(b + B)}{2b + (n + 1)B} \ . \tag{16}$$

It follows directly from (11), (13), (16) that $\hat{p} > p^{**} > p^*$, as shown in figure 1. Thus the Nash equilibrium in quantities results in higher profits and prices for all firms than would be the case for a Nash equilibrium in prices. This is a well known result, especially for the homogeneous product case when the Cournot solution yields positive profits and the Bertrand solution yields zero profits with $p^* = c$.

If firms or subjects in an experiment actually choose price and sell whatever is demanded at their prices, then is the Nash equilibrium in quantities irrelevant? Spence has argued that the use of a Nash equilibrium in quantities for price-setting firms may be a useful way of modelling tacit collusion: "...the quantity version captures a part of the tacit coordination to avoid all-out price competition, that I believe characterizes most industries."[9] Oliver Hart (1979, p. 28) makes a similar argument:

> "It should be noted that we are implicitly adopting the Cournot assumption that each firm takes the quantity decisions of other firms as given. If, on the other hand, we adopt the Bertrand assumption that price decisions of other firms are taken as given, then in the homogeneous case the only possible equilibrium is the perfectly competitive solution. We reject the Bertrand approach because it has the implausible implication that perfect competition is established even under duopoly."

IV. Conjectural Variations

The notion of a "conjectural variation" can be used to indicate the strategic difference between the price-setting and quantity-setting formulations. The conjectural variation for a Nash equilibrium in prices is that a firm's price increase or decrease will not be followed; other firms will stick to their equilibrium prices. The Cournot conjectural variation (for a Nash equilibrium in quantities) implies that one firm's price changes will be followed to some extent by other firms. To see this, note that if one duopolist raises price and thereby drives customers to the other firm, then the only way for the other firm to maintain its output is to raise its price. Similarly, one firm's price cut would induce the other to lower its price somewhat.[10]

It is straightforward to compute the Cournot conjectural variation for a change in one firm's price. Suppose that firm number one is considering a price change dp_1 with the knowledge that firms $2,\dots n$

will simultaneously adjust their prices so that their outputs $x_2, \dots x_n$ are maintained at their initial levels. Therefore, consider the total differential of the ith firm's demand function in (6) with $dx_i = 0$ for $i = 2, \dots n$:

$$Bdp_1 + B\sum_{j \neq 1} dp_j - (b + nB)dp_i = 0 \tag{17}$$

for $i = 2, 3, \dots n$. Notice that dp_1 represents an arbitrary, exogeneous change in the first firm's price, and the n-1 equations in (17) determine the changes in the other firms' prices which will restore output to its initial level for each of these other firms. It follows from the symmetry of the equations in (17) that

$$\frac{dp_j}{dp_1} = \frac{B}{b + B} \quad \text{for } j = 2, \dots n. \tag{18}$$

Of course, the same analysis applies to an exogeneous price change made by any of the firms.

In general, suppose that firms maximize the profit expression in (10) with the conjecture that dp_j/dp_i equals any constant $\lambda \in [0, 1]$. When $c_i = c$ for $i = 1, \dots n$, it is straightforward to show that the total derivative of the ith firm's profit with respect to p_i is:

$$\frac{d\pi_i}{dp_i} = \frac{1}{b(b + nB)} \left[Ab + B\Sigma_j p_j - (2b + 2nB - B)p_i + (b + nB - B)c \right]$$
$$+ \frac{1}{b(b + nB)} \left[(p_i - c)B(n - 1)\lambda \right], \tag{19}$$

for $i = 1, \ldots n$. The first term on the right side of (19) is the partial derivative: $\partial\pi_i/\partial p_i$. The total derivative of profit takes the conjectured reactions into account, and this results in the second term on the right side of (19). It follows from the symmetry of the equations in (19) that the common equilibrium price, denoted by p_λ, is:

$$p_\lambda = c + \frac{b(A - c)}{2b + (n - 1)B(1 - \lambda)} \quad . \tag{20}$$

When $\lambda = B/(b + B)$ as in (18), then p_λ equals the Cournot price p^{**} in (16). When $\lambda = 0$, $p_\lambda = p^*$. When $\lambda = 1$, each firm can lead the other to the joint profit maximizing price: $p_\lambda = \hat{p}$.

Note that the Cournot conjectural variation in (18) is positive but less than one, so price changes are only partially followed. Loosely speaking, the conjecture that price deviations from p^* are partially followed makes price increases more attractive than price decreases, and this has the effect of raising the symmetric equilibrium price from p^* to p^{**}. Of course, profits are greater for all firms at the p^{**} price than at the p^* price, and this is why Spence has concluded that the Cournot formulation involves some tacit collusion. An alternative model of tacit collusion is presented in the next section.

V. The Consistent Conjectures Equilibrium

There is, in my opinion, nothing self-evident about the Cournot conjectural variation when firms actually choose prices. If one firm raises its price, it is not unreasonable to expect other firms to respond to the increased demand for their products by allowing both prices and output quantities to rise. Bresnahan (1980) has recently proposed an alternative conjecture implied by a consistency condition, i.e. that conjectured reactions be equal to the actual profit-maximizing reactions of firms to the exogeneous deviation of one firm from the equilibrium. Bresnahan proposed this equilibrium concept in the context of a duopoly with quantity-setting firms, but the analysis in the previous sections suggests a natural way to apply this concept in the context of an oligopoly with price-setting firms.

As before, the determination of the equilibrium price in the symmetric cost model involves two steps: the determination of the conjecture λ which satisfies the consistency condition and the computation of the corresponding price p_λ.

Consider an equilibrium in which the total derivative of profit in (19) is equal to zero for each firm. These total derivatives are denoted by $d\pi_i/dp_i$. Suppose that firm number one is considering a price deviation from its current price by an amount dp_1, and that all other firms will simultaneously adjust their prices $p_2,\ldots p_n$ to maintain a zero total derivative of profit ($d\pi_i/dp_i = 0$ for $i = 2,\ldots n$). Then the consistency requirement is that the price reactions dp_i/dp_1 which maintain the zero total derivative of profit be equal to the λ which appears on theright side of (19). To impose this requirement, the total differential of the right side of (19) can be equated to zero, and the resulting equation can be expressed:

$$Bdp_1 + B\sum_{j\neq 1} dp_j - (2b + 2nB - B)dp_i + B(n-1)\lambda dp_i = 0 \qquad (21)$$

for i = 2,...n. Treating λ as a constant in this differentiation involves an assumption that the reaction functions are linear, i.e. that they have constant slopes. Note that the equation for firm i in (21) determines the ith firm's profit maximizing response to the exogenous price change made by firm 1 when all of the first firm's rivals are reacting simultaneously. Note that the n-1 equations in (21) are analogous to the n-1 equations in (17) which were used to compute the Cournot (constant output) conjecture.

If both sides of (21) are divided by dp_1 and the consistency condition that $dp_j/dp_1 = \lambda$ is imposed, then the resulting equation is quadratic in λ with roots:

$$\lambda = \frac{2b + nB \pm \left[(2b + nB)^2 - 4(n-1)B^2\right]^{1/2}}{2B(n-1)} \quad . \qquad (22)$$

The largest root in (25), denoted by λ^+, can be ruled out on the basis of economic plausibility. Note that λ^+ is an increasing function of b, so it follows from (25) that if b > 0, then

$$\lambda^{+} > \frac{nB + (n^2B^2 - 4nB^2 + 4B^2)^{1/2}}{2B(n - 1)}$$

$$= \frac{n + (n^2 - 4n + 4)^{1/2}}{2n - 2}$$

$$= \frac{n + (n - 2)}{2n - 2} = 1 \quad .$$

But a value of λ which exceeds one implies that the corresponding price p_λ exceeds the joint profit maximizing level $\hat{p}$. Therefore, the smallest root in (22) should be substituted into the formula for p_λ in (20) to compute the consistent conjectures equilibrium (CCE) price. This price will be denoted by $\bar{p}$. It can be shown that if $n \geq 2$, then the smallest root in (22) is between 0 and 1, and therefore, the consistent conjectures equilibrium price $\bar{p}$ is less than the perfectly collusive price $\hat{p}$ determined by (20) with $\lambda = 1$.

The stability condition for both the NEQ and the CCE models implies that firms ignore transitory profits which accrue before rivals react. Marschak and Selten (1978) maintain that this is a reasonable assumption when time periods are short and adjustments are costly. Nevertheless, the analysis of tacit collusion in this paper is not explicitly dynamic. One of my primary concerns is with explaining experimental data based on repetitions of the same static oligopoly game. Charles Plott, Vernon Smith, and others have had remarkable success in using static models of perfect competition to explain experimental behavior in repeated market games.[11] In their experiments, trades occur sequentially, so there is a dynamic structure within each game which is so complex that it seems to be impossible to analyze it explicitly as a dynamic, noncooperative game. If static competitive equilibrium theory works so well in this complicated context, then a static or pseudodynamic approach may also be satisfactory for some repeated oligopoly games. Of course, it would be even better to have an explicitly dynamic theory which yields good predictions.

VI. Experimental Evidence

Plott (1979, p. 24) makes a good case for using laboratory experiments to discriminate among alternative theoretical models:

> "Experimental methods allow a type of flexibility in situations that history frequently fails to provide. Situations can be created for which competing models give substantially different predictions. Thus the opportunity is presented for using an empirical basis for pruning the theoretical tree of less successful models."

Surely if there is a part of our theoretical tree which needs pruning, it is in the area of imperfect competition!

Dolbear, Lave, Lieberman, Prescott, Rueter, and Sherman (1968) did some oligopoly experiments which are still of interest. The firms in their experiments were college students who made simultaneous, independent price decisions. The total costs for all subjects were determined by the same function: $15 + 6x_i$. After price decisions were made, subjects' sales were computed by the experimenters using the following demand functions:

$$x_i = \frac{42}{9} + \frac{1}{9(n-1)} \sum_{j \neq i} p_j - \frac{2}{9} p_i$$

for $i = 1, \ldots n$. These demand functions can be expressed in the form of (2) with $\alpha = 42/9$, $\beta = 1/9(n-1)$, and $\gamma = (2n-1)/9(n-1)$. It follows from these formulas and from the conversion formulas in (3), (4), and (5) that the experiment is parameterized:

$$A = 42,\ B = \frac{9}{2n-1},\ b = \frac{9(n-1)}{2n-1},\ \text{and } c = 6. \tag{23}$$

The effects of n on the demand function parameters were deliberately chosen so that the Nash equilibrium in prices would be independent of n. This can be checked by substituting the parameter values in (23) into the formula for p^* in (13) to show that $p^* = 18$ for all n. Similarly, equation (11) can be used to show that the joint profit maximizing price is 24. Thus changes in n did not distort the range between the perfectly cooperative and noncooperative prices.

Two types of experiments were conducted. In the incomplete information experiments, subjects were given complete cost information, but the only demand information provided was that higher prices will reduce sales. In the complete information experiments, subjects were given payoff tables which summarized all cost and demand information. In each experiment, the subjects made simultaneous price decisions 15 times, but they were not told the number of repetitions in advance.

The average price for each experiment was obtained by averaging all prices for trials 8 through 12. There were 12 duopoly experiments with complete information, and the average price <u>across experiments</u> was 19.5. The corresponding average price for the 6 duopoly experiments with incomplete information was 19.3. Recall that $p^* = 18$, so these results indicate some tacit collusion in the duopoly cases. With 4 "firms", the average prices were 17.8 with complete information (6 experiments) and 17.9 with incomplete information (3 experiments). These results for $n = 4$ are remarkably consistent with predictions of the Nash equilibrium in prices ($p^* = 18$). Only one experiment was done with $n = 16$; this was an incomplete information experiment in which the average price was 16.9.[12]

The authors concluded: "These results suggest that tacit collusion is possible and more likely to occur in an oligopoly market with a small number of firms."[13] No models of tacit collusion were discussed. The experimental parameters in (23) can be used to show that $p_\lambda = 6 + 36/(3 - \lambda)$ for this experiments. Note that p_λ is independent of n in this experiment. The fact that observed average prices decrease as n increases implies that the λ which provides the best fit when $n = 2$ is larger than the λ which provides the best fit when $n = 4$, etc. This use of a constant conjectural variation parameter λ may be a valuable descriptive device, but it merely pushes the problem of explanation and prediction to a different level. What is lacking is a theoretical link between the model parameters (A, B, b, c, and n) and the level of tacit collusion which is measured by λ in this case. The NEQ, and CCE equilibria discussed in the previous sections provide alternative ways of determining the relationship between the model parameters and λ. Substituting the experimental parameters in (23) into the formulas in (16), (22), and (20) one can compute the NEQ, and CCE prices for various values of n. These computations are presented in table 1. The prices for all three equilibrium assumptions decrease as the number of firms increases, and this is consistent with the decreasing pattern of average experimental prices. The NEQ prices are consistently too high; the CCE prices provide a better explanation of the experimental data in my opinion.

One of the most interesting implications of the data is that the observed level of tacit collusion is less than the level implied by <u>noncooperative</u>, <u>quantity-setting behavior</u>. Nevertheless, these experimental data are not obviously inconsistent with the NEQ price pattern. For $n > 2$, the NEQ, NEP, and CCE price predictions are all reasonably good, but none of these deterministic equilibria explain the significant dispersion of average prices which was reported for both the $n = 2$ and then $n = 4$ experiments.[14] One purpose of this paper is to stimulate more theoretical work in this area.

VII. Summary

Tacit collusion among price-setting oligopolists is indicated when prices and profits exceed competitive levels determined by a Nash equilibrium in prices. Such tacit collusion is likely to be present in tight oligopoly situations with a small number of firms, and it is likely to be absent when the number of firms is large. A satisfactory equilibrium theory of tacit collusion should be able to explain both the degree of tacit collusion when the number of firms is small and the absence of tacit collusion when the number of firms is large.

This paper presents several equilibrium models of tacit collusion. In each equilibrium considered, firms are assumed to react to a rival firm's price change by altering prices in a manner which maintains a constant value of a target variable. For a Nash equilibrium in quantities, of course, the target variable for each firm is its output. For the consistent conjectures equilibrium discussed in section V, the target variable is the total derivative of a firm's profit with respect to its price. In each case, the equilibrium is stable in the sense that no firm can increase its profit by deviating from its equilibrium price when rivals are conjectured to maintain the equilibrium levels of their target variables.

These equilibrium notions are discussed in the context of a linear/quadratic oligopoly model with price-setting firms which sell heterogeneous products. All three equilibrium notions predict prices and profits which are greater than would be the case for a Nash equilibrium in prices and less than would be the case with perfect collusion. The price predictions for these alternative theories are computed for the parameters which were used by Dolbear, _et al_. (1968) to construct the payoff tables for a series of laboratory experiments. The consistent conjectures equilibrium provides the best explanation of the pattern and approximate level of the average prices observed in these experiments. Nevertheless, the predictions for the two types of Nash equilibria are fairly good, especially for experiments with four or more firms.

FIGURE I

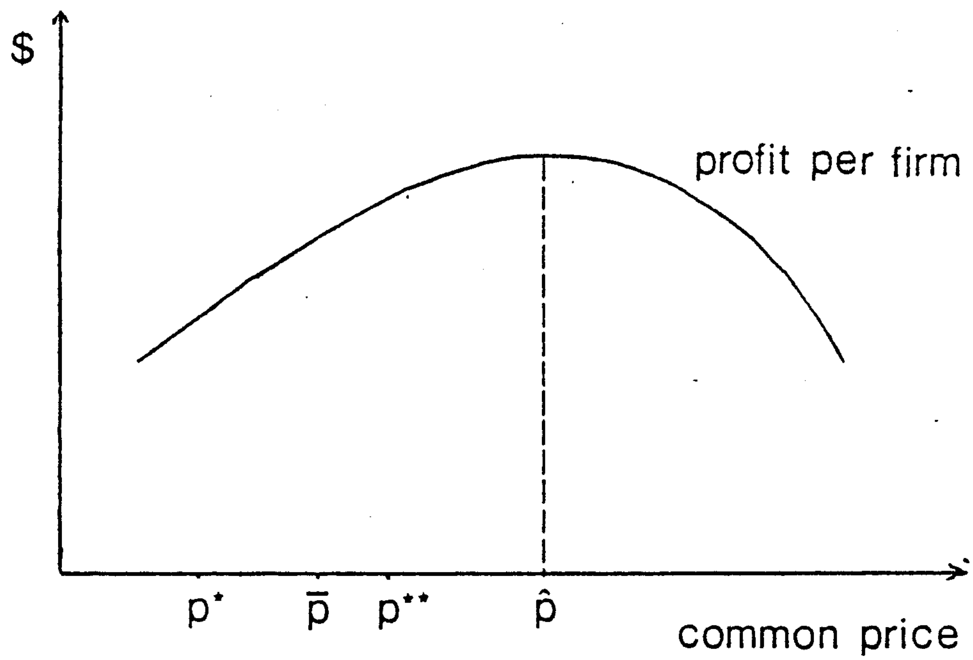

Table 1. Comparison of Equilibrium Predictions and Average Prices in the Dolbear, <u>et. al.</u> Experiments

	Average Prices		Equilibrium Predictions			
n	complete information	incomplete information	p^*	$\bar{p}$	p^{**}	$\hat{p}$
2	19.5	19.3	18.0	19.2	20.4	24.0
4	17.8	17.9	18.0	18.4	19.1	24.0
16	--	16.9	18.0	18.1	18.3	24.0

n - number of firms (subjects)

p^* - Nash equilibrium in prices

$\bar{p}$ - consistent conjectures equilibrium

p^{**} - Nash equilibrium in quantities

$\hat{p}$ - joint profit maximization

Footnotes

1. The term "Nash equilibrium" in this paper will always indicate a Nash noncooperative equilibrium for a single period oligopoly game.

2. In a symmetric model with no product differentiation and constant average costs, the Nash (Bertrand) equilibrium in prices is precisely the competitive equilibrium.

3. The notion of a consistent conjecture has been proposed by Timothy Bresnahan (1980) in the context of a duopoly with firms which choose quantities, and John Laitner (1980) has also discussed a similar notion of consistency. The consistent conjectures equilibrium which I consider is an oligopoly generalization of Bresnahan's concept for the case of price-setting firms. Morton Kamien and Nancy Schwartz (1980) and Martin Perry (1980) have analyzed the implications of the consistent conjectures assumption when industry demand is nonlinear. Also the notion of a consistent conjectures equilibrium was developed independently by Dennis Capozza and Robert Van Order (1980) in the context of a spatial model of monopolistic competition.

4. The most obvious discontinuity problem occurs when there is no source of "friction" which prevents the firm offering the lowest price from capturing the entire market. The analysis of such markets depends critically on institutional and informational assumptions. For example, see Holt (1980a) and (1980b) for a theoretical discussion of auction markets in which a procurement contract is awarded to the firm submitting the lowest sealed bid.

5. The parameters A and B in (1) are determined by the population distribution of consumers' reservation prices for the service, and the parameter b is determined by consumers' aversions to congestion. See Holt (1980c) for a more careful discussion of the relationship between consumer preferences and the resulting demand function which is a general, nonlinear version of (1).

6. For example, Bruce Owen and Michael Spence (1977) derive demand functions for heterogeneous commodities by assuming that consumers choose to purchase commodities which maximize a "net benefit function": $B(x_1, x_2, \ldots x_n) - \Sigma_i p_i x_i$. Thus the inverse demand functions are: $\partial B/\partial x_i = p_i$, and there is a quadratic benefit function which generates the inverse demand functions in (1).

7. It is, of course, possible to solve the equations in (12) for the asymmetric cost case. The easiest way to proceed is to sum the left side of (12) from $i = 1$ to $i = n$ and to set the resulting sum equal to zero. This makes it possible to solve for $\Sigma_j p_j^*$ as a function of the parameters. Then the equations in (12) can be used to determine each individual firm's equilibrium price.

8. The equations in (14) can be solved explicitly for the asymmetric cost case by following the procedure outlined in footnote 7.

9. See Spence (1976), p. 235. Spence (1978) presents a more general approach to modelling tacit collusion.

10. Spence and Owen (1977) note that the Cournot conjecture is more plausible that the Bertrand conjecture because one firm's price cut is likely to be followed by others.

11. These games are "double oral auctions" in which, at the beginning of each trading period, buyers (sellers) are free at any time to make an oral offer to buy (sell) one unit of a homogeneous commodity with a value (cost) for each trader which is known only to that trader. These values and costs are used to induce theoretical demand and supply curves. Accepted bids or offers constitute binding contracts. A good summary of double oral auction experiments can be found in Smith (1976).

12. The authors seemed surprised by prices below the p* level of 18. In my opinion, prices slightly below 18 are not surprising considering the probable effect of round-off errors in computing payoffs. For example, the payoff table which was used for the complete information experiments actually has two Nash equilibria in prices, one at 17 and one at 18.

13. Dolbear, et. al. (1968), p. 259.

14. James Friedman and Austin Hoggatt (1980) have reported results for experiments with experimental designs and payoff structures which are not comparable to the Dolbear et. al. experiments discussed here.

Nevertheless, Friedman and Hoggatt also observed that NEP prices were common when $n > 2$ and that significant dispersion existed across experiments for each value of n. They also observed that experience with oligopoly experiments tended to make subjects more cooperative in later experiments.

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