THE EQUILIBRIUM ALLOCATIONS OF WALRAS AND LINDAHL MANIPULATION GAMES

by

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Introduction

The satisfactory performance of an allocation mechanism requires that all the agents involved in its operation reliably carry out the various tasks assigned to them. It is often the case, however, that by behaving selfishly, an agent can secure an outcome that he prefers to the one that he would otherwise obtain. If everyone engages in such "manipulative behavior," some agents will gain, others lose, and the resulting outcome will not in general be the one that the mechanism was designed to achieve.

The literature devoted to the problem of manipulation has been mainly concerned with the characterization of environments for which "manipulation-free" or "incentive-compatible" mechanisms exist and of constructing such mechanisms (Clarke [1], Green and Laffont [5], Groves [6], Groves and Ledyard [7], Groves and Loeb [8], Vickrey [28]). Such environments are unfortunately narrow and a number of general impossibility theorems have been established (Hurwicz [10], Ledyard and Roberts [15]). In view of these negative results, it becomes more and more urgent to turn to the descriptive task of studying the extent to which existing mechanisms can be adversely affected by manipulative behavior. It is with this problem, to which much less effort has been so far devoted (see, however, Hurwicz [12], [11]) that the present paper is concerned.

Indeed, if for a given class of economies (or environments) it has been shown that no mechanism is immune to manipulation, there arises the second-best issue of which mechanism(s) are "less manipulable." As a

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criterion of manipulability of a mechanism, we consider the "size" of the set of allocations attainable as equilibria of the game of misrepresentation that one can associate to the mechanism, in a way that will be made clear later. How far from the desired allocations can a mechanism be led through manipulation should be seen as one of its important features.

A natural framework in which to study the joint attempts of various individuals to manipulate a mechanism is game theory. Formulating how an agent, confronted with a given mechanism, can evaluate his strategic possibilities and select an optimal strategy is the question to which Section I is devoted. To take the price mechanism in private good economies as an example, it is natural to think of offer curves as strategies, if preferences cannot be observed directly. However, one encounters almost immediately the difficulty that the Walrasian allocations associated to a given list of strategies are not unique, so that the Walras mechanism does not give rise to a well defined outcome function, but rather to what could be called an outcome correspondence. Because in general, mechanisms implement performance correspondences (more on this terminology in Section I), we will refer to quasi-games of manipulation. For simplicity, we could of course focus on mechanisms implementing outcome functions, or on environments over which the mechanism under examination happens to be single-valued. However, this would eliminate from consideration too many mechanisms or environments of interest, including the price mechanism, over otherwise perfectly well behaved environments.

The main objective of the paper is to analyze and compare the manipulability of the Walras and Lindahl mechanisms. For both theoretical and practical reasons, the Walras (or price) mechanism is the mechanism that deserves this attention the most. The Lindahl mechanism, as the natural counterpart of the Walras mechanism, has almost equal claim to the economist's attention. If the free rider problem in public good economies is the most often chosen to exemplify incentive issues, it is now well understood that private good economies are not immune to manipulation either. But is there a sense in which the Lindahl mechanism can be said to be more manipulable then the Walras mechanims?

The possibilities of manipulation open to a given individual depend on what is a priori known of his private characteristics - endowments and preferences in the environments we will be studying - or what could be found out after some investigation (audit). Since preferences can never be observed directly, they are the easiest to misrepresent. In Section II, we examine the preference quasi-games associated with the Walras and Lindahl mechanisms for certain classes of environments and derive the conclusion that the manipulability of the two mechanisms over these environments is in some sense identical.

In the next Section, we investigate the problem of misrepresentation of endowments. Destroying crops or falsely reporting on oil reserves are important and well-known examples that illustrate the vulnerability of the price mechanism to this kind of manipulation. In addition to its practical relevance, the study of endowment quasi-games offers the theoretical interest of involving strategy spaces that are considerably less complex than in the preceding Section, which, paradoxically, makes a characterization of equilibrium allocations more delicate. Nevertheless, we will be able to conclude that, here too, both mechanisms are equally manipulable.

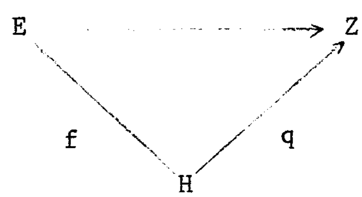
Finally, in the last Section, we focus on the case of large economies. Large economies deserve a special study since the action that each individual takes has an asymptotically vanishing effect on the rest of the economy and strategic possibilities are therefore quite different from what they are in small economies. In fact, it is in large economies that one would expect the greatest difference to show up between the Walras and Lindahl mechanisms. However, and perhaps most surprisingly, the results presented here support the opposite conclusion.

I. Mechanisms and Their Associated Quasi-games.

In this Section, a general definition of an economic mechanism and a unified framework in which the manipulability of any mechanism can be studied are provided. We show how to associate with every mechanism a manipulation quasi-game, the equilibria of which it will be our purpose to characterize for the Walras and Lindahl mechanisms. This preliminary conceptual work will also be useful in Section IV to compare our results with those developed in some current literature addressing similar issues.

An economy $e = (e_1,\ldots,e_n; Y)$ of cardinality n $(\# e = n)$ is composed of n consumers denoted A_i with $i = 1,\ldots,n$ and characterized by a list $e_i = (C_i, \omega_i, \succsim_i)$ where C_i , agent i's consumption set, is a subset of R^{ℓ}, the commodity space, ω_i is his initial endowment, a point of the commodity space, and $\precsim_i$ is his preference relation. After a subset Y of R^{ℓ} has been specified representing the production possibilities of e , the set of feasible allocation Z(e) , a subset of $R^{\ell(n+1)}$ is determined. An *environment* E is a class of economies sharing some common features. Thus, we will refer to "private good" or "public good" environments.

The following definitions are taken from Hurwicz [9] and Reiter [21]. A *mechanism* specifies procedures to generate, exchange and process information on the basis of which a decision is made. More precisely, a mechanism involves the specification of a *response correspondence* $f\colon\ E \to H$ associating to every economy e in the class of economies E , an element f(e) of a space H of *messages*, and an *outcome function* $q\colon\ H \to Z$ from the message space to the allocation space. It will be assumed here that the response correspondence is *privacy respecting* i.e., that there exist n correspondences $f_i\colon\ E_i \to H$, where E_i is the set of possible characteristics of A_i , such that $f(e) = \bigcap_i f_i(e_i)$. The composition $q \circ f\colon\ E \to Z$ is the *performance correspondence* implemented by the mechanism.



Manipulation of a Mechanism. An agent's characteristics $e_i = (C_i, \omega_i, \precsim_i)$ are in general known to him alone, although various components of e_i

may, in some situations, be observable by the agency operating the mechanism. A_i will be said to misrepresent his characteristics if he announces an element h_i of H different from $f_i(e_i)$. By doing so, a self-interested agent may affect the final outcome in his favor. Manipulation will go undetected only if his strategy choices are consistent with what is a priori known of him. Formally, if E_i' designates the subset of E_i in which e_i is known to belong, the effective strategy space of agent i is $f_i(E_i')$. For simplicity, in what follows, we take $E_i = E_i'$ for all i .

It remains to derive the principles that a rational individual can be expected to follow in the selection of his optimal strategies. The simplest situation is of course when the performance correspondence is single-valued over E . Then no agent ever has any doubt about the consequences of any particular strategy choice: to the original mechanism can be associated a game with $f_i(E_i)$ as A_i's strategy space for all i , and q as the outcome function. Although there exists a wide class of single-valued performance correspondences, many correspondences of interest do now have this property, except sometimes on some restricted environments. In particular, the Walras and Lindahl performance correspondences, on which this paper is focused, are not single-valued, even on otherwise well-behaved environments and alternative approaches should be considered:

1) A selection of the performance correspondence is specified: this amounts to redefining the original mechanism so that it yields a unique outcome in all of those cases where multiple outcomes had previously been

possible. Several such rules could be given to remove the ambiguities of the Walras and Lindahl mechanisms (e.g., order the agents, select the allocation the most favorable to the first agent; if there are several such allocations, select among those the one the most favorable to the second agent, etc.). There may indeed be sometimes good reasons to focus on sub-correspondences of the original correspondence - for instance, the sub-correspondence of the Walras correspondence containing the stable Walras allocations is a natural candidate. However, a complete resolution of all multiplicities cannot be in general expected on the basis of such or similar considerations. Any single-valued selection procedure would be somewhat arbitrary and in violation of the descriptive spirit of the present paper.

Many selection procedures have the additional drawback of requiring the prior determination of the whole equilibrium set, which from an informational viewpoint is much more demanding then hitting an arbitrary outcome in the equilibrium set. For operational simplicity, dynamic procedures involving an iteration of simple messages instead of the one-shot communication of complicated messages are often used. Such procedures do not usually guarantee convergence to a particular point of the equilibrium set but simply ensure convergence to some point in the set. Criterion 1) could not be applied to such procedures.

2) <u>Only strategy lists yielding a unique outcome are considered</u>: then the strategies available to any one agent depend on the strategies selected by the other agents, and we can speak of <u>generalized games</u> (see Debreu [ ]). Appealing to this concept is the most convincing

when the concern is with ensuring feasibility of the outcome as we will see in the Section on misrepresentation of endowments. Otherwise, the method may seem artificial. In addition, and more seriously, it may prevent truth-telling itself from being an admissible strategy.

3) If neither one of the above methods is used, multiplicities are unescapable, but partial orderings can still be defined on the strategy spaces that may be of some help to the agents in the optimal selection of their strategies. Given two strategies h_i and h_i' open to agent i, and given a fixed list h_j, $j \neq i$ of strategies of everyone else, Figure I1 illustrates the possible configurations of the utility levels, measured in some arbitrary utility representation of agent i's preferences, associated to the various outcomes in $S = h_i \cap (\bigcap_{j \neq i} h_j)$ and $S' = h_i' \cap (\bigcap_{j \neq i} h_j)$. By a slight abuse of notation, the elements of S (resp S') are denoted by the same symbols $x, y \ldots$ (resp $x', y' \ldots$) as their utilities for agent i.

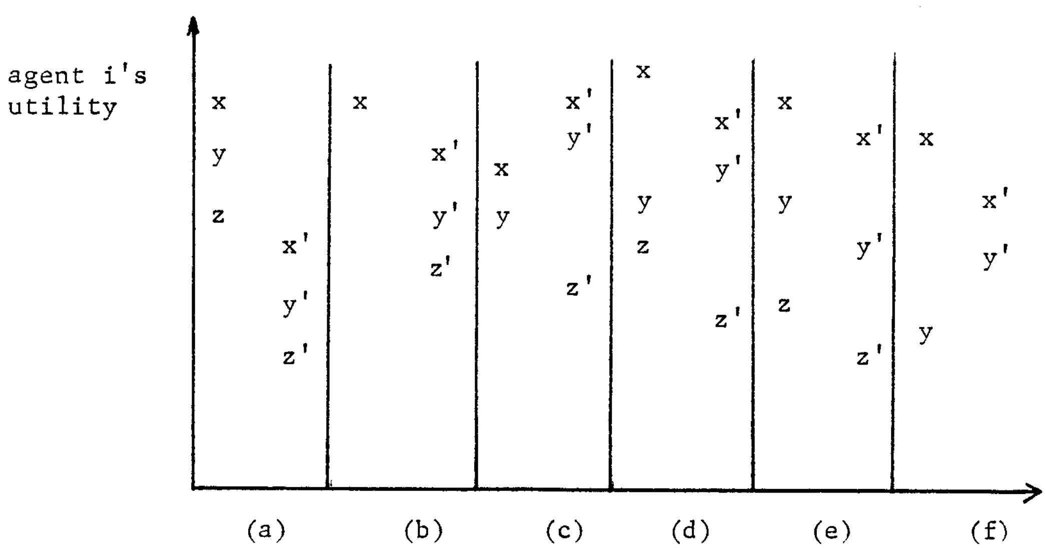


Figure I1

Several cases can now be distinguished.

(α) $\forall x \in S$, $\forall y = S'$, $q(x) \succsim_i q(y)$ (columns (a), (b)). It is then irrelevant to agent i how multiplicities are resolved. In that sense, h_i can be said to dominate h_i' . (Of course, this does not mean that h_i dominates h_i' for all lists $(h_j, j \neq i)$). As argued in Section I, the principle alone is sufficient to completely resolve all issues of multiplicity in the games of misrepresentation of preferences associated to the Walras and Lindahl games.

(β) There exist $x_o \in S$ and $x_o' \in S'$ such that $q(x) \succsim_i q(x_o)$, $\forall x \in S$ and $q(x') \succsim_i q(x_o')$, $\forall x' \in S'$, and $q(x_o) \succsim_i q(x_o')$. The worst outcome of S is preferred to the worst outcome of S' . If agent i is pessimistic, h_i will be a better strategy than h_i' according to the <u>maximin criterion</u> (column (c)).

(γ) There exist $x_o \in S$ and $x_o' \in S'$ such that $q(x_o) \succsim_i q(x)$, $\forall x \in S$ and $q(x_o') \succsim_i q(x')$, $\forall x' \in S'$, and $q(x_o) \succsim_i q(x_o')$. The best outcome of S is preferred to the best outcome of S' . If agent i is optimistic, h_i will be a better strategy than h_i according to the <u>maximax criterion</u>. This way of resolving multiplicities is used by Otani and Sicilian [17] and implicitly by Roberts and Postlewaite [23]. Some use of this criterion is also made in Section IV of the present paper. (Column (d).)

(δ) Without fully specifying a selection procedure, the assumption that the way outcomes are eventually chosen, whatever it may be, satisfies certain regularity conditions, may be of great help to an agent in the determination of his optimal strategy. If, for instance, there is a "natural" way of associating to each element x of S an element $a(x)$ of S' - in the sense that if x is selected from S, then $x' = a(x)$ can legitimately be expected to be selected from S' - then a comparison of each element x of S to the corresponding element $a(x)$ of S' may allow to rank h_i and h_i'. In column (e), assume that $x' = a(x)$, $y' = a(y)$ and $z' = a(z)$. Then, since $x \succsim_i x'$, $y \approx_i y'$, and $z \succsim_i z'$, h_i is a better strategy than h_i'. This is a criterion of <u>dominance outcome per outcome</u>. As an example, suppose that, in a 2-commodity exchange economy, the equilibrium price is determined through the use of a dynamic procedure with $p_x = 0$ or $p_y = 0$ as possible initial values. Then an agent observing that the outcome with the lowest (resp highest) price of x is selected from S could conclude that $p_x = 0$ $(p_y = 0)$ was the initial value and expect that the outcome with the lowest (resp highest) price of x would also be selected from S'.

(ϵ) A <u>rule of association</u> as appears in (δ) can lead to a weaker criterion: if x is selected from S, and if $x' = a(x)$ is such that $x \approx_i x'$, then h_i is a better strategy than h_i'. Under this criterion, h_i and h_i' may be ranked differently depending upon which outcome is selected from S. In column (f), assuming that $x' = a(x)$ and $y' = a(y)$, h_i is better than h_i' if x is selected from S, and h_i' is better than h_i if y is selected from S.

The example of the Walras mechanism is useful to motivate (δ) and (ϵ). Figure I2 abstractly represents the market clearing prices as a function of the strategy h_i chosen by agent i, assuming h_j

to be fixed for all $j \neq i$. When A_i changes his strategy from h_i to h_i' , the set of market clearing prices changes from $\{p_1, p_2, p_3\}$ to $\{p_1', p_2', p_3'\}$. Under a continuity assumption on the selection procedure,

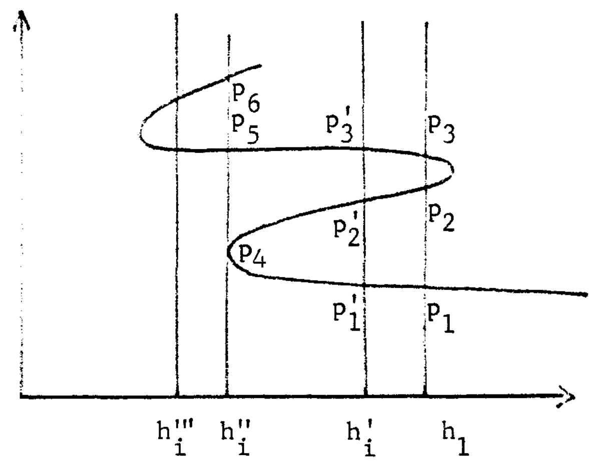


Figure I2

it is natural to associate p_1' to p_1 , p_2' to p_2 , and p_3' to p_3 . If the Walras allocations corresponding to p_1, p_2, p_3 are respectively preferred by A_i to the Walras allocations corresponding to p_1', p_2', p_3' , h_i is better for A_i than h_i' under the criterion formulated in (γ).

If the Walras allocation corresponding to p_1 is preferred by A_i to the Walras allocation corresponding to p_1' , h_i is better for A_i than h_i' if p_1 is selected from $\{p_1, p_2, p_3\}$ under the criterion formulated in (ϵ), independently of how the Walras allocation corresponding to p_2, p_2', p_3 and p_3' compare. An agent may not know beforehand which outcome will be selected from the outcome correspondence, but if he happens to find himself in a situation where some particular outcome (say the allocation corresponding to p_1) has been selected, he will expect that a change in his strategy that can be followed along the market clearing correspondence from p_1 to p_1', will be such that the allocation corresponding to p_1' will be selected for the new strategy list. Note, however, that this criterion would not allow to rank h_i'' and h_i''' if p_4 is selected from $\{p_4, p_5, p_6\}$.

Criterion (ϵ) will be extensively used in Section III on endowment misrepresentation. Since in any real world situation, no agent has complete information about the others, manipulation typically takes the form of local experimentations with strategies, each agent trying out small variations of his strategy on the assumption that the mechanism will respond discontinuously only in exceptional situations. This criterion is therefore particularly appropriate to model the behavior of somewhat myopic agents, and to formulate conditions for local optima.

The above pages have shown that there is no natural and complete method of resolving all multiplicities. Because $\bigcap_i h_i$ may not be single-valued for all lists of strategies $(h_1, \ldots, h_n)$, we will consequently speak of quasi-games of misrepresentations. The quasi-games of Section II will fortunately be equivalent (in a sense to be made precise later) to regular games, and we will be able to study their Nash (non-cooperative) equilibria. The quasi-games of Section III will not and, in the spirit of the remarks made in (ϵ), we will examine their local Nash equilibria. To clarify the terminology, the term equilibrium allocations is reserved to allocations in $\bigcap_i h_i$ for a list (h_i) of equilibrium strategies. Given an arbitrary list (h_i) of strategies, the elements of $\bigcap_i h_i$ will simply be called the Walras or Lindahl allocations for the list (h_i).

II. Misrepresentation of Preferences

Preferences can be easily misrepresented since they are never directly observable. In this Section, we examine the equilibria of the quasi-games associated with the Walras (subsection 1) and Lindahl (subsection 2) mechanisms. In subsection 3, the relative manipulability of the two mechanisms is studied.

1) The private commodity case: We consider the class E_1 of pure exchange economies $e = ((X_1,\omega_1,\precsim_1), (X_2,\omega_2,\precsim_2))$ with two agents and two commodities. $X_i = R_+^2$ is the consumption space of both agents; ω_i is an element of R_+^2 representing the initial endowment of agent i, and the preferences $\precsim_i$ of agent i are assumed to be closed, strictly convex and monotone. Let $Z(\omega) = \{(z_1,z_2) \in R_+^4 | z_1 + z_2 = \omega_1 + \omega_2\}$.

The manipulability of the Walras mechanism in E_1 was analyzed by Hurwicz. (His results were originally presented in Berkeley, Summer 1974; a statement of the theorem quoted below appears in [11] and a proof was given in a classroom presentation.) Initial endowments are assumed to be known, so that the strategy space of each agent is the set of offer curves consistent with strictly convex preferences and his fixed endowment. Since an arbitrary pair of such strategies may have multiple intersections, Hurwicz examined what, in accordance with subsection II.2, can be called the Walras preference generalized game, obtained by eliminating from consideration all strategy pairs yielding multiple Walras allocations.

Given an economy e of E_1 , let L(e) be the set of allocations defined by

$$L(e) = \{z = (z_1, z_2) \in Z(\omega) \,|\, z_i = \omega_i + k_i t_i^o(p) \quad \text{for } k_i \text{ such that } 0 \leq k_i \leq 1 \text{ and } t_i^o(p) \text{ being the true Walras trade at price } p \quad i = 1,2\}.$$

$L(e)$ has a lens shape if the true offer curves intersect in a unique point. For convenience, the first theorem is numbered Theorem 1'. (Later on, we will state a slightly different theorem, numbered Theorem 1.)

Theorem 1' (Hurwicz). For every economy e in E_1, the set of equilibrium allocations of the Walras preference generalized games of e is equal to $L(e)$.

Similar results have been independently obtained by Otani and Sicilian who dealt with multiplicities by assuming that the agents would be optimistic (see I 3 γ).

If preferences are assumed to be strictly convex, instead of simply convex, it is merely for ease of exposition. This assumption is made throughout the paper. Relaxing it would but marginally affect the results.

2) The public good case: Our analysis will be confined to the class E_2 of economies $e = ((X_1, \omega_1, \precsim_1), (X_2, \omega_2, \precsim_2), \overline{Y})$ with two agents and two commodities; $X_i = R_+^2$ is the consumption space of both agents; a vector (x_i, y) represents the consumption of x_i units of the (unique) private good by agent i, while y is the common consumption of the (unique) public good by both agents. Agent i's initial endowment ω_i is of the form $(\omega_{i1}, 0)$ where the initial level of the public good is taken equal to 0 without loss of generality. The public good is produced according to a linear production technology $\overline{Y}$, each unit of the input, the private good,

yielding one unit of the public good. It follows from these assumptions that a feasible allocation z is an element of R_+^3, $z = (x_1,x_2,y)$, satisfying $x_1 + x_2 + y \leqq \omega_{11} + \omega_{12} \equiv \omega_1$. The preferences $\succsim_i$ of both agents are assumed to be closed strictly convex and monotone so that only allocations where the above inequality is satisfied at equality will in fact be relevant to us. We will then be able to use the graphical representation introduced by Kolm [14]. Let ABC be an equilateral triangle of height ω_1. Given a point M in the triangle, let x_1, x_2 and y be the respective distances of M to sides AC, BC and AB; then $x_1 + x_2 + y = \omega_1$. This construction defines a 1-1 correspondence between points of the triangle and feasible allocations. Let $Z(\omega) = \{z = (z_1,x_2,y) \in R_+^3 | x_1 + x_2 + y = \omega_{11} + \omega_{12}\}$.

Let π_i be the individualized price of the public good in terms of the private good charged agent i. The geometric locus of the bundle maximizing utility as a function of π_i is called an offer curve by analogy with the pure exchange case. The intersections of the offer curves are the Lindahl allocations.

In the preference quasi-game associated with the Lindahl mechanism, the strategy space H_i of agent i is the space of all offer curves consistent with strictly convex preferences and his endowment $(\omega_{i1},0)$, assumed to be known in this Section.

Given a feasible allocation $z = (x_1,x_2,y) \in Z(\omega)$ different from ω, the slopes of the lines connecting $(\omega_{11},0)$ to (x_1,y) and $(\omega_{21},0)$ to (x_2,y) determine two prices $\pi_1(z)$ and $\pi_2(z)$.

Given $e \in E_2$, $L(e)$ is defined by

$$L(e) = \{(x_1,x_2,y) \in Z(\omega) \mid (x_i,z) = (\omega_{i1},0) + k_i t_i^o(\pi_i(z)) \text{ for } h_i \text{ such that } 0 \le k_i \le 1 \text{ and } t_i^o(\pi_i(z)) \text{ being the true Lindahl trade at price } \pi_i(z)\text{ , } i = 1,2\} .$$

This is the analogue of the lens defined in IIa.

Also, let $V(\omega)$ be defined by

$$V(\omega) = \{z = (x_1,x_2,y) \in Z(\omega) \mid x_i + y > \omega_{i1} \text{ for } i = 1,2\} .$$

This is the set of allocations such that both agents pay a non-negative price for the public good. Finally, let $\bar{L}(e) = L(e) \cap V(\omega)$.

Theorem 2. For every economy e of E_2 , the set of equilibrium allocations $N(e)$ of the Lindahl preference quasi-game of e coincides with $\bar{L}(e)$.

Before proving it, a few technical observations concerning the Kolm construction are in order. Let Ox and Oy be rectangular axes and let M be a point of coordinates (x,y) in the system of reference xOy . The images in the Kolm transformation of Ox , Oy and M are respectively denoted $O'x'$, $O'y'$ and M' . The axes $O'x'$ and $O'y'$ form a 60^o angle. Let $O'y''$ be an axis perpendicular to $O'x'$ and let (x',y') be the coordinates of M' in $x'O'y''$. It is easy to compute that

$$(x',y') = (\frac{1}{3} (y + 2x), y) .$$

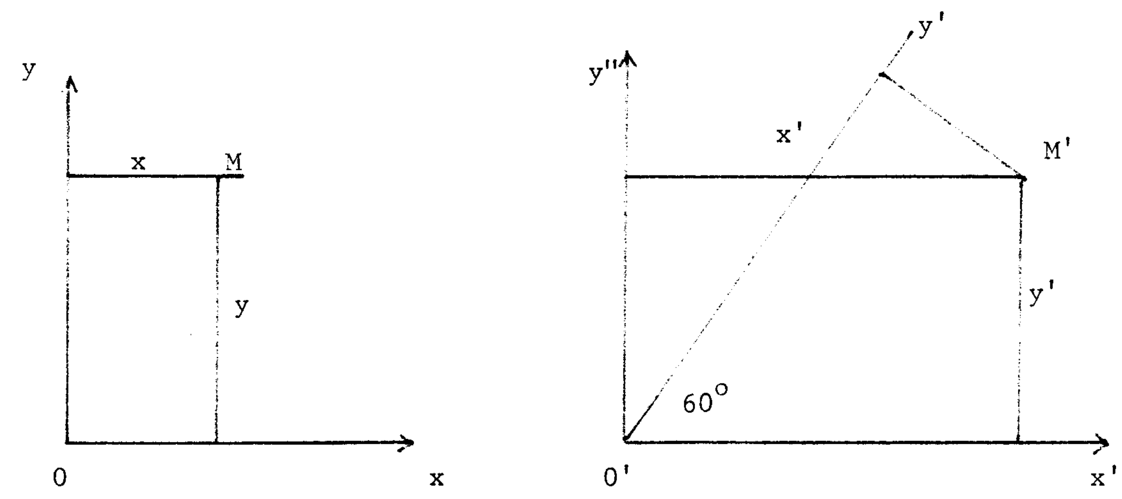


Figure II-1

From this algebraic relation, one can deduce the following facts that will be useful in this Section and/or in subsection III2: the transformation $M \to M'$ is a linear transformation; the image of a straight line is a straight line; the images of two parallel lines are two parallel lines; the Kolm transformation commutes with the translation operation; finally, the transformation preserves convexity.

Proof of Theorem 2: It will be organized in several steps.

(i) $\forall z = (x_1, x_2, y) \in N(e)$, $z \succsim_i^o \omega$ $i = 1,2$. Equilibrium allocations are preferred by both agents (according to their true preferences) to the initial allocation.

This follows from the non-coerciveness of the Lindahl mechanism. For any strategy h_i played by agent i $(i = 1,2)$, if agent j $(j \neq i)$ behaves truthfully, i.e., announces h_j^o, then any allocation z in $h_i \cap h_j^o$ is such that $z \succsim_j^o \omega$. This proves (i).

Until step (viii), we will ignore ω. The notation $h_1 \cap h_2$ will designate the allocations common to h_1 and h_2 other than ω.

(ii) $\forall\ (h_1,h_2) \in H_1 \times H_2$, $\forall\ z \in h_1 \cap h_2$, $\exists\ (h_1',h_2') \in H_1 \times H_2$ such that $h_1' \cap h_2 = h_1 \cap h_2' = \{z\}$. This means that if z is a Lindahl allocation for the strategy pair (h_1,h_2) , each agent can unilaterally change his strategy so as to make z the unique Lindahl allocation.

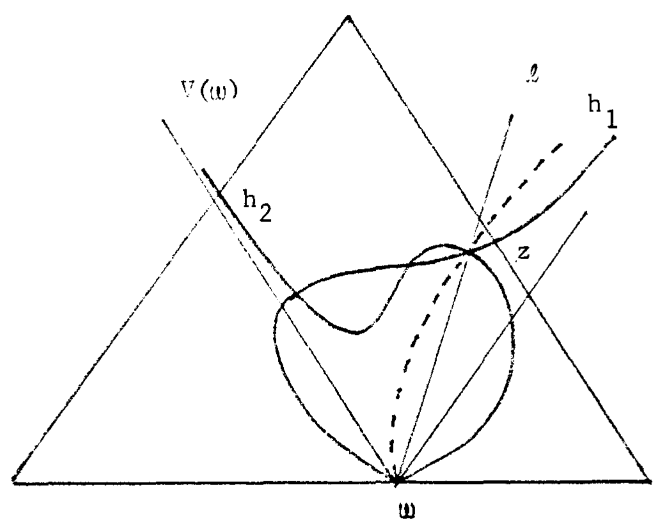


Figure II-2

Let $z \in h_1 \cap h_2$ for some element (h_1,h_2) of $H_1 \times H_2$, and let ℓ be the line determined by ω and z . By strict convexity of preferences, $h_1 \cap \ell = \{z\}$. It is then easy to see how, by picking h_1' sufficiently "flat" and close to ℓ , agent 1 can guarantee that z be the unique element of $h_1' \cap h_2$ (see Figure II-2). Similarly for agent 2. This is an important step since it will prevent multiplicities of $h_1 \cap h_2$ from causing problems; an agent need not worry about which allocation will be selected since a strategy is always available to him that eliminates the outcomes he likes the least. This argument is formalized in the next step.

Given $z \in Z(\omega)$, let $S_i^o(z)$ be the strict upper contour set at z of agent i, according to his true preferences.

Note that for any admissible strategy pair (h_1,h_2) in $H_1 \times H_2$, there exists a best (according to his true preferences) allocation in $h_1 \cap h_2$ for each agent. This follows from continuity of preferences and compactness of the set of feasible allocations. The set of best allocations in $h_1 \cap h_2$ for agent i is denoted $Z_i^o(h_1,h_2)$.

(iii) $\forall\ (h_1,h_2) \in H_1 \times H_2$, $\forall\ z \in Z_i^o(h_1,h_2)$, $S_i^o(z) \cap h_j \cap \text{int } V(\omega) = \emptyset$ for $i = 1,2;\ j \neq i$. Otherwise, for some agent, say agent 1, there is an allocation $\bar{z}$, that he strictly prefers to z and belongs to $h_2 \cap \text{int } V(\omega)$. By (ii), a strategy h_1' is available to him that makes $\bar{z}$ the unique Lindahl allocation for the pair (h_1',h_2). h_1' is clearly a better strategy than h_1.

(iv) $\forall\ (h_1,h_2) \in H_1 \times H_2$, card $(h_1 \cap h_2) \leq 2$. Suppose that for some pair $(h_1,h_2) \in H_1 \times H_2$, $h_1 \cap h_2$ contains at least three distinct allocations. These allocations would have to be indifferent to one another for both agents, since an agent can always eliminate the least preferred allocations by an appropriate strategy choice, which he should do independently of the selection procedure. However, three distinct allocations indifferent for agent 1 determine an arc strictly convex to agent 1's origin. By the same token, this arc should also be strictly convex to agent 2's orgin. This is not possible.

(v) $\forall$ $(h_1,h_2) \in H_1 \times H_2$, card $(h_1 \cap h_2) = 1$. By (iv), we know that card $(h_1 \cap h_2) \leqq 2$. Suppose that card $(h_1 \cap h_2) = \{z_1,z_2\}$ with $z_1 \neq z_2$, and let z_1 be the allocation with the highest level of the public good. (Since $z_1 \not\sim_1^o z_2$, they cannot have the same level of the public good without being identical, in violation of the assumption on preferences.) Let P_i be the supporting hyperplane at z_i to $S_i^o(z_2)$ with the smallest price of y for agent i. It intersects the horizontal axis in $\bar{\omega}_i$ (see Figure II-3).

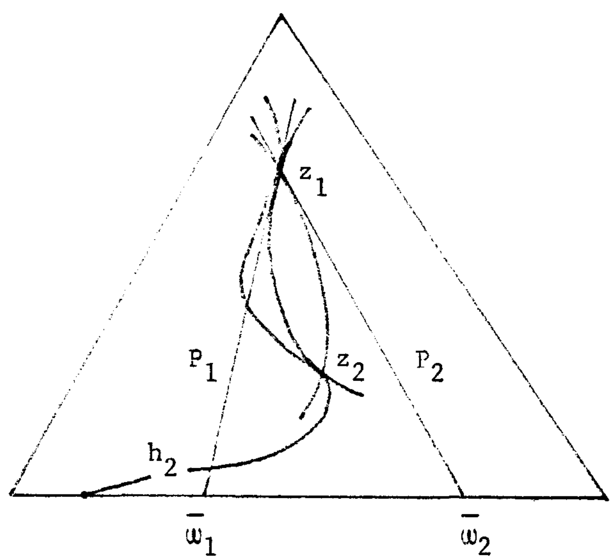


Figure II-3

By strict convexity of preferences and the fact that the indifference curves of both agents through z_2 also go through z_1 , $\bar{\omega}_1$ is to the left of $\bar{\omega}_2$ and distinct from $\bar{\omega}_2$. On the other hand, for h_2 to go through z_1,z_2 , and satisfy (iv), it is necessary that ω^o be to the left of $\bar{\omega}_1$. Similarly, ω^o should be to the right of $\bar{\omega}_2$. This is impossible.

(vi) $z \in \overline{L}(e) \Rightarrow z \notin N(e)$. Let $z \in Z(\omega)$ be outside of $\overline{L}(e)$. If $z \notin \text{int } V(\omega)$, there is no admissible strategy pair (h_1,h_2) in $H_1 \times H_2$ such that $z \in h_1 \cap h_2$. Suppose then that $z \in \text{int } V(\omega)$, but $z \notin L(e)$. This means that for one agent, say agent 1, the supporting hyperplanes P at z to $S_1^o(z)$ are all steeper than the line connecting ω_o to z . By strict convexity of preferences, h_2 would have to reach z from above, which would necessarily imply that $S_1^o(z) \cap h_2 \cap \text{int } V(\omega) \neq \emptyset$, in contradiction with (iii).

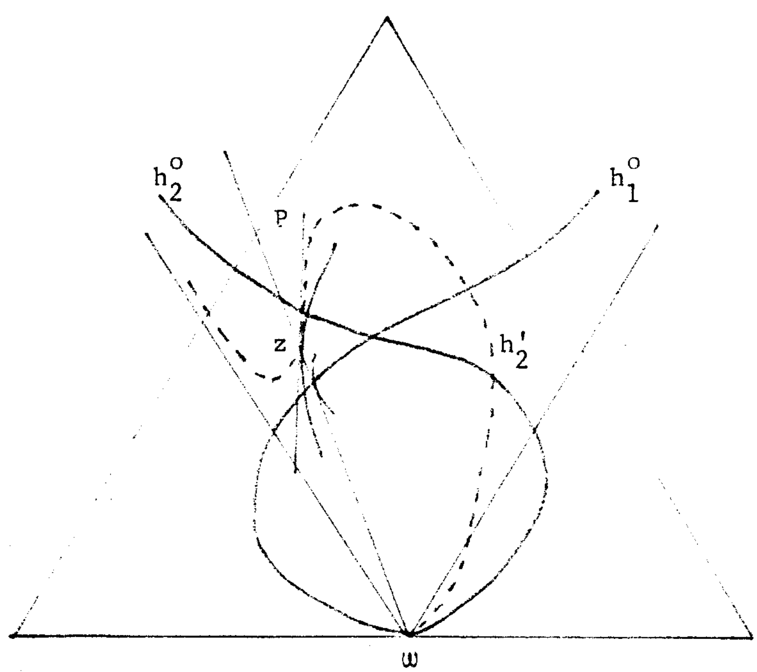


Figure II-4

(vii) $z \in \overline{L}(e) \Rightarrow z \in N(e)$. Let z be in allocation in $\overline{L}(e)$. $S_1^o(z)$ and $S_2^o(z)$ admits of supporting hyperplane at z that intersect the horizontal axis to the left and to the right of ω respectively. It is then easy (see Figure II-5) to find a pair of admissible strategies h_1 and h_2 satisfying all of the above conditions.

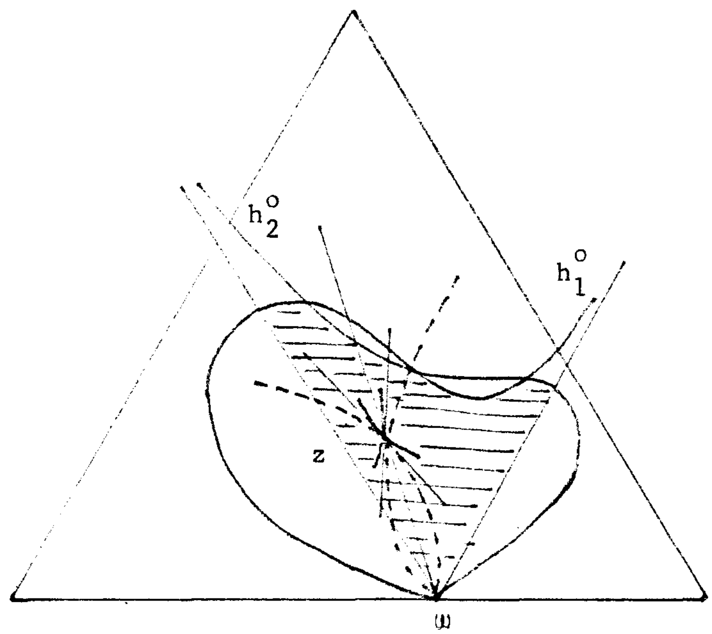


Figure II-5

(viii) Finally, note that step (vii) also applies to ω , which is therefore an equilibrium allocation.

The techniques used in the proof of Theorem 2 apply to the private good case as well. The only difference is that, in an economy e of E_1 , each agent i can for every strategy choice h_j , $j \neq i$, of the other agent, make any point of h_j (and in particular, the one he prefers) a unique Walras allocation, through an appropriate choice of his own strategy. For the purposes of comparison, it is then convenient to consider :

<u>Theorem 1</u>. For every economy e of E_1 , the set of equilibrium allocations N(e) of the Walras preference quasi-game of e coincides with L(e) .

Proof: It is omitted here. Multiplicities would be dealt with in the manner of Theorem 2, and the remainder of the argument would follow Theorem 1'.

3) Comparison of the manipulability of the Walras and Lindahl mechanisms. We would like to use Theorems 1 and 2 to evaluate the relative vulnerability of the Walras and Lindahl mechanisms to preference misrepresentation by examining the set of equilibrium allocations of their associated preference quasi-games. Unfortunately, since these sets do not lie in the same spaces, the comparison cannot be done directly. Nevertheless, a number of economically meaningful statements can be made concerning both cases, that point to a great similarity:

1) Truthful behavior does not constitute an equilibrium.
2) The true Walras and Lindahl allocations are equilibrium allocations.
3) The initial allocation is an equilibrium allocation.
4) Every equilibrium allocation Pareto-dominates (in terms of the true preferences) the initial allocation.
5) There are a continuum of equilibrium allocations compatible with the correct Walras or Lindahl prices.
6) The true Walras or Lindahl trades corresponding to the implicit price(s) given by the equilibrium allocations are always smaller than the trades required to reach it (them).
7) The equilibrium allocations constitute a two-dimensional continuum.

On the other hand, note that if agent i behaves truthfully and agent j cheats optimally, the resulting allocation is the monopoly point in the private good case; i.e., the allocation in agent i's offer curve that agent j prefers while the allocation similarly defined in the public good case would not necessarily be achievable. In addition, (what is really of concern to us here) is that the monopoly-allocations are equilibrium allocation in the private exchange case but not necessarily in the public good case.

In spite of the difference just noted, it seems legitimate to conclude that the manipulability of the Walras and Lindahl mechanisms is very similar when preferences are the strategy choices.

III. Misrepresentation of endowments

We now turn to the issue of misrepresentation of endowments. Even if preferences are known, it is easy to see how the Walras and Lindahl mechanisms can be so manipulated and to convince oneself of the practical importance of this problem, which can take two forms. The situation exemplified by farmers burning their crops was analyzed by Hurwicz[11] and Postlewaite [18] under the name of "destruction games": such manipulative behavior is accompanied by an actual change in the aggregate resources available to the economy. On the other hand, when an oil company gives false reports on its reserves, real resources are not affected, they are simply not accounted for correctly: we will then refer to "no-destruction games." An important class of no-destruction games are termed "withholding games" by Hurwicz and Postlewaite: such games involve only downward mispresentation; in many cases of interest, upward misrepresentation is indeed not possible; part or all of the agent's initial resources are simply hidden for private consumption or later marketing. The analysis that follows will be limited to no-destruction games.

It will here be assumed that preferences are known and that endowments only can be misrepresented. This assumption, which is clearly not always warranted, is made for analytical simplicity; it allows to clearly differentiate the present Section from the preceding one. However, any concrete situation would typically involve partial misrepresentation of both preferences and endowments, and would have to be analyzed in the light of both Sections II and III.

Certain components of an individual's endowment vector may be known or easily measurable (size of a lot), other components may be determined through a procedure that is too costly to enforce for all agents (audit), and some components may be observable only indirectly and unreliably (abilities). Knowing a component of an agent's endowment amounts to reducing the effective dimensionality of his strategy space. Finally, it is worth noting that, as compared to Section II, strategy spaces are here much less complex since a strategy is simply a point in a Euclidean space, instead of an element of a functional space. In addition to its practical relevance, this Section will therefore offer some insight into the theoretical significance of dealing with strategy spaces of such different natures.

a) Walras endowment games

As in IIa, let E_1 be the class of exchange economies $e = ((X_i, \omega_i^o, \succsim_i)\ i = 1,2)$ with two goods and two agents with consumption set $X_i = R_+^2$, $i = 1,2$, endowments ω_1^o and ω_2^o, elements of R_{++}^2, and preferences $\precsim_1$ and $\precsim_2$. Only endowments are unknown here, and manipulating one's endowments is the only strategy open to the agents. The strategy space of agent i will be denoted as $\Omega_i(\omega_i^o)$, where a possible dependence on the true endowments is indicated. To every pair (ω_1, ω_2) in $\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ can be associated a pair $(h_1(\omega_1), h_2(\omega_2))$ of offer curves derived from the knowledge of the agent's preferences. An outcome correspondence is defined by realizing from the true endowments the net trades corresponding to the Walras allocations given by $h_1(\omega_1) \cap h_2(\omega_2)$. This completes the specification of a Walras endowment quasi-game.

(α) Several difficulties arise. First, the trades required by the Walras mechanism may not be feasible from the true endowments (they clearly are always feasible from the announded endowments). One way of preventing this would be to allow for downward misrepresentation only, and to specify:

$$(1) \qquad \Omega_i(\omega_i^o) = \{\omega_i \in R_2^{++} | \omega_i \leqq \omega_i^o\} \quad \text{for} \quad i = 1,2 \ .$$

Limiting our attention to withholding games can be justified on several grounds. To start with, one could conceive of an exchange game where the agents are required to bring to the market what they want to transact. In such circumstances, it is possible for someone to bring less but not more than what he owns. Second, to preserve his credibility in future trading, it is important for an agent to avoid being in the position to have to deliver more than physically possible. The main argument against this way of dealing with the possibility of unfeasible outcomes is that it violates the privacy-respecting properties of the Walras mechanism. If endowments are not known, strategy spaces should not be made to depend on their true values. Because proofs of ownerships are required only when deals are concluded and not before, this would be more in keeping with the descriptive aim of this paper. However, the violation of privacy implied by (1) is mild since strategies are only constrained by inequalities.

The following alternative specification of the strategy spaces is more general.

$$(2) \qquad \Omega_i(\omega_i^o) = \{\omega_i \in R_{++}^2 | \omega_i \leqq \bar{\omega}_i\}$$

where $\bar{\omega}_i$ is some "natural" upper bound (possibly ∞) that may or may not depend on each individual (example of a physical constraint that would be equally relevant for all agents: there are 24 hours in a day).

When the Walras allocations corresponding to a pair (ω_1, ω_2) in $\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ happens to be unfeasible given the true endowments, one may i) impose the initial allocation as outcome, ii) specify a rationing scheme, iii) give the agents another chance. Option i) is the simplest; option ii) is somewhat more difficult to justify given our desire to describe the Walras mechanisms as faithfully as possible. In addition, ensuring the feasibility of any such rationing scheme would require the knowledge of the true endowments, again in violation of the privacy requirement.

It is the last option that we will follow. The process by which equilibria are reached is not detailed in this paper, but in a more explicitly formulated dynamic procedure, this option would appear quite natural. Formally, it amounts to limiting our analysis to strategy pairs leading to feasible trades. Consequently, the strategy choices available to an agent depend on the strategies chosen by the other agent. Since multiplicities of Walras allocations remain possible (more on this soon), we will therefore be dealing with a generalized quasi-game.

It should be pointed out that for destruction games, i.e., games in which agents attempt to influence the equilibrium price by destroying part of their endowment, the specification (1) of the strategy spaces would clearly be the appropriate one; the issue of feasibility would not appear there as the net trades corresponding to the Walras allocations associated to a pair of strategies $\omega = (\omega_1, \omega_2)$ would have to be measured from that same ω. In the quasi-games we are considering, the final allocation is always a point of an Edgeworth box whose dimensions are given by the true initial endowments. In destruction quasi-games, the final allocation belongs to an Edgeworth box whose dimensions are endogeneously determined.

(β) The second difficulty concerns the multiplicities of Walras allocations for an arbitrary pair of strategies (ω_1, ω_2) in $\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$. As was announced in Section I, we will assume here that agents expect a small change in strategy to lead to a small change in the selected outcome, unless the outcome set responds discontinuously in the neighborhood of the selected outcome: if for the strategy pair (ω_1, ω_2), the allocation $z \in h_1(\omega_1) \cap h_2(\omega_2)$ happens to be selected, and if small alternations in agent 1's strategy lead to small perturbations in the set of Walras allocations in the neighborhood of z, then agent 1 would expect the allocation of $h_1(\omega_1') \cap h_2(\omega_2)$ the closest to z (to be selected for the pair (ω_1', ω_2)). Similarly for agent 2.

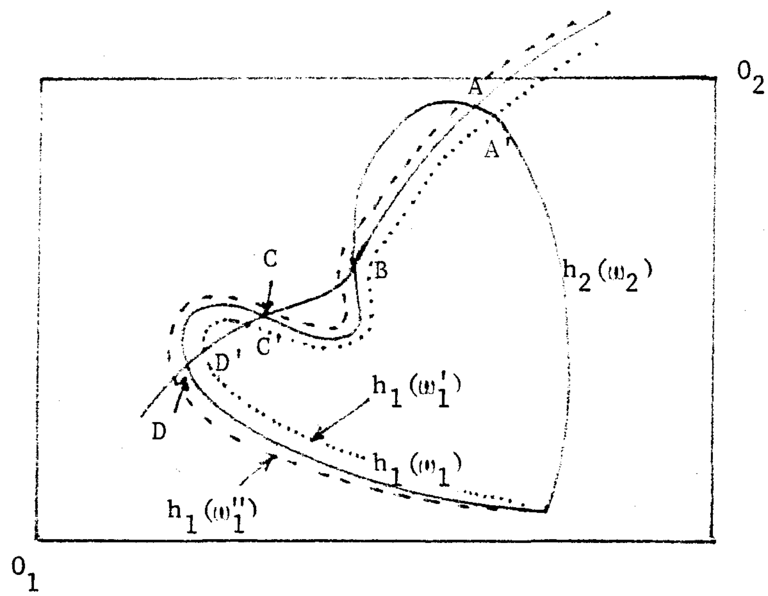


Figure III-1

This principle is illustrated in Figure III-1. A,B,C,D all belong to $h_1(\omega_1) \cap h_2(\omega_2)$. The dotted line represents $h_1(\omega_1')$ for ω_1' close to ω_1 . According to the principle, if A , C or D were selected for the pair (ω_1,ω_2) , agent 1 would expect A',C',D' respectively to be selected for the pair (ω_1',ω_2) . But if B were originally selected, agent 1 would not make any conjecture about which of A,C,D would be subsequently selected. For the alternative stragegy choice ω_1'' yielding an offer curve $h_1(\omega'')$ represented by the dashed line, agent 1 would still not be able to conjecture in every case, since in a neighborhood of B two new allocations appear, and it may be that one is preferred to B and the other not. Whether or not the offer curves are transversal at one of their common point candidate to be an equilibrium allocation is therefore crucial.

Given an economy e in E_1, we designate by $G(e)$ the Walras generalized quasi-game associated with it.

Definition: Given $e = ((X_i, \omega_i^o, \precsim_i), i = 1,2)$ in E_1, an allocation z in $Z(\omega^o)$ is a local equilibrium allocation of $G(e)$ if a) there exists a stragegy pair (ω_1, ω_2) in $\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ such that $z \in h_1'(\omega_1) \cap h_2'(\omega_2)$, where $h_i'(\omega_i) = h_i(\omega_i) + \{\omega_i^o - \omega_i\}$ (i.e., the announced offer curve after translation to ω^o) and b) there exist neighborhoods V, V_1, V_2 of z, ω_1, ω_2 in $Z(\omega^o)$, $\Omega_1(\omega_1^o)$ and $\Omega_2(\omega_2^o)$ respectively such that for all (ω_1', ω_2') in $V_1 \times V_2$, and for all z' in $h_i(\omega_i') \cap h_j(\omega_j) \cap V$, $i = 1,2$, $j \neq i$, $z' \precsim_i z'$.

This simply says that if z is selected from $h_1'(\omega_1) \cap h_2'(\omega_2)$, and V is a neighborhood of z, no agent can achieve anything better than z in V by a marginal change of strategy.

Theorem 3. Given $e = ((X_i, \omega_i^o, \precsim_i), i = 1,2)$ in E_1, a necessary and sufficient condition for z in $Z(\omega^o)$ to be a local equilibrium allocation of $G(e)$ for the strategy pair $(\omega_1, \omega_2) \in \text{int}\ (\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o))$ is that there exists a neighborhood V of z in $Z(\omega^o)$ such that $z \succsim_i z'$ for all z' in $V \cap h_j(\omega_j)$, $i = 1,2$ and $j \neq i$. (We will refer to this condition as the local separation condition.)

Proof: Figure III-2 represents the truthful economy. It is an Edgeworth box of dimension $\omega_1^o + \omega_2^o$. For $z \in Z(\omega^o)$ to be a local equilibrium allocation, it is necessary that there exist $(\omega_1, \omega_2) \in \Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ such that $h_1'(\omega_1) \cap h_2'(\omega_2) = z$. To prove the Theorem, suppose that in fact, $(\omega_1, \omega_2) \in \text{int}\ (\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o))$. If the local separation condition did not hold, there would be an agent, say agent 1, whose indifference curve through z would be transversal to $h_2'(\omega_2)$.

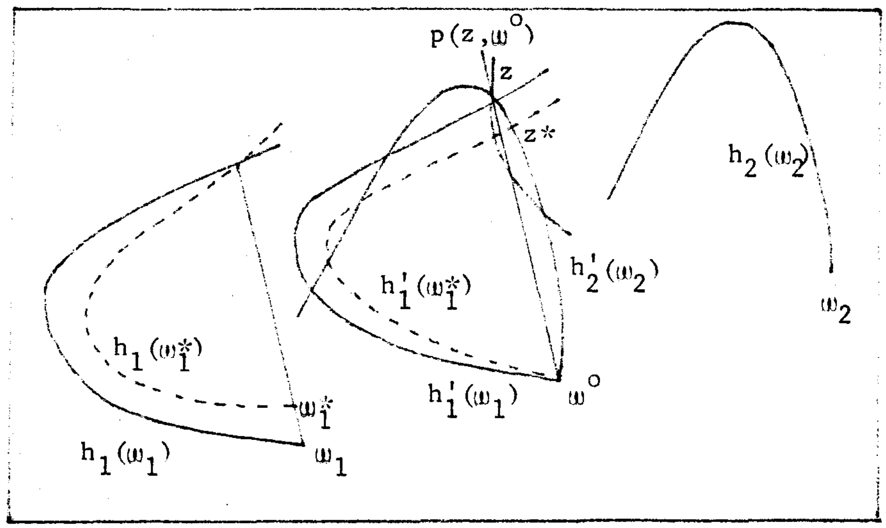


Figure III-2

Suppose $z \neq \omega^o$, and let $p(z,\omega^o)$ be the unique price defined by the line connecting ω^o to z . An examination of all the possible configurations of that indifference curve, $h_1'(\omega_1)$ and $h_2'(\omega_2)$ reveals that agent 1 could always change his strategy so as to achieve a local improvement. In Figure III-2, because $h_1'(\omega_1)$ is above $h_2'(\omega_2)$ to the right of z , choosing $\omega_1^* = \omega_1 + t(z - \omega^o)$ for small positive t will be sufficient. The dotted line represents $h_1(\omega_1^*)$ and its translate $h_1'(\omega_1^*)$ is seen to intersect $h_2'(\omega_2)$ at z^* that agent 1 strictly prefers to z . If $h_1'(\omega_1)$ were below $h_2'(\omega_2)$ to the right of z , choosing ω_1^* as above with t negative but small in absolute value would be appropriate. (The rare case in which $h_1'(\omega_1)$ and $h_2'(\omega_2)$ coincide over some interval is more delicate and may require choosing ω_1^* outside of the line connecting ω^o to z^*.)

Finally, if $z = \omega^o$ and if the local separation condition does not hold there we would consider a price p_1 between the smallest slope of a supporting hyperplane of agent 1's indifference curve at ω^o, and the greatest slope of a supporting hyperplane of $h_2'(\omega_2)$ at ω^o. For ω_1^* in an ε-neighborhood of agent 1's expansion path relative to the price p_1, there exists z in $h_1'(\omega_1^*) \cap h_2'(\omega_2)$ with $z \succ_1 \omega^o$. This completes the proof of Theorem 3. (See Figure III-3.)

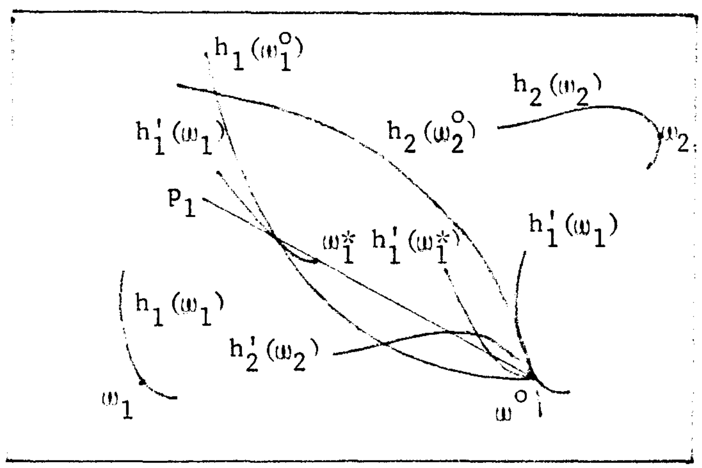


Figure III-3

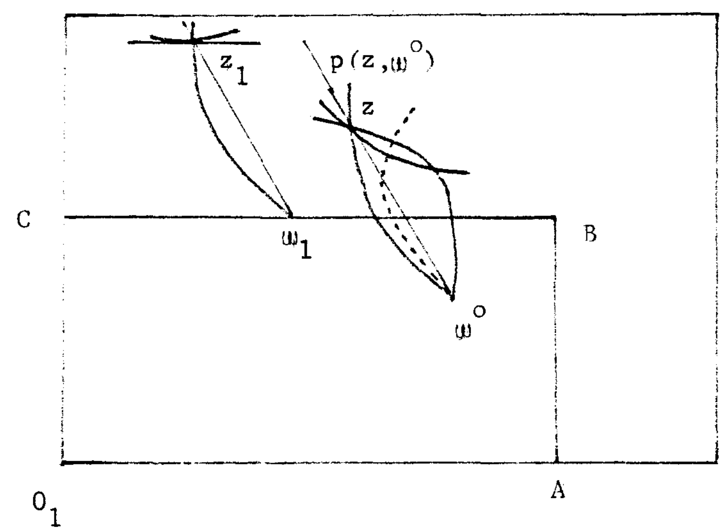


Figure III-4

Necessary and sufficient conditions for z in $Z(\omega^o)$ to be an equilibrium allocation for a $\omega = (\omega_1, \omega_2)$ on the boundary of the strategy space can also be derived. We will not write them out in full but will simply illustrate them with the use of Figure III-4 in which the strategies available to agent 1 are all points of the rectangle O_1 ABC . Let $z \in h_1'(\omega_1) \cap h_2'(\omega_2)$ with ω_1 on the boundary of $\Omega_1(\omega_1^o)$. As the Figure is drawn, agent 1 should "shrink" his offer curve so as to get something like the dotted line. If his expansion path relative to $p(z,\omega^o)$ is above a horizontal line through z_1 (defined by $z_1 - \omega_1 = z \cdot \omega^o$), this will not be feasible by marginal changes in ω_1 . The equilibrium condition would therefore relate the monotonicity of the expansion paths to those of the constraints defining the strategy spaces that are binding.

On the basis of Theorem 3 can be defined a systematic procedure to verify whether a given allocation of $Z(\omega^o)$ is a local equilibrium allocation for some pair ω of interior strategies: let z be given such that $z \neq \omega^o$, $z \in Z(\omega^o)$, and let $E_i(p(z,\omega^o))$ be the agent i's income expansion path relative to the price $p(z,\omega^o)$. More precisely,

$$E_i(p(z,\omega^o)) = \{z' \in Z(\omega^o) | \exists\ I \in [0,\infty[\ni z' \text{ is a best element for } \succsim_i \text{ in the budget set determined by } p(z,\omega^o) \text{ and } I\}.$$

Let then $t = z - \omega$ and $\Omega_i(z,\omega^o) = (E_i(p(z,\omega^o)) - \{t\}) \cap \Omega_i(\omega_1^o)$.

It is clear that for all (ω_1,ω_2) in $\Omega_1(z,\omega^o) \times \Omega_2(z,\omega^o)$, the derived offer curves $h_1(\omega_1)$ and $h_2(\omega_2)$ have the property that their translates $h_1'(\omega_1)$ and $h_2'(\omega_2)$ to ω^o are such that $z \in h_1'(\omega_1) \cap h_2'(\omega_2)$. If there exists (ω_1,ω_2) in $\Omega_1(z,\omega^o) \times \Omega_2(z,\omega^o)$ for which the local separation condition holds, then z is a local equilibrium allocation. (See Figure III-5.)

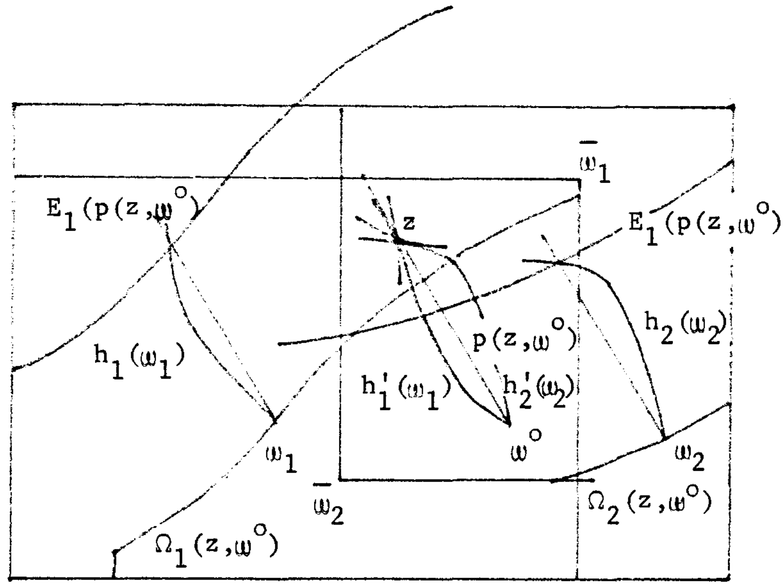


Figure III-5

If $z = \omega^o$, one should choose a supporting hyperplane P_i of each agent's indifference curve through ω^o at ω^o. Denoting p_i the slope of P_i, any pair $(\omega_1,\omega_2) \in E_1(p_1) \times E_2(p_2)$ will yield ω^o as a local equilibrium allocation.

Verifying whether an allocation is an equilibrium allocation for a pair of strategies or the boundary of the strategy space would involve the steps described in the paragraph that follows the proof of Theorem 3.

Remark 1. Since an individual always has the option of announcing the truth, every equilibrium allocation Pareto-dominates the (true) initial endowments.

Remark 2. It can be easily checked that truth-telling will not in general constitute an equilibrium.

Remark 3. Let e in E_1 be given, and assume that one agent (say agent j) behaves truthfully. Then one can find a large enough upper bound $\bar{\omega}_i$

on the other agent's strategy space for him to reach his monopoly point, i.e., the point of agent j's offer curve that he likes the best. This is in particular the case if $\bar{\omega}_i \geqq \omega_i^o$. Of course, in the case of multiple Walras allocations, nothing guarantees that the monopoly allocation will be selected, and agent i may or may not have a strategy ω_i^* that yields the monopoly allocation as the unique Walras allocation for the pair (ω_i^*, ω_j^o) .

Remark 4. An allocation outside L(e) may be a local equilibrium allocation. It is indeed possible to construct examples where the local separation condition of Theorem 3 holds at such points (see Figure III-6).

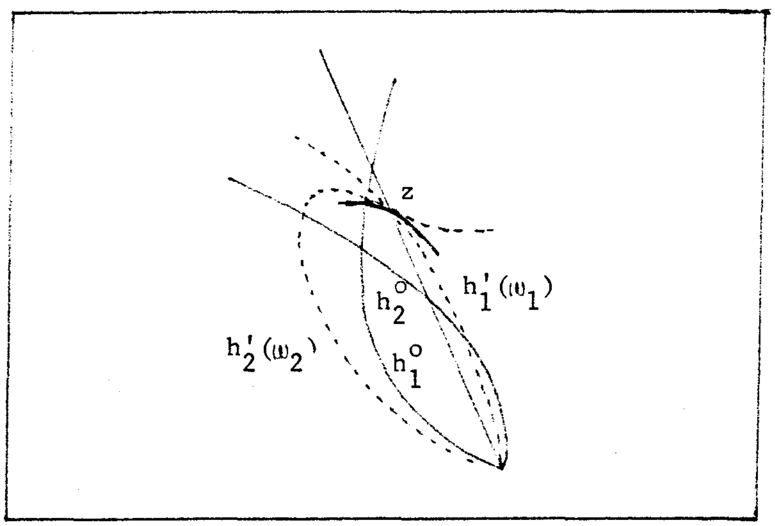


Figure III-6

(This possibility was eliminated in the preference quasi-games by global considerations.) Note however, that for the local separation condition to hold at z outside of L(e) , it is necessary that the offer curve of at least one agent (agent 1 in the above figure came "from above" the other agent's indifference curve through z). This phenomenon could not arise in economies with no Giffen goods. It follows that in such economies, the only candidates for local equilibrium are allocations in L(e) .

Remark 5. Let E be the class of Cobb-Douglas economies, and let $\Omega_i(\omega_i^o) = R^2_{++}$ for $i = 1,2$. Given e in E , let z* be the unique true Walras allocation of e and p* the price associated with it. Given V, a neighborhood of z* and z in $V \cap \text{int } L(e)$, let $p_1 = p(z,\omega^o)$ be defined as in the proof of Theorem 3. Given $\varepsilon_1, \varepsilon_2$ small, V can be chosen small enough so that the slopes of the supporting hyperplanes at z to $L_1(z)$ and $L_2(z)$ be $p_1 - \varepsilon_1$ and $p_1 + \varepsilon_2$. By going far enough on $\Omega_1(p_1,\omega^o)$ and $\Omega_2(p_2,\omega^o)$, one can find ω_1 and ω_2 such that the translated offer curves $h_1'(\omega_1)$ and $h_2'(\omega_2)$ be "flat enough" and have slopes $p_1 + \varepsilon_2$ and $p_1 - \varepsilon_1$ at z . This shows that with unbounded strategy spaces, allocations arbitrarily close to the true Walras allocations can be obrained as equilibria. Note that the class of differentiable economies with "flatter and flatter" offer curves $h(\omega)$ as ω goes to infinity contains the class of Cobb-Douglas economies, and is much wider: the same conclusion would apply to any element in that class.

Remark 6. Contrary to the preference quasi-games, it is not possible to give a geometric characterization of all the equilibrium allocations without being specific about the shape of the indifference curves. However, one can construct economies with equilibria at any preassigned location, close to the true Walrasian allocation as in Remark 5, close to the monopoly points or close to the initial allocation as the following example illustrates:

Example. Let agent i's indifference map be made of parallel straight lines of slope ℓ_i . Assume $\ell_i \neq \ell_j$. For all $\omega_i \in \Omega_i(\omega_i^o)$, $h_i(\omega_i)$ has the shape of a truncated right angle. Given $z \in Z(\omega^o)$, let $(\omega_1,\omega_2) \in \Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ be such that $z = h_1'(\omega_1) \cap h_2'(\omega_2)$. Let z_i^* be the point of $h_i'(\omega_i)$ that agent i prefers. It can be easily

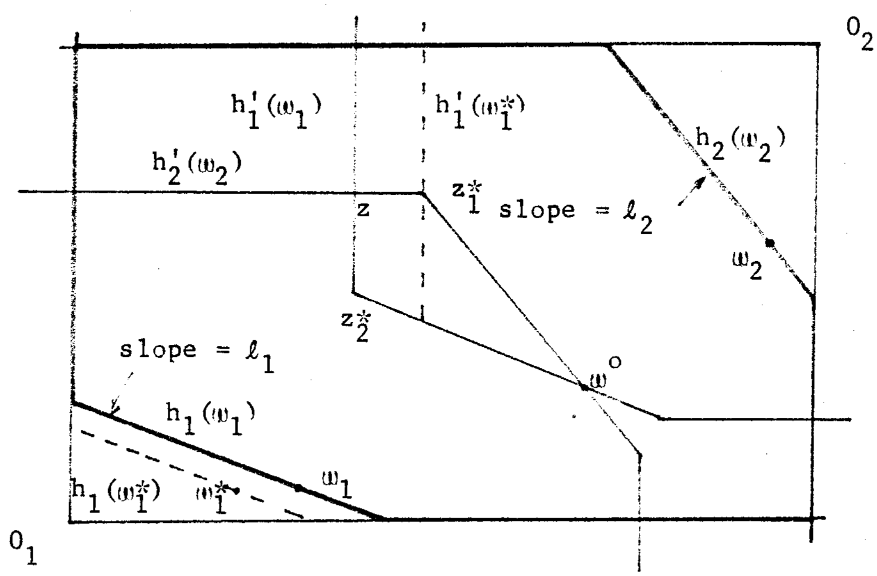


Figure III-7

checked that for each i , there is a strategy ω_i^* such that $\omega_i^* \leqq \omega_i$ and $z_i^* = h_i'(\omega_i^*) \cap h_j'(\omega_j)$ (with $j \neq i$). In addition, for at least one agent, z_i^* is strictly preferred to z , unless $z = \omega^o$. This means that only ω^o can be an equilibrium allocation. Indeed, ω^o is an equilibrium allocation for all admissible strategy pairs (ω_1,ω_2) such that $\omega_{1x} = 0$ and $\omega_{2y} = 0$. Note that in fact ω^o is a global equilibrium. Moreover, the indifference curves of the example could be approximated by strictly convex ones so as to yield an economy of E_1 whose only Nash equilibria could therefore be made arbitrarily close to ω^o .

Lindahl endowment games

Here again, preferences are assumed to be known, and endowments only are subject to manipulation. The analysis is confined to the class E_2 of public good economies defined in subsection II-2.

The difficulties concerning the definition of the strategy spaces and the issues of feasibility and multiplicities are resolved exactly as in the preceding Section, and no explicit treatment is needed here. Note, however, an important difference from the private good case: since public goods are consumed at the same level by all agents, the public good component of each ω_i cannot be unilaterally misrepresented by agent i. Strategy spaces will therefore be one-dimensional. For simplicity, the initial level of the public good is set equal to zero. Let $e = ((X_i, \omega_i^o, \precsim_i) \quad i = 1,2; Y)$ in E_2 be given. Agent i's strategy space is defined by:

$$\Omega_i(\omega_i^o) = \{\omega_i \in R_+ \times \{0\} | \omega_{i1} \leq \bar{\omega}_{i1}\}$$

where $\bar{\omega}_{i1}$ is some upper bound, possible infinite. In a withholding game, $\bar{\omega}_{i1} = \omega_{i1}^o$ for $i = 1,2$.

Geometrically, a strategy pair (ω_1, ω_2) in $\Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ can be represented by two points of the base of a Kolm triangle of height $\omega_{11}^o + \omega_{21}^o$. To the pair (ω_1, ω_2) is associated a pair of offer curves

$(h_1(\omega_1), h_2(\omega_2))$ from a knowledge of the agent's preferences. The intersection points of $h_1'(\omega_1)$ and $h_2'(\omega_2)$, obtained by translating $h_1(\omega_1)$ and $h_2(\omega_2)$ along the base of the triangle by the amounts $\omega^o - \omega_1$ and $\omega^o - \omega_2$, define the outcome correspondence of the Lindahl endowment generalized quasi-game $G(e)$ of e. $Z(\omega^o)$ is the set of feasible allocations of e. An allocation z in $Z(\omega^o)$ is a local equilibrium allocation of $G(e)$ if it satisfies the definition preceding the statement of Theorem 3.

Before stating Theorem 4, a number of definitions have to be introduced. (See Figure III-8.)

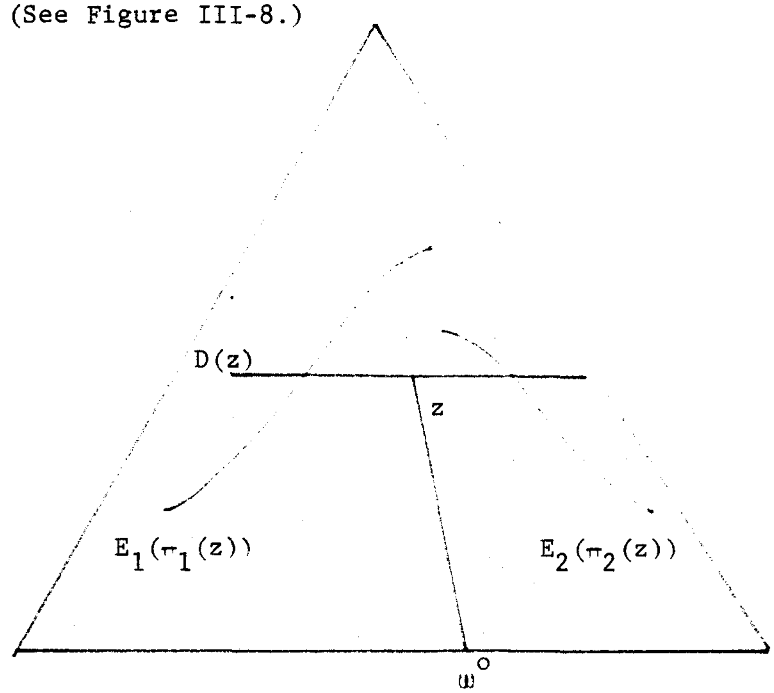


Figure III-8

Any allocation $z \neq \omega^o$ in $Z(\omega^o)$ implicitly defines two prices $\pi_1(z,\omega^o)$ and $\pi_2(z,\omega^o)$ adding up to one. Given a price π_i, $E_i(\pi_i)$

denotes agent i's income expansion path relative to π_i . Finally, D(z) denotes a horizontal line through z .

Theorem 4. Given $e = ((X_i,\omega_i^o,\succsim_i),\ i = 1,2;\ Y)$ in E_2 , it is necessary and sufficient for $z \in Z(\omega^o)$, $z \neq \omega^o$, to be a local equilibrium allocation of G(e) that conditions (i), (ii), (iii) below hold:

(i) There exists $(\omega_1,\omega_2) \in \Omega_1(\omega_1^o) \times \Omega_2(\omega_2^o)$ such that $z \in h_1'(\omega_1) \cap h_2'(\omega_2)$.

This simply says that z should be a Lindahl allocation for a pair of offer curves derived from some admissible pair of strategies.

(ii) The allocation $\bar{z}_i$ defined by $\bar{z}_i = \omega_i + z - \omega_i^o$ belongs to $D(z) \cap E_i(\pi_i(z,\omega^o))$ for $i = 1,2$.

This means that at the price $\pi_i(z,\omega^o)$ the trade maximizing agent i's preferences from ω_i is equal to the trade required to make z a Lindahl allocation with ω^o as the initial endowment. The statement follows from the fact that the Kolm transformation commutes with the operation of translation.

If the expansion paths E_1 and E_2 are strictly monotone in neighborhoods of $\bar{z}_1$ and $\bar{z}_2$, the equilibrium condition is simply that $h_j'(\omega_j)$ and $L_i(z)$ be locally separable at z . This is analogous to the condition of Theorem 3. If the local separation condition does not hold, equilibria can still exist if the only strategies available to the agents allow perturbations in the offer curves only in the "wrong" direction. More formally,

(iii) for each i , i = 1,2 ,

(a) either $E_i(\pi_i(z,\omega^o))$ is strictly monotone in a neighborhood of $\overline{z}_i$ and $h'_j(\omega_j)$ is locally separable from $L_i(z)$ at z ;

(b) or $E_i(\pi_i(z,\omega^o))$ is above (resp below) $D(z)$ in a neighborhood of z, and $h'_j(\omega_j)$ is below (resp above) $h'_i(\omega_i)$ in $L_i(z) \cap V$ for some neighborhood V of z.

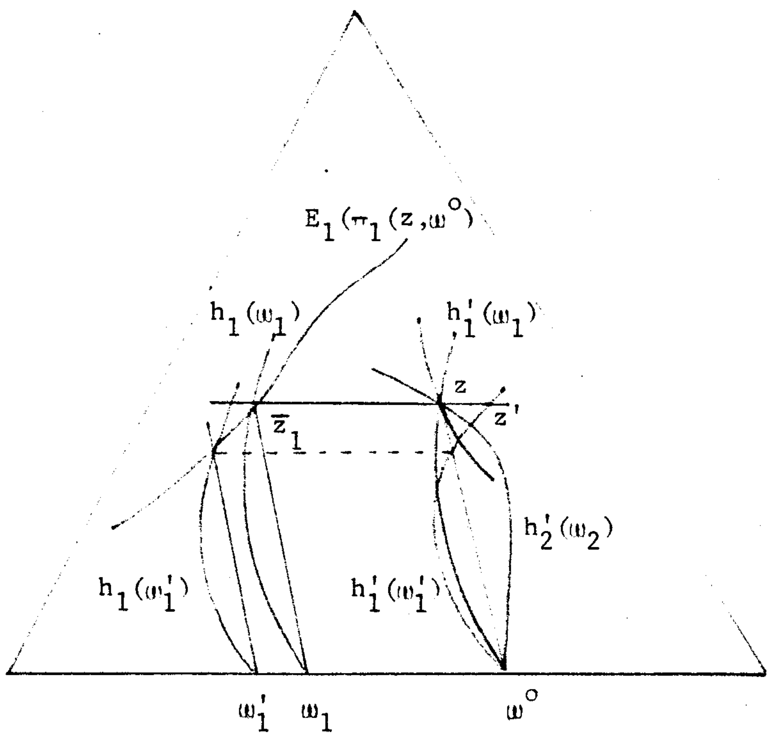


Figure III-9

To prove the statement, let us pick $i = 1$. If $h'_2(\omega_2)$ is transversal to agent 1's indifference curve through z, and $h'_2(\omega_2)$ is below $h'_1(\omega_1)$ in $L_1(z) \cap V$ for some neighborhood V of z, (as in Figure III-9), by the strict monotonicity of $E_1(\pi_1(z,\omega^o))$ it is possible to find $\omega'_1 \neq \omega_1$, to the left of ω_1 if the path is increasing (as in the Figure) or to the right of ω_1 if the path is decreasing, such that the new offer curve $h'_1(\omega'_1)$ would intersect $h'_2(\omega_2)$ at a point z' strictly preferred to z

by agent 1. The converse should be done if $h_2'(\omega_2)$ is above $h_1'(\omega_1)$ in $L_1(z) \cap V$.

Suppose now that $E_1(\pi_1(z,\omega^o))$ is above $D(z)$ in a neighborhood of z_1. Then by local changes in ω_1, the derived offer curve $h_1'(\omega_1')$ will require a larger trade at the price $\pi_1(z,\omega^o)$ than $h_1'(\omega_1)$. But if $h_2'(\omega_2)$ were transversal to agent 1's indifference curves through z at z and were below $h_1'(\omega_1)$ in $L_1(z) \cap V$, then smaller trades would be required for agent 1 to locally improve his welfare. Such a configuration is therefore compatible with equilibrium.

Similar conditions could be derived for (ω_1^o,ω_2^o) to be an equilibrium strategy pair.

In order to check whether an allocation z in $Z(\omega^o)$, $z \neq \omega^o$, is a local equilibrium allocation one should determine the individualized prices $\pi_1(z,\omega^o)$, $\pi_2(z,\omega^o)$, the expansion paths $E_1(\pi_1(z,\omega^o))$, $E_2(\pi_2(z,\omega^o))$ and determine their intersections with the horizontal line through z, $D(z)$. These intersections will be non-empty. Finally,

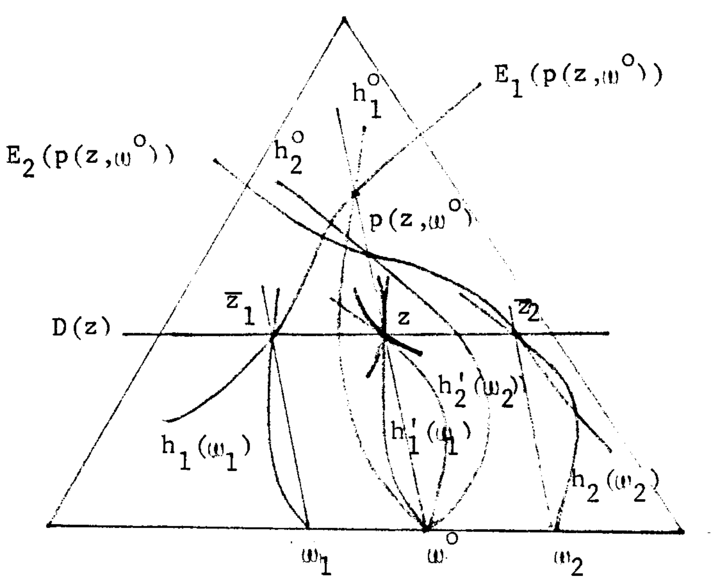


Figure III-10

condition (iii) should be checked. In Figure III-10, (iii)(a) holds for both agents for the pair ω_1, ω_2 and z is a local equilibrium allocation.

Remark 1'. Same as Remark 1 of preceding subsection.

Remark 2'. Same as Remark 2 of preceding subsection.

Remark 3'. If one agent (say agent j) behaves truthfully, it may not be possible for the other agent to reach the point of $h_j'(\omega_j^o)$ that he likes the best. In fact, no point outside of $V(\omega^o)$, as defined in subsection II-2 can ever be a Lindahl allocation and agent i's "monopoly point" may be outside of $V(\omega^o)$. However, if strategy spaces are large enough, and in particular, if $\bar{\omega}_{i1} \geq \omega_{i1}^o$, agent i can find a strategy yielding the point of $h_j'(\omega_j^o) \cap V(\omega^o)$ he likes the best (or a point arbitrarily close to it) as Lindahl allocations.

Remark 4'. Allocations in $V(\omega^o)\backslash L(e)$ may be local equilibrium allocations. However, if the public good is not a Giffen good, this will not happen and only allocations in $V(\omega^o) \cap L(e)$ will be possible local equilibria.

Remark 5'. Allocations very close to the true Lindahl allocations can be obtained as equilibria for preferences whose offer curves are flatter and flatter curves through the announced endowment ω_i as ω_i goes to infinity, which of course requires very large strategy spaces.

Remark 6'. Note that an allocation z in $h_1'(\omega_1) \cap h_2'(\omega_2)$ will be a local equilibrium allocation if $E_1(\pi_1(z,\omega^o))$ and $E_2(\pi_2(z,\omega^o))$ are horizontal in a neighborhood of their intersection with $D(z)$. Locally, the preferences are quasi-linear and the optimal level of the public good does not depend on the initial endowments, which explains why one agent will not gain by marginal changes in his strategy. More generally, if preferences are quasi-linear over the whole space, expansion paths are horizontal in the interior of the commodity space and the same result will hold. In such circumstances, global strategic considerations are much easier to make since offer curves are all translated of each other unless they are truncated by the boundary of the commodity space.

Remark 7'. It is not possible here to give a general geometric characterization of all equilibria as could be done in the preferences games. However, as in the private good endowment games, one can construct economies with equilibria at any preassigned location, close to the true Lindahl allocation as in Remark 5', close to the point of each agent's true offer curve that the other agent prefers provided this point is in $V(\omega^o)$, or close to the initial endowment. These various possibilities are illustrated in Examples 1 and 2.

Example 1.

Let agent i's indifference map be made of parallel straight lines with slope $\ell_i \neq 0$. Assume $1/\ell_i + 1/\ell_j \neq -1$. For every $\omega_i \in \Omega_i(\omega_i^o)$, $h_i(\omega_i)$ is the union of a line segment of slope ℓ_i and of a half line parallel to agent i's y axis.

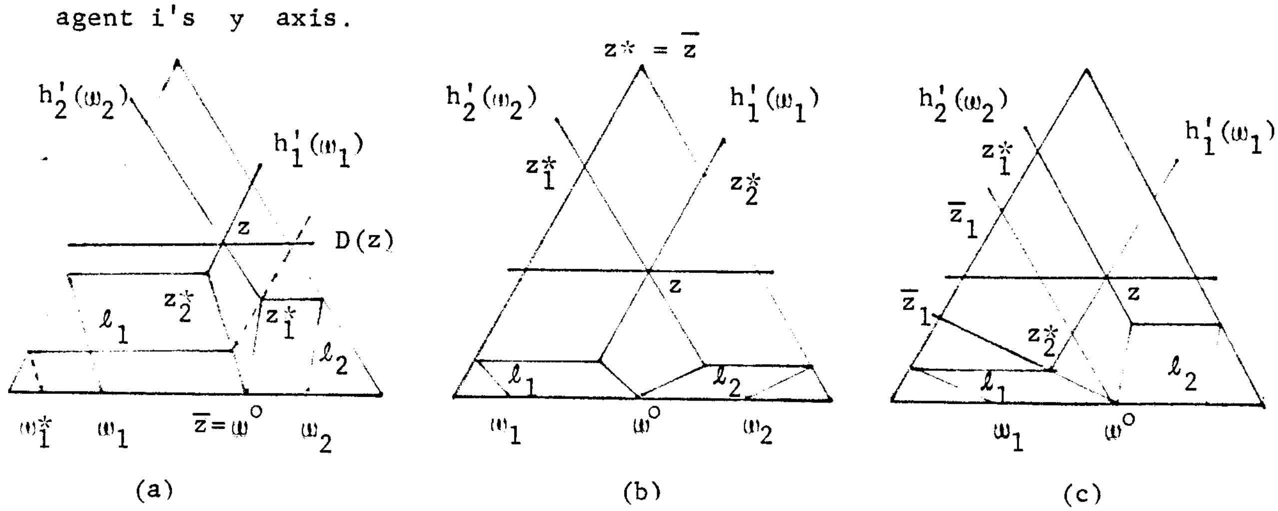


Figure III-11

Let $z \in Z(\omega^o)$ be given. In any no-destruction game, there is a strategy pair (ω_1, ω_2) such that $z = h_1'(\omega_1) \cap h_2'(\omega_2)$. Let z_i^* be the point of $h_j'(\omega_j)$ that agent i prefers. If $\ell_i < -1$, z_i^* is the kink of $h_j'(\omega_j)$. If $\ell_i > -1$, z_i^* is the intersection of $h_j'(\omega_j)$ with agent i's y axis. In both cases, agent i has a strategy $\bar{\omega}_i$ such that $z_i^* = h_i'(\bar{\omega}_i) \cap h_j(\omega_j)$.

a) suppose $\ell_i < -1$ for $i = 1,2$. Then for at least one agent, z_i^* is preferred to z, unless $z = \omega^o$. It follows that ω^o is the only equilibrium allocation. It can indeed be achieved by picking $\omega_i = 0$ for $i = 1,2$.

b) suppose $\ell_i > -1$ for $i = 1,2$. Then for at least one agent, z_i^* is preferred to z, unless $z = z^*$, the apex of the triangle, which also happens to be the truthful Lindahl allocation. It follows that z^* is the only equilibrium allocation. It can indeed be achieved by picking $\omega_i = \omega_i^o$ for $i = 1,2$, which are in fact dominant strategies.

c) suppose $\ell_1 > -1$ and $\ell_2 < -1$. Then $\omega_1 = \omega_1^o$ is a dominant strategy for agent 1 and $\omega_2 = \omega_2^o$ is a strategy for agent 2. The equilibrium allocation is the point $\bar{z}_1$ of $L(e) \cap V(\omega^o)$ that agent 2 likes the best. (The point $\bar{\bar{z}}_1$ of $h_1(\omega_1^o)$ that agent 2 likes the best is a Lindahl allocation for no strategy pair.)

Note that, for each of these cases, the unique local equilibrium allocation just identified is also a global equilibrium allocation, and that the preferences could be approximated by strictly convex ones without disturbing the equilibrium allocation by more than ε.

<u>Example 2.</u>

Let e be an economy in E_2 such that both agents' preferences admit of representations of the form xy. It is well known that, in orthogonal

likes the best is a Lindahl allocation for no strategy pair.)

coordinates, any offer curve derived from an endowment bundle on the horizontal axis is a vertical line with abscissa equal to half the abscissa of the endowment bundle. Also, expansion paths are straight lines through the origin. In the Kolm triangle, offer curves will be parallel to the slanted sides and expansion paths will still be straight lines through the origin.

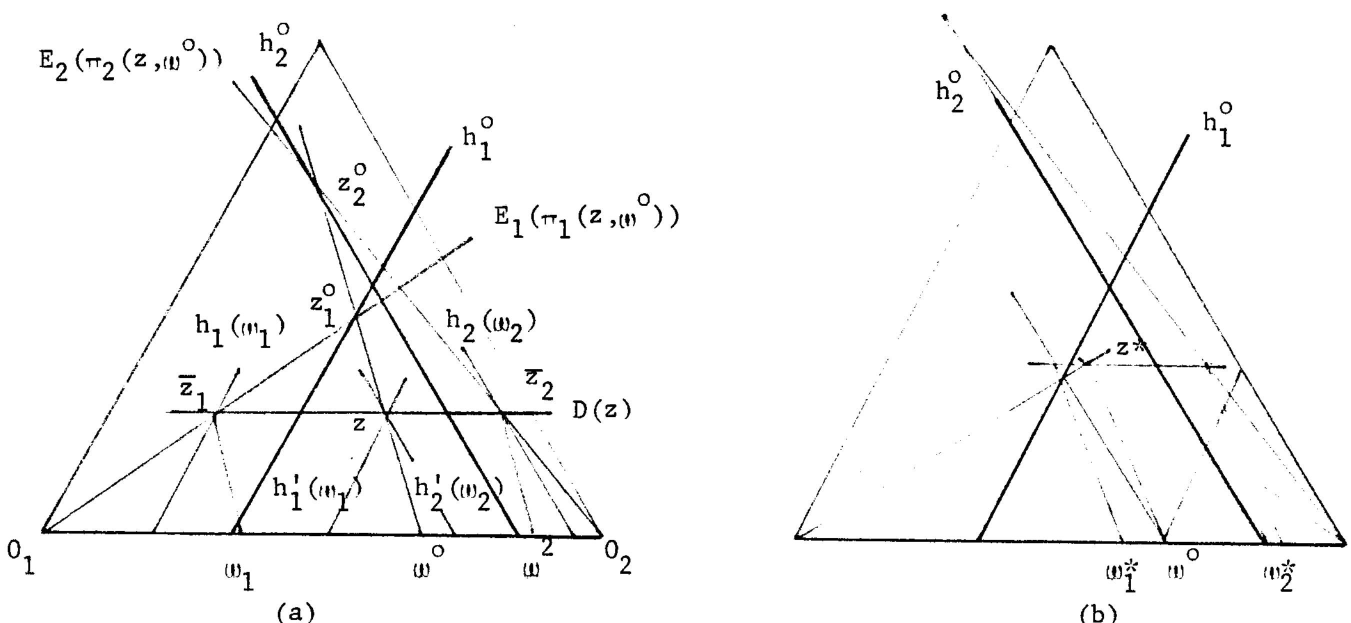


Figure III-12

Let z be an allocation in $Z(\omega^o)$, $z \neq \omega^o$. The line connecting ω^o and z intersect h_1^o and h_2^o in z_1^o and z_2^o . $E_1(\pi_1(z,\omega^o))$ and $E_2(\pi_2(z,\omega^o))$ are straight lines through (O_1,z_1^o) and (O_2,z_2^o) respectively. $D(z)$ intersects these lines in $\bar{z}_1$ and $\bar{z}_2$. This determines ω_1 and ω_2 , $h_1(\omega_1)$ and $h_2(\omega_2)$ and finally $h_1'(\omega_1)$ and $h_2'(\omega_2)$. Since the expansion paths are strictly increasing, it remains to check whether the indifference curves at z of agent i $(i = 1,2)$ is tangent to $h'_j(\omega_j)$ $j \neq i$. Since all the offer curves of agent j are parallel to each other, defining the implicit prices $\pi_i = 1$, $\pi_j = 0$, this is possible only if z belongs to $E_i(1)$, $i = 1,2$. But there is only one such point in $Z(\omega^o)$, z^*. Moreover, because of the convexity of the indifference curve and the fact that offer curves are straight lines, z^* is a global equilibrium as well. Finally, note that $z^* \in L(e)$ and that it is obtained for the unique pair (ω_1^*, ω_2^*) which is a strategy available in the withholding games when only downward misrepresentation is possible. Almost the same result would be available in economies with preferences representable by the function $x^{\alpha}y^{1-\alpha}$, $0 < \alpha < 1$.

3) Comparison of the manipulability of the Walras and Lindahl mechanisms. The comparison is more difficult here than in Section II since the exact location of the equilibrium allocations has to be determined separately in each individual case. Theorems 3 and 4, where conditions for an allocation to be an equilibrium allocation are stated, do involve the same kind of considerations though. Mainly, it is on the basis of Remarks 1-6 and 1'-7' and perhaps more specifically Remarks 6 and 7' that one can conclude that the manipulability of the two mechanisms is indeed quite similar.

IV. Large economies

The analysis of Section II is here extended to the case of large economies. The reasons for making a special study of large economies are well-known. For our purpose, it suffices to note that as an economy enlarges, the impact of each agent's decision or message on the "collective variables" is less and less significant, where this expression designates the variables whose equilibrium values are relevant to all the agents; these would include prices in private exchange economies and levels of public goods in public good economies. They should be distinguished from the "Individual variables," each of which affects directly only one agent, net trades and individualized prices respectively in the above two examples (if preferences are selfish). In any finite economy, a consumer will in general significantly affect both sets of variables, while in a large enough economy, he will but marginally influence the collective variables. The asymptotic lack of responsiveness of the collective variables to an agent's actions has very different implications in private and public good environments. An agent who has no impact on the equilibrium price is forced to competitive behavior, while complete free riding becomes optimal for an agent who has no impact on the public good levels.

Our program of comparing the manipulability of the Walras and Lindahl mechanisms would not be complete without an examination of large economies, and a formalization of the intuitive argument of the preceding paragraph. To further motivate this Section, a brief historical note might be useful. Unless some earlier references have escaped my attention, it is in Samuelson's article on optimality in public good economies [21] that the so-called "free-rider" problem was first stated. Samuelson remarked that one should not expect self-interested consumers to honestly reveal their marginal rates of substitution, information necessary to attain an optimal state. Until recently, and apart from a few contributions like that of Vickrey [28], the problem was thought to be mainly a public good problem. However, in [10], Hurwicz emphasized that manipulation was possible in private good economies as well, and showed that, in such environments, no mechanism would in general be manipulation free; the counterpart of this Theorem for public good economies was subsequently proved by Ledyard and Roberts [15]. In the preceding Sections, we have gone one step further and established that the two principal mechanisms for these two environments, are in some sense "equally" manipulable.

But what of large economies? The issue was addressed by Roberts and Postlewaite [23] and Roberts [22]. In [23], the authors introduced a definition of the "limit incentive-compatibility" of a mechanism and derived conditions under which the price-mechanism would satisfy it; on the other hand, Roberts [22] showed that no mechanism would enjoy this property in (non-trivial) public good economies. The distinction between private and public good economies reappears when the agents are numerous.

In what follows, we address the same question of the manipulability of the Walras and Lindahl mechanisms in large economies where, following the logic of the preceding Section, the criterion of manipulability is "the size" of the set of equilibrium allocations of the quasi-games associated with the mechanisms. Although only partial results are available, they point to a quite different conclusion.

In order to facilitate the comparisons of these various results, we first introduce a definition of limit incentive-compatibility close to the one used by Roberts and Postlewaite. It is based on examining what a given agent can achieve by unilateral misrepresentation, i.e., when everybody else is honest.

The symbol $\{e_k\}$ will always designate a sequence of economies of increasing cardinality.

Definition D1. A mechanism I defined on the environment E is limit incentive-compatible for the sequence $\{e_k\}$ in E, if given $\varepsilon > 0$, there is k^* such that $k > k^*$ implies that, for any agent A_i in e_k, and for any allocation x that A_i can attain through strategic behavior, while everyone else behaves truthfull, there exists an allocation x' indifferent for him to x, and an allocation y in $I(e_k)$ such that either $y \succsim_i x$ or $\|x_i' - y_i\| < \varepsilon$.

A number of comments will clarify this definition. First, note that it applies to misrepresentation of preferences as well as to misrepresentation of endowments (so will Definition 2).

Second, for an individual belonging to e_k for all k, it could be formulated as follows: given a continuous utility representation of his preferences, the mechanism would be said to be limit incentive-compatible for this particular agent in that particular sequence if his maximum utility gain achievable through misrepresentation would go to zero as k goes to infinity. A mechanism would be limit incentive-compatible for a replicated sequence of economies if it would be limit incentive-compatible for every type of agent.

Third, this Definition guarantees that an individual gains at most ε above what the best (for him) allocation associated to the truthful list of announcements would give him. But this concept is appropriate only for optimistic agents as explained in Section I3β. To each of the criteria formulated in I3 could in fact be associated a definition of a limit incentive-compatibility of a mechanism.

Fourth, what motivates D1 is the idea that an (optimistic) agent will announce the truth in a large economy, since he cannot gain much by misrepresentation and the fixed costs associated with the computation of an optimal strategy would not be recouped in large economies. (There does not seem to be any reason to believe that this computation would be easier as k increases.) However, this may be in contradiction with the dominance criterion formulated in I3α. For instance, if the mechanisms specifies three outcomes under truthful behavior and only one under strategic behavior, this unique outcome being indifferent to the best of the original three, misrepresentation may seem worthwhile to the agent. Of course, in large economies, one would not expect any individual to have such an impact on the cardinality of the set of outcomes.

If nobody gains more than ε by departing from honest behavior, telling the truth could be said to be an ε-equilibrium. (This concept appears in Radner [20] and Thomson [27].) A mechanism is therefore limiting incentive-compatible for $\{e_k\}$ if for every ε, there is k^* such that for all $k > k^*$, telling the truth is an ε-equilibrium in e_k. Naturally, there may be other ε-equilibria. In case of multiplicities, telling the truth would be a "prominent" or "natural" one to consider because of its ease of implementation.

The last point that deserves to be raised concerning D1 is that it considers departure from truthful behavior of one individual at a time. When everybody attempts to manipulate, the small influence each agent has on the mechanism, aggregated over a large number of participants, may significantly affect the final outcome and make strategic considerations definitely worthwhile.

In their analysis of the price mechanism, Roberts and Postlewaite do consider deviations from truth-telling by everybody and state conditions under which a list of strategies would guarantee truth-telling to be an ε-equilibrium. However, these conditions concern the strategies themselves and not the underlying economies.

They show that for replicated sequences, given any list of strategies for each type, possibly dependent on k, there is an order of replication for which no individual can gain more than ε by optimally departing from the strategy assigned to his type. However, for non-replicated sequences, the result may not hold. They provide an example of a general sequence, containing an individual who can by an appropriate strategy choice, guarantee himself a finite gain in utility, even for large k. Moreover, the strategy achieving this result can be chosen independently of k. This possibility results from the fact that the offer curve faced by that agent maintains a curvature bounded away from 0 as k increases. As pointed out by the authors, the example is non-pathological in the sense that preferences are well-behaved in every other way. They just "match up" in some peculiar way.

In what follows, we explicitly allow for joint manipulation of all the agents, and define the limit incentive-compatibility of a mechanism accordingly.

Definition D2. A mechanism I defined on the environment E is limit incentive compatible for the sequence $\{e_k\}$ in E, if given $\varepsilon > 0$, there is k^* such that $k > k^*$ implies that for every z in $N(e_k)$, the set of equilibrium allocations of the quasi-game associated with I, there exists $\bar{z} \in I(e_k)$ such that $\|z - \bar{z}\| < \varepsilon$.

This definition could be strenghthened by demanding that, conversely, for every $\varepsilon > 0$, there exists k^* such that $k > k^*$ implies that for every $\bar{z} \in I(e_k)$, there exists $z \in N(e_k)$ such that $\|z - \bar{z}\| < \varepsilon$. For replicated sequences, each of these two variants could be weakened by requiring closeness in utility terms only (after choosing an arbitrary (but invariant with k) utility representation of the preferences of each type).

If a mechanism satisfies D2 for some sequence $\{e_k\}$, should honest behavior be expected for large cardinalities? Maybe not. It still could be the case that when everybody else behaves strategically, a given agent achieves a significant gain by also behaving strategically, as opposed to telling the truth. If, however, D2 and D1 hold, then truth-telling is likely since nobody can gain more than ε by unilateral strategic behavior, nor is there any equilibria that yield more than an ε gain.

Let us now apply Definition D2 to the Walras and Lindahl mechanisms.

d) Private goods

Let E_1 be the class of economies defined in Section IIa, and let e_1 be an element of E_1 . It was shown in IIa that $N(e_1) = L(e_1)$. Let now e_k designate the k^{th} order replica of e_1 . The following Theorem shows that even for a well behaved sequence such as $\{e_k\}$, the Walras mechanism does not satisfy D2. More specifically, $N(e_k)$, projected on e_1 , contains $L(e_1)$ for every k .

Theorem 5: In the k^{th} replica e_k of any economy e_1 of E_1 , any equal treatment allocation (i.e., giving the same consumption bundle to all agents of the same type) whose projection on e_1 is in $L(e_1)$, is an element of

$N(e_k)$.

Proof: It is provided by the following construction.

To make the argument more transparent, we call D the offer curve of an agent buying Y , and S the offer curve of an agent selling Y . Consider an allocation z in $L(e_1)$. From Theorem 1, we know that two strategies D_1 and S_1 can be found to yield z as an equilibrium Nash-allocation of e_1 D_1 and S_1 are chosen so as to be respectively tangent at ω to the true indifference curves through ω . Given a price p , a line of slope p through ω intersects D_1 and S_1 in two points A_1 and B_1 . Measuring distances algebraically along the oriented line ω_p , the net demand at price p can be expressed as $\omega A_1 - \omega B_1$.

In the k^{th} replica, we will now define a common strategy D_k for all the "true" demanders, and a common strategy S_k for all the "true" suppliers.

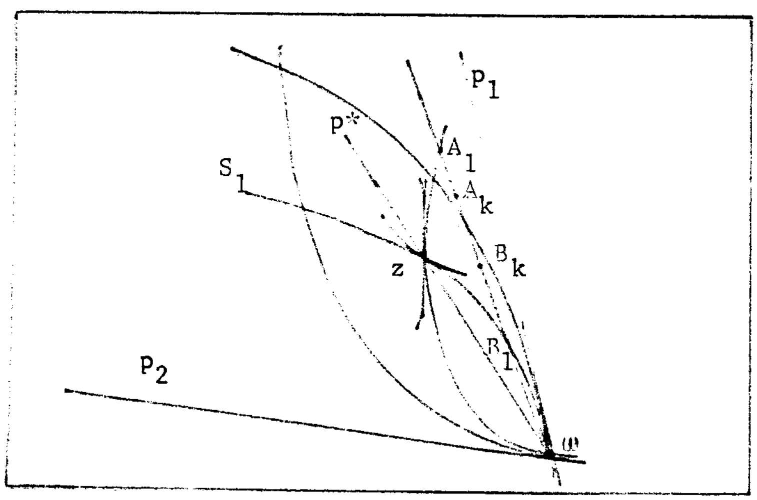


Figure IV-1

These strategies will be such that each agent will basically still face the same opportunity locus as the single agent of the corresponding type in e_1 .

Call p_1 and p_2 the slopes of the suppliers' and demanders' true offer curves at ω . For p between p_1 and p_2 , we will ensure that the net demand curve faced by any demander, S_k' , be the same as in e_1 , and similarly for the suppliers.

This is achieved by setting

$$(1) \quad S_k' = kS_k - (k-1)D_k = S_1$$

$$(2) \quad D_k' = kD_k - (k-1)S_k = D_1 \ .$$

The slope of the line joining ω to z is designated p^* . Solving (1) and (2) in S_k and D_k gives

$$(3) \quad D_k = \frac{1}{2k-1} (kD_1 + (k-1)S_1)$$

$$(4) \quad S_k = \frac{1}{2k-1} (kS_1 + (k-1)D_1) \ .$$

For p between p_1 and p_2 , $D_1(p)$ and $S_1(p)$ are both non-negative, and so are $D_k(p)$ and $S_k(p)$. In addition, $D_k(p^*) = S_k(p^*) = D_1(p^*) = S_1(p^*)$. Therefore, if the range of variation of p were $[p_1, p_2]$, any pair of offer curves whose restriction to that range were given by (3) and (4) would constitute an equilibrium.

However, for completeness, we will extend D_k and S_k as defined above to the whole price domain.

For p between 0 and p_1 , we should guarantee that D_k^1 does not reach the set of allocations strictly preferred by the suppliers to z .

One way to achieve this is to impose:

$$D_k' \geqq 0 \quad \text{for} \quad 0 \leqq p \leqq p_1 \ .$$

Also, we require:

$$D_k' \leqq 0 \quad \text{for} \quad p_2 \leqq p \leqq \infty \ .$$

Equivalent conditions for the demanders are

$$S_k' \leqq 0 \quad \text{for} \quad 0 \leqq p \leqq p_1$$

and

$$S_k' \geqq 0 \quad \text{for} \quad p_2 \leqq p \leqq \infty \ .$$

Restating D_k' and S_k' as functions of D_k and S_k yields:

$$\left.\begin{array}{ll} \text{(5a)} & kD_k - (k-1)S_k \geqq 0 \\ \text{(5b)} & kS_k - (k-1)D_k \leqq 0 \end{array}\right\} \quad 0 \leqq p \leqq p_1$$

$$\left.\begin{array}{ll} \text{(6a)} & kD_k - (k-1)S_k \leqq 0 \\ \text{(6b)} & kS_k - (k-1)D_k \geqq 0 \end{array}\right\} \quad p_2 \leqq p \leqq \infty$$

In addition to (3) and (4), we will complete the definition of D_k and S_k by:

(7) $D_k(p)$ increases as $p \to 0$ and $S_k(p)$ decreases as $p \to 0$

(8) $D_k(p)$ decreases as $p \to \infty$ and $S_k(p)$ increases as $p \to \infty$.

For $0 \leqq p \leqq p_1$, $D_k(p)$ is always non-negative; therefore (5b) $\Rightarrow$ (5a). Since, by (3) and (4),

$$D_k(p_1) = \frac{k}{2k-1} D_1(p_1) \quad \text{and} \quad S_k(p_1) = \frac{k-1}{2k-1} D_1(p_1) \ ,$$

(5b) holds at equality for $p = p_1$, (7) implies that (5b) and consequently (5a) holds for $0 \leqq p \leqq p_1$.

Similarly, it could be established that (8) implies that (6) holds for $p_2 \leqq p \leqq \infty$.

Finally, (3), (4), (7), and (8) can be simultaneously satisfied by well-defined demand and supply curves. This completes the proof.

Remark: It is important to note that an equilibrium list of strategies does not in general constitute an ε-equilibrium list of strategies. If an individual decided to behave truthfully, he would stand to lose a finite amount. In other words, $N(e_k)$ for large k is not (even approximately) the same thing as $N_\varepsilon(e_k)$.

In what follows, we reconcile this Theorem with the conclusions reached by Roberts and Postlewaite, and we analyze it in the light of recent contributions addressing closely related issues.

In a large replica economy, it is true that if all agents behave truthfully, everyone of them faces an almost flat net demand curve, and has a vanishing impact on the equilibrium price. Only a minimal gain can he achieve by strategic behavior and a small distortion of his offer curve will be sufficient for that purpose. If all agents do so, however, these small distortions, because they are aggregated over a large number of agents, may lead to a significant increase in the elasticity of the net demand curve faced by any one of them, justifying a second round of revisions, etc. The allocations reached at an equilibrium of this adjustment process may be quite far from the truthful allocations, in spite of the fact that truth-telling constitutes an ε-equilibrium (D1 is indeed satisfied, as proved by Roberts and

Postlewaite). It is worth noting that in order to preserve the impact of all agents on the equilibrium price, it is necessary that the strategies "match up" in some special way; the correspondence associating to each economy its set of Walras prices fails to be lower-semi-continuous for the sequence of "apparent" economies (once appropriate topologies are introduced). This is the technical reason why the present result is perfectly reconcilable with the Roberts and Postlewaite analysis since their main theorem states that any sequences of strategy lists for which continuity of the correspondence just defined was guaranteed, would constitute an ε-equilibrium list of strategies, for k large enough. What Theorem 5 indicates is that such sequences of strategies lists can precisely be equilibrium sequences in a quasi-game of misrepresentation. It is therefore not legitimate to disregard them.

That continuity is indeed essential to obtain truthful Walras allocations in the limit is confirmed by a characterization theorem of Mas-Colell's who, in a study of large exchange games, showed that any such game satisfying a certain list of axioms would have to yield Walras allocations as equilibria. One of the axioms is continuity.

Postlewaite and Schmeidler [19], Wilson [31], Walker [29] have constructed games whose Nash equilibria approximate the Walras allocations of large economies. In addition, Schmeidler [25], Hurwicz [13], and Walker [30] have devised games whose Nash equilibria yield Walras or Lindahl allocations even in finite private good and public good economies. Should these papers be interpreted as proving that strategic behavior (in the Nash sense) is perfectly compatible with the Walras and Lindahl mechanisms? Not quite. It really depends on which game the agents are assume to play. Each of the above papers assume that all agents take Nash position, i.e., truthfully maximize their utility, with the other agents' messages considered as fixed parameters. If the agents took into account the impact they had on the equilibrium values of the other agents' messages, i.e., if they behaved strategically in the sense of Section I and responded according to some function consistent with Nash behavior for some other set of preferences, the result would be quite different.

A given performance correspondence can in general be implemented by various mechanisms. The Walras performance correspondence can be implemented by the price mechanism, mechanism which involves an auctioneer and requires price taking behavior, and by various games (as set up by the above authors), which involve no auctioneer and requires "message-taking" behavior. In order to compare the manipulability of these various mechanisms, one should consider the quasi-games of misrepresentation associated with each of these mechanisms and compare their equilibrium sets. Performing this task (that we have undertaken here for the price mechanism) for these other mechanisms is beyond the limit of the present paper. However, there is no reason to expect that their equilibrium set would coincide or converge to the true Walras allocations.

Finally, two contributions should be mentioned where results very much in the spirit of ours are provided. Wilson [31] considers a replica of an economy of two agents with transferable utility bidding for some contract, and shows that the equilibria of a game in which strategies are demand schedules may be quite far away from the true Walras allocations even in large economies, although only one side of the market is allowed to engage in strategic considerations. A similar non-convergence result is proved by Green [4] for large dynamic markets.

2) Public goods

In a sequence of public good economies of increasing cardinality, it may happen that, under truthful behavior, the cost per capita of providing the good at an optimal level converges to zero. Revealing one's true preferences constitutes an ε-equilibrium in such sequences for sufficiently large cardinalities. In such economies, the incentive to be honest is not any stronger than the incentive to cheat. They should be contrasted with the alternative situation when, because of what are often called "crowding effects," the average cost of providing the public good optimally, is bounded away from zero. It is then that the free-rider problem is the worse.

We will therefore focus on that case and assume that the public good is produced according to a linear technology Y_k exhibiting crowding-effects of the simple form: $y = x/k$, where x is the input, y the output, and k the number of consumers.

Let $(X, \omega, \succsim)$ be any consumer as allowed in E_2 (see Section II-2). The economy e_k will contain k times that agent and have access to the technology Y_k. Let y^* be the uniquely optimal level of the public good in e_1. In e_k, there is a Lindahl allocation with y^* as public good level, where each agent contributes the same amount y^*. The total collected is therefore ky^* which is precisely what is needed to produce the good at that level given the technology available to e_k.

In Section II, it was shown that the set of equilibrium allocation of the preference quasi-game $G(e_2)$ of e_2 was equal to $L(e_2) \cap V(e_2)$, which meant that any level of the public good between 0 and y^* could be achieved in some equilibrium allocation. The next Theorem shows that this is still possible for any economy e_k (with $k > 1$) .

Theorem 6. Given any $\bar{y}$ for which $0 < \bar{y} < y^*$, there exists, in the k^{th} replica e_k of e_1 , a symmetric equilibrium strategy list $(h_k,\ldots,h_k)$ yielding a symmetric Lindahl allocation with $\bar{y}$ as public good level.

Proof: Given $\bar{y}$ as in the statement of the Theorem, we exhibit an offer curve h_k such that, if k-1 agents announce h_k , it will be in the last agent's best interest to also announce h_k .

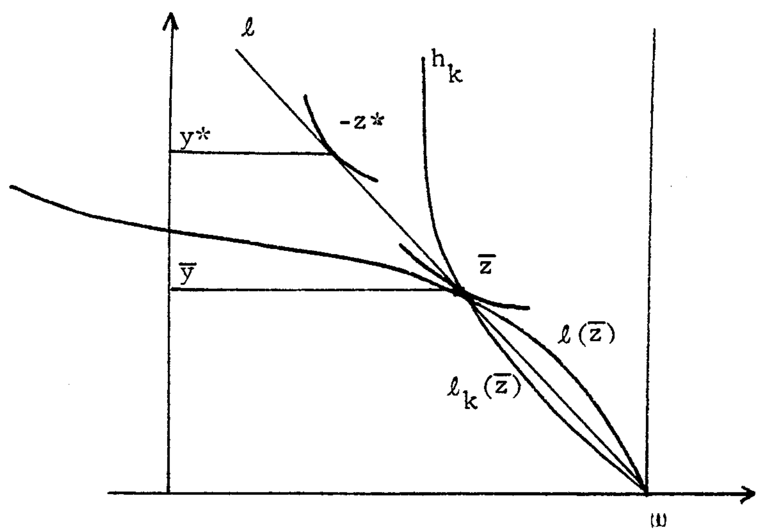


Figure IV-2

First, we consider e_1 . ω is initial endowment of the unique agent of e_1 . ℓ is a straight line of slope -1 through ω representing the consumption bundles available to this agent, and z^* , of ordinate y^* , is his optimal consumption. Given $0 < \bar{y} < y^*$, let z be the point of ℓ of ordinate $\bar{z}$. $\ell(\bar{z})$ supporting hyperplane at $\bar{z}$ to the agent's indifference curve through $\bar{z}$. By strict convexity of preferences, $\ell(z)$ is less steep than ℓ . Let $-1 + p(\bar{z})$ with $p(\bar{z}) > 0$ be its slope. Let $\ell_k(\bar{z})$ be a line of slope $-1 - p(\bar{z})/k-1$ through $\bar{z}$. Finally, let h_k be an arbitrary offer curve going through $\bar{z}$ and supported at that point by ℓ_k . It is easy to check that if k-1 agents announce h_k , the last agent will face an opportunity locus representable by a curve a_k going through $\bar{z}$, and supported at $\bar{z}$ by $\ell(\bar{z})$. It is therefore optimal for the agent to announce h_k too.

Comparison of the Walras and Lindahl mechanisms

Theorems 4 and 5 reveal that, according to the criterion of manipulability that we have been using throughout, private good and public good environments are much more similar than the criterion of unilateral manipulation formulated in Definition 1 would indicate. Indeed, in both cases, independently of k , we have been able to construct strategies yielding a continuum of allocations between the initial endowment and the true Walras or Lindahl allocations. Not only do we still get "bad" allocations in the private good case, but we also get "good" allocations in the public good case.

Concluding comments

The present paper has been concerned with characterizing the equilibrium allocations of several classes of misrepresentation games. The issue of how to reach these equilibria was not addressed and deserves to be investigated. Also, alternative equilibrium concepts could be considered. If not much should be expected of dominance equilibria, because they rarely exist (see, however, the Appendix), maximin, Bayesian, or perhaps other decision rules could be considered and for each of them, the set of equilibrium allocations could be determined. Such an analysis would allow a better understanding of the manipulability of various economic mechanisms. The vulnerability of a mechanism to manipulative behavior is an important criterion according to which to evaluate it, and the study of comparative economic systems needs more work in this direction.

Appendix

We consider here a different class of mechanisms which, strictly speaking, do not satisfy the definition of the first Section, because no feasibility constraint is imposed on their outcome functions. The absence of a feasibility constraint is meaningful in partial equilibrium analysis when surpluses or deficits can be financed from external funds. In environments characterized by "semi-linear" preferences (preferences having a numerical representation additively separable and linear in one commodity, the same for all agents, called "money"), Vickrey [28] for private good environments, and Groves and Loeb [8] for public good environments have constructed mechanisms inducing truthful elicitation of demand functions as dominant strategies. These schemes require that compensating payments be made to the agents, payments that do not generally add up to 0 ; Green and Laffont [5] have established that aggregate feasibility is in fact not possible, whenever the equilibrium strategies are required to be dominant strategies.

However, in certain sequences of economies of increasing cardinality, the aggregate net transfer evaluated on a per capita basis that these mechanisms require, converges to zero. For such sequences, the mechanisms can be said to be asymptotically feasible. In a closed economy, a certain share of the initial resources would have to be set aside for the purpose of incentive-compatible information gathering; this share, which can be thought of as the informational cost of the mechanisms, would become negligible in large economies. These observations motivate the following definitions:

Def: A partial equilibrium mechanism, a PE mechanism, satisfies all of the defining properties of a mechanism (see Section I) except perhaps aggregate feasibility.

Def: A PE mechanism I defined on the environment E is asymptotically feasible for the sequence $\{e_k\}$ of economies in E of increasing cardinality if, for every $\varepsilon > 0$, there exists k^* such that for every allocation y in $I(e_k)$, there exists an allocation z, feasible in e_k, such that $\|z - y\| < \varepsilon$.

We are now in a position to introduce two definitions of limiting incentive-compatibility of a PE mechanism, that are the counterparts of the ones (D1 and D2) appearing in Section IV, where the feasibility requirement for all economies is replaced by the weaker requirement of asymptotic feasibility.

D1': A PE mechanism I is l.i.c. for $\{e_k\}$, if given $\varepsilon > 0$, there is k^* such that $k > k^*$ implies that, for any agent A_i in e_k, and for any allocation x (feasible or not) that A_i can attain through strategic behavior, while everyone else behaves truthfully, there exist x' indifferent for him to x, and y in $I(e_k)$ such that either $y \succsim_i x$ or $\|x_i' - y_i\| < \varepsilon$, and there exists y', feasible in e_k, such that $\|y' - x\| < \varepsilon$. And, for every y in $I(e_k)$, there exist z, feasible in e_k such that $\|z-y\| < \varepsilon$.

D2': A PE mechanism I is l.i.c. for $\{e_k\}$, if given any $\varepsilon > 0$, there exists k^* such that $k > k^*$ implies for any $z \in N(e_k)$, there exist $\bar{z} \in I(e_k)$ such that $\|z - \bar{z}\| < \varepsilon$ and $\bar{\bar{z}}$, feasible in e_k, such that $\|z - \bar{\bar{z}}\| < \varepsilon$.

In the following pages, we investigate whether the Vickrey and

Groves-Loeb mechanisms satisfy D1' and D2'. Because telling the truth is a dominant strategy for both of them, characterizing their equilibrium allocation is trivial, and the two definitions are in fact equivalent. The mechanisms have indeed stronger properties than required by D1' and D2'. It is because the class of environments over which mechanisms with the dominance property of the Vickrey and Groves-Loeb mechanisms is limited that we felt the need to state D1' and D2' in the weaker form that they have above.

a) Private Goods

Let E_3 be the class of economies $e = ((x_i, \omega_i, \succsim_i),\ i=1,\ldots,n)$ with two commodities and n agents characterized by their consumption sets $X_i = R \times R_+$, their initial endowment $\omega_i \in X_i$, and their preferences $\succsim_i$ assumed to be closed; convex and to admit of a utility representation of the form "$x + v_i(y)$."

The Vickrey mechanism involves the communication of demand correspondences, the determination of the corresponding Walras allocation(s) and the computation of a side-payment to each agent whose precise expression is given in example a2. On E_3 , the Walras mechanism is essentially single-valued (see Hurwicz [9] in the sense that any two Walras allocations for a given list of demand correspondences are indifferent for all agents. In addition the Vickrey side-payments are well-defined as they do not depend on which one of the Walras allocations is selected. Multiplicities of Walras allocations are therefore inessential as evaluated with the implicit preferences corresponding to the reported demand but they may not be as evaluated with the true preferences. However, this eventually makes no difference since any truthful allocation dominates any non-truthful allocation. Truthtelling is a dominant strategy. By restricting ourselves to E_3' defined to be E_3 with the additional requirement that preferences be strictly convex, this minor non-uniqueness problem would disappear but the examples that follow would be slightly more difficult to build.

Example a1: This example is adapted from the one appearing in Roberts and Postlewaite [23] (see Section V), and establishing the existence of sequences of exchange economies $\{e_k\}$ in which a given individual can achieve the same gain through misrepresentation of his offer through a strategy choice independent of k . The present example shows that this phenomenon is also possible for sequences in E_3 .

In this example as well as in the next one, I designates the initial endowment. The term "buyers" (resp "sellers") refers to the agents who, at a Walrasian allocation, buy (resp sell) the second commodity. 0_1 (resp 0_2) designates the origin of the commodity space of the buyers (resp sellers).

The kth economy e_k of the sequence is composed of 4k agents, 2k - 1 buyers, and 2k sellers having the common offer curves IB_kC_k and ID_kA respectively (see Fig. A.1). In addition, a buyer (called agent 1) is singled out for special study: his truthful offer curve is IMM' and his announced offer curve is INN' . These various offer curves are drawn in such a way that

(a) The ordinate of A (resp C_k,M',N') is greater than the ordinate of D_k (resp B_k,M,N) .

(b) $2kIA = (2k - 1)IB_k + IM$

$2kID_k = (2k - 1)IC_k + IN'$.

(These conditions imply that B_k converges to A as $k \to \infty$. In addition, C_k and D_k are located on either side of B and converge to B as $k \to \infty$.)

Because of (b), in the true economy, the equilibrium price is p_1^* , given by the slope of IA , for all k , while in the "false" economy, it is p_2^* , given by the slope of IB . Because of (a), all of the offer curves involved are consistent with preferences admitting of a representation of the form "$x + v(y)$." In fact, only one such preference map (obtained by a simple integration) is consistent with any one of these offer curves. It follows that e_k belongs to E_3 for all k .

Finally, by misrepresenting his offer curve, agent 1 causes the equilibrium price to change by a finite amount $p_2^* - p_1^*$, and his own consumption to go from M to N' , a point strictly preferred to M according to the preference map that corresponds to the truthful offer

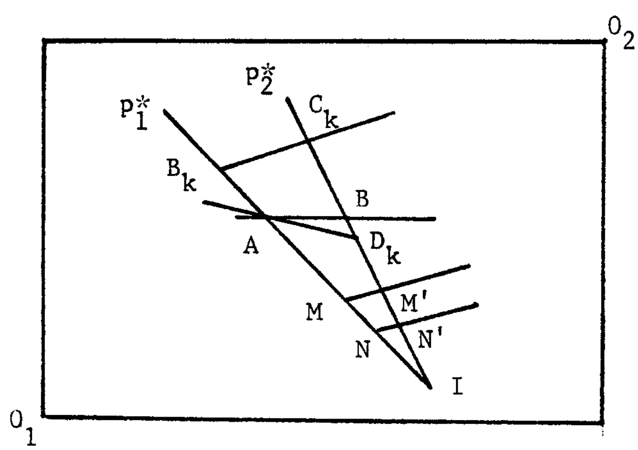


Figure A1

<u>Example a2</u>: The next example shows that there are sequences of economies for which the cost of the Vickrey scheme, evaluated on a per capita basis, does not go to zero as $k \to \infty$. Example a1 could have been used to this purpose too, but the present example has the advantage of a greater simplicity.

In e_k , there are k identical buyers and k identical sellers with IAC_k and ID_kA as offer curves respectively (see Figure A.2). These various points are chosen in such a way that:

(a) The ordinate of A is greater than the ordinate of D_k and smaller than the ordinate of C_k .

(b) $kID_k = (k - 1)IC_k \quad k \geqq 2$

(D_k and C_k converge to B as $k \to \infty$) .

As in example a1, (a) implies that e_k is in E_3 for all k , and (b) implies that p_1^*, given by the slope of IA , is the unique equilibrium price in e_k .

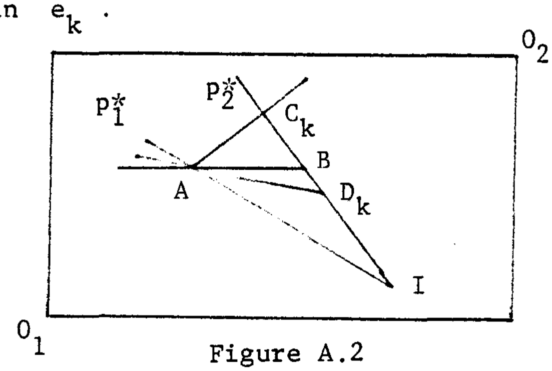


Figure A.2

In order to evaluate the cost of the Vickrey scheme, we compute the side-payment to each of the buyers and to each of the sellers (see Figure A.3).

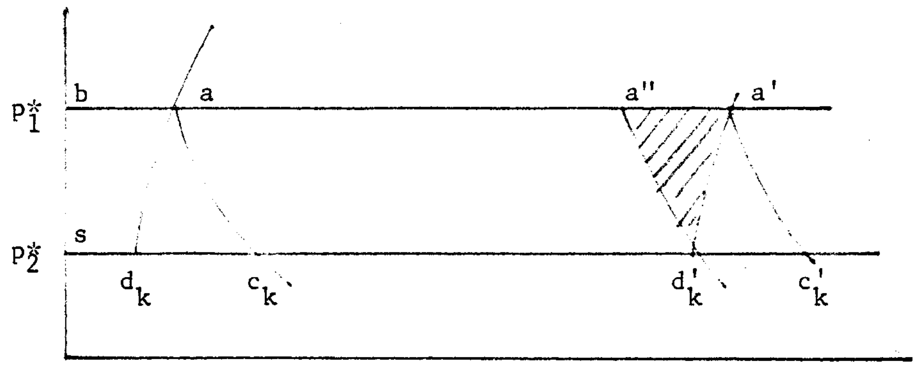


Figure A.3

bac_k and sd_ka represent the demand and supply curves of one buyer and one seller in e_k . Under the Vickrey scheme, the side-payment to each of the buyers is equal to the area $a'a''d_k'$ limited by the equilibrium price p_1^* , the aggregate supply curve $sd_k'a'$ and the demand curve of k-1 buyers, which goes through d_k' because of (b). It is represented by $ba''d_k'$ where a'a'' is equal to ba . The side-payment to a buyer can then be approximated by the area of the triangle $a'a''d_k'$ which is equal to $\frac{1}{2}ba \cdot (p_1^* - p_2^*)$, quantity that does not depend on k .

Because the deletion of one seller from e_k does not affect the equilibrium price, the side-payment to each seller is 0 .

Altogether, the Vickrey per capita side-payment is approximated by

$$\frac{1}{2k}\left[0 + k\,\tfrac{1}{2}(ba)\cdot(p_1^* - p_2^*)\right] = \frac{(ba)(p_1^* - p_2^*)}{4}$$

which does not vanish as $k \to \infty$.

Although the preferences used in these examples have nothing pathological, the sequences are built in such a way that the offer curves "match up" in a very special way.

As pointed out by Roberts and Postlewaite, a continuity condition on the correspondence associating to each economy, its set of equilibrium prices would guarantee that no agent would have more than an infinitesimal impact on the equilibrium price for economies of sufficiently large cardinalities. In most sequences of economies, it seems that this will be the case. However, formalizing this argument would go beyone the limits of this Appendix.

b) Public Goods

The object of this subsection is to develop examples of sequences of economies with public goods exhibiting the same features as examples a1 and a2.

Let E_4 be the class of economies $e = ((x_i, \omega_i, \precsim_i),\ i=1,\dots,n;\ y_n)$ with two commodities, a private good and a public good, n agents characterized by their consumptions sets $X_i = R \times R_+$, their initial endowment $\omega_i \in X_i$ satisfying $\omega_{i2} = \omega_{j2}$ for all i and j, and their preferences $\precsim_i$ assumed to admit of utility representations of the form "$x_i + v_i(y)$." The production set Y_n corresponds to a linear technology exhibiting crowding effects: in an economy of cardinality n, 1 unit of the input yields $1/n$ units of the output.

Example b1: This is a sequence of economies e_k in which a given agent can affect the equilibrium level of the public good by a given finite ($\neq 0$) amount through the use of a strategy that can be chosen independently of k, and in the process, gain a finite and constant ($\neq 0$) amount (see Figure A.4).

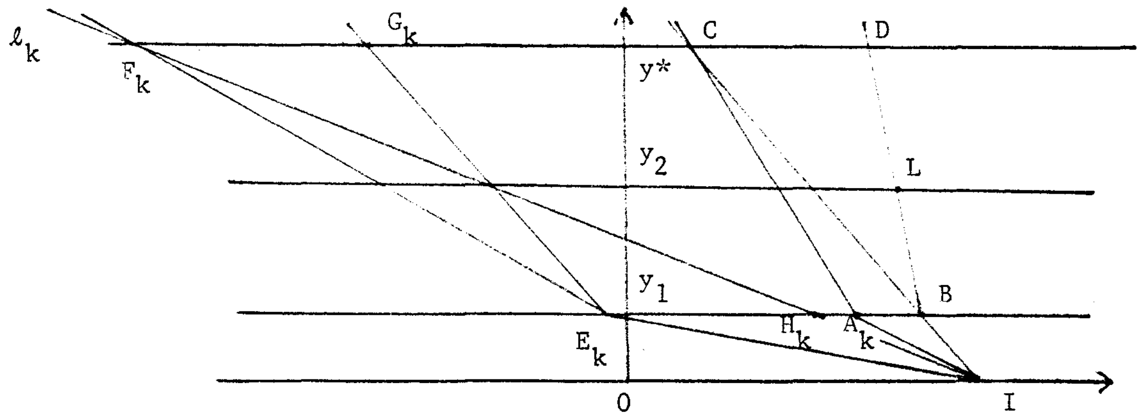


Figure A.4

I is the initial endowment of all agents. In e_k , there are k-1 identical agents whose offer curve is IA_kC . In addition, e_k contains an additional agent, agent 1, whose true offer curve is IBC and whose strategic offer curve is IBD. A_k is chosen so that

(a) $(k-1)A_kB = CD$.

Given ℓ_k , a line of slope -1/k through I , it follows that the aggregate demand curve for the truthful economy is IE_kF_k , and the unique Lindahl public good level is y^* , while in the false economy, the aggregate demand curve is IE_kG_k with $F_kG_k = CD = E_kH_k = (k-1)A_kB$ by (a), so that the unique Lindahl public good level is $y_2 = \frac{y_1 + y^*}{2}$. Then agent 1 can ensure that the public good level be y_2 for all k and that his consumption go from C to L , which is preferred to C according to the preference map that yields IBC as offer curve.

Example b2: We now construct a sequence of economies for which the Groves-Loeb side-payments do not go to 0 as $k \to \infty$. The above example could have been used to establish this point, but we prefer constructing a simpler one (see Figure A.5).

There are k identical agents in e_k with an offer curve given by IA_kC .

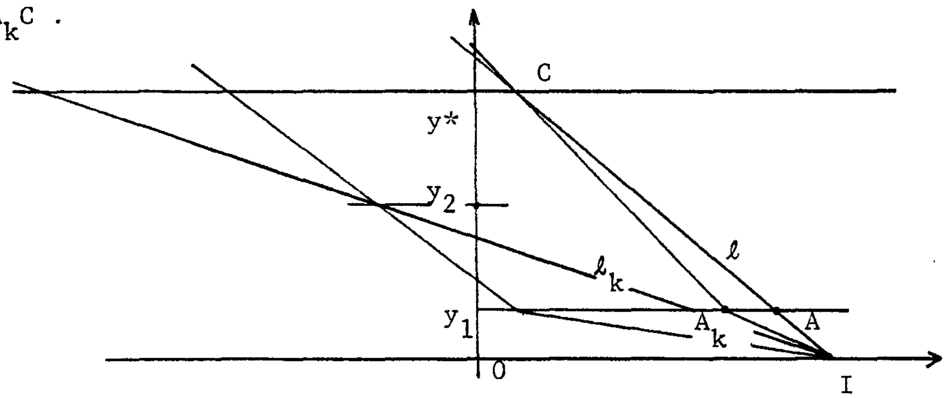


Figure A.5

ℓ and ℓ_k are lines of slope -1 and $-1/k$ passing through I .

A_k is chosen so that

(a) $$A_kA = y^*/(k-1) .$$

In e_k , the Lindahl public good level is y^* . After deleting any agent from e_k , the Lindahl public good level is $y_2 = (y_1 + y^*)/2$.

In Figure A.6, a_kc represents each agent's demand curve for the public good and $a_k'' c_k''$ is the aggregate demand in e_k . After deleting

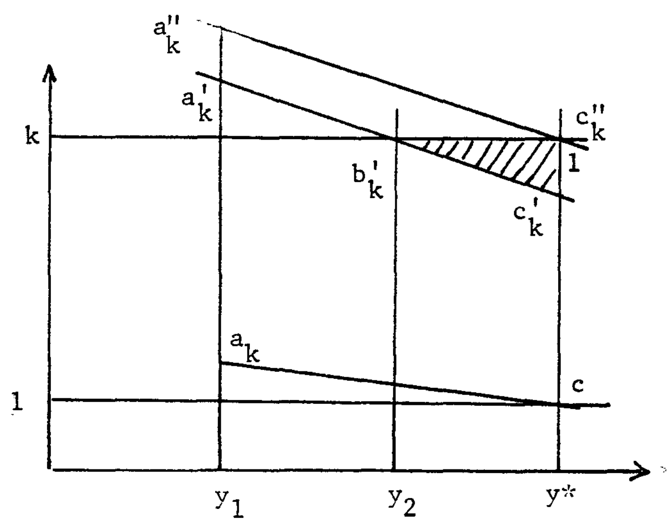


Figure A.6

one agent from e_k , the aggregate demand is $a'_k c'_k$ and the Groves-Loeb side-payment to agent i is given by the shaded area, limited by the optimal level y^* , the marginal cost curve, (horizontal line of ordinate k) and the aggregate demand of k-1 agents. This area is approximately equal to the area of the triangle $b'_k c'_k c''_k$ which is equal to $\frac{1}{2}(y^* - y_2)$.

Since all agents receive the same side-payments, this also gives the Groves-Loeb side-payment per capita. It is bounded away from 0 as $k \to \infty$.

The same comment appears here as in the provate commodity case, namely that the way the demand curves match up is in some sense exceptional. In general, one would not expect individuals to be able to affect the equilibrium level of the public good in economies of large cardinality. Conditions under which the Groves-Loeb side-payments do converge to 0 as k increases are given in Tideman and Tullock [26].

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