# A MODEL OF THE EURODOLLAR MARKET

by

Charles Freedman

Discussion Paper No. 25, February 1973

Center for Economic Research
Department of Economics
University of Minnesota
Minneapolis, Minnesota 55455

# A MODEL OF THE EURODOLLAR MARKET*

by

Charles Freedman

## 1. Introduction

In this paper we analyze the Eurodollar market[1] in the context of a system of fixed exchange rates using the Tobin-Brainard framework of portfolio selection and market clearing.[2] The model includes the following transactors in financial instruments in each of two countries (the United States and Europe) - nonbank lenders, nonbank borrowers, commercial banks, and central banks. The financial instruments available to lenders are demand deposits at U. S. banks, demand deposits at European banks, interest-bearing securities denominated in U. S. dollars, interest-bearing securities denominated in the European currency, and U. S. dollar deposits at European banks (Eurodollar deposits). Borrowers issue securities denominated in U. S. dollars, and securities denominated in the European currency. They also receive loans denominated in U. S. dollars from the European banks (Eurodollar loans). Explicit asset demand functions and liability supply functions are used for nonbank lenders and borrowers respectively. The supply of deposits by commercial banks in the U. S. is determined by the Federal Reserve's control over reserve assets. Similarly the supply of domestic demand deposits by European commercial banks is determined by the control over reserve assets by the European central bank.

---

* Helpful comments on an earlier version of this paper were received from participants at seminars at The Bank of Canada, The Board of Governors of the Federal Reserve System, Queen's University, The University of Chicago and the Minnesota Economics Association. I should like to thank the Graduate School of the University of Minnesota for its financial support.

In addition to transactions in their domestic assets, European banks accept interest-bearing deposits denominated in U. S. dollars and make loans denominated in U. S. dollars (Eurodollar deposits and Eurodollar loans respectively). In this part of their business they are not controlled by either central bank. We assume further that the banks act competitively in their Eurodollar business and are willing to accept deposits and extend loans for a given profit margin (determined by their costs and risks). In addition, the banks transform domestic assets into U. S. dollar assets and vice versa when such behavior is profitable. That is, they are willing to take on a nonzero net foreign asset position.

The emphasis in the model is on the short-run allocation of assets and liabilities by lenders, borrowers and banks and on the effects of changes in such allocations on interest rates and capital flows. In Section 2 the model is spelled out in detail and the effects on the dependent variables of a change in parameter are worked out for the general case.

Section 3 uses these general results on parameter changes to investigate the effects of the imposition of controls on the Eurodollar market. The two controls examined in detail are the setting of a ceiling on the net foreign asset position of the European banks and the imposition of a reserve ratio on the borrowing by U. S. banks in the Eurodollar market. For each of these, we examine the effect of the control on the Eurodollar rate, the U. S. balance of payments (official reserve transactions balance), and the size of the Eurodollar market. The model can also be used to determine the effect of the control on open-market operations in the U. S.

and Europe if central bank policy is to maintain the domestic interest rates at their target values.

In Section 4 we analyze the implications of the model for two interesting problems. These are: (i) the recycling phenomenon which occurs when central banks make deposits in the Eurodollar market; (ii) the so-called "Eurodollar multiplier". Finally, Section 5 provides a brief summary of the main points of the paper and indicates directions for empirical research implied by the paper.

## 2. The Model

The general approach of the Tobin-Brainard type of model is to treat the demands for and supplies of financial instruments as part of a general equilibrium system with market clearing in each market[3] and interdependence among markets because of the appearance of the same interest rates as arguments in many or all of the demand and supply functions. Thus individual decisions are based on the usual portfolio preference types of considerations in which the quantity demanded of a particular asset is a function of the rate of interest on that asset, rates of return on competing (and complementary) assets, and wealth. Similarly, the supply of debt instruments is a function of a vector of interest rates and total debt. Interest rates adjust so as to clear markets. A change in any independent variable is apt to affect all the dependent variables in such a model.

The world is assumed to be composed of two countries, the United States and Europe. In each country there are four participants in financial markets: the nonbank wealth owners, the nonbank borrowers (including

the government), the commercial banks, and the central bank. The nonbank borrowers borrow by issuing interest-bearing debt (domestic and foreign) and by getting Eurodollar loans from the European banks. The nonbank wealth owners hold domestic and foreign interest-bearing securities, Eurodollar deposits and domestic and foreign money.[4] The U. S. banks hold U. S. dollar interest-bearing securities as their earning asset.[5] Their liabilities are composed of U. S. dollar demand deposits and borrowings in the Eurodollar market. They also hold reserves at the Federal Reserve Bank in amounts determined by their liabilities and the reserve ratio imposed by the Federal Reserve.[6] European banks hold domestic interest-bearing securities (i.e., those denominated in the European currency) and Eurodollar loans as earning assets. Their liabilities are domestic demand deposits and Eurodollar deposits.[7] They hold reserves at the European central bank in amounts determined by their domestic liabilities and the reserve ratio imposed on these liabilities. They also maintain precautionary deposits at U. S. banks as reserves against their Eurodollar liabilities. The European central bank holds European interest-bearing securities, gold, and U. S. dollar deposits at the Federal Reserve.[8] The former asset is used for domestic open-market operations, the latter two assets are used to maintain fixed exchange rates whenever the net transactions of all other participants in the foreign exchange market do not add up to zero. The liabilities of the European central bank are the reserve assets of the European commercial banks. For some experiments we also allow the European central bank to hold Eurodollar deposits. Finally, the Federal Reserve holds U. S. interest-bearing instruments and

gold and it has liabilities to U. S. commercial banks and to the European central bank.

Throughout the analysis we assume that the exchange rate between the European currency and the U. S. dollar is 1:1. Furthermore, we assume that the exchange rate is expected (with certainty) to remain the same and therefore we need not take into consideration the market for forward currencies.[9]

The holdings of financial claims are summarized by the matrix in Table 1. The notation used throughout is as follows.

P - U. S. dollar interest-bearing securities

Q - European currency interest-bearing securities

M - U. S. dollar demand deposits

N - European currency demand deposits

E - Eurodollar deposits

F - Eurodollar loans

A - Central bank

B - Commercial bank

C - Nonbanks (wealth owners and borrowers)

1 - United States

2 - Europe

R - Deposits at Federal Reserve

U - Deposits at European central bank

G - Gold

D - Demand

S - Supply

Table 1

HOLDINGS OF FINANCIAL CLAIMS

| Financial Claim \ Transactor | U.S. Nonbank Lender | U.S. Nonbank Borrower | U.S. Banks | Federal Reserve | European Nonbank Lender | European Nonbank Borrower | European Banks | European Central Bank | Sum |
|---|---|---|---|---|---|---|---|---|---|
| U. S. Interest-Bearing Securities | $P^D_{C1}$ | $-P^S_{C1}$ | $P^D_{B1}$ | $P^D_{A1}$ | $P^D_{C2}$ | $-P^S_{C2}$ | - | - | 0 |
| European Interest-Bearing Securities | $Q^D_{C1}$ | $-Q^S_{C1}$ | - | - | $Q^D_{C2}$ | $-Q^S_{C2}$ | $Q^D_{B2}$ | $Q^D_{A2}$ | 0 |
| Eurodollar deposits | $E^D_{C1}$ | - | - | - | $E^D_{C2}$ | - | $-E^S_{B2}$ | $E^D_{A2}$ | 0 |
| Eurodollar loans | - | $-F^S_{C1}$ | $-F^S_{B1}$ | - | - | $-F^S_{C2}$ | $F^D_{B2}$ | - | 0 |
| U. S. Money | $M^D_{C1}$ | - | $-M^S_{B1}$ | - | $M^D_{C2}$ | - | $M^D_{B2}$ | - | 0 |
| European Money | $N^D_{C1}$ | - | - | - | $N^D_{C2}$ | - | $-N^S_{B2}$ | - | 0 |
| Reserves at Federal Reserve | - | - | $R_{B1}$ | $-R_{B1}-R_{A2}$ | - | - | - | $R_{A2}$ | 0 |
| Reserves at European Central Bank | - | - | - | - | - | - | $U_{B2}$ | $-U_{B2}$ | 0 |
| Gold | - | - | - | $G_{A1}$ | - | - | - | $G_{A2}$ | $\bar{G}$ |
| Sum | Total Wealth $_{C1}$ | -Total Debt $_{C1}$ | 0 | 0 | Total Wealth $_{C2}$ | -Total Debt $_{C2}$ | 0 | 0 | $\bar{G}$ |

Thus, for example, $M^D_{C1}$ is the demand for U. S. dollar demand deposits by nonbanks in the United States. A negative item denotes a liability of the transactor and a positive item denotes an asset. The columns of Table 1 give us the balance sheets of the transactors in the system, and the sum of assets or liabilities for each class of transactors is shown in the bottom row of the table.[10] The rows in the table are either market clearing equations or identities and the sum of the variables in each row is shown in the column on the right. For the system as a whole, the sum of all financial assets is equal to $\bar{G}$, the sum of the gold in the world.

We can now write out the equations for market clearing for each of the financial instruments -- P, Q, E, F, M, N .

$$P^D_{C1} + P^D_{C2} + P^D_{B1} + P^D_{A1} = P^S_{C1} + P^S_{C2} \tag{1}$$

$$Q^D_{C1} + Q^D_{C2} + Q^D_{B2} + Q^D_{A2} = Q^S_{C1} + Q^S_{C2} \tag{2}$$

$$E^D_{C1} + E^D_{C2} + E^D_{A2} = E^S_{B2} \tag{3}$$

$$F^D_{B2} = F^S_{C1} + F^S_{C2} + F^S_{B1} \tag{4}$$

$$M^D_{C1} + M^D_{C2} + M^D_{B2} = M^S_{B1} \tag{5}$$

$$N^D_{C1} + N^D_{C2} = N^S_{B2} \tag{6}$$

To investigate this system further we must make use of various balance sheet identities and technical relationships.

$$P^D_{C1} + Q^D_{C1} + E^D_{C1} + M^D_{C1} + N^D_{C1} \equiv W_{C1} \quad \text{(wealth owners in U.S.)} \tag{7}$$

$$P^D_{C2} + Q^D_{C2} + E^D_{C2} + M^D_{C2} + N^D_{C2} \equiv W_{C2} \quad \text{(wealth owners in Europe)} \quad (8)$$

$$P^S_{C1} + Q^S_{C1} + F^S_{C1} \equiv D_{C1} \quad \text{(borrowers in U.S.)} \quad (9)$$

$$P^S_{C2} + Q^S_{C2} + F^S_{C2} \equiv D_{C2} \quad \text{(borrowers in Europe)} \quad (10)$$

$$P^D_{B1} + R_{B1} \equiv M^S_{B1} + F^S_{B1} \quad \text{(U. S. banks)} \quad (11)$$

$$Q^D_{B2} + U_{B2} + F^D_{B2} + M^D_{B2} \equiv N^S_{B2} + E^S_{B2} \quad \text{(European banks)} \quad (12)$$

$$P^D_{A1} + G_{A1} \equiv R_{B1} + R_{A2} \quad \text{(American central bank)} \quad (13)$$

$$Q^D_{A2} + G_{A2} + E^D_{A2} + R_{A2} \equiv U_{B2} \quad \text{(European central bank)} \quad (14)$$

$$G_{A1} + G_{A2} \equiv \bar{G} \quad \text{(gold conservation)} \quad (15)$$

$$R_{B1} \equiv \rho_M \; M^S_{B1} \quad \text{(U. S. bank reserves)} \quad (16)$$

$$U_{B2} \equiv \rho_N \; N^S_{B2} \quad \text{(European bank reserves)} \quad (17)$$

$$M^D_{B2} \equiv \rho_E \; E^S_{B2} \quad \text{(reserves on Eurodollar deposits)} \quad (18)$$

$$NFA_{B2} \equiv F^D_{B2} + M^D_{B2} - E^S_{B2} \quad \text{(definition of net foreign asset position of European banks)} \quad (19)$$

$W$ is wealth and $D$ is total debt (both of which are assumed to be exogenous to the model).

Equations (7) to (14) are the balance sheet identities for the various participants in the market (as discussed above). Equation (15) states that the total amount of gold in the system is a constant, $\bar{G}$.[11] Finally, equations (16), (17) and (18) relate reserves held by the banks to their

deposit liabilities and the reserve ratios $\rho_M$, $\rho_N$, and $\rho_E$.[12] Equation (19) defines the net foreign asset position of the European banks as the difference between dollar assets and dollar liabilities of the banks.[13, 14]

Substituting from equations (7) to (18) into equations (1) to (6) we get the following market clearing equations.

$$P^D_{C1} + P^D_{C2} + \left(\frac{1}{\rho_M} - 1\right)\left(P^D_{A1} + (G_{A1} - R_{A2})\right) + F^S_{B1} + P^D_{A1} - P^S_{C1} - P^S_{C2} = 0 \tag{20}$$

$$Q^D_{C1} + Q^D_{C2} + \left(\frac{1}{\rho_N} - 1\right)\left(Q^D_{A2} + E^D_{A2} + \bar{G} - (G_{A1} - R_{A2})\right) - NFA_{B2} + Q^D_{A2} - Q^S_{C1} - Q^S_{C2} = 0 \tag{21}$$

$$E^D_{C1} + E^D_{C2} + E^D_{A2} - E^S_{B2} = 0 \tag{3}$$

$$F^D_{B2} - F^S_{C1} - F^S_{C2} - F^S_{B1} = 0 \tag{4}$$

$$M^D_{C1} + M^D_{C2} + \rho_E\left(E^D_{C1} + E^D_{C2} + E^D_{A2}\right) - \frac{1}{\rho_M}\left(P^D_{A1} + (G_{A1} - R_{A2})\right) = 0 \tag{22}$$

$$N^D_{C1} + N^D_{C2} - \frac{1}{\rho_N}\left(Q^D_{A2} + E^D_{A2} + \bar{G} - (G_{A1} - R_{A2})\right) = 0 \tag{23}$$

Now we have six equations, (20) to (23), (3) and (4), in five variables -- $r_P$, $r_Q$, $r_E$, $r_F$ and $(G_{A1} - R_{A2})$. The first four are the interest rates on U. S. dollar interest-bearing debt, European currency interest-bearing debt, Eurodollar deposits and Eurodollar loans. The last variable is the amount of net international reserves held by the Federal Reserve. In a fixed exchange system net movements in international capital by the banking and nonbanking public in response to changes in parameter must be offset by an opposite change in central bank holdings of net international reserves.

As we shall see shortly one of the equations is not independent. Hence we have a system of five equations in five variables. However, there is still one characteristic of the Eurodollar market that we have not investigated. In discussions of the market, it is generally asserted that the banks operating in the market are very competitive and that margins are small.[15] That is to say, the European banks are willing to act as intermediary between the depositor and the company borrowing Eurodollar loans (generally a prime name) for a small, competitively determined margin. This gives us an equation connecting the rates in the Eurodollar deposit and loan markets:

$$r_F = r_E + \alpha \tag{24}$$

where $\alpha$ is a fixed parameter of the system.[16]

We examine the economics of the notion of a competitive banking system in Figure 1. On the horizontal axis we represent the quantities of Eurodollar deposits and Eurodollar loans and on the vertical axis we represent the rates on interest on these instruments. For given domestic interest rates, we graph $EE$, the aggregate demand by nonbanks for Eurodollar deposits and $FF$, the aggregate supply of Eurodollar loans by nonbanks and by U. S. banks.[17] The quantity of Eurodollar deposits demanded by the nonbanks increases as $r_E$ increases. Similarly the quantity of Eurodollar loans desired by nonbanks and by U. S. banks decreases as $r_F$ increases. For simplicity we begin our analysis with the case of a zero net foreign asset position and zero reserves held by the banks against Eurodollar deposits (i.e., $\rho_E = 0$). We assume that competition will ensure that the profit margin $r_F - r_E$ will equal $\alpha$.

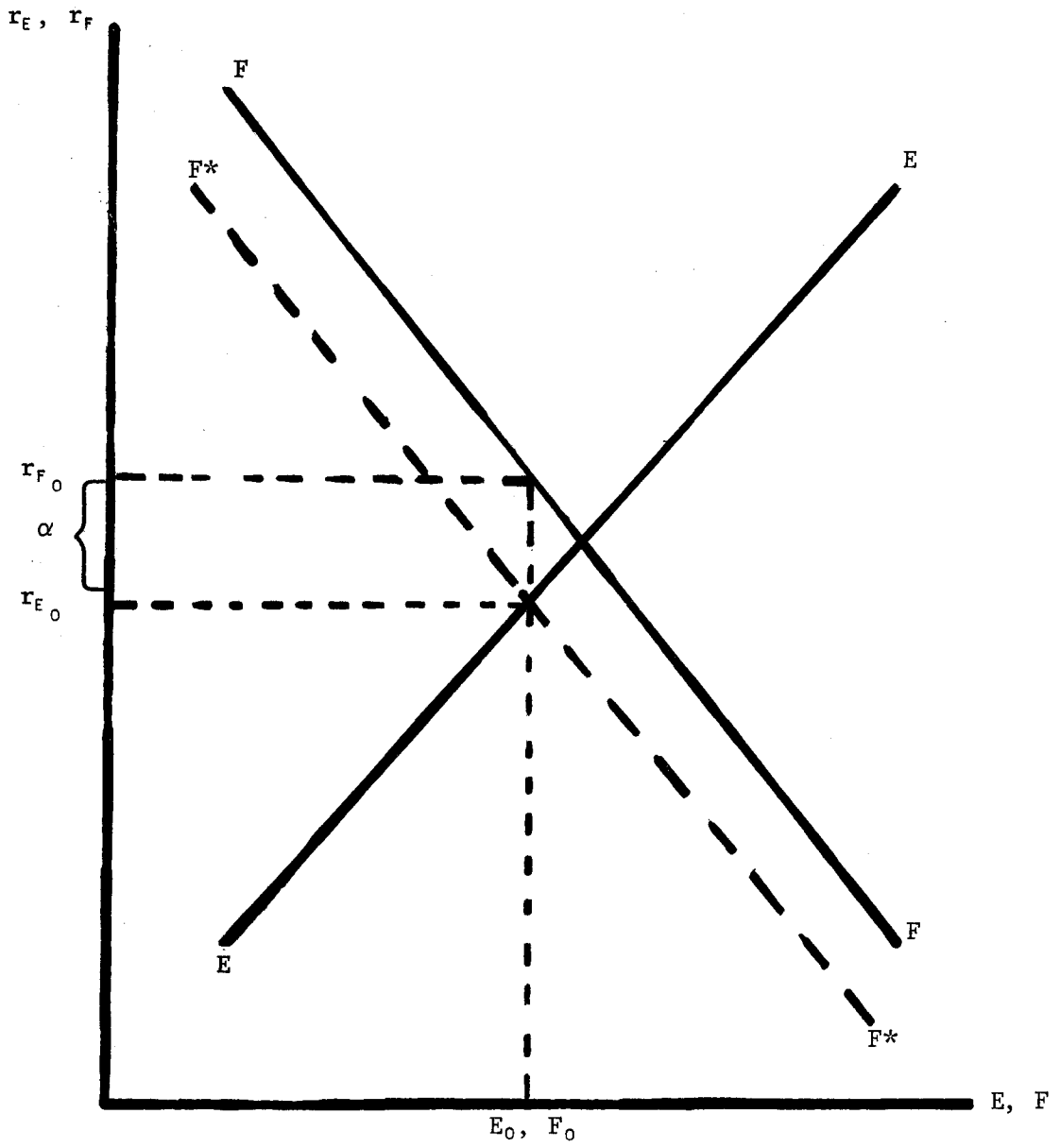


Figure 1

EQUILIBRIUM IN THE EURODOLLAR MARKET WITH ZERO NFA AND ZERO $\rho_E$

In the diagram this will result in interest rates $r_{E_0}$ and $r_{F_0}$ and deposit and loan levels $E_0$ and $F_0$. A margin greater than $\alpha$ (Eurodollar deposits and loans less than $E_0$) will lead to some banks increasing their participation in this lucrative business. They will do this by raising deposit rates slightly and reducing loan rates slightly. The process will continue until the profit margin $\alpha$ is reestablished. On the other hand, a margin less than $\alpha$ (Eurodollar deposits and loans greater than $E_0$) will induce some banks to reduce their participation in the market. Geometrically, it is useful to plot the loan supply curve against $r_E$ rather than against $r_F$. Since $r_F$ equals $r_E + \alpha$, this means that the FF curve is shifted downwards by the amount $\alpha$ to the dashed line shown in Figure 1. The intersection of the deposit demand curve and the F*F* curve (as we shall call the dashed curve) gives the equilibrium $r_E$ and equilibrium E and F in the case of zero NFA and zero $\rho_E$.

We now turn first to the algebraic treatment and then to the geometric treatment of our system of equations with nonzero $\rho_E$ and nonzero NFA. Adding equation (24) to the six equations (20), (21), (22), (23), (3) and (4) gives us seven equations in the five variables $r_P$, $r_Q$, $r_E$, $r_F$ and $(G_{A1} - R_{A2})$. However, one of the equations in the system is not independent. That is, if the five variables were to take on values which satisfied five of the six basic equations (20) to (23), (3) and (4), then the sixth equation would also be satisfied by these values.[18] Thus, we have six independent equations in five unknowns. However, the competitive nature of the banks in the Eurodollar system allows us to combine equations (3)

and (4) into a single equation.[19] Since the supply of Eurodollar deposits and demand for Eurodollar loans simply expand and contract in such a way as to bring about the required differential between $r_F$ and $r_E$ we can make use of (3), (4), (18) and (19) to write

$$(1 - \rho_E)(E^D_{C1} + E^D_{C2} + E^D_{A2}) - (F^S_{C1} + F^S_{C2} + F^S_{B1}) + NFA_{B2} = 0 \quad . \qquad (25)$$

We now have a system of six equations (20) to (25), of which only five are independent, in the five variables. We can simplify the system further by dropping equation (24) and writing $r_F$ as $r_E + \alpha$ each time it appears as an argument in a function. Thus we have reduced our system to four independent equations (four of (20) to (23) and (25)) in the four variables $r_P$, $r_Q$, $r_E$ and $(G_{A1} - R_{A2})$.[20, 21]

In Figure 2 we show the determination of equilibrium in the Eurodollar market with nonzero $\rho_E$ and nonzero NFA. The amount of funds available for Eurodollar loans is $(1 - \rho_E)$ times the amount of Eurodollar deposits. This is depicted by the E*E* curve which is derived by multiplying each abscissa of the EE curve by $(1 - \rho_E)$. Now the NFA position is directly related to $r_F$ for given $r_Q$. That is, as $r_F$ increases the European banks find it profitable to sell domestic assets in order to increase their holdings of Eurodollar loans. We plot the desired NFA position against $r_F$ to get the ZZ curve. Despite the nonzero NFA, it is apparent that the difference between $r_E$ and $r_F$ is still $\alpha$. If this were not so there would be an incentive for some banks to increase or reduce their Eurodollar business. Therefore we can shift the ZZ curve downward by $\alpha$ to obtain the Z*Z* curve which plots NFA against $r_E$. Now according to

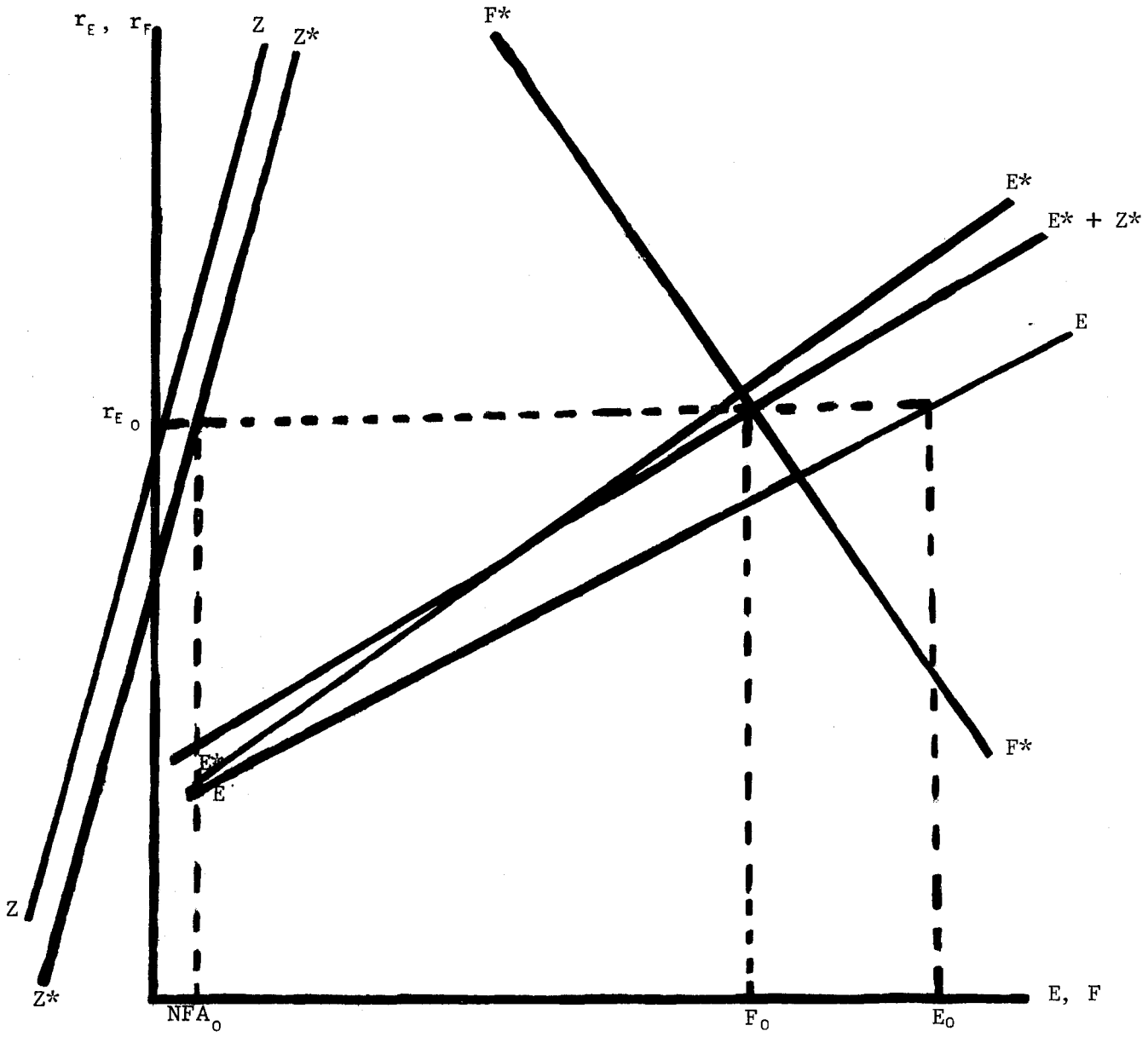


Figure 2

EQUILIBRIUM IN THE EURODOLLAR MARKET WITH NONZERO NFA AND POSITIVE $\rho_E$

equation (25), equilibrium is achieved at the intersection of the F*F* curve and the sum of the E*E* and Z*Z* curves. The latter is represented in the diagram as $E^* + Z^*$. Equilibrium is therefore achieved at a rate of interest $r_{E_0}$ with Eurodollar loans $F_0$, net foreign asset position $NFA_0$ and Eurodollar deposits $E_0$.

We can now go on to examine the arguments in the demand and supply functions. The demand for assets by the nonbanking public in each country is assumed to be a function of total wealth, and the three interest rates. Each asset is superior, i.e., as wealth increases, the quantity demanded of each asset increases (at constant interest rates). Furthermore, we make the assumption of weak gross substitutability; that is, the effect of an increase in one interest rate is to increase the quantity demanded of the asset with the higher interest rate and to reduce or leave unchanged the quantity demanded of the other assets. For example, as $r_E$ rises, $E^D_{C1}$ increases but $P^D_{C1}$, $Q^D_{C1}$, $M^D_{C1}$ and $N^D_{C1}$ fall or remain unchanged. Since total wealth is unaffected by changes in interest rates (because all assets are short-term) we have the following adding-up relationships (making use of (7) and (8)).

$$\frac{\partial}{\partial r_i}\left(P^D_{Cj} + Q^D_{Cj} + E^D_{Cj} + M^D_{Cj} + N^D_{Cj}\right) = 0 \qquad j = 1,\ 2 \qquad i = P,\ Q,\ E \tag{26}$$

The supply of liabilities by borrowers in each country is assumed to be a function of total debt, and all the relevant interest rates. Weak gross substitutability is assumed for borrowing also. That is, a rise in one interest rate leads to a decline in the amount of borrowing done via the instrument bearing that rate and a rise or no change in the amount of

borrowing done through the other instruments. All liabilities are superior, i.e., an increase in total debt leads to an increase in the amount supplied of each liability (at unchanged interest rates). Treating total debt as a parameter of the model (i.e., not a function of interest rates in the current period), we get the following adding-up relationships (making use of (9) and (10)).

$$\frac{\partial}{\partial r_i}\ (P^S_{Cj} + Q^S_{Cj} + F^S_{Cj}) = 0 \qquad j = 1,\ 2 \qquad i = P,\ Q,\ E \tag{27}$$

American banks are assumed to operate as follows. The domestic money supply is determined by available reserves and the required reserve ratio. On the asset side the banks hold domestic interest-bearing assets. In addition the banks will borrow in the Eurodollar loan market in order to buy more domestic interest-bearing assets if the interest rates on Eurodollar loans and domestic interest-bearing assets make such a transaction profitable.[22] From (11), (13) and (16), we can derive the following relationship.

$$P^D_{B1} \equiv (\frac{1}{\rho_M} - 1)(P^D_{A1} + (G_{A1}-R_{A2})) + F^S_{B1} \tag{28}$$

Now, from the banks' viewpoint $P^D_{A1}$ and $(G_{A1} - R_{A2})$ are exogenous. Hence we get the following relationships.

$$\frac{\partial P^D_{B1}}{\partial r_P} \equiv \frac{\partial F^S_{B1}}{\partial r_P} > 0 \tag{29}$$

$$\frac{\partial P^D_{B1}}{\partial r_F} \equiv \frac{\partial F^S_{B1}}{\partial r_F} < 0 \tag{30}$$

As interest rates go up on the domestic interest-bearing asset, it becomes profitable to increase Eurodollar borrowing in order to purchase these assets. Conversely, as Eurodollar rates rise, the amount of such borrowing is reduced.

A similar approach is taken to the operations of the Eurobanks. As argued above, the NFA position of the banks will change as the constellation of interest rates makes the shift from domestic to Eurodollar assets more or less profitable. From (12), (14), (17) and (19) we have

$$Q^D_{B2} \equiv (\frac{1}{\rho_N} - 1)(Q^D_{A2} + E^D_{A2} + \bar{G} - (G_{A1} - R_{A2})) - NFA_{B2} \tag{31}$$

Since the first term on the right-hand side is beyond the control of the commercial banks and is treated by them as given, we get the following relationships.

$$\frac{\partial NFA_{B2}}{\partial r_Q} \equiv - \frac{\partial Q^D_{B2}}{\partial r_Q} < 0 \tag{32}$$

$$\frac{\partial NFA_{B2}}{\partial r_F} \equiv - \frac{\partial Q^D_{B2}}{\partial r_F} > 0 \tag{33}$$

An increase in $r_Q$ causes an increase in the profitability of shifting from Eurodollar assets into domestic assets and therefore results in a decline in the net foreign asset position of the European banks. Similarly, an increase in $r_F$ increases the profitability of shifting from domestic assets into Eurodollar assets and leads to an increase in the net foreign asset position.

We now turn to the comparative statics of our system of equations.

Recall that we can use any four of (20), (21), (22), (23), and (25) to solve for the four variables $r_P$, $r_Q$, $r_E$ and $(G_{A1} - R_{A2})$. Parameters of this system include $P^D_{A1}$ and $Q^D_{A2}$, the central banks' holdings of market securities. However, in order to simplify the analysis and to bring the model closer to reality we will henceforward treat $r_P$ and $r_Q$ as parameters of the system and $P^D_{A1}$, $Q^D_{A2}$, $r_E$ and $(G_{A1} - R_{A2})$ as the four variables determined by the set of equations. That is, central banks are assumed to choose desired interest rates and to allow their holdings of market securities (and therefore the money supply) to adjust to maintain the chosen interest rate.[23]

To examine the effect of parameter changes on the dependent variables of the system, we differentiate totally four of the equations with respect to an arbitrary shift variable $\lambda$. Then we solve the resulting system of equations to get the results discussed in the next four paragraphs. It will simplify the exposition of these results if we use the notation $\frac{\partial EDP}{\partial \lambda}$ to represent the change in excess demand for asset P with respect to a change in the shift variable $\lambda$ where EDP is defined as the left-hand side of equation (20). Similarly EDQ, EDE, EDM and EDN are defined as the left-hand sides of equations (21), (25), (22) and (23) respectively. Further it will be clear on inspection that $\frac{\partial EDP}{\partial r_E} < 0$, $\frac{\partial EDQ}{\partial r_E} < 0$, $\frac{\partial EDE}{\partial r_E} > 0$, $\frac{\partial EDN}{\partial r_E} < 0$ and $\frac{\partial EDM}{\partial r_E} \gtrless 0$.[24]

The effect of a change in $\lambda$ on $r_E$ can be written as

$$\frac{dr_E}{d\lambda} = - \frac{\frac{\partial EDE}{\partial \lambda}}{\frac{\partial EDE}{\partial r_E}} \tag{34}$$

Initially, the change in $\lambda$ causes a positive or negative excess demand in the market for Eurodollar claims $\left(\frac{\partial EDE}{\partial \lambda}\right)$. The rates on Eurodollar deposits and loans must adjust in such a way as to clear the market once again. For example, if a one-unit increase in $r_E$ increases EDE by ten, and if the initial effect of the change in $\lambda$ is to create a negative excess demand of 20, then $r_E$ must rise by two units to eliminate the negative excess demand.[25] Perhaps more obviously one could write the equilibrium result as

$$\frac{\partial EDE}{\partial \lambda} + \frac{\partial EDE}{\partial r_E} \frac{dr_E}{d\lambda} = 0 \tag{35}$$

i.e., the combined effect of the initial change and the change in $r_E$ is to restore equilibrium in the Eurodollar market. The simplicity of this result derives from the fact that none of the other variables of the system ($P^D_{A1}$, $Q^D_{A2}$ and $(G_{A1} - R_{A2})$) impinges directly on the Eurodollar market.[26]

The effect of the change in $\lambda$ on net U. S. international reserves (which is equivalent to the change in the official reserve transactions balance of the U. S. balance of payments) can be written in two ways.

$$\begin{aligned} \frac{d(G_{A1}-R_{A2})}{d\lambda} &= -\left[\frac{\partial EDQ}{\partial \lambda} + \frac{\partial EDN}{\partial \lambda} + \left(\frac{\partial EDQ}{\partial r_E} + \frac{\partial EDN}{\partial r_E}\right) \frac{dr_E}{d\lambda}\right] \\ &= \left[\frac{\partial EDP}{\partial \lambda} + \frac{\partial EDM}{\partial \lambda} + \left(\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}\right) \frac{dr_E}{d\lambda}\right] \end{aligned} \tag{36}$$

In a fixed exchange rate system, the European central bank is committed to offsetting any excess supply of or excess demand for foreign exchange (U. S. dollars). We can envisage the system working in the following way. Suppose, for example, that the initial shift comes from a movement out of

Q, N, and E into P and M. The shift out of Q and N involves the purchase of dollars and the sale of the European currency by the public. Consequently, to prevent a rise in the value of the dollar the European central bank must intervene to sell $\left[-\frac{\partial EDQ}{\partial \lambda} - \frac{\partial EDN}{\partial \lambda}\right]$ dollars. Now the excess supply in the Eurodollar market leads to a rise in the Eurodollar rates, $r_E$ and $r_F$. This leads to a shift of lenders into Eurodollars and a shift of borrowers out of Eurodollars, reestablishing equilibrium in the Eurodollar market. The effect of this secondary shift on European reserves is contained in the term $-\left[\frac{\partial EDQ}{\partial r_E} + \frac{\partial EDN}{\partial r_E}\right] \frac{dr_E}{d\lambda}$. That is, there is a further decline in European reserves caused by the increase in $r_E$ which causes an excess supply of Q and N. The combined initial and secondary effects result in a decline in European net international reserves and a rise in the U. S. net international reserves.

We now turn to the change in central bank holdings of domestic assets. For the European central bank this can be written as

$$\frac{dQ^D_{A\,a}}{d\lambda} = -\left[\frac{\partial EDQ}{\partial \lambda} + (1-\rho_N)\frac{\partial EDN}{\partial \lambda}\right] - \left[\frac{\partial EDQ}{\partial r_E} + (1-\rho_N)\frac{\partial EDN}{\partial r_E}\right]\frac{dr_E}{d\lambda} \quad . \tag{37}$$

Continuing the example in the last paragraph, we see that the initial excess supply of Q would tend to raise the interest rate unless the bonds are purchased by the central bank. Hence there is an increase in the central bank holdings of domestic bonds equal to $-\frac{\partial EDQ}{\partial \lambda}$. Note that this transaction involves no change in the reserves of the banking system since the foreign exchange holdings of the central bank fall by exactly the same amount as bond holdings rise. Now, the initial excess supply of European money would lead to an equivalent decline in bank reserves if no action

were taken by the central bank. To carry out its constant interest rate policy, however, the central bank must cause the reserves of the banking system to fall just enough to be consistent with the desired decline in holdings of money. That is, because of the outflow of foreign exchange bank reserves would tend to fall by $-\frac{\partial EDN}{\partial \lambda}$, but the central bank wishes reserves to fall only by $\left(-\rho_N \frac{\partial EDN}{\partial \lambda}\right)$. It must therefore carry out open market purchases of $-(1-\rho_N) \frac{\partial EDN}{\partial \lambda}$. An identical analysis could be carried out for the required central bank policy with regard to shifts following the rise in $r_E$, i.e., $-\left[\frac{\partial EDQ}{\partial r_E} + (1-\rho_N) \frac{\partial EDN}{\partial r_E}\right]\frac{dr_E}{d\lambda}$. Combining the initial effects and secondary effects gives us the total central bank portfolio change.

The analysis of the effects of a change in $\lambda$ on the Federal Reserve's holding of domestic bonds proceeds in identical fashion. We get

$$\frac{dP^D_{A1}}{d\lambda} = -\left[\frac{\partial EDP}{\partial \lambda} + (1-\rho_M) \frac{\partial EDM}{\partial \lambda}\right] - \left[\frac{\partial EDP}{\partial r_E} + (1-\rho_M) \frac{\partial EDM}{\partial r_E}\right] \frac{dr_E}{d\lambda} \quad . \tag{38}$$

The interpretation is the same as that for the European central bank. Note that the demand for precautionary deposits at U. S. banks arising from increases in Eurodollar deposits is incorporated in the terms $\frac{\partial EDM}{\partial \lambda}$ and $\frac{\partial EDM}{\partial r_E} \frac{dr_E}{d\lambda}$.

One variant of the model that can be introduced at this stage is that in which the European central bank holds its international reserves in the form of U. S. interest-bearing securities ($P^D_{A2}$) instead of deposits at the Federal Reserve Bank. If all European surpluses lead to an increase in $P^D_{A2}$ and all declines lead to a reduction in $P^D_{A2}$ we get the same comparative static results as before for all variables except the holdings

of domestic bonds by the Federal Reserve. In this variant of the model instead of (38) we get

$$\frac{dP^{D}_{A1}}{d\lambda} = \rho_{M}\left[\frac{\partial EDM}{\partial \lambda} + \frac{\partial EDM}{\partial r_{E}}\frac{dr_{E}}{d\lambda}\right] \quad . \tag{39}$$

The reason for the change is that any net shift from P is automatically offset by the increased holdings of U. S. bonds of the European central bank. Similarly, any net shift from M leads to increased holdings of U. S. bonds by the European central bank. In the latter case, the Federal Reserve must reduce the reserves of the commercial banks by an amount consistent with the decline in money demanded as shown in (39). To put the result another way, if the reduction in U. S. net international reserves does not affect the balance sheet of the Federal Reserve Bank because the European central bank does not hold deposits at the Federal Reserve, then the problem of offsetting the effects of international flows on bank reserves does not arise.

3. Controls on the Eurodollar Market

A large number of different types of controls have been proposed or implemented which would affect the operation of the Eurodollar market. Such controls can be divided into three groups -- those on the lenders, those on the borrowers and those on the European banks. An alternative way of classifying the controls is by the form they take -- maximum or minimum quantities, imposition of reserve ratios, change in effective interest rate and others. In Table 2 we present a classification of some of the more interesting controls in terms of the above categories.[27]

Table 2

TYPES OF CONTROLS

| | Lenders | Borrowers | European Banks |
|---|---|---|---|
| Quantity Restrictions | Restrictions on holdings by U. S. lenders of Eurodollar deposits | Restrictions on amount of borrowing via Eurodollar loans | Maximum or minimum NFA |
| Reserve Ratios | | (a) Reserve ratios on borrowings by U. S. banks<br>(b) German reserve ratios on foreign borrowing | (a) Reserves in form of deposits at U. S. banks<br>(b) Reserves at European central bank |
| Interest Rate | | | (a) Swap arrangements by European central banks<br>(b) Ceiling on interest rate on Eurodollar deposits |
| Other | | U. S. government borrowing | |

To illustrate the usefulness of the model in investigating the effects of such controls we will analyze in detail two of the controls - (i) the imposition of a ceiling on the NFA position; (ii) the imposition of reserve ratios on the Eurodollar loans of U. S. banks. In each case we will look at the changes in the basic equations needed to incorporate the controls, the effect of the controls on the rates of interest on Eurodollar deposits and loans, the effect on the net international reserve position of the U. S., and the effect on the size of the Eurodollar market.

(a) Ceiling on the Net Foreign Asset Position

Let us assume that the European government is concerned about the magnitude of its net international reserve position and decides to limit the maximum NFA to an amount $X$.[28] The government informs the banks that this limit has been set on the amount of net dollar assets, i.e., on the magnitude of the conversion from domestic assets to Eurodollar assets. Of course, if the unconstrained equilibrium position is such that the NFA is less than the limit, the constraint is not binding and consequently has no effect. If, however, the unconstrained equilibrium position is such that the NFA exceeds the governmentally imposed ceiling, then the constraint will be binding and will have effects on the domestic monetary situation and on international capital flows.

Let us examine the situation in which the constraint is binding. The banks will then settle in a position with the maximum NFA allowed by law, i.e., $X$. The equations in the model remain unchanged except for the replacement in equations (21) and (25) of the variable $NFA_{B2}(r_Q, r_F)$ by

the constant $X$. This has the effect of reducing the absolute values of $\frac{\partial EDQ}{\partial r_E}$ and $\frac{\partial EDE}{\partial r_E}$ by the same amount in any comparative statics calculations under the system of controls.[29]

The effect on the Eurodollar market of the imposition of a ceiling on the NFA position of the European banks is illustrated in Figure 3. Suppose that the maximum NFA, X, is equal to 0A. Then the $Z^*Z^*$ curve (NFA plotted against $r_E$) follows the path of the uncontrolled curve up to the level 0A and is vertical from that point on. The dotted curve shows the desired NFA position without controls. As can be seen by comparing the unconstrained position T with the constrained position W, the constraint tends to increase $r_E$ (and therefore $r_F$). At the higher rates, there are more Eurodollar deposits and fewer Eurodollar loans. Thus the reduction in the NFA is brought about in part by an increase in Eurodollar liabilities and in part by a reduction in Eurodollar assets.

We can now examine the effect of a small decrease in the NFA ceiling under conditions when the constraint is binding. That is, the government reduces the value X of the maximum NFA the banks are permitted to hold. We find that a decrease in the ceiling results in an increase in $r_E$ and $r_F$, an increase in European net international reserves, open-market sales in Europe and open-market purchases in the United States. Furthermore, Eurodollar deposits increase and Eurodollar loans fall.

Under the usual assumption that $\left(\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}\right)$ is negative, we obtain the result that the decrease in NFA leads to an increase in the European net international reserves or, equivalently, to a decline in U. S. net international reserves. Furthermore the increase in European international

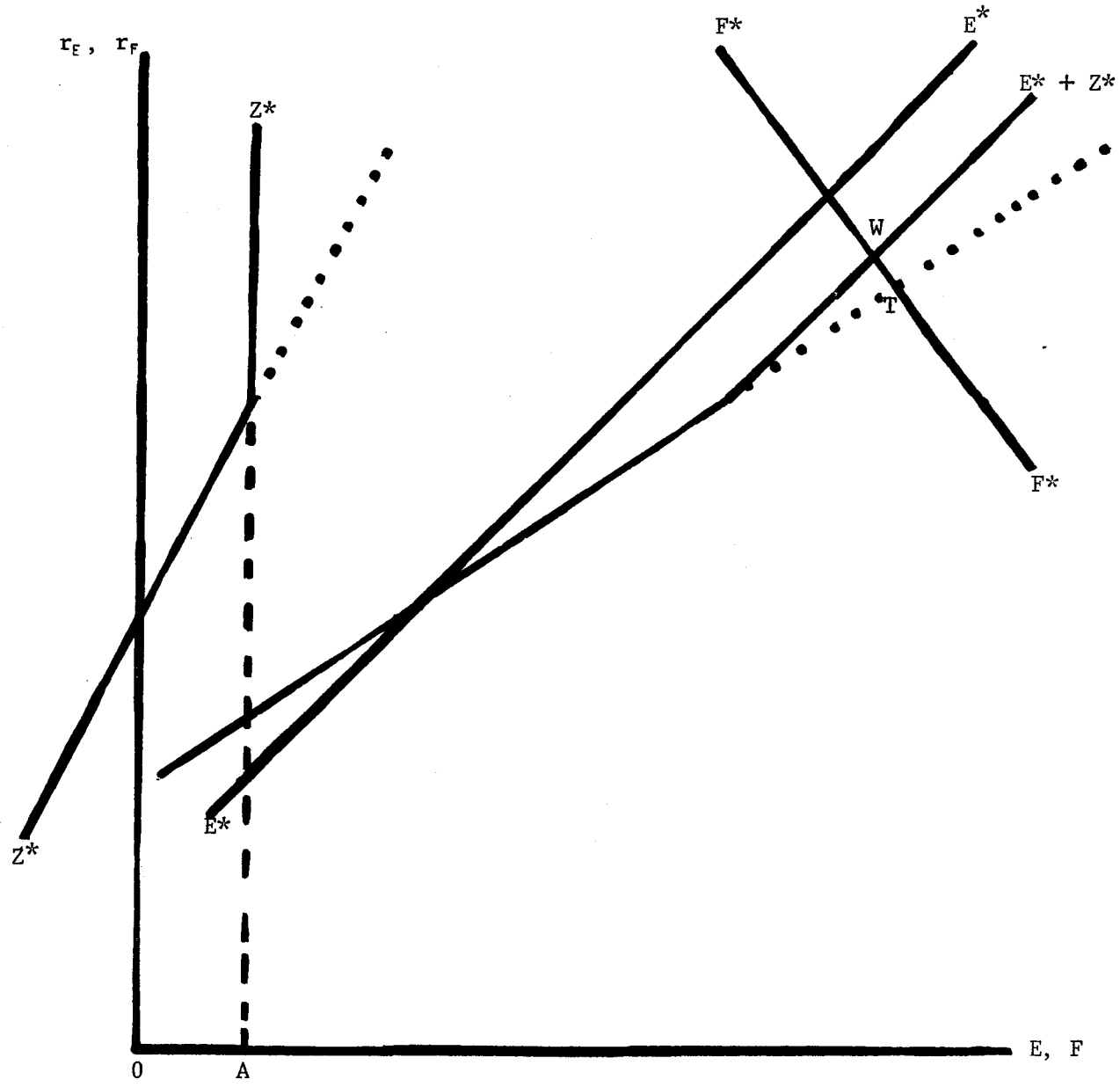


Figure 3

EFFECT OF A CEILING ON NFA

reserves is a fraction of the change in $X$, the fraction being equal to the expression

$$\frac{\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}}{-\frac{\partial EDE}{\partial r_E}} \quad \text{or} \quad \frac{\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}}{\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E} + \frac{\partial EDQ}{\partial r_E} + \frac{\partial EDN}{\partial r_E}} \quad . \tag{40}$$

The reduction in $X$ leads to an increase in $r_E$ and $r_F$. As $r_E$ increases, there are shifts into Eurodollar deposits from all the other assets in the system. Similarly, as $r_F$ increases there are shifts out of Eurodollar loans into U. S. and European domestic bonds. The larger is the magnitude of the movements between U. S. assets and the Eurodollar market relative to the magnitude of the movements between European assets and the Eurodollar market, the larger is the impact of the change in $X$ on U. S. net reserves. Note that the shift in assets and liabilities being discussed above is by nationality of financial instrument and not by nationality of transactor. Thus it does not matter whether it is an American or European who shifts from deposits at U. S. banks into Eurodollar deposits as $r_E$ rises. Nor does it matter whether it is a U. S. or European corporation that shifts its borrowings from the Eurodollar market to the U. S. market as $r_F$ increases. All that is relevant for the analysis is the size of net shifts from U. S. instruments to the Eurodollar market compared to the size of net shifts from European instruments to the Eurodollar market.

The economics of this result is straightforward. As the banks reduce their NFA, the net effect on European international reserves depends on the source of the increase in Eurodollar deposits and the destination of the decrease in Eurodollar loans. To the extent that the increase in $r_E$ draws funds from European domestic assets and to the extent that the

increase in $r_F$ causes borrowers to increase their borrowings in the form of an issue of European domestic interest-bearing securities (or, what is equivalent in our model, European domestic bank loans), then there will be little or no increase in European international reserves.[30]

In the "normal" case in which the decline in X has some effect on European international reserves, the net shift out of American assets and the net shift into European assets result in a tendency for U. S. interest rates to increase and European interest rates to decline. These tendencies will be offset by open-market purchases in the U. S. and open-market sales in Europe.

(b) Reserve Ratios on U. S. Bank Borrowing in the Eurodollar Market

In 1969, the Federal Reserve imposed a reserve ratio on the borrowing of U. S. banks in the Eurodollar market. Now the banks have to hold $\rho_F$ of their borrowings $(F^s_{B1})$ in the form of reserves at the Federal Reserve.[31] The introduction of this reserve ratio has two effects on our model - (i) a reduction in the net return to American banks from borrowing in the Eurodollar market; (ii) an increase in required reserves held against the liabilities of U. S. banks.

The net return to U. S. banks from Eurodollar borrowing used to purchase domestic assets is reduced from $r_P - r_F$ to $(1-\rho_F)\, r_P - r_F$ since only a fraction $(1-\rho_F)$ of any borrowing can be used to purchase domestic bonds. For example, suppose $\rho_F = 10\%$, $r_F = 5\%$ and $r_P = 6\%$. Before the reserve ratio was imposed, the bank could make a gross profit of $r_P - r_F$ or 1% by borrowing in the Eurodollar market and investing in the U. S. Now

for every one dollar borrowed, only 90 cents can be invested in interest-earning bonds; the other 10 cents must be deposited at the Federal Reserve and earns no return. Hence the effective return is $(1 - \rho_F)\, r_P - r_F$ or .4 percent since 90 cents at 6 percent yields 5.4 cents and therefore the net return to a dollar of borrowing is only 0.4 cents. Thus the arguments in the $F^S_{B1}$ function are $(1 - \rho_F)\, r_P$ and $r_F$ after the reserve ratio is imposed.

The effect of the increase in required reserves can best be seen by rewriting equations (16), (28), (20) and (22) to take account of the new requirements.

$$R_{B1} \equiv \rho_M\, M^S_{B1} + \rho_F\, F^S_{B1} \tag{16'}$$

$$P^D_{B1} \equiv \left(\frac{1}{\rho_M} - 1\right)\left(P^D_{A1} + (G_{A1} - R_{A2})\right) + \left(1 - \frac{\rho_F}{\rho_M}\right) F^S_{B1} \tag{28'}$$

$$P^D_{C1} + P^D_{C2} + \left(\frac{1}{\rho_M} - 1\right)\left(P^D_{A1} + (G_{A1} - R_{A2})\right) + \left(1 - \frac{\rho_F}{\rho_M}\right) F^S_{B1} + P^D_{A1} - P^S_{C1} - P^S_{C2} = 0 \tag{20'}$$

$$M^D_{C1} + M^D_{C2} + \rho_E\, (E^D_{C1} + E^D_{C2} + E^D_{A2}) - \frac{1}{\rho_M}\,(P^D_{A1} + (G_{A1} - R_{A2})) + \frac{\rho_F}{\rho_M}\, F^S_{B1} = 0 \tag{22'}$$

If the reserve base of U. S. banks is held constant an increase in $F^S_{B1}$ now leads to a decline in the U. S. domestic money supply (last two terms in (22')) and either an increase $(\rho_F < \rho_M)$ or a decrease $(\rho_F > \rho_M)$ in domestic bonds held by the banks.[32, 33]

We can now examine the effect of an increase in $\rho_F$ on the variables of the model. The initial impact of the rise in $\rho_F$ occurs in the two ways

discussed above. The first effect is the decrease in the net return to the banks from borrowing in the Eurodollar market and investing in U. S. interest-bearing securities. This has the effect of reducing the desired amount of borrowing in the Eurodollar market and reducing or increasing the amount of domestic assets held by banks depending on whether $\rho_F < \rho_M$ or $\rho_F > \rho_M$. As a by-product, it involves an increase in the supply of domestic money since with constant reserves a reduction in Eurodollar borrowings implies an increase in the supply of domestic money. The second effect, or required reserves effect, occurs via the overall increase in reserves required to back Eurodollar liabilities at the new higher reserve ratio. This leads to a decline in the desired money supply and in domestic bond holdings. The sum of these two effects is that, initially, there is an excess demand for Eurodollar claims, an excess supply of U. S. domestic interest-bearing assets (if $\rho_F < \rho_M$) or an indeterminate situation in the market for P (if $\rho_F > \rho_M$) and an indeterminate situation in the market for U. S. money.

The excess demand for Eurodollar claims leads to a decrease in $r_E$ and $r_F$. In Figure 4 we treat the increase in $\rho_F$ as reducing $F^S_{B1}$ at given $r_E$ and therefore shifting the $F^*F^*$ curve to the left. The consequent reduction in the Eurodollar rate (from $r_{E_0}$ to $r_{E_1}$) leads to a decrease in the amount of Eurodollar deposits and the reduction in $r_F$ relative to $r_Q$ leads to a reduction in NFA (from $NFA_0$ to $NFA_1$). Since both deposits and NFA have declined, the amount of Eurodollar loans (which is equal to $(1 - \rho_E)$ times the Eurodollar deposits plus NFA) must also have declined (from $F_0$ to $F_1$). The reduction in $r_F$ does, however,

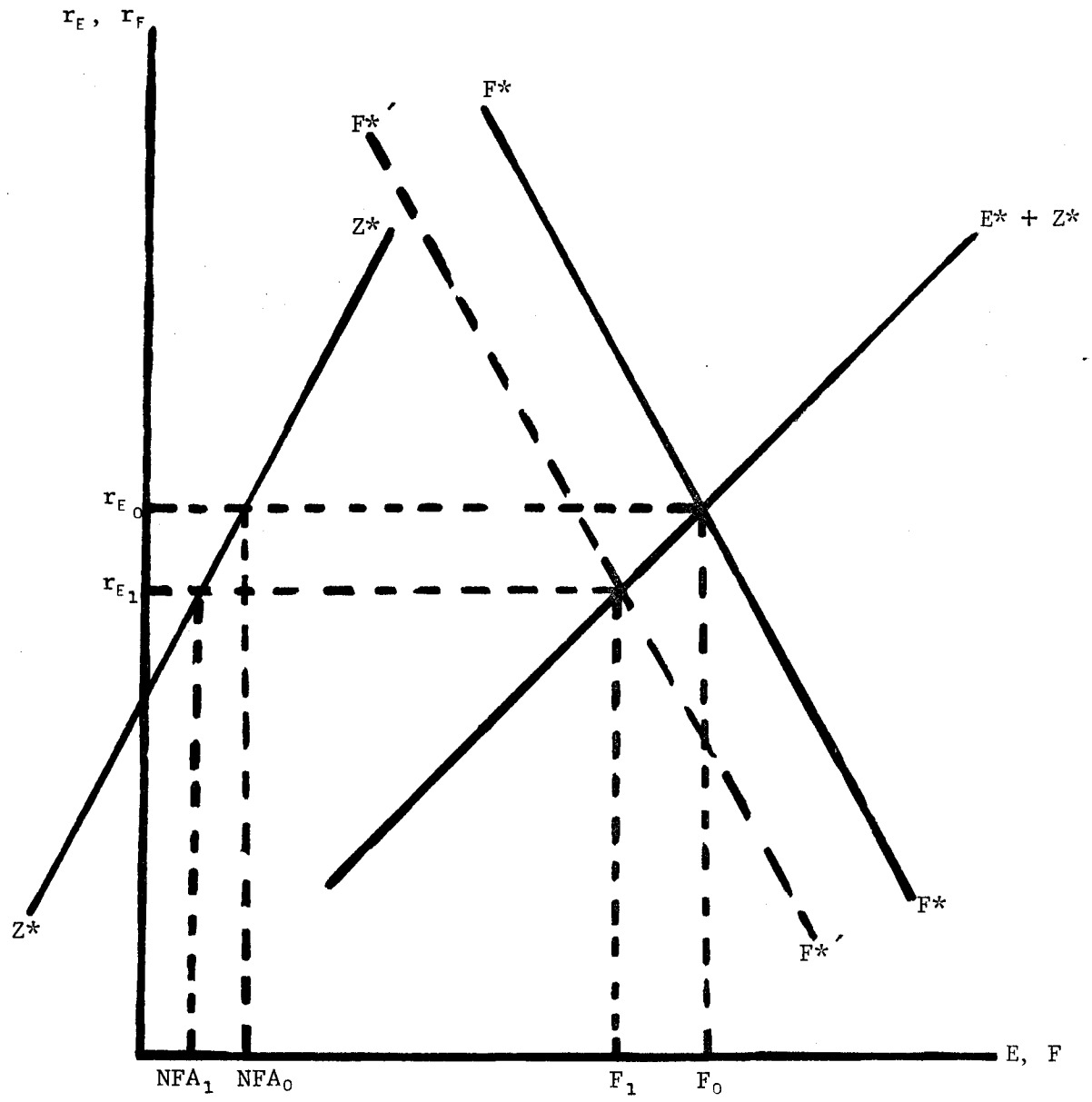


Figure 4

EFFECT OF RESERVE RATIO ON BORROWINGS BY U. S. BANKS

induce some borrowers to shift to Eurodollar loans from domestic loans and thus results in the equilibrium decline in Eurodollar loans being less (in absolute value) than the initial decline.

As expected the increase in $\rho_F$ has a negative effect on U. S. net international reserves. The reduction in Eurodollar loans by U. S. banks leads to a decline in $r_E$ and $r_F$ which in turn leads to a reduction of Eurodollar deposits and to an increase in Eurodollar loans by borrowers other than U. S. banks. To the extent that these depositors shift to European assets or these other borrowers shift out of European domestic loans there will be an increase in European international reserves. The absolute value of the magnitude of the effect on net international reserves is thus inversely related to the fraction shown in expression (40).

The net shift of funds to Europe results in a tendency for $r_Q$ to decline which necessitates open-market sales by the European central bank. The increase in the required reserve ratio against Eurodollar borrowings probably (but not certainly) leads to open-market purchases by the Federal Reserve to offset the tendency for $r_P$ to rise. It is worth pointing out that the required reserves effect enters into the determination of open-market purchases by the Federal Reserve but not into the determination of the changes in $r_E$, $(G_{A1} - R_{A2})$, or $Q^D_{A2}$. Because of the desired constancy of domestic interest rates, the Federal Reserve acts to offset the required reserves effect entirely leaving only the net return effect to influence the other three variables of the system.

It is interesting to note that the sale of bonds by the U. S. government or its agencies to the European banks would have precisely the same

effects on three of the variables of the system as a decrease in $\rho_F$ . In terms of the model, the increased borrowing can be treated as an increase in $F^s_{C1}$ matched by an equal decrease in $P^s_{C1}$ . This can be depicted by a shift to the right of the $F^*F^*$ curve in Figure 4. The effect of the shift is to increase $r_E$ and $r_F$ , to increase the net international reserves of the U. S., and to require open-market purchases by the European central bank. Furthermore, by judicious choice of the magnitude of the shift, the U. S. government could precisely offset the effects on these three variables of a given increase in $\rho_F$ . The shift in borrowing requires open-market sales in the U. S. to offset the tendency to a decline in $r_P$ .

We can use the above results to account for some of the policy changes in the period 1969 to 1971 by the U. S. government. The imposition of the reserve ratio on U. S. banks was responsible in part for the large reduction in 1970 of the borrowing by U. S. banks from their European branches. This reduction resulted in an increase in the deficit in the official reserve transactions balance of the U. S. To offset this in part, the U. S. government and the Export-Import Bank placed substantial amounts of special securities with the foreign branches of U. S. banks. One can also postulate that the retirement of these special securities late in 1971 which caused a deterioration of the U. S. balance of payments position was part of the U. S. policy to apply pressure to other countries to appreciate their currencies relative to the U. S. dollar.

## 4. Some Applications of the Model

In this section of the paper we use the model to examine two interesting questions regarding the Eurodollar market. First, what is the effect on international reserves of Eurodollar deposits by the European central bank? Secondly, what does the model have to say about the so-called Eurodollar multiplier?

### (a) Eurodollar Deposits by the European Central Bank

There has been much recent discussion in academic and governmental circles about the effect of European central banks' depositing some of their reserves in the Eurodollar market either directly or via the B.I.S.[34] and in June 1971 the European central banks agreed not to place any additional funds in the Eurodollar market.[35] We will show that the shift of reserves to the Eurodollar market by the European central banks leads to shifts by other depositors and by borrowers which in turn lead to an increase in total dollar reserves of the European central banks. Or, to put it another way, by virtue of their own actions, European central banks seem to create international reserves. Since the massive holdings of dollars by European central banks is one of the causes of instability in the international monetary sphere, this type of behavior can lead to difficulties.

Let us examine a once-and-for-all shift by European central banks from deposits at the Federal Reserve to Eurodollar deposits. This movement can be treated as a shift to the right of the $E^* + Z^*$ curve in Figure 5 and therefore leads to a reduction in $r_E$ and $r_F$, an increase in $E$ and $F$ and a reduction in the NFA position of the banks. The equilibrium

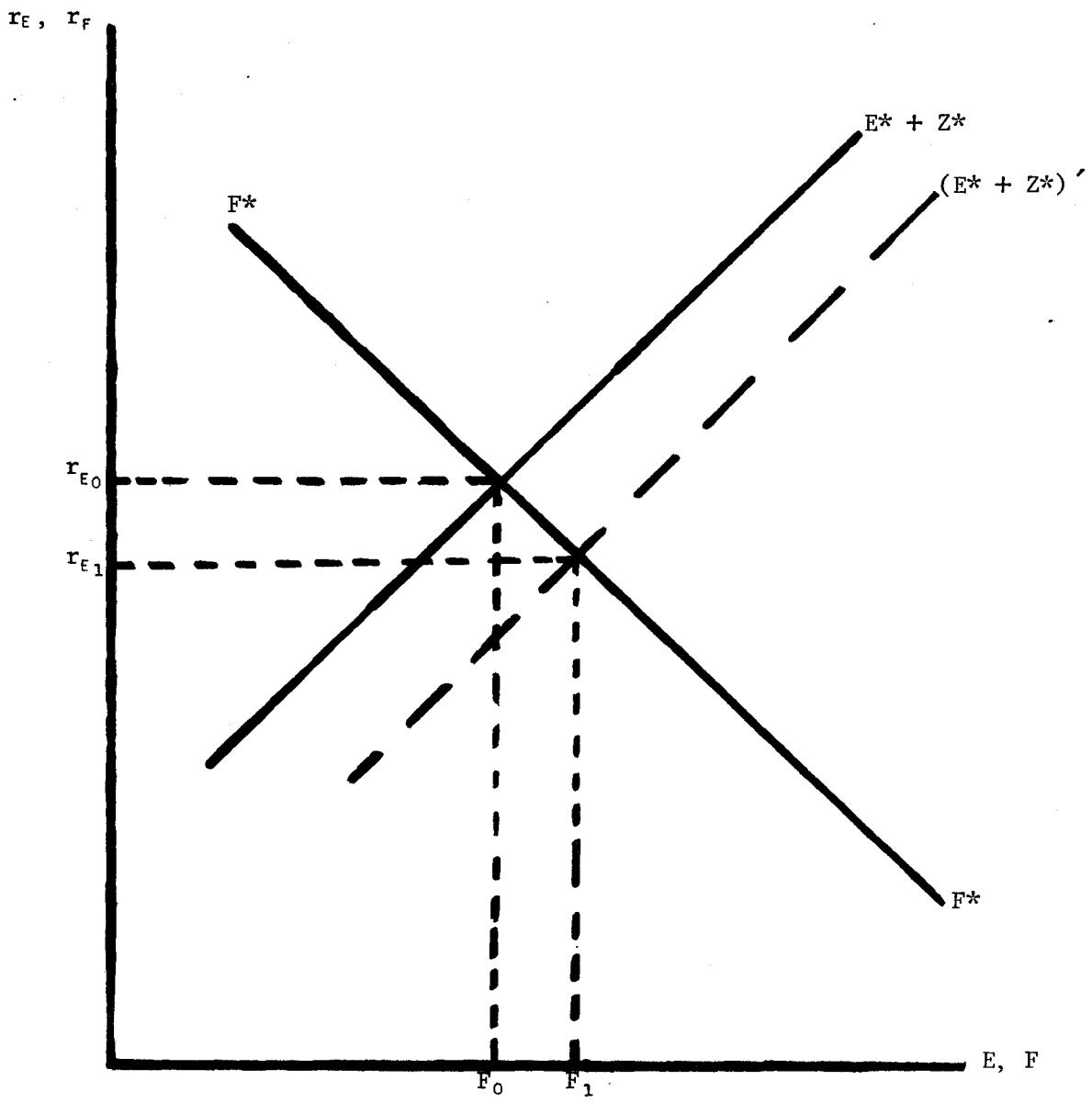


Figure 5

EFFECT OF EURODOLLAR DEPOSIT BY CENTRAL BANK

increase in $E$ is less than the initial shift because of the movement of depositors other than the central bank out of Eurodollar deposits as $r_E$ falls.

We now turn to the effect on international reserves of the shift in funds by the European central bank. First note that there is an asymmetry in the effects on U. S. net international reserves and European net international reserves because of differences in definition. The U. S. definition of the change in reserves is $\frac{d}{d\lambda}(G_{A1}-R_{A2})$ whereas the European definition is $\frac{d}{d\lambda}(G_{A2}+R_{A2}+E^D_{A2})$.[36] So long as $E^D_{A2}$ is a constant, then the sum of the changes in reserves of the two central banks is zero (because $G_{A2}$ can be written as $\bar{G} - G_{A1}$). But if $E^D_{A2}$ increases by one dollar because of a change in allocation of funds by the European central bank then the ultimate effect is that the sum of world reserves as perceived by the two central banks increases by one dollar. Furthermore, as we shall see, the ultimate effect (after equilibrating changes) of this shift is to increase perceived reserves of both central banks (i.e., the official reserve transactions balance will be positive in both countries). And the divergence between central bank dollar accumulation as perceived by European central banks and dollar liabilities as perceived by the U. S. will increase by one dollar. Thus there could be a divergence of views as to the magnitude of the problem of central bank accumulation of dollars and as to what steps are needed to deal with it.[37]

We can now examine more closely the effects of the central bank shift of reserves from a deposit at the Federal Reserve to a Eurodollar deposit. The initial shift causes a decline in $r_E$ and $r_F$. The decline in $r_E$ leads to a movement by depositors other than the European central bank from

Eurodollar deposits to the other assets in the system. And the decline in $r_F$ leads to a movement by borrowers out of borrowing in domestic currencies and into borrowing via Eurodollar loans. It also leads to a reduction in NFA by the banks, i.e., a shift from Eurodollar loans to domestic European assets. To the extent that these shifts by depositors and borrowers are to and from European financial instruments there will be a sale of dollars on the foreign exchange market which will lead to an increase in total dollar reserves of the European central bank. Thus, the result of the shift in allocation of the reserves of the European central bank is an increase in total dollar holdings by the European central bank. However, to the extent that the shifts by lenders and borrowers are mainly to and from American financial instruments there will be little or no increase in the dollar reserves of the European central bank. In the general case in which the shifts are between Eurodollars and both European and U. S. financial instruments, then the effect of the change in allocation of European reserves is to increase total dollar holdings by the European central bank by a fraction of the initial shift. Furthermore the net international reserves of the U. S. also rise by a fraction of the initial shift, the sum of the two fractions being one, since total perceived reserves rise by the amount of the shift. More formally, the increase in European dollar reserves is equal to the magnitude of the shift of the central bank funds to the Eurodollar market times $(1 - \rho_E)$ times one minus the fraction in expression (40). And the increase in U. S. reserves is the magnitude of the shift minus the increase in European reserves.

We complete this section by noting that the movement of funds by the European central bank from the U. S. to the Eurodollar market will, in the absence of open-market operations, tend to lead to a decline in both $r_P$ and $r_Q$. The former is a result of the decline in Federal Reserve liabilities to the European central bank leading to an increase in U. S. bank reserves. The latter is a result of the capital inflow into Europe. Thus both central banks must conduct open-market sales to prevent interest rates from falling.

(b) The Eurodollar Multiplier

There has recently been a series of articles on the magnitude of the so-called Eurodollar multiplier.[38] The framework in which the dispute has taken place is very much like the simple model of the domestic financial system and the main point of argument can be summarized as follows - is the Eurodollar system more like a commercial banking system or like a system of nonbank financial intermediaries? To put it another way, are the leakages from the Eurodollar system relatively small or relatively large?

None of the articles cited has taken into account the fact that in the Eurodollar market changes in the interest rates $r_E$ and $r_F$ play the crucial role of equilibration. And, to use the language of money multiplier analysis, in such a system the leakage caused by changes in interest rates is probably larger than any of the other leakages.

More formally, we wish to investigate the effect on the magnitude of Eurodollar deposits $(E^D_{C1} + E^D_{C2} + E^D_{A2})$ of an autonomous shift of one dollar from a U. S. bank deposit to a Eurodollar deposit. The literature on the subject treats the process of the multiplier in virtually the same way as

the domestic money multiplier. That is, the dollar of new deposits gives rise to a loan of $(1 - \rho_E)$ which in part is redeposited, and thus gives rise to a further loan, etc. Our model in contrast treats the system as one of short-run general equilibrium in which the sum of world assets and liabilities is given and the only question is one of allocation. With this in mind, we must inquire as to the source of the demand for funds (loan supply) which the Eurobank meets with the new funds at its disposal. The answer is that the Eurobank must reduce $r_F$ to attract borrowers into the market from the domestic markets. But the decline in $r_F$ is coupled to a decline in $r_E$ which will cause other depositors to reduce their Eurodollar deposits and shift to domestic assets.[39]

We can again use Figure 5 to show the effect of the autonomous shift from U. S. deposits to Eurodollar deposits. This can be depicted by a shift to the right of the $E^* + Z^*$ curve as a result of the shift in the $E^*E^*$ curve. As can be seen the new equilibrium results in a lower rate of interest and larger deposits than the old equilibrium, but the equilibrium increase in deposits is less than the initial shift.

Algebraically, the shift parameter $\lambda$ is an increase in $E^D_{C1}$ (or $E^D_{C2}$) which is balanced by a decrease in $M^D_{C1}$ (or $M^D_{C2}$). Now

$$\frac{dr_E}{d\lambda} = - \frac{(1-\rho_E)}{\frac{\partial EDE}{\partial r_E}} < 0 \tag{41}$$

and

$$\frac{d(E^D_{C1}+E^D_{C2}+E^D_{A2})}{d\lambda} = 1 - \frac{(1-\rho_E)\frac{\partial \Sigma E^D_1}{\partial r_E}}{(1-\rho_E)\frac{\partial \Sigma E^D_1}{\partial r_E} - \frac{\partial \Sigma F^S_1}{\partial r_F} + \frac{\partial NFA_{B2}}{\partial r_F}} \tag{42}$$

The second term in expression (42) is a positive fraction between 0 and 1 and therefore the value of the Eurodollar multiplier is a positive fraction (where we include the initial shift in the value of the multiplier). The multiplier reaches a maximum of one when $\frac{\partial \Sigma E_i^D}{\partial r_E} = 0$ . In this case there is no reduction in Eurodollar deposits induced by the decline in $r_E$ .[40] The multiplier reaches a minimum of zero when $\frac{\partial \Sigma F_i^S}{\partial r_F} = \frac{\partial NFA_{B2}}{\partial r_F} = 0$ . In this case, the decline in $r_F$ does not induce borrowers to enter the Eurodollar loan market nor does it induce the European banks to reduce their NFA position. Therefore, the interest rate on deposits must fall sufficiently to cause total deposits to decline to their earlier level.

There are two main exceptions to the basic argument we have presented above. The first covers the case in which the European government will not allow Europeans to shift from European domestic assets to dollars via the exchange market but will allow them to invest in dollar assets any funds they receive in the form of dollars. Then if a European receives a dollar in payment for services he might well redeposit it in the Eurodollar market whereas if he received payment in the form of the European domestic currency he would not be permitted to hold the asset as a dollar deposit. In such a case, the Eurodollar multiplier may be greater than one. We can think of the system as one in which the amount of Eurodollar deposits by Europeans is controlled but the level varies with receipts of dollars. The Eurodollar multiplier will then incorporate the "easing" of controls resulting from the autonomous shift from a U. S. deposit to a Eurodollar deposit.

The second main exception involves the case in which the European central bank redeposits in the Eurodollar market its acquisitions of dollars in the

foreign exchange market and draws down its Eurodollar deposits to meet dollar payments in the foreign exchange market. That is, the European central bank is assumed to leave unchanged its dollar deposits (and dollar bonds) in the U. S. and to increase (decrease) its Eurodollar deposits with reserve gains (losses). The system of equations corresponding to such a world is exactly the same as that described above with $E^D_{A2}$ replacing $(G_{A1} - R_{A2})$ as a variable of the system.

The effect on total Eurodollar deposits of a unit shift from a deposit at a U. S. bank to a Eurodollar deposit in such a model is the sum of an infinite number of terms. First, there is the initial shift; secondly, the effect of this shift on other depositors $(E^D_{C1} + E^D_{C2})$ via the decline in the Eurodollar interest rate; thirdly, the effect of the first and second shifts on European international reserves and hence on holdings of Eurodollar deposits by the European central bank. The latter increase in turn causes $r_E$ to change and thus induces further changes in $(E^D_{C1} + E^D_{C2})$ and in $E^D_{A2}$ itself of the sort outlined in Section 4(a). At each round the change in deposits by the central bank leads to a change in deposits by other depositors and a further change in central bank deposits. The final result is as follows:

$$\frac{d(E^D_{C1} + E^D_{C2} + E^D_{A2})}{d\lambda} = 1 - \frac{(1-\rho_E)\left(\frac{\partial(E^D_{C1}+E^D_{C2})}{\partial r_E} + \frac{\partial EDQ}{\partial r_E} + \frac{\partial EDN}{\partial r_E}\right)}{\frac{\partial EDE}{\partial r_E} + (1-\rho_E)\left(\frac{\partial EDQ}{\partial r_E} + \frac{\partial EDN}{\partial r_E}\right)} \tag{43}$$

where $\lambda$ is a one dollar shift from $M^D_{C1}$ to $E^D_{C1}$. Under our usual assumptions, the denominator of the second term in this expression is positive but the numerator may be positive or negative. If the numerator is positive, the second term is a positive fraction and the multiplier is

therefore a positive fraction. The economics of this situation is that the response of depositors to a decline in $r_E$ is substantial and that much of the movement in reaction to the decline in $r_E$ is between the Eurodollar market and American financial assets. The first condition causes a direct decline in the magnitude of the multiplier and the second condition causes an indirect decline by reducing the effect of the shift on dollar holdings of the European central bank.

If the numerator of the second term of expression (43) is negative then the multiplier is greater than one. The multiplier will reach its maximum when $\frac{\partial(E^D_{C1} + E^D_{C2})}{\partial r_E}$ is zero and $\frac{\partial EDE}{\partial r_E}$ equals $\left(-\frac{\partial EDQ}{\partial r_E} - \frac{\partial EDN}{\partial r_E}\right)$. That is, as $r_E$ declines there are no induced declines in holdings of Eurodollar deposits; and there are no shifts between American financial assets and the Eurodollar market by borrowers as $r_F$ declines. Therefore, the initial dollar deposited leads to $\rho_E$ in precautionary deposits in U. S. banks and $(1 - \rho_E)$ in loans to borrowers shifting out of European domestic loans. This in turn leads to a deposit of $(1 - \rho_E)$ by the European central bank. Of the latter the fraction $\rho_E$ is deposited in U. S. banks and $(1 - \rho_E)$ is lent to borrowers shifting out of European domestic loans, etc. The only "leakage" in this special case is into deposits at U. S. banks. Hence the multiplier reaches the classic value of $\frac{1}{\rho_E}$, the same as the value of the money multiplier in the simplest banking model. Needless to say, the assumptions of this special case are very far from reality.[41]

## 5. Summary and Conclusions

In this paper, we developed a model of the Eurodollar market under fixed exchange rates. The model emphasized the allocation of funds by lenders and the allocation of borrowing by corporations and governments. We assumed that domestic money supplies are controlled by the central banks but that the quantities of Eurodollar deposits and loans are determined by a competitive banking system. We solved the model for the dependent variables of the system as a function of a general shift variable. Using these results we investigated a number of interesting problems regarding the market.

The model also has implications for empirical research. First, if the assumption of competitive banking in the Eurodollar market is correct, it will be inappropriate to try to estimate $E^S_{B2}$ or $F^D_{B2}$ as a function of interest rates since the coefficients will be infinite.[42] Secondly, the significance of expression (40) in most of the comparative statics results implies that it is important to know the sources of shifts to the Eurodollar market as well as the amounts of such shifts. Thus attention must be paid to the effect of changes in $r_E$ on domestic instruments as well as on Eurodollar deposits. Thirdly, the effect of interest rate changes on deposits is clearly only one of three effects that enter into expressions like $\frac{\partial EDE}{\partial r_E}$. The effect on borrowers may be equally significant and the effect on the NFA position of the banks, although probably smaller in magnitude than the other two aspects, may still be too large to ignore.[43] Fourthly, if the conclusion is correct that the Eurodollar multiplier cannot explain the rapid growth of the Eurodollar market, other explanations will have to be sought. To test these will require careful specification of deposit demand functions and loan supply functions.

## FOOTNOTES

1. Good discussions of the institutional characteristics of the Eurodollar market can be found in [7, 8, 13]. For more recent theoretical and empirical studies see [5, 16, 17, 19].

2. See Tobin and Brainard [21] and Tobin [20].

3. This is not precisely accurate since in some markets the supplier of the financial instrument stands ready to supply the amount demanded at a fixed rate of interest. This case can be easily handled in this model. See, for example, [20, page 28].

4. It would not be difficult to introduce certificates of deposits paying a rate of interest fixed by the authorities into the model.

5. Thus bank loans are treated as securities issued by borrowers and held by the banks.

6. Excess reserves and borrowed reserves could be introduced into the model without difficulty.

7. We could treat the banks which engage in Eurodollar transactions as being separate from the banks engaged in European domestic banking business with no change in results.

8. The case in which the European central bank holds its international reserve asset in the form of U. S. interest-bearing securities is discussed briefly at the end of this section.

9. The certainty assumption is equivalent to an infinitely elastic speculative demand curve for the forward currency at the fixed exchange rate for spot currency.

10. It is easiest to think of American companies having liabilities of interest-bearing debt equal to their capital stock, American governments having liabilities of interest-bearing debt equal to their accumulated deficit, and American individuals holding assets equal to their net worth or wealth. A similar statement can be made about the European transactors.

11. It would be simple to introduce gold mining into the system. Gold purchase by speculators could also be handled.

12. The first two reserve ratios are imposed by the two central banks whereas the third is simply precautionary. Note that we assume that commercial banks hold no excess reserves and that they do not borrow reserves from the central bank. It would not be difficult to allow for such behavior on the part of the banks. However, the change in assumptions would make little difference to the results that we obtain.

13. Note that our definition of net foreign asset position is on the basis of currency and not of nationality. Thus it is equivalent to Mills' [18] notion of net foreign currency position. From the banks' viewpoint it is only the currency of the transaction and not the nationality of the transactor that is relevant for decision making. Thus the notion of net position vis-à-vis nonresidents is of importance to the banks only if it becomes the subject of controls.

14. We believe that the extra effort expended in building a model which allows the net foreign asset position to play a role is worthwhile because of the potential importance of the net foreign asset position in the real world. For example, the NFA of the Canadian banks declined

by $741 million in the six months between December 1971 and June 1972 [1, Table 11].

15. Little [13, page 3] asserts that the margin between the deposit and loan rates is about one-half percent on loans to prime industrial or commercial firms.

16. The publication "World Financial Markets" published by the Morgan Guaranty Trust Company gives a time series for rates on Eurodollar deposits and rates on Eurodollar loans to prime borrowers. The difference between the two series is 88 basis points over a fairly long time period. This indicates that at least one of the major participants in the Eurodollar market believes in the constancy of $\alpha$ over time. It is likely that $\alpha$ is related to the costs of running the Eurodollar business and to risks perceived by the banks in acting as an intermediary between lenders and borrowers in the market.

17. There is some terminological difficulty over loans and deposits. We talk about banks supplying deposits and demanding loans. Companies demand deposits and supply loans.

18. This is the analogue of Walras' Law for a Tobin-Brainard type of system [21, page 75 or 20, pages 18-19]. To see that the statement is true, we can assume for example that (20) to (22) and (3) and (4) hold and then, making use of the identities, we can show that this implies that (23) also holds.

19. The use of the notion of the competitive structure of the system to eliminate $E^S_{B2}$ and $F^D_{B2}$ is very similar to the procedure implicitly used in Regimes IIa and IIIa of Tobin and Brainard [21] to eliminate the behavior functions of the banks from the model. In their model the banks operate simply as an intermediary between the depositors and the ultimate borrowers. There is no equivalent to the NFA in their system.

20. We can treat our system as a special case of a system in which $E^S_{B2}$ and $F^D_{B2}$ are functions of $r_E$ and $r_F$. Our system results from assuming that the partial derivatives of $E^S_{B2}$ with respect to $r_E$ and $r_F$ approach $-\infty$ and $\infty$ respectively and that the partial derivatives of $F^D_{B2}$ with respect to $r_E$ and $r_F$ approach $-\infty$ and $\infty$ respectively. Thus our system may serve as an approximation to the real world if the partial derivatives of $E^S_{B2}$ and $F^D_{B2}$ with respect to $r_E$ and $r_F$ are very large in absolute value even if they are not infinite.

21. Hendershott [10] developed a pair of equations for the Eurodollar market which are similar to (3) and (4). However, he set $F^D_{B2}$ equal to $E^S_{B2}$ which means that he set $NFA_{B2}$ to zero in a no-reserve world. In the Hendershott model, $F^D_{B2}$ and $E^S_{B2}$ are functionally related to $r_E$ and $r_F$ and the partial derivatives are finite. Hence in his model the two equations do not collapse to one and the profit margin $r_F - r_E$ is a variable which must increase with the size of the Eurodollar market.

22. Recall that the purchase of domestic interest-bearing assets is equivalent in our model to extending more loans. In the real world the return on domestic loans may include some element of goodwill.

23. Another way of using the model is to assume that the domestic money supply is held fixed by the central bank. This is equivalent to assuming complete sterilization of capital flows. In such an approach, the variables determined in the model are the three interest rates and the net international reserve position. Open-market sales are constrained to be equal to the increase in net international reserves in each country.

24. The indeterminacy of $\frac{\partial EDM}{\partial r_E}$ occurs because the increase in $r_E$ leads to two conflicting effects on the demand for deposits at U. S. banks. First, it reduces $M^D_{C1}$ and $M^D_{C2}$ which reduces the quantity of deposits demanded. But by increasing the demand for Eurodollar deposits it indirectly increases the demand for U. S. deposits held as precautionary balances ($\rho_E$ times the increase in Eurodollar deposits demanded). This result is precisely analogous to that in Regime III of Tobin-Brainard [21, pages 78-80]. Fortunately, in our model all the more important results have $\frac{\partial EDM}{\partial r_E}$ appearing as part of a sum with $\frac{\partial EDP}{\partial r_E}$. Although it is theoretically possible that $\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}$ is positive, in practice $\rho_E$ is sufficiently small that such a result is extremely unlikely. Consequently, we shall assume throughout that $\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}$ is negative, referring occasionally in footnotes to the results in the situation in which this assumption is not true.

25. Geometrically $\frac{\partial EDE}{\partial r_E}$ is the sum of the absolute values of the slopes of $F^*F^*$, and $E^* + Z^*$ in Figure 2. The larger are these slopes (in absolute value), the smaller (in absolute value) will be the effect on $r_E$ of a given lateral shift of either of the curves.

26. Technically, the system can be dichotomized into the Eurodollar equation and the rest of the system.

27. For a discussion of actual controls imposed in the 1960s including many that affected the Eurodollar market see Mills [18].

28. See Mills [18, pages 30-33 and pages 49-50] for a discussion of postwar controls on banks' net foreign currency asset positions.

29. Note that it is necessary to check whether the constraint remains bending after a parameter change. To do this, one would have to calculate the equilibrium unconstrained NFA position after the change in parameter value and compare it with the ceiling value.

30. In fact, if there are no shifts at all by depositors or borrowers from U. S. instruments, then $\left(\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}\right)$ will be positive and we will have the "perverse" result that the decline in X leads to a reduction in European international reserves. This occurs because of the increase in deposits in U. S. banks held as reserves against the increase in Eurodollar deposits.

31. In reality the regulations were more complicated than this. For a discussion of the regulations, see [8, pages 25-26] or [2, pages 656-657] and [3, pages 941 and 963].

32. Note that even if $\rho_F > \rho_M$ it would still pay a single bank to borrow in the Eurodollar market although it would not pay the system as a whole to do so.

33. Note that if $\rho_F > \rho_M$, it is possible but very unlikely given the empirical magnitudes that $\frac{\partial EDP}{\partial r_E}$ will become positive. This will not affect our results in any way since $\frac{\partial EDP}{\partial r_E} + \frac{\partial EDM}{\partial r_E}$ is unchanged and it is always the sum that enters our calculations.

34. See, for example, [22, page 29] and [15].

35. See [4, page 813].

36. See [22, page 29].

37. Machlup [15, pages 12-13] estimates that "more than $5 billion of the dollars lodged in official reserves of central banks are dollars created by their own actions in the Euromarket." He also calculates the recorded difference between dollar reserves of central banks and U. S. net liabilities to central banks to be about $10.7 billion as of December 1970.

38. See, for example, [6, 9, 11, 12, 14].

39. Implicitly we are arguing that the ordinary money multiplier has merit only because the asset with which it is concerned bears a fixed (zero) rate of interest. Therefore an increase in the supply of that asset cannot lead to an equilibrating process in which the rate on that asset adjusts to the change in supply. Instead the excess supply leads to a decline in rates on the other assets in the system which will induce wealth owners to hold the increased amount of money.

In the case of the Eurodollar market, the equilibrating role of the changes in $r_E$ cannot be ignored.

40. Compare this result with Makin's estimates of the short-run and long-run deposit expansion multipliers as 10.31 and 18.45 [16, page 389] or Klopstock's estimate of between 1.5 and 1.9 [11, page 8]. Both estimates assume that the central banks deposit some of their reserve increases in the Eurodollar market. We shall discuss below the implications of such deposits for the Eurodollar multiplier.

41. If the multiplier is at most one (or slightly above one if there are redeposits in the market by central banks), other reasons must be sought to explain the phenomenal growth of the Eurodollar market in the 1960s. Formally, we can treat this problem by adding scale variables to the $E^D$ and $F^S$ functions and examining the effects on $r_E$ and the size of the market of the increase in one or both of the scale variables. This still leaves unanswered the question of the determinants of the rate of growth of the scale variables. The usual answer to this question in the literature is based on the general attractiveness of the investment medium offered by the Euro-banks [11, pages 8-9]. See also [6] and [16, page 382 and pages 388-390] for discussion of this question.

42. For an attempt to estimate the supply of Eurodollar deposits by the banks see Makin [16].

43. Unpublished empirical work by the author on the net foreign asset position of the Canadian banks supports this conclusion.

## LITERATURE CITED


[1] Bank of Canada Review (August, 1972).

[2] Board of Governors of the Federal Reserve System. Federal Reserve Bulletin (August, 1969).

[3] _____ . Federal Reserve Bulletin (December, 1970).

[4] _____ . Federal Reserve Bulletin (October, 1971).

[5] Clendenning, E. Wayne. The Euro-Dollar Market. Oxford: Clarendon Press, 1970.

[6] _____ . "Euro-dollars and Credit Creation," International Currency Review, III (March/April, 1971), 12-19.

[7] Einzig, Paul. The Euro-dollar System. London: Macmillan Company, 1970.

[8] Federal Reserve Bank of Cleveland. "The Eurodollar Market," Economic Papers (June, 1970).

[9] Friedman, Milton. "The Euro-Dollar Market: Some First Principles," The Morgan Guaranty Survey (October, 1969), 4-14.

[10] Hendershott, Patric H. "The Structure of International Interest Rates: The U. S. Treasury Bill Rate and the Eurodollar Deposit Rate," The Journal of Finance 22 (September, 1967), 455-465.

[11] Klopstock, Fred H. "The Euro-dollar Market: Some Unresolved Issues," International Finance Section, Princeton University, Essays in International Finance (March, 1968).

[12] _____ . "Money Creation in the Euro-Dollar Market - A Note on Professor Friedman's Views," Federal Reserve Bank of New York, Monthly Review (January, 1970), 12-15.

[13] Little, Jane Sneddon. "The Euro-dollar Market: Its Nature and Impact," Federal Reserve Bank of Boston, New England Economic Review (May/June, 1969).

[14] Machlup, Fritz. "Euro-dollar Creation: A Mystery Story," Banca Nazionale del Lavoro Quarterly Review, No. 94 (September, 1970), 219-260.

[15] _____ . "The Magicians and Their Rabbits," The Morgan Guaranty Survey (May, 1971), 3-13.

[16] Makin, John H. "Demand and Supply Functions for Stocks of Euro-dollar Deposits: An Empirical Study," The Review of Economics and Statistics, 54 (November, 1972), 381-391.

[17] Mikesell, Raymond F. "The Eurodollar Market and the Foreign Demand for Liquid Dollar Assets," Journal of Money, Credit and Banking, 4 (August, 1972), 643-683.

[18] Mills, Rodney H., Jr. "The Regulation of Short-Term Capital Movements in Major Industrial Countries," Board of Governors of the Federal Reserve System, Staff Economic Studies, No. 74 (October, 1972).

[19] Rich, Georg. "A Theoretical and Empirical Analysis of the Eurodollar Market," Journal of Money, Credit and Banking, 4 (August, 1972), 616-635.

[20] Tobin, James. "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit and Banking, 1 (February, 1969), 15-29.

[21] _____ , and William C. Brainard. "Financial Intermediaries and the Effectiveness of Monetary Controls," in Donald D. Hester and James Tobin, Financial Markets and Economic Activity. New York, John Wiley and Sons, Inc., 1967, 55-93.

[22] U. S. Department of Commerce. Survey of Current Business (June, 1971).